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Bob Smith
letters

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23 Feb 79

N. J. A. Sloane
Mathematics Research Center
Bell Telephone Laboratories, Inc.
Murray Hill, NJ 07974

Dear Sir,

I'm writing to you to ask for a copy of any supplements you might have to your delightful Handbook of Integer Sequences (my copy is the 1973 edition, first printing).

I also have an interesting sequence which I've been unable to find in any form in your handbook. The sequence stems from permutations, and answers the question, how many pemutations of any given order have a square root? That is, how many permutations Q can be represented as Q = P x P for some permutation P? The first few terms are contained in the enclosed table. Essentially, the sequence is a count of permutations with an even number of cycles of each even length (the permutations may have any number of odd length cycles).

The way I enumerated these values was to partition N, the order of the permutation, into N parts (some of which may be zero) where the parts are weighted by 1, 2, 3, ..., N, respectively. For example, five is partitioned as follows:

	1	2	3	4	5
a	0	0	0	0	1
b	1	0	0	1	0
c	0	1	1	0	0
d	2	0	1	0	0
e	1	2	0	0	0
f	3	1	0	0	0
g	5	0	0	0	0

Each partition counts the number of cycles whose length is the associated weight. Next, throw out those partitions which have an odd number in a column headed by an even cycle length. In particular, throw out partitions labeled b, c, and f. For each of the remaining partitions, count how many permutations have that cycle structure. To do this, I applied the formula found in Knuth's The Art of Computer Programming, Vol. 1, exercise 1.3.3-21. Sum over the partitions to yield the answer.

Using the approach mentioned by Knuth in exercise 1.3.3-22, I found a generating function (as an infinite product) for this series as follows:

$$\sqrt{\frac{1+x}{1-x}} \prod_{n \geq 1} \cosh \frac{x^{2n}}{2n} = \sum_{n \geq 0} f(n) \frac{x^n}{n!}$$

The corresponding generating function as a sum involves weighted partitions of N, and is a statement of the algorithm I used to enumerate the sequence.

Sincerely,

Bob

Bob Smith

enc.

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f(n)	n
~~1~~	1
1	2
3	3
12	4
60	5
270	6
1,890	7
14,280	8
128,520	9
1,096,200	10
12,058,200	11
139,043,520	12
1,807,565,760	13
22,642,139,520	14
339,632,092,800	15
5,237,183,952,000	16
89,032,127,184,000	17
1,475,427,973,219,200	18
28,033,131,491,164,800	19
543,494,606,861,606,400	20

Please enter

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Name: Permutations ~~which have~~ with a square root.

Bell Laboratories

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July 31, 1979

Dr. Bob Smith
Scientific Time Sharing Corp.
21243 Ventura Boulevard
Woodland Hills, CA 91364

Dear Dr. Smith:

Thank you very much indeed for your letter of February 23. I apologize for taking so long to reply. I'm glad you like the Handbook. A copy of Supplement I is enclosed, as well as a couple of other things. This is the only one issue so far: another is long overdue.

Your new sequence is very interesting, and new to me. Thank you for sending it.

Yours sincerely,

MH-1216-NJAS-dh N. J. A. Sloane

Enclosure
As Above