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147 Austin St.
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February 7, 1974

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Dear Neil:

Your letter finally arrived yesterday and I will try to answer your question. First of all, there is a basic problem. If K is a quadratic field, there is a unique square-free integer d such that $K = \mathbb{Q}(\sqrt{d})$. However, K is often also written as $\mathbb{Q}(\sqrt{4d})$. The reason has to do with ~~disc~~ discriminants; namely, if d is as above, the discriminant is d for $d \equiv 1\ (4)$ and $4d$ for $d \equiv 2, 3\ (4)$. You have to be careful, because ~~my often~~ quadratic fields are sometimes listed by their d's and sometimes by their discriminants. Now back to our subject:

(1) An interesting finite sequence: N1539.5 = 3246

new $\cancel{d}$ (= discriminant) = 5, 8, 12, 13, 17, 21, 24, 28, 29, 33, 37, 41, 44, ~~44~~, 57, 73, 76

cdy II 57 These are exactly the discriminants of real quadratic fields $\mathbb{Q}(\sqrt{d})$ $(d > 0)$ which are Euclidean under the metric given by the usual norm. One of the interesting things about this sequence is that for a while it was thought that 97 belonged to it also, but this has been disproved.

References:

E. S. Barnes & H. P. F. Swinnerton-Dyer "The inhomogeneous minima of binary quadratic forms I, II, III", Acta Math. 87 (1952) 259–323, 88 (1952) 279–316, 92 (1954) 199–234

V. Ennola "On the first inhomogeneous minimum of indefinite binary quadratic forms and Euclid's algorithm in real quadratic fields," Ann. Univ. Turku A I 28 (1958)

It is also known that if enough Riemann hypotheses are ~~true~~ true, then every real quadratic field of class number one is Euclidean under some norm.

(2) As far as Stark, pp. 306, 296 is concerned, here is a table of the smallest positive ~~discriminant~~ D such that $\mathbb{Q}(\sqrt{D})$ has class number one (i.e. is a UFD): ($D \neq$ discriminant in general)

D = 2, 3, 5, 6, 7, 11, 13, 14, 17, 19, 21, 22, 23, 29, 31, 33, 37,
38, 41, 43, 46, 47, 53, 57, 59, 61, 62, 67, 69, 71, ~~X~~
73, 77, 83, 86, 89, 93, 94, 97,

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Later note: Sorry about this. I was going to look up the result in Ince, but MIT's copy has been either stolen or misplaced. As a result, what I have above is taken from Stark (& the first few have been verified by comparison with Ince: I have a Xerox of a portion of his table). If you cannot ~~at~~ get access to a copy of Ince, write me a note, & I will try to find a copy at Harvard.

Reference: E. L. Ince *Cycles of Reduced Ideals in Quadratic Fields*, Bristol Assoc. for the Adv. of Science Math. Tables, vol. IV.

(Ince goes up to $2025 = 45^2$ with his tables).

To get a continuation of the list on p. 306 of Stark, just take the primes D above.

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but this would include 23??

Stark (p. 306) does not include $p = 2$, since

$$\mathbb{Q}(e^{\frac{2\pi i}{2}}) = \mathbb{Q}(-1) = \mathbb{Q},$$

but this is only a matter of personal preference.

It is ~~thought~~ conjectured that the ~~list~~ sequence on p. 306 (& hence the one on p. 296) is infinite, but this has not been proved.

(3) Euler's suitable or idoneal numbers:

The 65 known idoneal numbers are obtained by taking from the two lists in Borevich & Shafarevich (one of 65 and one of 36) those numbers which are even and dividing them by 4. (As far as anybody knows, the recurrence of 65 is purely coincidental.) It is thought that these are all, but has not been proven. It is known, however, that both sequences in B. & S. are finite, and in fact there is an effective bound such that there is at most one number larger than that bound which belongs to the larger table (i.e. the one which has 65 now) in B. & S., & at most one beyond that same bound which belongs to the smaller table in B. & S. The range up to that bound, however, has not been completely covered.

References:

E. Grosswald "Negative discriminants of binary quadratic forms with one class in each genus", Acta Arith. 8 (1963), 295–306

S. Chowla & W. E. Briggs "On discriminants of binary quadratic forms with a single class in each genus", Canad. J. Math. 6 (1954), 463–470.

With best wishes,

Andrew

A. M. Odlyzko

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Addendum to yesterday's letter:

Instead of using Ince, you can refer to Table 1 ([illegible] 422-424) of Borevich & Shafarevich (who got their information from Ince). Just look for the fields with $h=1$, and for the sequence on p. 306 of Stark take those for which d is prime.

Andrew