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October 16, 1990

Dr. Neil J. A. Sloane
Mathematics Research Center
Bell Telephone Laboratories, Inc.
Murray Hill
New Jersey 07974
U. S. A.

Dear Dr. Sloane:

I came across the article about your work in a recent issue of Discover. Here is an integer sequence that is not included in your Handbook (1973 edition):

1, 3, 4, 7, 9, 12, 13, 16, 19, 21, 25, 27, 28, 31, 36, 37, 39, 43, 48, 49, 52, 57, 61, 63, 64, 67, 73, 75, 76, 79, 81, 84, 91, 93, 97, 100, 103, 108, 109, 111, ...

These are the Löschian numbers.

The Löschian numbers are used when modeling the structure of settlement in agricultural regions. Imagine a homogeneous plain with farmsteads occupying the points of a uniform triangular lattice. (See enclosed diagram.) Different types of retail stores and consumer services will appear on the plain. For any such type of store or service, an entire network of supply points together with space-filling market areas is required, thus giving every customer access to every commodity.

There are two conventional constraints. First, all market areas are hexagonal. (Among space-filling regular polygons, the hexagon departs least from the circular market area that would surround a lone supplier.) Second, the centre of every hexagon in every net of market areas must lie exactly on a farmstead point.

The "size" of a market area is the number of farmstead points it includes. If the edges of a hexagon pass exactly through a farmstead point, such a point is counted as one-half or one-third of a farmstead, as appropriate.

Question: what sizes of market areas are possible? Answer: the Löschian numbers.

The enclosed papers provide further information, including the generating function and a characterization of these numbers in terms of their prime factors.

Yours sincerely,

Dr. John U. Marshall
Associate Professor

JUM/kp
encl.

LÖSCHIAN MARKET AREAS OF SIZE $N = 109$.

$x = 5$; $y = 7$; $N = x^2 + xy + y^2$.

DISTANCE $AB = \sqrt{N} = \sqrt{109}$.