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INTERIM REPORT ON ALIQUOT SERIES

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To D. H. Lehmer on his two-thirds century.

0. INTRODUCTION

We denote the sum of the *aliquot parts* of n , i.e. the divisors of n other than n itself, by $s(n)$, so that $s(n) = \sigma(n) - n$, where $\sigma(n)$ is the sum of the divisors of n . As usual, n is called *deficient*, *perfect* or *abundant* according as $s(n)$ is less than, equal to, or greater than, n . Catalan [1] conjectured that if the function $s(n)$ is iterated, then there is a number k such that $s^k(n) = 1$ for each n . For example, if n is prime, then $k=1$. Dickson [3] modified the conjecture to except those cases where $s^k(n)$ is perfect, or a member of an *amicable pair*, i.e. a pair of numbers n_1, n_2 such that $s(n_1) = n_2$ and $s(n_2) = n_1$, or a member of a longer cycle. Such cycles seem to occur rarely; for more than 50 years the only known ones were those discovered by Poulet [4]:

$$s^5(12496) = 12496, \quad s^{28}(14316) = 14316.$$

Recently Cohen [2] has found nine cycles of period 4; i.e. $s^4(n) = n$ for $n = 1264460, 2115324, 2784580, 4938136, 7169104, 18048976, 18656380, 28158165$ and 46722700 . He states that these are the only cycles with period at most 10 having least member at most 6×10^7 . Dickson [3] had earlier examined all abundant numbers not exceeding 6232 (the lesser of the fifth amicable pair) in an attempt to find cycles of period 8

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or less.

1. DEFINITIONS AND NOTATION

We call the sequence $s^k(n)$, $k = 0, 1, 2, \dots$ the *aliquot series* n . It is *terminating* if a value of k is known for which $s^k(n) = 1$; the value of k in this case is called the *length* of the series and is denoted by $l = l(n)$. The largest term in a terminating series is denoted by $m = m(n)$, and the value of k for which this maximum is attained is denoted by $r(m)$. The penultimate term $s^{l-1}(n)$ is (the unique) prime, $p = p(n)$, of the series, and $l-1$ is also written $r(p)$. An aliquot series is *periodic* if there are values of k and $c > 0$ such that

$$s^{k+c}(n) = s^k(n),$$

i.e. if there are terms which are perfect or members of an amicable pair or longer cycle. The least k with this property is the *preperiod*, $l' = l'(n)$, and the least positive c is the *period* or cycle length, $c = c(n)$. The *length*, $l = l(n)$ of a periodic series is the cardinality of the set of (distinct) members of the series, i.e. $l = l' + c$.

We assign, as the *name* of a series, the least number which leads to a given maximum. Such names are, of course, tentative in the case of incomplete series; the only such series whose name is certain is 276.

To facilitate typing, printing and computer output we denote the term of rank k of the series n , i.e. $s^k(n)$ by $n:k$, except that n

is written for $n:0$. Also, factorizations of numbers into prime factors are given with exponents parenthesized, e.g. $2^5 \times 3 \times 5^2 \times 17 \times 211$ is written $2(5)3.5(2)17.211$.

2. RESULTS AND MAIN TABLE

Dickson [3,p.272] gave $138:k$ for $k \leq 73$, but did not factor $138:73 = 2(2)2053.49253$. D. H. Lehmer, Poulet and others showed that $\mathcal{L}(138) = 177$. Dickson gave $\mathcal{L}(318) = 34$ and $\mathcal{L}(480) = 65$, but omitted the term $480:53 = 1334 = 2.23.29$. More recently the Lehmers have shown that $600:69 = 702:300 = 720:194 = 858:167 = 864:59 = 936:184 = 1$.

We have examined all series with $n < 10^4$. Each incomplete series has been taken beyond $133499710216 = m(9840)$; all such series with $n < 4500$ have been taken beyond $m(4500)$ and those with $n < 3192$ beyond $m(3192)$. The Lehmers have told us that for $n = 702$ the series terminates; we have not yet verified their computed values of its maximum, prime and length. Since they took all incomplete series with $n < 10^3$ beyond $m(702)$, we can be sure of the names of all of our terminating series.

We consider a terminating series to be trivial if it contains no term with a prime factor greater than $\frac{1}{3} \cdot 10^5$; these are not further considered in this report. All other series with $n < 10^4$ are indexed in Table 1. Entries are of nine types, exemplified as follows:

(T) $600:69 = 1$, or generally $n:l = 1$, means that $n = 600$ is the name of a terminating series of length $l = 69$.

(T') $1938:8 = 138:13$, or generally $n':k' = n:k$

means that the series $n' = 1938$ merges with the named terminating series $n = 138$, the first common term being that of rank $k' = 8$ in the former and that of rank $k = 13$ in the latter.

(P) $496 = 496P$ means that 496 is perfect.

(P') $3528:7 = 6P$, or generally $n':k' = nP$, means that the term of rank $k' = 7$ (and no earlier term) of the series $n' = 3528$ is the perfect number $n = 6$.

(A) $1210 = 1210A$ means that 1210 is a member of an amicable pair.

(A') $1380:11 = 2924A$, or generally, $n':k' = nA$, means that the term of rank $k' = 11$ of the series $n' = 1380$ is the member $n = 2924$ of an amicable pair, and no earlier term is such a member.

(C) $5916:2 = 14316C$, or generally, $n':k' = nC$, means that the term of rank $k' = 2$ of the series $n' = 5916$ is the member $n = 14316$ of a cycle of period greater than 2 (here the Poulet 28-cycle).

(I) $552:56$ B, or generally $n:r(b)$ B, means that the series with (tentative) name $n = 552$ is incomplete, and that $r(b) = 56$ is the rank of its intermediate bound (see section 5. below).

(I') $1218:9$ $1134:3$, or generally $n':k'$ $n:k$, means that the series $n' = 1218$ merges with the incomplete

series with tentative name $n = 1134$, the first common term being that of rank $k' = 9$ in the former and that of rank $k = 3$ in the latter.

Note that entries of types T and T' refer to terminating series, those of types P, P', A, A' and C to periodic series, and those of types I and I' to incomplete series. Entries of types I and I' do not carry an equals sign.

3. PERIODIC SERIES AND DIGRAPHS

We have not attached names to the periodic series, but have displayed the relevant digraphs in Figures 1 to 10. The italic numerals indicate the distance of the terms from a member of the cycle, i.e. the length of the preperiod, l' . Every number less than 10^4 for which the series is known to be periodic appears in one of the Figures. Figures 1, 2, 3 are for series having the perfect numbers 6, 496 and 8128 for their periodic part. There are no values of n such that $s(n) = 28$, other than $n = 28$ itself. Figures 4 to 8 are for the first five amicable pairs (220,284), (1184,1210), (2620,2924), (5020,5564) and (6232,6368). Figures 9 and 10 are for the two Poulet cycles.

4. TERMINATING SERIES

Table 2 lists the name, n , the rank of the prime, $r(p)$, the prime, p , the length, l , the rank, $r(m)$, of the maximum, and the maximum itself, for each non-trivial terminating series presently known.

5. INCOMPLETE SERIES

To- ~~The~~ assist in detecting the merging of two incomplete series (in which case the larger name is suppressed) we have arbitrarily defined a term in each incomplete series, called the *intermediate bound*, an abundant term greater than 10^{10} ($10^{11}, 10^{12}$) having two prime factors p, q with $1000 < p < q < p^2$ and $pq > 10^9$ (resp. $10^8, 10^7$), except that the abundancy condition was dropped in the third case. Table 3 lists the intermediate bound for each incomplete series.

6. CONCLUDING REMARKS

Of over one hundred series reaching the intermediate bound, only seven have so far terminated: namely 702, 720, 858, 936, 3192, 4500 and 5760. We are tempted to conjecture, contrary to Catalan and Dickson, that infinitely many series go to infinity. The series 276 has been pursued by the Lehmers to its 331st term, which has 32 decimal digits. The longest known terminating series are exemplified by

$$702:300 = 6240:231 = 5430:213 = 3192:209 = 4500:203 = 9840:198 = 1044:195$$

The names of these series are 702, 1316, 1860, 3192, 4500, 9840 and 720.

The largest term we have so far factored completely is
 $1074:241 = 8629159519816727389344 = 2(5)3(3)7.11.137.211.4673.960207793$
and the series for which we have the most terms has reached
 $3774:305 = 51928246590440388 = 2(2)3.7. \text{ composite.}$

The computing was all done on Olivetti-Underwood Programma

101 machines, using the methods described in [5]. Additional programs were developed, specific to the problem in hand; for help with these, we thank E. F. Ecklund. Details may be obtained from the authors.

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Table 1

6 = 6P	828:1	660:1	1350	966:2	1734:87	B
25:1 = 6P	840:24	B	1356	660:1	1758:1	1134:1
28 = 28P	858:167 = 1		1372 = 1316:1		1770	1134:1
95:2 = 6P	864:59 = 1		1380:11=2924A		1772:2 = 1184A	
119:2 = 6P	870:7 = 138:13		1392	552:2	1778:4 = 496P	
138:177 = 1	888	552:1	1398:4	966:4	1794 = 936:1	
143:2 = 6P	909:4 = 6P		1410:3	966:4	1806:5	1134:3
150 = 138:1	913:3 = 6P		1420:2 = 1210A		1816:2 = 1210A	
168:1 = 138:4	936:184 = 1		1422:1 = 720:1		1818 = 702:3	
220 = 220A	960 = 138:6		1428 = 1316:2		1820:9 = 1316:7	
222 = 138:2	966:50 B		1440:6 = 702:9		1830:1 = 318:7	
234 = 138:3	978 = 702:1		1441:3 = 6P		1836	660:2
276:52 B	990 = 702:2		1443:4 = 6P		1842:5	564:4
284 = 284A	996:1	660:1	1464:29 B		1848:85	B
306:1	1032 = 480:1		1470 = 318:6		1854:4	564:4
312 = 138:4	1044:2 = 720:1		1476:52 B		1860:149 = 1	
318:34 = 1	1062:8 = 702:9		1482:6 = 138:13		1866:6 = 138:13	
330 = 318:1	1064:2 = 1184A		1488:36 B		1872	276:4
396	1074:180 B		1490:1 = 1210A		1876:3	276:10
417:3 = 6P	1086	1074:1	1506:10 = 138:13		1878:5 = 138:13	
445:3 = 6P	1098	1074:2	1512:38 B		1890:4 = 138:13	
480:65 = 1	1104	276:3	1518:9 = 138:13		1896:1	1632:1
496 = 496P	1122 = 318:5		1521:1 = 858		1898:1 = 1210A	
498:3 = 318:4	1134:66 B		1530:6 = 702:9		1902:7	1134:3
510:2 = 318:4	1146:1 = 858:1		1542:3 = 858:3		1914:6	1134:3
528 = 138:5	1148:2 = 600:1		1554:2 = 858:3		1920:123 B	
534 = 318:2	1158 = 858:1		1560:33 B		1932:2	276:10
546 = 318:3	1170 = 858:2		1572	564:2	1938:8 = 138:13	
552:56 B	1177:3 = 6P		1574:4 = 496P		1944:1 = 1860:1	
562:1 = 284A	1184 = 1184A		1578:55 B		1950	1734:1
564:43 B	1188:2 = 2924A		1590	1578:1	1962:1	552:3
565:3 = 6P	1204:1 = 600:1		1595:4 = 6P		1974:1 = 318:7	
570:8 = 138:13	1210 = 1210A		1596:9 = 1316:7		1986:3	966:4
600:69 = 1	1218:9	1134:3	1604:1 = 1210A		1992:25 B	
608:2 = 496P	1230:1 = 936:1		1608 = 480:2		1998:2	966:4
650:2 = 496P	1235:4 = 6P		1620:7 = 702:9		2008:3 = 1184A	
652:1 = 496P	1248:36 B		1632:60 B		2010:2	1476:3
660:67 B	1260 = 600:1		1633:3 = 6P		2040	840:1
675:4 = 6P	1266:8 = 702:9		1650:4	660:4	2046:2	966:4
685:3 = 6P	1278:7 = 702:9		1656 = 864:1		2058:78	B
696	1290:6 = 138:13		1662:8	1134:3	2082:2 = 858:3	
702:300 = 1	1294:3 = 496P		1674:7	1134:3	2088 = 138:7	
720:194 = 1	1302:1	1134:1	1686:1 = 720:1		2094:1 = 858:3	
726:8 = 138:13	1308:3 = 1184A		1690:2 = 1210A		2106 = 858:3	
750:1 = 318:5	1314:1	564:2	1692:4 = 2924A		2122:3 = 1184A	
780	1316:163 = 1		1698 = 720:1		2124	564:3
783:4 = 6P	1320	1074:3	1710 = 720:2		2136:74 = 1	
786:1 = 318:4	1326:1 = 720:1		1715:4 = 6P		2142:6 = 702:9	
790:3 = 496P	1336:1 = 1184A		1717:3 = 6P		2152:2 = 1210A	
798 = 318:4	1338	966:1	1722:1	1578:3	2154:7	1134:3

Table 1 (continued)

2160 = 702:4	2526 2514:1	2886:1 1476:3	3198:5 = 138:13
2162:4 = 496P	2538 2514:2	2898:1 2850:1	3202:2 =1210A
2166:6 1134:3	2541:5 = 6P	2904 1632:1	3204 660:3
2172:1 =2924A	2542:2 =1210A	2910:1 1134:3	3210:4 = 936:6
2173:3 = 6P	2544:23 1488:1	2912:2 = 702:5	3222 2514:3
2190:189 = 1	2550 1134:2	2922:8 = 702:9	3234:2 660:4
2195:4 = 6P	2562:1 966:4	2924 =2924A	3246:1 1734:2
2202:3 564:4	2568:1 1848:1	2930:2 =1184A	3258 1734:2
2208:1 1734:3	2580:46 B	2934:7 = 702:9	3264 =2136:1
2212:63 = 1	2581:3 = 6P	2940:7 =1316:7	3270:48 B
2214:2 564:4	2582:4 = 496P	2946:4 1134:3	3276:2 =2212:5
2225:4 = 6P	2604 =1316:3	2950:5 =1184A	3277:3 = 6P
2226:4 1134:3	2620 =2620A	2952:2 =2190:3	3278:4 =1184A
2232:89 B	2622:1 =2190:1	2958:3 1134:3	3282:2 1134:3
2238 = 936:2	2630:4 =1184A	2970 = 720:3	3286:2 =1210A
2250 = 936:3	2634 = 318:7	2974:2 =1210A	3288 1512:1
2256 1464:1	2640:22 1488:1	2976 = 858:4	3294:1 1134:3
2268 =2212:1	2646 = 318:8	2982:73 B	3306:2 2664:1
2280 1248:1	2652:10=2924A	2996:2 = 600:2	3311:4 = 6P
2292:21 1134:9	2658:9 1134:8	3000 1074:4	3312:1 1074:5
2298 1578:2	2660:3 =2212:5	3018:11 1734:6	3318:4 1578:5
2310 1578:3	2664:51 B	3024 = 864:2	3330:2 =3078:3
2322:4 1134:3	2670:8 1134:8	3025:1 1074:2	3332:5 =1210A
2328 552:3	2676:3 =2924A	3030:10 1734:6	3336 564:4
2334:1 1476:1	2678:3 =1210A	3036:81 = 1	3337:3 = 6P
2340:55 B	2682:5 = 702:9	3040 2360:1	3350:3 =1210A
2346 1476:1	2712:34 B	3048 1992:1	3360:28=376736C
2352:1 =1860:2	2716 276:8	3050 276:7	3366:48 B
2356:1 564:3	2718:5 = 936:6	3052:1 = 600:2	3378:1 966:4
2360:61 B	2724:9 1134:9	3060:12 1734:58	3388:3 =2212:5
2362:1 =1184A	2725:4 = 6P	3078:63 = 1	3390 966:4
2370 966:3	2742 2058:1	3080:66 = 1	3408:44 B
2382:7 = 138:13	2754 2058:2	3084:20 1134:9	3430:1 276:5
2387:4 = 6P	2760:1 =2136:2	3102:8 1134:8	3432:247 B
2394:6 = 138:13	2772 276:9	3108 = 600:2	3444:1 276:10
2402:2 = 600:1	2784:8 1134:9	3114:40 = 1	3450:1 1578:5
2406:5 1134:3	2802:4 660:4	3120:1 =1860:3	3456:30 B
2418:4 1134:3	2808:37 = 1	3124:1 =2924A	3462:6 = 702:9
2424:1 1464:2	2814:3 660:4	3126:1 =2190:1	3472 1488:2
2430:3 = 936:7	2826:1 564:4	3138 =2190:1	3474:5 = 702:9
2436:2 =2212:6	2828:9 =1316:7	3142:5 = 496P	3480 1560:1
2442:11 1734:6	2838 1476:2	3144:8 1134:9	3486:3 = 936:6
2448:1 = 936:5	2850:61 B	3150 =2190:2	3492:2 276:8
2472 = 480:3	2856:29=376736C	3156:1 2484:1	3498 1476:3
2478:3 1134:3	2862:84 = 1	3162:14=1184A	3510:1 = 138:10
2480 1488:1	2863:4 = 6P	3168:4 = 702:9	3516 =1860:1
2484:48 B	2864:1 276:8	3172:1 1632:1	3522:2 1134:3
2490:54 = 1	2874:2 1476:3	3174:7 = 702:9	3528:7 = 6P
2502:4 1134:3	2880:120 B	3186:5 =3078:3	3534:1 1134:3
2514:69 B	2884:8 =1316:7	3192:209 = 1	3552 552:4

Table 1 (continued)

3556:42	B	3864:1	564:6	4170:32	B	4518:12=3192:19	
3558	=2490:1	3870:3	= 138:13	4172:3	=2212:6	4520	2360:2
3564:34	B	3876:132	B	4180:9	=2924A	4522:3	=1184A
3570	=2490:2	3882:2	2664:1	4182	=3078:1	4530:4	= 138:13
3575:4	= 6P	3888:6	= 6P	4194	= 318:9	4535:4	= 6P
3582:10	1734:6	3892:5	=1316:7	4200	1920:1	4542	=2862:2
3588:3	3366:3	3894:1	2664:1	4206:10	1734:6	4554	=2862:3
3594:2	=2862:1	3899:4	= 6P	4208:10	=2212:5	4560:5	= 702:12
3596:2	=2924A	3906:46	B	4218:9	1734:6	4564	=1316:4
3606:1	=2862:1	3912	1848:1	4224:65	B	4566:3	= 936:6
3608:34	= 1	3918:1	2982:1	4228:2	=2212:6	4578:2	= 936:6
3612	3556:1	3930	2982:1	4230:2	1464:3	4584:2	=4428:2
3618	=2862:1	3936:1	3432:1	4236	2484:1	4590	2850:1
3630:41	B	3938:3	=1210A	4242:77	= 1	4602	1578:4
3636:9	1734:6	3942:1	1476:5	4278	1476:4	4614:7	1488:1
3642:47	= 1	3944:3	=2924A	4284:1	=2212:6	4620	=1316:5
3654	=3642:1	3948:4	=1316:7	4290:74	B	4626:6	1488:1
3660:8	1134:9	3952	=3608:1	4296:2	2340:2	4632	1992:2
3672	=3114:1	3960:21	1488:1	4302:1	3366:1	4638:1	3270:1
3678:40	B	3976:9	=2212:5	4314:9	1734:6	4640:11	=1316:7
3690	3678:1	3990:6	1734:6	4317:5	= 6P	4644:3	3366:2
3693:5	= 6P	3999:4	= 6P	4320:9	=1860:19	4648:9	2484:9
3696	1464:2	4002:2	3270:1	4326:8	1734:6	4650	3270:1
3702:3	660:4	4004:1	= 600:3	4344:48	= 1	4662:8	=3078:10
3712:4	=1210A	4008	2232:1	4350:51	B	4664:8	=2212:5
3714:2	660:4	4014:4	=3078:3	4362:3	1578:5	4668:2	1632:3
3720:1	= 936:7	4016:2	3456	4372:3	=1210A	4674:7	1134:8
3726:1	660:4	4026:9	1734:6	4374:2	1578:5	4680:5	=2212:5
3738:9	1734:6	4032:6	=2212:5	4380:98	B	4686:5	3270:5
3744:26	=2212:3	4060:2	=2212:5	4386:161	= 1	4704:1	2058:4
3746:4	276:10	4068:7	=2190:10	4392:4	3630:5	4710:8	=3192:12
3750:13	=1184A	4080:4	=3608:6	4396:3	660:25	4714:1	2360
3762	= 138:8	4086:1	= 936:5	4404:9	=2924A	4716	=1860:2
3768	= 480:4	4088:12	=2212:5	4422:4	3906:5	4717:3	= 6P
3770	276:5	4092:4	= 702:9	4428:32	= 1	4722:3	=3078:3
3774:113	B	4098:3	966:6	4434	3774:1	4728	=3608:2
3780	2058:3	4107:2	= 858	4440	840:2	4734:2	=3078:3
3786:1	2514:4	4110:2	966:6	4446	3774:2	4736:2	1488:1
3790	276:6	4116:62	B	4448:4	=1210A	4739:4	= 6P
3796:1	3456	4118:2	=1184A	4452:2	660:25	4746	2664:1
3798	2514:4	4122:2	= 936:7	4458:1	2514:5	4758:8	1734:6
3810:7	1134:8	4128	2712:1	4464	1488:3	4763:4	= 6P
3822:83	= 1	4134:1	1476:5	4470	2514:5	4770:3	= 138:13
3828:2	1632:3	4140:19	1134:9	4482:1	= 138:9	4775:4	= 6P
3834	= 936:4	4141:3	= 6P	4488:27	B	4776:7	1134:9
3836:6	=1316:7	4144:1	= 702:5	4494:13	2514:12	4782:2	2982:2
3840	1734:3	4146	1134:3	4500:203	= 1	4788:36	B
3846:5	= 138:13	4152:22	1488:1	4506:13	=3192:19	4790:5	=1210A
3850:4	=1210A	4156:2	=2924A	4508	=2212:2	4792:11	=2212:5
3858:4	= 138:13	4158	1134:4	4512:12	1734:58	4794:1	2982:2

Table 1 (continued)

4798:3 = 600:1	5088 = 858:5	5442:7 1734:10	5724:68 = 1
4800:31 B	5092:1 =4428	5448:27 B	5730:1 2982:4
4806 = 936:5	5094:2 = 936:6	5454:6 1734:10	5736:32 B
4812 2580:1	5104:6 =2212:5	5456:6 2484:9	5740 2360:3
4818 966:5	5106:1 966:6	5460 = 600:4	5742:11 660:29
4830:32 B	5118 =4386:1	5466:1 1578:5	5748:43 B
4840:6 =1210A	5124 =2212:4	5478 1578:5	5754 =3822:1
4842:29 B	5130 =4386:2	5480:4 3366:2	5760:161 = 1
4848:1 = 936:7	5148:44 B	5490:2 =3192:2	5766:4 1578:10
4854:4 2982:4	5150:4 =1184A	5502:8 3270:9	5778:43 B
4860:9 3366:2	5158:5 = 496P	5508:1 5250:2	5784:28=376736C
4866:3 2982:4	5160:1 1920:2	5510:10=2212:5	5790:1 =2490:4
4878:2 2982:4	5178:18 1578:18	5514:19 1578:18	5796:2 276:13
4890 =3078:2	5180:3 660:25	5520 3408:1	5808:8 660:25
4896:54 = 1	5190:17 1578:18	5526:18 1578:18	5816:7 =2212:5
4897:3 = 6P	5208:157 B	5536:2 276:8	5818:3 = 702:5
4902:8 1734:6	5214:7 =2190:10	5538:2 = 138:10	5820:2 3366:3
4914:4 =3192:6	5220:20 1632:37	5544:28 = 1	5826:1 966:6
4920 1248:2	5226:1 2664:2	5548:1 2580:1	5838 966:6
4926:1 1476:5	5232:8 660:25	5550 =4242:1	5840:58 2484:9
4930:6 =1210A	5234:1 =2620A	5560 =3080:1	5844:4 =2212:5
4932 = 318:10	5238:1 =2190:3	5562:1 =3078:3	5852:3 660:25
4938 1476:5	5243:4 = 6P	5564 =5564A	5854:3 =1184A
4950 1476:6	5250:27 B	5568:26=3080:9	5855:4 = 6P
4954:1 1488:1	5256:41 = 1	5574 2982:2	5856 660:5
4956:3 3432:66	5280 = 702:5	5586 2982:3	5862:13 2514:12
4962:2 660:4	5286:13=3192:19	5592 =2808:1	5870:2 2360
4974:1 660:4	5290:9 =2212:5	5598 = 138:9	5872:3 276:8
4980:33 = 1	5292:2 1464:5	5603:4 = 6P	5874:12 2514:12
4984:8 =2212:5	5298:12=3192:19	5605:5 = 6P	5880 =2136:2
4986 660:4	5304 2340:1	5610:12 966:17	5886:5 3906:4
4992 1512:2	5310:11=3192:19	5622:12=1184A	5890:5 =1184A
4998:5 3906:4	5314:4 =2212:5	5630:4 =1184A	5898:1 4170:1
5010:12 2514:12	5340:41 = 1	5634:11=1184A	5900:8 =2924A
5012:1 =2212:3	5348:1 = 600:3	5640:1 1992:4	5904:10 966:17
5020 =5020A	5352:47 B	5646:8 1734:6	5908:1 276:10
5022:3 =4242:2	5364:12 660:29	5658:7 1734:6	5910 4170:1
5024:7 =1210A	5370:3 3906:5	5664:3 3456:4	5916:2=14316C
5028 =3036:1	5376:1 =5340:11	5670 = 720:4	5922:12=3192:19
5034:8 1134:8	5380:10=1210A	5676 2484:2	5928 1848:2
5040:1 = 864:12	5382:26=2212:3	5680:6 276:8	5934:3 3270:5
5044:6 = 702:12	5394:2 3678:2	5682:4 3270:5	5940 3366:2
5046:7 1134:8	5400:25 B	5688 4842:1	5942:3 =1210A
5056:7 =2212:5	5404 = 600:3	5694:3 3270:5	5946 3630:1
5058 3366:1	5406:6 1134:8	5704:8 =2212:5	5958 3630:2
5064 564:5	5418:5 1734:6	5706 =3642:2	5960:9 =1210A
5068 =2212:3	5426:1 276:8	5712 = 480:5	5964 276:10
5076 1632:2	5430:69=1860:5	5718:2 2982:4	5970:71 B
5078:3 =1210A	5432:8 2484:9	5720:42 = 1	5982:1 = 936:6
5082:68=1860:5	5436:5 1488:1	5722:2 276:8	5994 = 936:6

Table 1 (continued)

6006:33 = 1	6276:5 1488:1	6594:2 =4242:2	6870:60 = 1
6018:2 =2490:4	6280:8 =1210A	6600 3564:1	6872:2 =5564A
6020:1 =2212:5	6282:4 5448:3	6604:1 3366:2	6876:10=1184A
6024 552:5	6288:21 1488:1	6612:10=1184A	6882:3 3630:5
6028:1 =5564A	6294:7 =2190:10	6618 1578:6	6894:20 1134:9
6030:31 = 1	6300 3556:3	6624:9 1578:15	6896 = 864:3
6036:8 =1184A	6304:3 1488:1	6630 1578:7	6902:5 276:8
6042:1 =3078:3	6306:6 =2190:10	6632:4 = 702:5	6904:6 =2212:5
6056:5 =2212:5	6318:5 =2190:10	6636:6 =1316:7	6906:1 =3078:3
6058:4 276:8	6328:7 2484:9	6648 3432:1	6908:2 =5340
6068:7 =2212:5	6330 2514:6	6654:8 =3192:12	6916:1 =2212:5
6072 2232:2	6338:2 1632:1	6656:8 =1316:7	6918 =3078:3
6074:1 2360:1	6342:3 3906:4	6658:6 =1210A	6930 =3078:4
6076:6 =1316:7	6348:5 660:25	6660:3 = 702:9	6936:2 =4428:2
6078 3906:1	6360 1074:5	6663:5 = 6P	6938:1 1488:2
6090 3906:2	6364:1 =5340	6666:7 =3192:12	6940:3 3366:2
6098:2 = 600:2	6366:11=3192:19	6672:13= 6P	6942:1 =2490:4
6102:22 4788:6	6368 =6368A	6678 =2862:4	6948:5 = 702:5
6108:12 1734:58	6372:9 3366:2	6680:55 B	6960 2712:2
6114:2 3678:2	6378:10=3192:19	6688:1 1488:4	6966:1 4830:2
6123:5 = 6P	6388:4 = 600:1	6690:1 =2190:6	6972:2 4800:6
6126:1 3678:2	6390:9 =3192:19	6692:5 =1316:7	6978:1 3630:3
6132:1 2360:5	6396:57 B	6694:4 =1210A	6984:50 B
6138 3678:2	6402:3 3630:5	6696 =3642:3	6990 3630:3
6140:8 =2212:5	6408 =3192:1	6700:10=1316:7	6996:19=2212:5
6145:5 = 6P	6414:3 = 138:13	6720:5 4350:11	7002:1 1464:3
6150:3 1578:10	6420:18=1210A	6726:69=1860:5	7006:4 =1210A
6160:40 B	6424:1 = 864:3	6732 =3036:2	7008 1992:3
6168:4 =3608:6	6426:2 = 138:13	6738:2 3270:5	7014:13 966:17
6170:2 1488:1	6438:6 1734:6	6744 3456:1	7020:105 = 1
6174:1 =2190:5	6444 2580:2	6748:4 =1316:7	7025:4 = 6P
6180:4 =3608:6	6448:5 2484:9	6750:1 3270:5	7026 2880:1
6184:2 276:8	6450:46 B	6752:2 3366:2	7028:1 4116:1
6186:1 2664:2	6462:22 4788:6	6756:7 1134:9	7032:3 1632:40
6188:58 2484:9	6474 3774:3	6768:11 1734:58	7038 2880:2
6198 2664:2	6480:1 4830:5	6774:6 1734:10	7040 =3080:2
6204 3876:1	6486:17 1578:18	6780:40 = 1	7044:182 B
6210 2664:3	6496 = 864:4	6786:5 1734:10	7050:33 = 1
6222:8 3270:9	6504:1 2340:2	6792:86 B	7060:4 = 600:3
6228:1 4350:2	6522 =2190:3	6796:7 =2212:5	7062:2 3270:4
6232 =6232A	6528:49 = 1	6798 =2490:3	7074:5 4350:4
6234:11=3192:19	6532:1 =5564A	6804:3 =1316:7	7080:46 B
6240:75=1316:7	6534 =2190:4	6808:3 =5564A	7084 4116:1
6242:2 =2924A	6546:2 = 138:10	6810 4350:1	7086:11 2514:12
6244 3556:2	6552:32 B	6822:78 B	7092:12 966:17
6246:10=3192:19	6558:1 = 138:10	6828:1 =4980:1	7098:10 2514:12
6252:1 1632:3	6570 = 138:10	6832:143 B	7106:4 =1184A
6258:5 1134:8	6576 =4344:1	6834:12 966:17	7110:117 B
6264:55 660:25	6582:3 =4242:2	6846:1 6822:2	7120:7 =2924A
6270:3 5778:4	6590:10=2212:5	6858:1 = 318:12	7122:8 1734:10

Table 1 (continued)

7128 =3114:2	7388:2 2580:1	7644:1 660:25	7890:62 B
7130:5 =1210A	7392:60 B	7656 564:6	7900:25 = 1
7134:7 1734:10	7396:5 = 6P	7662:69=1860:5	7902:2 =4386:3
7140:42 = 1	7398 =7374:2	7664:9 660:25	7908:4 6396:4
7144:3 =5340	7403:4 = 6P	7666:7 =1316:7	7920:52 B
7146:2 =6030:2	7410:38 = 1	7668:46 = 1	7922:2 2360
7149:5 = 6P	7412:6 2484:9	7674:68=1860:5	7923:4 = 6P
7152 =3608:3	7416:1 = 702:6	7676:2 3366:2	7924:57 2484:9
7158:8 3270:9	7418:5 =1210A	7680:1 2232:4	7930:6 =1210A
7170:7 3270:9	7420:4 1464:5	7684:1 6680	7940:7 =1210A
7172:2 3366:2	7422:52 B	7686:67=1860:5	7950:3 4170:9
7180:8 =1210A	7434:1 7422	7690:3 1488:1	7956:4 2514:12
7182 5778:1	7440:4 2484:9	7692 5748:1	7960:2 =3080:9
7186:3 =2924A	7446:12=3192:19	7694:5 =1210A	7962:33=1316:7
7192:8 660:25	7458:6 =2190:10	7698:3 3630:5	7968:1 5400:1
7194:7 =3078:10	7464:6 1134:16	7704:1 = 600:5	7974:32=1316:7
7206:7 4350:11	7470:2 =4344:19	7710:2 3630:5	7980:56 2484:9
7208:7 660:25	7482:5 4170:9	7712:2 276:5	7982:2 1488:1
7216:8 660:25	7494 =3822:2	7722:25=2212:3	7986:6 1734:10
7218:6 4350:11	7504:21=1316:7	7728:1 =2808:3	7992:4 6792:4
7224:6 1134:9	7506 =3822:3	7740:14=5720:9	7998 6822:1
7225:22 1134:9	7514:11=1210A	7752:34 B	8002:2 = 600:3
7227:6 = 6P	7518:22 1632:37	7756:6 =1316:7	8010:29 = 1
7230:1 4830:3	7520:8 =1184A	7764:29 = 1	8015:2 1074:2
7242:5 1734:6	7524:1 =14316C	7770:4 =2190:10	8016 4224:1
7254 3270:2	7527:4 = 6P	7778:6 =1316:7	8020:6 660:25
7256:2 =5340	7530:5 1734:6	7782:2 =4980:2	8022:6 =3192:12
7260:3 =3608:6	7532:3 660:25	7788:1 =6780:1	8024:2 3456:2
7266:1 =2190:6	7534:1 276:5	7794:1 =4980:2	8026:3 3456
7272:65 = 1	7540:8 =1210A	7796:4 =1184A	8028:14=1316:7
7294:2 =2620A	7542:1 3678:3	7800 = 936:7	8034:1 =2190:6
7296 =1860:3	7543:4 = 6P	7806 4290:1	8040:38 B
7302:5 3906:4	7554:2 6822:2	7808:3 = 600:3	8046:11 966:17
7308:2 2360:7	7560:40 B	7812:5 =1316:7	8052 4380:1
7314:4 3906:4	7566:1 6822:2	7818 4290:2	8056:9 =1316:7
7320 1560:2	7570:2 2360:1	7820:3 =2212:5	8058:2 3270:3
7326:9 =3192:19	7572:60 = 1	7824:15=2212:5	8064:58 B
7332:3 =4428:4	7578:21 4788:6	7828:1 =3036:2	8068:5 276:8
7338:16 1578:18	7584:11 1734:58	7830 4290:3	8070:10 2880:7
7344:4 2514:12	7586:2 3456	7842:12 966:17	8076:7 =1184A
7350:15 1578:18	7587:6 = 6P	7848:6 = 600:24	8080:2 4800:1
7352:6 2484:9	7588:2 660:25	7852:1 7044	8082:19 1134:9
7360:57 660:25	7590:2 3906:5	7854:11 966:17	8084:11=1316:7
7362 1134:5	7596:1 6160:1	7856:6 = 6P	8088 5352:1
7364:5 1464:5	7600 =7572:2	7860:1=14316C	8092:8 4788:6
7368:3 5448:3	7602 966:7	7864:1 = 864:3	8094 2982:4
7372:6 660:25	7614:1 2880:3	7866:2 = 138:13	8096:4 =2212:3
7374:121 =1	7626 = 318:11	7868:58 2484:9	8104:5 =1184A
7380:13=1210A	7632:1 =5760:1	7878:2 =4386:3	8112:5 7920:2
7386 =7374:1	7638 3774:4	7885:5 = 6P	8114:3 =2212:5

Table 1 (continued)

8118:31=1316:7	8382:1 =2490:6	8610:3 =3192:7	8904:60 B
8120:1 7044:135	8388:1 =2190:7	8622:4 3270:6	8910 6822:3
8124:2 =4980:3	8394:2 =4896:1	8624 = 864:5	8922:1 2514:7
8128 =8128P	8396:4 1488:1	8628 1134:6	8924:9 =1210A
8130:12 966:17	8406:1 =4896:1	8632 6160:1	8932:7 =1316:7
8135:4 = 6P	8408:7 660:25	8634:7 =3078:10	8934 2514:7
8136:2 3678:5	8412:10 3366:2	8640:43 = 1	8942:5 =2212:5
8142:4 1134:8	8418:1 =2190:6	8646:6 =3078:10	8944:1 5208:1
8144:8 =1316:7	8424 1734:4	8648:1 6160:1	8946 2514:8
8148:5 =1316:7	8427:2 1074:2	8650:4 660:25	8952:4 1920:5
8154:58 B	8430 5970:1	8652 660:23	8958:5 =2190:10
8160:12 4788:6	8432 1488:4	8658:65 B	8964:18 1134:9
8164:1 1560:2	8440 6680:1	8664 5736:1	8970:4 =2190:10
8166:1 =2490:4	8442:12 966:17	8682 3774:5	8976:48 B
8172:11 1734:58	8448 =2808:2	8694 3774:6	8988:7 4788:6
8176:1 3456:2	8454:3 =6870:1	8706:12=3192:19	8994 4830:1
8178 =2490:4	8456:3 =2212:5	8708:1 =2212:5	8996:10=1316:7
8184:57 B	8460:5 4350:11	8718:11=3192:19	9000:1 6160:5
8190:1 3630:8	8466:2 =6870:1	8720:10 1134:9	9006 4830:2
8204:4 =1316:7	8470:2 =8128P	8730:10=3192:19	9008:3 6160:1
8208 1464:3	8472 4488:1	8736:27=376736C	9018:1 =3192:2
8232 5448:1	8476:2 6160:1	8742:1 =6870:1	9024:1 2712:3
8238:3 3906:4	8477:4 = 6P	8744:8 =1316:7	9028:7 1134:16
8250:2 3906:4	8480:11=1184A	8748:4 3876:2	9030:5 1488:8
8260:3 =1316:7	8484:2 3432:66	8754:5 4350:4	9034:1 2360:2
8262:30 B	8490:1 3270:4	8760:120 B	9035:5 = 6P
8274:2 = 138:11	8496:28 B	8764 =2212:5	9036:6 1134:9
8280:17 1632:37	8502 = 318:12	8766:4 4350:4	9038:4 =1184A
8286:11 660:29	8512:3 7890:9	8776:6 =1210A	9042:7 =8592:4
8288:53 B	8514:19 1632:37	8778:1 =4896:4	9044 4116:2
8298:10 660:29	8520 = 858:6	8784:7 =1184A	9048:4 6552:4
8300:10=1316:7	8526:3 =3192:6	8785:5 = 6P	9052:2=14316C
8306:3 =2924A	8532:4 =1316:4	8790:7 =3078:10	9054:11=3192:19
8310:4 1734:6	8538:11=3192:19	8802:1 3630:5	9064:4 =2212:5
8316:2 2058:6	8540:24 4380:12	8814:2 3678:4	9066:7 =3078:10
8320:9 =1184A	8544 6832:1	8816:3 =2212:5	9068:4 =5564A
8322:1 =2190:6	8545:5 = 6P	8820 =2212:6	9078:6 =3078:10
8328:2 4842:3	8550:10=3192:19	8826:1 3678:3	9096 552:6
8334:22 1632:37	8556:22 = 1	8832:56=1210A	9100:7 4788:6
8340:10 5208:6	8562:2 =4242:2	8838 3678:3	9102 =2490:5
8344:4 =2212:5	8568:1 6792:2	8844:216 B	9108 2484:3
8346 4170:2	8574:1 =4242:2	8850:10=3192:19	9114:12 966:17
8352:45 B	8576:1 =2212:5	8862:61 B	9120:24 B
8354:10=2924A	8580:100 = 1	8864:5 660:25	9122:1 =1316:4
8358:4 4170:9	8586 =4242:2	8872:7 =1316:7	9126:65=1860:5
8360:13=2212:3	8588:7 660:25	8874:2 =4344:19	9132 =4980:1
8364 1632:3	8592:65 = 1	8876:8 =1316:7	9136 660:21
8370 2850:2	8593:5 = 6P	8880:20 4788:6	9138:3 1134:8
8372 2360:4	8596 660:22	8886:1 6822:2	9144:35 = 1
8376:1 =6030:2	8598:4 =3192:7	8898 6822:2	9150:2 1134:8

Table 1 (concluded)

9156:5 =1316:7	9400 =5720:1	9680:5 660:25	9930:12=3192:19
9162 =7374:3	9402:11 966:17	9683:4 = 6P	9936 2580:3
9168:1 =2136:3	9404:5 = 600:3	9684:50 B	9942:11 966:17
9174 =5256:1	9412:1 1734:4	9690 =6870:1	9950:5 660:25
9176:5 =2212:5	9414:10 966:17	9696 =4500:1	9954:10 966:17
9180:75 = 1	9420 7044:1	9699:6 = 6P	9958:3 1488:1
9186 2982:5	9424 1488:5	9702:3 9282:4	9960:43 = 1
9192:1 7752:1	9426 =2190:5	9708:31 B	9978 = 318:13
9196:1 1488:5	9436:62 B	9712:1 660:21	9980:16 3876:11
9198 2982:6	9438 =2190:6	9714:1 5250:1	9990 = 318:14
9200:9 =1316:7	9440:13=2212:3	9716:1 4788:1	9996:52 = 1
9204:1=14316C	9450:27 4116:21	9720:9 660:29	9997:5 = 6P
9210:1 1578:9	9456:2 3456:4	9726 5250:1	
9222:1 3270:3	9460 =7900:1	9728:11=1316:7	
9234:5 2982:8	9462:66 B	9738 5250:2	
9240 840:3	9464:1=12496C	9750:3 =2190:10	
9244:4 3366:2	9466:3 1488:1	9762:21 1632:37	
9246:2 3678:4	9468:18 1134:9	9768 660:6	
9252:6 3270:9	9474:2 1578:10	9772 4788:1	
9258:1 =4386:3	9480:33 B	9774:20 1632:37	
9268:59 = 1	9484:8 =2924A	9780 =5340:1	
9270 =4386:3	9486:1 1578:10	9782:5 =2212:5	
9274:12=1316:7	9492 9436:1	9784:2 =2212:5	
9280:3 =1316:7	9498:102 = 1	9786:3 4170:9	
9282:57 B	9510 =9498:1	9796:3 =4980:3	
9288 1512:3	9526:3 = 600:2	9798 4170:3	
9294 =3822:4	9534:57 3876:5	9810 2880:3	
9296 =4344:17	9540:58 B	9816 2340:2	
9299:4 = 6P	9550:4 =2924A	9822:7 4350:9	
9306 =3822:5	9558 1476:7	9828 4788:2	
9312:3 =3608:6	9564:1 4380:2	9834:6 4350:9	
9318:4 7890:7	9570:2 8154:7	9840:198 = 1	
9320:10 1134:9	9572:4 =2924A	9844:11=1316:7	
9322:4 =1210A	9574:6 =1210A	9846 =4896:1	
9324 =9268:1	9576:161 = 1	9848:1 6160:1	
9330:3 7890:7	9578:12=2212:5	9852:48 B	
9332:5 =1210A	9582:5 1734:10	9856:12=2212:5	
9336:41 B	9588:32 B	9858 3630:4	
9342:31=1316:7	9592:59 660:25	9870 966:8	
9344:4 = 600:2	9594:4 1734:10	9882 =6030:1	
9352:59 660:25	9606 4350:2	9888:7 =9180:8	
9354:4 4170:9	9614:8 =1316:7	9894:2 7422:2	
9360:20=1316:7	9618 4350:3	9902:2 1488:1	
9366:3 4170:9	9620:6 =2924A	9906:2 7110:1	
9370:12=1210A	9630:40=2212:3	9912 4842:2	
9372:24 2360:4	9653:4 = 6P	9916:12 1734:58	
9378:23 B	9656:3 =2212:5	9918 6450:1	
9380:2 4800:3	9660 2058:4	9921:5 = 6P	
9390:1 6984:2	9672:25=3080:9	9924 3876:2	
9396 =5724:1	9678:1 =6870:1	9928:9 =1316:7	

A3023

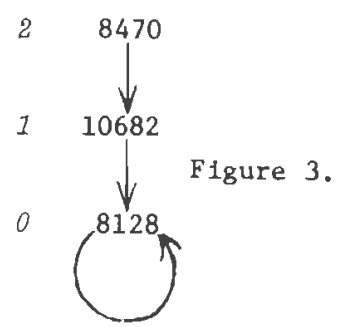
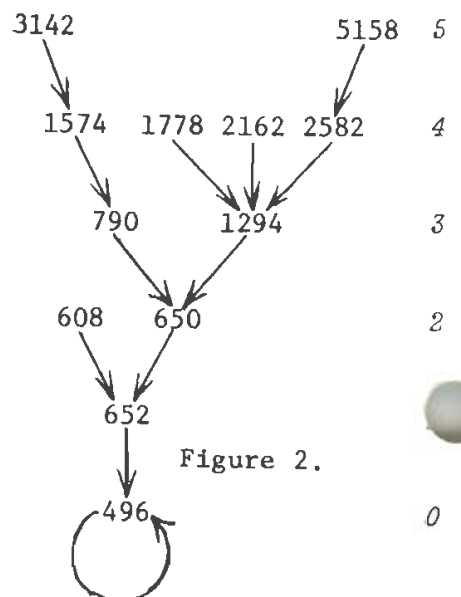
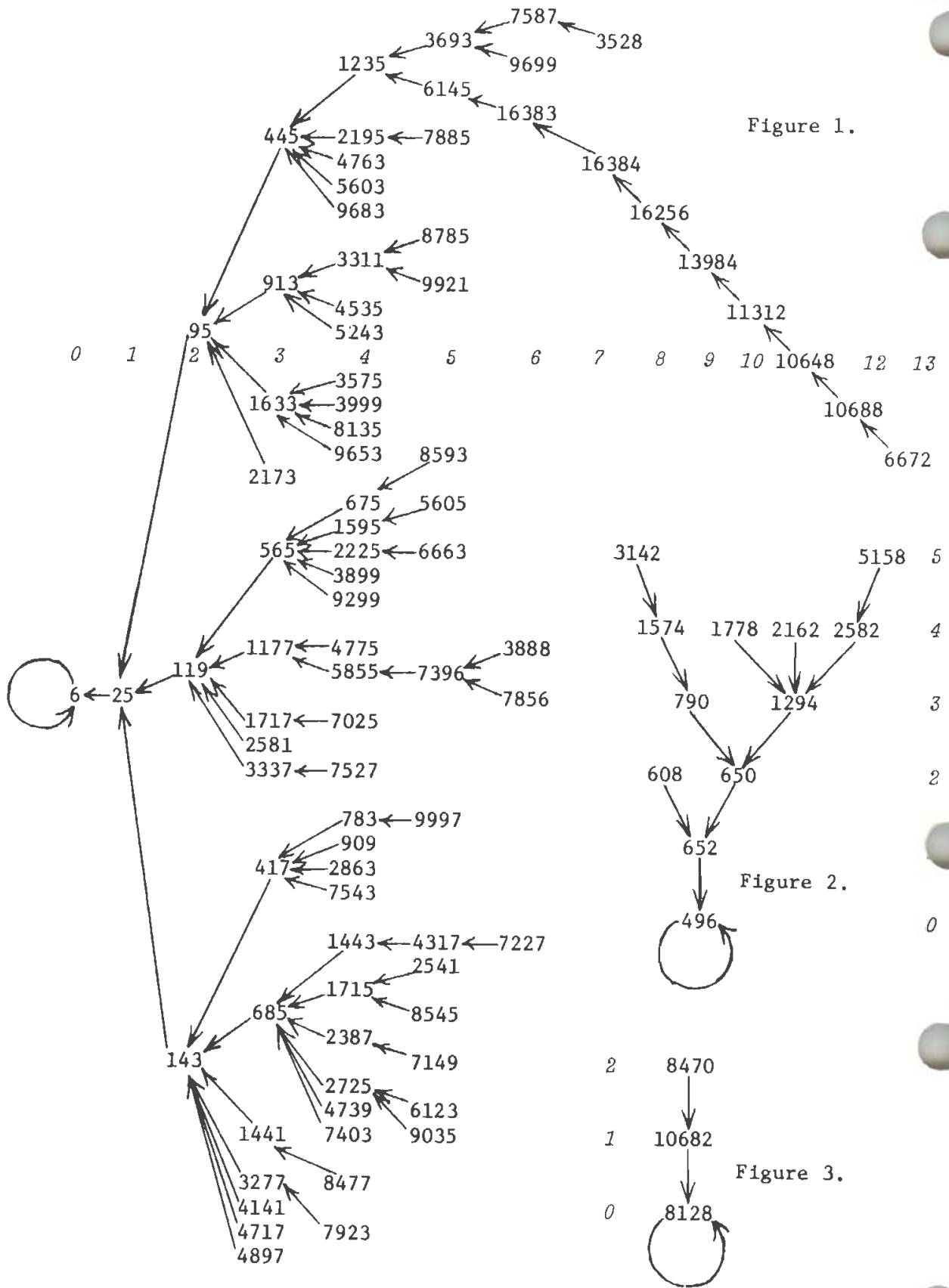




Figure 4.

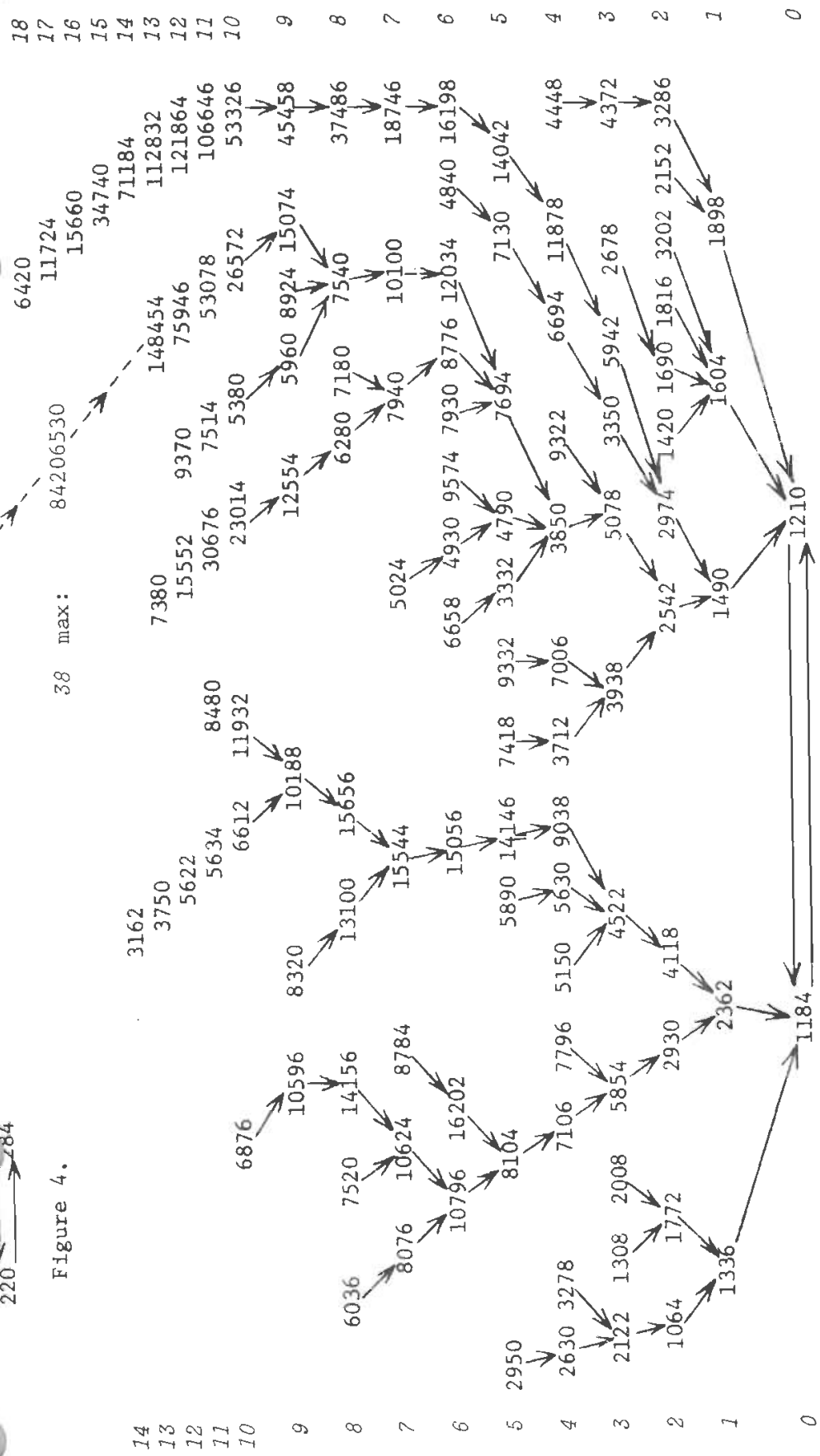


Figure 5.

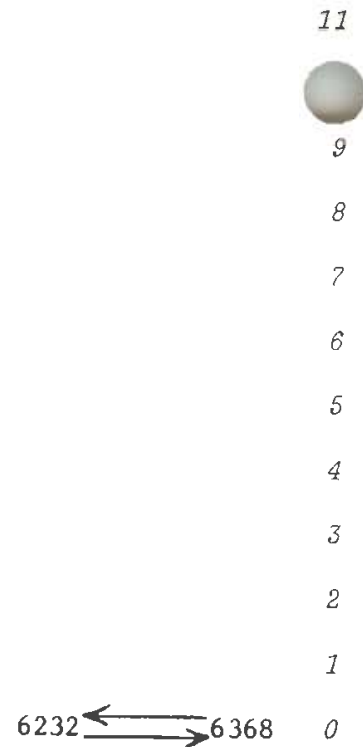
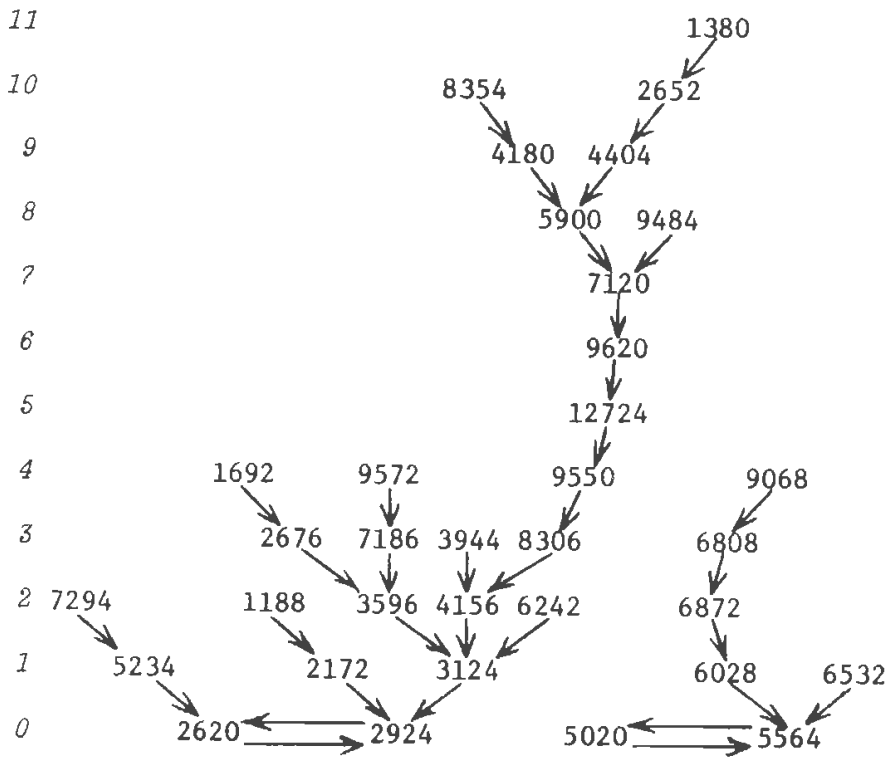


Figure 6.

Figure 7.

Figure 8.

29 2856
 28 3360 5784
 27 8736
 26 19488
 25 40992
 24 84000
 23 230496
 22 475356
 21 792484
 20 1013852
 19 1013908
 18 1058092
 17 1264340
 16 2049964
 15 2123576
 14 2778664
 13 3492536
 12 3077104
 11 2884816
 10 3391568
 9 3775384
 8 3303476
 7 3003244
 6 2288756
 5 1773484
 4 1558964
 3 1440718
 2 720362
 1 360184

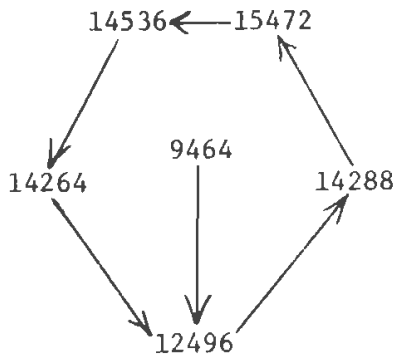


Figure 9.

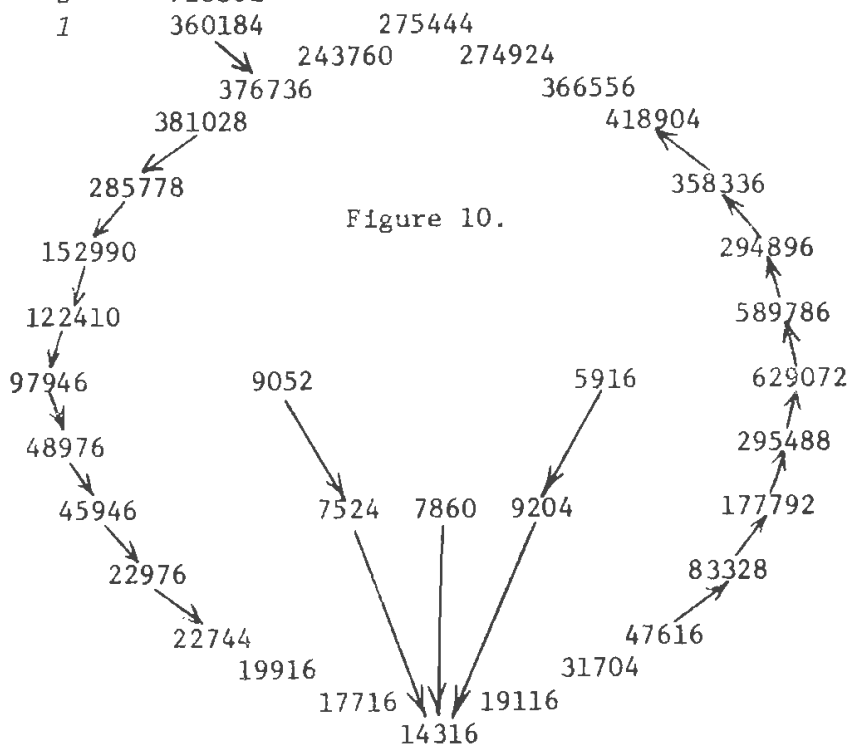


Figure 10.

NON-TRIVIAL TERMINATING SERIES

$n:r(p)$	p	l	$r(m)$	Maximum
138:176	59	177	117	179931895322 = 2.61.929.1587569
318:33	321329	34	32	722961 = 3(2)80329
480:64	41	65	12	321836 = 2(2)61.1319
600:68	601	69	29	594884 = 2(2)148721
702:299	191	300	158	517339938566442996382 = 2.7.107.179.706753.2729879657
720:193	277	194	125	36445367869087816 = 2(3)4555670983635977
858:166	59	167	61	41683781196584 = 2(3)25153.207151141
864:58	41	59	19	565832 = 2(3)70729
936:183	43	184	113	33289162099526 = 2.25943.641582741
1316:162	37	163	39	205780934846 = 2.71.1231.1177223
1860:148	59	149	66	956557384 = 2(3)179.667987
2136:73	15559	74	24	74733199568 = 2(4)4670824973
2190:188	7	189	45	14653555856 = 2(4)915847241
2212:62	7	63	25	4365364 = 2(2)19.41.809
2490:53	43	54	18	262084 = 2(2)65521
2808:36	73	37	10	387976 = 2(3)48497
2862:83	113	84	22	551692 = 2(2)107.1289
3036:80	761	81	37	1132877468 = 2(2)293.966619
3078:62	601	63	17	564196 = 2(2)17.8297
3080:65	41	66	21	911870 = 2.5.67.1361
3114:39	43	40	15	417896 = 2(3)52237
3192:208	59	209	128	58044272769664 = 2(7)453470881013
3608:33	1093	34	11	292616 = 2(3)79.463
3642:46	41	47	10	221030 = 2.5.23.31(2)
3822:82	601	83	42	210021938 = 2.7(2)2143081
4242:76	73	77	27	2470768 = 2(4)154423
4344:47	59	48	29	333634 = 2.7.23831
4386:160	7	161	59	7535462602 = 2.17.29(2)263533
4428:31	41	32	6	73126 = 2.36563
4500:202	61	203	123	2222039156768 = 2(5)16001.4339649

Table 2

$n:r(p)$	p	l	$r(m)$	Maximum
4896:53	7	54	23	1952108 = 2(2)107.4561
4980:32	59	33	14	250946 = 2.271.463
5256:40	43	41	15	500824 = 2(3)62603
5340:40	193	41	21	3249184 = 2(5)101537
5544:27	257	28	6	290908 = 2(2)72727
5720:41	113	42	19	288722 = 2.7.41.503
5724:67	12451	68	36	82508726 = 2.1319.31277
5760:160	41	161	58	98359352128 = 2(6)1536864877
6006:32	59	33	13	269914 = 2.19.7103
6030:30	36637	31	10	1296376 = 2(3)131.1237
6528:48	3	49	7	470012 = 2(2)117503
6780:39	1153	40	11	1118498 = 2.17.67.491
6870:59	41	60	12	819824 = 2(4)51239
7020:104	7	105	25	359386892 = 2(2)107.179.4691
7050:32	59	33	13	111278 = 2.55639
7140:41	59	42	19	275752 = 2(3)34469
7272:64	41	65	12	595406 = 2.7.71.599
7374:120	191	121	35	93259952 = 2(4)191.30517
7410:37	59	38	11	478264 = 2(3)191.313
7572:59	41	60	18	1020632 = 2(3)127579
7668:45	43	46	8	840016 = 2(4)52501
7764:28	7	29	7	101044 = 2(2)25261
7900:24	29959	25	9	218732 = 2(2)149.367
8010:28	281	29	21	591773 = 7(2)13.929
8556:21	72937	22	11	920204 = 2(2)31.41.181
8580:99	601	100	40	229526216 = 2(3)28690777
8592:64	1483	65	19	8897606 = 2.29.153407
8640:42	43	43	4	201422 = 2.13.61.127
9144:34	59	35	6	179356 = 2(2)44839
9180:74	59	75	28	835518724 = 2(2)43(2)173.653
9268:58	263	59	43	97317813 = 3.137.236783
9498:101	43	102	58	8875096 = 2(3)1109387
9576:160	41	161	62	11937719378 = 2.13.149.3081497
9840:197	7	198	66	133499710216 = 2(3)89(2)216737
9960:42	41	43	7	2114512 = 2(4)132157
9996:51	11	52	13	6645238 = 2.523.6353

Table 2 (concluded)

INCOMPLETE SERIES

$n:r(b)$	Intermediate bound
276:52	220578719452 = 2(2)7.13.1223.495491
552:56	405894291460 = 2(2)5.7.7993.362723
564:43	26419407576 = 2(3)3.5443.202243
660:67	212809257084 = 2(2)3.3361.5276437
840:24	191723155392 = 2(6)3.5581.178921
966:50	124794509588 = 2(2)7.27073.164627
1074:180	2478974179836 = 2(2)3.7.11(2)11677.20887
1134:66	304645135320 = 2(3)3(3)5.6287.44867
1248:36	101907983448 = 2(3)3(2)2153.657403
1464:29	989941190112 = 2(5)3(2)7.4079.120383
1476:52	13574152986 = 2.3.11467.197293
1488:36	39600629604 = 2(2)3(2)32987.33347
1512:38	3582166158840 = 2(3)3.5.139.2239.95917
1560:33	6875289292128 = 2(5)3.70009.1022977
1578:55	32848875546 = 2.3.1973.2774867
1632:60	374571381000 = 2(3)3.5(3)2579.48413
1734:87	104516519202 = 2.3(2)39623.146543
1848:85	1525857209544 = 2(3)3.16487.3856213
1920:123	140173209036 = 2(2)3(2)7.1667.333679
1992:25	164283163080 = 2(3)3.5.34651.39509
2058:78	115085277702 = 2.3(2)53.7937.15199
2232:89	1854307659480 = 2(3)3.5.49789.310361
2340:55	3117736513444 = 2(2)7.29.37.8387.12373
2360:61	86114683386 = 2.3(3)17123.93133
2484:48	45348987912048 = 2(4)3(2)31.7639.1329863
2514:69	60728960514 = 2.3.3691.2742209
2580:46	5865843954480 = 2(4)3.5.8963.2726879
2664:51	10788868626 = 2.3.7057.254803
2712:34	266403416848 = 2(4)31.10357.51859
2850:61	712934354580 = 2(2)3.5.7.22871.74219
2880:120	70327178908 = 2(2)7.22079.113759
2982:73	4552519505636 = 2(2)277.2503.1641539
3270:48	1434065254826 = 2.673.6121.174061
3366:48	160793817810 = 2.3.5.2971.1804037
3408:44	1502099831160 = 2(3)3(4)5.10039.46181

Table 3

$n:r(b)$	Intermediate bound
3432:247	97956747006 = 2.3.59441.274661
3456:30	322138579560 = 2(3)3.5.7.5801.66109
3556:42	1533695223900 = 2(2)3.5(2)7.2039.358181
3564:34	300074840328 = 2(3)3.98387.127081
3630:41	14309148161196 = 2(2)3(2)7.27529.2062637
3678:40	1328517972840 = 2(3)3.5.12011.921737
3774:113	12267186324 = 2(2)3.3169.322583
3876:132	546951209364 = 2(2)3(2)7.19.5309.21517
3906:46	31182877410 = 2.3.5.18211.57077
4116:62	862926219446304 = 2(5)3.13(2)27673.1922027
4170:32	220044748920 = 2(3)3.5.17.5479.19687
4224:65	433921126548 = 2(2)3.7.13.2351.169019
4290:74	809972572890 = 2.3.5.13.1487.1396673
4350:51	195460355104 = 2(5)7.11813.73867
4380:98	9028171047420 = 2(2)3(4)5.7.23017.34589
4488:27	2138163784584 = 2(3)3.23.197.1787.11003
4788:36	949932696780 = 2(2)3(2)5.17341.304331
4800:31	143870596324 = 2(2)7.13.1511.261581
4830:32	27854323368 = 2(3)3.3853.301219
4842:29	636460714680 = 2(3)3.5.7.5737.132071
5148:44	289917661590 = 2.3(4)5.5351.66889
5208:157	986270520846 = 2.3.13.10853.1165069
5250:27	1089646580040 = 2(3)3.5.73.5443.22853
5352:47	175466928360 = 2(3)3.5.3517.415759
5400:25	20250672643488 = 2(5)3.7.29.1811.573791
5448:27	108767076460 = 2(2)5.7.15107.51427
5736:32	223163372628 = 2(2)3(2)7.1283.690233
5748:43	1413647866520 = 2(3)5.347.3779.26951
5778:43	16500700716 = 2(2)3.9391.146423
5970:71	382891527264 = 2(5)3.21419.186211
6160:40	400715783412 = 2(2)3.7.64327.74159
6396:57	795384851880 = 2(3)3.5.4007.1654157
6450:53	31987842302928 = 2(4)3.47.32783.432511
6552:32	288391101138 = 2.3(3)7.18793.40597
6680:46	214727191584 = 2(5)3.19.2663.44207

Table 3 (continued)

$n:r(b)$	Intermediate bound
6792:86	184401993894 = 2.3.4621.6650869
6822:78	6693765458130 = 2.3(6)5.8273.110989
6832:143	51591336072 = 2(3)3.19037.112919
6984:50	30790380318 = 2.3.11321.453293
7044:182	1513482717068 = 2(2)7.509.8741.12149
7080:46	128369960910 = 2.3(2)5.8951.159349
7110:117	30123043119408 = 2(4)3.31.76231.265561
7392:60	184373541168 = 2(4)3.6373.602717
7422:52	570457341040 = 2(4)5.9791.728293
7560:40	775004722020 = 2(2)3(2)5.17.2297.110261
7752:34	326667059728 = 2(4)31.2753.239231
7890:62	10025736658512 = 2(4)3.19.31.13291.26681
7920:52	20026075100640 = 2(5)3.5.7.1949.3058051
8040:38	2833036225608 = 2(3)3.33113.3564859
8064:58	83931543954 = 2.3(3)4283.362897
8154:58	82449521394 = 2.3.3727.3687037
8184:57	1090270726972 = 2(2)7.71.2957.185467
8262:30	245736526254 = 2.3.3541.11566249
8288:53	399358281042 = 2.3(3)7.14557.72577
8352:45	507954462786 = 2.3(2)11779.2395763
8496:28	166392530612 = 2(2)7.11.6781.79669
8658:65	143381221722 = 2.3(2)45863.173683
8760:120	1834812100146 = 2.3.17.5981.3007583
8844:216	124575365472 = 2(5)3.18691.69427
8862:61	24755870778 = 2.3.7727.533969
8904:60	4178203366208 = 2(6)127.7069.72719
8976:48	305661525738 = 2.3.26713.1907071
9120:24	1694514822144 = 2(10)3.8971.61487
9282:57	67250306328 = 2(3)3.27943.100279
9336:41	10122623140032 = 2(6)3(2)13.127.1231.8647
9378:23	106515409056 = 2(5)3.7.1297.122209
9436:62	410340106720 = 2(5)5.46099.55633
9462:66	6240250640112 = 2(4)3.49627.2619647
9480:33	61026906660 = 2(2)3.5.1447.702913
9540:58	93157320116 = 2(2)7.4337.767131
9588:32	1444419167016 = 2(3)3.7(2)5413.226907
9684:50	857629165910496 = 2(5)3.7.19.167.1999.201209
9708:31	116461066446 = 2.3(2)9791.660817
9852:48	1998913535056 = 2(4)31(2)1627.79903

Table 3 (concluded)