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On ninth order knottiness

by

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This paper is an exercise in applied algebraic topology. Our purpose is to examine the known knots, up to a certain level of complexity, and try to decide which of them are in fact topologically inequivalent. There is at present no general algorithm for determining if two knots* are different (Figure 1) or the same (Figure 2) and empirical methods, although persuasive to some [3,pp.341-342], are demonstrably unreliable [8,Fig.1]. Thus, we search on an *ad hoc* basis for invariants which suffice to distinguish certain knots in the hope that we will be able to show the equivalence of those which then remain. If successful, we will have proven (not just guessed) the number of different knot types in a given table. *Cf.* [8].

In the present paper we direct our attention to the tabled knots with eleven crossings and discuss the extent to which we fall short of our goal of complete classification. An invaluable tool in distinguishing those knots is D. A. Lombardero's table of the Alexander polynomials and signatures of virtually all known (and presumably prime) knots with eleven or fewer crossings [7]. We are grateful to Professor H. F. Trotter for funishing us with a copy of it.

* There do exist generally applicable methods for deciding if a given knot is trivial [17,pp.72-73,171].

1. Background

In the late nineteenth century knot tables were compiled by P. G. Tait [15; 16] and C. N. Little [4; 5; 6] in an effort to list as many as possible of the prime knot types with ten or fewer crossings and alternating primes with eleven crossings. Both authors seem to have recognized that, in the absence of mathematical proofs, their tables were not necessarily complete and might contain duplications which they had not empirically detected [15,pp.327-328; 16,pp.502,505-506; 6,p.776]. In 1967 J. H. Conway made some corrections to one of Little's tables, expanded it to include a number of (supposedly) non-alternating knots with eleven crossings, and announced that he had proven that the tables, as corrected and expanded, were complete [3,pp.330,341].

Conway also expressed the belief that his tables were free of duplications [3,pp.341-342]. Notwithstanding his "very strong" evidence for this proposition, the tables contain a duplication in the portion originally compiled by Little which Conway tells us he was able to re-check in "just one afternoon's work" [3,p.329]. Little's attention was no doubt diverted from this duplication by the "theorem" in [6,p.774] to which it is an apparent* counterexample. Regrettably, this error in Conway's table has recently been perpetuated [14,p.415].

In 1927 Alexander and Briggs distinguished the tabled knots with eight or fewer crossings by reference to the homology

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* We have not proven that it cannot be projected with fewer than ten crossings.

invariants of their branched cyclic covering spaces [2]. In the subsequent paper which introduced his famous polynomial, Alexander noted the inability of cyclic invariants to cope with certain knots in the range of nine crossings [1,p.306]. Classification of those knots was completed by Reidemeister by use of dihedral linking invariants [11] which also suffice to classify the tabled knots with ten crossings [8; 10].

2. Lombardero's results

It appears from Lombardero's calculations [7] that Conway's table omits at least two knot types: 8*210:-20 (Figure 3) and 8*210:.2 (Figure 4). One may be explained by the fact that Conway lists knot 8*210:.20 twice [3,p.357]. The other casts doubt on Conway's claim that his tables are complete.

This is especially regrettable from the standpoint of establishing the minimal crossing number of tabled knot types. Were the tables complete, it would then suffice to prove a knot prime and distinguish it from the tabled primes with fewer crossings. In the absence of a proof of completeness, however, we really do not know that the tabled knots cannot be projected with fewer crossings -- after, say, several billion deformations which raise and then lower the crossing number. Note, in this regard, that the equivalence of the 10-crossing knots ${}_{5\mathrm{II}}$ and ${}_{6\mathrm{VI}}$ (Figure 2) can only be demonstrated by an interim increase in the number of crossings [8,Fig.1].

Lombardero also lists the following five knots which are equivalent to knots in Conway's table: 9*-3(8*2.-20.20), 6*-30:210:20 (6*-2110:2:20), 6*-210:30:20 (6*-310:20:20), 6*-210:-30:2 (mirror image of 6*-30:21:-20) and 8*-210:-20 (mirror image of 6*2.-3.-20.20). The latter two are further apparent counterexamples to the "theorem" in [6]. Suppressing these duplications, we arrive at a total of 550 supposedly distinct knots with eleven crossings, listed by either Conway [3] or Lombardero [7]. Cf. [5].

At least* 272 of these 550 knots possess Alexander polynomials which are unique among all 800 tabled knots. The remaining 278 give rise to (at least) 68 more new polynomials which do not belong to any of the 250 known prime knot types with ten or fewer crossings. Thus, while the Alexander polynomial remains a central knot theoretic invariant, the need for stonger (or, at least, radically different) methods of proving knot types inequivalent becomes more pronounced as the crossing number increases.

	Tabled knots	Distinct knot types	Knot types not distinguishable by the Alexander polynomial
0-8 crossings	36	36	0
9 crossings	49	49	6
10 crossings	166	165	33
11 crossings	550	?	210 (or more)

* Five knots are missing from Lombardero's table and we do not know the polynomials of four of them.

3. Remarks on polynomial duplicates

Appended hereto is a table of invariants which distinguish certain knot types with the same Alexander polynomial. The notation is that of [8], [9] and [10]. Included are knot types distinguished by other invariants and other authors, including the signature σ as tabulated by Lombardero [7], the torsion numbers τ of the first integral homology group of the double branched cyclic covering [2] and homology invariants of the coverings studied by S. W. Reyner [12] and R. Riley [13]. In comparing 2-bridged knots either with each other, or with a knot known not to be 2-bridged [10,p.77,fn.], we merely note these facts and rely on Schubert's theorems.

As predicted by Reyner [12,p.37] there are knots in this range which, although presumably distinct, possess identical dihedral linking invariants. A couple of representative examples are 3,21,3,2- $\overset{?}{=}$ 3,3,21,2- and 6**.2.-(21,2) $\overset{?}{=}$ 6**.-(21,2).2. In this regard, it is noteworthy that Riley has successfully distinguished 6**.-(3,2).2 from 6**.2.-(3,2) [13,pp.615-616].

The signature seems to us to be remarkably inefficient in distinguishing polynomial duplicates and we have no explanation for the frequency with which such knots share the numerator, but not the denominator, of a dihedral linking number.

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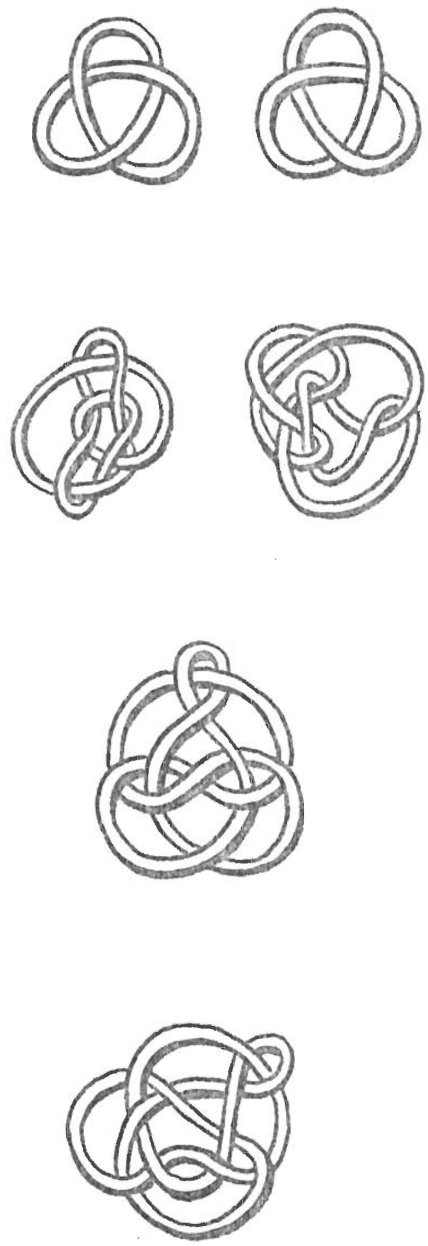

TABLE

5 -3 1 — 6_3 amph., 2-bridged
$23, 22, 2-$ — $V^{13}_{0i} = 6, 6, -2, -6, -2, -2$

9 -5 1 — 7_7 ∄ simple S_4 rep.
$24, 21, 2-$ — ∃ simple S_4 rep.

9 -7 2 — 8_{11} $V_3 = 6$
$10_{2\mathrm{II}}$ $V_3 = 6/5$
$32, 3, 21-$ $V_3 = 6/7$

11 -7 1 — 9_{48} $\tau = 3, 9$
$23, 211, 2-$ $\tau = 27$

11 -8 3 — 8_{15} $\sigma = 4$
$(3,3-)(21,2)$ $\sigma = 0$

13 -9 2 — 9_{12} M_5 simply connected
$22, 22, 21-$ $\pi M_5 = Z_3 * Z_3$

13 -10 3 — $10_{2\mathrm{III}}$ ∃ simple S_4 rep.
$(3,3-)(3,2)$ ∄ simple S_4 rep

11 -9 3 — 10_{77} $V_5 = 4$
$10_{2\mathrm{IX}}$ $V_5 = 52/11$
$-40:20:20$ $V_5 = 12$

13 -10 4 — 9_{18} 2-bridged
11_{106} 2-bridged

19 -11 2 — 9_{37} $\tau = 3, 15$
$221, 3, 21-$ $\tau = 45$

19 -12 3 — 9_{41} $\tau = 7, 7$
$31, 22, 21-$ $\tau = 49$

21 -14 3 — 9_{39} M_5 simply connected
$8^* 2 :. -20 :. 2$ $\pi M_5 = Z_{11}$

21 -15 4 — 10_{49} 2-bridged
11_{248} 2-bridged

23 -16 4 — 10_{63} $V_3 = 14/5$
10_{72} $V_3 = -, 10/3, 14/3, 14/$
$(3,21)(21,2-)$ $V_3 = 14$

25 -16 4 — 10_{22} 2-bridged
11_{21} 2-bridged

25 -17 4 — 10_{60} 2-bridged
11_{293} 2-bridged

25 -18 6 — 10_{57} not 2-bridged
11_{136} 2-bridged

29 -21 7 — 10_{90} } distinguished
$22, 22, 3$ } by Reyner

3 -3 3 -1 — 8_2 $\sigma = 4$
$(211,2)-(21,2)$ $\sigma = 0$

3 -4 4 -1 — $10_{4\mathrm{V}}$ ∄ simple S_4 rep
$20.-3.-20.2$ ∃ simple S_4 re

5 -5 3 -1 — 8_7 2-bridged
$(21,2+)(3,2-)$ $V^{23}_{0i} = -2, 6, -$
$2, -2, 2$
$-2, 2, -2, -2, 2$
∴ not 2-bridged

7 -5 3 -1 — 8_9 $V_5 = 0$
$10_{2\mathrm{VII}}$ $V_5 = 4/5$ or $-4/5$
$(3,2+)(3,2-)$ $V_5 = 4$

7 -6 3 -1 — 8_{10} $V_4 = 2$
$10_{3\mathrm{III}}$ $V_4 = 4$
$212, 3, 21-$ $V_4 = 0$

7 -6 4 -1 $10_{4\text{I}}$ $\sigma = 4$
$10_{4\text{VI}}$ $\sigma = 4$
213, 21, 2- $\sigma = 0$

9 -9 7 -2 10_{66} $V_9 = 0$
(3,21)(3,2-) $V_9 = 6$

11 -9 5 -1 9_{20} $\sigma = 4$
$10_{4\text{VIII}}$ $\sigma = 4$
(3,2+)-(21,2) $\sigma = 0$

15 -12 5 -1 9_{28} $V_3 = 4$ $V_4 = 2$
9_{29} $V_3 = 10$
$10_{3\text{VI}}$ $V_3 = 2$
212, 21, 21- $V_3 = 4$ $V_4 = 2$

15 -14 8 -2 10_{110} $V_3 = 2$
8* 3:-20 $V_3 = 2/5$

19 -14 6 -1 9_{33} $V_{61}(6,-6) = 4$
21111, 21, 2- $V_{61}(6,-6) = 32$

[illegible] -16 9 -3 10_{56} $V_4 = 8$
3, 3, 2+++ $V_4 = 6$

17 -14 8 -2 10_{61} 2-bridged
10_{64} not 2-bridged
11_{37} 2-bridged

23 -16 6 -1 9_{34} $\nexists$ simple S_4 rep.
(22,2)(21,2-) $\exists$ simple S_4 rep.

23 -18 7 -1 9_{40} $V_3 = 2$
10_{42} $V_3 = 4$
(21,21-)(21,2) $V_3 = 26/7$

25 -19 9 -2 10_{54} not 2-bridged
11_{123} 2-bridged

27 -21 10 -2 10_{93} $V_3 = 2$
11_{141} $V_3 = 10$

29 -20 7 -1 10_{39} $\pi M_5 = Z_2 * Z_2$
8* 2.20.-20 $\pi M_5 = Z_{29}$

31 -23 10 -2 10_{85} not 2-bridged
11_{335} 2-bridged

31 -24 11 -2 10_{87} $\sigma = 0$
9*.-3 $\sigma = 4$

5 -5 4 -3 1 10_{123} $\sigma = 6$
3111, 3, 2- $\sigma = 2$

23 -19 12 -5 1 10_{26} $V_{97}(6,-6) = 0$
3111, 3, 2 $V_{97}(6,-6) = 260$

29 -24 15 -6 1 10_{38} $\tau = 11, 11$
.3.21.20 $\tau = 121$

1 TRIVIAL KNOT
.-(3,2).2
.2.-(3,2)
} distinguished by Riley

5 -2 6_1 $V_3 = 2$
9_{46} $V_3 = 2/3, 2/3, -2/3, -$
5, 3, 21- $V_3 = 2/5$ $V_9 = 4/5$
(3,21)-(21,2) $V_3 = 2/7$
(3,21)-(3,2) $V_3 = 2/5$ $V_9 = -6/5$

13 -8 2 $10_{1\text{III}}$ $V_3 = 14/5$
311, 22, 2- M_3 simply connected
42, 21, 2- $\exists$ simple S_4 rep

15 -10 2 9_{15} $V_3 = 6$ M_3 simply connected
$10_{4\text{IV}}$ $V_3 = 6$ $V_4 = 4$
23, 21, 21- $\pi M_3 = Z_7$
411, 21, 2- $V_3 = 0$ $\pi M_3 = Z_2$

15 -11 3 10_{78} 2-bridged
11_{32} 2-bridged
221, 22, 2- $\{V_{0i}^{43}\} =$ 2 (8 times), -2 (6 "), 6 (3 "), -6 (4 ")
$\therefore$ not 2-bridged

19 -13 3
- 10_{51} 2-bridged
- 11_{80} 2-bridged
- 231, 21, 2- ∃ simple S_4 rep ∴ not 2-bridged

9 -8 4 -1
- 8_{16} $V_5 = 12$
- 10_{3X} $V_5 = 8$
- 21111, 3, 2- $V_5 = 16$
- 312, 21, 2- $V_5 = 4$
- 41, 22, 2- $\pi M_5 = Z_2 * Z_2$

13 -10 5 -1
- 8_{18} $\sigma = 0$ $V_3 = 2, 2, -2, -2$
- 9_{24} $\sigma = 0$ $V_3 = 0$ $V_4 = 2$
- 2111, 3, 21- $\sigma = 0$ $V_3 = 0$ $V_4 = 0$
- 2.2.-2.20.20 $\sigma = 4$

13 -12 8 -2
- 10_{68} $V_3 = 2$ $V_{0i}^{19} = 6, 2, -2, -2, -6, -2, -2, 2, 6$
- 221, 21:, 2- $V_{0i}^{19} = 2, -6, -2, -2, 10, 2, -10, 2, 2$ $V_3 = 2$
- 24, 3, 2 $V_3 = 4$

15 -11 5 -1
- 9_{27} $V_7 = 2$ M_7 s.c.
- (211, 2)(3, 2-) $V_7 = 14$ M_7 s.c.
- (21, 2+)(21, 2-) $V_7 = 6$ M_7 s.c.
- 8^*20:20:-20 $\pi M_7 = Z_{29}$

17 -13 5 -1
- 9_{31} $V_5 = 12$ M_5 s.c.
- -2110:2:2 $\pi M_5 = Z_{31}$
- -211:-20:-20 $V_5 = 76$ M_5 s.c.
- (22, 2-)(3, 2) $\pi M_5 = Z_2 * Z_2$
- (21, 2+)-(21, 2) $V_5 = 8$ M_5 s.c.

21 -16 7 -1
- 10_{45} $V_3 = 4$ $V_4 = 6$
- 2111, 21, 2- $V_3 = 4$ $V_4 = 4$
- (21, 21-)(3, 2) ∄ simple S_4 rep.

23 -18 9 -2
- 10_{46} $V_3 = 0$ $V_4 = 2(3, -1)$
- 10_{92} $V_3 = 4, 4, 4, 6$
- 3, 21, 2+++ $V_3 = 0$ $V_4 = 2(1, 1)$
- 4, 3, 21+ $V_3 = 0$ $V_4 = 4$

25 -18 7 -1
- 2.-3.2.20 $\pi M_7 = Z_{17}$
- 10^*-20 $H_1 M_7 = Z_{29}$
- 10_7 M_7 SIMPLY CONNECTED

25 -20 9 -2
- 10_6 $V_4 = 2(3, -1)$
- 9^*.2:.-2 ∄ simple S_4 rep
- 21, 21, 2+++ $V_4 = 2(1, 1)$

31 -24 10 -2
- 10_{25} not 2-bridged; ∃ a period 6 S_5 rep.
- 11_{143} 2-bridged; ∃ a period 6 S_5 rep.
- 23, 21, 2+ ∄ period 6 S_5 rep.

33 -26 11 -2
- 10_{30} $V_3 = 26/7$
- 11_{182} $V_3 = 26$
- 11_{129} $V_3 = 4$

35 -27 11 -2
- 10_{28}, 11_{341}, 11_{334}, 11_{183}, 11_{111} — distinguished by Riyner

21 -19 12 -5 1
- 10_{88} $V_5 = 56/11$
- 11_{113} $V_5 = 8$ M_5 s.c.
- 11_{131} $V_5 = 4(10, -6)$ M_5 s.c.
- 11_{269} $V_5 = 4(4, 0)$ $\pi M_5 = Z_2 * Z$

11 -8 4 -1
- 8_{17} amphicheiral
- 2121, 3, 2- not amphicheiral because, although $V_{37} = \{V_{i0}^{37}\} =$ 6 (four times) -6 (three times) 2 (four times) & -2 (7 times)

11 -10 5 -1
- 9_{22} $H_1(\mathcal{M}_4^*) = Z_6$
- 8^*-210...2 $H_1(\mathcal{M}_4) = Z$

* 4-FOLD TETRAHEDRAL COVER

13 -11 5 -1
- 9_{26} 2-bridged
- (3, 2+)(21, 2-) not 2-bridged because some $V_{i0}^{47} = 6$

13 -10 4 -1
- 10_{3IX} $V_{43} = 10$
- 2112, 21, 2- $V_{43} = 2$

1 1 -1   24,3,2- $\exists$ simple S_4 rep
23,3,3- $\nexists$ simple S_4 rep

3 -4 2   311,3,3- $\nexists$ simple S_4 rep
42,3,2- $\exists$ simple S_4 rep

21 -16 6   -40:2:2 $\pi M_5 = Z_{19}$
11_{105} M_5 SIMPLY CONNECTED

27 -18 4 11_{103} 2-bridged
11_{284} 2-bridged

31 -19 4 11_{127} 2-bridged
11_{272} 2-bridged

33 -20 4 11_{230} $V_3 = 18/5$
11_{100} $V_3 = 18/7$

1 0 -2 1   22:-20:-20 $\pi M_5 = Z_{31}$
5,22,2- $\pi M_5 = Z_2 * Z_2$

11 -10 6 -1   41,21:,2- $\nexists$ simple S_4 rep
-22:-20:-20 $\exists$ simple S_4 rep

13 -11 7 -3   -310:-20:-20 $\pi M_5 = Z_{19}$
11_{251} M_5 SIMPLY CONNECTED

19 -15 6 -1   -211:20:20 $V_3 = 6$
8* 2:-20:20 $V_3 = 6/7$

19 -15 7 1   (211,2-)(3,2) $\pi M_5 = Z_2 * Z_2$
211,211,21- $\pi M_5 = Z_3 * Z_3$
-2110:-20:-20 $\pi M_5 = Z_{31}$

19 -17 9 -2   8*-40 $\nexists$ simple S_4 rep
11_{15} $\exists$ simple S_4 rep

21 -15 5 -1   8* 210:-20 $\tau = 63$
2.2.-2.2.20 $\tau = 3, 21$

23 -20 10 -2 11_{268} $V_3 = 0$
11_{135} $V_3 = 6$

23 -20 12 -4 11_{300} $V_5 = 8$
11_{310} $V_5 = 12$

25 -21 +11 -2 11_{282} $V_{0i}^{31} =$ 2,-2,10,-6,-2,
2,-10,-2,-2,-10,
6,2,-2,10,2
$\therefore$ not 2-bridged
11_{298} 2-bridged

27 -23 11 -2 11_{234} $V_3 = 22/5$
11_{107} $V_3 = 22/7$
11_{312} $V_3 = 22/3, 22/3, 26/3, -$

29 -23 11 -2 11_{124} $\sigma = 0$
11_{324} $\sigma = 4$

31 -26 12 -2 11_{69} $V_3 = 8$
11_{281} $V_3 = 34/7$

35 -27 12 -2 11_{88} $V_3 = 0$
11_{351} $V_3 = 6$

35 -28 14 -3 11_{329} $\tau = 125$
11_{71} $\tau = 5, 25$

37 -27 11 -2 11_{337} $V_3 = 26/5$
11_{266} $V_3 = 26/7$
11_{133} $V_3 = 2$
11_{61} $V_3 = 22/9, 2/3, 2/3, -2/3$

37 -29 12 -2 11_{222} $V_3 = 4$
11_{325} $V_3 = 34/5$

41 -32 13 -2 11_{210} $V_3 = 30$ $\nexists$ simple S_4 rep
11_{342} $V_3 = 6$ $\nexists$ simple S_4 rep
11_{409} $\exists$ simple S_4 rep

41 -32 15 -3 11_{302} $\exists$ simple S_4 rep
11_{404} $\nexists$ simple S_4 rep

15 -14 10 -5 1 11_{26} $\tau = 75$
9*.-21 $\tau = 5, 15$

17 -15 10 -5 1 11_{36} $\sigma = 6$
11_{249} $\sigma = 2$

21 -18 12 -5 1 11_7 $V_3 = 14/5$
11_{82} $V_3 = 4$ $V_4 = 4$
11_{270} $V_3 = 4$ $V_4 = 0$

25 -21 13 -5 1 11_{332} 2-bridged
11_{128} 2-bridged

27 -24 +15 -6 1 $\left.\begin{matrix}11_{86}\\11_{215}\end{matrix}\right\}$ distinguished by Reyner

29 -25 14 -5 1 $\left.\begin{matrix}11_{45}\\11_{308}\end{matrix}\right\}$ distinguished by Reyner

31 -25 14 -5 1 11_{228} $v_{11} = 6$
11_{330} $v_{11} = 2$

31 -28 17 -6 1 11_{65} $v_3 = 2$
11_{171} $v_3 = 2/3, 26/9, -, -$

33 -28 16 -6 1 11_{211} $v_4 = 4$
11_{264} $v_4 = 2$ M_5 s.c.
11_{403} $v_4 = 2$ $H_1 M_5 = \mathbb{Z}_{29}$

35 -30 17 -6 1 11_{174}, 11_{261} } distinguished by Reyner

37 -30 17 -6 1 11_{209} $v_5 = 208$ M_5 s.c.
11_{260} $v_5 = 4$ M_5 s.c.
11_{164} $\pi M_5 = \mathbb{Z}_{11}$

37 -31 17 -6 1 11_{40} $\tau = 147$ $H_1 M_7 = \mathbb{Z}_{83}$
11_{205} $\tau = 7, 21$
11_{262} $\tau = 147$ $\pi M_7 = \mathbb{Z}_{13}$

39 -29 12 -2 11_{216} M_5 IS SIMPLY CONNECTED
11_{326} $\pi M_5 = \mathbb{Z}_3 * \mathbb{Z}_3$
11_{66} $\pi M_5 = \mathbb{Z}_2 * \mathbb{Z}_2$

3 0 -1   -30:21:-20   ∃ A UNIQUE TETRAHEDRAL REP.
4, 22, 3 -   ∃ FIVE DISTINCT TERAHEDRAL REPS.

29 -24 11 -2 11_{97} $\mathcal{V}_4^{*} = 2$
11_{339} $\mathcal{V}_4 = 14$

* LINKING OF BRANCH CURVES IN 4-FOLD TETRAHEDRAL COVER

35 -26 11 -2 11_{263} $H_1(\mathcal{M}_4) = \mathbb{Z}$
11_{356} $H_1(\mathcal{M}_4) = \mathbb{Z}_6$

37 -28 12 -2 11_{87} $H_1(M_{11}) = \mathbb{Z}_{23}$
11_{350} M_{11} IS SIMPLY CONNECTED

EQUIVALENCE OF KNOTS 10_{161} AND 10_{162}

AS TABULATED IN D. ROLFSEN'S BOOK

<u>KNOTS AND LINKS</u>, P. 415