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*73T-G76. ROBERT F. BROWN, University of California, Los Angeles, California 90024. Fixed points of endomorphisms of compact groups.

For a function $f: X \rightarrow X$, let $\Phi(f) = \{x \in X \mid f(x) = x\}$. If X is a group (Lie group, vector space) and h is a homomorphism, then $\Phi(f)$ is a group (Lie group, vector space). Let $h: G \rightarrow G$ be an endomorphism of a topological group, let $H^*(G)$ denote real Čech cohomology, h^* the induced endomorphism of $H^*(G)$, and $h^{*,1}$ its restriction to $H^1(G)$. Use the symbol $\Phi_0(h)$ for the identity component of $\Phi(h)$. Theorem. If G is a compact, connected abelian topological group and h is an endomorphism of G , then the dimension of the topological group $\Phi_0(h)$ is equal to the dimension of the vector space $\Phi(h^{*,1})$. Now assume that G is a compact Lie group and let $\underline{H}^*(G)$ denote the subspace of $H^*(G)$ spanned by a set of algebra generators for $H^*(G)$. Let h be an automorphism of G and let \underline{h}^* be the restriction of h^* to $\underline{H}^*(G)$. Theorem. If G is a compact, connected Lie group and h is an automorphism of G , then the dimension of a maximal torus of the Lie group $\Phi_0(h)$ is equal to the dimension of the vector space $\Phi(\underline{h}^*)$.
(Received March 12, 1973.)

*73T-G77. DALE P. ROLFSEN, University of Wyoming, Laramie, Wyoming 82070 and University of British Columbia, Vancouver 8, British Columbia, Canada. Localized Alexander invariants and isotopy of links.

The Alexander invariant of a link of two n -spheres in R^{n+2} is the homology $H_*(\tilde{X})$ of the universal abelian cover of the complement, considered as a module over the ring Λ of finite Laurent polynomials in two variables with integer coefficients. Let $\Sigma = \{f(x)g(y)\}$ be the set of nonzero members of Λ which factor into terms involving only one variable. Form the localization Λ/Σ and the corresponding localization $H_*(\tilde{X})/\Sigma$ of the Alexander invariant. Theorem (PL category). If two links are isotopic, then their localized Alexander invariants are isomorphic as Λ/Σ -modules. Here isotopy refers to nonambient isotopy (= continuous family of embeddings), a relation under which, for instance, PL knot theory in all dimensions becomes trivial. The localized invariant settles the corresponding question for link theory, hitherto known only for $n = 1$. Corollary. For each $n \geq 1$, there are infinitely many PL isotopy classes of links of two n -spheres in R^{n+2} . (Received March 5, 1973.) (Author introduced by Professor Joseph M. Martin.)

*73T-G78. ELIZABETH B. CHANG, Hood College, Frederick, Maryland 21701. Characterizations of some zero-dimensional spaces.

A Hausdorff space is collectionwise ultranormal if for every locally finite collection $\{A_i : i \in I\}$ of mutually disjoint closed subsets there is a collection $\{C_i : i \in I\}$ of pairwise disjoint clopen sets such that $A_i \subseteq C_i$ for each i in I . It is shown that collectionwise spaces are characterized by the existence of extensions for continuous functions defined on closed subsets with values in complete metric spaces. They are also characterized by the existence of continuous selections for certain types of set-valued functions (specifically, l.s.c. carriers with values which are generally compact subsets of complete metric spaces). Analogues of these results are presented for some further classes of zero-dimensional spaces. (Received March 26, 1973.)

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*73T-G79. KENNETH A. PERKO, JR., One Chase Manhattan Plaza, New York, New York 10005. On the classification of knots.

The Tait-Little tables of 166 presumably prime, 10-crossing knots were recently rechecked for completeness and republished along with algebraic invariants which distinguished all but 31 pairs. [See J. H. Conway's paper in "Computational problems in abstract algebra" (Pergamon, Oxford, 1970) but beware of false "beliefs".] Linking numbers between the branch curves of appropriate noncyclic covering spaces of these examples newly distinguish all but the pair $10_{5II} 10_{6VI}$ which turns out to be a duplication in Little's table that Conway overlooked; thus these tables contain precisely 165 distinct, new knot types. Such linking numbers also solve the amphicheirality problem for the 6 remaining examples in Reidemeister's table, proving that the only amphicheirals up to nine crossings

are those identified as such by Tait when this table first appeared, [Compare "Knotentheorie," vol. III, p. 15, and the results reported in *Canad. J. Math.*, 22(1970), 200-201.] (Received February 26, 1973.)

73T-G80. SUKHJIT SINGH, Pennsylvania State University, University Park, Pennsylvania 16802. A 3-dimensional compact absolute retract which contains no 2-dimensional compact absolute retract.

By an AR we understand a compact absolute retract for the category of metric spaces. Let E^3 denote the 3-dimensional Euclidean space and B^3 denote the closed ball of unit radius in E^3 . Let n and k be positive integers with $2 \leq k \leq n$. An n -dimensional AR X will be called (irreducible)^k if and only if X does not contain any proper AR of dimension $k, (k-1), \dots, 2$. Theorem. There exists an upper semicontinuous decomposition G of B^3 whose nondegenerate elements form a countable null family of arcs such that the decomposition space B^3/G is a 3-dimensional AR which is (irreducible)³. This solves a problem posed by Bing and Borsuk (*Fund. Math.* 54(1964), 159-175). Whether there exists an n -dimensional AR X with $n > 3$ such that X is (irreducible)^k, with $2 \leq k \leq n$, is an open problem. Corollary. There is an upper semicontinuous decomposition G of E^3 whose nondegenerate elements form a countable null collection of arcs such that the decomposition space E^3/G does not contain any 2-dimensional AR. (Received March 30, 1973.)

*73T-G81. SIDNEY A. MORRIS and E. T. ORDMAN, University of New South Wales, Kensington, New South Wales 2033, Australia. Remarks on free products of topological groups. Preliminary report.

We are able to prove the following, which settle several previously announced questions: Theorem 1. If the abstract group $G * H$ has a topology making it a Hausdorff, nondiscrete topological group, such that the natural maps $G * H \rightarrow G$ and $G * H \rightarrow H$ are continuous, then $G * H$ is not locally compact. Corollary. If the free product of two or more topological groups is locally compact, all factors and the product are discrete. Theorem 2. If G and H are Hausdorff topological groups and k_w -spaces, the subgroup of their free product $G * H$ generated by $[G, H] = \{g^{-1}h^{-1}gh \mid g \in G, h \in H\}$ is the (Graev) free topological group on $[G, H]$. Theorem 3. If G and H are Hausdorff topological groups admitting continuous monomorphisms into locally invariant (SIN) groups, then $G * H$ also admits such a continuous monomorphism. Example. The free product of two groups which are k -spaces need not be a k -space. Let $G = S * S$, the free product of two circle groups, and let $H =$ the rationals. Then $G, H, G \times G, H \times H$ are k -spaces but $G \times H$ and $G * H$ are not k -spaces. (Received March 30, 1973.)

*73T-G82. RONALD H. ROSEN, University of Michigan, Ann Arbor, Michigan 48104. An annulus theorem for suspension spheres. Preliminary report.

A suspension sphere of dimension $n - 1$ is a space X whose suspension, S^*X , is homeomorphic to S^n . From Kirby's proof that orientation preserving homeomorphisms of S^n are stable for $n \geq 5$ (*Ann. of Math.* (2) 89(1969), 575-582) we now know that the ordinary annulus conjecture is true if $n \neq 4$. The author uses Kirby's work to prove the following annulus type theorem. Theorem. Let X be a suspension sphere of dimension $n - 1$ and let $n \geq 5$. Let $X_i = f_i(X)$, $i = 1, 2$, so that $f_i: X \rightarrow S^n$ is an embedding and X_i is bicollared in S^n . If X_1 and X_2 are disjoint and U is the region between them then the closure of U is homeomorphic to $X \times I$. The proof is not completely straightforward since if $X \neq S^{n-1}$ there is no single canonic family of embeddings of X in S^n (similar to the round $(n - 1)$ -spheres in S^n). (Received April 2, 1973.)

*73T-G83. JOHN M. ATKINS, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A note on metacompact developable spaces.

We assume all spaces are at least T_1 . Theorem 1. X is a metacompact developable space if and only if