Winter Fruits

New Problems from OEIS

December 2016 - January 2017

Neil J. A. Sloane

The OEIS Foundation, and
Rutgers University

Experimental Mathematics Seminar
Rutgers University, January 26 2017

[Several slides have been updated since the talk - Neil Sloane, Jan 30 2017]

Outline

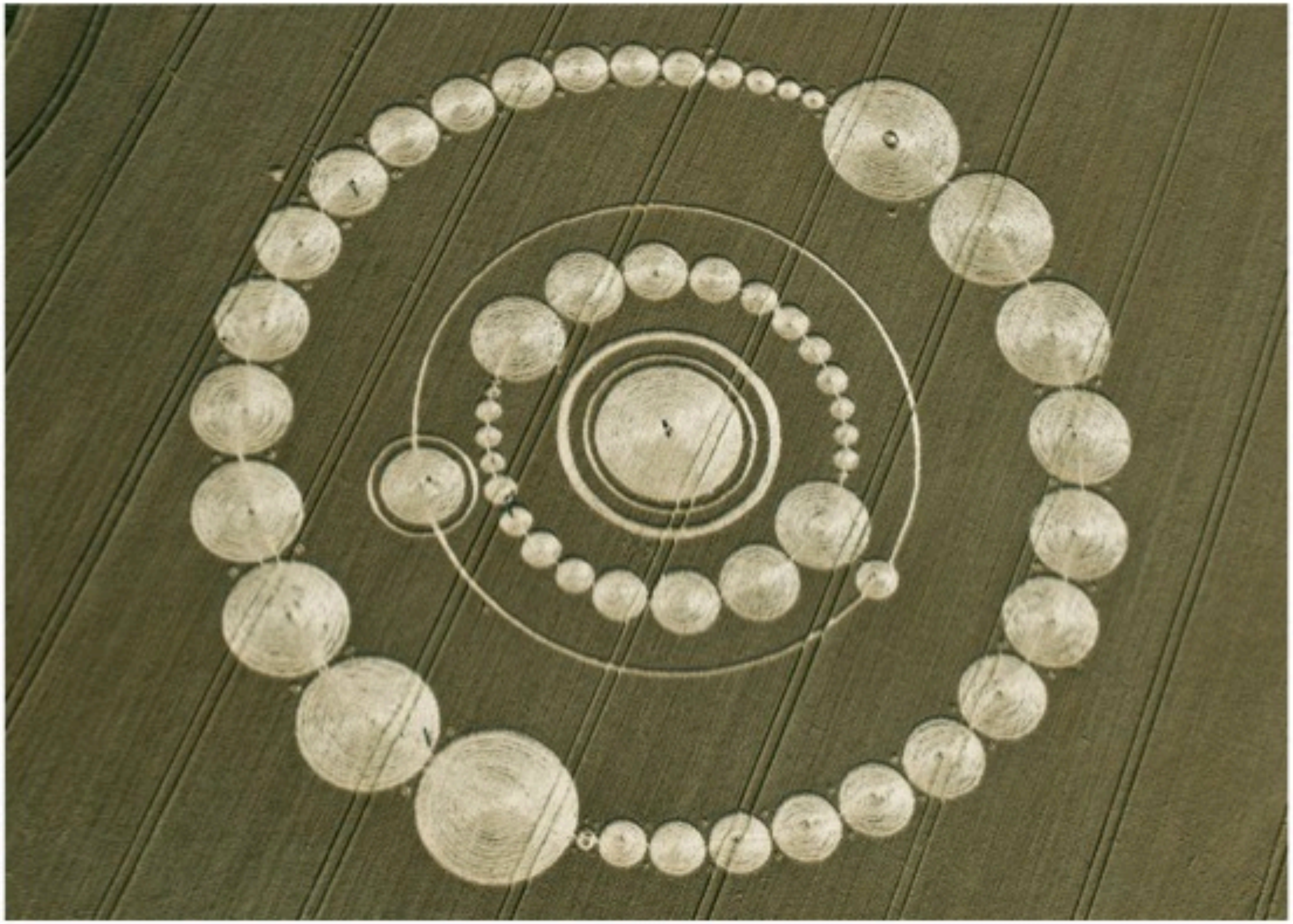
- Crop circles / What not to submit / pau
- Graphs of Chaotic Cousin of Hofstadter-Conway
- Richard Guy's 1971 letter
- Fibonachos
- Fibonacci digital sums
- Carryless problems
- Tisdale's sieve
- Square permutations, square words
- Remy Sigrist's new recurrences
- Michael Nyvang's musical compositions based on OEIS

Dec 28 2016, Question in email:

What is the significance of 11,12,14,18?

Prime numbers?
The Windmill Hill crop circle, July 26, 2011

This elegant and massive display (over 400 feet in diameter) is composed of circles that ascend from small to large or descend from large to small. In the inner ring there are 11 circles followed by 12 circles. In the outer ring there are 14 circles followed by 18.



Answer:
Sorry, I cannot help you

WHAT NOT TO SUBMIT

A-numbers of sequences contributed by [your name]

```
271***, 275***, 275***, 275***,
276***, 276***, 276***, 276***,
276***, 276***, 278***, ....
```

Added immediately to OEIS Wiki page
"Examples of What Not to Submit"

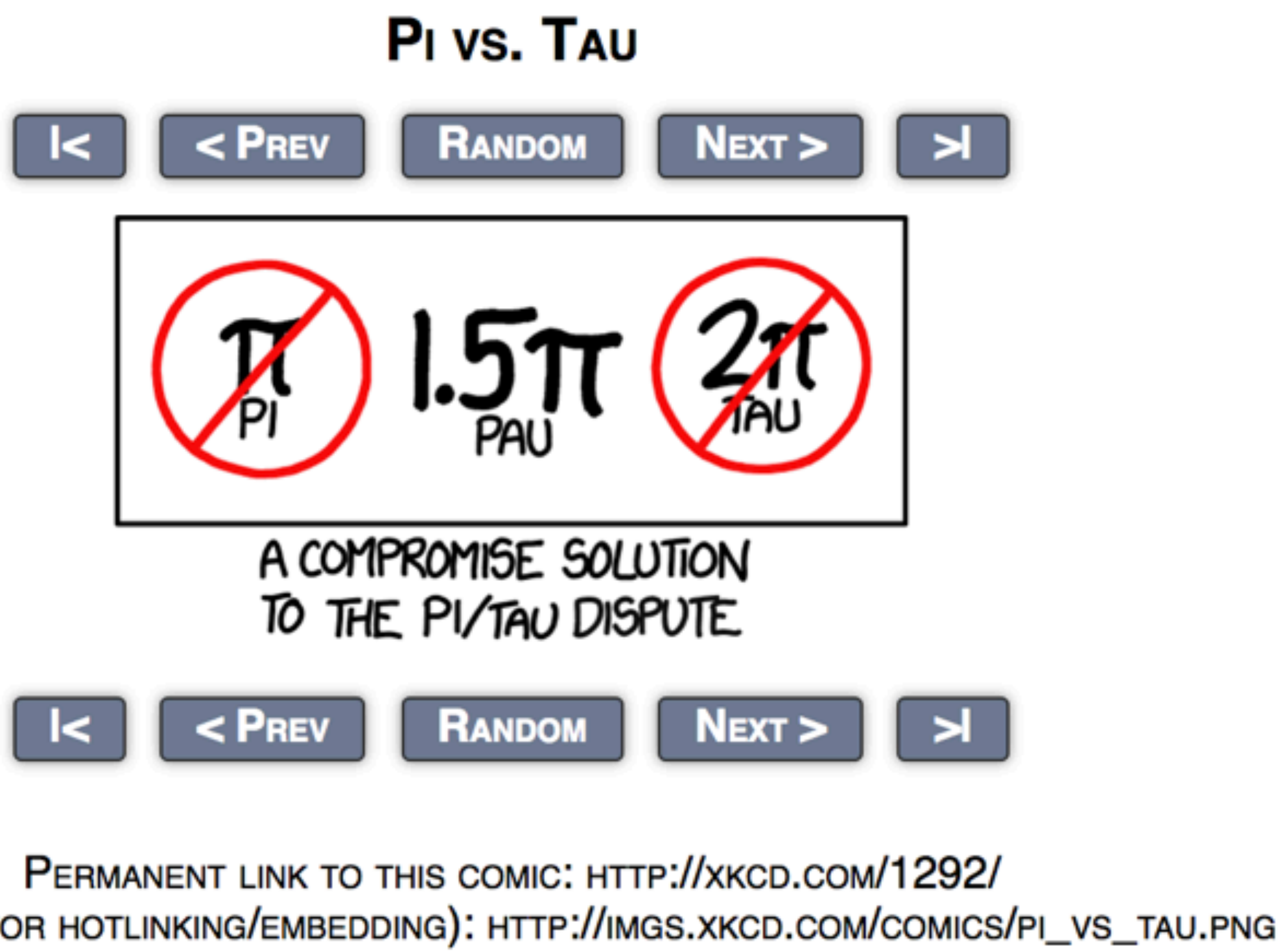
"NOGI" = Not of General Interest

The number pau

Comment on A197723, Jan 8 2017:

Decimal expansion of 3 Pi / 2 = `4.712388980384...`

Randall Munroe suggests the name pau as a compromise between pi and tau.

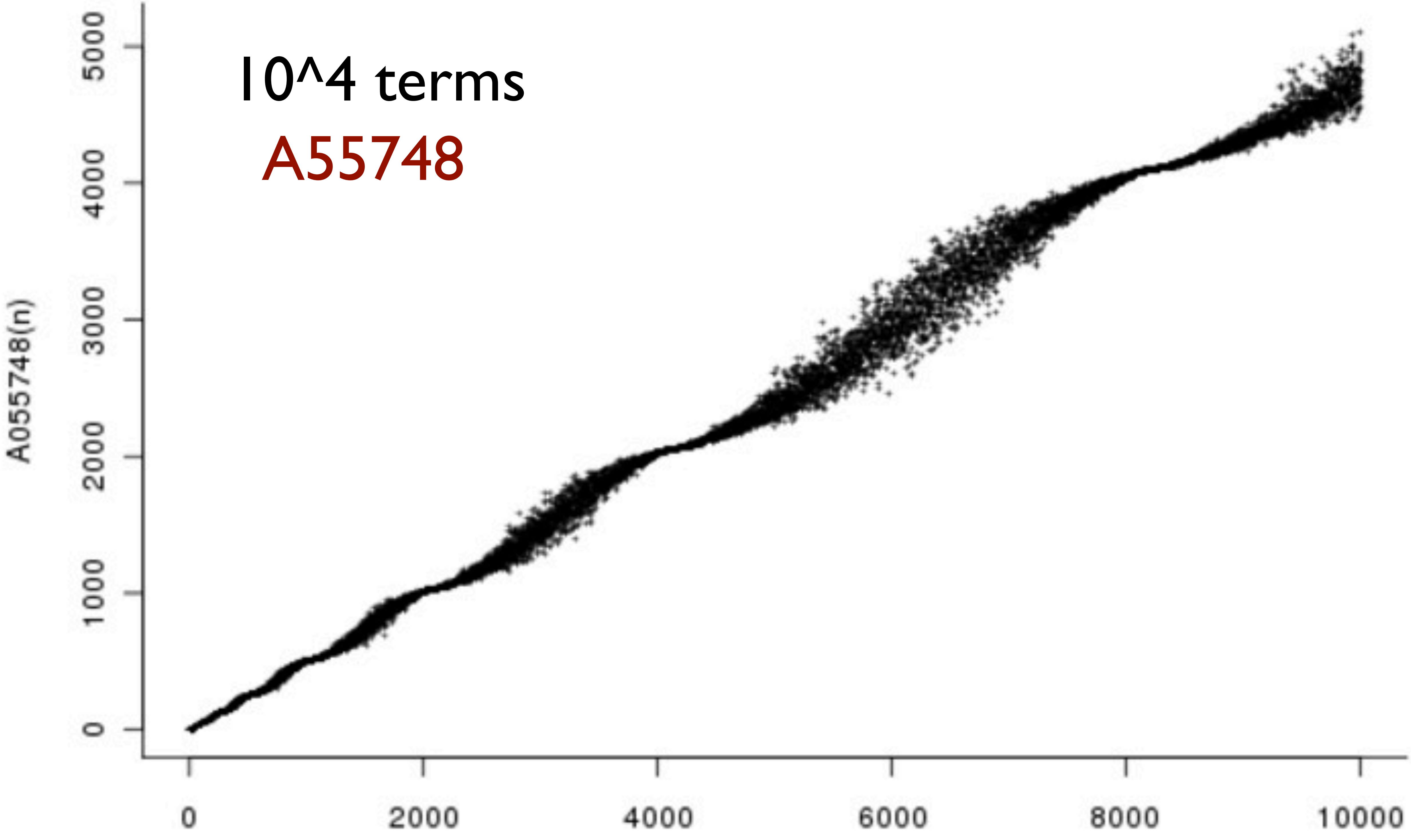


New Graphs of A55748 Chaotic Cousin of Hofstadter-Conway

A4001 (the $10,000 sequence):

$$a(n) = a(a(n-1)) + a(n-a(n-1))$$

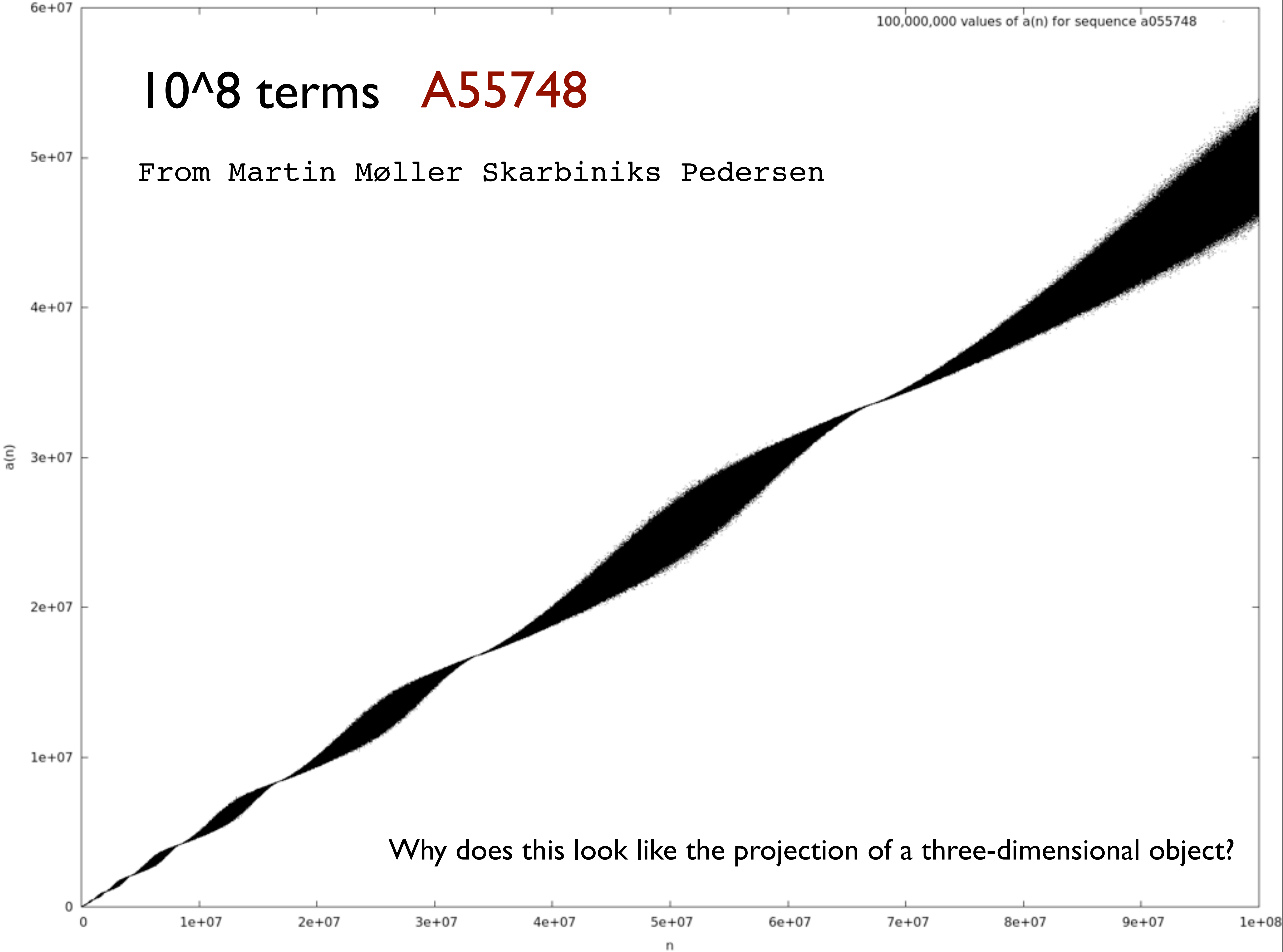
A55748: $a(n) = a(a(n-1)) + a(n-a(n-2)-1)$



From Martin Møller Skarbiniks Pedersen

10^8 terms A55748

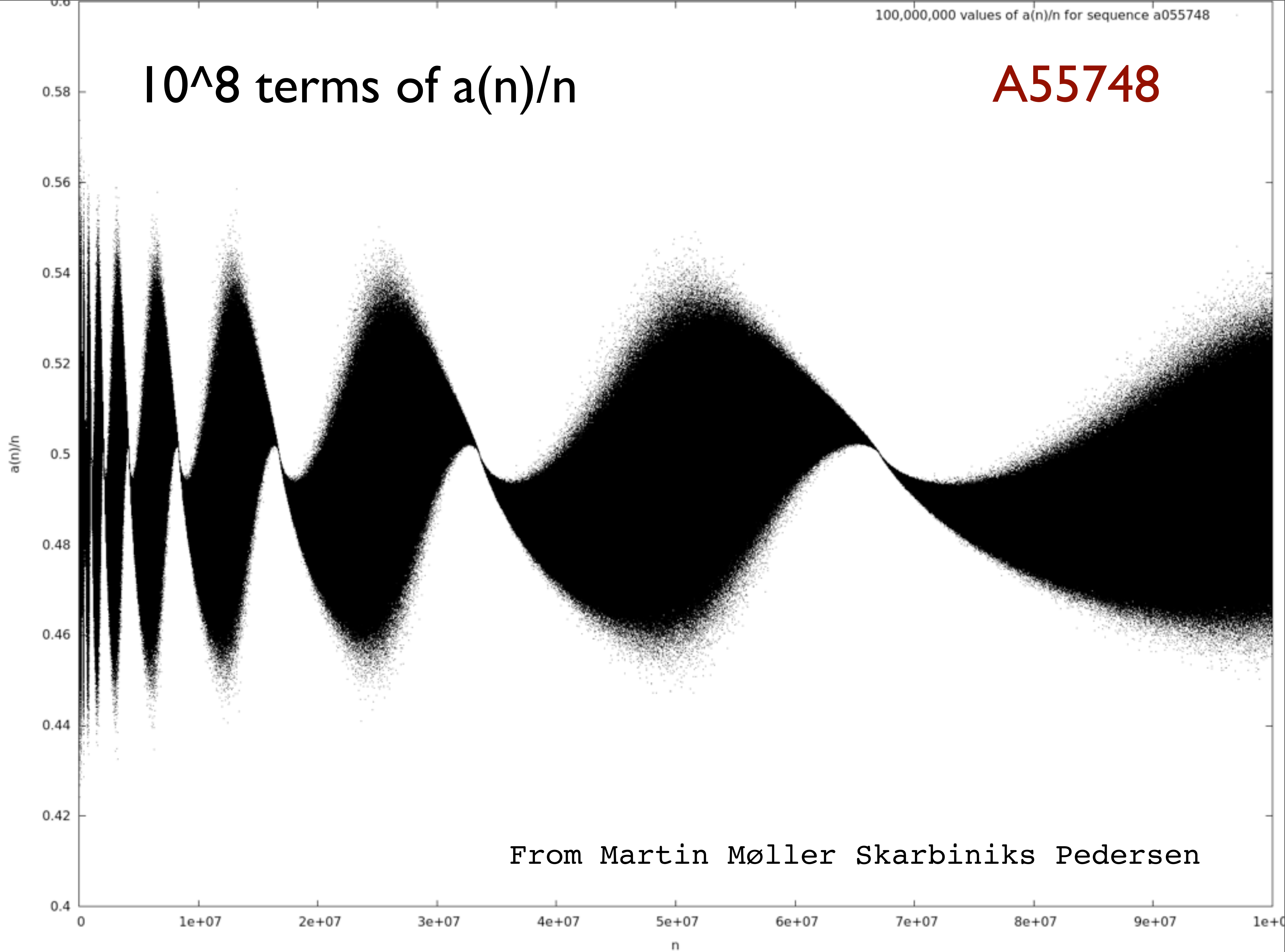
From Martin Møller Skarbiniks Pedersen



Why does this look like the projection of a three-dimensional object?

10^8 terms of a(n)/n

A55748



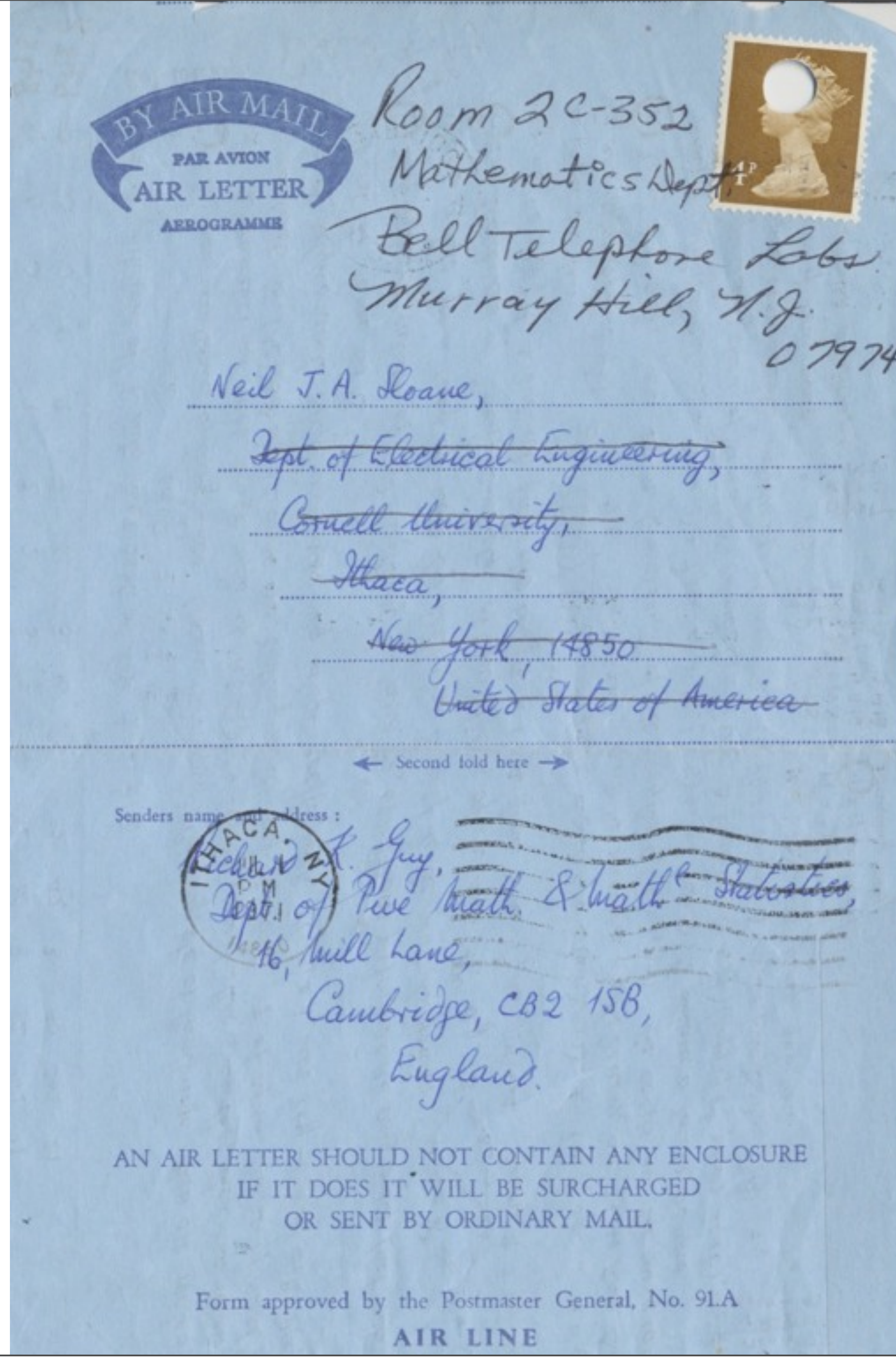
From Martin Møller Skarbiniks Pedersen

Richard Guy’s letter June 24 1971

(15 sequences, many still need extending, 46 years later)

One of many letters from Richard Guy

June 24 1971



BY AIR MAIL
PAR AVION
AIR LETTER
AEROGRAMME

Room 2C-352
Mathematics Dept.
Bell Telephone Labs
Murray Hill, N.J.
07974

Neil J.A. Sloane,
~~Dept of Electrical Engineering,~~
~~Cornell University,~~
~~Ithaca,~~
~~New York 14850,~~
~~United States of America~~

← Second fold here →

Senders name and address :
Richard K. Guy,
Dept. of Pure Math. & Math^l. Statistics,
16, Mill Lane,
Cambridge, CB2 1SB,
England.

ITHACA, N.Y.

AN AIR LETTER SHOULD NOT CONTAIN ANY ENCLOSURE
IF IT DOES IT WILL BE SURCHARGED
OR SENT BY ORDINARY MAIL.

Form approved by the Postmaster General, No. 91A

AIR LINE

no longer at Oxford, now at Cambridge, but only until July 3.
address from July 4–9: Dept. of Math., Royal Holloway College, Englefield Green, Surrey, England. From July 12–22, c/o R.L. Graham, Bell Labs., 600 Mountain Ave., Murray Hill, NJ, 07974, U.S.A. From July 23 onwards, Dept of Math., Statistics & Computing Science, The Univ. of Calgary, Calgary 44, Alberta, Canada. 403 2845202
−3 hrs

UNIVERSITY OF OXFORD
Mathematical Institute
24-29 St Giles
Oxford OX1 3LB
Telephone 0865 54295

June 24
1971

Dear Neil, Some sequences I have come across recently which you may not have (until recently I had access to a 1st edition; now no access; in Calgary I have editions 1, 2 & 4.

#	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
2572 ✓	✓ A	1	1	2	3	5	9	16	28	50	89	159	285					
2843	✓ B	1	1	2	4	7	13	24	43	78	141	253	456					
2844	✓ C	1	1	2	5	13	36	102	296	871	2599							
	✓ D	1	1	2	5	13	36	102	295	864		A279196						
2845	/ E	1	1	1	2	4	8	17	36	78	171	379						
2846	✓ F	1	1	1	2	4	11	33	116	435	1832	8167	39700	201785	1099449	6237505		
bound	G	1	1	1	2	4	18	~~72~~ 54	~~288~~ 288	~~1440~~ 1152	7200	36000	311040					
2847	H	1	2	3	7	15	43	131	468	1776	7559	34022	166749	853823	4682358			
	I	1	1	2	2	11	11	50?	A279197									
	J	0	0	0	2	7	52		A279198									
	K	1	1	2	6	25	115		A202705									
	L	0	1	3	9	30	117	512	A279199									
	M	1	2	5	15	55	232	A104429		(9)			(12)			(15)		
2848	N	0	0	1	1	2	2	3	7	15	12	30	8	32	162	21		
2849	P	0	0	1	2	4	6	3	10	25	12	42	8	40	202	21		

Here are ma[ny] other numbers but they are a[ll] integer value[d] polynomials, e.g. $s(43^n2^m) = (n+1)\left\{\binom{m+2n+4}{n+2} - \left(\right.\right.$

[illegible] parts of size $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Monday, January 30, 17

e.g. $s(4\,3^n 2^m) =$
$(n+1)\left\{\binom{m+2n+4}{n+2} - \binom{m+2n+4}{n+1}\right\}$

										(9)			(12)			(15)
	I	1	1	2	2	11	11	50?	A279197							
	J	0	0	0	2	7	52		A279198							
	K	1	1	2	6	25	115		A202705							
	L	0	1	3	9	30	117	512	A279199							
	M	1	2	5	15	55	232	A104429								
48	N	0	0	1	1	2	2	3	7	15	12	30	8	32	162	21
49	P	0	0	1	2	4	6	3	10	25	12	42	8	40	202	21

Best wishes,

✓A # of partitions of 1 into $n+1$ parts of size $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

✓B — — — — — — — n " parts $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \dots$

(C) — — — "non-isentropic binary trees" (Helen Alderson, JHConway, etc at Cambridge. They are rooted trees with 2 branches at each stage and if A, B, C, D (v. fig. rt.) are further growths, then one treats (AB)(CD) as equivalent to (AC)(BD) — otherwise one distinguishes left & right. # of equiv. classes of such trees.

·D # of polynomials $P(x,y)$ with non-neg. integer coeffs with $P(x,y) \equiv 1$, mod $x+y-1$ and with $P(1,1) = n$. (almost the same as C!)

(E) # of distinct values of $2^{2^{2^{2}}}$ with n 2's and the $n-1$ operations performed in any order (not necessarily "nested") (have asked for a copy of note by Selfridge & self to be sent to you).

(F) # of sequences of "refinements" of partitions of n into 1^n e.g. $5 \to 41 \to 31^2 \to 21^3 \to 1^5$, $5 \to 32 \to 2^21 \to 21^3$ (with $41 \to 2^21$, $32 \to 31^2$) e.g. in the fig. ∃ 4 distinct paths of length $n-1 = 4$ from 5 to 1^5, so $s(5) = 4$. Moon & I are writing a paper on this.

·G an upper bound for F: $\dfrac{\left(\left[\frac{1}{2}n\right]!\right)^2 \left[\frac{1}{2}n\right]^{m-\varepsilon}}{2.5.\dots.\frac{1}{2}(m+1)(m+2)}$ where $m(m+1) \le n < (m+1)(m+2)$, $\varepsilon = 1$ or 2 acc. as n is odd or even and the numbers in the denom are 1 less than Δar #s.

(H) Total # of paths from n towards 1^n of all lengths $0, 1, 2, \dots, n-1$. The coeffs. in the inequality
$s(n) \ge s(n-1) + 2s(n-2) + 3s(n-3) + 7s(n-4) + 15s(n-5) + \dots$ used to obtain a lower bound.

I In the generalization of Sedláček's conjecture (Roger B. Eggleton & self) (copy of 1st paper sent), the # of "self-conjugate inseparable" solutions of $x+y = 2z$ (integer, disjoint triples from $\{1,2,3,\dots,3n\}$) e.g.

2 4 3
5 7 6
1 15 8
9 11 10
12 14 13

J # of pairs of "conj. insep" " , e.g.

2 4 3 — 2 6 4
5 7 6 — 3 7 5
1 15 8 — 1 15 8
9 13 11 — 9 11 10
10 14 12 — 12 14 13

K = I + 2J, # of "insep" solutions

M = K + L, # of solutions.

L # of "separable" solutions, e.g.

1 3 2
4 8 6
5 9 7

(N) # of solutions of $x+y = z$ chosen from $\{1, 2, \dots n\}$ counting only " which include n. (P) counting other solutions.

Sequences C and D from Guy's letter need more terms and clearer definition

C: A2844

Number of non-isentropic binary rooted trees with n nodes.

1, 1, 2, 5, 13, 36, 102, 296, 871, 2599

Studied by Helen Alderson, J. H. Conway, etc. at Cambridge. These are rooted trees with two branches at each stage and if A,B,C,D (see drawing in letter) are further growths, then one treats (AB)(CD) as equivalent to (AC)(BD) - otherwise one distinguishes left and right. The sequence gives the number of equivalence classes of such trees.

D: A279196

December 15 2016

Number of polynomials P(x,y) with non-negative integer coefficients such that
P(x,y) == 1 mod x+y-1 and P(1,1) = n.

1, 1, 2, 5, 13, 36, 102, 295, 864

(both have offset 1)

Postscript, Jan 28 2017: Doron Zeilberger informs me he has a Maple program that implements the definition of sequence C, and he is extending the sequence.

See A002844 for details.

Guy's sequences I, J, K, L, M also need more terms

M: A202705

Number of irreducible ways to split 1...3n into n 3-term arithmetic progressions

1, 1, 2, 6, 25, 115, 649, 4046, 29674, ...

Offset 1. Only 14 terms known, extended by Alois Heinz in 2011.

3 papers by Richard Guy, 1971-1976
Calgary thesis by Richard Nowakowski 1975, not online

Are there any applications here of modern "additive combinatorics" (Gowers et al.)?

Definition not clear, need better examples, formulas?

I: A279197

Number of self-conjugate inseparable solutions of X + Y = 2Z (integer, disjoint triples from {1,2,3,...,3n}).

1, 1, 2, 2, 11, 11, 50 (offset 1)

```
Example of solutions X,Y,Z for n=5:
2,4,3
5,7,6
1,15,8
9,11,10
12,14,13
```

Fibonachos
and generalizations

Fibonachos Numbers

Based on Reddit page created by “Teblefer”

A280521, contributed by Peter Kagey, Jan 4 2017

Fibonachos, cont.

Start with pile of n nachos.

Successively remove 1,1,2,3,5,8,...,F_i until number left is < F_{i+1}

Then successively remove 1,1,2,3,5,8,...,F_j until number left is < F_{j+1}

Repeat until no nachos left. a(n) = number of stages.

n=23	subtract	leaves
	1	22
	1	21
	2	19
	3	16
	5	11
	8	3
	1	2
	1	1
	1	0

3 stages, so a(23) = 3

1, 1, 2, 1, 2, 2, 1, 2, 2, 3, 2, 1, 2, 2, 3, ...

A280521

Fibonachos, cont.

A280523: When do we first see n?

1, 3, 10, 30, 84, 227, 603, 1589, 4172, 10936, ..., 20365011049 (25 terms known)

Conjecture: This is bisection of A215004:

$$c(0)=c(1)=1;$$

$$c(n) = c(n-1) + c(n-2) + \mathrm{floor}(n/2).$$

Why?

Postscript: Only a few hours after I gave this talk, Nathan Fox pointed out that the Reddit web page also mentioned a simpler formula, namely

$$c(n) = \mathrm{Fib}(n+3) - \mathrm{floor}((n+3)/2).$$

Using this Nathan was able to prove the conjecture. See A280523 for details.

Fibonachos, cont.

Generalize: Nachos based on S,
where S = 1,... is a sequence of positive numbers.

S	a(n)	records at	
Fib.	A280521	A280523	
n	A057945	A006893	No. of triangular nos. needed to represent n by greedy alg.
n(n+1)/2	A281367	A281368	(*) Error in talk: see below
2^n	A100661	A000325	2^n-n
n^2	A280053	A280054	New

(*) In the talk I said the nachos sequence based on triangular numbers was A104246 and was conjectured to be unbounded. This was nonsense, as Matthew Russell pointed out.

Fibonachos, cont.

A28053 and A280054

Nachos based on Squares

n=36 subtract leaves

1	35
4	31
9	22
16	6
1	5
4	1
1	0

3 stages, so a(36)=3

Smallest number with nachos value n

```
1 1
2 2
3 3
4 4
5 9
6 23
7 53
8 193
9 1012
10 11428
11 414069
12 89236803
13 281079668014
14 49673575524946259
15 3690344289594918623401179
16 2363083530686659576336864121757607550
17 1210869542685904980187672572977511794639836071291151196
18 444145001054590209463353573888030904503184365398155859130743499369619675545966466
```

24 terms from Lars Blomberg

What are these numbers?

Digital sums of Fibonacci numbers

A67182

Smallest Fibonacci number with digital sum n

Dec 26 2016: A067182 was in a deplorable state:

a(n) = smallest Fib. no. with digital sum n, or -1 if none exists

0, 1, 2, 3, 13, 5, -1, 34, 8, 144, 55, -1, -1, -1, 4181, -1, -1, 89, ...

All ◯ were conjectures!

Me to Seq. Fans.: No progress since 2002 !

First reply: Not gonna happen!

Me: F_n mod 100 has period 300, might tell us something

Joseph Myers, Don Reble (indep.): You were close!

Look at F_n mod 9999, period 600,

no value is 6 mod 9999, so a(6) = -1 is true.

But all other -1 entries are still conjectures.

Even in base 2 this is hard - see next slide

A222296 Digital sums in base 2

Row n: All Fibonacci numbers with Hamming weight n:

{0},
{1, 1, 2, 8}, (Carmichael's Th.)
{3, 5, 34, 144}, (Elkies)
{13, 21, ...}. Conj. Full!

It is conjectured that the previous (n=3) row is complete, and that the subsequent rows are:

{89, 610, 2584},
{55, 233, 4181},
{377, 10946, 46368, 75025},
{1597},

Charles Greathouse (Q) and Noam D. Elkies (replies) on MathOverflow, 2014:

The Hamming weight $w(n)$ is the number of 1s in n when written in binary. Is there some effective bound on Fibonacci numbers F_n with $w(F_n) \leq x$ for a given x?

Since you specify "effective" in the question I guess you know this already, but just in case: there are only finitely many such n, because $2^{e_1} + \cdots + 2^{e_x} = (\varphi^n - \varphi^{-n})/\sqrt{5}$ is an S-unit equation in $x + 2$ variables over $\mathbf{Q}(\sqrt{5})$; but in general no effective proof is known for such a result (though the *number* of solutions of $w(F_n) \leq x$ may be effectively bounded). – Noam D. Elkies Mar 2 '14 at 6:28

A222296

Theorem:

Noam D. Elkies, Mar 2 2014

3, 5, 34, 144 are the only Fib. nos. with wt 2.

The case $x = 2$ is still tractable. If $F_n = 2^e + 2^f$ with $e < f$ then $e < 5$, else $F_n \equiv 0 \bmod 2^5$, which happens **iff** $n \equiv 0 \bmod 24$, and then $7 \mid 21 = F_8 \mid F_{24} \mid F_n$, which is impossible because $2^e + 2^f$ is never a multiple of 7. So we have only a few candidates for e, and we can deal with each of them separately, possibly even by elementary means, to show that $(n, e, f) = (12, 4, 7)$ is the last solution.

⟨ **EDIT** ⟩ Here's such an elementary proof. For each e (other than the trivial $e = 2$), we choose some $f_0 > e$, try each f with $e < f_0 < f$, and then once $f \geq f_0$ we use the condition $F_n = 2^e + 2^f \equiv 2^e \bmod 2^f$ to get a congruence condition on n, and then reach a contradiction by considering F_n modulo some odd prime (usually 3, but with one much larger exception).

$e = 0$: We take $f_0 = 4$. Trying $f = 1$ and $f = 2$ yields the Fibonacci numbers $F_4 = 3$ and $F_5 = 5$, and $f = 3$ yields the non-Fibonacci number 9. Once $f \geq 4$ we have $F_n \equiv 1 \bmod 16$. But $F_n \bmod 16$ is periodic with period 24, and it turns out that the remainder is 1 only for $n \equiv 1, 2, 23 \bmod 24$. But $F_n \bmod 3$ has period 8, which is a factor of 24; and $F_1 = F_2 = F_{-1} = 1$. We deduce $F_n \equiv 1 \bmod 3$. Hence $2^f \equiv 0 \bmod 3$, which is impossible.

$e = 1$: The Fibonacci numbers F_n congruent to $2 \bmod 4$ are those with $n \equiv 3 \bmod 6$, and these always turn out to be $2 \bmod 32$. Thus $f \geq 5$, and $f = 5$ yields the Fibonacci number $34 = F_9$. We claim that this is the only possibility, using $f_0 = 6$. Once $f \geq 6$ we have $F_n \equiv 2 \bmod 64$, and then $n \equiv \pm 3 \bmod 24$. But (again thanks to 8-periodicity mod 3) this implies $F_n \equiv 2 \bmod 3$, so once more we reach a contradiction from the congruence $2^f \equiv 0 \bmod 3$.

$e = 2$: impossible because F_n is never $2 \bmod 4$.

$e = 3$: We take $f_0 = 5$. Since $2^3 + 2^4 = 24$ is not a Fibonacci number, we may assume $f \geq 5$, and then $F_n \equiv 8 \bmod 32$. This is equivalent to $n \equiv 6 \bmod 24$, which again yields a contradiction mod 3 since $2^f = F_n - 2^e$ would have to be a multiple of 3.

$e = 4$: This is the hardest case: because $f = 7$ yields $144 = F_{12}$, it is not enough to use congruences that can be deduced from $F_n \equiv 2 \bmod 2^7$, and we must take $f_0 > 7$. It turns out that $f_0 = 9$ works. Then $f = 5, 6, 8$ yield the non-Fibonacci $48, 80, 272$. Once $f \geq 9$ we must have $F_n \equiv 16 \bmod 2^9$. Now $F_n \bmod 2^9$ has period 768, but the condition $F_n \equiv 16 \bmod 2^9$ determines $n \bmod 384$ (half of 768), and we compute $n \equiv -84 \bmod 384$. Now $n \bmod 384$ determines F_n modulo the prime 4481 (the period is 128), and we find $F_n \equiv 2284 \bmod 4481$, whence $2^f = F_n - 2^e \equiv 2284 - 16 = 2268 \bmod 4481$. But this is impossible because 2 is a fourth power (even an 8th power) mod 4481, and 2268 is not.

⟨ **/EDIT** ⟩

But I doubt that one can prove that such a technique can work for all x...

Carryless Stuff

(No caries)

Recall!

Carryless Arithmetic

Dedicated to Martin Gardner

No carries in the Carryless Islands!

(former penal colony - prisons have excellent dental care)

6 + 7 = 3

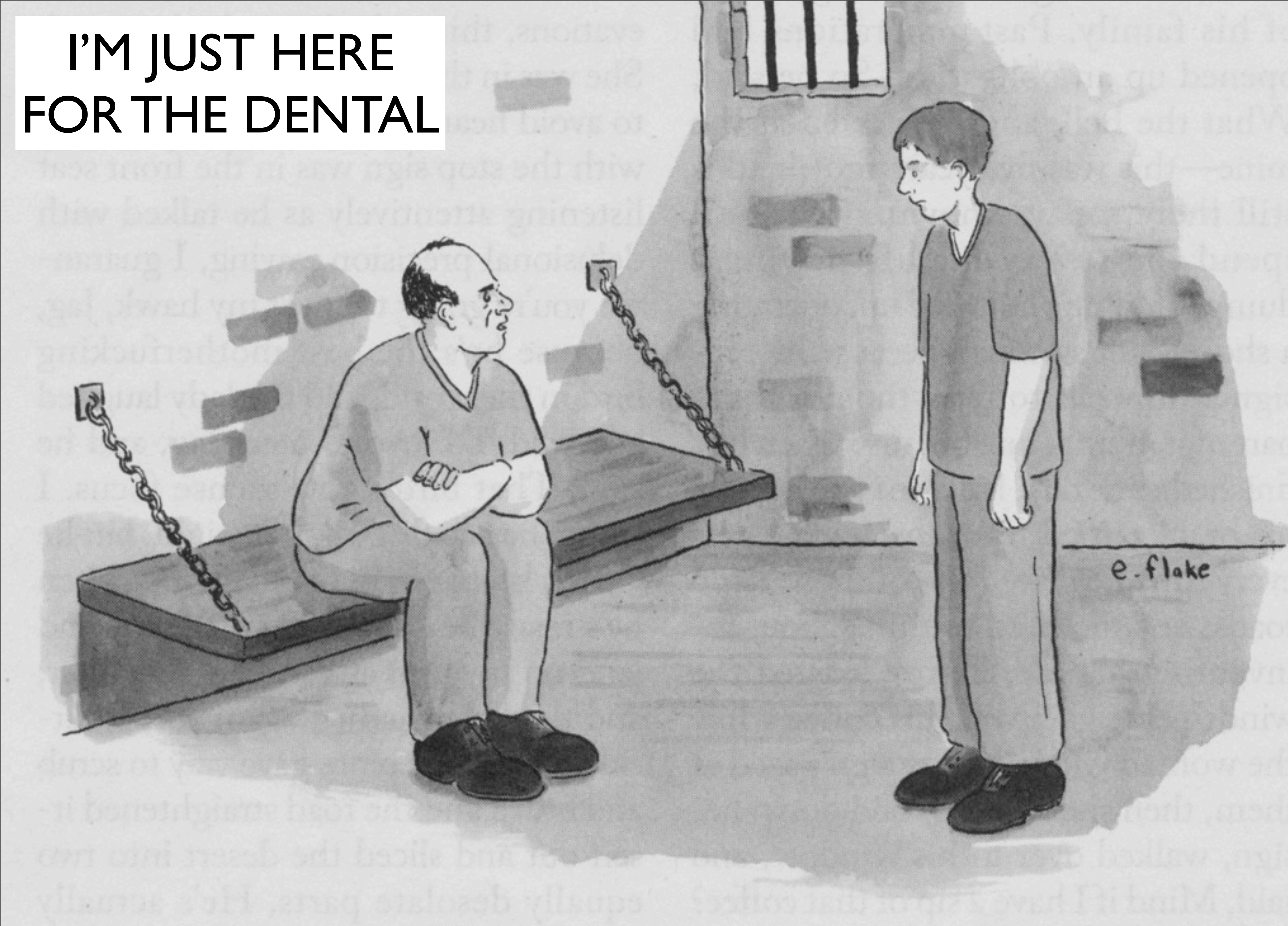
6 x 7 = 2

```
  785
 +376
------
= 051
```

```
  643
 x 59
------
  467
 0050
------
= 417
```

Applegate, LeBrun, Sloane
College Math. J. 2012
George Polya Prize

I'M JUST HERE FOR THE DENTAL



"I'm just here for the dental."

Recall!

What are the carryless primes?

First try fails!

Any number is divisible by 9,
e.g. 9 x 99 = 11, so no primes exist

Better: Note that 3 x 7 = 1, 9 x 9 = 1

So 1, 3, 7, 9 are UNITS, and don't count.

p is prime if only factorization is p = u x p',
where u is 1, 3, 7, 9

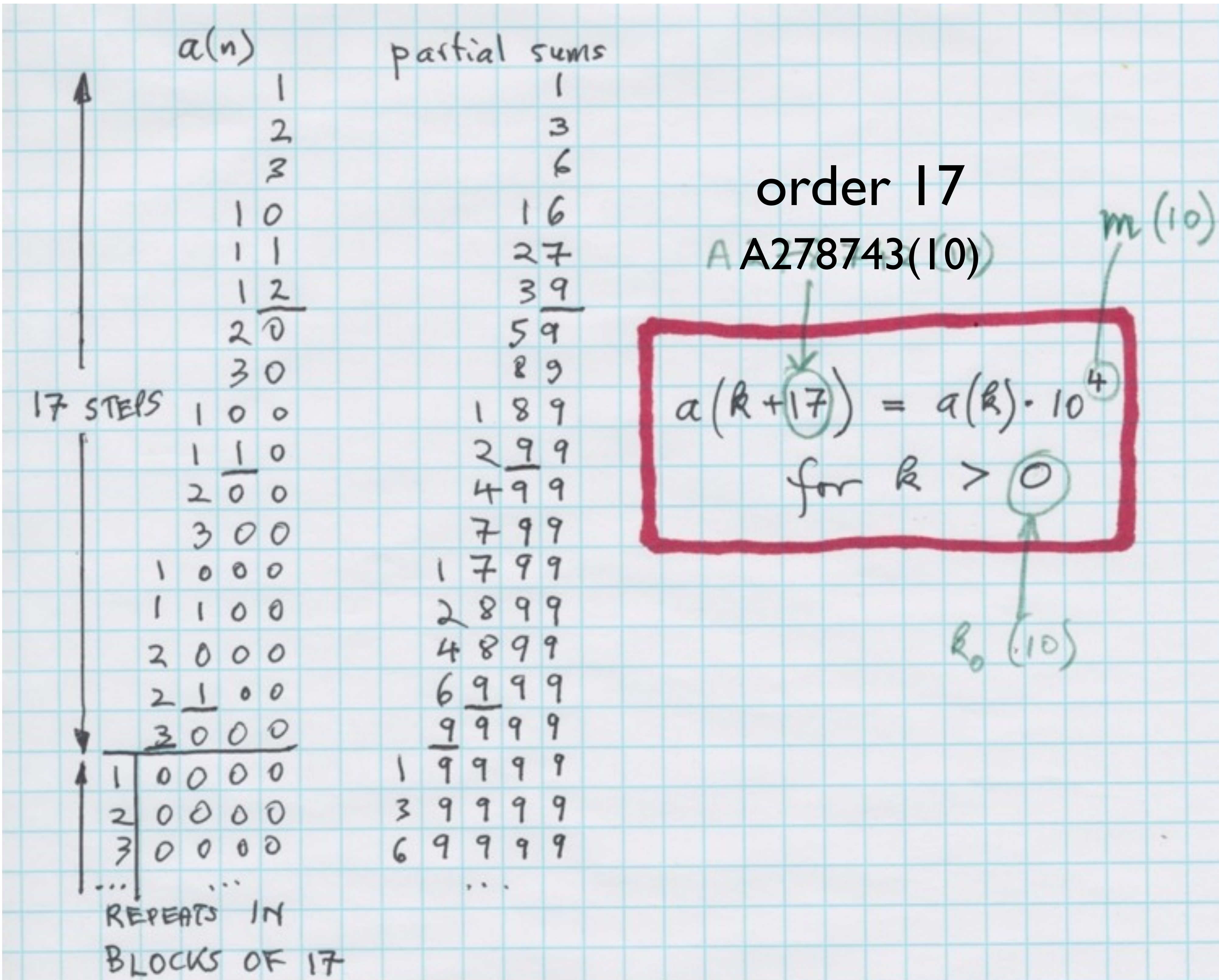
Carryless primes are 21, 23, 25, 27, 29, 41, 43, 45, ...

Sequence A169887 in OEIS

(Be careful: 2 = 4 X 5005555503 !)

a(n) = smallest s.t. a(1)+...+a(n) has no carries.

A278742 Rémy Sigrist Nov 27 2016



BASE 9

A281366, A280731

	base 9	base 10
1	1	1
2	2	2
3	3	3
4	10	9
5	11	10
6	20	18
7	21	19
8	100	81
9	110	90
10	200	162
11	210	171
12	1000	729
13	1100	810
14	2000	1458
15	2100	1539
16	10000	6561

A278743(9) m(9)

$$a(k+4) = a(k) \cdot 9^{1}$$

for $k > 3$

$k_0(9)$

Sigrist's Conjecture

A278743, A280051, A280052

For any base b, $\exists$ d, k_0, m such that

$$a(k+d) = a(k) \cdot b^m \quad \text{for } k > k_0$$

(d ↑ order)

b	d	k_0	m
2	1	0	1
3	3	0	2
4	2	0	1
5	5	0	2
6	9	0	3
7	3	0	1
8	7	0	2
9	4	3	1
10	17	0	4
11	4	0	1
12	9	0	2

A278743 : d
A280051 : k_0
A280052 : m

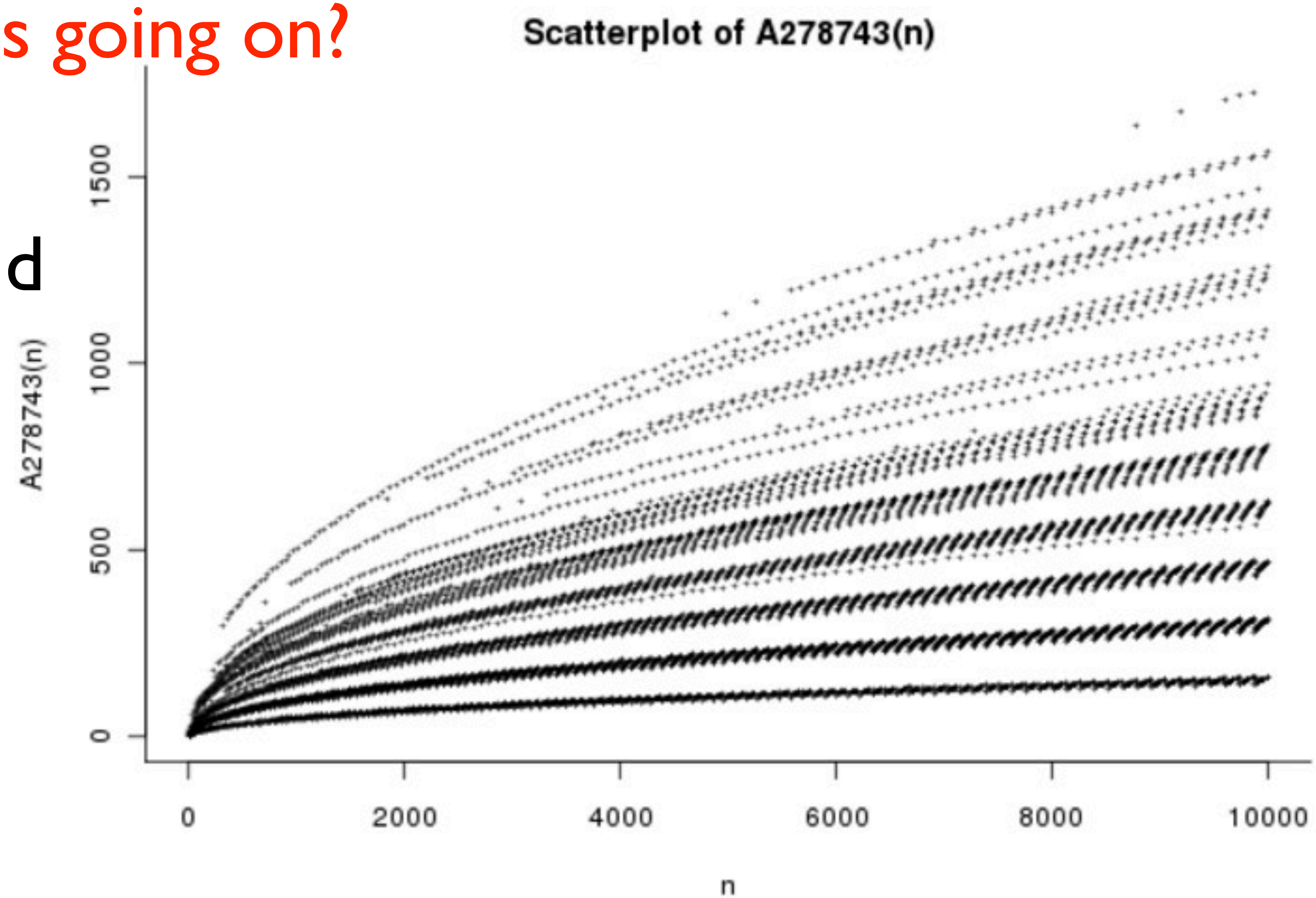
A278743

From Sigrist's conjecture:

Order of recurrence for greedy carryless sequence in base b

What is going on?

Order d



Base b

The Tisdale Sieve

A141436

The Tisdale Sieve
A141436

Dec 25 2016: Editor J.E.S. said: A141436 is a mess!
Me: I will edit it! And discovered a diamond.....

Let P = primes 2, 3, 5, 7, 11, 13, ...
N = nonprimes 1, 4, 6, 8, 9, 10, 12, ...

Define ∞ set of sequences $S_1, S_2, S_3, \ldots$ by

$S_i(1)$ = smallest number not yet used
$S_i(j+1)$ = either $P(S_i(j))$ or $N(S_i(j))$ so that primes and nonprimes alternate in S_i.

S_1 = 1, 2 → 4 → 7 → 12 → 37
(1st prime, 2nd nonprime, 4th prime, 7th nonprime, 12th prime)

S_2 = 3, 6, 13, 21, ...

S_3 = 5, 9, 23, ...

A141436 = 1, 3, 5, 8, 10, 11, ...

Conjecture (R. J. Mathar): This is union of

A6450 = p ∉ P(N) = P(P) and
A102615 = n ∉ N(P) = N(N)

Proof of Mathar's conjecture by David Applegate

Lemma Given $f: \mathbb{Z}^+ \to \mathbb{Z}^+$ with $f(k) > k$ for all k

Define $S_1, S_2, S_3, \ldots$ by:

$S_i(1) =$ smallest number not yet used

$S_i(j+1) = f(S_i(j))$

Then $S_1(1), S_2(1), S_3(1), \ldots = k \notin Im(f)$.

Proof If $k \notin Im(f)$ then $k \neq S_i(j), j > 1$

$\therefore k = S_i(1)$ for some i

Conversely: If $k \in Im(f)$, then $\exists\, m$ with $f(m) = k$.

- if $m \in Im(f)$ then by IH $m = S_i(j), j > 1$

  so $k = S_i(j+1)$

- if $m \notin Im(f)$ then $m = S_i(1)$, $k = S_i(2)$. □

Apply Lemma with

$$f(k) = P(k) \text{ if } k \in N, \quad N(k) \text{ if } k \in P.$$

QE[D]

Square Permutations
and
Square Binary Words

Number of Square Permutations

A279200 Dec. 15 2016

based on S. Giraudo, arXiv 2016

Samuele Giraudo and Stephane Vialette

$\sigma \in S_{2n}$ s.t. $\sigma = \pi ⧢ \pi$, $\pi \in S_n$

$⧢$ = imperfect shuffle

$(n \geq 0)$ 1, 2, 20, 504, 21032, 1293418.

Number of Square Binary Words of Length 2n

A191755

$u \in \{0,1\}^{2n}$ s.t. $u = v ⧢ v$, $v \in \{0,1\}^n$

$(n \geq 0)$ 1, 2, 6, 22, 82, 320, 1268, ...

Known for $n \leq 15$ (Joerg Arndt)

No theory, formulas, ...

Remy Sigrist's New Sequences

A280864, A280866

Rémy Sigrist   Two new sequences   Jan 9 2017

A280864 Distinct, earliest; for any prime p any run of consecutive multiples of p has length 2.

	1	2	4	3	6	8	5	10	12	9	7	14	16	11
↙	–	–	2	–	3	2	–	5	2	3	–	7	2	–
↗	–	2	–	3	2	–	5	2	3	–	7	2	–	11
	↑		↑			↑				↑			↑	

"FREE": NEXT TERM IS SMALLEST MISSING NO.

A280866 Same except ... has length ≥ 2.

(They agree for 41 terms)

A280866

Th.: This is a perm of natural numbers

Proof

1. clearly infinite
2. Any m is either in sequence, or $\exists\, n_0$ s.t.

$$n > n_0 \Rightarrow a(n) > m$$

3. For any prime p, $\exists$ term divisible by p

(If p never appears, then no prime $> p$ can appear.
$\therefore$ all terms are ~~mult~~ products of $2\ 3\ 5 \cdots p-1$
Go out past $p!$. Then candidates for next term
are p and $p \cdot \{$any product of distinct primes $< p\}$
& these are $< p!$ so will appear next)

4. ~~If~~ For a prime p, let $a(n)$ be
first multiple of p. Either $a(n) = p$, & $a(n-1)$ was
free, or $a(n) = kp$, $a(n+1) = p$ and is free.

$\therefore \infty$ many free terms

$\therefore$ every number appears $\square$

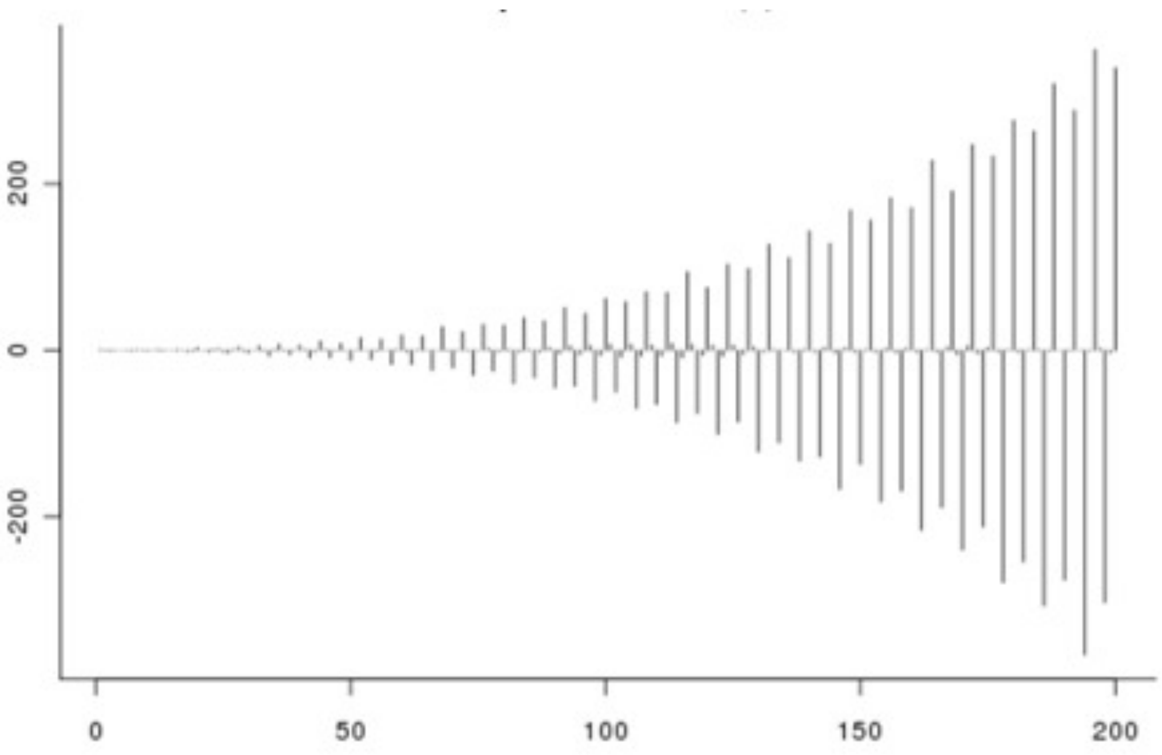
New A281488 with key-words "look" and "hear"

A281488 from Andrey Zabolotskiy
January 22 2017

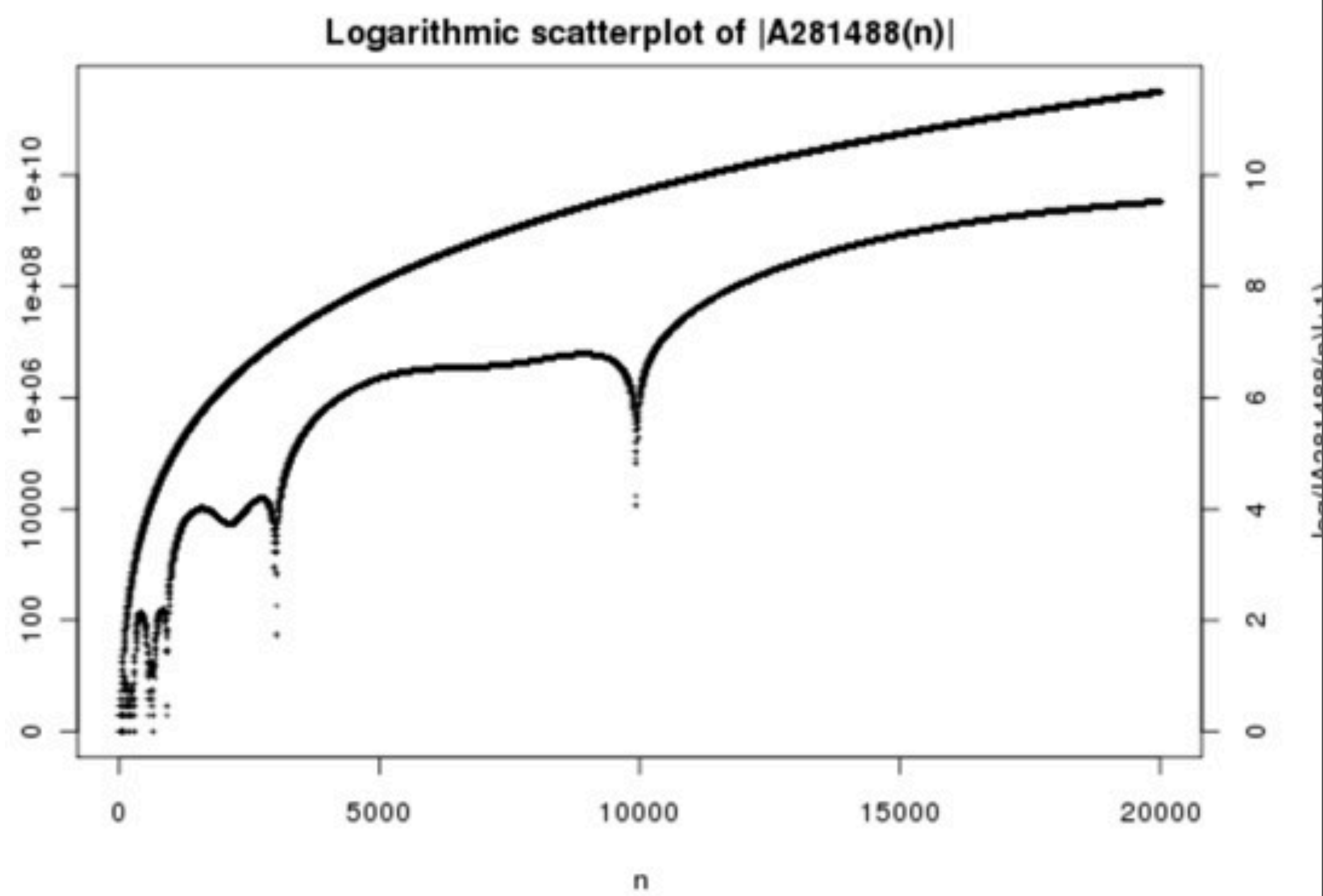
$$a(n) = -\sum_{\substack{d|(n-2) \\ 1 \le d \le n-1}} a(d)$$

1, -1, -1, 0, 0, 0, -1, 1, 0, -1, 0, ...

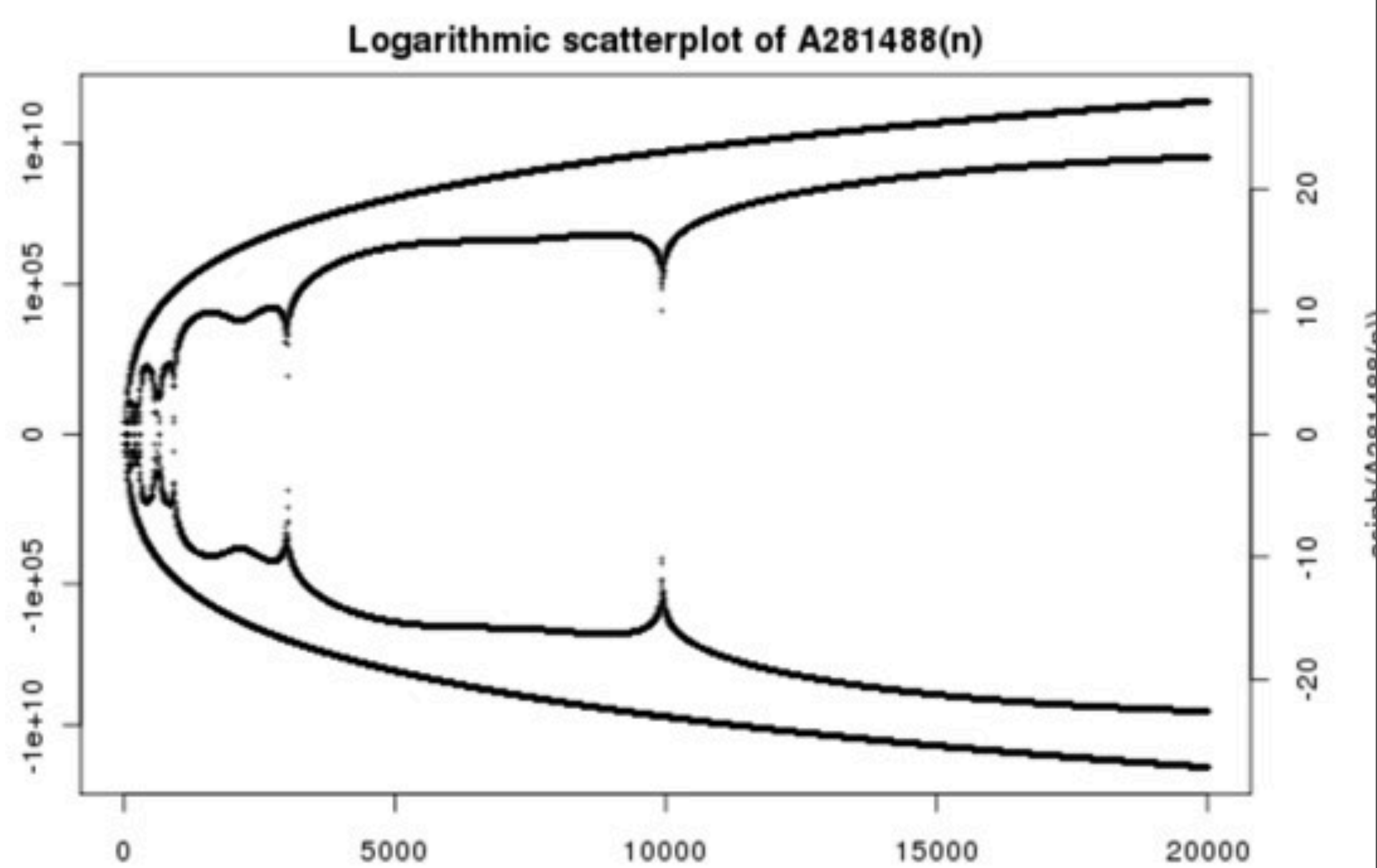
A281488



200
0
-200
0
50
100
150
200



Logarithmic scatterplot of |A281488(n)|
1e+10
1e+08
1e+06
10000
100
0
10
8
6
4
2
0
0
5000
10000
15000
20000
n



Logarithmic scatterplot of A281488(n)
1e+10
1e+05
0
-1e+05
-1e+10
20
10
0
-10
-20
0
5000
10000
15000
20000

Two compositions from Michael Nyvang (Copenhagen) based on OEIS sequences

- surreal-cantata--final.mp3
- A276207-and-neighbors-music-forNJ.mp3

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Become a volunteer OEIS editor!

Contact Neil Sloane, njasloane@gmail.com