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Karlson - Sloane
correspondence
1974
(add to 2 seqs)

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N.J.A. Sloane
Mathematics Research Center
Bell Telephone Labortories
Murray Hill, New Jersey

Dear Sir:

I have recently gotten a copy of your book, "A HANDBOOK OF INTEGER SEQUENCES." Sequence #608, i.e., "Denominat~~ion~~ors of Generalized Bernoulli Numbers" seems to be identical to the sequence of denominators of the formulae for the sums of powers of consecutive numbers to N.

If the sequences are indeed identical, perhaps it might be so stated in an ensuing supplement. If not, perhaps the sequence of denominators of the formulae for power sums might be added.

Please list me to receive any supplements which may be issued.

Sincerely,

C.H. Karlson
545 West 126th St.
New York City 10027

Sent
Add to list

2790

②

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

October 1, 1974

Mr. C. H. Karlson
545 West 126th Street
New York, New York 10027

Dear Mr. Karlson:

Thank you very much for your interesting letter. The two sequences are <u>not</u> the same:

	n	1	2	3	4	5	6	7	8	9	10
a_n =	seq. 608	2	6	4	30	12	84	24	90	20	132
	b_n	2	6	4	30	12	42	24	?	?	?

N608=2790 ~~608~~

The definition of a_n is (from Milne-Thompson)

$$\frac{t}{(1+t)\log(1+t)} = \sum_{n=0}^{\infty} \frac{t^n}{n!} a_n . \qquad (1)$$

Whereas b_q is the denominator of the right-hand side of

$$\sum_{n=1}^{n-1} n^q = \frac{11}{q+1} \sum_{k=0}^{q} \binom{q+1}{k} B_k N^{q-k+1} , \qquad (2)$$

considered as a polynomial in N, where B_k are the usual Bernoulli numbers given by

$$\frac{t}{e^t - 1} = \sum_{r=0}^{\infty} \frac{t^r}{r!} B_r . \qquad (3)$$

so there is really no reason why they should be the same.

I would like to include $\{b_n\}$ in later supplements. Can you supply any more values?

Supplement I is enclosed.

Thank you again for your letter.

Yours sincerely,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above

JOINT SCHOOLS COMMITTEE FOR ACADEMIC EXCELLENCE NOW, INC.
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October 14,1974

Dear Mr. Sloane:

Your letter dated October 1st was very welcome. Thank you for the supplement. It is an expensive issue obviously and I think I should be billed for it.

$Snx = 1^x + 2^x + \cdots + n^x$

For typing purposes Snx will mean the sum 1 & 2 & n each to exponent x.

It is curious to me that Sn4 and Sn5 can be derived from lesser Sn but higher Sn cannot i.e. Snx = Sn2+12a = (Sn2)(Sn1) + 2a and Sn5 = Sn3 + 12b = (Sn3) (Sn1) + 3b.

At least I have been unable to find similar equalities for higher Sn.

It seems to me that the denominators of Snx are always equal to twice the coefficient of nx. If this is so, and if you wish I will write the necessary simultaneities for solution of Sn8,9and 10. But I do not have acess to a computer nor the ability to program one.

Thank you again for your supplement

Very Sincerely,

Carl Harlson

$$Snx = \sum_{i=1}^{n} i^x$$

$$Sn4 = \sum_{1}^{N-1} n^4$$

$$= \frac{1}{5}\left[1\cdot N^5 - \frac{5}{2}\cdot N^4 + \frac{10}{6}\cdot N^3 \quad - \frac{5}{30} N\right]$$

$$= \frac{1}{5}\left(N^5 - \frac{5}{2}N^4 + \frac{5}{3}N^3 - \frac{1}{6}N\right)$$

$$\begin{array}{r} 32 \\ 13 \\ \hline 45 \\ 40 \end{array}$$

$N=2$: 1 = ans

$$\frac{1}{5}\left(32 - 5.8 + \frac{5.8}{3} - \frac{1}{3} \quad 13\right) = 1$$

$N=3$: 17

$$\frac{1}{5}\left(243 - \frac{5}{2}.81 + 45. - \frac{1}{2}\right.$$

$\frac{406}{2}$ -203

$$\begin{array}{r} 243 \\ 45 \\ \hline 288 \\ 203 \\ \hline 5)85 \\ 17 \end{array}$$

$$\boxed{Sn4\Big|_{1}^{N-1} = \frac{N}{30}\left(N^4 - 15N^3 + 10N^2 - 1\right)} \text{ —— } (1)$$

Sn2 $\dfrac{(N-1)(N)(2N-1)}{6} \cdot \dfrac{(N-1)N}{2}$

$$\frac{N}{30} \cdot 5(N-1)(2N-1)$$

$$10N^2 - 15N + 5$$

$$\frac{N}{30}\cdot\left(N^4 - 15N^3 + 15N - 6\right)$$

N^2 $(N^2 - 2N + 1)(2N - 1)$

$$\begin{array}{rrrr} & 1 & -2 & 1 \\ \hline & -1 & 2 & -1 \\ 2 & -4 & 2 & \\ \hline \boxed{2} & \boxed{-5} & \boxed{4} & \boxed{-1} \end{array}$$

(4)

Bell Laboratories

600 Mountain Avenue
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Phone (201) 582-3000

November 6, 1974

Mr. C. H. Karlson
545 West 126th Street
New York, New York 10027

Dear Mr. Karlson:

Thanks for your letter of October 14. But I don't understand what you mean by

$$\begin{aligned} Sn4 &= Sn2 + 12a \\ &= (Sn2)(Sn1) + 2a \end{aligned}$$

Please explain!

Regards,

MH-1216-NJAS-mv

N. J. A. Sloane

KARLSON

(5)

11/13 74

Mr. N.J.A. Sloane
Bell Laboratories
Murray Hill, New Jersey

Dear Mr. Sloane:

SN4=SN2 +12a = (SN2)(SN1) + 2a

1. (2n 3 + 3n2 +n)/6 +12a=((2n3+3n2+n)/6)((n2+n)/2)+2a
whence
2. (2n5+5n4-5n2-2n)/10=12a
substuting for 12a in 1 we get
(6n5+15n4 +10n3-n)/30=Sn4

The values of a for n=1,2,3,------------- are0,1,7,27,77,182,etc. A5585
Similarly for Sn5=Sn3+12b=(Sn3)(Sn1)+3b

It might be noted that Sierpinski gives a second definition of seq. 100 .
seq.100 "Partitions into distinct parts". Sierpinski gives that and also "the number of decompositions of n into a sum of non decreasing odd numbers". ref.sll pp400-401.
An interesting sequence is related to an inequality often found in tests.
(x)--n +(x+1)--n≠(x+2)--n for n greater than 2.

It gives highest and lowest values related to the balance of the inequality. But it does not meet the requirement that it be found in the literature(as far as I know).
2, 47, 64, 657, 2002, 12219, 59264, 28357, etc. Omit

Cordially yours,

Carl Karlson

Carl Karlson

EK?mr

(6)

Bell Laboratories

600 Mountain Avenue
Murray Hill, New Jersey 07974
Phone (201) 582-3000

November 21, 1974

Mr. C. H. Karlson
545 W. 126 Street
New York, New York 10027

Dear Mr. Karlson:

Thanks for your letter of November 13. The last Sequence you mention is new to me, although I don't really understand how it is defined, or calculated. For the past year or so a colleague (Mrs. MacWilliams) and I have been writing a book on error-correcting codes. A copy of Chapter 1 is enclosed in case you are interested. Any criticisms would be appreciated.

Best regards,

MH-1216-NJAS-mv

N. J. A. Sloane

Enc.
As above