Math Gazette 46 (1962)

Please enter 2717

the restriction limiting B to lie tween A and Q is removed. The s the parallel to PQ through A in nd solution can be obtained by corresponding to M, B, and C. ed, with B' on AP produced and y homothetic to the quadrangle of the lines QH, QH' is parallel to solution is possible.

by L' an AQ produced such that le with centre L' and radius AP. through A, parallel to PQ, in two s or in no real points according as o or greater than AP. If $\angle PAQ =$ H' is parallel to AP, and again only here are two, three, or four possible $AQP - AP$ is negative, zero or $Q = 60°$ or $120°$ when the number of The re only two solutions when P 120°.

H. E. TESTER

t on a chess-board so that no two are r answer.

I. J. GOOD

s

l obtained by stopping the exponential real linear factors if p is even and only dd.

) then (i)

$= f_{p-1}(x)$ (ii)

$= f_{p-1}(x) + (x^p/p!)$

all real values of x, $f'_{2n+1}(x)$ is positive n have only one real linear factor. Then only one real x and hence $f_{2n+2}(x)$ can Also by (ii) with $p = 2n + 2$, it follows $f_{2n+2}(x) = (x^{2n+2}/(2n+2)!)$, which is hat this point is a minimum point and itive for all real x. But $f_2(x)$ is positive induction, $f_{2n}(x)$ is also and $f_{2n+1}(x)$ has proves both results.

due to Mr W. Martyn, Glasgow Technical ebted for it.

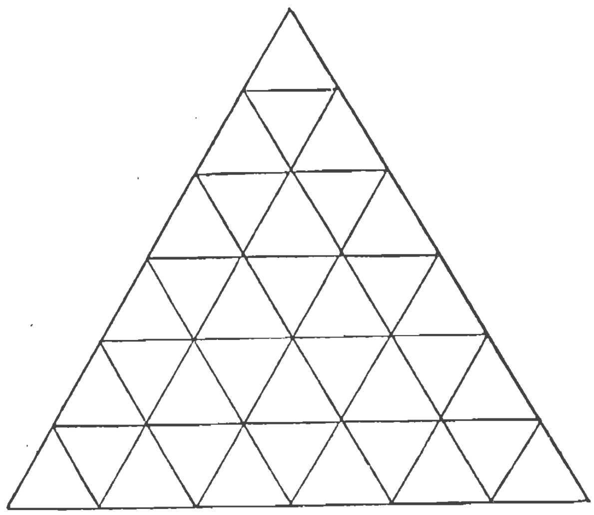
H. V. LOWRY

CLASS ROOM NOTES

83. An interesting series

Even in the Primary School, something of exceptional interest crops up from time to time. Such an occasion was when the Puzzles Editor of my classroom wall magazine received the following contribution:

How many triangles does this figure contain?



I pointed out that it was his duty to ensure that the answer given in the following week's issue was the correct one, and suggested that he might find it interesting to try and discover a rule that would give the answer for similar figures with any number of rows.

He discovered, by counting, that the first six cases were:

Rows of triangles:	1	2	3	4	5	6
Number of triangles:	1	5	13	27	48	78

2717 ✓

He analysed the intervals as follows, and was able to continue the series:

1	5	13	27	48	78	*118*	*170*	*235*	*315*	*411*	~~525~~
4	8	14	21	30	*40*	*52*	*65*	*80*	*96*	*114*	
4	6	7	9	*10*	*12*	*13*	*15*	*16*	*18*	19	21
2	1	2	*1*	*2*	*1*	*2*	*1*	*2*	*1*	2	

~~2718~~

This was as far as the Puzzles Editor could go, as the series was not a simple arithmetical one. I found, however, that it was related in a curious way to the well-known series:

1	3	6	10	15	21	28	36	45	55	66	78
	1	3	6	10	15	21	28	36	45	55	66
	1	3	6	10	15	21	28	36	45	55	66
		1	3	6	10	15	21	28	36	45	55
			1	3	6	10	15	21	28	36	45
			1	3	6	10	15	21	28	36	45
				1	3						
1	5	13	27	48	etc						

A formula for calculating the number of triangles (T) in an odd number of rows (R) is:

$$T = \frac{4x^3 + 11x^2 + 9x + 2}{2} \quad \text{when} \quad x = \frac{R-1}{2}.$$

The formula for an even number of rows is:

$$T = \frac{4x^3 + 5x^2 + x}{2} \quad \text{when} \quad x = \frac{R}{2};$$

in terms of R itself the number of triangles is

$$\frac{4R^3 + 10R^2 + 4R - 1 + (-1)^R}{16}.$$

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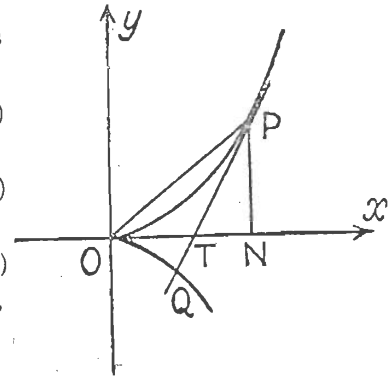
J. HALSALL

84. The semi-cubical parabola

This curve (the evolute of the parabola) has been introduced into the Advanced Level Mathematics syllabus of the N.U.J.M.B. in the form $y^2 = x^3$. This seems a rather uninspired choice as the curve is lacking in geometrical properties. No examples occur in the Specimen Questions issued by the Board and covering most of their newly introduced topics. The following is a survey of the basic equations of the curve and a few geometrical properties. The point $[t]$ has the co-ordinates (t^2, t^3), where the parameter t is tan $\angle$POx and the letters refer to the diagram.



The chord joining $[p]$ and $[q]$ is

$$(p^2 + pq + q^2)x - (p+q)y - p^2q^2 = 0 \quad \text{(i)}$$

The tangent at $[p]$ is

$$3px - 2y - p^3 = 0 \quad \text{(ii)}$$

The normal at $[p]$ is

$$2x + 3py = 2p^2 + 3p^4 \quad \text{(iii)}$$

From (i), the condition for $[p]$, $[q]$ and $[r]$ to be collinear is

$$pq + qr + rp = 0$$