

Mats Gazette

46 (1962)

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ICAL GAZETTE

the restriction limiting B to lie between A and Q is removed. The parallel to PQ through A in solution can be obtained by corresponding to M , B , and C . with B' on AP produced and homothetic to the quadrangle of the lines QH , QH' is parallel to solution is possible.

by L' an AQ produced such that le with centre L' and radius AP . through A , parallel to PQ , in two s or in no real points according as o or greater than AP . If $\angle PAQ = H'$ is parallel to AP , and again only here are two, three, or four possible $AQP - AP$ is negative, zero or $Q = 60^\circ$ or 120° when the number of The are only two solutions when $\angle P = 120^\circ$.

H. E. TESTER

it on a chess-board so that no two are r answer.

I. J. GOOD

obtained by stopping the exponential real linear factors if p is even and only odd.

then (i)

(ii)

for all real values of x , $f'_{2n+1}(x)$ is positive n have only one real linear factor. Then r only one real x and hence $f_{2n+2}(x)$ can

Also by (ii) with $p = 2n + 2$, it follows $f_{2n-2}(x) = (x^{2n+2}/(2n+2)!)$, which is hat this point is a minimum point and itive for all real x . But $f_2(x)$ is positive induction, $f_{2n}(x)$ is also and $f_{2n+1}(x)$ has proves both results.

due to Mr W. Martyn, Glasgow Technical elted for it.

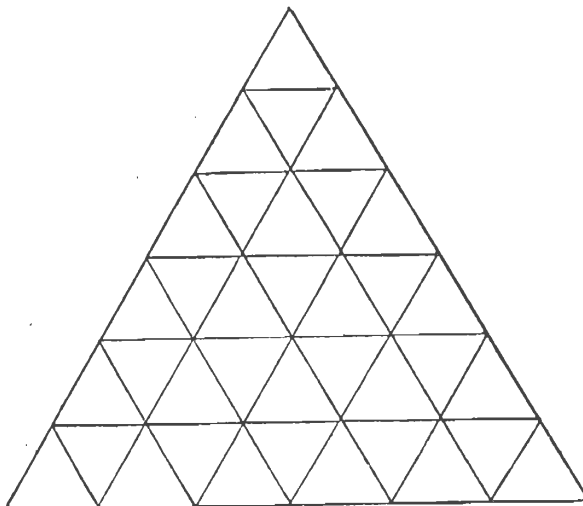
H. V. LOWRIE

CLASS ROOM NOTES

83. An interesting series

Even in the Primary School, something of exceptional interest crops up from time to time. Such an occasion was when the Puzzles Editor of my classroom wall magazine received the following contribution:

How many triangles does this figure contain?



I pointed out that it was his duty to ensure that the answer given in the following week's issue was the correct one, and suggested that he might find it interesting to try and discover a rule that would give the answer for similar figures with any number of rows.

He discovered, by counting, that the first six cases were:

Rows of triangles: 1 2 3 4 5 6
Number of triangles: 1 5 13 27 48 78

He analysed the intervals as follows, and was able to continue the series:

1	5	13	27	48	78	118	170	235	315	411	525
4	8	14	21	30	40	52	65	80	96	114	
4	6	7	9	10	12	13	15	16	18	19	21
2	1	2	1	2	1	2	1	2	1	2	

This was as far as the Puzzles Editor could go, as the series was not a simple arithmetical one. I found, however, that it was related in a curious way to the well-known series:

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1	3	6	10	15	21	28	36	45	55	66	78
	1	3	6	10	15	21	28	36	45	55	66
	1	3	6	10	15	21	28	36	45	55	66
		1	3	6	10	15	21	28	36	45	55
			1	3	6	10	15	21	28	36	45
				1	3	6	10	15	21	28	36
					1	3	6	10	15	21	28
						1	3	6	10	15	21
							1	3	6	10	15
								1	3	6	10
									1	3	6
										1	3
											1
1	5	13	27	48	etc						

A formula for calculating the number of triangles (T) in an odd number of rows (R) is:

$$T = \frac{4x^3 + 11x^2 + 9x + 2}{2} \quad \text{when } x = \frac{R-1}{2}$$

The formula for an even number of rows is:

$$T = \frac{4x^3 + 5x^2 + x}{2} \quad \text{when } x = \frac{R}{2}$$

in terms of R itself the number of triangles is

$$\frac{4R^3 + 10R^2 + 4R - 1 + (-1)^R}{16}$$

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84. The semi-cubical parabola

This curve (the evolute of the parabola) has been introduced into the Advanced Level Mathematics syllabus of the N.U.J.M.B. in the form $y^2 = x^3$. This seems a rather uninspired choice as the curve is lacking in geometrical properties. No examples occur in the Specimen Questions issued by the Board and covering most of their newly introduced topics. The following is a survey of the basic equations of the curve and a few geometrical properties. The point $[t]$ has the co-ordinates (t^2, t^3) , where the parameter t is $\tan \angle POx$ and the letters refer to the diagram.

The chord joining $[p]$ and $[q]$ is

$$(p^2 + pq + q^2)x - (p + q)y - p^2q^2 = 0 \quad (i)$$

The tangent at $[p]$ is

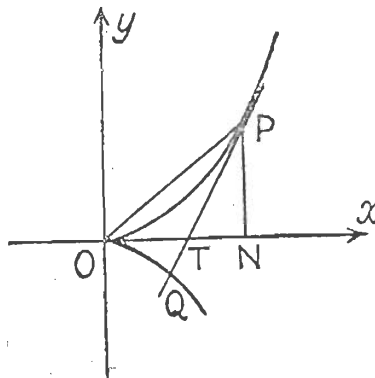
$$3px - 2y - p^3 = 0 \quad (ii)$$

The normal at $[p]$ is

$$2x + 3py = 2p^2 + 3p^4 \quad (iii)$$

From (i), the condition for $[p]$, $[q]$ and $[r]$ to be collinear is

$$pq + qr + rp = 0$$



i.e. the chord $[p][q]$ meet tangent at $[p]$ meets the

From (ii), if the tangent ordinate, then $3.OT =$ the property: $n \times$ sub tangents can be drawn their points of contact, p of the curve, all three are at $[p]$ and $[q]$ meet at corresponding result for two

From (iii), "in general given point and, if $[p]$, $r + s = 0$ and $pq + rs =$ be proved that only two

Geometrical Properties PQ is the tangent at P .

1. The locus of the parabola which touches which is also a tangent.

2. $PT:TQ = 8:1$

3. If the perpendicular the locus of the intersect touching the semi-cubic

4. For a set of parab. three intersections with the axis Ox .

5. If PQ is divided at cubical parabola.

6. If the chord OS $OS = 3.PQ$.

7. If the ordinate of Q of the intersection of t parabola.

8. A, B, C and D are C are collinear and the (i) If $\angle AOx = \angle BOy$, then $\angle COD = 90^\circ$.

9. If chords are drawn the vertex then the locus with the curve is a semi-vertex.

10 For a set of parab triangle formed by the curve is a straight line

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