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I. *Numerical Differentiation near the Limits of a Difference Table.*

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1. *Introduction.*

This paper has its immediate origin in the implications of a remark by Bradfield and Southwell (1). From the values of a function calculated (approximately) for eleven equally spaced arguments they wished to calculate derivatives of the function for the same arguments, using the known finite difference formulæ. They remark that "the order of the highest derivative which can be calculated is different at different sections"—implying that this order is that of the highest difference which can be used. If the finite difference formulæ are restricted to those containing only forward, backward, or central differences respectively, the implication is correct—but there must exist formulæ of a "mixed"

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type which enable the whole ten differences to be utilized at any argument, *i. e.*, formulæ commencing as central difference formulæ, switching over to forward (or backward) differences when the edge of the difference table is reached *. The route through the difference table would then be $\longrightarrow\nearrow$ or $\longrightarrow\searrow$. Similar considerations apply near the beginning or end of any difference table. We are not aware that the necessary coefficients have hitherto been published, and the object of this paper is to supply them.

In section 2 the main results, in the form of tables of coefficients, are described, together with (it is hoped sufficient) discussion and examples to enable those who so wish to use the tables without mastering or reading the underlying theory. In the last section the theory, calculation, and checking of the formulæ are considered.

Numerical differentiation—by *any* formula—is of necessity relatively inaccurate when the values of the function are not exact (*e. g.*, rounded-off or experimental), and the smaller the tabular interval the greater is the relative effect of such inexactitude. It is therefore desirable to use a large tabular interval, even though this entails the use of differences of highish order; we give coefficients for differences up to the twelfth. This inaccuracy increases as one proceeds to higher derivatives, when it is usually combined with a progressive loss of significant figures; we have therefore refrained from giving complete tables for derivatives of higher order than the fourth, although they have been computed for all orders up to the twelfth. Should they be needed, they can be constructed by applying the rules developed in section 3.3 to the entries of the basic Tables I. and IV.

The possible and/or probable errors in the derivatives calculated from the "mixed" type formulæ vary with the argument. A discussion of them is desirable, and is in hand.

2. *The Tables and the Method of Use.*

2.1. *Notation.*—The usual † notations for forward, backward, and central differences, as exhibited in the following schemes, will be used;

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* Formulæ starting diagonally, and finishing horizontally, will also exist. They are less convenient and more laborious in use, and will not be considered here.

† These differ from astronomical usage (Comrie (2)); there forward, backward, and central differences are not distinguished by different symbols, $\triangle$ being used for central differences, while mean differences are identifiable by difference of parity between index and doubled suffix.

the actual numbers in the same relative positions will, of course, be identical.

x.	Forward.			Backward.			Central.		
a	y_0			y_0			y_0		
		Δy_0			∇y_1			$\delta y_{\frac{1}{2}}$	
$a+w$	y_1		$\Delta^2 y_0$	y_1		$\nabla^2 y_2$	y_1		$\delta^2 y_1$
		Δy_1			∇y_2			$\delta y_{1\frac{1}{2}}$	
$a+2w$	y_2		$\Delta^2 y_1$	y_2		$\nabla^2 y_3$	y_2		$\delta^2 y_2$
		Δy_2			∇y_3			$\delta y_{2\frac{1}{2}}$	
$a+3w$	y_3		$\Delta^2 y_2$	y_3		$\nabla^2 y_4$	y_3		$\delta^2 y_3$
		Δy_3			∇y_4			$\delta y_{3\frac{1}{2}}$	
$a+4w$	y_4			y_4			y_4		

along with $$\mu\delta^n y_m = \tfrac{1}{2}(\delta^n y_{m+\frac{1}{2}} + \delta^n y_{m-\frac{1}{2}}).$$

The tabular interval is denoted by w. The differential operator will be denoted by D, and we shall use the abbreviation

$$(\mathrm{D}^q y)_{x=a+mw} = \mathrm{D}^q y_m, \text{ or sometimes } \mathrm{D}^q_m.$$

We abbreviate $\delta^n y_m$ similarly to δ^n_m.

2.2. *Description of the Tables.*—Table I. (p. 11) gives the Gregory–Newton coefficients $\mathrm{N}_{q,n}$ in the formulæ

$$w^q \mathrm{D}^q y_m = \begin{cases} \sum\limits_{n=q}^{\infty} (-)^{n-q} \mathrm{N}_{q,n} \Delta^n y_m \\ \sum\limits_{n=q}^{\infty} \mathrm{N}_{q,n} \nabla^n y_m \end{cases}$$

for $q=0(1)12$ and $n \not> 12$.

Table II. (p. 12) gives the corresponding central difference coefficients $\mathrm{C}_{q,n}$ in the formulæ

$$w^q \mathrm{D}^q y_m = \begin{cases} \sum\limits_{r=0}^{\infty} (-)^r \mathrm{C}_{q,q+2r} \delta^{q+2r} y_m & (q \text{ even}) \\ \sum\limits_{r=0}^{\infty} (-)^r \mathrm{C}_{q,q+2r} \mu\delta^{q+2r} y_m & (q \text{ odd}) \end{cases}$$

Tables III., 1–4 (facing p. 12), give the coefficients in the mixed type formulæ for the first four derivatives at the tabulated values of the argument. Their arrangement is such that the coefficient to be used as a multiplier occupies the same relative position as the difference which it is to multiply. The "route" is indicated by the arrows; central difference coefficients are given in bolder type; the central difference coefficient (always unity) on the left of the table for D^q multiplies $\delta^q y_0$ if q is even and $\mu\delta^q y_0$ if q is odd. In order further to clarify the mode of usage of the tables, illustrative examples are worked out in detail (see pp. 4–6).

Tables IV. (p. 13) and V. (p. 14), for arguments at mid-interval, correspond to Tables I. and II., and give $N'_{q,n}$ and $C'_{q,n}$ for the formulæ

$$w^q D^q y_{m-\frac{1}{2}} = \begin{cases} \sum\limits_{n=q}^{\infty} (-)^{n-q} N'_{q,n} \Delta^n y_m \\ \sum\limits_{n=q}^{\infty} N'_{q,n} \nabla^n y_{m-1.} \\ \sum\limits_{r=0}^{\infty} (-)^r C'_{q,q+2r} \mu\delta^{q+2r} y_{m-\frac{1}{2}} \quad (q \text{ even}) \\ \sum\limits_{r=0}^{\infty} (-)^r C'_{q,q+2r} \delta^{q+2r} y_{m-\frac{1}{2}} \quad (q \text{ odd}) \end{cases}$$

Tables VI., 0–3 (facing p. 14), give the coefficients in the mixed type formulæ for the function and its first three derivatives at mid-interval.

2.3. *Numerical Examples.*—As material we employ values of the function cos x (x in radians) at interval 0·1, to ten decimals. The values and differences are given on p. 5.

We will use these differences, and the coefficients from Tables III. 1, III. 2, and VI. 1, to calculate y' (0·2), y'' (0·2), and y' (0·25) respectively.

(i) y' (0·2)

Coefficient (Table III., 1)	×	difference (p. 5)	=	term in result
+1		$\mu\delta_2 =$		−1983 38381
−1/6		$\mu\delta_2^3 = 19\ 81732$		− 3 30288 7
+1/30		$\Delta_0^5 = \delta_{2\frac{1}{2}}^5 = -\ 24686$		− 822 9
−1/60		$\Delta_0^6 = \delta_3^6 = -\ 9533$		+ 158 9
+1/105		$\Delta_0^7 = \delta_{3\frac{1}{2}}^7 = +\ 347$		+ 3 3
−1/168		$\Delta_0^8 = \delta_4^8 = +\ 81$		− 5
				−1986 69330 9

Since $w=0{\cdot}1$, this means that

$$y'\ (0{\cdot}2) = -0{\cdot}19866\ 9331.$$

The exact value is −sin 0·2=−0·19866 93307 95 . . .

(ii) y'' (0·2)

Coefficient (Table III., 2)	×	difference (p. 5)	=	term in result
+ 1		$\delta_2^2 =$		−979 25012
− 1/12		$\delta_2^4 = +9\ 78432$		− 81536 0
+ 1/90		$\Delta_0^6 = \delta_3^6 = -\ 9533$		− 105 9
− 1/90		$\Delta_0^7 = \delta_{3\frac{1}{2}}^7 = +\ 347$		− 3 9
+47/5040		$\Delta_0^8 = \delta_4^8 = +\ 81$		+ 8
−19/2520		$\Delta_0^9 = \delta_{4\frac{1}{2}}^9 = +\ 25$		− 2
				−980 06657 2

x	$\cos x$	δ	δ^2	δ^3	δ^4	δ^5	δ^6	δ^7	δ^8	δ^9
0·0	1·00000 00000									
		− 499 58347								
0·1	0·99500 41653		−994 17528							
		−1493 75875		+14 92516						
0·2	·98006 65778	**−1983 38381**	−979 25012	**+19 81732**	+9 78432					
		−2473 00887		+24 70948		−24686				
0·3	·95533 64891		−954 54064		+9 53746		−9533			
		−3427 54951		+34 24694		−34219		+347		
0·4	·92106 09940		−920 29370		+9 19527		−9186		+ 81	
		−4347 84321		+43 44221		−43405		+428		+25
0·5	·87758 25619		−876 85149		+8 76122		−8758		+106	
		−5224 69470		+52 20343		−52163		+534		−59
0·6	·82533 56149		−824 64806		+8 23959		−8224		+ 47	
		−6049 34276		+60 44302		−60387		+581		
0·7	·76484 21873		−764 20504		+7 63572		−7643			
		−6813 54780		+68 07874		−68030				
0·8	·69670 67093		−696 12630		+6 95542					
		−7509 67410		+75 03416						
0·9	·62160 99683		−621 09214							
		−8130 76624								
1·0	·54030 23059									

The terms in bold type are mean central differences.

Since $w^2=0{\cdot}01$, this means that

$$y''\,(0{\cdot}2)=-0{\cdot}98006\ 657.$$

This may be compared with the true value,

$$-\cos 0{\cdot}2=-0{\cdot}98006\ 65778\ldots$$

(iii) $y'\,(0{\cdot}25)$

Coefficient (Table VI., 1)	×	difference (p. 5)	=	term in result
$+1$		$\delta_{2\frac{1}{2}}=$		$-2473\ 00887$
$-1/24$		$\delta^3_{2\frac{1}{2}}=+24\ 70948$		$-\quad 1\ 02956\ 2$
$+3/640$		$\delta^5_{2\frac{1}{2}}=-\quad 24686$		$-\quad 115\ 7$
$-5/7168$		$\Delta^7_0=\delta^7_{3\frac{1}{2}}=+\quad 347$		$-\quad 2$
$+5/7168$		$\Delta^8_0=\delta^8_4=+\quad 81$		$+\quad 1$
				$-2474\ 03959\ 0$

Thus $$y'\,(0{\cdot}25)=-0{\cdot}24740\ 3959,$$

while $$-\sin 0{\cdot}25=-0{\cdot}24740\ 39592\ 5\ldots$$

Scrutiny of (i) and (ii) will show how and why decimals, and hence significant figures, are progressively lost as we reduce the interval or as we calculate higher derivatives, whenever the tabular interval is less than unity.

3. *Derivation of Formulæ and Calculation of Coefficients.*

3.1. *Gregory–Newton Coefficients.*—Denoting as usual by E the operation of adding w to the argument, or its equivalent of increasing any suffix by unity,

$$y_1=\mathrm{E}y_0=(1+\Delta)y_0=e^{w\mathrm{D}}y_0$$

where the last expression is the symbolic form of Taylor's series. Thus

$$\mathrm{E}=(1+\Delta)=e^{w\mathrm{D}}$$

so that $$w\mathrm{D}=\log\,(1+\Delta)$$

$$=\Delta-\tfrac{1}{2}\Delta^2+\tfrac{1}{3}\Delta^3-\tfrac{1}{4}\Delta^4+\ldots$$

Consequently

$$w^q\mathrm{D}^q y_m=(\Delta-\tfrac{1}{2}\Delta^2+\tfrac{1}{3}\Delta^3-\tfrac{1}{4}\Delta^4+\ldots)^q y_m$$

$$=\sum_{n=q}^{\infty}(-)^{n-q}\mathrm{N}_{q,\,n}\Delta^n y_m$$

Similarly $$\mathrm{E}^{-1}=(1-\nabla)=e^{-w\mathrm{D}}$$

and $$w\mathrm{D}=-\log\,(1-\nabla)$$

$$=\nabla+\tfrac{1}{2}\nabla^2+\tfrac{1}{3}\nabla^3+\tfrac{1}{4}\nabla^4+\ldots$$

so that
$$w^q \mathrm{D}^q y_m = (\nabla + \tfrac{1}{2}\nabla^2 + \tfrac{1}{3}\nabla^3 + \tfrac{1}{4}\nabla^4 + \ldots)^q y_m$$
$$= \sum_{n=q}^{\infty} \mathrm{N}_{q,n} \nabla^n y_m$$

The coefficients $\mathrm{N}_{q,n}$ can thus be calculated by involution of the logarithmic series. Other—and less laborious—methods exist. They depend on the Gregory–Newton interpolation formula
$$y(a+rw) = y_r = \Sigma \frac{r(r-1)(r-2)\ldots(r-n+1)}{n!} \Delta^n y_0$$

Differentiating q times with respect to r gives
$$w^q \mathrm{D}^q y_r = \Sigma \frac{d^q\{r(r-1)(r-2)\ldots(r-n+1)\}/dr^q}{n!} \Delta^n y_0$$

so that
$$n!\,\mathrm{N}_{q,n} = (-)^{n-q}[d^q\{r(r-1)(r-2)\ldots(r-n+1)\}/dr^q]_{r=0}$$
$$= [d^q\{r(r+1)(r+2)\ldots(r+n-1)\}/dr^q]_{r=0}$$

which is essentially Markoff's formula. The values of the derivatives when $r=0$ can evidently be found as simple multiples of the coefficients, and this leads to a second method of calculating $\mathrm{N}_{q,n}$.

From the last result we can also derive a third method. Thus
$$(n+1)!\,\mathrm{N}_{q,n+1} = [(d/dr)^q\{r(r+1)(r+2)\ldots(r+n)\}]_{r=0}$$
$$= [(r+n)(d/dr)^q\{r(r+1)\ldots(r+n-1)\}$$
$$+ q(d/dr)^{q-1}\{r(r+1)(r+2)\ldots(r+n-1)\}]_{r=0}$$

or, on reduction,
$$(n+1)\mathrm{N}_{q,n+1} = n\mathrm{N}_{q,n} + q\mathrm{N}_{q-1,n}$$

This recurrence formula, coupled with the fact that $\mathrm{N}_{q,q}=1$, enables Table I. to be built up column by column, and provides what is probably the quickest and least laborious way of doing so. Actually all three methods have been employed, and they check one another.

It has seemed worth while also to provide formulæ for mid-interval, since in numerical processes the time often comes when a reduction of the interval is imperative, and the simplest method of so doing is to halve it. The corresponding coefficients are obtained from formulæ very similar to those above.

Putting $r=s-\frac{1}{2}$, the Gregory–Newton formula becomes
$$y_{s-\frac{1}{2}} = \sum_{n=0}^{\infty} \frac{(s-\frac{1}{2})(s-\frac{3}{2})\ldots(s-n+\frac{1}{2})}{n!} \Delta^n y_0$$

whence we derive, as in the case of $\mathrm{N}_{q,n}$
$$n!\,\mathrm{N}'_{q,n} = [(d/ds)^q\{(s+\tfrac{1}{2})(s+\tfrac{3}{2})\ldots(s+n-\tfrac{1}{2})\}]_{s=0}$$

and
$$2(n+1)\mathrm{N}'_{q,n+1} = (2n+1)\mathrm{N}'_{q,n} + 2q\mathrm{N}'_{q-1,n}$$

This recurrence formula, coupled with $N'_{q,q}=1$, $N'_{q,0}=0$ $(q>0)$, enables Table IV. to be constructed. From the above it is also evident that the N' are multiples of the coefficients in the polynomial

$$(x+1)(x+3)\ldots(x+2n-1),$$

which provides a second method of calculating them. A third method depends on the fact that

$$w^qD^qy_{-\frac{1}{2}}=(\Delta-\tfrac{1}{2}\Delta^2+\tfrac{1}{3}\Delta^3-\ldots)^q(1+\Delta)^{-\frac{1}{2}}y_0$$

3.2. *Central Difference Formulæ.*—These have been given, up to the terms involving the twelfth difference, by Comrie (2). It will transpire in our next subsection that the coefficients automatically emerge in the process of calculating the coefficients for the "mixed" formulæ. To calculate them independently we have

$$\delta=E^{\frac{1}{2}}-E^{-\frac{1}{2}}=2\sinh\tfrac{1}{2}wD$$

so that

$$wD=2\sinh^{-1}(\delta/2)$$

Also

$$\mu=\tfrac{1}{2}(E^{\frac{1}{2}}+E^{-\frac{1}{2}})=\cosh\tfrac{1}{2}wD=\sqrt{(1+\tfrac{1}{4}\delta^2)}$$

Consequently

$$w^qD^q=\begin{array}{ll}\{2\sinh^{-1}(\delta/2)\}^q & \begin{cases}q\text{ even, interval points}\\ q\text{ odd, mid-interval}\end{cases}\\ \mu\{2\sinh^{-1}(\delta/2)\}^q(1+\tfrac{1}{4}\delta^2)^{-\frac{1}{2}} & \begin{cases}q\text{ odd, interval points}\\ q\text{ even, mid-interval}\end{cases}\end{array}$$

For further information and details concerning differentiation formulæ see Steffensen (4) §7, and §18.

3.3. *Mixed Formulæ.*—Although the general rule by which the coefficients in the "mixed" formulæ are derived is the same in all cases, its deduction will probably be more easily followed if we analyse the particular case of the first derivative.

We have

$$\begin{aligned}wDy_1&=(\Delta-\tfrac{1}{2}\Delta^2+\tfrac{1}{3}\Delta^3-\tfrac{1}{4}\Delta^4\ldots)y_1\\&=\Delta y_1-(\tfrac{1}{2}\Delta^2-\tfrac{1}{3}\Delta^3+\tfrac{1}{4}\Delta^4\ldots)(1+\Delta)y_0\\&=\Delta y_1-\tfrac{1}{2}\Delta^2y_0+\{(\tfrac{1}{3}-\tfrac{1}{2})\Delta^3-(\tfrac{1}{4}-\tfrac{1}{3})\Delta^4\ldots\}y_0\\&=\mu\delta y_1-\tfrac{1}{6}\Delta^3y_0+\tfrac{1}{12}\Delta^4y_0-\ldots\end{aligned}$$

since

$$\mu\delta y_1=\tfrac{1}{2}(\Delta y_1+\Delta y_0)=\Delta y_1-\tfrac{1}{2}\Delta^2y_0$$

Similarly

$$\begin{aligned}wDy_2&=\mu\delta y_2-(\tfrac{1}{6}\Delta^3-\tfrac{1}{12}\Delta^4+\tfrac{1}{20}\Delta^5\ldots)y_1\\&=\mu\delta y_2-\tfrac{1}{6}\Delta^3y_1+(\tfrac{1}{12}\Delta^4-\tfrac{1}{20}\Delta^5\ldots)(1+\Delta)y_0\\&=\mu\delta y_2-\tfrac{1}{6}\mu\delta^3y_1+\{(\tfrac{1}{12}-\tfrac{1}{20})\Delta^5-(\tfrac{1}{20}-\tfrac{1}{30})\Delta^6+\ldots\}y_0\end{aligned}$$

$$
\begin{array}{ccccc}
1 & & d_2 & & \\
 & c_1 & & d_3 & \\
1 & & c_2 & & d_4 \\
 & b_1 & & c_3 & \\
1 & & b_2 & & c_4 \\
 & a_1 & & b_3 & \\
 & & a_2 & & b_4 \\
 & & & a_3 & \\
 & & & & a_4
\end{array}
$$

The process can be indefinitely continued, for the law of formation of the coefficients is now clear. If the accompanying table represents the coefficients, with

$$a_n = (-)^n \mathrm{X}_{q,\, n+q}$$

then

$$b_n = a_{n-1} + a_n$$

$$c_n = b_{n-1} + b_n$$

and so on. The upward progress in any column ceases when the corresponding backward Gregory–Newton coefficient is reached. The reproduction of the known central difference coefficients, and later of the backward Gregory–Newton coefficients, provides a check (practically conclusive and complete) upon the accuracy of the calculations.

Modifications are still necessary to convert a table so constructed into the corresponding member of the set III. or VI. In the case of odd derivatives at tabular points, or of even derivatives at mid-interval—that is, where the central difference formula involves mean differences—the central line of coefficients in the original table is repeated three times, at interval $\frac{1}{2}w$, with $\delta_{-\frac{1}{2}}$, $\mu\delta_0$ and $\delta_{\frac{1}{2}}$ respectively in III., 1, 3, or with δ_0, $\mu\delta_{\frac{1}{2}}$ and δ_1 in VI., 0, 2. In the case of even derivatives at tabular points, or of odd derivatives at mid-interval, the original table has a central line of zeros, and the entries in the lines above and below this are equal. In Tables III. 2, 4, these entries are repeated three times, with δ_{-1}, δ_0, and δ_1 respectively (or with $\delta_{-\frac{1}{2}}$, $\delta_{\frac{1}{2}}$ and $\delta_{1\frac{1}{2}}$ in Tables VI. 1, 3), instead of twice, at interval w; the zeros are not printed in the tables.

The suffixes also need consideration. In Tables III. the sum of the outermost pair in any column is zero. In Tables VI., however, an asymmetry appears, owing to the fact that the tables relate to suffix $+\frac{1}{2}$; the sum of the outermost suffixes in any column is, in this case, unity.

It may also be mentioned that the number of 1's in the left-hand column *as computed* is equal to the order of the derivative, for the interval point schemes, but one more than this for mid-interval schemes.

To illustrate these points we reproduce a few columns of the tables for D^2y and D^3y as computed; these should be compared with the corresponding portions of Tables III. 2, and III. 3.

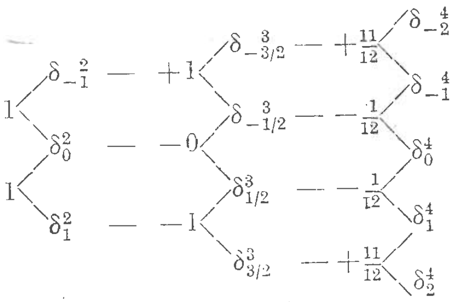
The tables could also have been exhibited in the form of "hexagon" diagrams (Fraser (3)), and portions corresponding to parts of the tables just mentioned are given, to exhibit this possibility. In the hexagon diagrams horizontal lines indicate addition or subtraction, sloping lines indicate multiplication, and any continuous route from left to right may be followed.

$D^2 y_0$

Original Table.

				137/180
			5/6	
		11/12		−13/180
	1		−1/12	
1		−1/12		1/90
	0		0	
1		−1/12		1/90
	−1		1/12	
		11/12		−13/180
			−5/6	
				137/180

Hexagon Diagram.



$D^3 y_0$

				29/15
			15/8	
		7/4		7/120
	3/2		1/8	
1		1/4		−1/15
	1/2		−1/8	
1		−1/4		7/120
	−1/2		1/8	
1		1/4		−1/15
	−3/2		−1/8	
		7/4		7/120
			−15/8	
				29/15

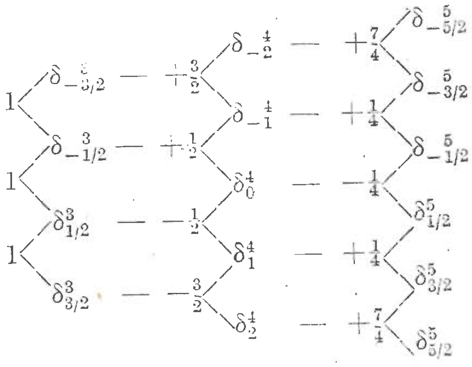


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TABLE I. $N_{q,n}$

q \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	q
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
1		1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$	1
2	$\frac{2547}{2548}$		1	$\frac{1}{1}$	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{137}{180}$	$\frac{7}{10}$	$\frac{363}{560}$	$\frac{761}{1260}$	$\frac{7129}{12600}$	$\frac{671}{1260}$	$\frac{83711}{1\ 66320}$	2
3	$\frac{2545}{2546}$			$\frac{1}{1}$	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{15}{8}$	$\frac{29}{15}$	$\frac{469}{240}$	$\frac{29531}{15120}$	$\frac{1303}{672}$	$\frac{16103}{8400}$	$\frac{1\ 90553}{1\ 00800}$	3
4					1	2	$\frac{17}{6}$	$\frac{7}{2}$	$\frac{967}{240}$	$\frac{89}{20}$	$\frac{4523}{945}$	$\frac{7645}{1512}$	$\frac{3\ 41747}{64800}$	4
5						1	$\frac{5}{2}$	$\frac{25}{6}$	$\frac{35}{6}$	$\frac{1069}{144}$	$\frac{285}{32}$	$\frac{31063}{3024}$	$\frac{1\ 39381}{12096}$	5
6							1	3	$\frac{23}{4}$	9	$\frac{3013}{240}$	$\frac{781}{48}$	$\frac{2\ 42537}{12096}$	6
7								1	$\frac{7}{2}$	$\frac{91}{12}$	$\frac{105}{8}$	$\frac{4781}{240}$	$\frac{13321}{480}$	7
8									1	4	$\frac{29}{3}$	$\frac{55}{3}$	$\frac{10831}{360}$	8
9										1	$\frac{9}{2}$	12	$\frac{99}{4}$	9
10											1	5	$\frac{175}{12}$	10
11												1	$\frac{11}{2}$	11
12													1	12

I have already

$$w^q D^q y_m = \sum_{n=q}^{\infty} (-)^{n-q} N_{q,n} \triangle^n y_m = \sum_{n=q}^{\infty} N_{q,n} \nabla^n y_m$$

$N_{q,q}=1 \qquad N_{0,n}=0 \quad (n>0) \qquad N_{1,n}=1/n \qquad (n+1)N_{q,n+1}=nN_{q,n}+qN_{q-1,n}$

TABLE II. $C_{q,n}$

q \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	q
0	1		0		0		0		0		0		0	0
1		1		$\frac{1}{6}$		$\frac{1}{30}$		$\frac{1}{140}$		$\frac{1}{630}$		$\frac{1}{2772}$		1
2			1		$\frac{1}{12}$		$\frac{1}{90}$		$\frac{1}{560}$		$\frac{1}{3150}$		$\frac{1}{16632}$	2
3				1		$\frac{1}{4}$		$\frac{7}{120}$		$\frac{41}{3024}$		$\frac{479}{1\ 51200}$		3
4					1		$\frac{1}{6}$		$\frac{7}{240}$		$\frac{41}{7560}$		$\frac{479}{4\ 53600}$	4
5						1		$\frac{1}{3}$		$\frac{13}{144}$		$\frac{139}{6048}$		5
6							1		$\frac{1}{4}$		$\frac{13}{240}$		$\frac{139}{12096}$	6
7								1		$\frac{5}{12}$		$\frac{31}{240}$		7
8									1		$\frac{1}{3}$		$\frac{31}{360}$	8
9										1		$\frac{1}{2}$		9
10											1		$\frac{5}{12}$	10
11												1		11
12													1	12

$$w^q D^q y_m = \begin{cases} \sum_{r=0}^{\infty} (-)^r C_{q,q+2r}\, \delta^{q+2r} y_m & (q \text{ even}) \\ \sum^{\infty} (-)^r C_{q,q+2r}\, \mu\delta^{q+2r} y_m & (q \text{ odd}) \end{cases}$$

2457

12a

TABLE III. 1. Dy_0

										$+\frac{1}{10}\delta_{-}^{1}$
									$+\frac{1}{9}\delta_{-4\frac{1}{2}}^{9}$	
								$+\frac{1}{8}\delta_{-4}^{8}$		$-\frac{1}{90}\delta_{-}^{1}$
							$+\frac{1}{7}\delta_{-3\frac{1}{2}}^{7}$		$-\frac{1}{72}\delta_{-3\frac{1}{2}}^{9}$	
						$+\frac{1}{6}\delta_{-3}^{6}$		$-\frac{1}{56}\delta_{-3}^{8}$		$+\frac{1}{360}\delta_{-}^{1}$
					$+\frac{1}{5}\delta_{-2\frac{1}{2}}^{5}$		$-\frac{1}{42}\delta_{-2\frac{1}{2}}^{7}$		$+\frac{1}{252}\delta_{-2\frac{1}{2}}^{9}$	
				$+\frac{1}{4}\delta_{-2}^{4}$		$-\frac{1}{30}\delta_{-2}^{6}$		$+\frac{1}{168}\delta_{-2}^{8}$		$-\frac{1}{840}\delta_{-}^{1}$
			$+\frac{1}{3}\delta_{-1\frac{1}{2}}^{3}$		$-\frac{1}{20}\delta_{-1\frac{1}{2}}^{5}$		$+\frac{1}{105}\delta_{-1\frac{1}{2}}^{7}$		$-\frac{1}{504}\delta_{-1\frac{1}{2}}^{9}$	
		$+\frac{1}{2}\delta_{-1}^{2}$		$-\frac{1}{12}\delta_{-1}^{4}$		$+\frac{1}{60}\delta_{-1}^{6}$		$-\frac{1}{280}\delta_{-1}^{8}$		$+\frac{1}{1260}\delta_{-}^{1}$
	$+1\,\delta_{-\frac{1}{2}}$		$-\frac{1}{6}\delta_{-\frac{1}{2}}^{3}$		$+\frac{1}{30}\delta_{-\frac{1}{2}}^{5}$		$-\frac{1}{140}\delta_{-\frac{1}{2}}^{7}$		$+\frac{1}{630}\delta_{-\frac{1}{2}}^{9}$	
→ 2457 $\rightarrow$	$+1\mu\delta_0$	$\rightarrow$	$-\frac{1}{6}\mu\delta_0^3$	$\rightarrow$	$+\frac{1}{30}\mu\delta_0^5$	$\rightarrow$	$-\frac{1}{140}\mu\delta_0^7$	$\rightarrow$	$+\frac{1}{630}\mu\delta_0^9$	$\rightarrow$
	$+1\,\delta_{\frac{1}{2}}$		$-\frac{1}{6}\delta_{\frac{1}{2}}^{3}$		$+\frac{1}{30}\delta_{\frac{1}{2}}^{5}$		$-\frac{1}{140}\delta_{\frac{1}{2}}^{7}$		$+\frac{1}{630}\delta_{\frac{1}{2}}^{9}$	
		$-\frac{1}{2}\delta_{1}^{2}$		$+\frac{1}{12}\delta_{1}^{4}$		$-\frac{1}{60}\delta_{1}^{6}$		$+\frac{1}{280}\delta_{1}^{8}$		$-\frac{1}{1260}\delta_{1}^{10}$
			$+\frac{1}{3}\delta_{1\frac{1}{2}}^{3}$		$-\frac{1}{20}\delta_{1\frac{1}{2}}^{5}$		$+\frac{1}{105}\delta_{1\frac{1}{2}}^{7}$		$-\frac{1}{504}\delta_{1\frac{1}{2}}^{9}$	
				$-\frac{1}{4}\delta_{2}^{4}$		$+\frac{1}{30}\delta_{2}^{6}$		$-\frac{1}{168}\delta_{2}^{8}$		$+\frac{1}{840}\delta_{2}^{10}$
					$+\frac{1}{5}\delta_{2\frac{1}{2}}^{5}$		$-\frac{1}{42}\delta_{2\frac{1}{2}}^{7}$		$+\frac{1}{252}\delta_{2\frac{1}{2}}^{9}$	
						$-\frac{1}{6}\delta_{3}^{6}$		$+\frac{1}{56}\delta_{3}^{8}$		$-\frac{1}{360}\delta_{3}^{10}$
							$+\frac{1}{7}\delta_{3\frac{1}{2}}^{7}$		$-\frac{1}{72}\delta_{3\frac{1}{2}}^{9}$	
								$-\frac{1}{8}\delta_{4}^{8}$		$+\frac{1}{90}\delta_{4}^{10}$
									$+\frac{1}{9}\delta_{4\frac{1}{2}}^{9}$	
										$-\frac{1}{10}\delta_{5}^{10}$

12b

[*To face* p. 12.

TABLE III. 1. Dy_0

										$+\frac{1}{12}\delta^{12}_{-6}$
									$+\frac{1}{11}\delta^{11}_{-5\frac12}$	
								$+\frac{1}{10}\delta^{10}_{-5}$		$-\frac{1}{132}\delta^{12}_{-5}$
							$+\frac{1}{9}\delta^{9}_{-4\frac12}$		$-\frac{1}{110}\delta^{11}_{-4\frac12}$	
						$+\frac{1}{8}\delta^{8}_{-4}$		$-\frac{1}{90}\delta^{10}_{-4}$		$+\frac{1}{660}\delta^{12}_{-4}$
					$+\frac{1}{7}\delta^{7}_{-3\frac12}$		$-\frac{1}{72}\delta^{9}_{-3\frac12}$		$+\frac{1}{495}\delta^{11}_{-3\frac12}$	
				$+\frac{1}{6}\delta^{6}_{-3}$		$-\frac{1}{56}\delta^{8}_{-3}$		$+\frac{1}{360}\delta^{10}_{-3}$		$-\frac{1}{1980}\delta^{12}_{-3}$
			$+\frac{1}{5}\delta^{5}_{-2\frac12}$		$-\frac{1}{42}\delta^{7}_{-2\frac12}$		$+\frac{1}{252}\delta^{9}_{-2\frac12}$		$-\frac{1}{1320}\delta^{11}_{-2\frac12}$	
		$+\frac{1}{4}\delta^{4}_{-2}$		$-\frac{1}{30}\delta^{6}_{-2}$		$+\frac{1}{168}\delta^{8}_{-2}$		$-\frac{1}{840}\delta^{10}_{-2}$		$+\frac{1}{3960}\delta^{12}_{-2}$
	$+\frac{1}{3}\delta^{3}_{-1\frac12}$		$-\frac{1}{20}\delta^{5}_{-1\frac12}$		$+\frac{1}{105}\delta^{7}_{-1\frac12}$		$-\frac{1}{504}\delta^{9}_{-1\frac12}$		$+\frac{1}{2310}\delta^{11}_{-1\frac12}$	
δ^{2}_{-1}		$-\frac{1}{12}\delta^{4}_{-1}$		$+\frac{1}{60}\delta^{6}_{-1}$		$-\frac{1}{280}\delta^{8}_{-1}$		$+\frac{1}{1260}\delta^{10}_{-1}$		$-\frac{1}{5544}\delta^{12}_{-1}$
	$-\frac{1}{6}\delta^{3}_{-\frac12}$		$+\frac{1}{30}\delta^{5}_{-\frac12}$		$-\frac{1}{140}\delta^{7}_{-\frac12}$		$+\frac{1}{630}\delta^{9}_{-\frac12}$		$-\frac{1}{2772}\delta^{11}_{-\frac12}$	
	$-\frac{1}{6}\mu\delta^{3}_{0}$		$+\frac{1}{30}\mu\delta^{5}_{0}$		$-\frac{1}{140}\mu\delta^{7}_{0}$		$+\frac{1}{630}\mu\delta^{9}_{0}$		$-\frac{1}{2772}\mu\delta^{11}_{0}$	
	$\delta^{3}_{\frac12}$		$+\frac{1}{30}\delta^{5}_{\frac12}$		$-\frac{1}{140}\delta^{7}_{\frac12}$		$+\frac{1}{630}\delta^{9}_{\frac12}$		$-\frac{1}{2772}\delta^{11}_{\frac12}$	
δ^{2}_{1}		$+\frac{1}{12}\delta^{4}_{1}$		$-\frac{1}{60}\delta^{6}_{1}$		$+\frac{1}{280}\delta^{8}_{1}$		$-\frac{1}{1260}\delta^{10}_{1}$		$+\frac{1}{5544}\delta^{12}_{1}$
	$+\frac{1}{3}\delta^{3}_{1\frac12}$		$-\frac{1}{20}\delta^{5}_{1\frac12}$		$+\frac{1}{105}\delta^{7}_{1\frac12}$		$-\frac{1}{504}\delta^{9}_{1\frac12}$		$+\frac{1}{2310}\delta^{11}_{1\frac12}$	
		$-\frac{1}{4}\delta^{4}_{2}$		$+\frac{1}{30}\delta^{6}_{2}$		$-\frac{1}{168}\delta^{8}_{2}$		$+\frac{1}{840}\delta^{10}_{2}$		$-\frac{1}{3960}\delta^{12}_{2}$
			$+\frac{1}{5}\delta^{5}_{2\frac12}$		$-\frac{1}{42}\delta^{7}_{2\frac12}$		$+\frac{1}{252}\delta^{9}_{2\frac12}$		$-\frac{1}{1320}\delta^{11}_{2\frac12}$	
				$-\frac{1}{6}\delta^{6}_{3}$		$+\frac{1}{56}\delta^{8}_{3}$		$-\frac{1}{360}\delta^{10}_{3}$		$+\frac{1}{1980}\delta^{12}_{3}$
					$+\frac{1}{7}\delta^{7}_{3\frac12}$		$-\frac{1}{72}\delta^{9}_{3\frac12}$		$+\frac{1}{495}\delta^{11}_{3\frac12}$	
						$-\frac{1}{8}\delta^{8}_{4}$		$+\frac{1}{90}\delta^{10}_{4}$		$-\frac{1}{660}\delta^{12}_{4}$
							$+\frac{1}{9}\delta^{9}_{4\frac12}$		$-\frac{1}{110}\delta^{11}_{4\frac12}$	
								$-\frac{1}{10}\delta^{10}_{5}$		$+\frac{1}{132}\delta^{12}_{5}$
									$+\frac{1}{11}\delta^{11}_{5\frac12}$	
										$-\frac{1}{12}\delta^{12}_{6}$

12c

TABLE III. 2. D^2y_0

								$+\frac{7129}{12600}\delta^{10}_{-5}$
							$+\frac{761}{1260}\delta^{9}_{-4\frac{1}{2}}$	
						$+\frac{363}{560}\delta^{8}_{-4}$		$-\frac{481}{12600}\delta^{10}_{-4}$
					$+\frac{7}{10}\delta^{7}_{-3\frac{1}{2}}$		$-\frac{223}{5040}\delta^{9}_{-3\frac{1}{2}}$	
				$+\frac{137}{180}\delta^{6}_{-3}$		$-\frac{29}{560}\delta^{8}_{-3}$		$+\frac{17}{2800}\delta^{10}_{-3}$
			$+\frac{5}{6}\delta^{5}_{-2\frac{1}{2}}$		$-\frac{11}{180}\delta^{7}_{-2\frac{1}{2}}$		$+\frac{19}{2520}\delta^{9}_{-2\frac{1}{2}}$	
		$+\frac{11}{12}\delta^{4}_{-2}$		$-\frac{13}{180}\delta^{6}_{-2}$		$+\frac{47}{5040}\delta^{8}_{-2}$		$-\frac{37}{25200}\delta^{10}_{-2}$
	$+1\,\delta^{3}_{-1\frac{1}{2}}$		$-\frac{1}{12}\delta^{5}_{-1\frac{1}{2}}$		$+\frac{1}{90}\delta^{7}_{-1\frac{1}{2}}$		$-\frac{1}{560}\delta^{9}_{-1\frac{1}{2}}$	
$+1\,\delta^{2}_{-1}$		$-\frac{1}{12}\delta^{4}_{-1}$		$+\frac{1}{90}\delta^{6}_{-1}$		$-\frac{1}{560}\delta^{8}_{-1}$		$+\frac{1}{3150}\delta^{10}_{-1}$
$+\frac{1}{1}\delta^{2}_{0}$		$-\frac{1}{12}\delta^{4}_{0}$		$+\frac{1}{90}\delta^{6}_{0}$		$-\frac{1}{560}\delta^{8}_{0}$		$+\frac{1}{3150}\delta^{10}_{0}$
$+1\,\delta^{2}_{1}$		$-\frac{1}{12}\delta^{4}_{1}$		$+\frac{1}{90}\delta^{6}_{1}$		$-\frac{1}{560}\delta^{8}_{1}$		$+\frac{1}{3150}\delta^{10}_{1}$
	$-1\,\delta^{3}_{1\frac{1}{2}}$		$+\frac{1}{12}\delta^{5}_{1\frac{1}{2}}$		$-\frac{1}{90}\delta^{7}_{1\frac{1}{2}}$		$+\frac{1}{560}\delta^{9}_{1\frac{1}{2}}$	
		$+\frac{11}{12}\delta^{4}_{2}$		$-\frac{13}{180}\delta^{6}_{2}$		$+\frac{47}{5040}\delta^{8}_{2}$		$-\frac{37}{25200}\delta^{10}_{2}$
			$-\frac{5}{6}\delta^{5}_{2\frac{1}{2}}$		$+\frac{11}{180}\delta^{7}_{2\frac{1}{2}}$		$-\frac{19}{2520}\delta^{9}_{2\frac{1}{2}}$	
				$+\frac{137}{180}\delta^{6}_{3}$		$-\frac{29}{560}\delta^{8}_{3}$		$+\frac{17}{2800}\delta^{10}_{3}$
					$-\frac{7}{10}\delta^{7}_{3\frac{1}{2}}$		$+\frac{223}{5040}\delta^{9}_{3\frac{1}{2}}$	
						$+\frac{363}{560}\delta^{8}_{4}$		$-\frac{481}{12600}\delta^{10}_{4}$
							$-\frac{761}{1260}\delta^{9}_{4\frac{1}{2}}$	
								$+\frac{7129}{12600}\delta^{10}_{5}$

I have

~~2544~~

~~1, 12, 90, 560, etc~~

~~– the denominators~~

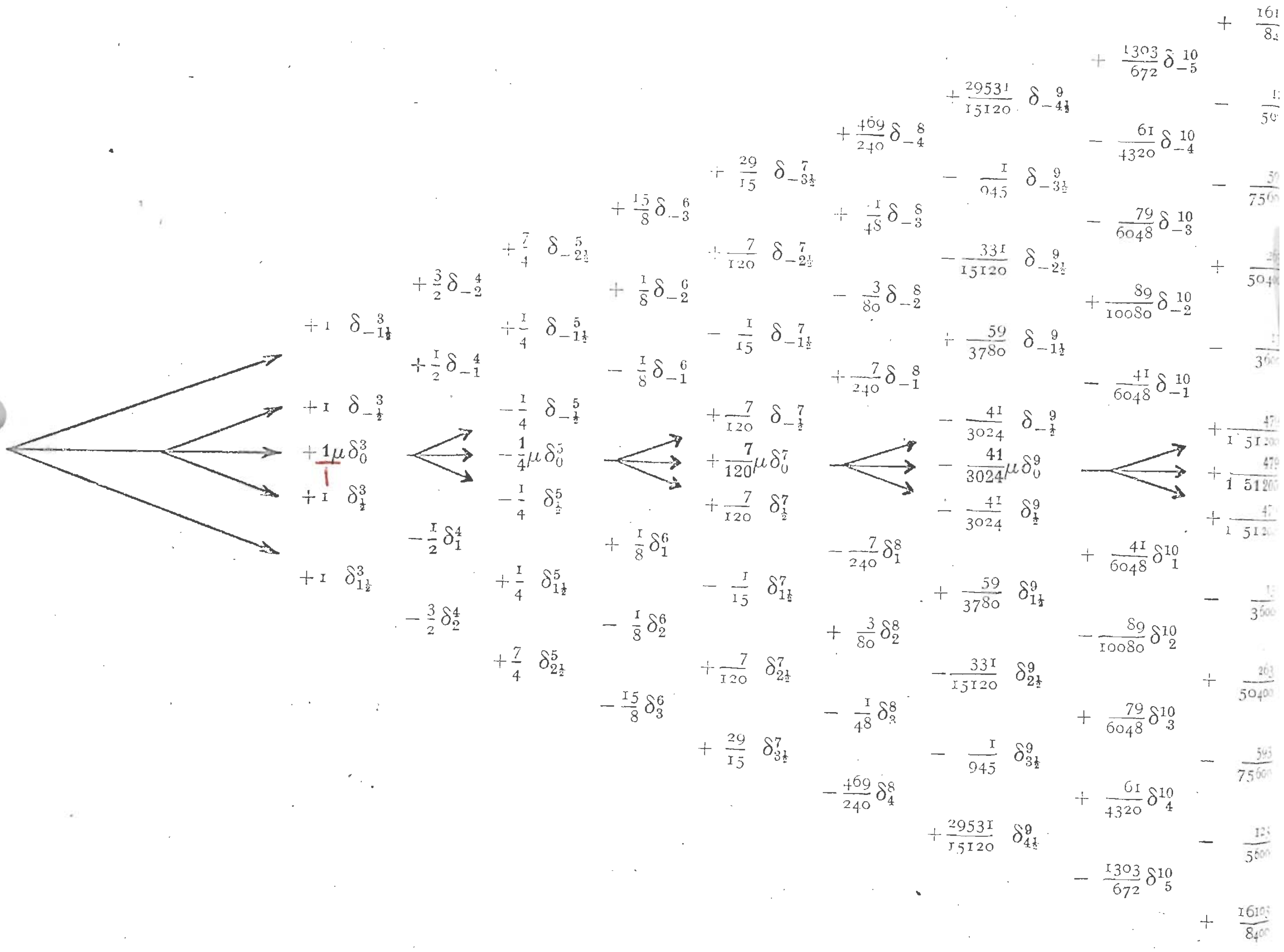
12d

TABLE III. 2. D^2y_0

									$+\frac{83711}{1\,66320}\delta^{12}_{-6}$
								$+\frac{671}{1260}\delta^{11}_{-5\frac12}$	
							$+\frac{7129}{12600}\delta^{10}_{-5}$		$-\frac{4861}{1\,66320}\delta^{12}_{-5}$
						$+\frac{761}{1260}\delta^{9}_{-4\frac12}$		$-\frac{419}{12600}\delta^{11}_{-4\frac12}$	
					$+\frac{363}{560}\delta^{8}_{-4}$		$-\frac{481}{12600}\delta^{10}_{-4}$		$+\frac{3349}{8\,31600}\delta^{12}_{-4}$
				$+\frac{7}{10}\delta^{7}_{-3\frac12}$		$-\frac{223}{5040}\delta^{9}_{-3\frac12}$		$+\frac{31}{6300}\delta^{11}_{-3\frac12}$	
			$+\frac{137}{180}\delta^{6}_{-3}$		$-\frac{29}{560}\delta^{8}_{-3}$		$+\frac{17}{2800}\delta^{10}_{-3}$		$-\frac{743}{8\,31600}\delta^{12}_{-3}$
		$+\frac{5}{6}\delta^{5}_{-2\frac12}$		$-\frac{11}{180}\delta^{7}_{-2\frac12}$		$+\frac{19}{2520}\delta^{9}_{-2\frac12}$		$-\frac{29}{25200}\delta^{11}_{-2\frac12}$	
	$+\frac{11}{12}\delta^{4}_{-2}$		$-\frac{13}{180}\delta^{6}_{-2}$		$+\frac{47}{5040}\delta^{8}_{-2}$		$-\frac{37}{25200}\delta^{10}_{-2}$		$+\frac{107}{4\,15800}\delta^{12}_{-2}$
$+1\delta^{3}_{-1\frac12}$		$-\frac{1}{12}\delta^{5}_{-1\frac12}$		$+\frac{1}{90}\delta^{7}_{-1\frac12}$		$-\frac{1}{560}\delta^{9}_{-1\frac12}$		$+\frac{1}{3150}\delta^{11}_{-1\frac12}$	
	$-\frac{1}{12}\delta^{4}_{-1}$		$+\frac{1}{90}\delta^{6}_{-1}$		$-\frac{1}{560}\delta^{8}_{-1}$		$+\frac{1}{3150}\delta^{10}_{-1}$		$-\frac{1}{16632}\delta^{12}_{-1}$
	$-\frac{1}{12}\delta^{4}_{0}$		$+\frac{1}{90}\delta^{6}_{0}$		$-\frac{1}{560}\delta^{8}_{0}$		$+\frac{1}{3150}\delta^{10}_{0}$		$-\frac{1}{16632}\delta^{12}_{0}$
	$-\frac{1}{12}\delta^{4}_{1}$		$+\frac{1}{90}\delta^{6}_{1}$		$-\frac{1}{560}\delta^{8}_{1}$		$+\frac{1}{3150}\delta^{10}_{1}$		$-\frac{1}{16632}\delta^{12}_{1}$
$-1\delta^{3}_{1\frac12}$		$+\frac{1}{12}\delta^{5}_{1\frac12}$		$-\frac{1}{90}\delta^{7}_{1\frac12}$		$+\frac{1}{560}\delta^{9}_{1\frac12}$		$-\frac{1}{3150}\delta^{11}_{1\frac12}$	
	$+\frac{11}{12}\delta^{4}_{2}$		$-\frac{13}{180}\delta^{6}_{2}$		$+\frac{47}{5040}\delta^{8}_{2}$		$-\frac{37}{25200}\delta^{10}_{2}$		$+\frac{107}{4\,15800}\delta^{12}_{2}$
		$-\frac{5}{6}\delta^{5}_{2\frac12}$		$+\frac{11}{180}\delta^{7}_{2\frac12}$		$-\frac{19}{2520}\delta^{9}_{2\frac12}$		$+\frac{29}{25200}\delta^{11}_{2\frac12}$	
			$+\frac{137}{180}\delta^{6}_{3}$		$-\frac{29}{560}\delta^{8}_{3}$		$+\frac{17}{2800}\delta^{10}_{3}$		$-\frac{743}{8\,31600}\delta^{12}_{3}$
				$-\frac{7}{10}\delta^{7}_{3\frac12}$		$+\frac{223}{5040}\delta^{9}_{3\frac12}$		$-\frac{31}{6300}\delta^{11}_{3\frac12}$	
					$+\frac{363}{500}\delta^{8}_{4}$		$-\frac{481}{12000}\delta^{10}_{4}$		$+\frac{3349}{8\,31600}\delta^{12}_{4}$
						$-\frac{761}{1260}\delta^{9}_{4\frac12}$		$+\frac{419}{12600}\delta^{11}_{4\frac12}$	
							$+\frac{7129}{12600}\delta^{10}_{5}$		$-\frac{4861}{1\,66320}\delta^{12}_{5}$
								$-\frac{671}{1260}\delta^{11}_{5\frac12}$	
									$+\frac{83711}{1\,66320}\delta^{12}_{6}$

12e

TABLE III. 3. D^3y_0



$+1\,\delta^{3}_{-1\frac{1}{2}}$
$+1\,\delta^{3}_{-\frac{1}{2}}$
$+1\mu\delta^{3}_{0}$
$+1\,\delta^{3}_{\frac{1}{2}}$
$+1\,\delta^{3}_{1\frac{1}{2}}$

$+\frac{3}{2}\delta^{4}_{-2}$
$+\frac{1}{2}\delta^{4}_{-1}$
$-\frac{1}{2}\delta^{4}_{1}$
$-\frac{3}{2}\delta^{4}_{2}$

$+\frac{7}{4}\delta^{5}_{-2\frac{1}{2}}$
$+\frac{1}{4}\delta^{5}_{-1\frac{1}{2}}$
$-\frac{1}{4}\delta^{5}_{-\frac{1}{2}}$
$-\frac{1}{4}\mu\delta^{5}_{0}$
$-\frac{1}{4}\delta^{5}_{\frac{1}{2}}$
$+\frac{1}{4}\delta^{5}_{1\frac{1}{2}}$
$+\frac{7}{4}\delta^{5}_{2\frac{1}{2}}$

$+\frac{15}{8}\delta^{6}_{-3}$
$+\frac{1}{8}\delta^{6}_{-2}$
$-\frac{1}{8}\delta^{6}_{-1}$
$+\frac{1}{8}\delta^{6}_{1}$
$-\frac{1}{8}\delta^{6}_{2}$
$-\frac{15}{8}\delta^{6}_{3}$

$+\frac{29}{15}\delta^{7}_{-3\frac{1}{2}}$
$+\frac{7}{120}\delta^{7}_{-2\frac{1}{2}}$
$-\frac{1}{15}\delta^{7}_{-1\frac{1}{2}}$
$+\frac{7}{120}\delta^{7}_{-\frac{1}{2}}$
$+\frac{7}{120}\mu\delta^{7}_{0}$
$+\frac{7}{120}\delta^{7}_{\frac{1}{2}}$
$-\frac{1}{15}\delta^{7}_{1\frac{1}{2}}$
$+\frac{7}{120}\delta^{7}_{2\frac{1}{2}}$
$+\frac{29}{15}\delta^{7}_{3\frac{1}{2}}$

$+\frac{469}{240}\delta^{8}_{-4}$
$+\frac{1}{48}\delta^{8}_{-3}$
$-\frac{3}{80}\delta^{8}_{-2}$
$+\frac{7}{240}\delta^{8}_{-1}$
$-\frac{7}{240}\delta^{8}_{1}$
$+\frac{3}{80}\delta^{8}_{2}$
$-\frac{1}{48}\delta^{8}_{3}$
$-\frac{469}{240}\delta^{8}_{4}$

$+\frac{29531}{15120}\delta^{9}_{-4\frac{1}{2}}$
$-\frac{1}{945}\delta^{9}_{-3\frac{1}{2}}$
$-\frac{331}{15120}\delta^{9}_{-2\frac{1}{2}}$
$+\frac{59}{3780}\delta^{9}_{-1\frac{1}{2}}$
$-\frac{41}{3024}\delta^{9}_{-\frac{1}{2}}$
$-\frac{41}{3024}\mu\delta^{9}_{0}$
$-\frac{41}{3024}\delta^{9}_{\frac{1}{2}}$
$+\frac{59}{3780}\delta^{9}_{1\frac{1}{2}}$
$-\frac{331}{15120}\delta^{9}_{2\frac{1}{2}}$
$-\frac{1}{945}\delta^{9}_{3\frac{1}{2}}$
$+\frac{29531}{15120}\delta^{9}_{4\frac{1}{2}}$

$+\frac{1303}{672}\delta^{10}_{-5}$
$-\frac{61}{4320}\delta^{10}_{-4}$
$-\frac{79}{6048}\delta^{10}_{-3}$
$+\frac{89}{10080}\delta^{10}_{-2}$
$-\frac{41}{6048}\delta^{10}_{-1}$
$+\frac{41}{6048}\delta^{10}_{1}$
$-\frac{89}{10080}\delta^{10}_{2}$
$+\frac{79}{6048}\delta^{10}_{3}$
$+\frac{61}{4320}\delta^{10}_{4}$
$-\frac{1303}{672}\delta^{10}_{5}$

Table III. 3. D^3y_0

									$+\frac{1\,90553}{1\,00800}\delta^{12}_{-6}$
								$+\frac{16103}{8400}\delta^{11}_{-5\frac12}$	
							$+\frac{1303}{672}\delta^{10}_{-5}$		$-\frac{2683}{1\,00800}\delta^{12}_{-5}$
						$+\frac{29531}{15120}\delta^{9}_{-4\frac12}$		$-\frac{123}{5600}\delta^{11}_{-4\frac12}$	
					$+\frac{469}{240}\delta^{8}_{-4}$		$-\frac{61}{4320}\delta^{10}_{-4}$		$-\frac{67}{14400}\delta^{12}_{-4}$
				$+\frac{29}{15}\delta^{7}_{-3\frac12}$		$-\frac{1}{945}\delta^{9}_{-3\frac12}$		$-\frac{593}{75600}\delta^{11}_{-3\frac12}$	
			$+\frac{15}{8}\delta^{6}_{-3}$		$+\frac{1}{48}\delta^{8}_{-3}$		$-\frac{79}{6048}\delta^{10}_{-3}$		$+\frac{193}{60480}\delta^{12}_{-3}$
		$+\frac{7}{4}\delta^{5}_{-2\frac12}$		$+\frac{7}{120}\delta^{7}_{-2\frac12}$		$-\frac{331}{15120}\delta^{9}_{-2\frac12}$		$+\frac{263}{50400}\delta^{11}_{-2\frac12}$	
	$+\frac{3}{2}\delta^{4}_{-2}$		$+\frac{1}{8}\delta^{6}_{-2}$		$-\frac{3}{80}\delta^{8}_{-2}$		$+\frac{89}{10080}\delta^{10}_{-2}$		$-\frac{613}{3\,02400}\delta^{12}_{-2}$
$\delta^{3}_{-1\frac12}$		$+\frac{1}{4}\delta^{5}_{-1\frac12}$		$-\frac{1}{15}\delta^{7}_{-1\frac12}$		$+\frac{59}{3780}\delta^{9}_{-1\frac12}$		$-\frac{13}{3600}\delta^{11}_{-1\frac12}$	
	$+\frac{1}{2}\delta^{4}_{-1}$		$-\frac{1}{8}\delta^{6}_{-1}$		$+\frac{7}{240}\delta^{8}_{-1}$		$-\frac{41}{6048}\delta^{10}_{-1}$		$+\frac{479}{3\,02400}\delta^{12}_{-1}$
$\delta^{3}_{-\frac12}$		$-\frac{1}{4}\delta^{5}_{-\frac12}$		$+\frac{7}{120}\delta^{7}_{-\frac12}$		$-\frac{41}{3024}\delta^{9}_{-\frac12}$		$+\frac{479}{1\,51200}\delta^{11}_{-\frac12}$	
δ^{3}_{0}	→	$-\frac{1}{4}\mu\delta^{5}_{0}$	→	$+\frac{7}{120}\mu\delta^{7}_{0}$	→	$-\frac{41}{3024}\mu\delta^{9}_{0}$	→	$+\frac{479}{1\,51200}\mu\delta^{11}_{0}$	
$\delta^{3}_{\frac12}$		$-\frac{1}{4}\delta^{5}_{\frac12}$		$+\frac{7}{120}\delta^{7}_{\frac12}$		$-\frac{41}{3024}\delta^{9}_{\frac12}$		$+\frac{479}{1\,51200}\delta^{11}_{\frac12}$	
	$-\frac{1}{2}\delta^{4}_{1}$		$+\frac{1}{8}\delta^{6}_{1}$		$-\frac{7}{240}\delta^{8}_{1}$		$+\frac{41}{6048}\delta^{10}_{1}$		$-\frac{479}{3\,02400}\delta^{12}_{1}$
$\delta^{3}_{1\frac12}$		$+\frac{1}{4}\delta^{5}_{1\frac12}$		$-\frac{1}{15}\delta^{7}_{1\frac12}$		$+\frac{59}{3780}\delta^{9}_{1\frac12}$		$-\frac{13}{3600}\delta^{11}_{1\frac12}$	
	$-\frac{3}{2}\delta^{4}_{2}$		$-\frac{1}{8}\delta^{6}_{2}$		$+\frac{3}{80}\delta^{8}_{2}$		$-\frac{89}{10080}\delta^{10}_{2}$		$+\frac{613}{3\,02400}\delta^{12}_{2}$
		$+\frac{7}{4}\delta^{5}_{2\frac12}$		$+\frac{7}{120}\delta^{7}_{2\frac12}$		$-\frac{331}{15120}\delta^{9}_{2\frac12}$		$+\frac{263}{50400}\delta^{11}_{2\frac12}$	
			$-\frac{15}{8}\delta^{6}_{3}$		$-\frac{1}{48}\delta^{8}_{3}$		$+\frac{79}{6048}\delta^{10}_{3}$		$-\frac{193}{60480}\delta^{12}_{3}$
				$+\frac{29}{15}\delta^{7}_{3\frac12}$		$-\frac{1}{945}\delta^{9}_{3\frac12}$		$-\frac{593}{75600}\delta^{11}_{3\frac12}$	
					$-\frac{469}{240}\delta^{8}_{4}$		$+\frac{61}{4320}\delta^{10}_{4}$		$+\frac{67}{14400}\delta^{12}_{4}$
						$+\frac{29531}{15120}\delta^{9}_{4\frac12}$		$-\frac{123}{5600}\delta^{11}_{4\frac12}$	
							$-\frac{1303}{672}\delta^{10}_{5}$		$+\frac{2683}{1\,00800}\delta^{12}_{5}$
								$+\frac{16103}{8400}\delta^{11}_{5\frac12}$	
									$-\frac{1\,90553}{1\,00800}\delta^{12}_{6}$

TABLE III. 4. D^4y_0

						$+\frac{4523}{945}\delta^{10}_{-5}$
					$+\frac{89}{20}\delta^{9}_{-4\frac{1}{2}}$	
				$+\frac{967}{240}\delta^{8}_{-4}$		$+\frac{1271}{3780}\delta^{10}_{-4}$
			$+\frac{7}{2}\delta^{7}_{-3\frac{1}{2}}$		$+\frac{101}{240}\delta^{9}_{-3\frac{1}{2}}$	
		$+\frac{17}{6}\delta^{6}_{-3}$		$+\frac{127}{240}\delta^{8}_{-3}$		$-\frac{1279}{15120}\delta^{10}_{-3}$
	$+2\,\delta^{5}_{-2\frac{1}{2}}$		$+\frac{2}{3}\delta^{7}_{-2\frac{1}{2}}$		$-\frac{13}{120}\delta^{9}_{-2\frac{1}{2}}$	
$+1\,\delta^{4}_{-2}$		$+\frac{5}{6}\delta^{6}_{-2}$		$-\frac{11}{80}\delta^{8}_{-2}$		$+\frac{359}{15120}\delta^{10}_{-2}$
	$+1\,\delta^{5}_{-1\frac{1}{2}}$		$-\frac{1}{6}\delta^{7}_{-1\frac{1}{2}}$		$+\frac{7}{240}\delta^{9}_{-1\frac{1}{2}}$	
$+1\,\delta^{4}_{-1}$		$-\frac{1}{6}\delta^{6}_{-1}$		$+\frac{7}{240}\delta^{8}_{-1}$		$-\frac{41}{7560}\delta^{10}_{-1}$
$+1\,\delta^{4}_{0}$		$-\frac{1}{6}\delta^{6}_{0}$		$+\frac{7}{240}\delta^{8}_{0}$		$-\frac{41}{7560}\delta^{10}_{0}$
$+1\,\delta^{4}_{1}$		$-\frac{1}{6}\delta^{6}_{1}$		$+\frac{7}{240}\delta^{8}_{1}$		$-\frac{41}{7560}\delta^{10}_{1}$
	$-1\,\delta^{5}_{1\frac{1}{2}}$		$+\frac{1}{6}\delta^{7}_{1\frac{1}{2}}$		$-\frac{7}{240}\delta^{9}_{1\frac{1}{2}}$	
$+1\,\delta^{4}_{2}$		$+\frac{5}{6}\delta^{6}_{2}$		$-\frac{11}{80}\delta^{8}_{2}$		$+\frac{359}{15120}\delta^{10}_{2}$
	$-2\,\delta^{5}_{2\frac{1}{2}}$		$-\frac{2}{3}\delta^{7}_{2\frac{1}{2}}$		$+\frac{13}{120}\delta^{9}_{2\frac{1}{2}}$	
		$+\frac{17}{6}\delta^{6}_{3}$		$+\frac{127}{240}\delta^{8}_{3}$		$-\frac{1279}{15120}\delta^{10}_{3}$
			$-\frac{7}{2}\delta^{7}_{3\frac{1}{2}}$		$-\frac{101}{240}\delta^{9}_{3\frac{1}{2}}$	
				$+\frac{967}{240}\delta^{8}_{4}$		$+\frac{1271}{3780}\delta^{10}_{4}$
					$-\frac{89}{20}\delta^{9}_{4\frac{1}{2}}$	
						$+\frac{4523}{945}\delta^{10}_{5}$

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TABLE III. 4. D^4y_0

								$+\frac{3\,41747}{64800}\delta^{12}_{-6}$
							$+\frac{7645}{1512}\delta^{11}_{-5\frac{1}{2}}$	
						$+\frac{4523}{945}\delta^{10}_{-5}$		$+\frac{98729}{4\,53600}\delta^{12}_{-5}$
					$+\frac{89}{20}\delta^{9}_{-4\frac{1}{2}}$		$+\frac{2041}{7560}\delta^{11}_{-4\frac{1}{2}}$	
				$+\frac{967}{240}\delta^{8}_{-4}$		$+\frac{1271}{3780}\delta^{10}_{-4}$		$-\frac{23731}{4\,53600}\delta^{12}_{-4}$
			$+\frac{7}{2}\delta^{7}_{-3\frac{1}{2}}$		$+\frac{101}{240}\delta^{9}_{-3\frac{1}{2}}$		$-\frac{167}{2520}\delta^{11}_{-3\frac{1}{2}}$	
		$+\frac{17}{6}\delta^{6}_{-3}$		$+\frac{127}{240}\delta^{8}_{-3}$		$-\frac{1279}{15120}\delta^{10}_{-3}$		$+\frac{6329}{4\,53600}\delta^{12}_{-3}$
	$+2\delta^{5}_{-2\frac{1}{2}}$		$+\frac{2}{3}\delta^{7}_{-2\frac{1}{2}}$		$-\frac{13}{120}\delta^{9}_{-2\frac{1}{2}}$		$+\frac{277}{15120}\delta^{11}_{-2\frac{1}{2}}$	
$+1\delta^{4}_{-2}$		$+\frac{5}{6}\delta^{6}_{-2}$		$-\frac{11}{80}\delta^{8}_{-2}$		$+\frac{359}{15120}\delta^{10}_{-2}$		$-\frac{283}{64800}\delta^{12}_{-2}$
	$+1\delta^{5}_{-1\frac{1}{2}}$		$-\frac{1}{6}\delta^{7}_{-1\frac{1}{2}}$		$+\frac{7}{240}\delta^{9}_{-1\frac{1}{2}}$		$-\frac{41}{7560}\delta^{11}_{-1\frac{1}{2}}$	
$+1\delta^{4}_{-1}$		$-\frac{1}{6}\delta^{6}_{-1}$		$+\frac{7}{240}\delta^{8}_{-1}$		$-\frac{41}{7560}\delta^{10}_{-1}$		$+\frac{479}{4\,53600}\delta^{12}_{-1}$
$+1\delta^{4}_{0}$		$-\frac{1}{6}\delta^{6}_{0}$		$+\frac{7}{240}\delta^{8}_{0}$		$-\frac{41}{7560}\delta^{10}_{0}$		$+\frac{479}{4\,53600}\delta^{12}_{0}$
$+1\delta^{4}_{1}$		$-\frac{1}{6}\delta^{6}_{1}$		$+\frac{7}{240}\delta^{8}_{1}$		$-\frac{41}{7560}\delta^{10}_{1}$		$+\frac{479}{4\,53600}\delta^{12}_{1}$
	$-1\delta^{5}_{1\frac{1}{2}}$		$+\frac{1}{6}\delta^{7}_{1\frac{1}{2}}$		$-\frac{7}{240}\delta^{9}_{1\frac{1}{2}}$		$+\frac{41}{7560}\delta^{11}_{1\frac{1}{2}}$	
$+1\delta^{4}_{2}$		$+\frac{5}{6}\delta^{6}_{2}$		$-\frac{11}{80}\delta^{8}_{2}$		$+\frac{359}{15120}\delta^{10}_{2}$		$-\frac{283}{64800}\delta^{12}_{2}$
	$-2\delta^{5}_{2\frac{1}{2}}$		$-\frac{2}{3}\delta^{7}_{2\frac{1}{2}}$		$+\frac{13}{120}\delta^{9}_{2\frac{1}{2}}$		$-\frac{277}{15120}\delta^{11}_{2\frac{1}{2}}$	
		$+\frac{17}{6}\delta^{6}_{3}$		$+\frac{127}{240}\delta^{8}_{3}$		$-\frac{1279}{15120}\delta^{10}_{3}$		$+\frac{6329}{4\,53600}\delta^{12}_{3}$
			$-\frac{7}{2}\delta^{7}_{3\frac{1}{2}}$		$-\frac{101}{240}\delta^{9}_{3\frac{1}{2}}$		$+\frac{167}{2520}\delta^{11}_{3\frac{1}{2}}$	
				$+\frac{967}{240}\delta^{8}_{4}$		$+\frac{1271}{3780}\delta^{10}_{4}$		$-\frac{23731}{4\,53600}\delta^{12}_{4}$
					$-\frac{89}{20}\delta^{9}_{4\frac{1}{2}}$		$-\frac{2041}{7560}\delta^{11}_{4\frac{1}{2}}$	
						$+\frac{4523}{945}\delta^{10}_{5}$		$+\frac{98729}{4\,53600}\delta^{12}_{5}$
							$-\frac{7645}{1512}\delta^{11}_{5\frac{1}{2}}$	
								$+\frac{3\,41747}{64800}\delta^{12}_{6}$

TABLE IV. $N'_{q,n}$

1790 →

$\frac{2549}{2550}$ →

q	n	0	1	2	3	4	5	6	7	8	9	10	11	12	q
0	→	1	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{5}{16}$	$\frac{35}{128}$	$\frac{63}{256}$	$\frac{231}{1024}$	$\frac{429}{2048}$	$\frac{6435}{32768}$	$\frac{12155}{65536}$	$\frac{46189}{2\,62144}$	$\frac{88179}{5\,24288}$	$\frac{6\,76039}{41\,94304}$	0
1	→		1	$\boxed{\frac{1}{1}}$	$\frac{23}{24}$	$\frac{11}{12}$	$\frac{563}{640}$	$\frac{1627}{1920}$	$\frac{88069}{1\,07520}$	$\frac{1423}{1792}$	$\frac{15\,93269}{20\,64384}$	$\frac{77\,59469}{103\,21920}$	$\frac{317\,30711}{432\,53760}$	$\frac{465\,22243}{648\,80640}$	1
2	$\frac{2551}{2552}$ →			$\boxed{\frac{1}{1}}$	$\frac{3}{2}$	$\frac{43}{24}$	$\frac{95}{48}$	$\frac{12139}{5760}$	$\frac{25333}{11520}$	$\frac{81227}{35840}$	$\frac{4\,98233}{2\,15040}$	$\frac{1215\,63469}{516\,09600}$	$\frac{2461\,83839}{1032\,19200}$	$\frac{3\,28081\,17961}{1\,36249\,34400}$	2
3					1	2	$\frac{23}{8}$	$\frac{29}{8}$	$\frac{8197}{1920}$	$\frac{2317}{480}$	$\frac{51\,42611}{9\,67680}$	$\frac{11\,11619}{1.93536}$	$\frac{3161\,11237}{516\,09600}$	$\frac{5005\,69373}{774\,14400}$	3
4						1	$\frac{5}{2}$	$\frac{101}{24}$	$\frac{287}{48}$	$\frac{14861}{1920}$	$\frac{12103}{1280}$	$\frac{107\,49419}{9\,67680}$	$\frac{245\,63869}{19\,35360}$	$\frac{65980\,23581}{4644\,86400}$	4
5							1	3	$\frac{139}{24}$	$\frac{55}{6}$	$\frac{14927}{1152}$	$\frac{19627}{1152}$	$\frac{41\,24677}{1\,93536}$	$\frac{8\,29385}{32256}$	5
6								1	$\frac{7}{2}$	$\frac{61}{8}$	$\frac{213}{16}$	$\frac{39209}{1920}$	$\frac{1\,10539}{3840}$	$\frac{370\,06861}{9\,67680}$	6
7									1	4	$\frac{233}{24}$	$\frac{445}{24}$	$\frac{58933}{1920}$	$\frac{44359}{960}$	7
8										1	$\frac{9}{2}$	$\frac{289}{24}$	$\frac{1199}{48}$	$\frac{2\,55751}{5760}$	8
9											1	5	$\frac{117}{8}$	$\frac{131}{4}$	9
10												1	$\frac{11}{2}$	$\frac{119}{24}$	10
11													1	6	11
12														1	12

$$w^q\,\mathrm{D}^q y_{m-\frac{1}{2}} = \sum_{n=q}^{\infty} (-)^{n-q}\,N'_{q,n}\,\Delta^n y_m = \sum_{n=q}^{\infty} N'_{q,n}\nabla^n y_{m-1}$$

$$N'_{q,q}=1 \qquad N'_{0,n}=1.3\ldots(2n-1)/2\,4\ldots 2n \qquad 2(n+1)N'_{q,n+1}=(2n+1)N'_{q,n}+2qN'_{q-1,n}$$

TABLE V. $_{,n}$

q	n	0	1	2	3	4	5	6	7	8	9	10	11	12	q
0		1		$\frac{1}{8}$		$\frac{3}{128}$		$\frac{5}{1024}$		$\frac{35}{32768}$		$\frac{63}{2\ 62144}$		$\frac{231}{41\ 94304}$	0
1	$\frac{m}{2553}$ →		$\frac{1}{1}$		$\frac{1}{24}$		$\frac{3}{640}$		$\frac{5}{7168}$		$\frac{35}{2\ 94912}$		$\frac{63}{28\ 83584}$		1
2	$\frac{2554}{2555}$ →			$\frac{1}{1}$		$\frac{5}{24}$		$\frac{259}{5760}$		$\frac{3229}{3\ 22560}$		$\frac{1\ 17469}{516\ 09600}$		$\frac{71\ 56487}{1\ 36249\ 34400}$	2
3					1		$\frac{1}{8}$		$\frac{37}{1920}$		$\frac{3229}{9\ 67680}$		$\frac{19679}{172\ 03200}$		3
4						1		$\frac{7}{24}$		$\frac{47}{640}$		$\frac{17281}{9\ 67680}$		$\frac{19\ 97021}{4644\ 86400}$	4
5							1		$\frac{5}{24}$		$\frac{47}{1152}$		$\frac{1571}{1\ 93536}$		5
6								1		$\frac{3}{8}$		$\frac{209}{1920}$		$\frac{28067}{9\ 67680}$	6
7									1		$\frac{7}{24}$		$\frac{133}{1920}$		7
8										1		$\frac{11}{24}$		$\frac{871}{5760}$	8
9											1		$\frac{3}{8}$		9
10												1		$\frac{13}{24}$	10
11													1		11
12														1	12

$$w^q \mathrm{D}^q y_{m-\frac{1}{2}} = \begin{cases} \sum\limits_{r=0}^{\infty} (-)^r C'_{q,q+2r}\, \mu\delta^{q+2r} y_{m-\frac{1}{2}} & (q \text{ even}) \\ \sum\limits_{r=0}^{\infty} (-)^r C'_{q,q+2r}\, \delta^{q+2r} y_{m-\frac{1}{2}} & (q \text{ odd}) \end{cases}$$

TABLE VI. 0. $y_{\frac{1}{2}}$

									$+\frac{12155}{65536}\delta$
								$+\frac{6435}{32768}\delta^{8}_{-4}$	
							$+\frac{429}{2048}\delta^{7}_{-3\frac{1}{2}}$		$-\frac{715}{65536}\delta$
						$+\frac{231}{1024}\delta^{6}_{-3}$		$-\frac{429}{32768}\delta^{8}_{-3}$	
					$+\frac{63}{256}\delta^{5}_{-2\frac{1}{2}}$		$-\frac{33}{2048}\delta^{7}_{-2\frac{1}{2}}$		$+\frac{143}{65536}\delta$
				$+\frac{35}{128}\delta^{4}_{-2}$		$-\frac{21}{1024}\delta^{6}_{-2}$		$+\frac{99}{32768}\delta^{8}_{-2}$	
			$+\frac{5}{16}\delta^{3}_{-1\frac{1}{2}}$		$-\frac{7}{256}\delta^{5}_{-1\frac{1}{2}}$		$+\frac{9}{2048}\delta^{7}_{-1\frac{1}{2}}$		$-\frac{55}{65536}\delta$
		$+\frac{3}{8}\delta^{2}_{-1}$		$-\frac{5}{128}\delta^{4}_{-1}$		$+\frac{7}{1024}\delta^{6}_{-1}$		$-\frac{45}{32768}\delta^{8}_{-1}$	
	$+\frac{1}{2}\delta_{-\frac{1}{2}}$		$-\frac{1}{16}\delta^{3}_{-\frac{1}{2}}$		$+\frac{3}{256}\delta^{5}_{-\frac{1}{2}}$		$-\frac{5}{2048}\delta^{7}_{-\frac{1}{2}}$		$+\frac{35}{65536}\delta$
$+1\ y_0$		$-\frac{1}{8}\delta^{2}_{0}$		$+\frac{3}{128}\delta^{4}_{0}$		$-\frac{5}{1024}\delta^{6}_{0}$		$+\frac{35}{32768}\delta^{8}_{0}$	
$+1\mu y_{\frac{1}{2}}$		$-\frac{1}{8}\mu\delta^{2}_{\frac{1}{2}}$		$+\frac{3}{128}\mu\delta^{4}_{\frac{1}{2}}$		$-\frac{5}{1024}\mu\delta^{6}_{\frac{1}{2}}$		$+\frac{35}{32768}\mu\delta^{8}_{\frac{1}{2}}$	
$+1\ y_1$		$-\frac{1}{8}\delta^{2}_{1}$		$+\frac{3}{128}\delta^{4}_{1}$		$-\frac{5}{1024}\delta^{6}_{1}$		$+\frac{35}{32768}\delta^{8}_{1}$	
	$-\frac{1}{2}\delta_{1\frac{1}{2}}$		$+\frac{1}{16}\delta^{3}_{1\frac{1}{2}}$		$-\frac{3}{256}\delta^{5}_{1\frac{1}{2}}$		$+\frac{5}{2048}\delta^{7}_{1\frac{1}{2}}$		$-\frac{35}{65536}\delta^{9}_{1\frac{1}{2}}$
		$+\frac{3}{8}\delta^{2}_{2}$		$-\frac{5}{128}\delta^{4}_{2}$		$+\frac{7}{1024}\delta^{6}_{2}$		$-\frac{45}{32768}\delta^{8}_{2}$	
			$-\frac{5}{16}\delta^{3}_{2\frac{1}{2}}$		$+\frac{7}{256}\delta^{5}_{2\frac{1}{2}}$		$-\frac{9}{2048}\delta^{7}_{2\frac{1}{2}}$		$+\frac{55}{65536}\delta^{9}_{2\frac{1}{2}}$
				$+\frac{35}{128}\delta^{4}_{3}$		$-\frac{21}{1024}\delta^{6}_{3}$		$+\frac{99}{32768}\delta^{8}_{3}$	
					$-\frac{63}{256}\delta^{5}_{3\frac{1}{2}}$		$+\frac{33}{2048}\delta^{7}_{3\frac{1}{2}}$		$-\frac{143}{65536}\delta^{9}_{3\frac{1}{2}}$
						$+\frac{231}{1024}\delta^{6}_{4}$		$-\frac{429}{32768}\delta^{8}_{4}$	
							$-\frac{429}{2048}\delta^{7}_{4\frac{1}{2}}$		$+\frac{715}{65536}\delta^{9}_{4\frac{1}{2}}$
								$+\frac{6435}{32768}\delta^{8}_{5}$	
									$-\frac{12155}{65536}\delta^{9}_{5\frac{1}{2}}$

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TABLE VI. 0. $y_{\frac{1}{2}}$

							$+\frac{6\,76039}{41\,94304}\delta^{12}_{-6}$
						$+\frac{88179}{5\,24288}\delta^{11}_{-5\frac{1}{2}}$	
					$+\frac{46189}{2\,62144}\delta^{10}_{-5}$		$-\frac{29393}{41\,94304}\delta^{12}_{-5}$
				$+\frac{12155}{65536}\delta^{9}_{-4\frac{1}{2}}$		$-\frac{4199}{5\,24288}\delta^{11}_{-4\frac{1}{2}}$	
			$+\frac{6435}{32768}\delta^{8}_{-4}$		$-\frac{2431}{2\,62144}\delta^{10}_{-4}$		$+\frac{4199}{41\,94304}\delta^{12}_{-4}$
		$+\frac{429}{2048}\delta^{7}_{-3\frac{1}{2}}$		$-\frac{715}{65536}\delta^{9}_{-3\frac{1}{2}}$		$+\frac{663}{5\,24288}\delta^{11}_{-3\frac{1}{2}}$	
	$+\frac{231}{1024}\delta^{6}_{-3}$		$-\frac{429}{32768}\delta^{8}_{-3}$		$+\frac{429}{2\,62144}\delta^{10}_{-3}$		$-\frac{1105}{41\,94304}\delta^{12}_{-3}$
$+\frac{63}{256}\delta^{5}_{-2\frac{1}{2}}$		$-\frac{33}{2048}\delta^{7}_{-2\frac{1}{2}}$		$+\frac{143}{65536}\delta^{9}_{-2\frac{1}{2}}$		$-\frac{195}{5\,24288}\delta^{11}_{-2\frac{1}{2}}$	
	$-\frac{21}{1024}\delta^{6}_{-2}$		$+\frac{99}{32768}\delta^{8}_{-2}$		$-\frac{143}{2\,62144}\delta^{10}_{-2}$		$+\frac{455}{41\,94304}\delta^{12}_{-2}$
$-\frac{7}{256}\delta^{5}_{-1\frac{1}{2}}$		$+\frac{9}{2048}\delta^{7}_{-1\frac{1}{2}}$		$-\frac{55}{65536}\delta^{9}_{-1\frac{1}{2}}$		$+\frac{91}{5\,24288}\delta^{11}_{-1\frac{1}{2}}$	
	$+\frac{7}{1024}\delta^{6}_{-1}$		$-\frac{45}{32768}\delta^{8}_{-1}$		$+\frac{77}{2\,62144}\delta^{10}_{-1}$		$-\frac{273}{41\,94304}\delta^{12}_{-1}$
$+\frac{3}{256}\delta^{5}_{-\frac{1}{2}}$		$-\frac{5}{2048}\delta^{7}_{-\frac{1}{2}}$		$+\frac{35}{65536}\delta^{9}_{-\frac{1}{2}}$		$-\frac{63}{5\,24288}\delta^{11}_{-\frac{1}{2}}$	
	$-\frac{5}{1024}\delta^{6}_{0}$		$+\frac{35}{32768}\delta^{8}_{0}$		$-\frac{63}{2\,62144}\delta^{10}_{0}$		$+\frac{231}{41\,94304}\delta^{12}_{0}$
→	$-\frac{5}{1024}\mu\delta^{6}_{\frac{1}{2}}$	→	$+\frac{35}{32768}\mu\delta^{8}_{\frac{1}{2}}$	→	$-\frac{63}{2\,62144}\mu\delta^{10}_{\frac{1}{2}}$	→	$+\frac{231}{41\,94304}\mu\delta^{12}_{\frac{1}{2}}$
	$-\frac{5}{1024}\delta^{6}_{1}$		$+\frac{35}{32768}\delta^{8}_{1}$		$-\frac{63}{2\,62144}\delta^{10}_{1}$		$+\frac{231}{41\,94304}\delta^{12}_{1}$
$-\frac{3}{256}\delta^{5}_{1\frac{1}{2}}$		$+\frac{5}{2048}\delta^{7}_{1\frac{1}{2}}$		$-\frac{35}{65536}\delta^{9}_{1\frac{1}{2}}$		$+\frac{63}{5\,24288}\delta^{11}_{1\frac{1}{2}}$	
	$+\frac{7}{1024}\delta^{6}_{2}$		$-\frac{45}{32768}\delta^{8}_{2}$		$+\frac{77}{2\,62144}\delta^{10}_{2}$		$-\frac{273}{41\,94304}\delta^{12}_{2}$
$+\frac{7}{256}\delta^{5}_{2\frac{1}{2}}$		$-\frac{9}{2048}\delta^{7}_{2\frac{1}{2}}$		$+\frac{55}{65536}\delta^{9}_{2\frac{1}{2}}$		$-\frac{91}{5\,24288}\delta^{11}_{2\frac{1}{2}}$	
	$-\frac{21}{1024}\delta^{6}_{3}$		$+\frac{99}{32768}\delta^{8}_{3}$		$-\frac{143}{2\,62144}\delta^{10}_{3}$		$+\frac{455}{41\,94304}\delta^{12}_{3}$
$-\frac{63}{256}\delta^{5}_{3\frac{1}{2}}$		$+\frac{33}{2048}\delta^{7}_{3\frac{1}{2}}$		$-\frac{143}{65536}\delta^{9}_{3\frac{1}{2}}$		$+\frac{195}{5\,24288}\delta^{11}_{3\frac{1}{2}}$	
	$+\frac{231}{1024}\delta^{6}_{4}$		$-\frac{429}{32768}\delta^{8}_{4}$		$+\frac{429}{2\,62144}\delta^{10}_{4}$		$-\frac{1105}{41\,94304}\delta^{12}_{4}$
		$-\frac{429}{2048}\delta^{7}_{4\frac{1}{2}}$		$+\frac{715}{65536}\delta^{9}_{4\frac{1}{2}}$		$-\frac{663}{5\,24288}\delta^{11}_{4\frac{1}{2}}$	
			$+\frac{6435}{32768}\delta^{8}_{5}$		$-\frac{2431}{2\,62144}\delta^{10}_{5}$		$+\frac{4199}{41\,94304}\delta^{12}_{5}$
				$-\frac{12155}{65536}\delta^{9}_{5\frac{1}{2}}$		$+\frac{4199}{5\,24288}\delta^{11}_{5\frac{1}{2}}$	
					$+\frac{46189}{2\,62144}\delta^{10}_{6}$		$-\frac{29393}{41\,94304}\delta^{12}_{6}$
						$-\frac{88179}{5\,24288}\delta^{11}_{6\frac{1}{2}}$	
							$+\frac{6\,76039}{41\,94304}\delta^{12}_{7}$

14c

TABLE VI. 1. $Dy_{\frac{1}{2}}$

							$+\frac{1423}{1792}\delta^{8}_{-4}$
						$+\frac{88069}{1\,07520}\delta^{7}_{-3\frac{1}{2}}$	
					$+\frac{1627}{1920}\delta^{6}_{-3}$		$-\frac{2689}{1\,07520}\delta^{8}_{-3}$
				$+\frac{563}{640}\delta^{5}_{-2\frac{1}{2}}$		$-\frac{3043}{1\,07520}\delta^{7}_{-2\frac{1}{2}}$	
			$+\frac{11}{12}\delta^{4}_{-2}$		$-\frac{31}{960}\delta^{6}_{-2}$		$+\frac{59}{17920}\delta^{8}_{-2}$
		$+\frac{23}{24}\delta^{3}_{-1\frac{1}{2}}$		$-\frac{71}{1920}\delta^{5}_{-1\frac{1}{2}}$		$+\frac{143}{35840}\delta^{7}_{-1\frac{1}{2}}$	
	$+1\,\delta^{2}_{-1}$		$-\frac{1}{24}\delta^{4}_{-1}$		$+\frac{3}{640}\delta^{6}_{-1}$		$-\frac{5}{7168}\delta^{8}_{-1}$
$+1\,\delta_{-\frac{1}{2}}$		$-\frac{1}{24}\delta^{3}_{-\frac{1}{2}}$		$+\frac{3}{640}\delta^{5}_{-\frac{1}{2}}$		$-\frac{5}{7168}\delta^{7}_{-\frac{1}{2}}$	
$+1\,\delta_{\frac{1}{2}}$		$-\frac{1}{24}\delta^{3}_{\frac{1}{2}}$		$+\frac{3}{640}\delta^{5}_{\frac{1}{2}}$		$-\frac{5}{7168}\delta^{7}_{\frac{1}{2}}$	
$+1\,\delta_{1\frac{1}{2}}$		$-\frac{1}{24}\delta^{3}_{1\frac{1}{2}}$		$+\frac{3}{640}\delta^{5}_{1\frac{1}{2}}$		$-\frac{5}{7168}\delta^{7}_{1\frac{1}{2}}$	
	$-1\,\delta^{2}_{2}$		$+\frac{1}{24}\delta^{4}_{2}$		$-\frac{3}{640}\delta^{6}_{2}$		$+\frac{5}{7168}\delta^{8}_{2}$
		$+\frac{23}{24}\delta^{3}_{2\frac{1}{2}}$		$-\frac{71}{1920}\delta^{5}_{2\frac{1}{2}}$		$+\frac{143}{35840}\delta^{7}_{2\frac{1}{2}}$	
			$-\frac{11}{12}\delta^{4}_{3}$		$+\frac{31}{960}\delta^{6}_{3}$		$-\frac{59}{17920}\delta^{8}_{3}$
				$+\frac{563}{640}\delta^{5}_{3\frac{1}{2}}$		$-\frac{3043}{1\,07520}\delta^{7}_{3\frac{1}{2}}$	
					$-\frac{1627}{1920}\delta^{6}_{4}$		$+\frac{2689}{1\,07520}\delta^{8}_{4}$
						$+\frac{88069}{1\,07520}\delta^{7}_{4\frac{1}{2}}$	
							$-\frac{1423}{1792}\delta^{8}_{5}$

14d

TABLE VI. 1. $Dy_{\frac{1}{2}}$

$$\frac{[illegible]}{[illegible]}\delta_{-3}^{6}$$
$$\frac{11}{[illegible]}\delta_{-2}^{6}$$
$$\frac{3}{640}\delta_{-1}^{6}$$
$$\frac{3}{640}\delta_{2}^{6}$$
$$\frac{11}{[illegible]}\delta_{3}^{6}$$
$$\frac{[illegible]}{[illegible]}\delta_{4}^{6}$$

$$+\frac{88069}{1\ 07520}\delta_{-3\frac{1}{2}}^{7}$$
$$-\frac{3043}{1\ 07520}\delta_{-2\frac{1}{2}}^{7}$$
$$+\frac{143}{35840}\delta_{-1\frac{1}{2}}^{7}$$
$$-\frac{5}{7168}\delta_{-\frac{1}{2}}^{7}$$
$$-\frac{5}{7168}\delta_{\frac{1}{2}}^{7}$$
$$-\frac{5}{7168}\delta_{1\frac{1}{2}}^{7}$$
$$+\frac{143}{35840}\delta_{2\frac{1}{2}}^{7}$$
$$-\frac{3043}{1\ 07520}\delta_{3\frac{1}{2}}^{7}$$
$$+\frac{88069}{1\ 07520}\delta_{4\frac{1}{2}}^{7}$$

$$+\frac{1423}{1792}\delta_{-4}^{8}$$
$$-\frac{2689}{1\ 07520}\delta_{-3}^{8}$$
$$+\frac{59}{17920}\delta_{-2}^{8}$$
$$-\frac{5}{7168}\delta_{-1}^{8}$$
$$+\frac{5}{7168}\delta_{2}^{8}$$
$$-\frac{59}{17920}\delta_{3}^{8}$$
$$+\frac{2689}{1\ 07520}\delta_{4}^{8}$$
$$-\frac{1423}{1792}\delta_{5}^{8}$$

$$+\frac{15\ 93269}{20\ 64384}\delta_{-4\frac{1}{2}}^{9}$$
$$-\frac{46027}{20\ 64384}\delta_{-3\frac{1}{2}}^{9}$$
$$+\frac{28009}{103\ 21920}\delta_{-2\frac{1}{2}}^{9}$$
$$-\frac{1195}{20\ 64384}\delta_{-1\frac{1}{2}}^{9}$$
$$+\frac{35}{2\ 94912}\delta_{-\frac{1}{2}}^{9}$$
$$+\frac{35}{2\ 94912}\delta_{\frac{1}{2}}^{9}$$
$$+\frac{35}{2\ 94912}\delta_{1\frac{1}{2}}^{9}$$
$$-\frac{1195}{20\ 64384}\delta_{2\frac{1}{2}}^{9}$$
$$+\frac{28009}{103\ 21920}\delta_{3\frac{1}{2}}^{9}$$
$$-\frac{46027}{20\ 64384}\delta_{4\frac{1}{2}}^{9}$$
$$+\frac{15\ 93269}{20\ 64384}\delta_{5\frac{1}{2}}^{9}$$

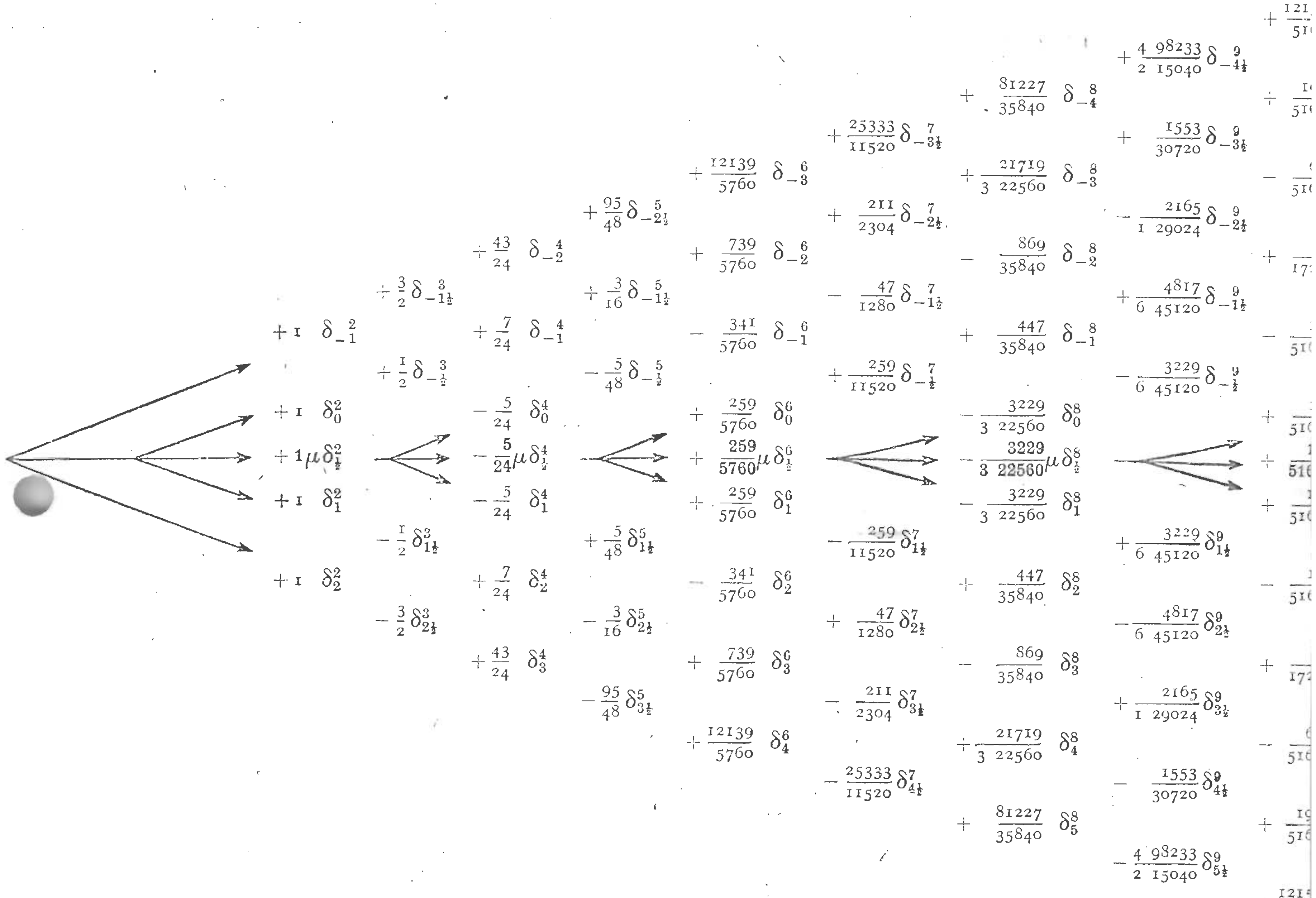
$$+\frac{77\ 59469}{103\ 21920}\delta_{-5}^{10}$$
$$-\frac{51719}{25\ 80480}\delta_{-4}^{10}$$
$$+\frac{7753}{34\ 40640}\delta_{-3}^{10}$$
$$-\frac{475}{10\ 32192}\delta_{-2}^{10}$$
$$+\frac{35}{2\ 94912}\delta_{-1}^{10}$$
$$-\frac{35}{2\ 94912}\delta_{2}^{10}$$
$$+\frac{475}{10\ 32192}\delta_{3}^{10}$$
$$-\frac{7753}{34\ 40640}\delta_{4}^{10}$$
$$+\frac{51719}{25\ 80480}\delta_{5}^{10}$$
$$-\frac{77\ 59469}{103\ 21920}\delta_{6}^{10}$$

$$+\frac{317\ 30711}{432\ 53760}\delta_{-5\frac{1}{2}}^{11}$$
$$-\frac{164\ 88341}{9083\ 28960}\delta_{-4\frac{1}{2}}^{11}$$
$$+\frac{5\ 72249}{3027\ 76320}\delta_{-3\frac{1}{2}}^{11}$$
$$-\frac{22003}{605\ 55264}\delta_{-2\frac{1}{2}}^{11}$$
$$+\frac{2513}{259\ 52256}\delta_{-1\frac{1}{2}}^{11}$$
$$-\frac{63}{28\ 83584}\delta_{-\frac{1}{2}}^{11}$$
$$-\frac{63}{28\ 83584}\delta_{\frac{1}{2}}^{11}$$
$$-\frac{63}{28\ 83584}\delta_{1\frac{1}{2}}^{11}$$
$$+\frac{2513}{259\ 52256}\delta_{2\frac{1}{2}}^{11}$$
$$-\frac{22003}{605\ 55264}\delta_{3\frac{1}{2}}^{11}$$
$$+\frac{5\ 72249}{3027\ 76320}\delta_{4\frac{1}{2}}^{11}$$
$$-\frac{164\ 88341}{9083\ 28960}\delta_{5\frac{1}{2}}^{11}$$
$$+\frac{317\ 30711}{432\ 53760}\delta_{6\frac{1}{2}}^{11}$$

$$+\frac{465\ 22243}{648\ 80640}\delta_{-6}^{12}$$
$$-\frac{21\ 47647}{1297\ 61280}\delta_{-5}^{12}$$
$$+\frac{3\ 63703}{2270\ 82240}\delta_{-4}^{12}$$
$$-\frac{52387}{1816\ 65792}\delta_{-3}^{12}$$
$$+\frac{973}{129\ 76128}\delta_{-2}^{12}$$
$$-\frac{63}{28\ 83584}\delta_{-1}^{12}$$
$$+\frac{63}{28\ 83584}\delta_{2}^{12}$$
$$-\frac{973}{129\ 76128}\delta_{3}^{12}$$
$$+\frac{52387}{1816\ 65792}\delta_{4}^{12}$$
$$-\frac{3\ 63703}{2270\ 82240}\delta_{5}^{12}$$
$$+\frac{21\ 47647}{1297\ 61280}\delta_{6}^{12}$$
$$-\frac{465\ 22243}{648\ 80640}\delta_{7}^{12}$$

14e

TABLE VI. 2. $D^2y_{\frac{1}{2}}$



$+1\ \delta_{-1}^{2}$
$+1\ \delta_{0}^{2}$
$+1\mu\delta_{\frac{1}{2}}^{2}$
$+1\ \delta_{1}^{2}$
$+1\ \delta_{2}^{2}$

$+\frac{3}{2}\delta_{-1\frac{1}{2}}^{3}$
$+\frac{1}{2}\delta_{-\frac{1}{2}}^{3}$
$-\frac{1}{2}\delta_{1\frac{1}{2}}^{3}$
$-\frac{3}{2}\delta_{2\frac{1}{2}}^{3}$

$+\frac{43}{24}\ \delta_{-2}^{4}$
$+\frac{7}{24}\ \delta_{-1}^{4}$
$-\frac{5}{24}\ \delta_{0}^{4}$
$-\frac{5}{24}\mu\delta_{\frac{1}{2}}^{4}$
$-\frac{5}{24}\ \delta_{1}^{4}$
$+\frac{7}{24}\ \delta_{2}^{4}$
$+\frac{43}{24}\ \delta_{3}^{4}$

$+\frac{95}{48}\delta_{-2\frac{1}{2}}^{5}$
$+\frac{3}{16}\delta_{-1\frac{1}{2}}^{5}$
$-\frac{5}{48}\delta_{-\frac{1}{2}}^{5}$
$+\frac{5}{48}\delta_{1\frac{1}{2}}^{5}$
$-\frac{3}{16}\delta_{2\frac{1}{2}}^{5}$
$-\frac{95}{48}\delta_{3\frac{1}{2}}^{5}$

$+\frac{12139}{5760}\ \delta_{-3}^{6}$
$+\frac{739}{5760}\ \delta_{-2}^{6}$
$-\frac{341}{5760}\ \delta_{-1}^{6}$
$+\frac{259}{5760}\ \delta_{0}^{6}$
$+\frac{259}{5760}\mu\delta_{\frac{1}{2}}^{6}$
$+\frac{259}{5760}\ \delta_{1}^{6}$
$-\frac{341}{5760}\ \delta_{2}^{6}$
$+\frac{739}{5760}\ \delta_{3}^{6}$
$+\frac{12139}{5760}\ \delta_{4}^{6}$

$+\frac{25333}{11520}\delta_{-3\frac{1}{2}}^{7}$
$+\frac{211}{2304}\delta_{-2\frac{1}{2}}^{7}$
$-\frac{47}{1280}\delta_{-1\frac{1}{2}}^{7}$
$+\frac{259}{11520}\delta_{-\frac{1}{2}}^{7}$
$-\frac{259}{11520}\delta_{1\frac{1}{2}}^{7}$
$+\frac{47}{1280}\delta_{2\frac{1}{2}}^{7}$
$-\frac{211}{2304}\delta_{3\frac{1}{2}}^{7}$
$-\frac{25333}{11520}\delta_{4\frac{1}{2}}^{7}$

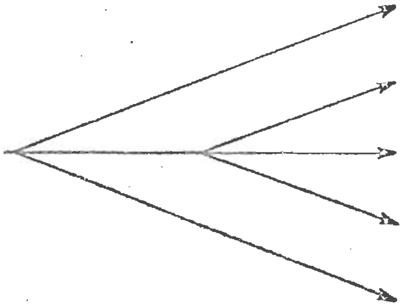
$+\frac{81227}{35840}\ \delta_{-4}^{8}$
$+\frac{21719}{3\,22560}\ \delta_{-3}^{8}$
$-\frac{869}{35840}\ \delta_{-2}^{8}$
$+\frac{447}{35840}\ \delta_{-1}^{8}$
$-\frac{3229}{3\,22560}\ \delta_{0}^{8}$
$-\frac{3229}{3\,22560}\mu\delta_{\frac{1}{2}}^{8}$
$-\frac{3229}{3\,22560}\ \delta_{1}^{8}$
$+\frac{447}{35840}\ \delta_{2}^{8}$
$-\frac{869}{35840}\ \delta_{3}^{8}$
$+\frac{21719}{3\,22560}\ \delta_{4}^{8}$
$+\frac{81227}{35840}\ \delta_{5}^{8}$

$+\frac{4\,98233}{2\,15040}\delta_{-4\frac{1}{2}}^{9}$
$+\frac{1553}{30720}\delta_{-3\frac{1}{2}}^{9}$
$-\frac{2165}{1\,29024}\delta_{-2\frac{1}{2}}^{9}$
$+\frac{4817}{6\,45120}\delta_{-1\frac{1}{2}}^{9}$
$-\frac{3229}{6\,45120}\delta_{-\frac{1}{2}}^{9}$
$+\frac{3229}{6\,45120}\delta_{1\frac{1}{2}}^{9}$
$-\frac{4817}{6\,45120}\delta_{2\frac{1}{2}}^{9}$
$+\frac{2165}{1\,29024}\delta_{3\frac{1}{2}}^{9}$
$-\frac{1553}{30720}\delta_{4\frac{1}{2}}^{9}$
$-\frac{4\,98233}{2\,15040}\delta_{5\frac{1}{2}}^{9}$

TABLE VI. 2. $D^2y_{\frac{1}{2}}$

						$+\frac{3\,28081\,17961}{1\,36249\,34400}\delta^{12}_{-6}$
					$+\frac{2461\,83839}{1032\,19200}\delta^{11}_{-5\frac{1}{2}}$	
				$+\frac{1215\,63469}{516\,09600}\delta^{10}_{-5}$		$+\frac{3118\,51213}{1\,36249\,34400}\delta^{12}_{-5}$
			$+\frac{4\,98233}{2\,15040}\delta^{9}_{-4\frac{1}{2}}$		$+\frac{10\,18967}{344\,06400}\delta^{11}_{-4\frac{1}{2}}$	
		$+\frac{81227}{35840}\delta^{8}_{-4}$		$+\frac{19\,87549}{516\,09600}\delta^{10}_{-4}$		$-\frac{916\,59719}{1\,36249\,34400}\delta^{12}_{-4}$
	$+\frac{25333}{11520}\delta^{7}_{-3\frac{1}{2}}$		$+\frac{1553}{30720}\delta^{9}_{-3\frac{1}{2}}$		$-\frac{1\,31171}{147\,45600}\delta^{11}_{-3\frac{1}{2}}$	
$\frac{139}{5760}\delta^{6}_{-3}$		$+\frac{21719}{3\,22560}\delta^{8}_{-3}$		$-\frac{6\,21491}{516\,09600}\delta^{10}_{-3}$		$+\frac{59\,08457}{27249\,86880}\delta^{12}_{-3}$
	$+\frac{211}{2304}\delta^{7}_{-2\frac{1}{2}}$		$-\frac{2165}{1\,29024}\delta^{9}_{-2\frac{1}{2}}$		$+\frac{64957}{206\,43840}\delta^{11}_{-2\frac{1}{2}}$	
$\frac{739}{5760}\delta^{6}_{-2}$		$-\frac{869}{35840}\delta^{8}_{-2}$		$+\frac{81503}{172\,03200}\delta^{10}_{-2}$		$-\frac{26\,65867}{27249\,86880}\delta^{12}_{-2}$
	$-\frac{47}{1280}\delta^{7}_{-1\frac{1}{2}}$		$+\frac{4817}{6\,45120}\delta^{9}_{-1\frac{1}{2}}$		$-\frac{1\,64233}{1032\,19200}\delta^{11}_{-1\frac{1}{2}}$	
$\frac{341}{5760}\delta^{6}_{-1}$		$+\frac{447}{35840}\delta^{8}_{-1}$		$-\frac{1\,40851}{516\,09600}\delta^{10}_{-1}$		$+\frac{83\,49421}{1\,36249\,34400}\delta^{12}_{-1}$
	$+\frac{259}{11520}\delta^{7}_{-\frac{1}{2}}$		$-\frac{3229}{6\,45120}\delta^{9}_{-\frac{1}{2}}$		$+\frac{1\,17469}{1032\,19200}\delta^{11}_{-\frac{1}{2}}$	
$\frac{259}{5760}\delta^{6}_{0}$		$-\frac{3229}{3\,22560}\delta^{8}_{0}$		$+\frac{1\,17469}{516\,09600}\delta^{10}_{0}$		$-\frac{71\,56487}{1\,36249\,34400}\delta^{12}_{0}$
$\mathbf{\frac{259}{5760}}\mu\delta^{6}_{\frac{1}{2}}$	$\overset{\nearrow}{\underset{\searrow}{\longrightarrow}}$	$-\mathbf{\frac{3229}{3\,22560}}\mu\delta^{8}_{\frac{1}{2}}$	$\overset{\nearrow}{\underset{\searrow}{\longrightarrow}}$	$+\mathbf{\frac{1\,17469}{516\,09600}}\mu\delta^{10}_{\frac{1}{2}}$	$\overset{\nearrow}{\underset{\searrow}{\longrightarrow}}$	$-\mathbf{\frac{71\,56487}{1\,36249\,34400}}\mu\delta^{12}_{\frac{1}{2}}$
$\frac{259}{5760}\delta^{6}$		$-\frac{3229}{3\,22560}\delta^{8}_{1}$		$+\frac{1\,17469}{516\,09600}\delta^{10}_{1}$		$-\frac{71\,56487}{1\,36249\,34400}\delta^{12}_{1}$
	$-\frac{259}{11520}\delta^{7}_{1\frac{1}{2}}$		$+\frac{3229}{6\,45120}\delta^{9}_{1\frac{1}{2}}$		$-\frac{1\,17469}{1032\,19200}\delta^{11}_{1\frac{1}{2}}$	
$\frac{341}{5760}\delta^{6}_{2}$		$+\frac{447}{35840}\delta^{8}_{2}$		$-\frac{1\,40851}{516\,09600}\delta^{10}_{2}$		$+\frac{83\,49421}{1\,36249\,34400}\delta^{12}_{2}$
	$+\frac{47}{1280}\delta^{7}_{2\frac{1}{2}}$		$-\frac{4817}{6\,45120}\delta^{9}_{2\frac{1}{2}}$		$+\frac{1\,64233}{1032\,19200}\delta^{11}_{2\frac{1}{2}}$	
$\frac{739}{5760}\delta^{6}_{3}$		$-\frac{869}{35840}\delta^{8}_{3}$		$+\frac{81503}{172\,03200}\delta^{10}_{3}$		$-\frac{26\,65867}{27249\,86880}\delta^{12}_{3}$
	$-\frac{211}{2304}\delta^{7}_{3\frac{1}{2}}$		$+\frac{2165}{1\,29024}\delta^{9}_{3\frac{1}{2}}$		$-\frac{64957}{206\,43840}\delta^{11}_{3\frac{1}{2}}$	
$\frac{139}{5760}\delta^{6}_{4}$		$+\frac{21719}{3\,22560}\delta^{8}_{4}$		$-\frac{6\,21491}{516\,09600}\delta^{10}_{4}$		$+\frac{59\,08457}{27249\,86880}\delta^{12}_{4}$
	$-\frac{25333}{11520}\delta^{7}_{4\frac{1}{2}}$		$-\frac{1553}{30720}\delta^{9}_{4\frac{1}{2}}$		$+\frac{1\,31171}{147\,45600}\delta^{11}_{4\frac{1}{2}}$	
		$+\frac{81227}{35840}\delta^{8}_{5}$		$+\frac{19\,87549}{516\,09600}\delta^{10}_{5}$		$-\frac{916\,59719}{1\,36249\,34400}\delta^{12}_{5}$
			$-\frac{4\,98233}{2\,15040}\delta^{9}_{5\frac{1}{2}}$		$-\frac{10\,18967}{344\,06400}\delta^{11}_{5\frac{1}{2}}$	
				$+\frac{1215\,63469}{516\,09600}\delta^{10}_{6}$		$+\frac{3118\,51213}{1\,36249\,34400}\delta^{12}_{6}$
					$-\frac{2461\,83839}{1032\,19200}\delta^{11}_{6\frac{1}{2}}$	
						$+\frac{3\,28081\,17961}{1\,36249\,34400}\delta^{12}_{7}$

TABLE VI. 3. $D^3y_{\frac{1}{2}}$



						+
					$+\frac{2317}{480}\delta^{8}_{-4}$	
				$+\frac{8197}{1920}\delta^{7}_{-3\frac12}$		+
			$+\frac{29}{8}\delta^{6}_{-3}$		$+\frac{357}{640}\delta^{8}_{-3}$	
		$+\frac{23}{8}\delta^{5}_{-2\frac12}$		$+\frac{1237}{1920}\delta^{7}_{-2\frac12}$		−
	$+2\,\delta^{4}_{-2}$		$+\frac{3}{4}\delta^{6}_{-2}$		$-\frac{83}{960}\delta^{8}_{-2}$	
$+1\,\delta^{3}_{-1\frac12}$		$+\frac{7}{8}\delta^{5}_{-1\frac12}$		$-\frac{203}{1920}\delta^{7}_{-1\frac12}$		+
	$+1\,\delta^{4}_{-1}$		$-\frac{1}{8}\delta^{6}_{-1}$		$+\frac{37}{1920}\delta^{8}_{-1}$	
$+1\,\delta^{3}_{-\frac12}$		$-\frac{1}{8}\delta^{5}_{-\frac12}$		$+\frac{37}{1920}\delta^{7}_{-\frac12}$		−
$+1\,\delta^{3}_{\frac12}$		$-\frac{1}{8}\delta^{5}_{\frac12}$		$+\frac{37}{1920}\delta^{7}_{\frac12}$		−
$+1\,\delta^{3}_{1\frac12}$		$-\frac{1}{8}\delta^{5}_{1\frac12}$		$+\frac{37}{1920}\delta^{7}_{1\frac12}$		−
	$-1\,\delta^{4}_{2}$		$+\frac{1}{8}\delta^{6}_{2}$		$-\frac{37}{1920}\delta^{8}_{2}$	
$+1\,\delta^{3}_{2\frac12}$		$+\frac{7}{8}\delta^{5}_{2\frac12}$		$-\frac{203}{1920}\delta^{7}_{2\frac12}$		+
	$-2\,\delta^{4}_{3}$		$-\frac{3}{4}\delta^{6}_{3}$		$+\frac{83}{960}\delta^{8}_{3}$	
		$+\frac{23}{8}\delta^{5}_{3\frac12}$		$+\frac{1237}{1920}\delta^{7}_{3\frac12}$		−
			$-\frac{29}{8}\delta^{6}_{4}$		$-\frac{357}{640}\delta^{8}_{4}$	
				$+\frac{8197}{1920}\delta^{7}_{4\frac12}$		+
					$-\frac{2317}{480}\delta^{8}_{5}$	
						+

TABLE VI. 3. $D^3y_{\frac{1}{2}}$

							$+\frac{5005\,69373}{774\,14400}\delta^{12}_{-6}$
						$+\frac{3161\,11237}{516\,09600}\delta^{11}_{-5\frac{1}{2}}$	
					$+\frac{11\,11619}{1\,93536}\delta^{10}_{-5}$		$+\frac{105\,61007}{309\,65760}\delta^{12}_{-5}$
				$+\frac{51\,42611}{9\,67680}\delta^{9}_{-4\frac{1}{2}}$		$+\frac{84\,34073}{221\,18400}\delta^{11}_{-4\frac{1}{2}}$	
			$+\frac{2317}{480}\delta^{8}_{-4}$		$+\frac{1\,03871}{2\,41920}\delta^{10}_{-4}$		$-\frac{15\,58369}{387\,07200}\delta^{12}_{-4}$
		$+\frac{8197}{1920}\delta^{7}_{-3\frac{1}{2}}$		$+\frac{4\,71539}{9\,67680}\delta^{9}_{-3\frac{1}{2}}$		$-\frac{24\,79643}{516\,09600}\delta^{11}_{-3\frac{1}{2}}$	
	$+\frac{29}{8}\delta^{6}_{-3}$		$+\frac{357}{640}\delta^{8}_{-3}$		$-\frac{3737}{64512}\delta^{10}_{-3}$		$+\frac{12\,05453}{1548\,28800}\delta^{12}_{-3}$
$\frac{23}{8}\delta^{5}_{-2\frac{1}{2}}$		$+\frac{1237}{1920}\delta^{7}_{-2\frac{1}{2}}$		$-\frac{13649}{1\,93536}\delta^{9}_{-2\frac{1}{2}}$		$+\frac{72851}{73\,72800}\delta^{11}_{-2\frac{1}{2}}$	
	$+\frac{3}{4}\delta^{6}_{-2}$		$-\frac{83}{960}\delta^{8}_{-2}$		$+\frac{1219}{96768}\delta^{10}_{-2}$		$-\frac{1\,62209}{774\,14400}\delta^{12}_{-2}$
$\frac{7}{8}\delta^{5}_{-1\frac{1}{2}}$		$-\frac{203}{1920}\delta^{7}_{-1\frac{1}{2}}$		$+\frac{15419}{9\,67680}\delta^{9}_{-1\frac{1}{2}}$		$-\frac{4\,20529}{1548\,28800}\delta^{11}_{-1\frac{1}{2}}$	
	$-\frac{1}{8}\delta^{6}_{-1}$		$+\frac{37}{1920}\delta^{8}_{-1}$		$-\frac{3229}{9\,67680}\delta^{10}_{-1}$		$+\frac{10679}{172\,03200}\delta^{12}_{-1}$
$\frac{1}{8}\delta^{5}_{-\frac{1}{2}}$		$+\frac{37}{1920}\delta^{7}_{-\frac{1}{2}}$		$-\frac{3229}{9\,67680}\delta^{9}_{-\frac{1}{2}}$		$+\frac{10679}{172\,03200}\delta^{11}_{-\frac{1}{2}}$	
$\frac{1}{8}\delta^{5}_{\frac{1}{2}}$		$+\frac{37}{1920}\delta^{7}_{\frac{1}{2}}$		$-\frac{3229}{9\,67680}\delta^{9}_{\frac{1}{2}}$		$+\frac{10679}{172\,03200}\delta^{11}_{\frac{1}{2}}$	
$\frac{1}{8}\delta^{5}_{1\frac{1}{2}}$		$+\frac{37}{1920}\delta^{7}_{1\frac{1}{2}}$		$-\frac{3229}{9\,67680}\delta^{9}_{1\frac{1}{2}}$		$+\frac{10679}{172\,03200}\delta^{11}_{1\frac{1}{2}}$	
	$+\frac{1}{8}\delta^{6}_{2}$		$-\frac{37}{1920}\delta^{8}_{2}$		$+\frac{3229}{9\,67680}\delta^{10}_{2}$		$-\frac{10679}{172\,03200}\delta^{12}_{2}$
$\frac{7}{8}\delta^{5}_{2\frac{1}{2}}$		$-\frac{203}{1920}\delta^{7}_{2\frac{1}{2}}$		$+\frac{15419}{9\,67680}\delta^{9}_{2\frac{1}{2}}$		$-\frac{4\,20529}{1548\,28800}\delta^{11}_{2\frac{1}{2}}$	
	$-\frac{3}{4}\delta^{6}_{3}$		$+\frac{83}{960}\delta^{8}_{3}$		$-\frac{1219}{96768}\delta^{10}_{3}$		$+\frac{1\,62209}{774\,14400}\delta^{12}_{3}$
$\frac{23}{8}\delta^{5}_{3\frac{1}{2}}$		$+\frac{1237}{1920}\delta^{7}_{3\frac{1}{2}}$		$-\frac{13649}{1\,93536}\delta^{9}_{3\frac{1}{2}}$		$+\frac{72851}{73\,72800}\delta^{11}_{3\frac{1}{2}}$	
	$-\frac{29}{8}\delta^{6}_{4}$		$-\frac{357}{640}\delta^{8}_{4}$		$+\frac{3737}{64512}\delta^{10}_{4}$		$-\frac{12\,05453}{1548\,28800}\delta^{12}_{4}$
		$+\frac{8197}{1920}\delta^{7}_{4\frac{1}{2}}$		$+\frac{4\,71539}{9\,67680}\delta^{9}_{4\frac{1}{2}}$		$-\frac{24\,79643}{516\,09600}\delta^{11}_{4\frac{1}{2}}$	
			$-\frac{2317}{480}\delta^{8}_{5}$		$-\frac{1\,03871}{2\,41920}\delta^{10}_{5}$		$+\frac{15\,58369}{387\,07200}\delta^{12}_{5}$
				$+\frac{51\,42611}{9\,67680}\delta^{9}_{5\frac{1}{2}}$		$+\frac{84\,34073}{221\,18400}\delta^{11}_{5\frac{1}{2}}$	
					$-\frac{11\,11619}{1\,93536}\delta^{10}_{6}$		$-\frac{105\,61007}{309\,65760}\delta^{12}_{6}$
						$+\frac{3161\,11237}{516\,09600}\delta^{11}_{6\frac{1}{2}}$	
							$-\frac{5005\,69373}{774\,14400}\delta^{12}_{7}$