

March 28, 1988

Professor R.K. Guy  
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Calgary, AB  
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COPY

Dear Professor Guy:

The (unlabelled, unordered) binary trees with  $n$  terminal nodes can indeed be interpreted as representing the "structure" of a knock-out tournament on  $n$  players. But this is not what the Narayana-Capell numbers  $T_n$  enumerate, their verbal description notwithstanding. It seems that  $T_n$  is the number of sequences of integers corresponding to the number of matches in the consecutive rounds of a knock-out tournament on  $n$  players. The same tree-structure can realize different sequences -  $\wedge$  realizes both (2,1) and (1,1,1) - and different trees can realize the same sequence -  $\wedge$  also realizes (1,1,1); so there doesn't seem to be any obvious relation between the two sets of numbers.

More specifically, the Narayana-Capell numbers  $T_n$  may be

$$\left[ \frac{1}{2} n \right]$$

defined as follows:  $T_2 = 1$  and  $T_n = \sum_{k=1}^{n-1} T_{n-k}$  for  $n \geq 3$ .

This implies that  $T_{2m} = 2T_{2m-1}$  for  $m \geq 2$ ; the Wedderburn-Etherington numbers do not satisfy this relation. Furthermore, Otter and Bender have shown that the  $n$ -th Wedderburn-Etherington number is asymptotic to  $n^{-3/2} \delta^{-n}$  where  $\delta = .402 \dots$ ; whereas Narayana and Capell assert that  $T_n$  is quite close to  $b2^n$  for large  $n$ .

Sincerely yours,

J.W. Moon

JWM/hh

P.S. Poor Narayane died last year, I don't know anything about Capell.

cc N.J.A. Sloane

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88-03-22

Dr. John W. Moon,  
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Dear John,

I don't know whether Narayana or Capell still exist, but no doubt you'll consult them if they do. I've just been looking at their paper, On knock-out tournaments, *Canad. Math. Bull.* 13 (1970) 108, and they enumerate the number,  $T_n$ , of "random (knock-out) tournaments" for  $n$  players as

$n$	(1)	2	3	4	5	6	7	8	9	10	11
$T_n$	(1)	1	1	2	3	6	11	22	42	84	165

My naive guess is that these should be the same as the number of binary trees (= Wedderburn-Etherington numbers = # of homogeneous dendrites: Melzak, *Canad. Math. Bull.* 11 (1968) 85-93 and see Comtet, *Advanced Combinatorics*, Dordrecht, 1974, p.54) and sequence 297 in Sloane's *Handbook of Integer Sequences* should coincide with sequence 298: ... 6, 11, 23, 46, 98, 207, 451, ....

Am I wrong?

Best wishes,

Yours sincerely,

RKG:1

Richard K. Guy.

cc: N.J.A. Sloane

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88-03-31

Professor John W. Moon,  
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Dear John,

Many thanks for clarifying Capell-Narayana. I was sorry to hear about Narayana. Was he already retired? (No need to answer: tell me in Kalamazoo.)

The first difference occurs for 6 players. They distinguish between



and



but not



i.e. between 221 and 311 (the heavy edge is the winner of a first round match having a bye during the second round: not usually done in practice. E.g. suppose you've only got two tennis courts, you could still carry out all varieties of tournaments). On the other hand, they don't distinguish between the two different 221 binary trees: the first & third of the above diagrams. Another way of putting it is to say that the first two are separate tournaments of 4 & 2 players, whose winners then meet, as opposed to two separate tournaments of 3 players. There's no difference in the actual numbers (11) for  $n=7$ , the first difference occurs at  $n=8$ : 22 as opposed to 23.

I've calculated the next several Capell-Narayana numbers, and I'll copy this to Neil Sloane, so he can put two full lines for sequence 227 in the *Handbook*:

- 1, 1, 2, 3, 6, 11, 22, 42, 84, 165, 330, 654, 1308, 2605, 5210,
- 10398, 20796, 41550, 83100, 166116, 332232, 664299, 1328598,
- 2656866, 5313732, 10626810, 21253620, 42505932, 85011864, 170021123,
- 340042246, 680079282, 1360158564, 2720306730, ...

Best wishes,

Yours sincerely,

*Richard*

Richard K. Guy.

RKG:l  
cc. NJA Sloane