

fa1

1861
1862
248
1858

Oct 19, 1970

Dear Neill:

1. I notice in the draft of chapter 2 the Bell numbers are designated by G_n - just when I have been having a little success in promoting B_n . (Altho G_n has been used it is nondescript) This reminded me of

$$B_n(z) = \sum_0^n \binom{n}{k} B_k B_{n-k} = \sum_0^n S(n,k) 2^k$$

and I extended my table to

✓ (1861)

n	0	1	2	3	4	5	6	7	8	9	10
$B_n(z)$	1	2	6	22	94	454	2436	14214	89918	610182	4412798

2. Also in Chap 2 I notice that the simplest recurrence for the Catalan has $(n+2)C_{n+1} = (4n+2)C_n$ is absent, as well as my favorite CI p. 157 $C_{n+1} = \sum_0^n \binom{n}{2k} 2^{n-2k} C_k$. Also in the applications & pictures section, 1 was noticed by Catalan (1838), according to Netto, and the reference to Motzkin in 5 is not as good as W.A. Whitworth Messenger of Math 8 (1879) 105-114, the pioneering paper in ballot problems. The mapping of 6 on 5 is given in A.M. Yaglom and I.M. Yaglom (the twin brothers) Challenging Math Problems with Elementary Solutions, Holden-Day 1964. The mapping of plane trees on lattice paths is due to T.E. Harris (independently by Ed Gilbert) - I don't have the ~~reference~~ Trans Amer Math Soc 73 (1952), 471-486.

3. The Wedderburn-Etherington Nos are the ~~series reduced~~ planted trees with points of degree 1 or 3, having n endpoints other than the root, which might be noted

4. Comtet I, p 102 Ex. 43 gives the numbers of ^{forests} rooted of rooted trees of least height ~~with~~ according to number of (labeled) points. In the notation of Forests of Labeled Trees, there are $L_n(1)$. The corresponding number for free trees is $L_n^*(1)$; here are both nos

n	0	1	2	3	4	5	6	7	8	9	10	
(sequence 248)	$L_n(1)$	1	1	3	10	41	196	1057	6322	41393	293608	2237921
✓ 1862	$L_n^*(1)$	1	1	2	7	26	111	562	3151	19252	128449	925226
(1858 again)	$A_n^*(1)$	1	1	2	7	38	291	2932	36961	561948	10026505	205608536

I have added $A_n^*(1)$ = no of forests of trees with n labeled pts (without restrictions) $A_n^*(1) = (n+1)^{n-1}$ = no of forests of rooted trees, n labeled pts, no restriction

5. Talking about acknowledgements (as I was some letters back), the difficulty about "I should like to thank A and B for helpful comments" is that should is conditional, requiring an added statement specifying the conditions e.g. "but my publisher begrudges the space" or "neither A nor B likes gratitude". And in any case by making the statement so conditionally "I" have not thanked them. An alternate "Thanks to his persistent interest in this catalog, John Riordan has forced me to do many things I do not like".

Yours
John