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88-07-18

Dr. Neil J.A. Sloane,
AT&T Bell Laboratories, Room 2C-376
600 Mountain Avenue,
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Dear Neil,

S.700 and S.1420 are well-known, of course, but perhaps alternate members of the former deserve a place of their own, as they crop up in their own right. See enclosed.

No!

Off to A.C.C. G.M.C. on Thursday.

Best wishes,

Yours sincerely,

Richard.

RKG:1

Richard K. Guy.

encl: E.3302

E 3302. Proposed by Jim Delany. The mean and standard deviation of any 7 consecutive integers are both integers. What natural numbers greater than 1 share this property with 7?

Solution by Richard K. Guy, The University of Calgary, Alberta, Canada, T2N 1N4. We may translate the problem so that the mean is zero. Then the number of integers must be odd, say $2k + 1$, and the standard deviation, $s$, of $0, \pm 1, \pm 2, \ldots, \pm k$, is given by

$$2(1^2 + 2^2 + \ldots + k^2) = (2k + 1)s^2$$

$$3s^2 = k(k + 1).$$

As $k$ and $k + 1$ are coprime, they are $3a^2$ and $b^2$ in some order, and $3a^2 - b^2 = \pm 1$, whose solutions are given by even ranked convergents to the continued fraction for $\sqrt{3}$ : $b/a =$

$$\left(\frac{1}{0}\right) \quad \frac{2}{1} \quad \frac{7}{4} \quad \frac{26}{15} \quad \frac{97}{56} \quad \frac{362}{209} \quad \ldots$$

both numerators and denominators satisfying the recurrence $a_{n+1} = 4a_n - a_{n-1}$. Then $s_n = a_n b_n$ and $(2k+1)_n = 6a_n^2 + 1 = b_{2n}$.

So the required numbers are alternate numerators, $b_{2n}$,

$$(1),\ 7,\ 97,\ 1351,\ 18797,\ 262087,\ \ldots$$

satisfying the recurrence $b_{2n+2} = 14b_{2n} - b_{2n-2}$ and the Binet-type formula $\frac{1}{2}\{(7 + 4\sqrt{3})^n + (7 - 4\sqrt{3})^n\}$.