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But the side, $s$, of a regular decagon inscribed in a circle of radius is $r(\sqrt{5}-1)/2$ so $x=s$ and the problem is proved.

Other solutions using synthetic geometry were submitted by *Danny Coop- and Annette Mayhew (Jointly), Alexander Hamilton High School, Los Angele California; Elaine Hemenway, Catholic Girls High School, Los Angel California and Philip Maclasky, Philadelphia, Pennsylvania.*

## A Quadrilateral with Maximum Integral Area

**156.** [January 1953] *Proposed by E. P. Starke, Rutgers University.*

A quadrilateral has sides of length 2, 3, 4, and 5 in that orde: Determine the angle between the sides of length 4 and 5 such that t area shall be the largest possible integer.

*Solution by L. A. Ringenberg, Eastern Illinois State College.* Let denote the angle between the sides of length 4 and 5. Let $B$ denote th interior angle opposite to $A$. Note that $A$ is a positive acute angle. Fro the Cosine Law we obtain (1) $10\cos A - 3\cos B = 7$. The area of the quadri- lateral is given by (2) $K = 10\sin A + 3\sin B$. Then $\frac{dK}{DA} = 3\sin B(\cot A + \cot B$ $K$ is a maximum when $\cot A + \cot B = 0$. That is, when the angles $A$ and $B$ ar supplementary (and hence the quadrilateral is cyclic). From equations ( and (2) we have $K_{max} = 13\sin A$ where $13\cos A = 7$. Thus $K_{max} = \sqrt{120}$. Th largest integral $K$ is 10 so from (1) and (2) we obtain $3\sin B = 10 - 10\sin$ and $3\cos B = 10\cos A - 7$. There are two solutions:

$$A = \arcsin(120 \pm 7\sqrt{5})/149$$

which gives the approximate values of:

$$A = 44°27'09''$$

and

$$A = 65°33'47''$$

Several solvers started with the fact that a quadrilateral with fix sides has a maximum area when it is cyclic. Thus Heron's Formula $S$ $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ leads to $S = \sqrt{5\cdot4\cdot3\cdot2}$ or $S = \sqrt{120}$ as the max imum area.

Also solved by *Leon Bankoff, Los Angeles, California; W. B. Carve Cornell University; R. Huck, Baltimore, Maryland; H. I. James, Hampt Institute; Sam Kravitz, East Cleveland, Ohio; C. W. Trigg, Los Ange City College and the proposer.*

## A Factorial Ending In Zeros

**158.** [January 1953] *Proposed by Leo Moser, University of Alberta, Canada.*

Find all values of $r$ such that no $n!$ (written in decimal notation) ca end in exactly $r$ zeros.

*Solution by C. W. Trigg, Los Angeles City College.* If $2^p$ and $5^q$ are the highest powers of 2 and 5 which divide $n!$, clearly $p > q$ and $n!$ ends in $q$ zeros. Furthermore $q = [n/5] + [n/5^2] + [n/5^3] + \cdots$, where $[x]$ denotes the greatest integer in $x$.

Now as $n$ takes on successive integer values, $q$ also takes on successive integer values, but more slowly, with the exception of those values $r$ which are skipped when $[n/5^i] = (n/5^i)$. Hence we have $r = (6k - 1) + p_1[k/5] + p_2[k/5^2] + p_3[k/5^3] + \cdots$, where $k = 1, 2, 3, \cdots$ and $p_i = 1$ except when $[k/5^i] = (k/5^i)$, when $p_i$ also has the value $(k - 5^i)/k$. Thus the first 31 values of $r$ are: 5, 11, 17, 23, 29, 30, 36, 42, 48, 54, 60, 61, 67, 73, 79, 85, 91, 92, 98, 104, 110, 116, 122, 123, 129, 135, 141, 147, 153, 154, 155.

Also solved by *B. A. Hausman, West Baden College, Indiana and C. R. Perisho, Nebraska Wesleyan University.*

## The Broken Stick

159. [January 1953] *Proposed by A/2C D. L. Silverman, Patrick A. F. B., Florida.*

A man breaks a stick in two places. What is the probability that he will be able to form a triangle with the three segments?

*Solution by W. Funkenbusch, Michigan College of Mining and Technology, Sault Ste. Marie Branch.* Let the lengths of the three segments be given by $x$, $y$, $L - x - y$, ($L$ a constant). Then clearly:

$$p = \frac{\int_0^{L/2} \int_{L/2 - x}^{L/2} dy\,dx}{\int_0^{L} \int_0^{L - x} dy\,dx} = \frac{1}{4}.$$

It should be noted that the problem is not new. It is found in *Uspensky's "Introduction to Mathematical Probability" and also in Ball's Mathematical Recreations and Essays".*

Bankoff found the problem in the *Hall and Knight, "Higher Algebra", London 1948 and also in "Solutions of the Problems and Riders proposed in the Senate-House Examinations for 1854", Cambridge: Macmillan and Co. (1854), pages 49-52.*

Also solved by *A. L. Epstein, Cambridge Research Center; Sam Kravitz, East Cleveland, Ohio; B. I. Pfeiffer, Outremont, Quebec; L. A. Ringenberg, Eastern Illinois State College; Milton Scharf, Brooklyn, New York; Dmitri Thoro, University of Florida and C. W. Trigg, Los Angeles City College.*

## Data For Triskaidekaphobes

160. [January 1953] *Proposed by Victor Thebault, Tennie, Sarthe, France.*

In which of the remaining years of the twentieth century will Friday-the-thirteenth occur most (or least) frequently?