112
608
→ 798

October 25, 1970

Dr N. J. A. Sloane
Room 2C-363, Bell Labs.

Dear Dr. Sloane,

The "work" McEliece mentioned to you on counting posets was simply a brute-force listing of 6-posets, with a resulting total of 318. Of these 238 are connected (i.e. their Hasse diagrams are connected) and 80 are disconnected.

You may be interested in an abstract by John A. Wright, # 70T-A106 in the June 1970 Notices. He agrees with my figure for the connected 6-posets but disagrees on the total, giving 336. However, the disconnected ones are easy to count directly from the lower order posets, and Dr. Richard P. Stanley of MIT claims Wright's own recursion formula gives 80 disconnected 6-posets, hence I'm confident that the total of 318 is correct. Perhaps Wright can provide you with further information, including totals for higher orders. If you would like a listing of the 318 6-posets I've found, let me know and I'll be glad to oblige. My list is not currently in xerox-able condition; otherwise I would have sent it along.

I hope I was able to be of some help to you.

Sincerely,
Neil L. White

D of M
Harvard
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Dept. of Mathematics
Harvard University
Cambridge, Mass. 02138
November 13, 1970

Dr. N. J. A. Sloane
Bell Labs, 600 Mountain Ave.
Murray Hill, N.J. 07974

Dear Dr. Sloane:

Thank you for your letter of Nov. 6. I have managed to get my list of the 6-posets into reasonable shape, and a copy is enclosed. I have attempted to provide a partial classification of the posets, though it is somewhat haphazard. Each poset is listed next to its dual (except for 74+76 - I goofed). The posets in a given classification consist of all posets (and their duals) that satisfy the given conditions, but such that neither the poset nor its dual is in a preceeding class. For example, #167-190, having exactly one top element and one co-atom (an element covered by the top element), cannot have a unique bottom element which is covered by more than one element.

I have also listed the number of

labeled posets corresponding to each of the 318 (unlabeled) posets. The total for labelled is 130,023. For any n-poset P, the number of corresponding labeled posets is just $\frac{n!}{\text{order of } Aut(P)}$

The method I used in constructing this table was to add a new bottom element in all possible ways to all 5-posets. Then I checked for duplication. I completed this in 1967, as a second-year graduate student.

I have recently obtained a copy of the work of J.A. Wright, of the U. of Rochester Math Dept., whom I mentioned in my last letter. He has corrected his total to 318 and has provided a listing of the connected 6-posets, which agrees with mine. He also agrees with me on the number of labelled 6-posets. He also points out the equivalence of n-posets with T_0 topologies on n points and enumerates the total number of topologies up to $n=6$. I suggest you write him if you haven't already.

I hope I've been of assistance to you.

Sincerely,

Neil L. White

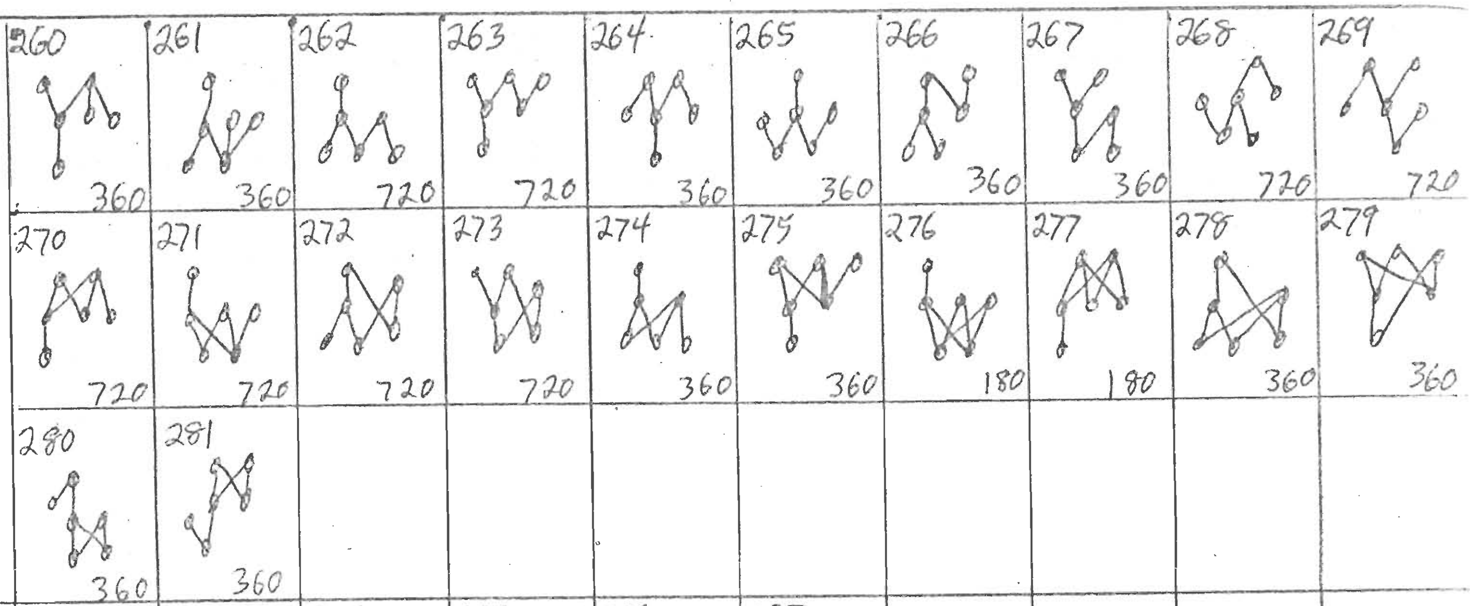
6-posets

disconnected with one component (or more) a single point.	1: * 1	2: * 30	3: 60	4: 60	5: * 120	6: * 360	7: 360	8: 360	9: 180	10: 18
one-to-one with 5-posets	11: * 180	12: * 360	13: * 180	14: 60	15: 60	16: * 90	17: 360	18: 360	19: * 720	20: 360
KEY:	21: 360	22: 30	23: 30	24: 360	25: 360	26: 720	27: 720	28: * 720	29: 360	30: 36
* = self-dual; lower right corner is number of labeled posets corresponding	31: 720	32: 720	33: * 720	34: 360	35: 360	36: 360	37: 360	38: 720	39: 720	40: 36
to given po[set]	41: 360	42: 120	43: 120	44: 180	45: 180	46: * 120	47: 360	48: 360	49: 360	50: 360
	51: 60	52: 60	53: 720	54: 720	55: 720	56: 720	57: * 180	58: * 720	59: * 720	60: 36
	61: 360	62: 360	63: 360							
other disconnected 2-4	64: * 720	65: 720	66: 720	67: 360	68: 360	69: * 360	70: * 720	71: 120	72: 120	73: * 180
3-3 and 2-2-2	74: 90	75: * 180	76: 90	77: 360	78: 360	79: * 360	80: * 120			

one top elt ≥4 co-atoms (and duals)	81 6	82 6	83 120	84 120	85 180	86 180	87 120	88 120	89 * 30	
one top 3 co-atoms	90 360	91 360	92 180	93 180	94 360	95 360	96 120	97 120	98 720	99 720
	100 * 360	101 360	102 360	103 720	104 720	105 180	106 180	107 360	108 360	109 360
	110 360	111 360	112 360	113 60	114 60	115 360	116 360	117 360	118 360	
one top 2 co-atoms	119 720	120 720	121 360	122 360	123 720	124 720	125 360	126 360	127 120	128 120
	129 720	130 720	131 360	132 360	133 360	134 360	135 720	136 720	137 * 720	138 * 360
	139 720	140 720	141 * 720	142 180	143 180	144 360	145 360	146 * 180	147 720	148 720
	149 720	150 720	151 720	152 720	153 720	154 720	155 720	156 720	157 360	158 360
	159 360	160 360	161 360	162 360	163 360	164 360	165 60	166 60		
one top one co-atom	167 * 720	168 720	169 720	170 720	171 720	172 360	173 360	174 360	175 360	176 120

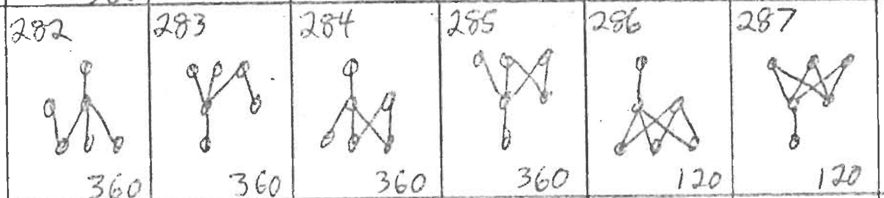
one top one co-atom continued	177 — 120	178 — 360	179 — 360	180 — 360	181 — 360	182 — 30	183 — 30	184 — * 360	185 — 360	186 — 360
	187 — 720	188 — 720	189 — 180	190 — 180						
2 tops 2 bottoms chain of 4	191 — 720	192 — 720	193 — 720	194 — 720	195 — * 720	196 — 360	197 — 360	198 — * 720	199 — * 180	200 — 720
	201 — 720	202 — 720	203 — 720	204 — 360	205 — 360	206 — * 720	207 — 720	208 — 720	209 — * 720	
2 tops 2 bottoms 2 disjoint chains of 3	210 — 720	211 — 720	212 — * 720	213 — 180	214 — 180	215 — * 720	216 — 720	217 — 720	218 — * 720	219 — * 360
	220 — * 720	221 — 360	222 — 360	223 — * 90						
remaining 2 tops + 2 bottoms	224 — 720	225 — 720	226 — * 360	227 — 720	228 — 720	229 — 360	230 — 360	231 — 720	232 — 720	233 — * 360
	234 — 180	235 — 180	236 — 360	237 — 360	238 — 720	239 — 720	240 — 360	241 — 360		
2 tops + 3 bottoms unique chain of 3	242 — 720	243 — 720	244 — 360	245 — 360	246 — 360	247 — 360	248 — 720	249 — 720	250 — 720	251 — 72[illegible]
	252 — 720	253 — 720	254 — 720	255 — 720	256 — 360	257 — 360	258 — 360	259 — 360		

2 tops
3 bottoms
2 chains of 3



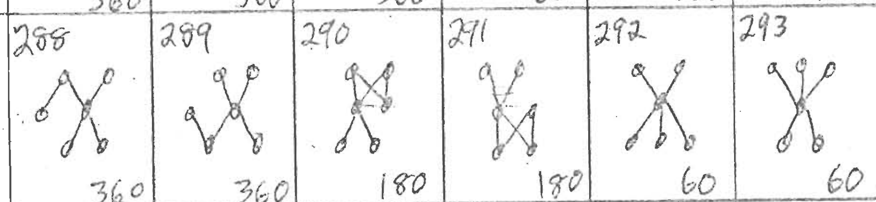
260	261	262	263	264	265	266	267	268	269
360	360	720	720	360	360	360	360	720	720
270	271	272	273	274	275	276	277	278	279
720	720	720	720	360	360	180	180	360	360
280	281								
360	360								

2 tops
3 bottoms
3 chains of 3



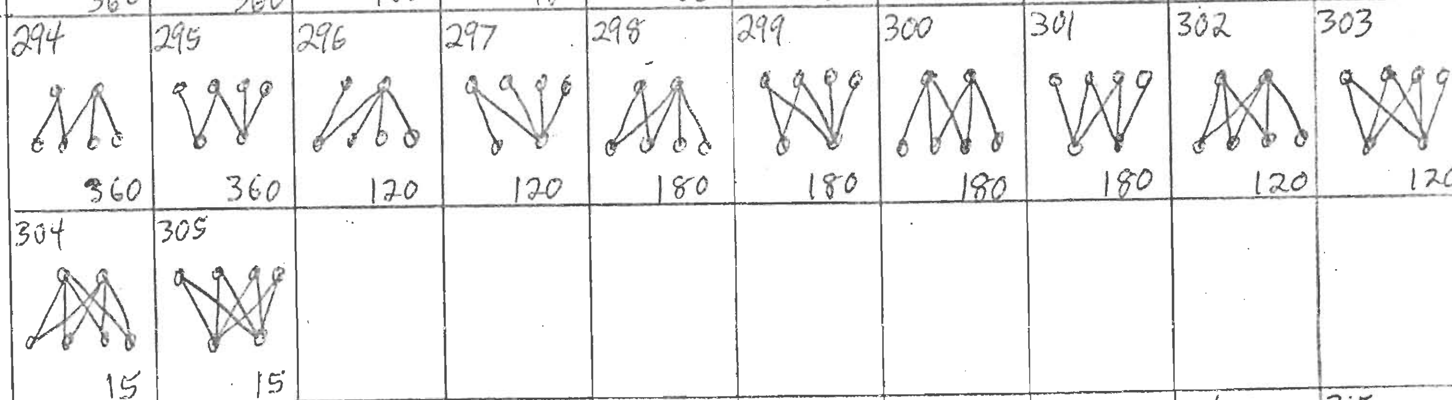
282	283	284	285	286	287
360	360	360	360	120	120

2 tops
3 bottoms
4 or 6 ch[illegible] of 3



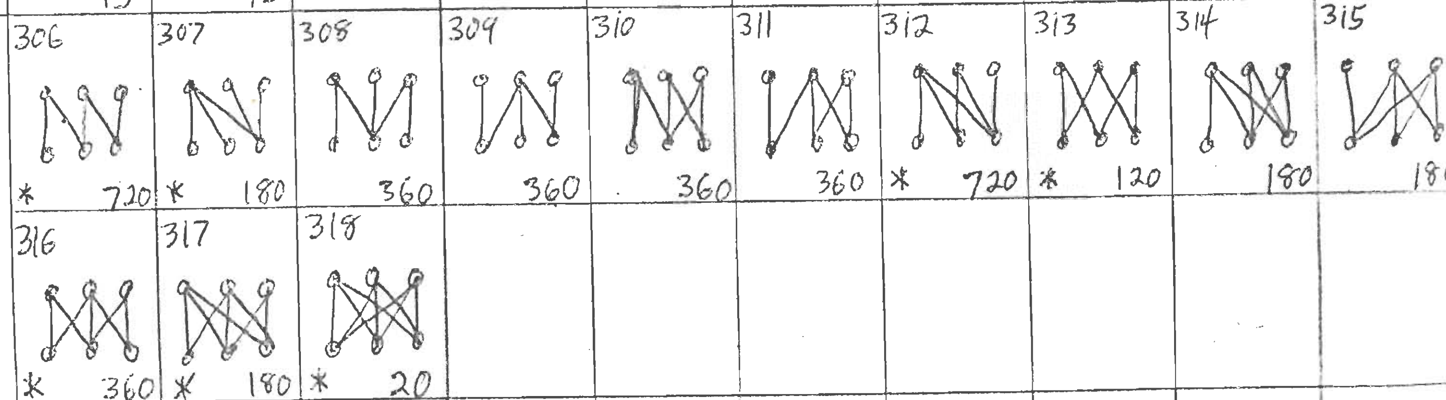
288	289	290	291	292	293
360	360	180	180	60	60

2 tops
4 bottoms



294	295	296	297	298	299	300	301	302	303
360	360	120	120	180	180	180	180	120	120
304	305								
15	15								

3 tops
3 bottoms



306	307	308	309	310	311	312	313	314	315
* 720	* 180	360	360	360	360	* 720	* 120	180	180
316	317	318							
* 360	* 180	* 20							