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THE TRANSITIVE GROUPS OF DEGREE UP TO ELEVEN[+]

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INTRODUCTION

Biggs in [14] states that Kirkman [see ibid. ref. 63.2] had by 1863 a recursive method for determining permutation groups of low degree. In this he has clear priority over others. Ignorant of this, we did not use his methods which may yet be the best. Groups are omitted in his list because of early errors and the recursive nature of his method. He does not distinguish groups with the same numbers of elements of each cycle type. An account of activity at the beginning of this century is in [2].

We have relied on Sims [9] for primitive groups. Our list stops at degree 11 since Miller [7] lists 298 groups of degree 12.

Our approach [5,10,11,15] to computing Galois groups of polynomials needs a list of transitive groups and orbit lengths of the group action on ordered and unordered sets of roots. The latter is found [15] and we hope the former will fulfil a need.

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THEORY AND BACKGROUND

We will use the notation and definitions of [12]. Let G be a transitive permutation group on Ω of size n. Then either G is primitive - the primitive groups of degree up to twenty are given in [9] - or G stabilizes a partition of Ω where each of the m sets of the partition has the same size k. The stabilizer of the partition in the symmetric group Σ_n is isomorphic to the semidirect product

$$(\bigotimes_{i=1}^{m} \Sigma_k)\,\Sigma_m .$$

The transitive imprimitive groups are determined by investigating the subgroup structure of the partition stabilizer. This requires a knowledge of the transitive groups of degree k and m. Recourse is also made to the following theory on the subgroups of a direct product. The first result is a reformulation of Goursat-Lambek's result [13, p.237, Ex.30]. We have not found this approach in the literature, although it is probably not new.

Theorem

Let G_1 and G_2 be finite groups. The subgroups H of $G_1 \times G_2$ in one-to-one correspondence with the tuples (H_1,H_2,H_3,ψ) where $H_1 \leq G_1$, $H_2 \leq G_2$, $H_3 \trianglelefteq H_2$, and ψ: $H_1 \to H_2/H_3$ is a surjective homomorphism.

Proof

We give the correspondence and its inverse. For a subgroup H, the tuple is given by $H_1 = \{a|(a,b) \in H\}$, $H_2 = \{b|(a,b) \in H\}$, $H_3 = \{b|(1,b) \in H\}$, and $\psi(a) = bH_3$ where $(a,b) \in H$. Given a tuple the subgroup is $H = \{(a,b)|a \in H_1,\ b \in \psi(a)\}$. □

Let $(g_1,g_2) \in G_1 \times G_2$ then $(H_1,H_2,H_3,\psi)^{(g_1,g_2)} =$ $(H_1^{g_1},H_2^{g_2},H_3^{g_2},g_1^{-1}\psi g_2)$ where the homorphism $g_1^{-1}\psi g_2$ is defined

by $a \to (\psi(a^{g_1^{-1}}))^{g_2}$. Hence a unique

of subgroups of $G_1 \times G_2$ is formed as

a) choose a representative of H

b) choose a representative H_2 c

c) choose a normal subgroup H_3

class, and

d) there is an action $\psi \to g_1^{-1}\psi g_2$

$N_{G_1}(H_1) \times (N_{G_2}(H_2) \cap N_{G_2}(H_3)$

ψ from each orbit which cont

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Representing subgroups by their

$(K_1,K_2,K_3,\tau) \leq (H_1,H_2,H_3,\psi)$ holds if

$K_3 = K_2 \cap H_3$, and the diagram

$$\begin{array}{ccc} K_1 & \xrightarrow{\tau} & K_2/K_3 \\ \downarrow i & & \\ H_1 & \xrightarrow[\psi]{} & \end{array}$$

commutes. □

METHOD

When the degree is prime then ev

primitive. For degree 4 and 6 the su

by $a \to (\psi(a^{g_1^{-1}}))^{g_2}$. Hence a unique representative of each class of subgroups of $G_1 \times G_2$ is formed as follows:

a) choose a representative of H_1 of each G_1 - class,

b) choose a representative H_2 of each G_2 - class,

c) choose a normal subgroup H_3 of H_2 from each $N_{G_2}(H_2)$ - class, and

d) there is an action $\psi \to g_1^{-1}\psi g_2$ on Hom $(H_1, H_2/H_3)$ by $N_{G_1}(H_1) \times (N_{G_2}(H_2) \cap N_{G_2}(H_3))$. Choose one homomorphism ψ from each orbit which contains a surjective homomorphism.

For completeness we state the following result:

Theorem

Let G_1 and G_2 be finite groups. The normal subgroups H of $G_1 \times G_2$ are in one-to-one correspondence with the tuples (H_1, H_2, H_3, ψ) where $H_1 \trianglelefteq G_1$, $H_2 \trianglelefteq G_2$, $H_3 \trianglelefteq H_2$, $H_3 \trianglelefteq G_2$, and $\psi\colon H_1 \to H_2/H_3$ is a surjective homomorphism with im $\psi \le Z(G_2/H_3)$.

Representing subgroups by their tuples, the inclusion $(K_1, K_2, K_3, \tau) \le (H_1, H_2, H_3, \psi)$ holds if and only if $K_1 \le H_1$, $K_2 \le H_2$, $K_3 = K_2 \cap H_3$, and the diagram

$$\begin{array}{ccc} K_1 & \xrightarrow{\ \tau\ } & K_2/K_3 \cong K_2H_3/H_3 \\ \int i & & \int i \\ H_1 & \xrightarrow[\ \psi\ ]{} & H_2/H_3 \end{array}$$

commutes. □

METHOD

When the degree is prime then every transitive group is primitive. For degree 4 and 6 the subgroup lattice of the symmetric group can be easily determined. For degree 8 and 9, an implementation of the subgroup lattice program [8] was kindly modified by Felsch and Neubuser to indicate the transitive subgroups. It was applied to the stabilizers of the partitions $[4^2]$, $[2^4]$, and $[3^3]$. The algorithm [1] determined the groups which preserved both partitions of type $[4^2]$ and $[2^4]$ thus allowing us to avoid duplication. The conjugacy classes of elements of the groups were then determined, and any cases of possible conjugacy in the symmetric group were investigated using the algorithm [3] which determines conjugacy of subgroups in a permutation group.

We used the implementation of the algorithms [1,3] in CAYLEY [4]. The system also determined the conjugacy classes of elements of the groups, except that for large groups we used the character tables [6] or theoretical considerations.

For degree 10 the transitive subgroups of the partition stabilizers were determined by hand. We then proceeded as in the degree 8 case.

Let G be the stabilizer of the partition $[\{1,2,\ldots,5\}, \{6,7,\ldots,10\}]$ and let $f = (1,6)(2,7)..(5,10)$. Then G is isomorphic to $(\Sigma_5 \times \Sigma_5)Z_2$. Let L be the subgroup of G stabilizing one set of the partition. Then $L \cong \Sigma_5 \times \Sigma_5$. Let H be a transitive subgroup of G and let $K = H \cap L$. As there is an element g in H-K interchanging the two sets of the partition, $K = (K_1, K_2, K_3, \psi)$ where $K_1 = K_2^{\,f}$ is transitive on 5 letters. Hence K_1 is Σ_5, A_5, $F_5^{\,4}$, $F_5^{\,2}$ or Z_5. In Σ_5 there is one class of each isomorphism type and their normalizers are Σ_5, Σ_5, $F_5^{\,4}$, $F_5^{\,4}$, and $F_5^{\,4}$ respectively. Their normal subgroups are obvious. There is one orbit (of surjective homomorphisms) on $\mathrm{Hom}(K_1, K_2/K_3)$ in each case, except for $\mathrm{Hom}(F_5^{\,4}, F_5^{\,4}/Z_5)$ which has two orbits with representatives $a \to Z_5 a$ and $a \to Z_5 a^{-1}$. Having determined K, we determine H by

considering the classes of involuti

representative we ask whether $\langle K,g\rangle$

interchanges the two sets of the par

$\langle N_L(K), f\rangle$.

We list the results in Table 1.

Let G be the stabilizer of the

$\{7,8\},\{9,10\}]$. Define the elements

5, and $f = (1,3,5,7,9)(2,4,6,8,10)$.

$L_2 = \langle a_i a_{i+1} \mid i = 1,2,\ldots,4\rangle$, $L_3 =$

Then L_1 is an elementary abelian sub

normal in G and G/L_1 is isomorphic t

subgroups of L_1 are L_i, $i = 1,2,3$ an

Let H be a transitive subgroup

can assume F is a Sylow 5-subgroup o

1,2,3 or K = 1. Using the canonical

choose $\overline{H}$ to be a transitive subgroup

$\langle(1,2,3,4,5)\rangle$. Then for each pair (

extending the action of $\overline{H}$ on the 5 s

action on the 10 letters.

For example, let $\overline{H} = \langle \overline{f}, \overline{g}\rangle \cong F_5^{2}$

let $K = L_2$. Then g has order 2 or 4

assume $g = (1)(2)(3,9)(4,10)\ldots$ Usi

necessary we can assume $g = (3,9)(4,$

$(4,10)(5,7,6,8)$.

The group H is also imprimitive

if K = 1 or L_3.

Tables

For each degree we present the

of that degree in a set of tables.

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we determine H by

considering the classes of involutions of $N_{\Sigma_{10}}(K)/K$. If $\bar{g}$ is a representative we ask whether $\langle K,g\rangle$ is transitive (that is, if g interchanges the two sets of the partition). Note that $N_{\Sigma_{10}}(K) = \langle N_L(K), f\rangle$.

We list the results in Table 1.

Let G be the stabilizer of the partition [{1,2},{3,4},{5,6},{7,8},{9,10}]. Define the elements $a_i = (2i-1,2i)$ for $i = 1,2,\ldots,5$, and $f = (1,3,5,7,9)(2,4,6,8,10)$. Let $L_1 = \langle a_i \mid i=1,2,\ldots,5\rangle$, $L_2 = \langle a_i a_{i+1} \mid i = 1,2,\ldots,4\rangle$, $L_3 = \langle a_1a_2a_3a_4a_5\rangle$ and $F = \langle f\rangle$. Then L_1 is an elementary abelian subgroup of order 2^5 which is normal in G and G/L_1 is isomorphic to Σ_5. The only F-invariant subgroups of L_1 are L_i, i = 1,2,3 and the identity.

Let H be a transitive subgroup of G and let $K = H \cap L_1$. We can assume F is a Sylow 5-subgroup of H and hence $K = L_i$, i = 1,2,3 or K = 1. Using the canonical homomorphism $\bar{\ }: G \to G/L_1$ we choose $\bar{H}$ to be a transitive subgroup of Σ_5 containing $\bar{F} = \langle(1,2,3,4,5)\rangle$. Then for each pair $(K,\bar{H})$ we determine H by extending the action of $\bar{H}$ on the 5 sets of imprimitivity to an action on the 10 letters.

For example, let $\bar{H} = \langle\bar{f},\bar{g}\rangle \cong F_5^2$ where $\bar{g} = (2,5)(3,4)$ and let $K = L_2$. Then g has order 2 or 4. As $(1,2)(3,4) \in L_2$ we can assume $g = (1)(2)(3,9)(4,10)\ldots$ Using other elements of L_2 if necessary we can assume $g = (3,9)(4,10)(5,7)(6,8)$ or $g = (3,9)(4,10)(5,7,6,8)$.

The group H is also imprimitive of type [5^2] if and only if K = 1 or L_3.

Tables

For each degree we present the information about the transitive groups of that degree in a set of tables. The groups are named T1, T2, etc..., for

TABLE 1

	(K_1,K_3)	generators of K	$N_L(K)$	$N_{\Sigma_{10}}(K)/K$	g
1.	$(\Sigma_5,1)$	$a_1a_2,\ d_1d_2$	1	Z_2	f
2.	(Σ_5,A_5)	a_1,a_2,c_1,c_2,d_1d_2	3	$Z_2 \times Z_2$	f, d_1f
3.	(Σ_5,Σ_5)	a_1,d_1,a_2,d_2	3	Z_2	f
4.	$(A_5,1)$	a_1a_2,c_1c_2	1	$Z_2 \times Z_2$	f, d_1d_2f
5.	(A_5,A_5)	a_1,c_1,a_2,c_2	3	D_8	f
6.	$(F_5^4,1)$	a_1a_2,b_1b_2	6	Z_2	f
7.	(F_5^4,Z_5)	a_1,a_2,b_1b_2	10	D_8	f b_1f
8.	"	$a_1,a_2,b_1b_2^{-1}$	10	$Z_2 \times Z_4$	f b_1^2f
9.	(F_5^4,F_5^2)	$a_1,b_1^2,a_2,b_2^2,b_1b_2$	10	$Z_2 \times Z_2$	f, b_1f
10.	(F_5^4,F_5^4)	a_1,b_1,a_2,b_2	10	Z_2	f
11.	$(F_5^2,1)$	$a_1a_2,b_1^2b_2^2$	6	$Z_2 \times Z_2$	f , b_1b_2f
12.	(F_5^2,Z_5)	$a_1,a_2,b_1^2b_2^2$	10	W	f , b_1b_2f
13.	(F_5^2,F_5^2)	a_1,b_1^2,a_2,b_2^2	10	D_8	f
14.	$(Z_5,1)$	a_1a_2	6	$Z_2 \times Z_4$	f , $b_1^2b_2^2f$
15.	(Z_5,Z_5)	a_1,a_2	10	$(Z_4 \times Z_4)\ Z_2$	f

$a_1 = (1,2,3,4,5)$ $b_1 = (2,3,5,4)$ $c_1 = (1,5)(2,3)$ $d_1 = (1,2)$

$f = (1,6)(2,7)(3,8)(4,9)(5,10)$ $a_2 = a_1^f,\ b_2 = b_1^f,\ c_2 = c_1^f,\ d_2 = d_1^f$

$W = \langle b_1,b_2,\ f \mid b_1^2 = b_2^2,\ b_1^f = b_2,\ f^2 = b_1^4 = (b_1b_2)^2 = 1\rangle$

TABLE II: Groups

$(K,\bar{H})$
$(1,\ F_5^2)$
$(1,F_5^4)$
$(1,\Sigma_5)$
(L_3,Z_5)
(L_3,F_5^2)
(L_3,F_5^4)
(L_3,A_5)
(L_3,Σ_5)
(L_2,Z_5)
(L_2,F_5^2)
(L_2,F_5^4)
$(L_2,\ A_5)$
$(L_2,\ \Sigma_5)$
(L_1,Z_5)
(L_1,F_5^2)
(L_1,F_5^4)
(L_1,A_5)
(L_1,Σ_5)

TABLE II: Groups imprimitive of type $[2^5]$

$(K,\bar{H})$	g
$(1, F_5^2)$	(1,2)(3,10)(4,9)(5,8)(6,7)
$(1,F_5^4)$	(1,2)(3,6,9,8)(4,5,10,7)
$(1,\Sigma_5)$	(1,4)(2,3)(5,6)(7,8)(9,10)
(L_3,Z_5)	-
(L_3,F_5^2)	g_1 = (3,9)(4,10)(5,7)(6,8)
(L_3,F_5^4)	g_2 = (3,5,9,7)(4,6,10,8)
(L_3,A_5)	(1,9)(2,10)(3,5)(4,6)
(L_3,Σ_5)	(1,3)(2,4)
(L_2,Z_5)	-
(L_2,F_5^2)	g_1 or (3,9)(4,10)(5,7,6,8)
(L_2,F_5^4)	g_2 or (1,2)(3,5,9,7)(4,6,10,8)
(L_2, A_5)	g_3 = (1,9)(2,10)(3,5)(4,6)
(L_2, Σ_5)	g_4 = (1,9)(2,10) or (1,9)(2,10)(3,4)
(L_1,Z_5)	-
(L_1,F_5^2)	g_1
(L_1,F_5^4)	g_2
(L_1,A_5)	g_3
(L_1,Σ_5)	g_4

convenience, and if there may be confusion about the degree of the group we te n T i to mean the i-th group of degree n.

In Table A we list the order of the group, whether it contains only even permutations, the number of inequivalent minimal sets of imprimitivity of each possible type, and the number of conjugacy classes of elements. If the group has a faithful representation of smaller degree this is given in the column headed 'Other Representations', and if the group is known by a common name this name is given in the column headed 'Name'.

In Table B we give a set of generators for each group.

Table C sets out the number of elements of each group with each cycle type.

The notation for the group names is as follows: n denotes the cyclic group of order n; p^n denotes an elementary abelian group of order p^n, where p is a prime; D_n denotes the dihedral group of order n; Q_8 is the quaternion group of order 8; A_n is the alternating group of degree n; Σ_n is the symmetric group of degree n. If A and B are names for groups then A·B denotes a group with a normal subgroup isomorphic to A such that (A·B)/A is isomorphic to B; while AxB denotes the direct product.

For n=11, to save space the cycle types of elements occurring only in alternating and symmetric groups are omitted. An element of cycle type $1^{a_1}2^{a_2}\ldots k^{a_k}$ occurs $n!/\Pi\, i^{a_i}a_i!$ times or not at all. Only those permutations with Σa_{2j} even occur in the alternating group.

Let N_n denote the number of transitive groups of degree n. We find

n =	1	2	3	4	5	6	7	8	9	10	11
N_n =	1	1	2	5	5	16	7	50	34	45	8

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Table 3A: groups of degree 3

Group	Order	Even
T1	3	+
T2	6	

Table 3B: group generators

a = (1,2,3)

T1 = <a>

Table 3C: cycle type distribution

	1^3	$1\,2$
T1	1	.
T2	1	3

Table 4A: groups of degree 4

Group	Order	Even
T1	4	
T2	4	+
T3	8	
T4	12	+
T5	24	

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8

Table 3A: groups of degree 3

Group	Order	Even	Number of classes	Name
T1	3	+	3	A_3
T2	6		3	Σ_3

Table 3B: group generators

a = (1,2,3)   b = (1,2)

T1 = <a>   T2 = <a,b>

Table 3C: cycle type distribution

	1^3	$\overset{2}{1}$	3
T1	1	.	2
T2	1	3	2

Table 4A: groups of degree 4

Group	Order	Even	Imprimitive $[2^2]$	Number of classes	Name
T1	4		✓	4	4
T2	4	+	3	4	2^2
T3	8		✓	5	D_8
T4	12	+		4	A_4
T5	24			5	Σ_4

Table 4B: group generators

a = (1,3,4) c = (2,4)

b = (1,3) d = (1,2)(3,4)

T1	=	$\langle ac \rangle$
T2	=	$\langle bc, d \rangle$
T3	=	$\langle ac, bc \rangle$
T4	=	$\langle a, d \rangle$
T5	=	$\langle ac, b \rangle$

Table 4C: cycle type distribution

	1^4	$2\,1^2$	2^2	$3\,1$	4
T1	1	.	1	.	2
T2	1	.	3	.	.
T3	1	2	3	.	2
T4	1	.	3	8	.
T5	1	6	3	8	6

Table 5A: groups of degree 5

Group	Order	Even	Number of Classes	Name
T1	5	+	5	5
T2	10	+	4	5·2
T3	20		5	5·4
T4	60	+	5	A_5
T5	120		7	Σ_5

Table 5B: group generators

a = (1,2,3,4,5)

b = (1,2)

T1	=	$\langle a \rangle$
T2	=	$\langle a, c^2 \rangle$
T3	=	$\langle a, c \rangle$

Table 5C: cycle type distribution

	1^5	$2\,1^3$	$2^2\,1$	$3\,2$
T1	1	.	.	.
T2	1	.	5	.
T3	1	.	5	.
T4	1	.	15	.
T5	1	10	15	20

= (2,4)

= (1,2)(3,4)

= <a,d>

= <ac,b>

$^3_1$	4
.	2
.	.
.	2
8	.
8	6

of Classes	Name
5	5
4	5·2
5	5·4
5	A_5
7	Σ_5

Table 5B: group generators

a = (1,2,3,4,5) c = (2,3,5,4)

b = (1,2)

T1	=	<a>	T4	=	<a,bab>
T2	=	$\langle a,c^2\rangle$	T5	=	<a,b>
T3	=	<a,c>			

Table 5C: cycle type distribution

	1^5	$2\ 1^3$	$2^2\ 1$	$3\ 2$	$3\ 1^2$	$4\ 1$	5
T1	1	.	.	.	.	.	4
T2	1	.	5	.	.	.	4
T3	1	.	5	.	.	10	4
T4	1	.	15	.	20	.	24
T5	1	10	15	20	20	30	24

Table 6A: groups of degree 6

Group	Order	Even	Imprimitive $[2^3]$	$[3^2]$	Number of Classes	Other Representations	Name
T1	6		✓	✓	6		6
T2	6		3	✓	3	3T2	Σ_3
T3	12		✓	✓	6		D_{12}
T4	12	+	✓		4	4T4	A_4
T5	18			✓	9		$3x\Sigma_3$
T6	24		✓		8		$2xA_4$
T7	24	+	✓		5	4T5	$\Sigma_4/2^2$
T8	24		✓		5	4T5	$\Sigma_4/4$
T9	36			✓	9		$3^2{\cdot}2^2$
T10	36	+		✓	6		$3^2{\cdot}4$
T11	48		✓		10		$2x\Sigma_4$
T12	60	+			5	5T4	L(2,5)
T13	72			✓	9		$3^2{\cdot}D_8$
T14	120				7	5T5	PGL(2,5)
T15	360	+			7		A_6
T16	720				11		Σ_6

Table 6B: group generators

a = (1,2,3)
b = (1,4)(2,5)(3,6)
c = (1,5,2,4)(3,6)
d = ab
e = bc^2
f = (1,2)
g = (1,3,5)(2,4,6)
h = $fgfg^2$

T1 = <d>
T2 = <e,j>
T3 = <d,e>
T4 = <g,h>
T5 = <a,b>
T6 = <g,f>
T7 = <g,h,i>
T8 = <g,h,j>
T9 = <a,b,e>
T10 = <a,c>

r of ses	Other Representations	Name
		6
	3T2	Σ_3
		D_{12}
	4T4	A_4
		$3x\Sigma_3$
		$2xA_4$
	4T5	$\Sigma_4/2^2$
	4T5	$\Sigma_4/4$
		$3^2 \cdot 2^2$
		$3^2 \cdot 4$
		$2x\Sigma_4$
	5T4	L(2,5)
		$3^2 \cdot D_8$
	5T5	PGL(2,5)
		A_6
		Σ_6

Table 6B: group generators

a = (1,2,3)

b = (1,4)(2,5)(3,6)

c = (1,5,2,4)(3,6)

d = ab

e = bc^2

f = (1,2)

g = (1,3,5)(2,4,6)

h = $fgfg^2$

i = (1,3)(2,4)

j = (1,6)(2,5)(3,4)

k = (1,2,3,4,5)

l = (1,6)(2,5)

m = (2,3,5,4)

T1 = <d>

T2 = <e,j>

T3 = <d,e>

T4 = <g,h>

T5 = <a,b>

T6 = <g,f>

T7 = <g,h,i>

T8 = <g,h,j>

T9 = <a,b,e>

T10 = <a,c>

T11 = <f,g,i>

T12 = <k,l>

T13 = <a,b,c>

T14 = <k,l,m>

T15 = <c,k>

T16 = <d,k>

Table 6C: cycle type distribution

	1^6	$2\ 1^4$	$2^2\ 1^2$	2^3	$3\ 1^3$	$3\ 2\ 1$	3^2	$4\ 1^2$	$4\ 2$	$5\ 1$	6
T1	1	.	.	1	.	.	2	.	.	.	2
T2	1	.	.	3	.	.	2	.	.	.	.
T3	1	.	3	4	.	.	2	.	.	.	2
T4	1	.	3	.	.	.	8	.	.	.	.
T5	1	.	.	3	4	.	4	.	.	.	6
T6	1	3	3	1	.	.	8	.	.	.	8
T7	1	.	9	.	.	.	8	.	6	.	.
T8	1	.	3	6	.	.	8	6	.	.	.
T9	1	.	9	6	4	.	4	.	.	.	12
T10	1	.	9	.	4	.	4	.	18	.	.
T11	1	3	9	7	.	.	8	6	6	.	8
T12	1	.	15	.	.	.	20	.	.	24	.
T13	1	6	9	6	4	12	4	.	18	.	12
T14	1	.	15	10	.	.	20	30	.	24	20
T15	1	.	45	.	40	.	40	.	90	144	.
T16	1	15	45	15	40	120	40	90	90	144	120

Table 7A: groups of degree

Group	Order	Ev
T1	7	
T2	14	
T3	21	
T4	42	
T5	168	
T6	2520	
T7	5040	

Table 7B: group generators

a = (1,2,3,4,5,6,

b = (2,4,3,7,5,6)

T1 = <a>

T2 = <a,b

T3 = <a,b

T4 = <a,b

T5 = <a,c

3^2	$4\,1^2$	$4\,2$	$5\,1$	6
2	.	.	.	2
2	.	.	.	.
2	.	.	.	2
8	.	.	.	.
4	.	.	.	6
8	.	.	.	8
8	.	6	.	.
8	6	.	.	.
4	.	.	.	12
4	.	18	.	.
8	6	6	.	8
20	[illegible]	.	24	.
4	.	18	.	12
20	30	.	24	20
40	.	90	144	.
40	90	90	144	120

Table 7A: groups of degree 7

Group	Order	Even	Number of Classes	Name
T1	7	+	7	7
T2	14		5	D_{14}
T3	21	+	5	7·3
T4	42		7	7·6
T5	168	+	6	L(3,2)
T6	2520	+	9	A_7
T7	5040		15	Σ_7

Table 7B: group generators

a = (1,2,3,4,5,6,7)

b = (2,4,3,7,5,6)

c = (2,3)(4,7)

d = (1,2,3)

T1 = $\langle a \rangle$

T2 = $\langle a, b^3 \rangle$

T3 = $\langle a, b^2 \rangle$

T4 = $\langle a, b \rangle$

T5 = $\langle a, c \rangle$

T6 = $\langle a, d \rangle$

T7 = $\langle b, d \rangle$

Table 7C: cycle type distribution

	1^7	$2\ 1^5$	$2^2\ 1^3$	$2^3\ 1$	$3\ 1^4$	$3\ 2\ 1^2$	$3\ 2^2$	$3^2\ 1$	$4\ 1^3$	$4\ 2\ 1$	$4\ 3$	$5\ 1^2$	$5\ 2$	$6\ 1$	7
T1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	6
T2	1	.	.	7	.	.	.	.	.	.	.	.	.	.	6
T3	1	.	.	.	.	.	.	14	.	.	.	.	.	.	6
T4	1	.	.	7	.	.	.	14	.	.	.	.	.	14	6
T5	1	.	21	.	.	.	.	56	.	42	.	.	.	.	48
T6	1	.	105	.	70	.	210	280	.	630	.	504	.	.	720
T7	1	21	105	105	70	420	210	280	210	630	420	504	504	840	720

Table 8A: groups of degree 8

Group	Order	Even	Imprimiti $[2^4]$	$[4^2$
T1	8		√	√
T2	8	+	3	√
T3	8	+	7	√
T4	8	+	5	√
T5	8	+	√	√
T6	16		√	√
T7	16		√	√
T8	16		√	√
T9	16	+	3	√
T10	16	+	3	√
T11	16	+	√	√
T12	24	+	√	
T13	24	+	√	√
T14	24	+	√	√
T15	32		√	√
T16	32		√	√
T17	32		√	√
T18	32	+	3	√
T19	32	+	√	√
T20	32	+	√	√
T21	32		√	√
T22	32	+	√	√
T23	48		√	
T24	48	+	√	√

Table 8A: groups of degree 8

Group	Order	Even	Imprimitive $[2^4]$	Imprimitive $[4^2]$	Number of Classes	Other Representations	Name
T1	8		✓	✓	8		8
T2	8	+	3	✓	8		2x4
T3	8	+	7	✓	8		2^3
T4	8	+	5	✓	5	4T3	D_8
T5	8	+	✓	✓	5		Q_8
T6	16		✓	✓	7		
T7	16		✓	✓	10		
T8	16		✓	✓	7		
T9	16	+	3	✓	10		
T10	16	+	3	✓	10		
T11	16	+	✓	✓	10		
T12	24	+	✓		7		SL(2,3)
T13	24	+	✓	✓	8	6T6	$2xA_4$
T14	24	+	✓	✓	5	4T5	Σ_4
T15	32		✓	✓	11		
T16	32		✓	✓	11		
T17	32		✓	✓	14		
T18	32	+	3	✓	14		
T19	32	+	✓	✓	11		
T20	32	+	✓	✓	11		
T21	32		✓	✓	11		
T22	32	+	✓	✓	17		
T23	48		✓		8		
T24	48	+	✓	✓	10	6T11	$2x\Sigma_4$

Table 8A (continued)

Group	Order	Even	Imprimitive $[2^4]$	$[4^2]$	Number of Classes	Other Representations	Name
T25	56	+			8		$2^3 \cdot 7$
T26	64		✓	✓	16		
T27	64		✓	✓	13		
T28	64		✓	✓	13		
T29	64	+	✓	✓	16		
T30	64		✓	✓	13		
T31	64		✓	✓	16		
T32	96	+	✓		11		
T33	96	+		✓	10		
T34	96	+		✓	10		
T35	128		✓	✓	20		
T36	168	+			8		$2^3 \cdot (7 \cdot 3)$
T37	168	+			6	7T5	L(2,7)
T38	192		✓		16		
T39	192	+	✓		13		
T40	192		✓		13		
T41	192	+		✓	14		
T42	288	+		✓	14		
T43	336				9		PGL(2,7)
T44	384		✓		20		
T45	576	+		✓	16		
T46	576			✓	13		
T47	1152			✓	20		
T48	1344	+			11		$2^3 \cdot L(3,2)$
T49	20160	+			14		A_8
T50	40320				22		Σ_8

Table 8B: group generators

a = (1,4,6,8,2,3,5.7)

b = (1,3,5,7)(2,4,6,8)

c = (1,6)(2,5)(3,8)(4,7

d = (1,8)(2,7)(3,6)(4,5

e = (1,7)(2,8)(3,5)(4,6

f = (1,7)(2,8)(3,6)(4,5

g = (1,7,2,8)(3,5,4,6)

h = (3,4)(7,8)

i = (1,6)(2,5)(3,4)

j = (1,6)(2,5)(3,7)(4,8

k = (1,6)(2,5)

l = (1,3)(2,4)(5,8)(6,7

m = (1,5)(2,6)(3,7)(4,8

n = (3,5,7)(4,6,8)

o = (1,4)(2,3)(5,6)(7,8

p = (1,2)(7,8)

T1 = <a>

T2 = <b,c>

T3 = $<b^2,e,c>$

T4 = <b,d>

T5 = $<a^2,g>$

T6 = <a,f>

T7 = <a,h>

T8 = <a,i>

T9 = <b,e,c>

...er of ...sses	Other Representations	Name
8		$2^3\cdot 7$
6		
3		
3		
6		
3		
6		
1		
0		
0		
0		
8		$2^3\cdot(7\cdot 3)$
6	7T5	L(2,7)
6		
3		
3		
4		
4		
9		PGL(2,7)
0		
6		
3		
0		
1		$2^3\cdot L(3,2)$
4		A_8
2		Σ_8

Table 8B: group generators

a = (1,4,6,8,2,3,5.7)

b = (1,3,5,7)(2,4,6,8)

c = (1,6)(2,5)(3,8)(4,7)

d = (1,8)(2,7)(3,6)(4,5)

e = (1,7)(2,8)(3,5)(4,6)

f = (1,7)(2,8)(3,6)(4,5)

g = (1,7,2,8)(3,5,4,6)

h = (3,4)(7,8)

i = (1,6)(2,5)(3,4)

j = (1,6)(2,5)(3,7)(4,8)

k = (1,6)(2,5)

l = (1,3)(2,4)(5,8)(6,7)

m = (1,5)(2,6)(3,7)(4,8)

n = (3,5,7)(4,6,8)

o = (1,4)(2,3)(5,6)(7,8)

p = (1,2)(7,8)

q = (1,6,2,5)(3,7)(4,8)

r = (5,6)

s = (1,3)(2,4)

t = (1,2)

u = (1,5)(2,6)

v = (3,4)

w = (1,3)(2,4)(7,8)

x = (2,4,3)(6,8,7)

y = (1,8)(2,5)(3,6)(4,7)

z = (6,8,7)

A = (1,2,3,4,5,6,7)

B = (2,4,3,7,5,6)

C = (2,3)(4,7)

D = (1,8)(2,4)(3,7)(5,6)

E = (1,8)(2,7)(3,4)(5,6)

F = (1,7,3,5)(2,8,4,6)

T1 = <a>

T2 = <b,c>

T3 = <b^2,e,c>

T4 = <b,d>

T5 = <a^2,g>

T6 = <a,f>

T7 = <a,h>

T8 = <a,i>

T9 = <b,e,c>

T26 = <a,f,b^2>

T27 = <a,t>

T28 = <a,u>

T29 = <b,e,f>

T30 = <b,p,iku>

T31 = <q,e,t>

T32 = <e,j,n>

T33 = <F,x>

T34 = <vsv,x,y>

Table 8B (continued)

T10 = <b,j>	T35 = <a,f,t>
T11 = <a^2,b^2,l>	T36 = <A,D,B^2>
T12 = <g,n>	T37 = <A,B^2,E>
T13 = <hj,n>	T38 = <v,e,n>
T14 = <n,o>	T39 = <j,n,s>
T15 = <a,f,h>	T40 = <j,n,shv>
T16 = <a,b^2>	T41 = <F,x,y>
T17 = <a,e>	T42 = <s,z,m>
T18 = <b,e,j>	T43 = <A,B,E>
T19 = <b,f>	T44 = <t,b,s>
T20 = <b,p>	T45 = <s,z,m,y>
T21 = <q,e>	T46 = <s,z,q>
T22 = <a^2,b^2,j,e>	T47 = <$vsxz^{-1}$,t,m>
T23 = <n,w>	T48 = <A,C,D>
T24 = <c,n,s>	T49 = <A,z>
T25 = <A,D>	T50 = <A,z,t>

Table 8C: cycle type distributio

	1^8	$2\ 1^6$	$2^2\ 1^4$	$2^3\ 1^2$	2^4
T1	1	.	.	.	1
T2	1	.	.	.	3
T3	1	.	.	.	7
T4	1	.	.	.	5
T5	1	.	.	.	1
T6	1	.	.	4	5
T7	1	.	2	.	1
T8	1	.	.	4	1
T9	1	.	2	.	9
T10	1	.	2	.	5
T11	1	.	2	.	5
T12	1	.	.	.	1
T13	1	.	.	.	7
T14	1	.	.	.	9
T15	1	.	2	8	5
T16	1	.	6	.	5
T17	1	.	2	.	5
T18	1	.	6	.	13
T19	1	.	2	.	9
T20	1	.	6	.	5
T21	1	.	6	.	5
T22	1	.	6	.	13

$T35 = \langle a,f,t \rangle$

$T36 = \langle A,D,B^2 \rangle$

$T37 = \langle A,B^2,E \rangle$

$T38 = \langle v,e,n \rangle$

$T39 = \langle j,n,s \rangle$

$T40 = \langle j,n,shv \rangle$

$T41 = \langle F,x,y \rangle$

$T42 = \langle s,z,m \rangle$

$T43 = \langle A,B,E \rangle$

$T44 = \langle t,b,s \rangle$

$T45 = \langle s,z,m,y \rangle$

$T46 = \langle s,z,q \rangle$

$T47 = \langle vsxz^{-1},t,m \rangle$

$T48 = \langle A,C,D \rangle$

$T49 = \langle A,z \rangle$

$T50 = \langle A,z,t \rangle$

Table 8C: cycle type distribution

	1^8	$2\,1^6$	$2^2\,1^4$	$2^3\,1^2$	2^4	$3\,1^5$	$3\,2\,1^3$	$3\,2^2\,1$	$3^2\,1^2$	$3^2\,2$	$4\,1^4$	$4\,2\,1^2$	$4\,2^2$
T1	1	.	.	.	1	.	.	.	.	.	.	.	.
T2	1	.	.	.	3	.	.	.	.	.	.	.	.
T3	1	.	.	.	7	.	.	.	.	.	.	.	.
T4	1	.	.	.	5	.	.	.	.	.	.	.	.
T5	1	.	.	.	1	.	.	.	.	.	.	.	.
T6	1	.	.	4	5	.	.	.	.	.	.	.	.
T7	1	.	2	.	1	.	.	.	.	.	.	.	.
T8	1	.	.	4	1	.	.	.	.	.	.	.	.
T9	1	.	2	.	9	.	.	.	.	.	.	.	.
T10	1	.	2	.	5	.	.	.	.	.	.	.	.
T11	1	.	2	.	5	.	.	.	.	.	.	.	.
T12	1	.	.	.	1	.	.	.	8	.	.	.	.
T13	1	.	.	.	7	.	.	.	8	.	.	.	.
T14	1	.	.	.	9	.	.	.	8	.	.	.	.
T15	1	.	2	8	5	.	.	.	.	.	.	.	.
T16	1	.	6	.	5	.	.	.	.	.	.	.	.
T17	1	.	2	.	5	.	.	.	.	.	4	.	4
T18	1	.	6	.	13	.	.	.	.	.	.	.	.
T19	1	.	2	.	9	.	.	.	.	.	.	8	.
T20	1	.	6	.	5	.	.	.	.	.	.	.	.
T21	1	.	6	.	5	.	.	.	.	.	.	.	16
T22	1	.	6	.	13	.	.	.	.	.	.	.	.

Table 8C (continued)

	1^8	$2\ 1^6$	$2^2\ 1^4$	$2^3\ 1^2$	2^4	$3\ 1^5$	$3\ 2\ 1^3$	$3\ 2^2\ 1$	$3^2\ 1^2$	$3^2\ 2$	$4\ 1^4$	$4\ 2\ 1^2$	$4\ 2^2$
T23	1	.	.	12	1	.	.	.	8	.	.	.	.
T24	1	.	6	.	13	.	.	.	8	.	.	.	.
T25	1	.	.	.	7	.	.	.	.	.	.	.	.
T26	1	.	6	8	13	.	.	.	.	.	4	.	4
T27	1	4	6	4	5	.	.	.	.	.	.	.	8
T28	1	.	10	.	9	.	.	.	.	.	.	8	16
T29	1	.	10	.	17	.	.	.	.	.	.	8	.
T30	1	.	6	8	5	.	.	.	.	.	4	.	20
T31	1	4	6	4	13	.	.	.	.	.	.	.	24
T32	1	.	6	.	13	.	.	.	32	.	.	.	.
T33	1	.	6	.	13	.	.	.	32	.	.	.	.
T34	1	.	6	.	21	.	.	.	32	.	.	.	.
T35	1	4	10	12	17	.	.	.	.	.	4	8	28
T36	1	.	.	.	7	.	.	.	56	.	.	.	.

Table 8C (continued)

	1^8	$2\ 1^6$	$2^2\ 1^4$	$2^3\ 1^2$	2^4	$3\ 1^5$
T37	1	.	.	.	21	.
T38	1	4	6	4	13	.
T39	1	.	18	.	25	.
T40	1	.	6	24	13	.
T41	1	.	18	.	25	.
T42	1	.	6	.	21	16
T43	1	.	.	28	21	.
T44	1	4	18	28	25	.
T45	1	.	42	.	33	16
T46	1	.	42	.	9	16
T47	1	12	42	36	33	16
T48	1	.	42	.	49	.
T49	1	.	210	.	105	112
T50	1	28	210	420	105	112

3 2^2 1	3^2 1^2	3^2 2	4 1^4	4 2 1^2	4 2^2
.	8	.	.	.	.
.	8	.	.	.	.
.	.	.	.	.	.
.	.	.	4	.	4
.	.	.	.	.	8
.	.	.	.	8	16
.	.	.	.	8	.
.	.	.	4	.	20
.	.	.	.	.	24
.	[illegible]	.	.	.	.
.	[illegible]	.	.	.	.
.	32	.	.	.	.
.	.	.	4	8	28
.	56	.	.	.	.

Table 8C (continued)

	1^8	$2\ 1^6$	$2^2\ 1^4$	$2^3\ 1^2$	2^4	$3\ 1^5$	$3\ 2\ 1^3$	$3\ 2^2\ 1$	$3^2\ 1^2$	$3^2\ 2$	$4\ 1^4$	$4\ 2\ 1^2$	$4\ 2^2$
T37	1	.	.	.	21	.	.	.	56	.	.	.	.
T38	1	4	6	4	13	.	.	.	32	32	.	.	24
T39	1	.	18	.	25	.	.	.	32	.	.	24	.
T40	1	.	6	24	13	.	.	.	32	.	12	.	12
T41	1	.	18	.	25	.	.	.	32	.	.	24	.
T42	1	.	6	.	21	16	.	48	64	.	.	.	.
T43	1	.	.	28	21	.	.	.	56	.	.	.	.
T44	1	4	18	28	25	.	.	.	32	32	12	24	36
T45	1	.	42	.	33	16	.	48	64	.	.	72	.
T46	1	.	42	.	9	16	.	48	64	.	.	72	144
T47	1	12	42	36	33	16	96	48	64	.	12	72	180
T48	1	.	42	.	49	.	.	.	224	.	.	168	.
T49	1	.	210	.	105	112	.	1680	1120	.	.	2520	.
T50	1	28	210	420	105	112	1120	1680	1120	1120	420	2520	1260

Table 8C (continued)

	4 3 1	4^2	$5\ 1^3$	5 2 1	5 3	$6\ 1^2$	6 2	7 1	8
T1	.	2	.	.	.	.	.	.	4
T2	.	4	.	.	.	.	.	.	.
T3	.	.	.	.	.	.	.	.	.
T4	.	2	.	.	.	.	.	.	.
T5	.	6	.	.	.	.	.	.	.
T6	.	2	.	.	.	.	.	.	4
T7	.	4	.	.	.	.	.	.	8
T8	.	6	.	.	.	.	.	.	4
T9	.	4	.	.	.	.	.	.	.
T10	.	8	.	.	.	.	.	.	.
T11	.	8	.	.	.	.	.	.	.
T12	.	6	.	.	.	.	8	.	.
T13	.	.	.	.	.	.	8	.	.
T14	.	6	.	.	.	.	.	.	.
T15	.	8	.	.	.	.	.	.	8
T16	.	4	.	.	.	.	.	.	16
T17	.	8	.	.	.	.	.	.	8
T18	.	12	.	.	.	.	.	.	.
T19	.	12	.	.	.	.	.	.	.
T20	.	20	.	.	.	.	.	.	.
T21	.	4	.	.	.	.	.	.	.
T22	.	12	.	.	.	.	.	.	.

Table 8C (continued)

	4 3 1	4^2	$5\ 1^3$	5 2 1
T23	.	6	.	.
T24	.	12	.	.
T25	.	.	.	.
T26	.	12	.	.
T27	.	20	.	.
T28	.	4	.	.
T29	.	28	.	.
T30	.	20	.	.
T31	.	12	.	.
T32	.	12	.	.
T33	.	12	.	.
T34	.	36	.	.
T35	.	28	.	.
T36	.	.	.	.
T37	.	42	.	.
T38	.	12	.	.
T39	.	60	.	.
T40	.	12	.	.
T41	.	60	.	.
T42	.	36	.	.
T43	.	42	.	.
T44	.	60	.	.

6 2	7 1	8
.	.	4
.	.	.
.	.	.
.	.	.
.	.	.
.	.	4
.	.	8
.	.	4
.	.	.
.	.	.
.	.	.
8	.	.
8	.	.
.	.	.
.	.	8
.	.	16
.	.	8
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.

Table 8C (continued)

	4 3 1	4^2	5 1^3	5 2 1	5 3	6 1^2	6 2	7 1	8
T23	.	6	.	.	.	.	8	.	12
T24	.	12	.	.	.	.	8	.	.
T25	.	.	.	.	.	.	.	48	.
T26	.	12	.	.	.	.	.	.	16
T27	.	20	.	.	.	.	.	.	16
T28	.	4	.	.	.	.	.	.	16
T29	.	28	.	.	.	.	.	.	.
T30	.	20	.	.	.	.	.	.	.
T31	.	12	.	.	.	.	.	.	.
T32	.	12	.	.	.	.	32	.	.
T33	.	12	.	.	.	.	32	.	.
T34	.	36	.	.	.	.	.	.	.
T35	.	28	.	.	.	.	.	.	16
T36	.	.	.	.	.	.	56	48	.
T37	.	42	.	.	.	.	.	48	.
T38	.	12	.	.	.	32	32	.	.
T39	.	60	.	.	.	.	32	.	.
T40	.	12	.	.	.	.	32	.	48
T41	.	60	.	.	.	.	32	.	.
T42	.	36	.	.	.	.	96	.	.
T43	.	42	.	.	.	56	.	48	84
T44	.	60	.	.	.	32	32	.	48

Table 8C (continued)

	4 3 1	4^2	5 1^3	5 2 1	5 3	6 1^2	6 2	7 1	8
T45	.	108	.	.	.	.	192	.	.
T46	.	36	.	.	.	.	.	.	144
T47	96	108	.	.	.	.	192	.	144
T48	.	252	.	.	.	.	224	384	.
T49	.	1260	1344	.	2688	.	3360	5760	.
T50	3360	1260	1344	4032	2688	3360	3360	5760	5040

Table 9A: groups of degree 9

Groups	Order	Even	Imprimitive $[3^3]$
T1	9	+	✓
T2	9	+	4
T3	18	+	✓
T4	18		2
T5	18	+	4
T6	27	+	✓
T7	27	+	✓
T8	36		2
T9	36	+	
T10	54	+	✓
T11	54	+	✓
T12	54		✓
T13	54		✓
T14	72	+	
T15	72	+	
T16	72		
T17	81	+	✓
T18	108		✓
T19	144		
T20	162		✓
T21	162	+	✓
T22	162		✓
T23	216	+	

5	6	6	7	
3	1^2	2	1	8
	.	192	.	.
	.	.	.	144
	.	192	.	144
	.	224	384	.
3	.	3360	5760	.
3	3360	3360	5760	5040

Table 9A: groups of degree 9

Groups	Order	Even	Imprimitive $[3^3]$	Numbers of Classes	Other Representations	Name
T1	9	+	✓	9		9
T2	9	+	4	9		3^2
T3	18	+	✓	6		D_{18}
T4	18		2	9	6T5	$3 \times \Sigma_3$
T5	18	+	4	6		$3^2 \cdot 2$
T6	27	+	✓	11		
T7	27	+	✓	11		
T8	36		2	9	6T9	$3^2 \cdot 2^2$
T9	36	+		6	6T10	$3^2 \cdot 4$
T10	54	+	✓	10		
T11	54	+	✓	10		
T12	54		✓	10		
T13	54		✓	10		
T14	72	+		6		$3^2 \cdot Q_8$
T15	72	+		9		$3^2 \cdot 8$
T16	72			9	6T13	$3^2 \cdot D_8$
T17	81	+	✓	17		
T18	108		✓	11		
T19	144			9		$3^2 \cdot (8 \cdot 2)$
T20	162		✓	22		
T21	162	+	✓	13		
T22	162		✓	13		
T23	216	+		10		$3^2 \cdot SL(2,3)$

Table 9A (continued)

Groups	Order	Even	Imprimitive $[3^3]$	Numbers of Classes	Others Representations	Name
T24	324		✓	17		
T25	324	+	✓	13		
T26	432			11		$3^2 \cdot GL(2,3)$
T27	504	+		9		L(2,8)
T28	648		✓	17		
T29	648		✓	17		
T30	648	+	✓	14		
T31	1296		✓	22		
T32	1512	+		11		$L(2,8) \cdot 3$
T33	$(\frac{1}{2})9!$	+		18		A_9
T34	9!			30		$\Sigma 9$

Table 9B: group generators

a = (1,5,8,2,6,9,3,4,7)

b = (1,4,7)(2,5,8)(3,6,9)

c = (1,9,5)(2,7,6)(3,8,4)

d = (1,8)(2,7)(3,9)(4,6)

e = (4,7)(5,8)(6,9)

f = (1,7)(2,9)(3,8)(5,6)

g = (2,3)(5,6)(8,9)

h = (1,3,2)(4,5,6)

i = (1,6,9)(2,4,7)(3,5,8)

j = (1,2,3)

T1 = $\langle a \rangle$

T2 = $\langle b,c \rangle$

T3 = $\langle a,d \rangle$

T4 = $\langle b,e,c \rangle$

T5 = $\langle b,c,f \rangle$

T6 = $\langle a,h \rangle$

T7 = $\langle b,c,i \rangle$

T8 = $\langle b,c,e,f \rangle$

T9 = $\langle a^3,o^2 \rangle$

T10 = $\langle a,d,h \rangle$

T11 = $\langle b,c,f,i \rangle$

T12 = $\langle b,c,i,e \rangle$

T13 = $\langle b,c,i,g \rangle$

T14 = $\langle a^3,o^2,oe \rangle$

T15 = $\langle a^3,o \rangle$

T16 = $\langle a^3,o^2,e \rangle$

T17 = $\langle a,h,j \rangle$

T18 = $\langle b,c,f,i,e \rangle$

...s of ...ses	Others Representations	Name
		$3^2 \cdot GL(2,3)$
		L(2,8)
		$L(2,8) \cdot 3$
		A_9
		$\Sigma 9$

Table 9B: group generators

a = (1,5,8,2,6,9,3,4,7)

b = (1,4,7)(2,5,8)(3,6,9)

c = (1,9,5)(2,7,6)(3,8,4)

d = (1,8)(2,7)(3,9)(4,6)

e = (4,7)(5,8)(6,9)

f = (1,7)(2,9)(3,8)(5,6)

g = (2,3)(5,6)(8,9)

h = (1,3,2)(4,5,6)

i = (1,6,9)(2,4,7)(3,5,8)

j = (1,2,3)

k = (2,3)(5,6)

l = (7,8,9)

m = (2,3)(8,9)

n = (2,3)

o = (2,6,4,9,3,8,7,5)

p = (2,4,9)(3,7,5)

q = (2,7)(3,6)(4,5)(8,9)

r = (1,2,3,4,5,6,7)

s = (2,4,3,7,5,6)

t = (1,8)(2,4)(3,7)(5,6)

T1 = $\langle a \rangle$

T2 = $\langle b,c \rangle$

T3 = $\langle a,d \rangle$

T4 = $\langle b,e,c \rangle$

T5 = $\langle b,c,f \rangle$

T6 = $\langle a,h \rangle$

T7 = $\langle b,c,i \rangle$

T8 = $\langle b,c,e,f \rangle$

T9 = $\langle a^3,o^2 \rangle$

T10 = $\langle a,d,h \rangle$

T11 = $\langle b,c,f,i \rangle$

T12 = $\langle b,c,i,e \rangle$

T13 = $\langle b,c,i,g \rangle$

T14 = $\langle a^3,o^2,oe \rangle$

T15 = $\langle a^3,o \rangle$

T16 = $\langle a^3,o^2,e \rangle$

T17 = $\langle a,h,j \rangle$

T18 = $\langle b,c,f,i,e \rangle$

T19 = $\langle a^3,o,e \rangle$

T20 = $\langle a,h,j,e \rangle$

T21 = $\langle a,h,j,d \rangle$

T22 = $\langle a,h,j,g \rangle$

T23 = $\langle a^3,o^2,p \rangle$

T24 = $\langle a,h,j,e,d \rangle$

T25 = $\langle j,hj^2,k,l,m,b \rangle$

T26 = $\langle a^3,o,p \rangle$

T27 = $\langle r,t,q \rangle$

T28 = $\langle j,n,hj^2,gn,l,mn,b \rangle$

T29 = $\langle j,hj^2,k,l,m,b,e \rangle$

T30 = $\langle j,hj^2,k,l,m,b,ne \rangle$

T31 = $\langle j,n,b,e \rangle$

T32 = $\langle r,s^2,t,q \rangle$

T33 = $\langle r,l \rangle$

T34 = $\langle r,l,n \rangle$

Table 9C: cycle type distribution

	1^9	$2\ 1^7$	$2^2\ 1^5$	$2^3\ 1^3$	$2^4\ 1$	$3\ 1^6$	$3\ 2\ 1^4$	$3\ 2^2\ 1^2$	$3\ 2^3$	$3^2\ 1^3$	$3^2\ 2\ 1$	3^3	$4\ 1^5$	$4\ 2\ 1^3$	$4\ 2^2\ 1$	$4\ 3\ 1^2$
T1	1	.	.	.	.	.	.	.	.	.	.	2	.	.	.	.
T2	1	.	.	.	.	.	.	.	.	.	.	8	.	.	.	.
T3	1	.	.	.	9	.	.	.	.	.	.	2	.	.	.	.
T4	1	.	.	3	.	.	.	.	.	.	.	8	.	.	.	.
T5	1	.	.	.	9	.	.	.	.	.	.	8	.	.	.	.
T6	1	.	.	.	.	.	.	.	.	6	.	2	.	.	.	.
T7	1	.	.	.	.	.	.	.	.	6	.	20	.	.	.	.
T8	1	.	.	6	9	.	.	.	.	.	.	8	.	.	.	.
T9	1	.	.	.	9	.	.	.	.	.	.	8	.	.	.	.
T10	1	.	.	.	9	.	.	.	.	6	.	2	.	.	.	.
T11	1	.	.	.	9	.	.	.	.	6	.	20	.	.	.	.
T12	1	.	.	9	.	.	.	.	.	6	.	20	.	.	.	.
T13	1	.	.	9	.	.	.	.	.	6	.	20	.	.	.	.
T14	1	.	.	.	9	.	.	.	.	.	.	8	.	.	.	.
T15	1	.	.	.	9	.	.	.	.	.	.	8	.	.	.	.
T16	1	.	.	12	9	.	.	.	.	.	.	8	.	.	.	.

T17	1	.	.	.	.	6	.	.	.	12	.	26	.	.	.	.
T18	1	.	.	18	9	.	.	.	.	6	.	20	.	.	.	.
T19	1	.	.	12	9	.	.	.	.	.	.	8	.	.	.	.
T20	1	.	.	9	.	6	.	.	18	12	.	26	.	.	.	.
T21	1	.	.	.	27	6	.	.	.	12	.	26	.	.	.	.
T22	1	.	.	27	.	6	.	.	.	12	.	26	.	.	.	.
T23	1	.	.	.	9	.	.	.	.	24	.	56	.	.	.	.
T24	1	.	.	36	27	6	.	.	18	12	.	26	.	.	.	.
T25	1	.	27	.	.	6	.	54	.	12	.	80	.	.	.	.

T8	1	.	.	6	9	.	.	.	.	.	.	8	.	.	.	.
T9	1	.	.	.	[illegible]	.	.	.	.	.	.	8	.	.	.	.
T10	1	.	.	.	[illegible]	.	.	.	.	6	.	2	.	.	.	.
T11	1	.	.	.	9	.	.	.	.	6	.	20	.	.	.	.
T12	1	.	.	9	.	.	.	.	.	6	.	20	.	.	.	.
T13	1	.	.	9	.	.	.	.	.	6	.	20	.	.	.	.
T14	1	.	.	.	9	.	.	.	.	.	.	8	.	.	.	.
T15	1	.	.	.	9	.	.	.	.	.	.	8	.	.	.	.
T16	1	.	.	12	9	.	.	.	.	.	.	8	.	.	.	.

T17	1	.	.	.	.	6	.	.	.	12	.	26	.	.	.	.
T18	1	.	.	18	9	.	.	.	.	6	.	20	.	.	.	.
T19	1	.	.	12	9	.	.	.	.	.	.	8	.	.	.	.
T20	1	.	.	9	.	6	.	.	18	12	.	26	.	.	.	.
T21	1	.	.	.	27	6	.	.	.	12	.	26	.	.	.	.
T22	1	.	.	27	.	6	.	.	.	12	.	26	.	.	.	.
T23	1	.	.	.	9	.	.	.	.	24	.	56	.	.	.	.
T24	1	.	.	36	27	6	.	.	18	12	.	26	.	.	.	.
T25	1	.	27	.	.	6	.	54	.	12	.	80	.	.	.	.
T26	1	.	.	36	9	.	.	.	.	24	.	56	.	.	.	.
T27	1	.	.	.	63	.	.	.	.	.	.	56	.	.	.	.
T28	1	9	27	27	.	6	36	54	.	12	36	80	.	.	.	.
T29	1	.	27	18	.	6	.	54	36	12	.	80	.	.	162	.
T30	1	.	27	.	54	6	.	54	.	12	.	80	.	54	.	.
T31	1	9	27	45	54	6	36	54	36	12	36	80	.	54	162	.
T32	1	.	.	.	63	.	.	.	.	168	.	56	.	.	.	.
T33	1	.	378	.	945	168	.	7560	.	3360	.	2240	.	7560	.	.
T34	1	36	378	1260	945	168	2520	7560	2520	3360	10080	2240	756	7560	11340	15120

Table 9C (continued)

	4 3 2	4^2 1	5 1^4	5 2 1^2	5 2^2	5 3 1	5 4	6 1^3	6 2 1	6 3	7 1^2	7 2	8 1	9
T1	.	.	.	.	.	.	.	.	.	.	.	.	.	6
T2	.	.	.	.	.	.	.	.	.	.	.	.	.	.
T3	.	.	.	.	.	.	.	.	.	.	.	.	.	6
T4	.	.	.	.	.	.	.	.	.	6	.	.	.	.
T5	.	.	.	.	.	.	.	.	.	.	.	.	.	.
T6	.	.	.	.	.	.	.	.	.	.	.	.	.	18
T7	.	.	.	.	.	.	.	.	.	.	.	.	.	.
T8	.	.	.	.	.	.	.	.	.	12	.	.	.	.
T9	.	18	.	.	.	.	.	.	.	.	.	.	.	.
T10	.	.	.	.	.	.	.	.	18	.	.	.	.	18
T11	.	.	.	.	.	.	.	.	18	.	.	.	.	.
T12	.	.	.	.	.	.	.	.	.	18	.	.	.	.
T13	.	.	.	.	.	.	.	.	.	18	.	.	.	.
T14	.	54	.	.	.	.	.	.	.	.	.	.	.	.
T15	.	18	.	.	.	.	.	.	.	.	.	.	36	.
T16	.	18	.	.	.	.	.	.	.	24	.	.	.	.
T17	.	.	.	.	.	.	.	.	.	.	.	.	.	36
T18	.	.	.	.	.	.	.	.	18	36	.	.	.	.
T19	.	54	.	.	.	.	.	.	.	24	.	.	36	.
T20	.	.	.	.	.	.	.	18	.	36	.	.	.	36
T21	.	.	.	.	.	.	.	.	54	.	.	.	.	36
T22	.	.	.	.	.	.	.	.	.	54	.	.	.	36
T23	.	54	.	.	.	.	.	.	72	.	.	.	.	.
T24	.	.	.	.	.	.	.	18	54	90	.	.	.	36
T25	.	.	.	.	.	.	.	.	.	.	.	.	.	144

T8	.	.	.	.	.	.	.	.	.	12	.	.	.	.
T9	.	18	.	.	.	.	.	.	.	.	.	.	.	.
T10	.	.	.	.	.	.	.	.	18	.	.	.	.	18
T11	.	.	.	.	.	.	.	.	18	.	.	.	.	.
T12	.	.	.	.	.	.	.	.	.	18	.	.	.	.
T13	.	.	.	.	.	.	.	.	.	18	.	.	.	.
T14	.	54	.	.	.	.	.	.	.	.	.	.	.	.
T15	.	18	.	.	.	.	.	.	.	.	.	.	36	.
T16	.	18	.	.	.	.	.	.	.	24	.	.	.	.
T17	.	.	.	.	.	.	.	.	.	.	.	.	.	36
T18	.	.	.	.	.	.	.	.	18	36	.	.	.	.
T19	.	54	.	.	.	.	.	.	.	24	.	.	36	.
T20	.	.	.	.	.	.	.	18	.	36	.	.	.	36
T21	.	.	.	.	.	.	.	.	54	.	.	.	.	36
T22	.	.	.	.	.	.	.	.	.	54	.	.	.	36
T23	.	54	.	.	.	.	.	.	72	.	.	.	.	.
T24	.	.	.	.	.	.	.	18	54	90	.	.	.	36
T25	.	.	.	.	.	.	.	.	.	.	.	.	.	144
T26	.	54	.	.	.	.	.	.	72	72	.	.	108	.
T27	.	.	.	.	.	.	.	.	.	.	216	.	.	168
T28	.	.	.	.	.	.	.	.	.	216	.	.	.	144
T29	.	.	.	.	.	.	.	36	.	72	.	.	.	144
T30	108	.	.	.	.	.	.	.	108	.	.	.	.	144
T31	108	.	.	.	.	.	.	36	108	288	.	.	.	144
T32	.	.	.	.	.	.	.	.	504	.	216	.	.	504
T33	15120	11340	3024	.	9072	24192	.	.	30240	.	25920	.	.	40320
T34	15120	11340	3024	18144	9072	24192	18144	10080	30240	20160	25920	25920	45360	40320

Table 10A: groups of degree 10

Group	Order	Even	Imprimitive $[2^5]$	Imprimitive $[5^2]$	Number of Classes	Other Representations	Name
T1	10		✓	✓	10		10
T2	10		5	✓	4	5T2	5.2
T3	20		✓	✓	8		2x5.2
T4	20		✓	✓	5		5.4
T5	40		✓	✓	10		2x5.4
T6	50			✓	20		
T7	60	+			5	5T4	A_5
T8	80	+	✓		8		
T9	100			✓	16		
T10	100			✓	10		
T11	120		✓	✓	10		
T12	120		✓	✓	7		
T13	120				7	5T5	Σ_5
T14	160		✓		16		
T15	160	+	✓		10		
T16	160		✓		10		
T17	200			✓	14		
T18	200	+		✓	11		
T19	200			✓	14		
T20	200			✓	8		
T21	200			✓	14		
T22	240		✓	✓	14		
T23	320		✓		20		
T24	320	+	✓		11		

Table 10A (continued)

Group	Order	Even	Imprimitive $[2^5]$	Imprimitive $[5^2]$
T25	320		✓	
T26	360	+		
T27	400			✓
T28	400	+		✓
T29	640		✓	
T30	720			
T31	720			
T32	720			
T33	800			✓
T34	960	+	✓	
T35	1440			
T36	1920		✓	
T37	1920	+	✓	
T38	1920		✓	
T39	3840		✓	
T40	7200			✓
T41	14400			✓
T42	14400	+		✓
T43	28800			✓
T44	($\frac{1}{2}$)10!	+		
T45	10!			

Table 10A (continued)

Group	Order	Even	Imprimitive $[2^5]$	Imprimitive $[5^2]$	Number of Classes	Other Representations	Name
T25	320		✓		11		
T26	360	+			7	6T15	L(2,9)
T27	400			✓	16		
T28	400	+		✓	13		
T29	640		✓		22		
T30	720				11		PGL(2,9)
T31	720				8		M_{10}
T32	720				11	6T16	Σ_6
T33	800			✓	20		
T34	960	+	✓		12		
T35	1440				13		PΓL(2,9)
T36	1920		✓		24		
T37	1920	+	✓		18		
T38	1920		✓		18		
T39	3840		✓		36		
T40	7200			✓	20		
T41	14400			✓	25		
T42	14400	+		✓	22		
T43	28800			✓	35		
T44	$(\frac{1}{2})10!$	+			24		A_{10}
T45	10!				42		Σ_{10}

Table 10B: group generators

a = (1,2,3,4,5)
b = (6,7,8,9,10)
c = (2,3,5,4)
d = (7,8,10,9)
e = (1,5)(2,3)
f = (6,10)(7,8)
g = (1,2)
h = (6,7)
i = (1,6)(2,7)(3,8)(4,9)(5,10)
j = (1,3,5,7,9)(2,4,6,8,10)
k = (1,2)(3,4)
l = (1,9)(2,10)(3,4)
m = (3,9)(4,10)(5,7,6,8)
n = (1,2)(3,5,9,7)(4,6,10,8)
o = (3,9)(4,10)(5,7)(6,8)
p = (3,5,9,7)(4,6,10,8)
q = (1,9)(2,10)(3,5)(4,6)
r = (1,9)(2,10)(3,5,4,6)
s = (1,2,3)(4,5,6)(7,8,9)
t = (2,6,4,9,3,8,7,5)
u = (4,7)(5,8)(6,9)
v = (1,8)(2,5,6,3)(4,9,7,10)
w = (1,5,7)(2,9,4)(3,8,10)
x = (1,10)(4,7)(5,6)(8,9)
y = (2,3,4,5,6,7,8,9,10)

T1 = $\langle ab,i \rangle$
T2 = $\langle ab,c^2d^2i \rangle$
T3 = $\langle ab,c^2d^2,i \rangle$
T4 = $\langle ab,c^2d^2,cdi \rangle$
T5 = $\langle ab,cd,i \rangle$
T6 = $\langle a,i \rangle$
T7 = $\langle v^2,w \rangle$
T8 = $\langle k,j \rangle$
T9 = $\langle a,c^2d^2,i \rangle$
T10 = $\langle a,c^2d^2,cdi \rangle$
T11 = $\langle ab,ef,i \rangle$
T12 = $\langle ab,ef,ghi \rangle$

Table 10B (continued)

T13 = $\langle v,w \rangle$
T14 = $\langle g,j \rangle$
T15 = $\langle k,j,o \rangle$
T16 = $\langle k,j,m \rangle$
T17 = $\langle a,cd,i \rangle$
T18 = $\langle a,cd,ci \rangle$
T19 = $\langle a,cd^{-1},i \rangle$
T20 = $\langle a,cd^{-1},c^2i \rangle$
T21 = $\langle a,c^2,i \rangle$
T22 = $\langle ab,gh,i \rangle$
T23 = $\langle g,j,o \rangle$
T24 = $\langle k,j,p \rangle$
T25 = $\langle k,j,n \rangle$
T26 = $\langle s,t^2,x \rangle$
T27 = $\langle a,c^2,cd,i \rangle$
T28 = $\langle a,c^2,cd,ci \rangle$
T29 = $\langle g,j,p \rangle$

n = (1,2)(3,5,9,7)(4,6,10,8)

o = (3,9)(4,10)(5,7)(6,8)

p = (3,5,9,7)(4,6,10,8)

q = (1,9)(2,10)(3,5)(4,6)

r = (1,9)(2,10)(3,5,4,6)

s = (1,2,3)(4,5,6)(7,8,9)

t = (2,6,4,9,3,8,7,5)

u = (4,7)(5,8)(6,9)

v = (1,8)(2,5,6,3)(4,9,7,10)

w = (1,5,7)(2,9,4)(3,8,10)

x = (1,10)(4,7)(5,6)(8,9)

y = (2,3,4,5,6,7,8,9,10)

7 = <v^2,w>

8

9 = <a,c^2d^2,i>

0 = <a,c^2d^2,cdi>

1 = <ab,ef,i>

2 = <ab,ef,ghi>

Table 10B (continued)

T13 = <v,w>

T14 = <g,j>

T15 = <k,j,o>

T16 = <k,j,m>

T17 = <a,cd,i>

T18 = <a,cd,ci>

T19 = <a,cd^{-1},i>

T20 = <a,cd^{-1},c^2i>

T21 = <a,c^2,i>

T22 = <ab,gh,i>

T23 = <g,j,o>

T24 = <k,j,p>

T25 = <k,j,n>

T26 = <s,t^2,x>

T27 = <a,c^2,cd,i>

T28 = <a,c^2,cd,ci>

T29 = <g,j,p>

T30 = <s,t,x>

T31 = <s,t^2,tu,x>

T32 = <s,t^2,u,x>

T33 = <a,c,i>

T34 = <k,j,q>

T35 = <s,t,u,x>

T36 = <g,j,q>

T37 = <k,j,gl>

T38 = <k,j,l>

T39 = <g,j,kl>

T40 = <a,e,i>

T41 = <a,e,gh,i>

T42 = <a,b,e,f,gh,gi>

T43 = <a,g,i>

T44 = <y,k>

T45 = <y,g>

Table 10C: cycle type distribution

	1^{10}	$2\ 1^{8}$	$2^{2}\ 1^{6}$	$2^{3}\ 1^{4}$	$2^{4}\ 1^{2}$	2^{5}	$3\ 1^{7}$	$3\ 2\ 1^{5}$	$3\ 2^{2}\ 1^{3}$	$3\ 2^{3}\ 1$	$3^{2}\ 1^{4}$	$3^{2}\ 2\ 1^{2}$	$3^{2}\ 2^{2}$
T1	1	.	.	.	.	1	.	.	.	.	.	.	.
T2	1	.	.	.	.	5	.	.	.	.	.	.	.
T3	1	.	.	.	5	6	.	.	.	.	.	.	.
T4	1	.	.	.	5	.	.	.	.	.	.	.	.
T5	1	.	.	.	5	6	.	.	.	.	.	.	.
T6	1	.	.	.	.	5	.	.	.	.	.	.	.
T7	1	.	.	.	15	.	.	.	.	.	.	.	.
T8	1	.	10	.	5	.	.	.	.	.	.	.	.
T9	1	.	.	.	25	10	.	.	.	.	.	.	.
T10	1	.	.	.	25	.	.	.	.	.	.	.	.
T11	1	.	.	.	15	16	.	.	.	.	20	.	.
T12	1	.	.	.	15	10	.	.	.	.	20	.	.
T13	1	.	.	10	15	.	.	.	.	.	.	.	.
T14	1	5	10	10	5	1	.	.	.	.	.	.	.
T15	1	.	10	.	25	.	.	.	.	.	.	.	.
T16	1	.	10	.	5	20	.	.	.	.	.	.	.
T17	1	.	.	.	25	10	.	.	.	.	.	.	.
T18	1	.	.	.	25	.	.	.	.	.	.	.	.
T19	1	.	.	.	25	20	.	.	.	.	.	.	.
T20	1	.	.	.	25	.	.	.	.	.	.	.	.
T21	1	.	10	.	25	10	.	.	.	.	.	.	.
T22	1	.	10	.	15	26	.	.	.	.	20	.	20
T23	1	5	10	10	25	21	.	.	.	.	.	.	.

T7	1	.	.	.	15	.	.	.	.	.	.	.	.
T8	1	.	10	.	[illegible]	.	.	.	.	.	.	.	.
T9	1	.	.	.	25	10	.	.	.	.	.	.	.
T10	1	.	.	.	25	.	.	.	.	.	.	.	.
T11	1	.	.	.	15	16	.	.	.	.	20	.	.
T12	1	.	.	.	15	10	.	.	.	.	20	.	.
T13	1	.	.	10	15	.	.	.	.	.	.	.	.
T14	1	5	10	10	5	1	.	.	.	.	.	.	.

T15	1	.	10	.	25	.	.	.	.	.	.	.	.
T16	1	.	10	.	5	20	.	.	.	.	.	.	.
T17	1	.	.	.	25	10	.	.	.	.	.	.	.
T18	1	.	.	.	25	.	.	.	.	.	.	.	.
T19	1	.	.	.	25	20	.	.	.	.	.	.	.
T20	1	.	.	.	25	.	.	.	.	.	.	.	.
T21	1	.	10	.	25	10	.	.	.	.	.	.	.
T22	1	.	10	.	15	26	.	.	.	.	20	.	20
T23	1	5	10	10	25	21	.	.	.	.	.	.	.
T24	1	.	10	.	25	.	.	.	.	.	.	.	.
T25	1	.	10	.	25	.	.	.	.	.	.	.	.
T26	1	.	.	.	45	.	.	.	.	.	.	.	.
T27	1	.	10	.	25	20	.	.	.	.	.	.	.
T28	1	.	10	.	25	.	.	.	.	.	.	.	.
T29	1	5	10	10	25	21	.	.	.	.	.	.	.
T30	1	.	.	.	45	36	.	.	.	.	.	.	.
T31	1	.	.	.	45	.	.	.	.	.	.	.	.

Table 10C (continued)

	1^{10}	$2\,1^8$	$2^2\,1^6$	$2^3\,1^4$	$2^4\,1^2$	2^5	$3\,1^7$	$3\,2\,1^5$	$3\,2^2\,1^3$	$3\,2^3\,1$	$3^2\,1^4$	$3^2\,2\,1^2$	$3^2\,2^2$
T32	1	.	.	30	45	.	.	.	.	.	.	.	.
T33	1	.	10	.	25	20	.	.	.	.	.	.	.
T34	1	.	10	.	65	.	.	.	.	.	80	.	80
T35	1	.	.	30	45	36	.	.	.	.	.	.	.
T36	1	5	10	10	65	61	.	.	.	.	80	160	80
T37	1	.	30	.	125	.	.	.	.	.	80	.	240
T38	1	.	10	60	65	20	.	.	.	.	80	.	80
T39	1	5	30	70	125	81	.	.	.	.	80	160	240
T40	1	.	30	.	225	60	40	.	600	.	400	.	.
T41	1	.	130	.	225	120	40	.	1000	.	400	.	400
T42	1	.	130	.	225	.	40	.	1000	.	400	.	400
T43	1	20	130	300	225	120	40	440	1000	600	400	800	400
T44	1	.	630	.	4725	.	240	.	25200	.	8400	.	25200
T45	1	45	630	3150	4725	945	240	5040	25200	25200	8400	50400	25200

Table 10C (continued)

	$3^3\,1$	$4\,1^6$	$4\,2\,1^4$	$4\,2^2\,1^2$
T1	.	.	.	.
T2	.	.	.	.
T3	.	.	.	.
T4	.	.	.	.
T5	.	.	.	.
T6	.	.	.	.
T7	20	.	.	.
T8	.	.	.	.
T9	.	.	.	.
T10	.	.	.	.
T11	.	.	.	.
T12	.	.	.	.
T13	20	.	.	.
T14	.	.	.	.
T15	.	.	.	.
T16	.	.	.	40
T17	.	.	.	.
T18	.	.	.	.
T19	.	.	.	.
T20	.	.	.	.
T21	.	.	.	.

Table 10C (continued)

	$3^3 1$	$4 1^6$	$4 2 1^4$	$4 2^2 1^2$	$4 2^3$	$4 3 1^3$	$4 3 2 1$	$4 3^2$	$4^2 1^2$	$4^2 2$
T1	.	.	.	.	.	.	.	.	.	.
T2	.	.	.	.	.	.	.	.	.	.
T3	.	.	.	.	.	.	.	.	.	.
T4	.	.	.	.	.	.	.	.	.	10
T5	.	.	.	.	.	.	.	.	10	10
T6	.	.	.	.	.	.	.	.	.	.
T7	20	.	.	.	.	.	.	.	.	.
T8	.	.	.	.	.	.	.	.	.	.
T9	.	.	.	.	.	.	.	.	.	.
T10	.	.	.	.	.	.	.	.	.	50
T11	.	.	.	.	.	.	.	.	.	.
T12	.	.	.	.	.	.	.	.	.	30
T13	20	.	.	.	.	.	.	.	.	30
T14	.	.	.	.	.	.	.	.	.	.
T15	.	.	.	.	40	.	.	.	20	.
T16	.	.	.	40	.	.	.	.	.	20
T17	.	.	.	.	.	.	.	.	50	50
T18	.	.	.	.	.	.	.	.	50	.
T19	.	.	.	.	.	.	.	.	50	.
T20	.	.	.	.	.	.	.	.	50	100
T21	.	.	.	.	.	.	.	.	.	50

Table 10C (continued)

	$3^3\ 1$	$4\ 1^6$	$4\ 2\ 1^4$	$4\ 2^2\ 1^2$	$4\ 2^3$	$4\ 3\ 1^3$	$4\ 3\ 2\ 1$	$4\ 3^2$	$4^2\ 1^2$	$4^2\ 2$
T22	.	.	.	.	.	.	.	.	30	30
T23	.	.	.	40	40	.	.	.	20	20
T24	.	.	.	.	40	.	.	.	100	.
T25	.	.	.	.	40	.	.	.	20	80
T26	80	.	.	.	.	.	.	.	90	.
T27	.	.	.	.	.	.	.	.	100	100
T28	.	.	.	.	.	.	.	.	100	.
T29	.	.	.	40	40	.	.	.	100	100
T30	80	.	.	.	.	.	.	.	90	.
T31	80	.	.	.	.	.	.	.	270	.
T32	80	.	.	.	.	.	.	.	90	90
T33	.	20	.	100	.	.	.	.	100	100
T34	.	.	.	.	120	.	.	.	60	.
T35	80	.	.	.	.	.	.	.	270	90
T36	.	.	.	120	120	.	.	.	60	60
T37	.	.	60	.	140	.	.	.	300	.
T38	.	20	.	60	120	.	.	160	60	240
T39	.	20	60	180	140	.	.	160	300	300
T40	.	.	.	.	.	.	.	.	.	900
T41	.	.	600	.	.	.	1200	.	900	1800
T42	.	.	600	.	1200	.	1200	.	900	.
T43	.	60	600	900	1200	1200	1200	.	900	1800
T44	22400	.	18900	.	18900	.	151200	.	56700	.
T45	22400	1260	18900	56700	18900	50400	151200	50400	56700	56700

Table 10C (continued)

	$5\ 1^5$	$5\ 2\ 1^3$	$5\ 2^2\ 1$	$5\ 3\ 1^2$
T1	.	.	.	.
T2	.	.	.	.
T3	.	.	.	.
T4	.	.	.	.
T5	.	.	.	.
T6	8	.	.	.
T7	.	.	.	.
T8	.	.	.	.
T9	8	.	.	.
T10	8	.	.	.
T11	.	.	.	.
T12	.	.	.	.
T13	.	.	.	.
T14	.	.	.	.
T15	.	.	.	.
T16	.	.	.	.
T17	8	.	.	.
T18	8	.	.	.
T19	8	.	.	.
T20	8	.	.	.
T21	8	.	40	.
T22	.	.	.	.

4 3 1^3	3 2 1	4 3^2	4^2 1^2	4^2 2
.	.	.	30	30
.	.	.	20	20
.	.	.	100	.
.	.	.	20	80
.	.	.	90	.
.	.	.	100	100
.	.	.	100	.
.	.	.	100	100
.	.	.	90	.
.	.	.	270	.
.	.	.	90	90
.	.	.	100	100
.	.	.	60	.
.	[illegible]	.	270	90
	[illegible]	.	60	60
	.	.	300	.
	.	160	60	240
	.	160	300	300
	.	.	.	900
	1200	.	900	1800
	1200	.	900	.
)	1200	.	900	1800
	151200	.	56700	.
)	151200	50400	56700	56700

Table 10C (continued)

	5 1^5	5 2 1^3	5 2^2 1	5 3 1^2	5 3 2	5 4 1	5^2	6 1^4	6 2 1^2	6 2^2
T1	.	.	.	.	.	.	4	.	.	.
T2	.	.	.	.	.	.	4	.	.	.
T3	.	.	.	.	.	.	4	.	.	.
T4	.	.	.	.	.	.	4	.	.	.
T5	.	.	.	.	.	.	4	.	.	.
T6	8	.	.	.	.	.	16	.	.	.
T7	.	.	.	.	.	.	24	.	.	.
T8	.	.	.	.	.	.	64	.	.	.
T9	8	.	.	.	.	.	16	.	.	.
T10	8	.	.	.	.	.	16	.	.	.
T11	.	.	.	.	.	.	24	.	.	20
T12	.	.	.	.	.	.	24	.	.	20
T13	.	.	.	.	.	.	24	.	.	.
T14	.	.	.	.	.	.	64	.	.	.
T15	.	.	.	.	.	.	64	.	.	.
T16	.	.	.	.	.	.	64	.	.	.
T17	8	.	.	.	.	.	16	.	.	.
T18	8	.	.	.	.	.	16	.	.	.
T19	8	.	.	.	.	.	16	.	.	.
T20	8	.	.	.	.	.	16	.	.	.
T21	8	.	40	.	.	.	16	.	.	.
T22	.	.	.	.	.	.	24	.	.	40

Table 10C (continued)

	5 1^5	5 2 1^3	5 2^2 1	5 3 1^2	5 3 2	5 4 1	5^2	6 1^4	6 2 1^2	6 2^2
T23	.	.	.	.	.	.	64	.	.	.
T24	.	.	.	.	.	.	64	.	.	.
T25	.	.	.	.	.	.	64	.	.	.
T26	.	.	.	.	.	.	144	.	.	.
T27	8	.	40	.	.	.	16	.	.	.
T28	8	.	40	.	.	.	16	.	.	.
T29	.	.	.	.	.	.	64	.	.	.
T30	.	.	.	.	.	.	144	.	.	.
T31	.	.	.	.	.	.	144	.	.	.
T32	.	.	.	.	.	.	144	.	.	.
T33	8	.	40	.	.	80	16	.	.	.
T34	.	.	.	.	.	.	384	.	160	.
T35	.	.	.	.	.	.	144	.	.	.
T36	.	.	.	.	.	.	384	80	160	80
T37	.	.	.	.	.	.	384	.	160	.
T38	.	.	.	.	.	.	384	.	160	160
T39	.	.	.	.	.	.	384	80	160	240
T40	48	.	720	960	.	.	576	.	.	1200
T41	48	.	720	960	.	.	576	.	.	2400
T42	48	.	720	960	.	.	576	.	.	.
T43	48	480	720	960	960	1440	576	.	.	2400
T44	6048		90720	120960	.	.	72576	.	151200	.
T45	6048	60480	90720	120960	120960	181440	72576	25200	151200	75600

Table 10C (continued)

	6 3 1	6 4	7 1^3	7 2 1
T1	.	.	.	.
T2	.	.	.	.
T3	.	.	.	.
T4	.	.	.	.
T5	.	.	.	.
T6	.	.	.	.
T7	.	.	.	.
T8	.	.	.	.
T9	.	.	.	.
T10	.	.	.	.
T11	.	.	.	.
T12	.	.	.	.
T13	20	.	.	.
T14	.	.	.	.
T15	.	.	.	.
T16	.	.	.	.
T17	.	.	.	.
T18	.	.	.	.
T19	.	.	.	.
T20	.	.	.	.
T21	.	.	.	.
T22	.	.	.	.

5 4 1	5^2	6 1^4	6 2 1^2	6 2^2
.	64	.	.	.
.	64	.	.	.
.	64	.	.	.
.	144	.	.	.
.	16	.	.	.
.	16	.	.	.
.	64	.	.	.
.	144	.	.	.
.	144	.	.	.
.	144	.	.	.
80	16	.	.	.
.	384	.	160	.
.	144	.	.	.
.	[illegible]	80	160	80
.	384	.	160	.
.	384	.	160	160
.	384	80	160	240
.	576	.	.	1200
.	576	.	.	2400
.	576	.	.	.
40	576	.	.	2400
.	72576	.	151200	.
40	72576	25200	151200	75600

Table 10C (continued)

	6 3 1	6 4	7 1^3	7 2 1	7 3	8 1^2	8 2	9 1	10
T1	.	.	.	.	.	.	.	.	4
T2	.	.	.	.	.	.	.	.	.
T3	.	.	.	.	.	.	.	.	4
T4	.	.	.	.	.	.	.	.	.
T5	.	.	.	.	.	.	.	.	4
T6	.	.	.	.	.	.	.	.	20
T7	.	.	.	.	.	.	.	.	.
T8	.	.	.	.	.	.	.	.	.
T9	.	.	.	.	.	.	.	.	40
T10	.	.	.	.	.	.	.	.	.
T11	.	.	.	.	.	.	.	.	24
T12	.	.	.	.	.	.	.	.	.
T13	20	.	.	.	.	.	.	.	.
T14	.	.	.	.	.	.	.	.	64
T15	.	.	.	.	.	.	.	.	.
T16	.	.	.	.	.	.	.	.	.
T17	.	.	.	.	.	.	.	.	40
T18	.	.	.	.	.	.	100	.	.
T19	.	.	.	.	.	.	.	.	80
T20	.	.	.	.	.	.	.	.	.
T21	.	.	.	.	.	.	.	.	40
T22	.	.	.	.	.	.	.	.	24

Table 10C (continued)

	6 3 1	6 4	7 1^3	7 2 1	7 3	8 1^2	8 2	9 1	10
T23	.	.	.	.	.	.	.	.	64
T24	.	.	.	.	.	.	80	.	.
T25	.	.	.	.	.	80	.	.	.
T26	.	.	.	.	.	.	.	.	.
T27	.	.	.	.	.	.	.	.	80
T28	.	.	.	.	.	.	200	.	.
T29	.	.	.	.	.	80	80	.	64
T30	.	.	.	.	.	180	.	.	144
T31	.	.	.	.	.	.	180	.	.
T32	240	.	.	.	.	.	.	.	.
T33	.	.	.	.	.	.	200	.	80
T34	.	.	.	.	.	.	.	.	.
T35	240	.	.	.	.	180	180	.	144
T36	.	.	.	.	.	.	.	.	384
T37	.	160	.	.	.	.	240	.	.
T38	.	160	.	.	.	240	.	.	.
T39	.	160	.	.	.	240	240	.	384
T40	.	.	.	.	.	.	.	.	1440
T41	.	.	.	.	.	.	.	.	2880
T42	.	2400	.	.	.	.	3600	.	.
T43	.	2400	.	.	.	.	3600	.	2880
T44	.	151200	86400	.	172800	.	226800	403200	.
T45	201600	151200	86400	259200	172800	226800	226800	403200	362880

Table 11A: groups of degree 11

Group	Order
T1	11
T2	22
T3	55
T4	110
T5	660
T6	7920
T7	(½)11!
T8	11!

Table 11B: group generators

a = (1,2,3,4,5,6,7,8,9,10,

b = (2,3,5,9,6,11,10,8,4,7

c = (1,11)(2,7)(3,5)(4,6)

d = (4,8)(5,9)(6,7)(10,11)

e = (1,6)(3,5)(4,7)(9,10)

T1 = <a>

T2 = <a,b^5>

T3 = <a,b^2>

T4 = <a,b>

T5 = <f,e,c>

8^{1^2}	8^{2}	9^{1}	10
.	.	.	64
.	80	.	.
80	.	.	.
.	.	.	.
.	.	.	80
.	200	.	.
80	80	.	64
180	.	.	144
.	180	.	.
.	.	.	.
.	200	.	80
.	.	.	.
180	180	.	144
[illegible]	.	.	384
.	240	.	.
240	.	.	.
240	240	.	384
.	.	.	1440
.	.	.	2880
.	3600	.	.
.	3600	.	2880
.	226800	403200	.
226800	226800	403200	362880

Table 11A: groups of degree 11

Group	Order	Even	Number of Classes	Name
T1	11	+	11	11
T2	22		7	11.2
T3	55	+	7	11.5
T4	110		11	11.10
T5	660	+	8	L(2,11)
T6	7920	+	10	M_{11}
T7	($\frac{1}{2}$)11!	+	31	A_{11}
T8	11!		56	Σ_{11}

Table 11B: group generators

a = (1,2,3,4,5,6,7,8,9,10,11)
b = (2,3,5,9,6,11,10,8,4,7)
c = (1,11)(2,7)(3,5)(4,6)
d = (4,8)(5,9)(6,7)(10,11)
e = (1,6)(3,5)(4,7)(9,10)
f = (1,5,7)(2,9,4)(3,8,10)
g = (1,2,3)(4,5,6)(7,8,9)
h = (2,4,3,7)(5,6,9,8)
i = (2,9,3,5)(4,6,7,8)
j = (1,10)(4,7)(5,6)(8,9)
k = (1,2,3)

T1 = <a>
T2 = $\langle a,b^5\rangle$
T3 = $\langle a,b^2\rangle$
T4 = <a,b>
T5 = <f,e,c>
T6 = <g,h,i,j,d>
T7 = <a,k>
T8 = <b,k>

Table 11C: cycle type distribution

	1^{11}	$2^4 1^3$	$2^5 1$	$3^3 1^2$	$4^2 1^3$	$5^2 1$	$6\,3\,2$	$8\,2\,1$	$10\,1$	11
T1	1	.	.	.	.	.	.	.	.	10
T2	1	.	11	.	.	.	.	.	.	10
T3	1	.	.	.	.	44	.	.	.	10
T4	1	.	11	.	.	44	.	.	44	10
T5	1	55	.	110	.	264	110	.	.	120
T6	1	165	.	440	990	1584	1320	1980	.	1440

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	6	8		
5^2	3	2	10	
1	2	1	1	11
.	.	.	.	10
.	.	.	.	10
44	.	.	.	10
44	.	.	44	10
64	110	.	.	120
84	1320	1980	.	1440

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