```
From masg2.epfl.ch!lafitte Wed May  3 21:25:04 0600 1995
From: Melvyn Lafitte <lafitte@dma.epfl.ch>
Date: Wed, 3 May 1995 21:25:04 -0600
To: njas@research.att.com
Subject: 1 0 1 1 1 1 2 0 2 2 2 1 3 1 2 1 2 2 4 1 4 3 3 1 4 2 4 2
```

Attn: NJA Sloane

I'm sorry not to have got back to you before. Like I told you, the sequence is related to a Self-Replicating tile. (I'm sending another message with an uuencoded image of it) The self-replicating tile may be obtained by the following iterated functions system:

$$\begin{pmatrix}\xi'\\ \eta'\end{pmatrix} = \frac{1}{2}(\begin{pmatrix}\xi\\ \eta\end{pmatrix} + \begin{pmatrix}0\\ \frac{\sqrt{3}}{2}\end{pmatrix})$$
$$\begin{pmatrix}\xi'\\ \eta'\end{pmatrix} = \frac{1}{2}(\begin{pmatrix}\xi\\ \eta\end{pmatrix} + \begin{pmatrix}0\\ \frac{\sqrt{3}}{6}\end{pmatrix})$$
$$\begin{pmatrix}\xi'\\ \eta'\end{pmatrix} = \frac{1}{2}(\begin{pmatrix}\xi\\ \eta\end{pmatrix} + \begin{pmatrix}\frac{1}{2}\\ 0\end{pmatrix})$$
$$\begin{pmatrix}\xi'\\ \eta'\end{pmatrix} = \frac{1}{2}(\begin{pmatrix}\xi\\ \eta\end{pmatrix} + \begin{pmatrix}-\frac{1}{2}\\ 0\end{pmatrix})$$

to the interval $\mathcal{I}$

It is the attractor of this IFS, a very interesting triangle shaped fractal of non-empty interior and of Hausdorff dimension equal to 2. (and that satisfies the open set condition without allowing a stronger condition)

This fractal has been discovered by myself in 1991. I later came to know that Gosper had encountered it at about the same time, that Mandelbrot also encountered an object, formed by the union of copies of this fractal, in 1975 and lately I read an article by C. Bandt exposing facts about self-replicating tiles and giving as example this fractal.

At that time, I made a thorough study of this fractal. I've been able to prove 2 very elegant laws of formation (aside from its selfsimilar structure) based on the way it is "filled", indeed this fractal is composed of filled equal triangles . (equal in the sense: by similitude)

Now congruent filled triangles can be found on lines perpendicular to a side of the triangle shape of the object (triangle convex hull).

As we consider smaller and smaller filled triangles, we get more and more of them on these lines.

Well my sequence 1 0 1 1 1 ... results from counting these filled congruent triangles on these lines, and so each term corresponds to a line, to the counting on that line. (Remark that on these lines, are also empty triangles, triangles congruent to the triangles which are filled)

Understand that this forms a sequence only because there happens (proved) to be an additional self-similarity given to the object, and that it assures that counts for a certain size of triangles will be the same than for a greater size of triangles, but with more terms (counts) added.

I came to prove that this sequence is closely related to the sequence A2487 M0141 N0056, in fact it is a transformation of this sequence by: $new(3n) = \frac{old(3n)+1}{2}$
$new(3n+1) = \frac{old(3n+1)-1}{2}$
$new(3n+2) = \frac{old(3n+2)}{2}$

I did not intend to send you this sequence but another one which also results from this triangle shaped fractal. It is by mistake that I copied this sequence in my message and not that other one. In fact, this sequence I sent you is pretty interesting, even if it is the transformation of another known sequence, because for me it rather is the way those filled triangles are distributed along these lines on this object before being a transformation of that already known sequence. So I can give you 60 terms: 1 0 1 1 1 1 2 0 2 2 2 1 3 1 2 1 2 2 4 1 4 3 3 1 4 2 4 2 3 2 3 0 3 3 4 2 6 3 5 2 5 4 7 2 6 4 4 1 5 3 6 3 6 4 6 1 5 4 5 2 5 2 3 ...
Now the intended integer sequence starts by: 1 0 2 1 1 2 5 0 10 6 3 2 18 2 10 ...
It is a consideration of the filled and empty triangles on a line as the binary expansion of an integer:0 for empty and 1 for filled:

```
                    0   1
                1   0   0   1
        0   1   0 1 0   0 0 0
    1   0 1 0   1 1 1 1 1 1 1
1 0 0 1 1 0 1 0 0 0 1 0 1 0 0 ...
```

↓

1;0;10;1;001;10;101;0;1010;110;00011;10;10011;010;1010;...

It describes these "fillings" on these lines much better than the rather incomplete description that the counting of the filled (first sequence) only gives us.
I will be sending you about 60 terms in the days coming.
There are'nt any references on this sequence. In fact, I plan to finally publish some of my results in the near future. For me this rep-tile (and the sequence related to it), and the study I undertook on it, are only indications of a general and elegant theory concerning "critical" cases of self-similarity, theory I started to develop precisely after I met this example.

Please consider including these (especially the second one) sequences in your table and continue this very useful work.
Best Regards,
MJL.

PS:Please refer me with the email adress:melvyn.lafitte@dma.epfl.ch instead of any other non constantly working adress. Thanks.


```
--------------------------------------------------------------------------------
 Melvyn Jeremie Lafitte            | Departement de Mathematiques
  dit Mandelvyn                    | Ecole Polytechnique Federale de Lausanne
 Email: melvyn.lafitte@dma.epfl.ch | ETH-Lausanne (Sister of ETH-Zurich)
--------------------------------------------------------------------------------
```


```
                         Syag LeHochma -- Shtika
                    ( Wisdom is delineated by silence.)
--------------------------------------------------------------------------------
```