[WE2] Wells

→ 170
ent → 2562 ✓
136
560 ✓
315 ✓

borrowed from folder @ A170

APPENDIX II

TABLES OF INTERESTING NUMBERS

We include here tables of certain combinatorial numbers computed by the author. We believe this is the first time that Tables II.6 and II.7 have appeared in print.

Table II.1. Excess Number of Multiples ($< 2^n$) of p with Even 1-bit Parity (§ 3.1.5)

n	$p = 3$	$p = 7$	$p = 11$	$p = 17$
1	1	1	1	1
2	2	1	1	1
3	3	0	1	1
4	6	−1	0	1
5	9	−3	−1	2
6	18	−6	−2	4
7	27	−7	−2	8
8	54	−7	0	16
9	81	0	5	21
10	162	7	10	29
11	243	21	11	37
12	486	42	11	37
13	729	49	11	74
14	1 458	49	0	140
15	2 187	0	−11	264
16	4 374	−49	−22	528
17	6 561	−147	−22	697
18	13 122	−294	0	969
19	19 683	−343	55	1 241
20	39 366	−343	110	1 241
21	59 049	0	121	2 482
22	118 098	343	121	4 692
23	177 147	1 029	121	8 840
24	354 294	2 058	0	17 680
25	531 441	2 401	−121	23 341
26	1 062 882	2 401	−242	32 453
27	1 594 323	0	−242	41 565
28	3 188 646	−2 401	0	41 565
29	4 782 969	−7 203	605	83 130
30	9 565 938	−14 406	1210	157 148
31	14 348 907	−16 807	1331	296 072
32	28 697 814	−16 807	1331	592 144
33	43 046 721	0	1331	781 745
34	86 093 442	16 807	0	1 086 929
35	129 140 163	50 421	−1331	1 392 113

mult by 3

double

dull

170
=N775

TABLE II.2. NUMBER OF SOLUTIONS TO QUEENS' PROBLEM
(EXERCISE 1 OF § 5.2)
(EXERCISE 4 OF § 6.5)

2562
=N68

n	q_n	Iq_n
1	1	1
2	0	0
3	0	0
4	2	1
5	10	2
6	4	1
7	40	6
8	92	12
9	352	46
10	724	92
11	2 680	341
12	14 200	1787
13	73 712	9233
14	365 596	45 752

For an $n \times n$ chessboard, q_n is the total number of ways to place n nonattacking queens on the board. Iq_n is the number of ways inequivalent under the group of the square.

have

15	2,279,184	285,053

Ed. M. Reingold
Univ of Illinois
Urbana
Dept of C.S.

TABLE II.3. FOLDING NUMBERS
(SEE EXERCISE 13 OF § 5.2)

n	f_n	g_n
1	1	
2	2	
3	6	1
4	16	2
5	50	5
6	144	12
7	462	33
8	1 392	87
9	4 536	252
10	14 060	703
11	46 310	2 105
12	146 376	6 099
13	485 914	18 689

136 560
have have

I
indis

TABLE II.4. ANSWERS TO A DISTRIBUTION PROBLEM FOR $n = 5$ (EXERCISE 14 OF § 5.5)

Partition p	$N(p)$
1111111111	945
211111111	525
22111111	300
31111111	210
2221111	177
3211111	125
222211	109
322111	77
4111111	75
22222	73
331111	56
32221	50
421111	47
33211	36
42211	31
511111	26
3322	25
4222	23
43111	23
3331	18
52111	17
4321	16
5221	12
4411	11
61111	10
442	9
433	9
5311	9
6211	7
532	7
622	6
541	5
631	4
7111	4
64	3
721	3
55	3
811	2
73	2
82	2
91	1
10	1

$N(p)$ is the number of distinct ways in which ten objects of specification p can be placed in five indistinguishable boxes, each box containing exactly two objects.

315

TABLE II.5. NUMBER OF REDUCED LATIN SQUARES (§ 7.1.3)

n	l_n	Il_n
3	1	1
4	4	2
5	56	2
6	9 408	12
7	16 942 080	147
8	535 281 401 856	

The quantity l_n is the number of reduced Latin squares of order n, while Il_n is the number of squares inequivalent under label, row, and/or column permutations and permutation of labels with rows, labels with columns, etc. (The group has order $6n!^3$.)

TABLE II.6. NUMBER OF ZERO–ONE MATRICES WITH VANISHING PERMANENT (EXERCISE 8 OF § 6.4)

2×2		3×3		4×4		5×5	
k	No.	k	No.	k	No.	k	No.
2	4	3	6	4	8	5	10
3	4	4	45	5	96	6	200
4	1	5	90	6	576	7	1 900
		6	78	7	2128	8	11 500
		7	36	8	4860	9	50 025
		8	9	9	6976	10	166 720
		9	1	10	6496	11	439 600
				11	4080	12	923 700
				12	1796	13	1 534 800
				13	560	14	1 994 200
				14	120	15	2 010 920
				15	16	16	1 571 525
				16	1	17	956 775
						18	458 500
						19	174 700
						20	53 010
						21	12 650
						22	2 300
						23	300
						24	25
						25	1

The column labeled k gives the number of zeros in the $n \times n$ matrix. The No. column then gives the number of $n \times n$ zero–one matrices containing k zeros (hence $n^2 - k$ ones) whose permanent is zero.