Practical Foundations for Topological Quantum Programming

Urs Schreiber on joint work with Hisham Sati

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slides hosted at: `ncatlab.org/nlab/show/CQTS#MathFacultyMeetingSep2022`

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[Atiyah & Hirzebruch 1959]

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direct sum of vector bundles

$$\begin{tikzcd} \mathcal{V}_x \oplus \mathcal{W}_x \arrow[r] \arrow[d] & \mathcal{V} \oplus \mathcal{W} \arrow[d] \\ \{x\} \arrow[r] & \mathbf{X} \end{tikzcd}$$

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$$= \left\{ \begin{array}{c} \text{“virtual” vector bundles} \\ \mathcal{V}_+ \ominus \mathcal{V}_- \end{array} \right\} \Big/ \begin{array}{rl} \mathcal{V}_+ \ominus \mathcal{V}_- & \sim \ \mathcal{W}_+ \ominus \mathcal{W}_- \\ \Leftrightarrow \ \mathcal{V}_+ \oplus \mathcal{W}_- \oplus \underline{\mathbb{C}}^n & \simeq \ \mathcal{W}_+ \oplus \mathcal{V}_- \oplus \underline{\mathbb{C}}^n \end{array}$$

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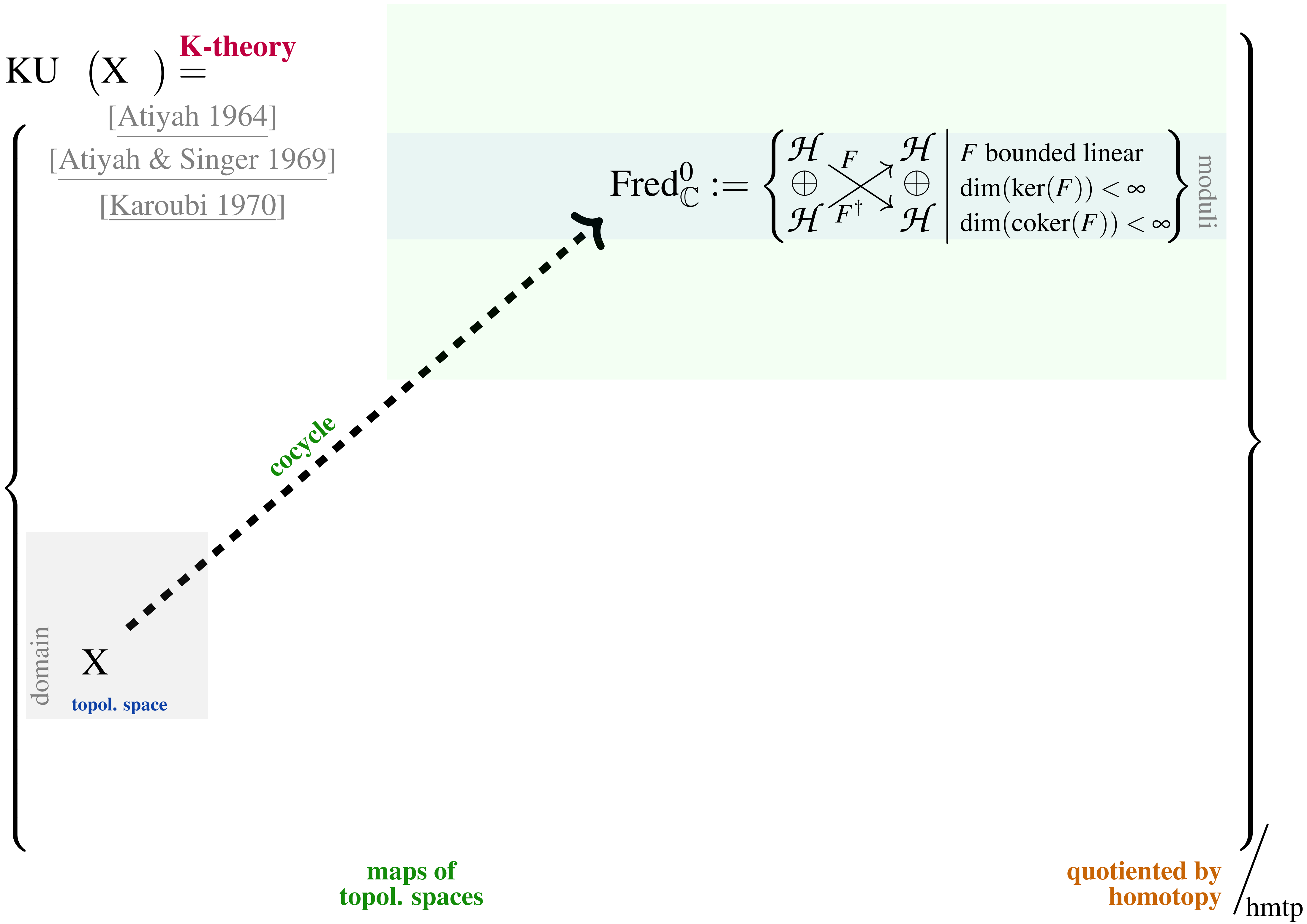

$KU(X) =$ **K-theory**

[Atiyah 1964]

[Atiyah & Singer 1969]

[Karoubi 1970]

$$\mathrm{Fred}^0_{\mathbb{C}} := \left\{ \begin{array}{c}\mathcal{H}\\ \oplus\\ \mathcal{H}\end{array} \underset{F^\dagger}{\overset{F}{\rightleftarrows}} \begin{array}{c}\mathcal{H}\\ \oplus\\ \mathcal{H}\end{array} \;\middle|\; \begin{array}{l} F \text{ bounded linear}\\ \dim(\ker(F)) < \infty \\ \dim(\mathrm{coker}(F)) < \infty\end{array}\right\}$$

moduli

cocycle

X

domain

topol. space

maps of topol. spaces

quotiented by homotopy

$/_{\mathrm{hmtp}}$

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such a map defines the virtual vector bundle $\ker(F) \ominus \mathrm{coker}(F)$

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moduli

cocycle

such a map defines the

virtual vector bundle

torus of Bloch momenta
in some crystal

valence

Bloch states

Fredholm operators

$$\widehat{\mathbb{T}}^d \dashrightarrow \mathrm{Fred}^0_{\mathbb{C}} = \left\{ \begin{tikzcd} \ker(F) \arrow[r, hook] & \mathcal{H} \arrow[ddr, "F"] & \mathcal{H} & \\ & \oplus & \oplus & \\ & \mathcal{H} \arrow[uur, "F^*"'] & \mathcal{H} \arrow[r, two heads] & \mathrm{coker}(F) \end{tikzcd} \right\}$$

electron
Bloch states

single electron
Hilbert space

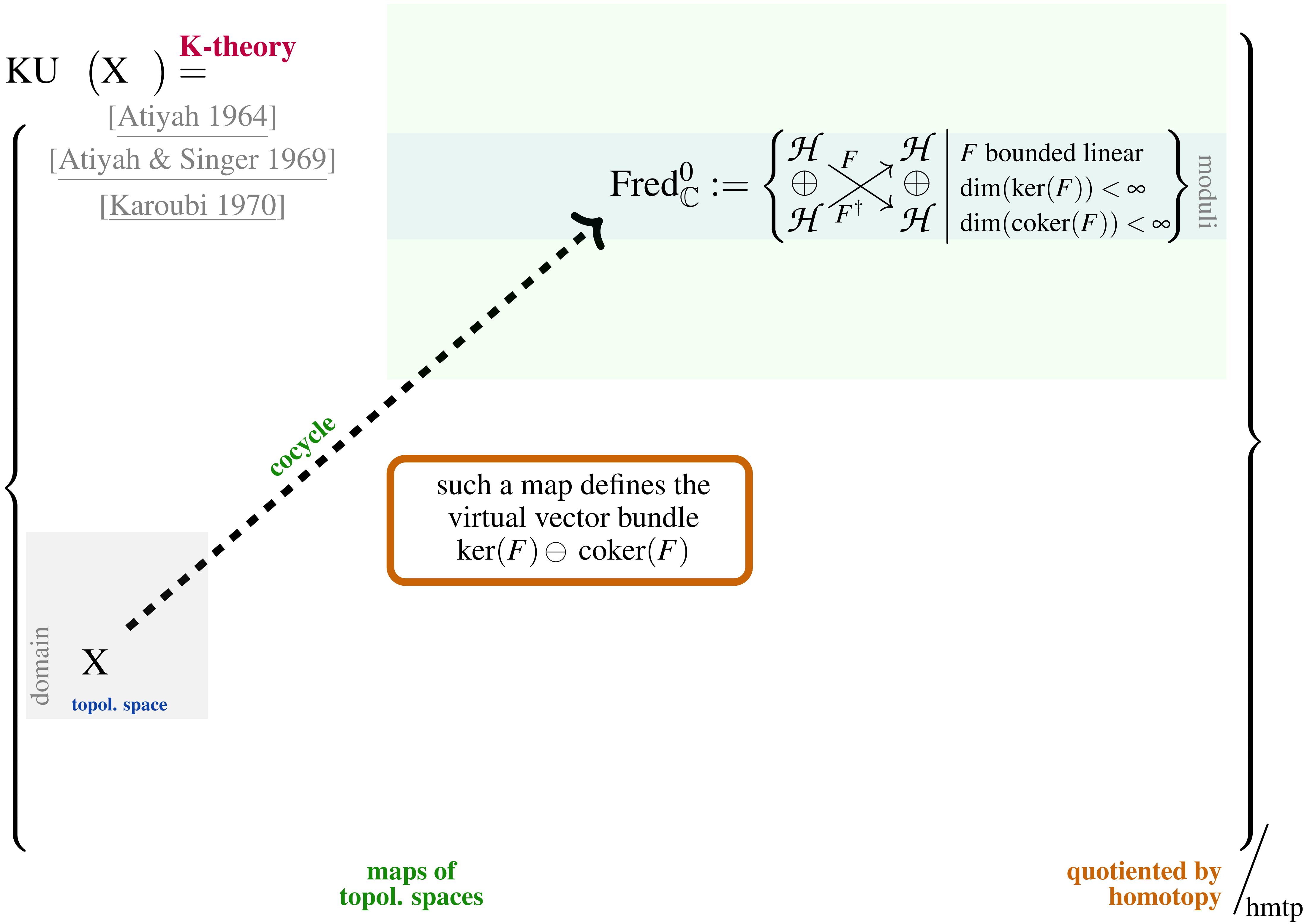
Fredholm operator

single positron
Hilbert space

positron
Bloch states

topol. spaces

homotopy $/_{\mathrm{hmtp}}$



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domain

X

topol. space

maps of topol. spaces

quotiented by homotopy

$/\mathrm{hmtp}$

twisted **K-theory**

$KU^\tau(X) =$

[Donovan & Karoubi 1970]
[Rosenberg 1989]
[Freed, Hopkins & Teleman 2002]

even projective unitary group:
acting by operator conjugation

$$\begin{tikzcd} & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H})\times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\right) \arrow[d] \\ X \arrow[ur, dashed, "\text{cocycle}"] \arrow[r, "\text{twist } \tau"] & \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H})\times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\right) \end{tikzcd}$$

moduli fibration

twist moduli

domain

topol. space

maps of topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by relative homotopy

/hmtp

twisted **K-theory**

$$\mathrm{KU}^{\tau}(\mathrm{X}) = \left\{ \begin{tikzcd} & \mathrm{Fred}^{0}_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, P\} \right)\Big) \arrow[d] \\ \mathrm{X} \arrow[ur, dashed, "\text{cocycle}"] \arrow[r, "\text{twist } \tau"] & \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, P\} \right)\Big) \end{tikzcd} \right\}_{/\mathrm{hmtp}}$$

[Parker 1988]

"**Parity**" involution P:
swapping $\mathcal{H}$-summands

moduli fibration

twist moduli

domain

topol. space

maps of
topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by
relative homotopy

twisted **KR-theory**

$$\mathrm{KR}^{\tau}(\mathrm{X}) = \left\{ \begin{tikzcd} & \mathrm{Fred}^{0}_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \overset{\{\mathrm{e}, P\}}{\underset{\{\mathrm{e}, T\}}{\times}} \right) \arrow[d] & \\ \mathrm{X} /\!\!/ \{\mathrm{e}, T\} \arrow[ur, dashed, "\text{cocycle}"] \arrow[r, "\text{twist } \tau"] \arrow[drr, "\text{real structure}"'] & \mathbf{B} \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \overset{\{\mathrm{e}, P\}}{\underset{\{\mathrm{e}, T\}}{\times}} \right) \arrow[dr] & \\ & & \mathbf{B} \left(\overset{\{\mathrm{e}, P\}}{\underset{\{\mathrm{e}, T\}}{\times}} \right) \end{tikzcd} \right\}_{/\mathrm{hmtp}}$$

[Atiyah 1966]
[Kriz 2001]

domain

orienti-fold

moduli fibration

"Time reversal" T:
complex conjugation

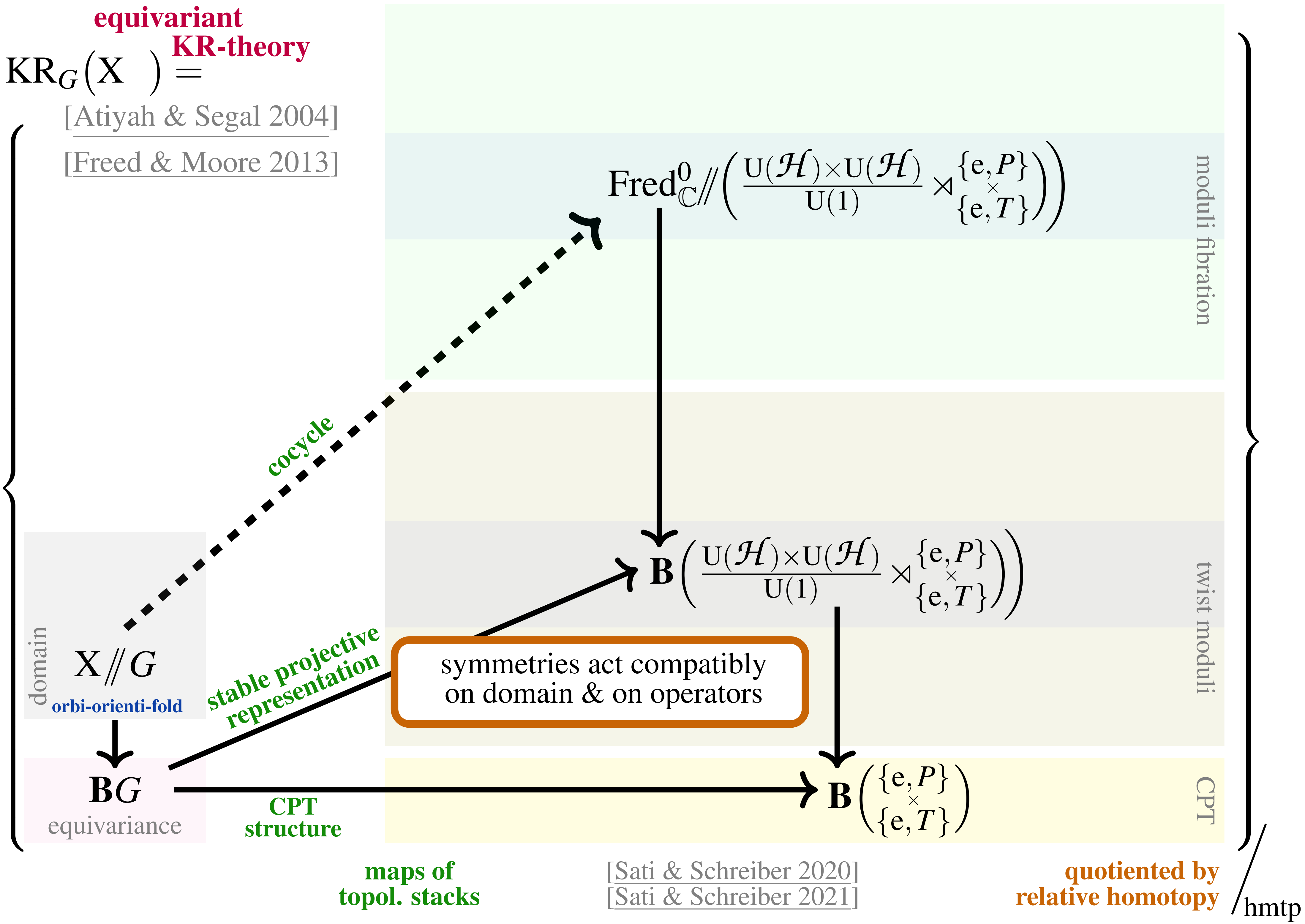
twist moduli

CPT

maps of
topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by
relative homotopy



$\mathrm{KR}_G(\mathrm{X}\quad) =$ **equivariant KR-theory**

[Atiyah & Segal 2004]

[Freed & Moore 2013]

$$\begin{tikzcd}
 & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix} \{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\} \end{smallmatrix} \right) \arrow[d] & \\
 & \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix} \{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\} \end{smallmatrix} \right) \arrow[ddr] & \\
\mathrm{X} /\!\!/ G \arrow[uur, dashed, "\text{cocycle}"] \arrow[d] & & \\
\mathbf{B}G \arrow[uur, "\text{stable projective representation}"] \arrow[rr, "\text{CPT structure}"'] & & \mathbf{B}\left(\begin{smallmatrix} \{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\} \end{smallmatrix} \right)
\end{tikzcd}$$

symmetries act compatibly
on domain & on operators

maps of
topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by
relative homotopy

$/_{\mathrm{hmtp}}$

twisted equivariant KR-theory

[Atiyah & Segal 2004]

[Uribe & Lück 2014]

$$\mathrm{KR}^{\tau}_{G}(\mathrm{X}) = \left\{ \begin{tikzcd} & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{matrix}\{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\}\end{matrix} \right) \arrow[d] \\ & \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{matrix}\{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\}\end{matrix} \right) \arrow[dd] \\ \mathrm{X} /\!\!/ G \arrow[uur, dashed, "\text{cocycle}"] \arrow[ur, "\text{equivariant twist } \tau"] \arrow[d] & \\ \mathbf{B}G \arrow[r, "\text{CPT structure}"'] & \mathbf{B}\left(\begin{matrix}\{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\}\end{matrix} \right) \end{tikzcd} \right\}_{\big/ \mathrm{hmtp}}$$

domain

orbi-orienti-fold

equivariance

moduli fibration

twist moduli

CPT

maps of topol. stacks

[Sati & Schreiber 2020]

[Sati & Schreiber 2021]

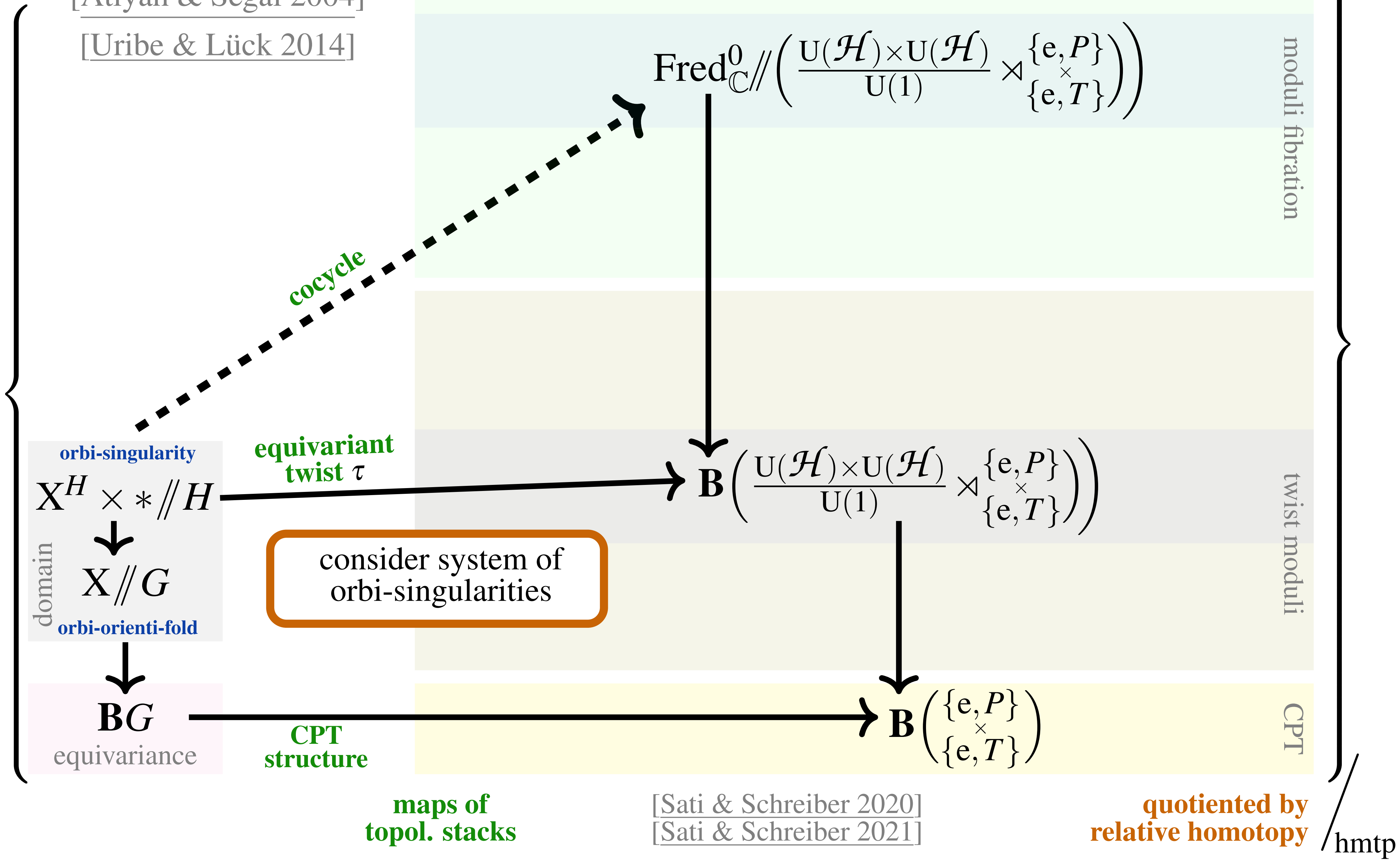
quotiented by relative homotopy

twisted equivariant KR-theory

$\mathrm{KR}^{\tau}_{H}\big(\mathrm{X}^{H}\big) =$

[Atiyah & Segal 2004]

[Uribe & Lück 2014]



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 & \mathrm{Fred}^{0}_{\mathbb{C}} /\!\!/ \left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix} \{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\} \end{smallmatrix} \right) \arrow[d] \\
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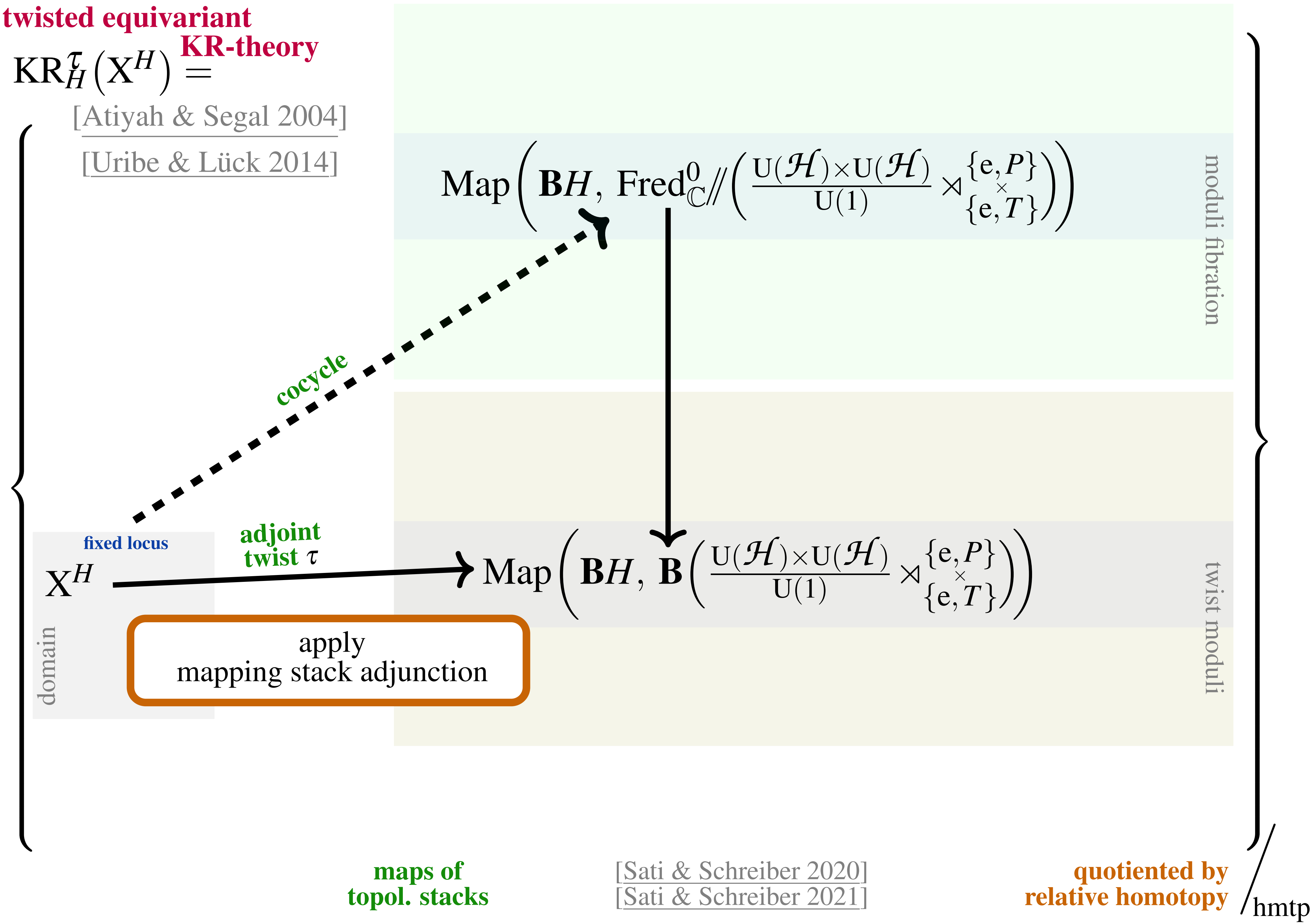
\mathrm{X} /\!\!/ G \arrow[d] & \\
\mathbf{B}G \arrow[r, "\text{CPT structure}"'] & \mathbf{B}\left(\begin{smallmatrix} \{\mathrm{e}, P\} \\ \times \\ \{\mathrm{e}, T\} \end{smallmatrix} \right)
\end{tikzcd}$$

consider system of orbi-singularities

maps of topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by relative homotopy



twisted equivariant KR-theory

$\mathrm{KR}^{\tau}_{H}\big(\mathrm{X}^{H}\big) =$

[Atiyah & Segal 2004]

[Uribe & Lück 2014]

$$\left\{\begin{tikzcd}
 & \mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^{0}_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big) \arrow[d] \\
\mathrm{X}^{H} \arrow[ur, dashed, "\text{cocycle}"] \arrow[r, "\substack{\text{adjoint}\\ \text{twist } \tau}"] & \mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)
\end{tikzcd}\right\}_{/\mathrm{hmtp}}$$

apply
mapping stack adjunction

maps of
topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by
relative homotopy

twisted equivariant KR-theory

$$\mathrm{KR}^{\tau}_{H}\big(\mathrm{X}^{H}\big) =$$

[Ruan 2000]
[Tu & Xu 2006]
[Freed, Hopkins & Teleman 2002]
[Sati & Schreiber 2022a]

$$\left\{\begin{tikzcd}
 & & \mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^{0}_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big) \arrow[d] \\
\mathrm{X}^{H} \arrow[r, "\text{inner local system}"'] \arrow[urr, dashed, "\text{cocycle}"] & \mathbf{B}H^{*} \arrow[r, "\text{automorphisms of stable representation}"'] & \mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)
\end{tikzcd}\right\}\Big/_{\mathrm{hmtp}}$$

fixed locus

domain

moduli fibration

twist moduli

the combination makes a subtle and deep twisting appear: by "*inner local systems*"

maps of topol. stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by relative homotopy

differential
K-theory

$$\widehat{\mathrm{KU}}\,(\mathrm{X}\,) = \left\{ \begin{tikzcd} & \Omega_{\mathrm{dR}}\big(-;\, \mathfrak{l}\mathrm{Fred}^0_{\mathbb{C}}\big) \arrow[ddr, "\text{de Rham theorem}"] & \\ & \mathrm{Fred}^0_{\mathbb{C}} \arrow[dr, "\text{rationalization}"'] & \\ & & L^{\mathbb{Q}}\mathrm{Fred}^0_{\mathbb{C}} \\ \mathrm{X} \arrow[uuur, dashed, "\text{differential}"] \arrow[uur, dashed, "\text{cocycle}"'] & & \end{tikzcd} \right\} \Big/ \mathrm{hmtp}$$

[Hopkins & Singer 2005]

[Bunke & Schick 2009]

[Fiorenza, Sati & Schreiber 2020]

L_∞-algebra valued
differential forms

moduli

domain

smooth space

**maps of
smooth stacks**

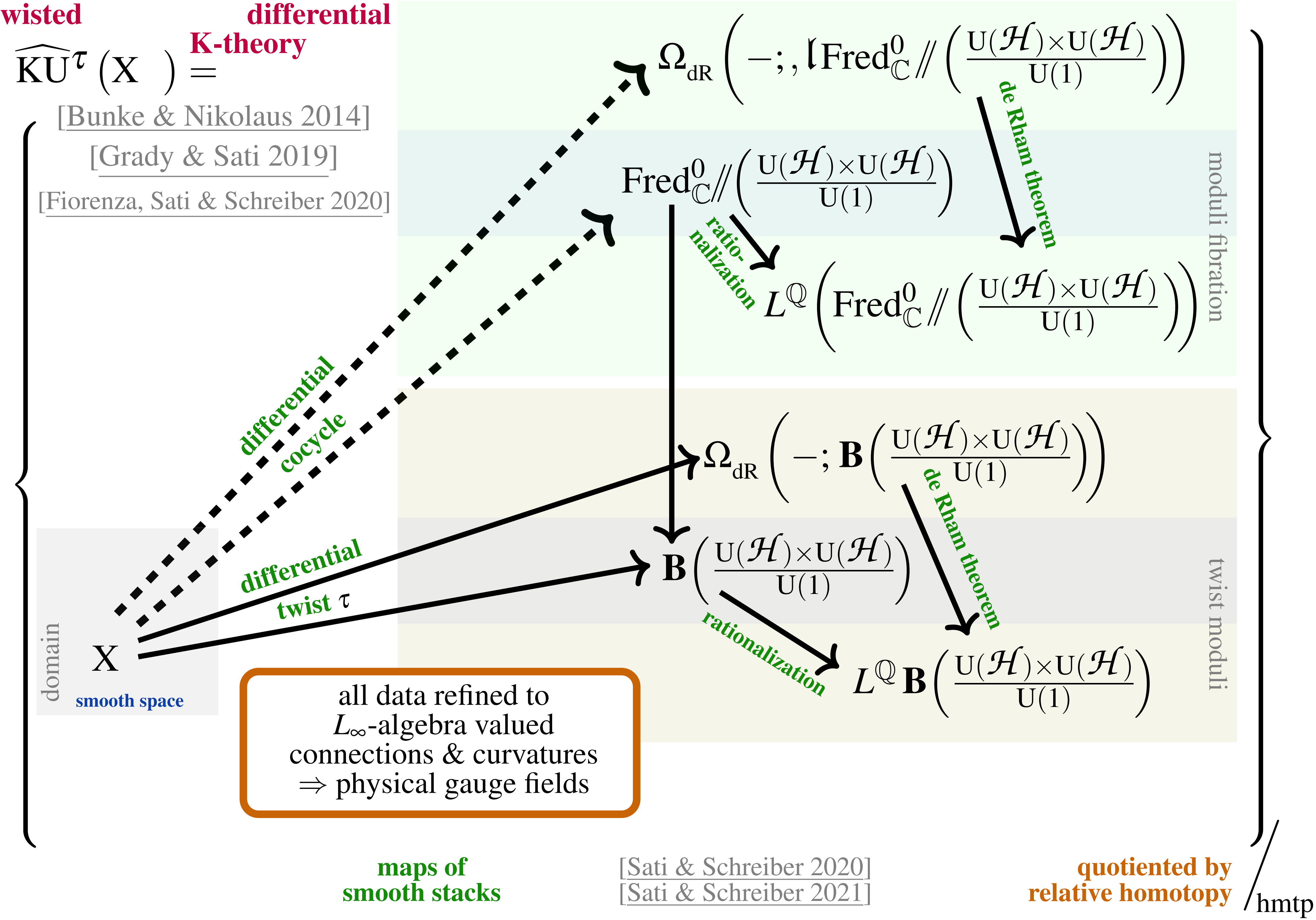
[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

**quotiented by
homotopy**

twisted **differential** **K-theory**

$\widehat{\mathrm{KU}}^{\tau}(\mathrm{X}\;\;) =$

[Bunke & Nikolaus 2014]
[Grady & Sati 2019]
[Fiorenza, Sati & Schreiber 2020]



$$\begin{tikzcd}
{} & {} & \Omega_{\mathrm{dR}}\Big(-;,\, \mathfrak{l}\,\mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\Big)\Big) \arrow[ddr, "\text{de Rham theorem}"] & {} \\
{} & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\Big) \arrow[dddd] \arrow[drr, "\text{ratio-nalization}"'] & {} & {} \\
{} & {} & {} & L^{\mathbb{Q}}\Big(\mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\Big)\Big) \\
{} & {} & \Omega_{\mathrm{dR}}\Big(-;\,\mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\Big)\Big) \arrow[dr, "\text{de Rham theorem}"] & {} \\
{} & \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\Big) \arrow[rr, "\text{rationalization}"'] & {} & L^{\mathbb{Q}}\mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)}\Big) \\
\mathrm{X} \arrow[uuuurr, dashed, "\text{differential}"] \arrow[uuuur, dashed, "\text{cocycle}"'] \arrow[urr, "\text{differential}"] \arrow[ur, "\text{twist } \tau"'] & {} & {} & {}
\end{tikzcd}$$

all data refined to
L_∞-algebra valued
connections & curvatures
⇒ physical gauge fields

maps of smooth stacks

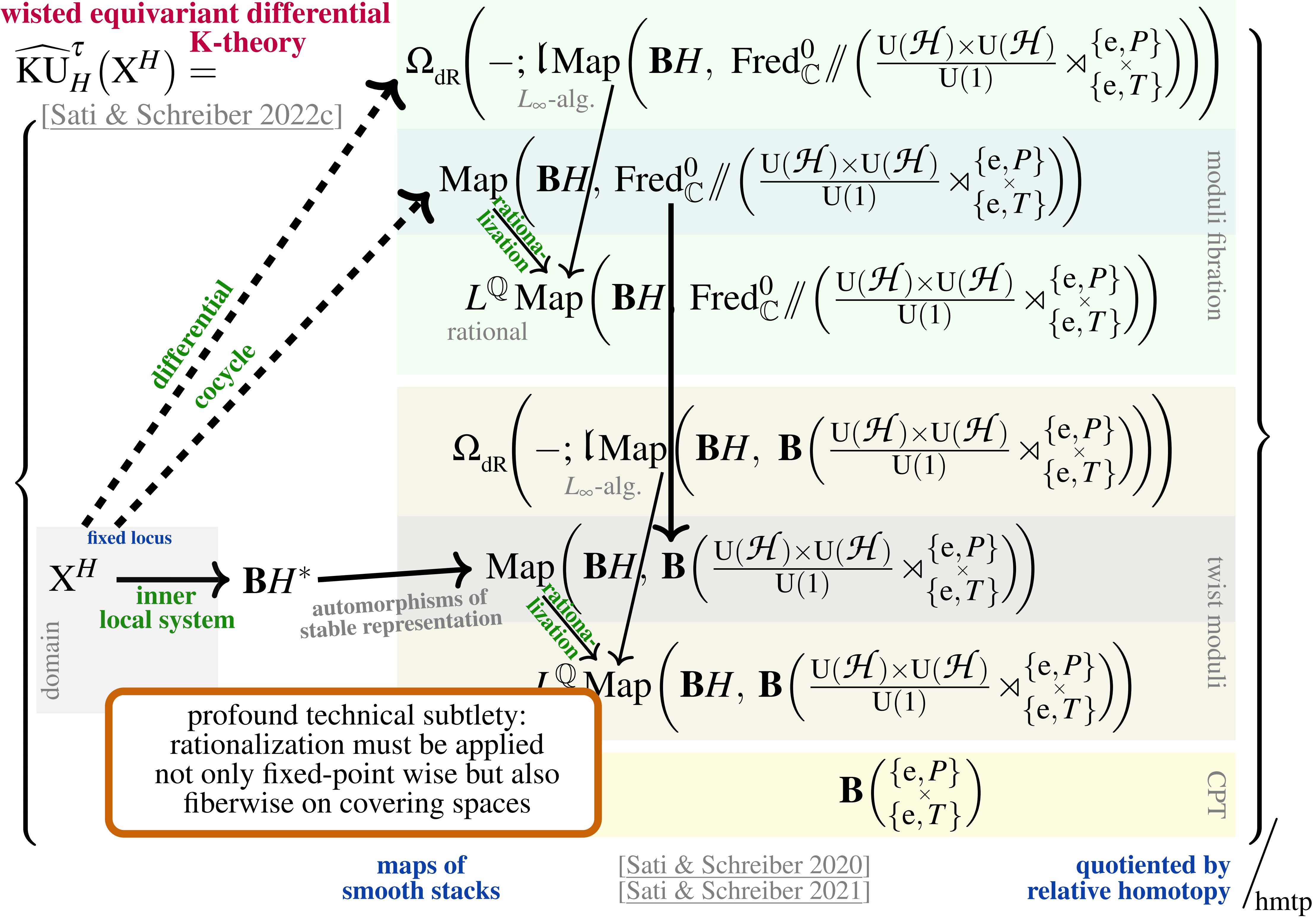
[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by relative homotopy $/_{\mathrm{hmtp}}$

twisted equivariant differential K-theory

$\widehat{\mathrm{KU}}{}^{\tau}_{H}\big(\mathrm{X}^{H}\big) =$

[Sati & Schreiber 2022c]



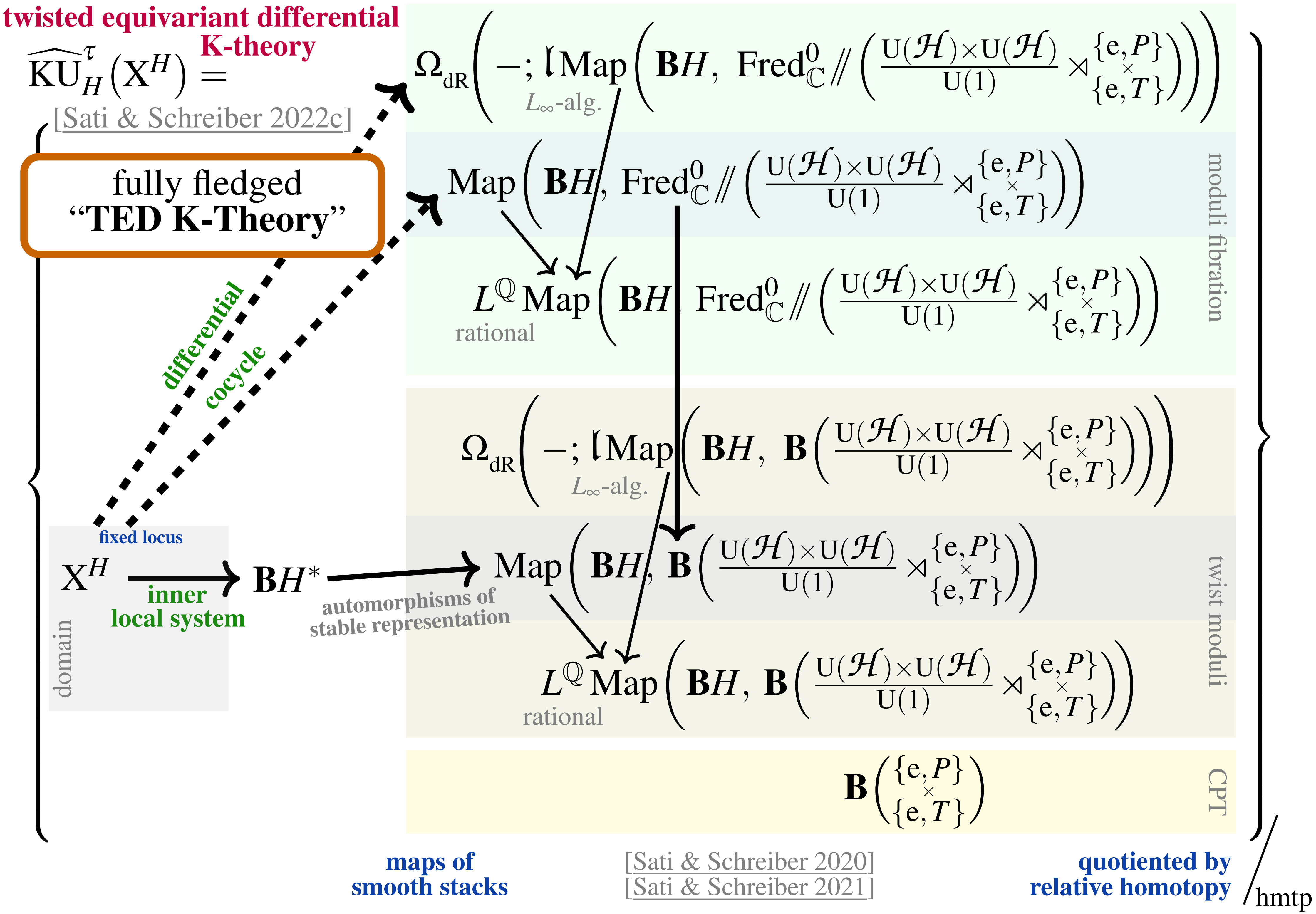
$$\begin{tikzcd}
 & & & \Omega_{\mathrm{dR}}\Big(-;\, \mathfrak{l}\mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)\Big) \arrow[dd] \\
 & & & \mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big) \arrow[d, "\text{rationalization}"'] \arrow[ddd, bend left] \\
 & & & L^{\mathbb{Q}}\mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big) \\
 & & & \Omega_{\mathrm{dR}}\Big(-;\, \mathfrak{l}\mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)\Big) \arrow[dd] \\
\mathrm{X}^{H} \arrow[r, "\substack{\text{inner}\\ \text{local system}}"'] \arrow[uuuurrr, dashed, "\text{differential}"] \arrow[uuurrr, dashed, "\text{cocycle}"'] & \mathbf{B}H^{*} \arrow[rr, "\substack{\text{automorphisms of}\\ \text{stable representation}}"'] & & \mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big) \arrow[d, "\text{rationalization}"'] \\
 & & & L^{\mathbb{Q}}\mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big) \\
 & & & \mathbf{B}\begin{pmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{pmatrix}
\end{tikzcd}$$

profound technical subtlety:
rationalization must be applied
not only fixed-point wise but also
fiberwise on covering spaces

maps of smooth stacks

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

quotiented by relative homotopy



twisted equivariant differential K-theory

$\widehat{\mathrm{KU}}{}^{\tau}_{H}\big(\mathrm{X}^H\big) =$

[Sati & Schreiber 2022c]

fully fledged
“TED K-Theory”

differential

cocycle

$$\Omega_{\mathrm{dR}}\Big(-;\, \mathfrak{l}\mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)\Big)$$

L_∞-alg.

$$\mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)$$

$$L^{\mathbb{Q}}\mathrm{Map}\Big(\mathbf{B}H,\ \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)$$

rational

moduli fibration

$$\Omega_{\mathrm{dR}}\Big(-;\, \mathfrak{l}\mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)\Big)$$

L_∞-alg.

$$\mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)$$

$$L^{\mathbb{Q}}\mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)$$

rational

twist moduli

$$\mathbf{B}\Big(\begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)$$

CPT

fixed locus

$$\mathrm{X}^H \xrightarrow[\text{local system}]{\text{inner}} \mathbf{B}H^* \longrightarrow \mathrm{Map}\Big(\mathbf{B}H,\ \mathbf{B}\Big(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \begin{smallmatrix}\{\mathrm{e},P\}\\ \times \\ \{\mathrm{e},T\}\end{smallmatrix}\Big)\Big)$$

automorphisms of
stable representation

domain

**maps of
smooth stacks**

[Sati & Schreiber 2020]
[Sati & Schreiber 2021]

**quotiented by
relative homotopy**

$/_{\mathrm{hmtp}}$

there is a curious dictionary

Condensed/Quantum Matter	**Alg. Topology/Geom. Homotopy**

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$ **String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$ **Alg. Topology/Geom. Homotopy**

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$	**String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$	**Alg. Topology/Geom. Homotopy**
Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

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$\mathcal{H}$

Hilbert space of quantum states

there is a curious dictionary

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Adiabatic transport of states		Moduli monodromy		Fibrations of vector spaces

$$\begin{tikzcd}
\mathcal{H}_1 \arrow[r, bend left, "U"'] & \mathcal{H}_2
\end{tikzcd}$$

Hilbert space of quantum states

unitary evolution

there is a curious dictionary

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$$\begin{tikzcd} \mathcal{H}_1 \arrow[r, bend left, "U"'] & \mathcal{H}_2 \end{tikzcd}$$

Hilbert space of quantum states

unitary evolution

$$P_1$$

external classical parameters at time t_1

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$$\begin{tikzcd}\mathcal{H}_1 \arrow[r, bend left, "U"'] & \mathcal{H}_2\end{tikzcd}$$

unitary evolution

Hilbert space of quantum states at parameter value P_1

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Adiabatic transport of states		Moduli monodromy		Fibrations of vector spaces

$$\begin{tikzcd} \mathcal{H}_1 \arrow[r, bend left, "U"] & \mathcal{H}_2 \end{tikzcd}$$

unitary evolution

Hilbert space of quantum states at parameter value P_1

$$\begin{tikzcd} P_1 \arrow[r, squiggly, "p_{12}"] & P_2 \end{tikzcd}$$

adiabatic flow of parameters

external classical parameters at time t_1

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$ **String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$ **Alg. Topology/Geom. Homotopy**

Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

$$\begin{tikzcd} \mathcal{H}_1 \arrow[r, bend left, "U_{p_{12}}"] & \mathcal{H}_2 \end{tikzcd}$$

Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

$$\begin{tikzcd} P_1 \arrow[r, squiggly, "p_{12}"] & P_2 \end{tikzcd}$$

external classical parameters at time t_1

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Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

$$\begin{tikzcd}
\mathcal{H}_1 \arrow[r, bend left, "U_{p_{12}}"] & \mathcal{H}_2
\end{tikzcd}$$

Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

$$\begin{tikzcd}
P_1 \arrow[r, squiggly, "p_{12}"] & P_2 \arrow[r, squiggly, "p_{23}"] & P_3
\end{tikzcd}$$

external classical parameters at time t_1

adiabatic flow of parameters

external classical parameters at time t_3

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Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

Hilbert space of quantum states at parameter value P_3

$$\begin{tikzcd} P_1 \arrow[r, squiggly, "p_{12}"] & P_2 \arrow[r, squiggly, "p_{23}"] & P_3 \end{tikzcd}$$

external classical parameters at time t_1

adiabatic flow of parameters

external classical parameters at time t_3

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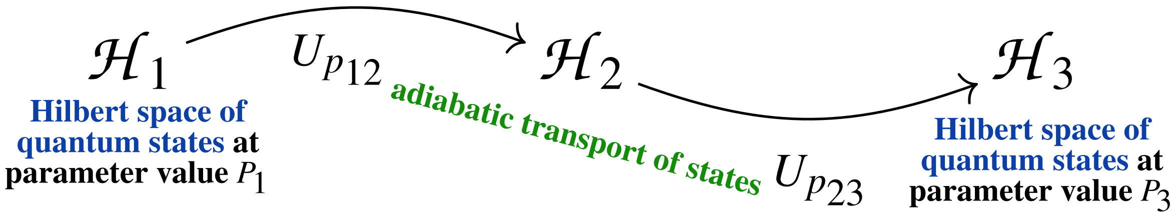
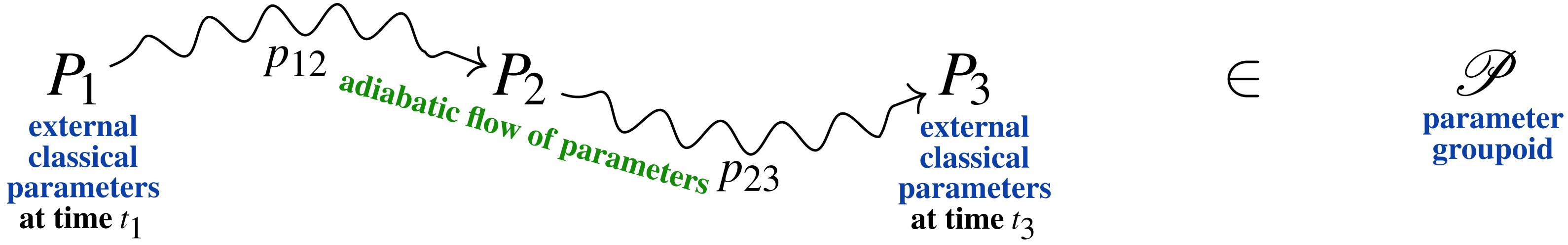


$$\begin{tikzcd}
\mathcal{H}_1 \arrow[r, bend left, "U_{p_{12}}"] & \mathcal{H}_2 \arrow[r, bend right, "U_{p_{23}}"'] & \mathcal{H}_3
\end{tikzcd}$$

Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

Hilbert space of quantum states at parameter value P_3



$$\begin{tikzcd}
P_1 \arrow[r, squiggly, "p_{12}"] & P_2 \arrow[r, squiggly, "p_{23}"'] & P_3 & \in & \mathscr{P}
\end{tikzcd}$$

external classical parameters at time t_1

adiabatic flow of parameters

external classical parameters at time t_3

parameter groupoid

there is a curious dictionary

Condensed/Quantum Matter	AdS/CMT ⟷	**String/M-Theory**	flux, charge quantization ⟷	**Alg. Topology/Geom. Homotopy**
Adiabatic transport of states		Moduli monodromy		Fibrations of vector spaces

$$\begin{tikzcd}
\mathcal{H}_1 \arrow[r, bend left, "U_{p_{12}}"'] & \mathcal{H}_2 \arrow[r, bend right, "U_{p_{23}}"] & \mathcal{H}_3 & \subset & \mathcal{H} \arrow[d, "\text{fibration of quantum state spaces}"] \\
P_1 \arrow[r, squiggly, "p_{12}"'] & P_2 \arrow[r, squiggly, "p_{23}"] & P_3 & \in & \mathscr{P}
\end{tikzcd}$$

Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

Hilbert space of quantum states at parameter value P_3

external classical parameters at time t_1

adiabatic flow of parameters

external classical parameters at time t_3

parameter groupoid

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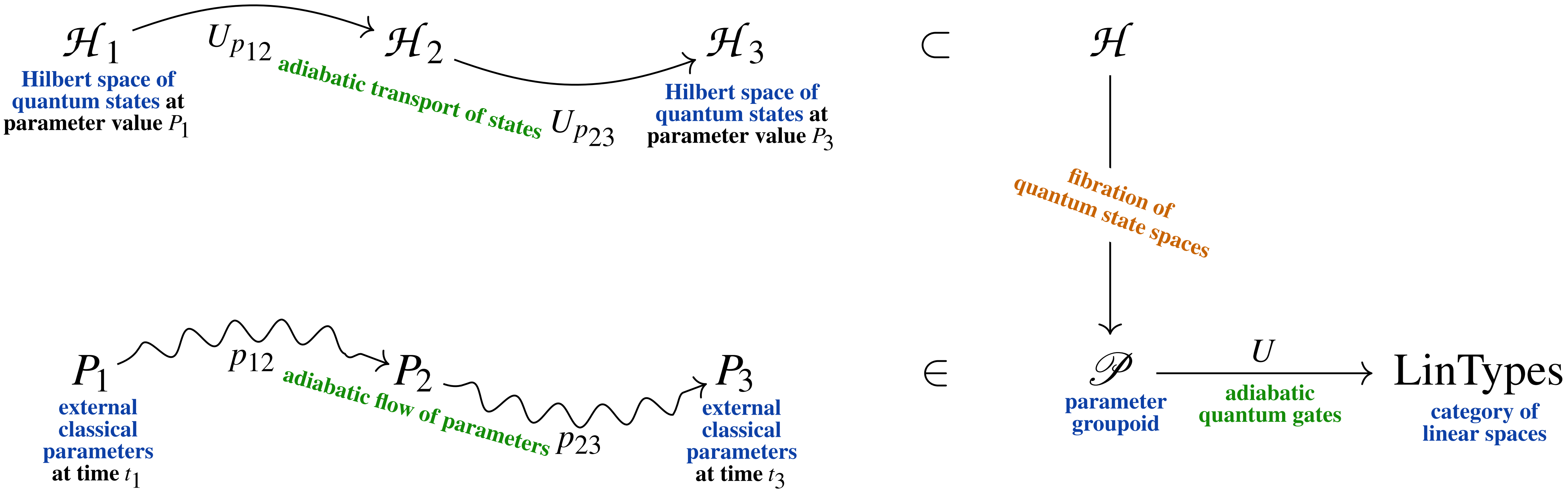
Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces



$$\begin{tikzcd}
\mathcal{H}_1 \arrow[r, bend left, "U_{p_{12}}"] & \mathcal{H}_2 \arrow[r, bend right, "U_{p_{23}}"] & \mathcal{H}_3 & \subset & \mathcal{H} \arrow[d] & \\
P_1 \arrow[r, squiggly, "p_{12}"] & P_2 \arrow[r, squiggly, "p_{23}"] & P_3 & \in & \mathscr{P} \arrow[r, "U"] & \mathrm{LinTypes}
\end{tikzcd}$$

Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

Hilbert space of quantum states at parameter value P_3

fibration of quantum state spaces

external classical parameters at time t_1

adiabatic flow of parameters

external classical parameters at time t_3

parameter groupoid

adiabatic quantum gates

category of linear spaces

there is a curious dictionary

Condensed/Quantum Matter	$\xleftrightarrow{\text{AdS/CMT}}$	**String/M-Theory**	$\xleftrightarrow{\text{flux, charge quantization}}$	**Alg. Topology/Geom. Homotopy**
Adiabatic transport of states		Moduli monodromy		Fibrations of vector spaces

$$\begin{tikzcd}
\mathcal{H}_1 \arrow[rr, bend left, "U_{p_{12}}"] & & \mathcal{H}_2 \arrow[rr, bend right, "U_{p_{23}}"'] & & \mathcal{H}_3 & \subset & \mathcal{H} \arrow[rr] \arrow[dd] \arrow[ddrr, phantom, "\text{(pb)}"] & & \widehat{\mathrm{LinTypes}} \arrow[dd] \\
\\
P_1 \arrow[rr, squiggly, "p_{12}"] & & P_2 \arrow[rr, squiggly, "p_{23}"'] & & P_3 & \in & \mathscr{P} \arrow[rr, "U"] & & \mathrm{LinTypes}
\end{tikzcd}$$

Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

Hilbert space of quantum states at parameter value P_3

fibration of quantum state spaces

univalent fibration of linear spaces

external classical parameters at time t_1

adiabatic flow of parameters

external classical parameters at time t_3

parameter groupoid

adiabatic quantum gates

category of linear spaces

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$	**String/M-Theory** $\xleftrightarrow[\text{quantization}]{\text{flux, charge}}$	**Alg. Topology/Geom. Homotopy**
Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

Topological Quantum Programming

Example. For TQC one takes:
parameters = sets of distinct points in plane
parameter paths = braids of their worldlines

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$ **String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$ **Alg. Topology/Geom. Homotopy**

Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

Topological Quantum Programming

Example. For TQC one takes:
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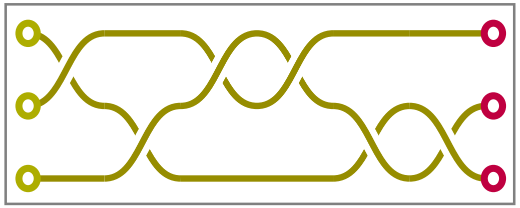
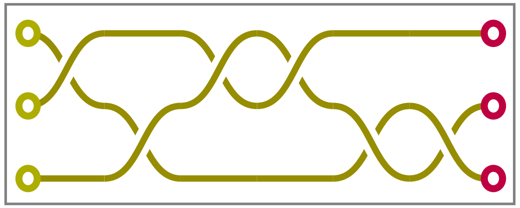


braid

there is a curious dictionary

Condensed/Quantum Matter	$\xleftrightarrow{\text{AdS/CMT}}$	**String/M-Theory**	$\xleftrightarrow{\text{flux, charge quantization}}$	**Alg. Topology/Geom. Homotopy**
Adiabatic transport of states		Moduli monodromy		Fibrations of vector spaces

Topological Quantum Programming



braid

there is a curious dictionary

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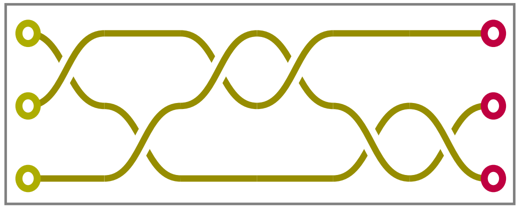
Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

Topological Quantum Programming

Adiabatic transprt along such parameters
is a unitary *braid representation*



braid

$\longmapsto$ **braid representation**

$$\mathcal{H}_3 \xrightarrow{U} \mathcal{H}_3$$

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$ **String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$ **Alg. Topology/Geom. Homotopy**

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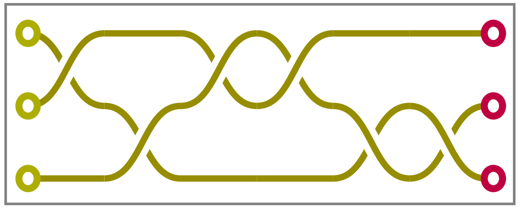
Topological Quantum Programming



braid

$\xmapsto{\text{braid representation}}$

$$\boxed{\mathcal{H}_3 \xrightarrow{U} \mathcal{H}_3}$$

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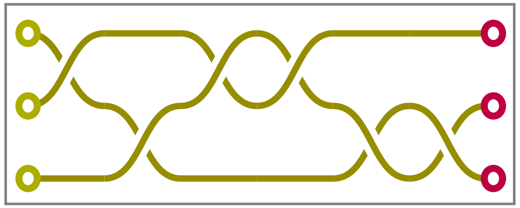
Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

Topological Quantum Programming

$$\begin{tikzcd} \mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3 \\ \lvert \psi_{\mathrm{in}} \rangle \arrow[u, phantom, "\rotatebox{90}{$\in$}"] \arrow[r, mapsto] & \lvert \psi_{\mathrm{out}} \rangle \arrow[u, phantom, "\rotatebox{90}{$\in$}"] \end{tikzcd}$$

$$\begin{tikzcd} {} \arrow[d, mapsto] \\ {} \end{tikzcd}$$



braid

$$\begin{tikzcd} {} \arrow[r, mapsto, "\text{braid representation}"'] & \mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3 \end{tikzcd}$$

there is a curious dictionary

Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$ **String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$ **Alg. Topology/Geom. Homotopy**

Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

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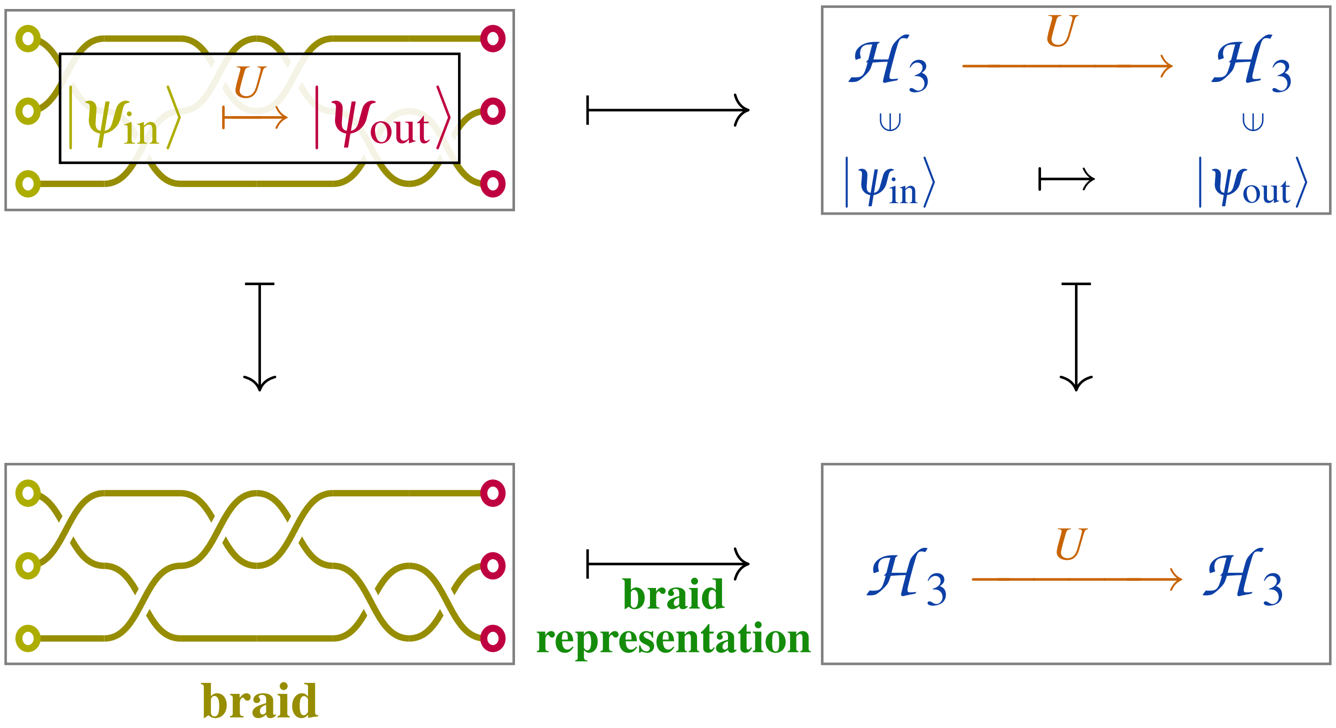

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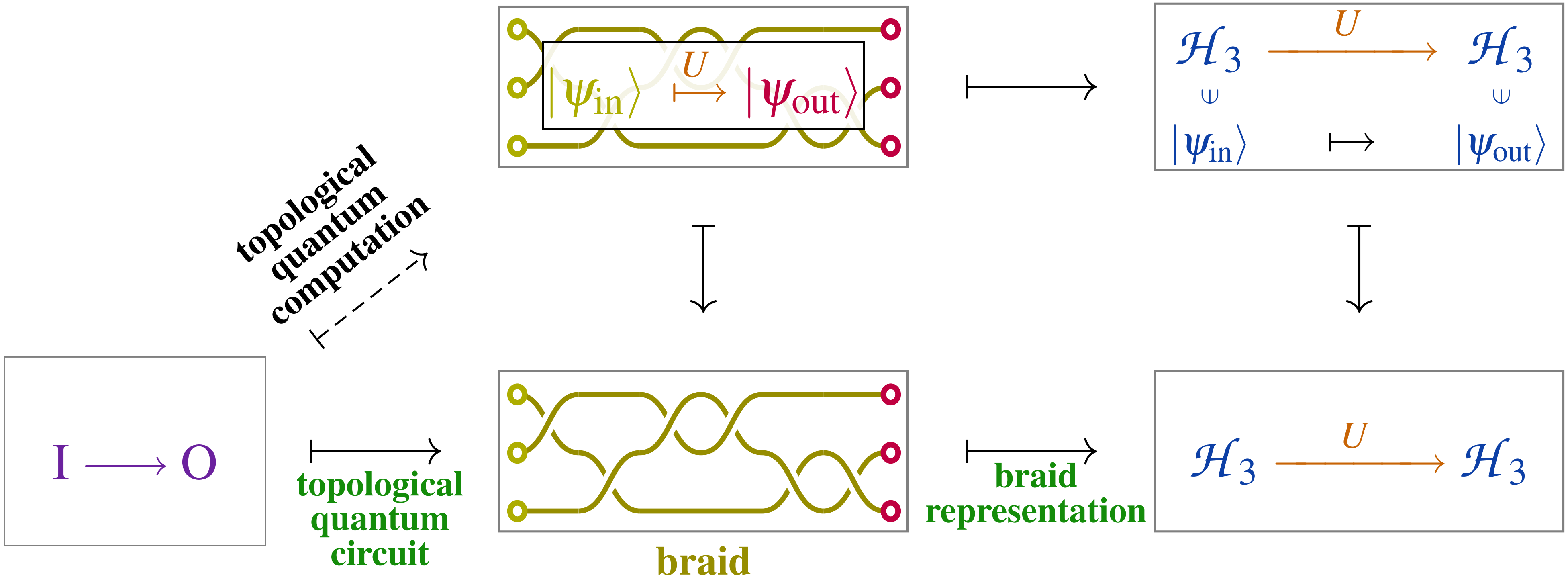
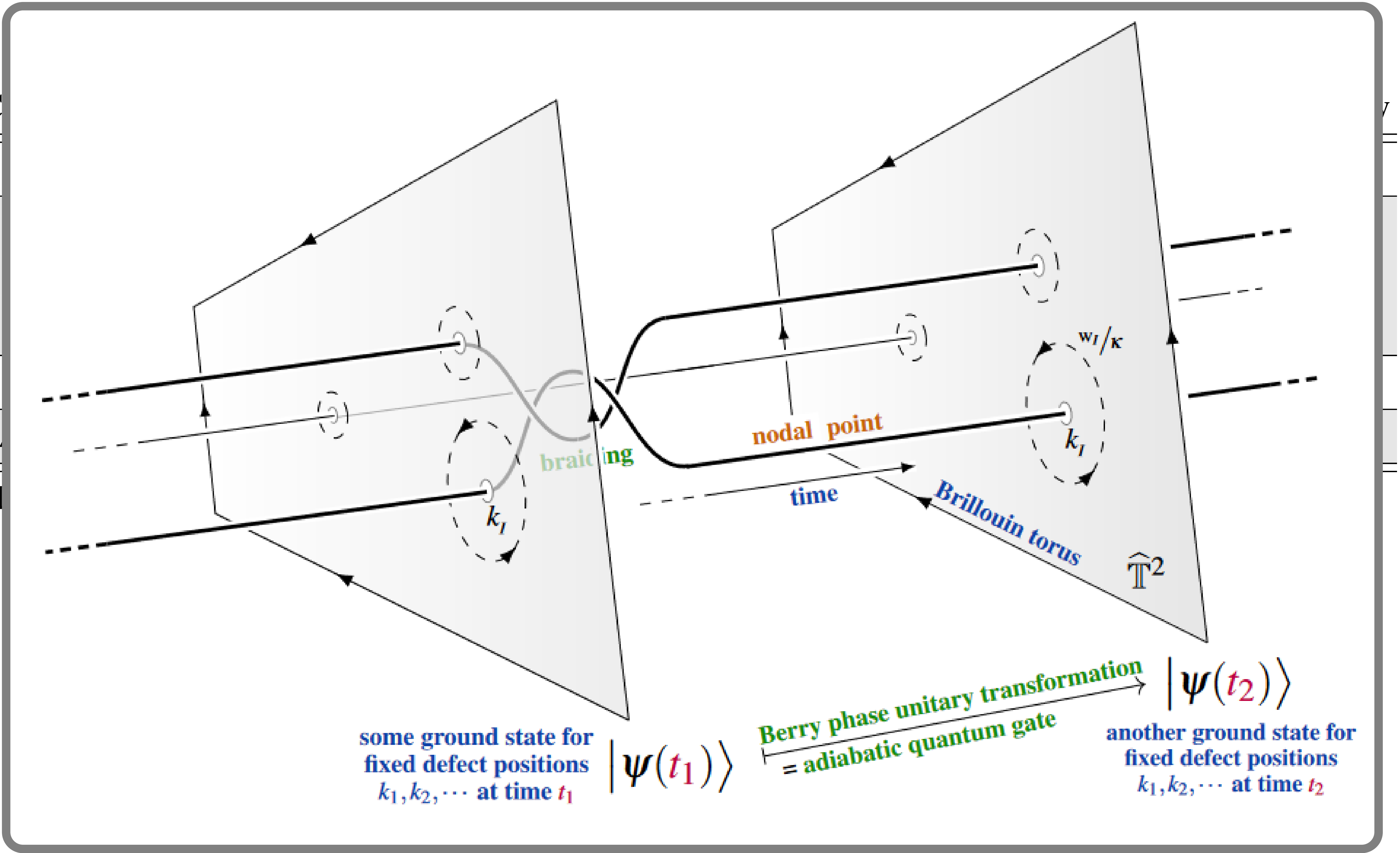
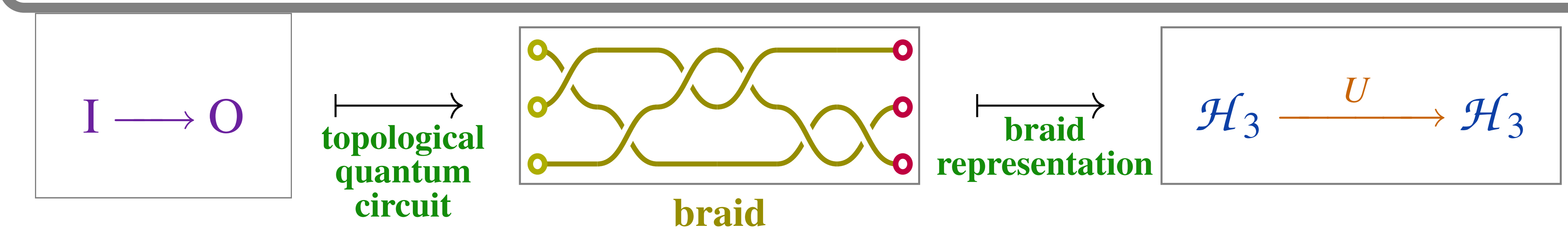



w_I/κ
nodal point
braiding
k_I
k_I
time
Brillouin torus
$\widehat{\mathbb{T}}^2$
some ground state for fixed defect positions $k_1, k_2, \cdots$ at time t_1
$|\psi(t_1)\rangle$
Berry phase unitary transformation
= adiabatic quantum gate
$|\psi(t_2)\rangle$
another ground state for fixed defect positions $k_1, k_2, \cdots$ at time t_2


I ⟶ O
topological quantum circuit
braid
braid representation
$\mathcal{H}_3 \xrightarrow{U} \mathcal{H}_3$

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Condensed/Quantum Matter	AdS/CMT ⟷	String/M-Theory	flux, charge quantization ⟷	Alg. Topology/Geom. Homotopy
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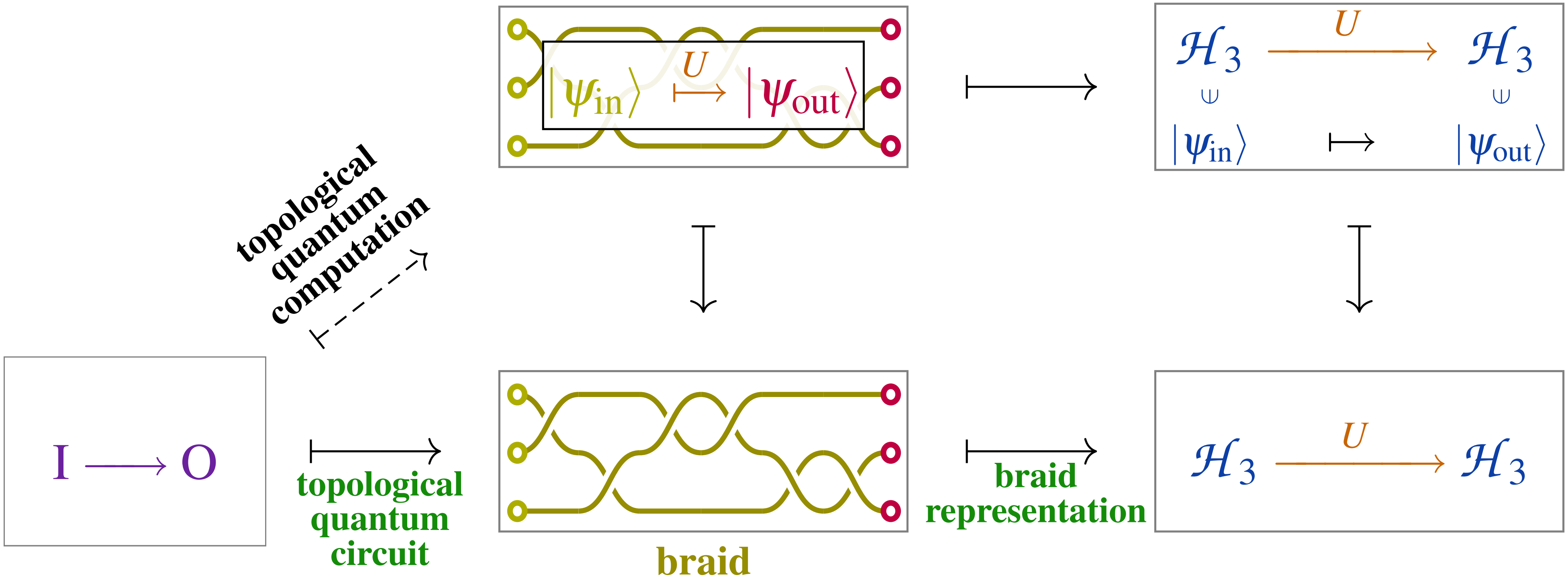

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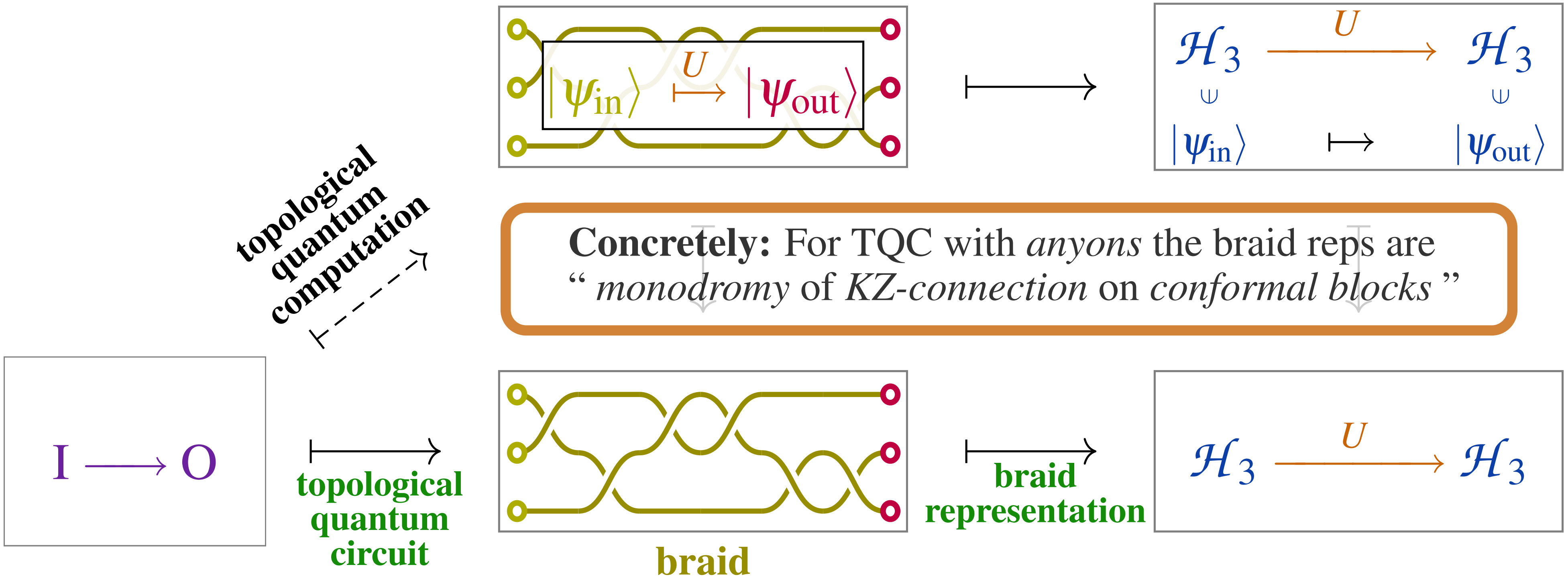

$$\begin{tikzcd} \mathrm{I} \arrow[r] & \mathrm{O} \end{tikzcd} \xmapsto{\text{topological quantum circuit}} \text{braid} \xmapsto{\text{braid representation}} \begin{tikzcd} \mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3 \end{tikzcd}$$

topological quantum computation

$|\psi_{\mathrm{in}}\rangle \overset{U}{\longmapsto} |\psi_{\mathrm{out}}\rangle$ $\longmapsto$

$$\begin{tikzcd} \mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3 \\ {|\psi_{\mathrm{in}}\rangle} \arrow[u, phantom, sloped, "\in"] \arrow[r, mapsto] & {|\psi_{\mathrm{out}}\rangle} \arrow[u, phantom, sloped, "\in"] \end{tikzcd}$$

Concretely: For TQC with *anyons* the braid reps are "*monodromy* of *KZ-connection* on *conformal blocks*"

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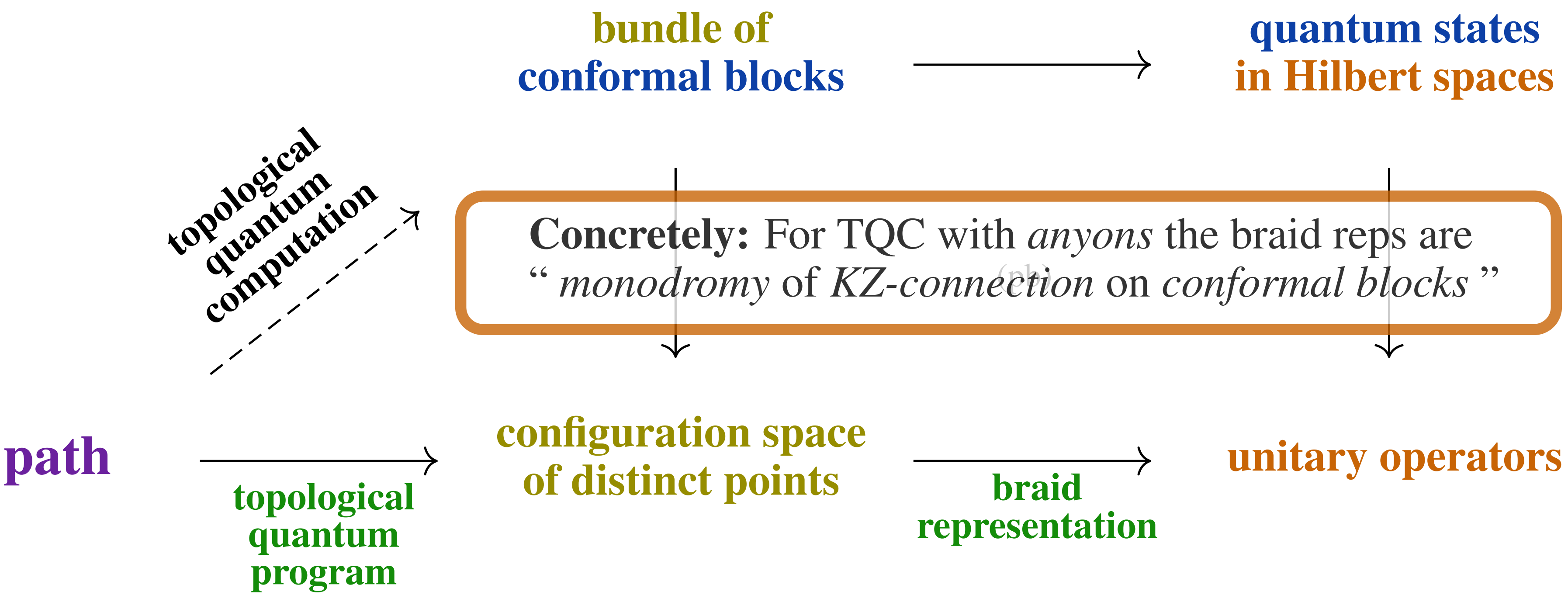

bundle of conformal blocks → **quantum states in Hilbert spaces**

topological quantum computation

path → **configuration space of distinct points** → **unitary operators**

topological quantum program

braid representation

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Topological Quantum Programming

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[d] & \\
\text{path} \arrow[r, "\text{topological quantum program}"'] \arrow[ur, dashed, "\text{topological quantum computation}"] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
\end{tikzcd}$$

This describes adiabatic braiding of *band nodes* of topol. ordered semi-metals classified in TED K-theory of config. space:

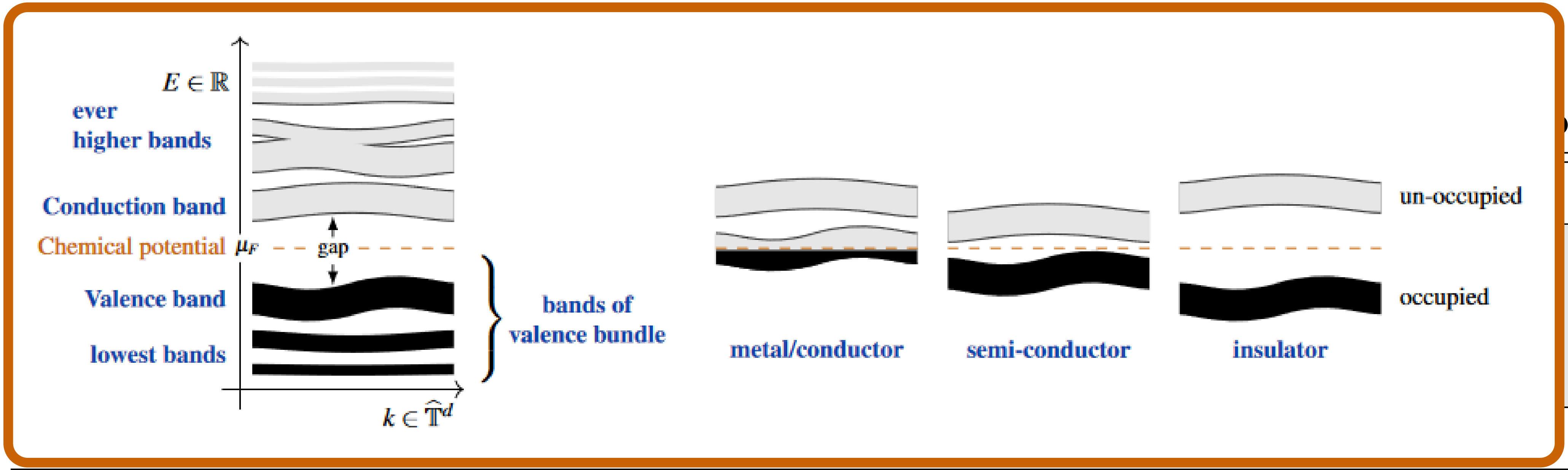
arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

Anyonic Topological Order in Twisted Equivariant Differential (TED) K-Theory

Hisham Sati, Urs Schreiber



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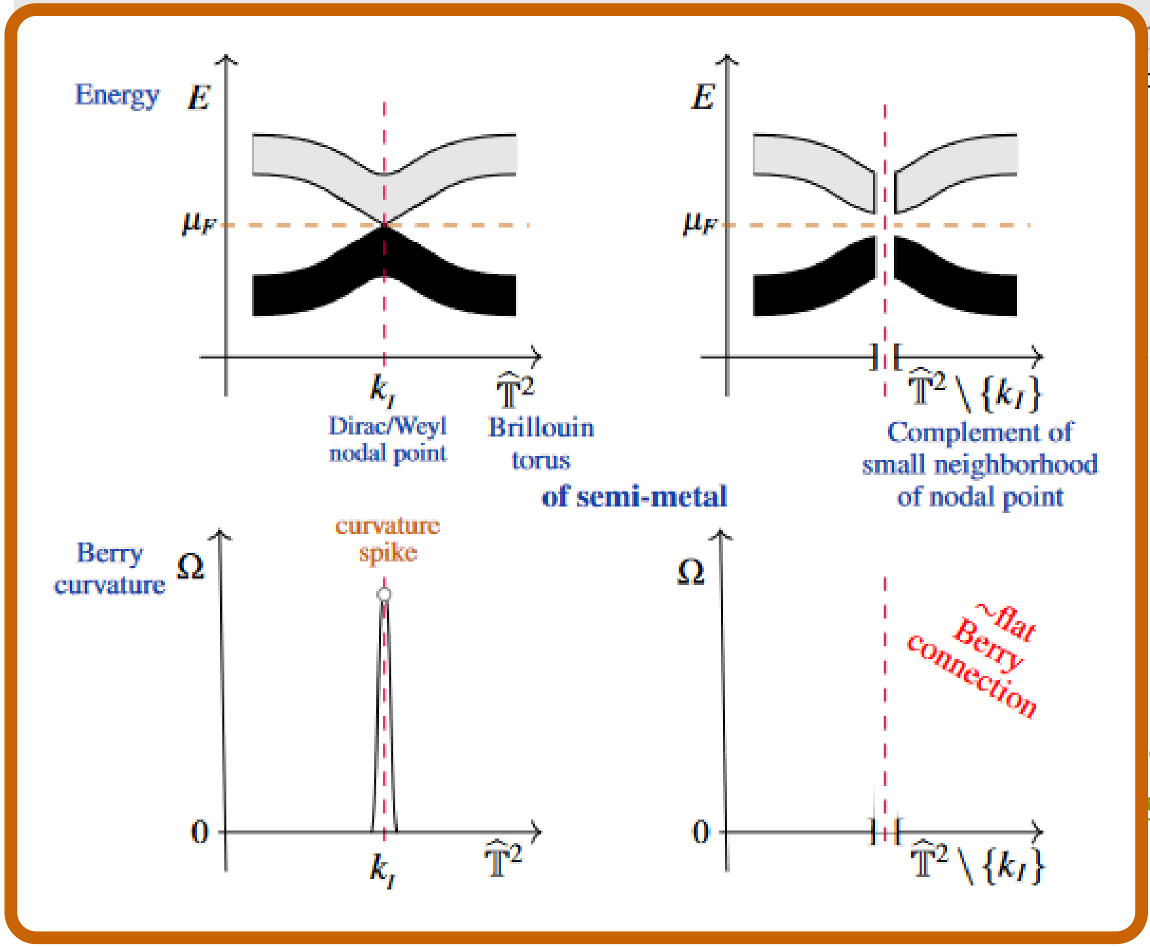

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$\xrightarrow[\text{braid representation}]{}$ **unitary operators**

arXiv > cond-mat > arXiv:1907.10611

nature physics

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 24 Jul 2019 (v1), last revised 7 Mar 2021 (this version, v4)]

Non-Abelian reciprocal braiding of Weyl points and its manifestation in ZrTe

Adrien Bouhon, QuanSheng Wu, Robert-Jan Slager, Hongming Weng, Oleg V. Yazyev, Tomáš Bzdušek

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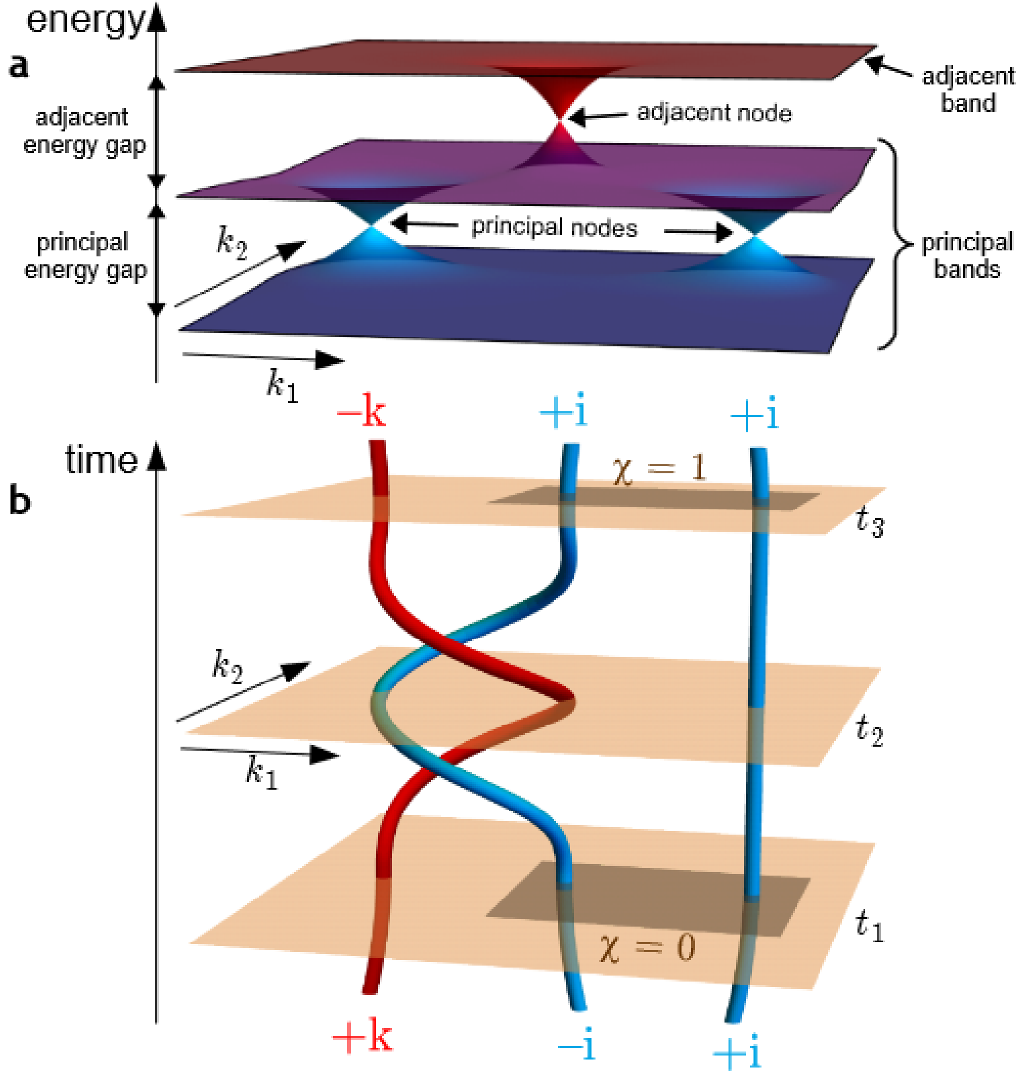

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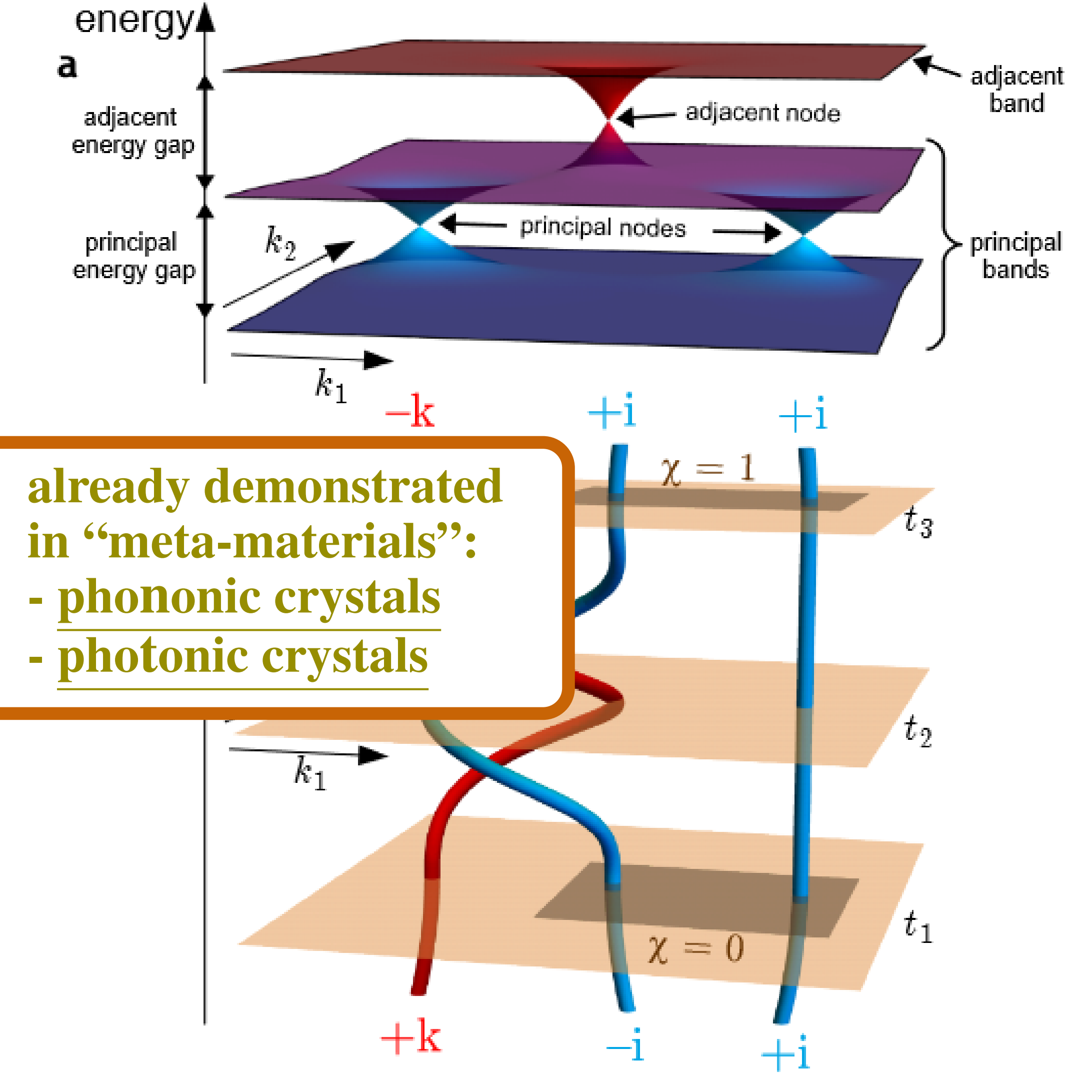

FIG. 1. **Reciprocal braiding of band nodes.**

already demonstrated in "meta-materials":
- **phononic crystals**
- **photonic crystals**

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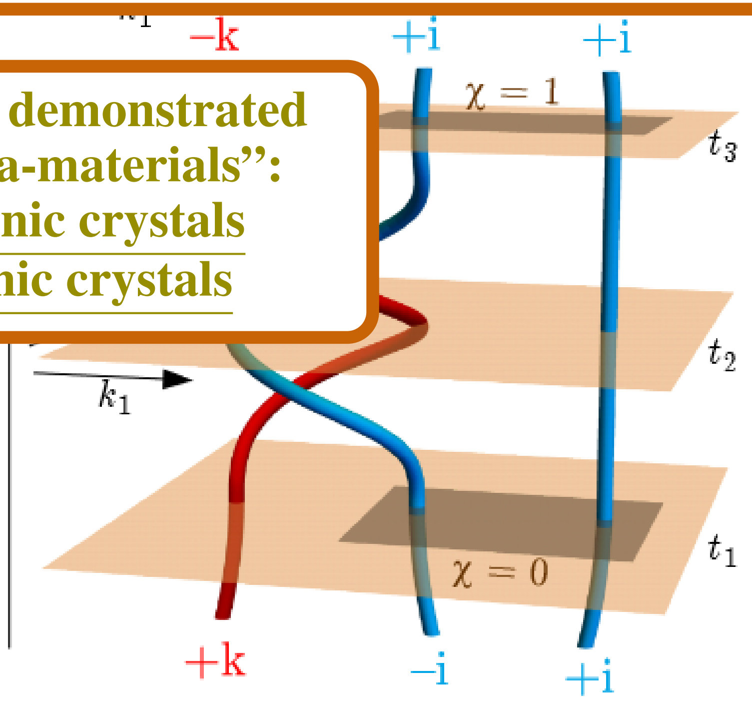
Experimental observation of non-Abelian topological acoustic semimetals and their phase transitions

Bin Jiang, Adrien Bouhon, Zhi-Kang Lin, Xiaoxi Zhou, Bo Hou, Feng Li, Robert-Jan Slager & Jian-Hua Jiang



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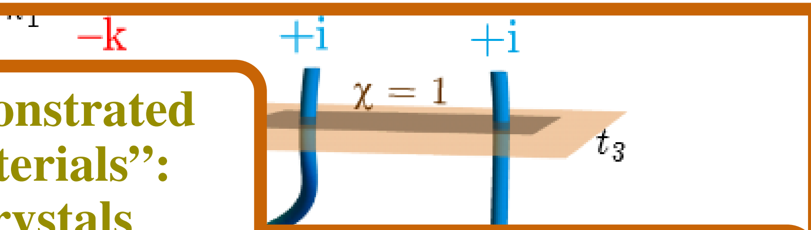

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ed by De Gruyter February 2, 2022

Nodal lines in momentum space: topological invariants and recent realizations in photonic and other

Haedong Park, Wenlong Gao, Xiao Zhang and Sang Soon Oh

From the journal Nanophotonics
https://doi.org/10.1515/nanoph-2021-0692

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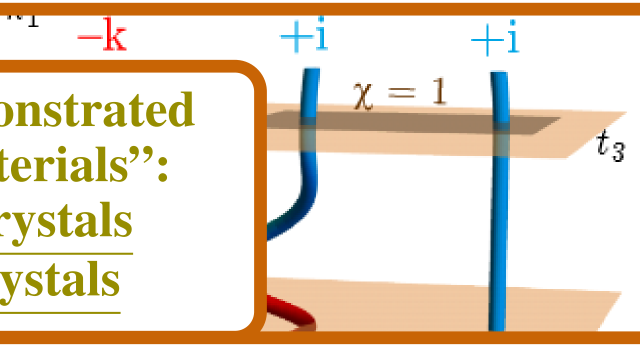

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Article | Published: 28 March 2022

Non-Abelian braiding on photonic chips

Xu-Lin Zhang, Feng Yu, Ze-Guo Chen, Zhen-Nan Tian, Qi-Dai Chen, Hon Ma

Nature Photonics **16**, 390–395 (2022) | Cite this article

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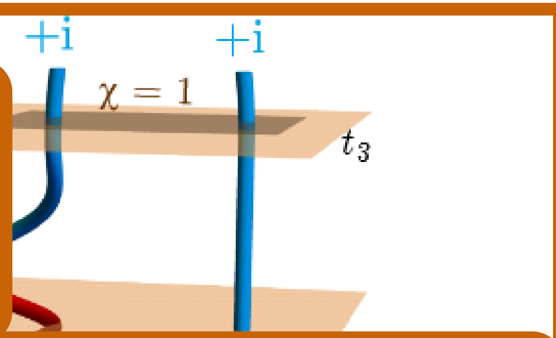

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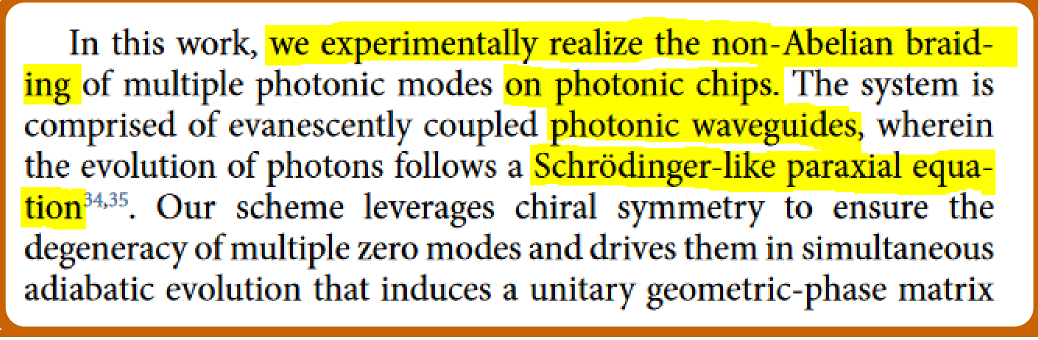
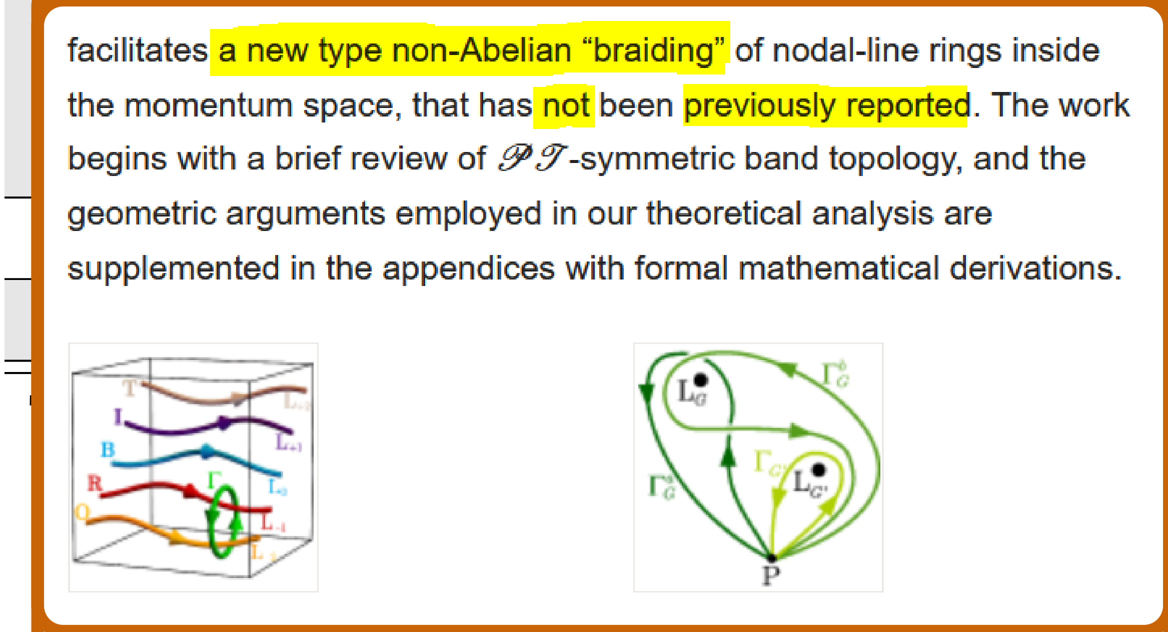

In this work, we experimentally realize the non-Abelian braiding of multiple photonic modes on photonic chips. The system is comprised of evanescently coupled photonic waveguides, wherein the evolution of photons follows a Schrödinger-like paraxial equation[34,35]. Our scheme leverages chiral symmetry to ensure the degeneracy of multiple zero modes and drives them in simultaneous adiabatic evolution that induces a unitary geometric-phase matrix

$$\xrightarrow[\text{braid representation}]{} \quad \text{unitary operators}$$

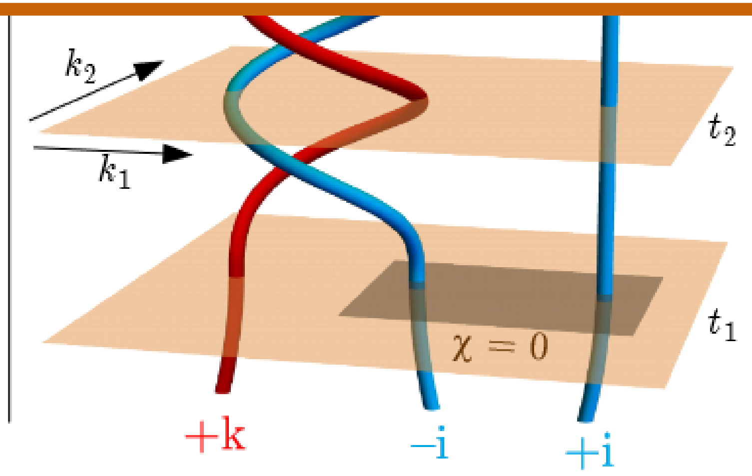
ictionary

Non-Abelian topology of nodal-line rings in $\mathscr{PT}$-symmetric systems

Apoorv Tiwari and Tomáš Bzdušek
Phys. Rev. B **101**, 195130 – Published 18 May 2020



facilitates a new type non-Abelian "braiding" of nodal-line rings inside the momentum space, that has not been previously reported. The work begins with a brief review of $\mathscr{PT}$-symmetric band topology, and the geometric arguments employed in our theoretical analysis are supplemented in the appendices with formal mathematical derivations.



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arXiv > hep-th > arXiv:2206.13563

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[Submitted on 27 Jun 2022]

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Hisham Sati, Urs Schreiber

ry ⟷ flux, charge quantization ⟷ **Alg. Topology/Geom. Homotopy**

my | Fibrations of vector spaces

braid representation ⟶ **unitary operators**

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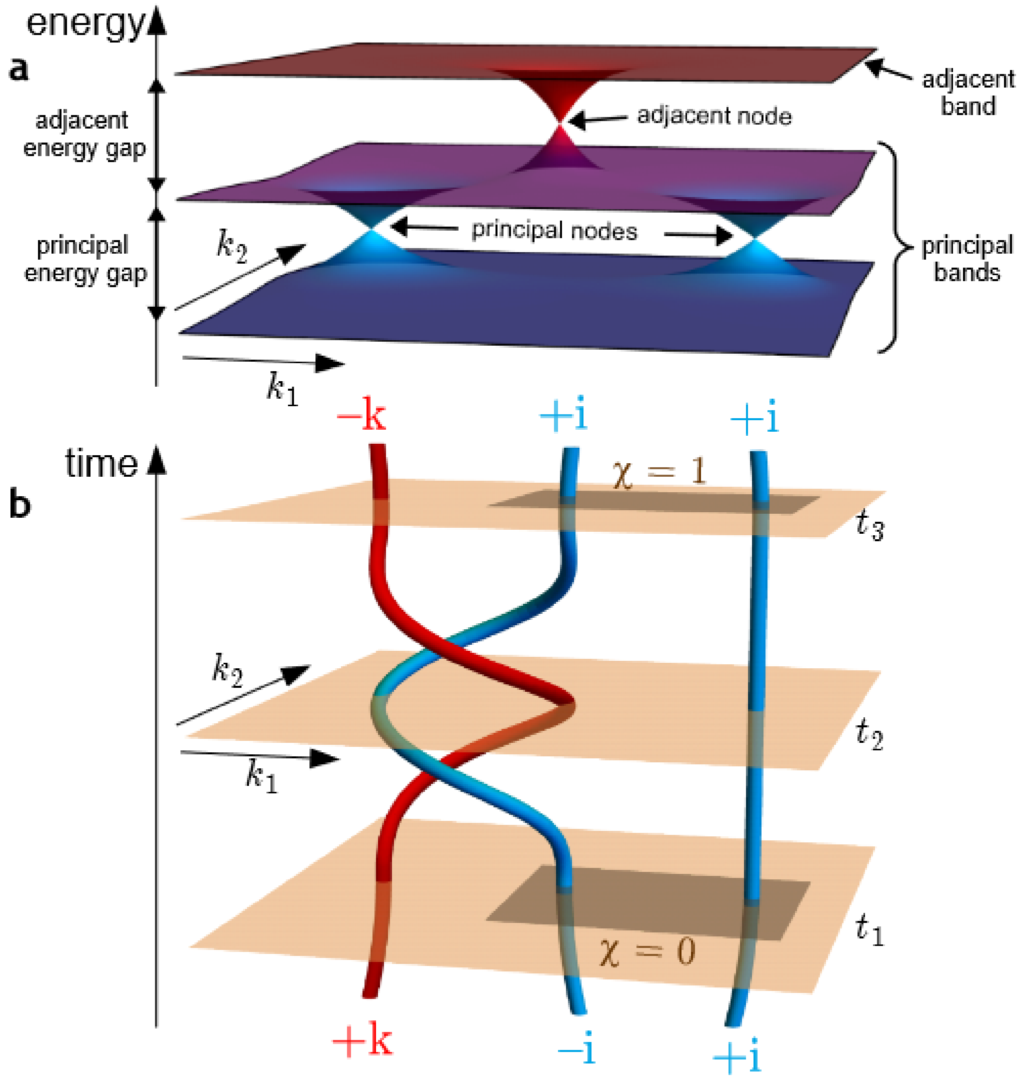

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Condense | **Theory** | flux, charge quantization | **Alg. Topology/Geom. Homotopy**

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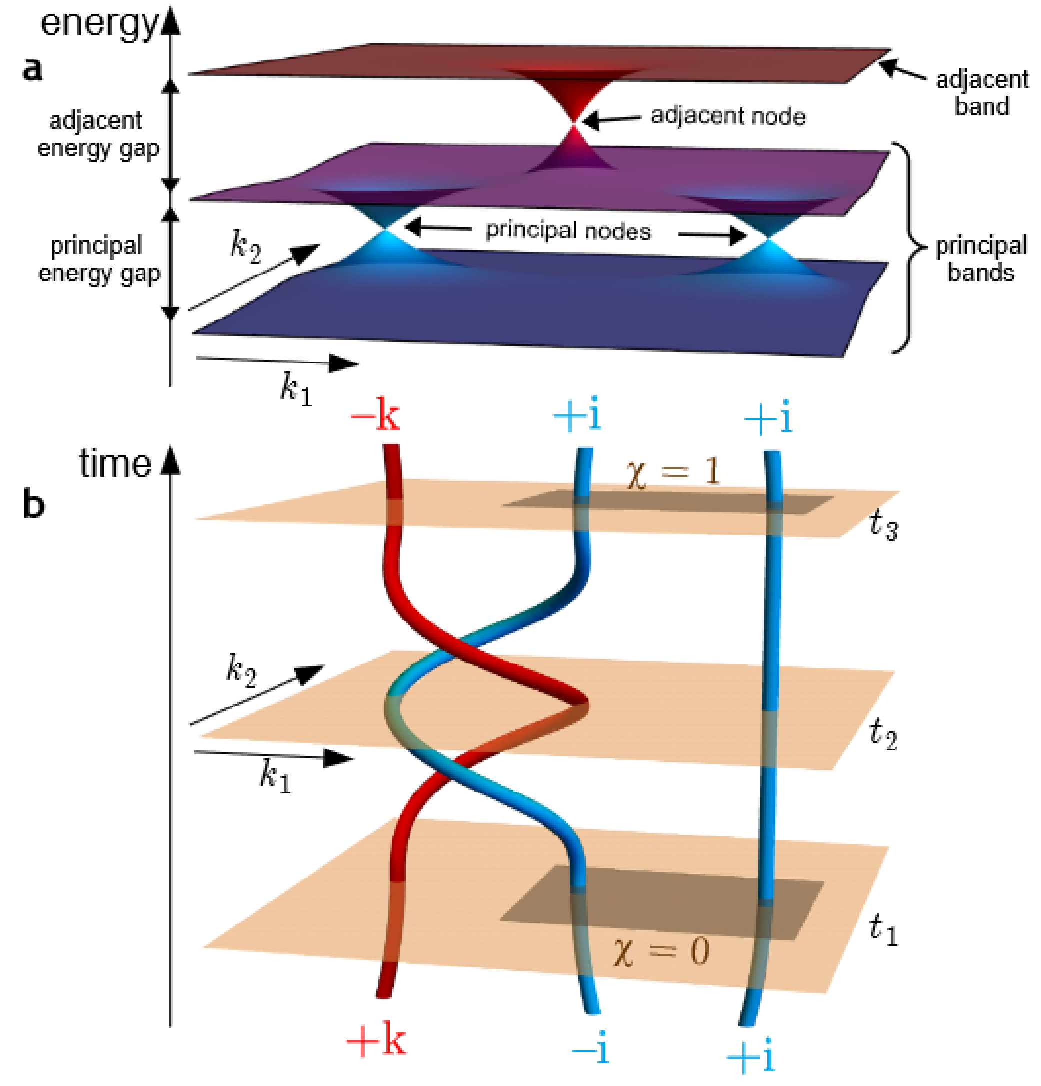
To do: Scrutinize further evidence that/when/how such band nodes indeed qualify as anyons in momentum space.



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$\longrightarrow$

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flux, charge

Condense

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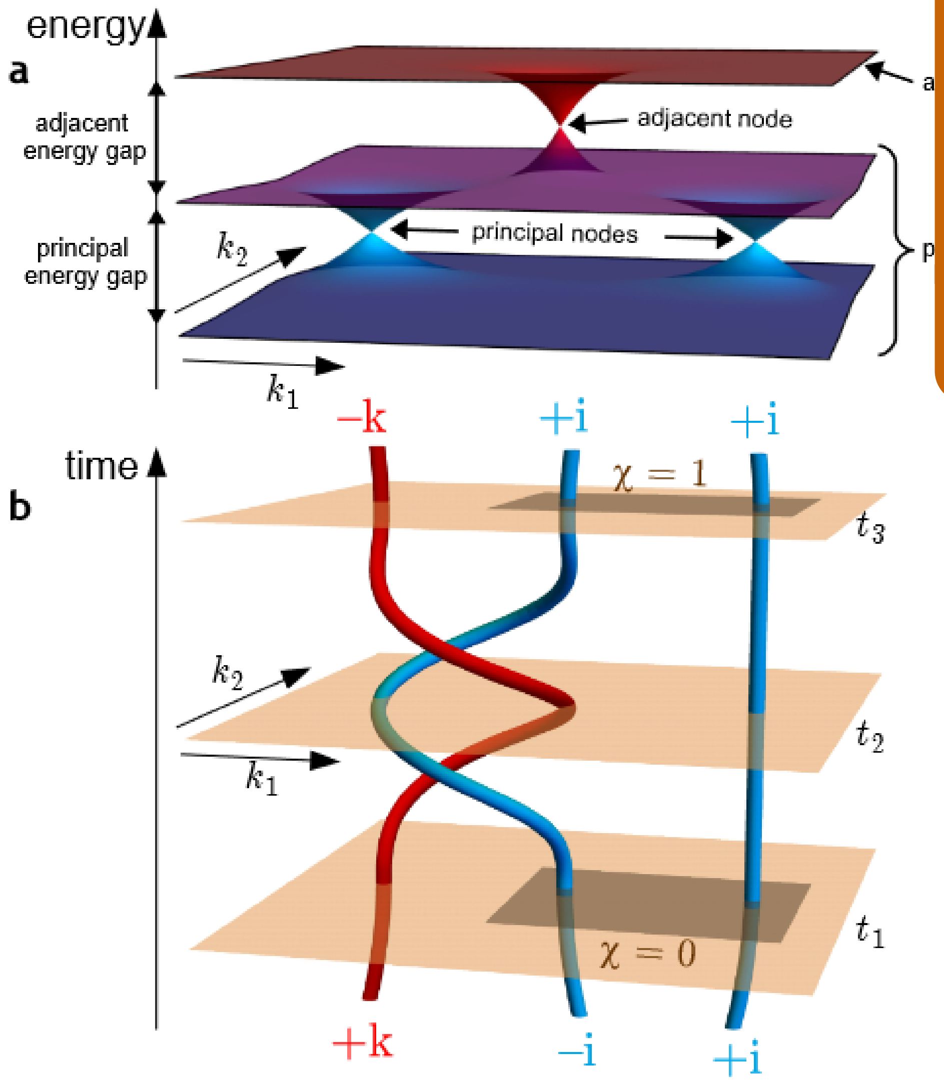

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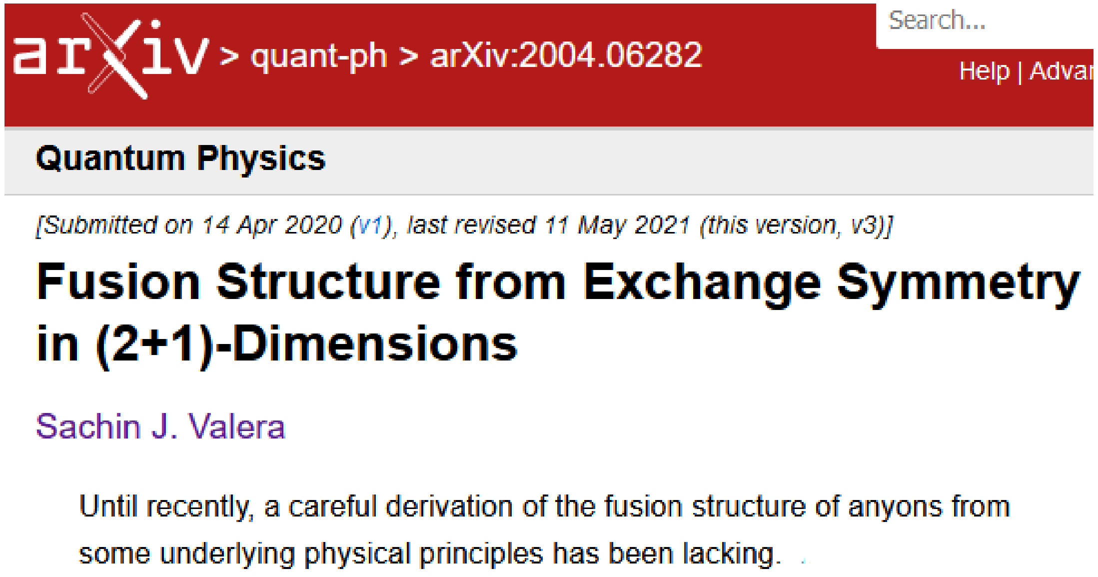

Quantum Physics

[Submitted on 14 Apr 2020 (v1), last revised 11 May 2021 (this version, v3)]

Fusion Structure from Exchange Symmetry in (2+1)-Dimensions

Sachin J. Valera

Until recently, a careful derivation of the fusion structure of anyons from some underlying physical principles has been lacking.

This describes adiabatic braiding of *band nodes* of topol. ordered semi-metals classified in TED K-theory of config. space:

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Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

Topological Quantum Programming

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[d] & \\
\text{path} \arrow[ur, dashed, "\text{topological quantum computation}"] \arrow[r, "\text{topological quantum program}"'] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
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gapped ground states		stable D-branes		topological KR-theory
Adiab				aces
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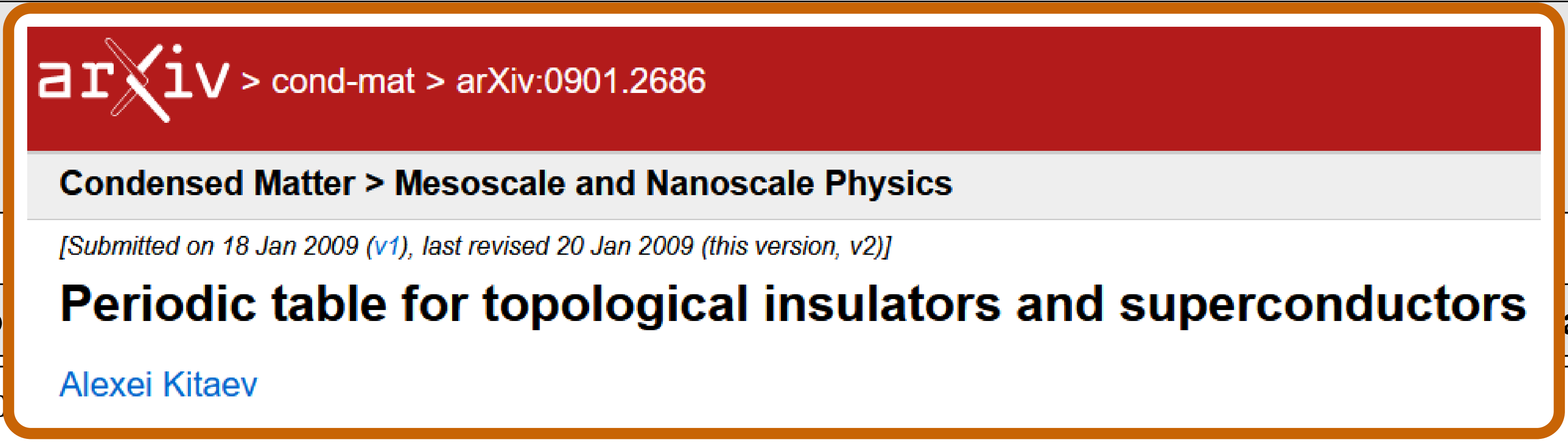

arXiv > cond-mat > arXiv:0901.2686

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 18 Jan 2009 (v1), last revised 20 Jan 2009 (this version, v2)]

Periodic table for topological insulators and superconductors

Alexei Kitaev

$$\begin{tikzcd}
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arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

Anyonic Topological Order in Twisted Equivariant Differential (TED) K-Theory

Hisham Sati, Urs Schreiber

there is a curious dictionary

Condensed/Quantum Matter	AdS/CMT ⟷	**String/M-Theory**	flux, charge quantization ⟷	**Alg. Topology/Geom. Homotopy**
gapped ground states		stable D-branes		topological KR-theory
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Topological Quantum Programming

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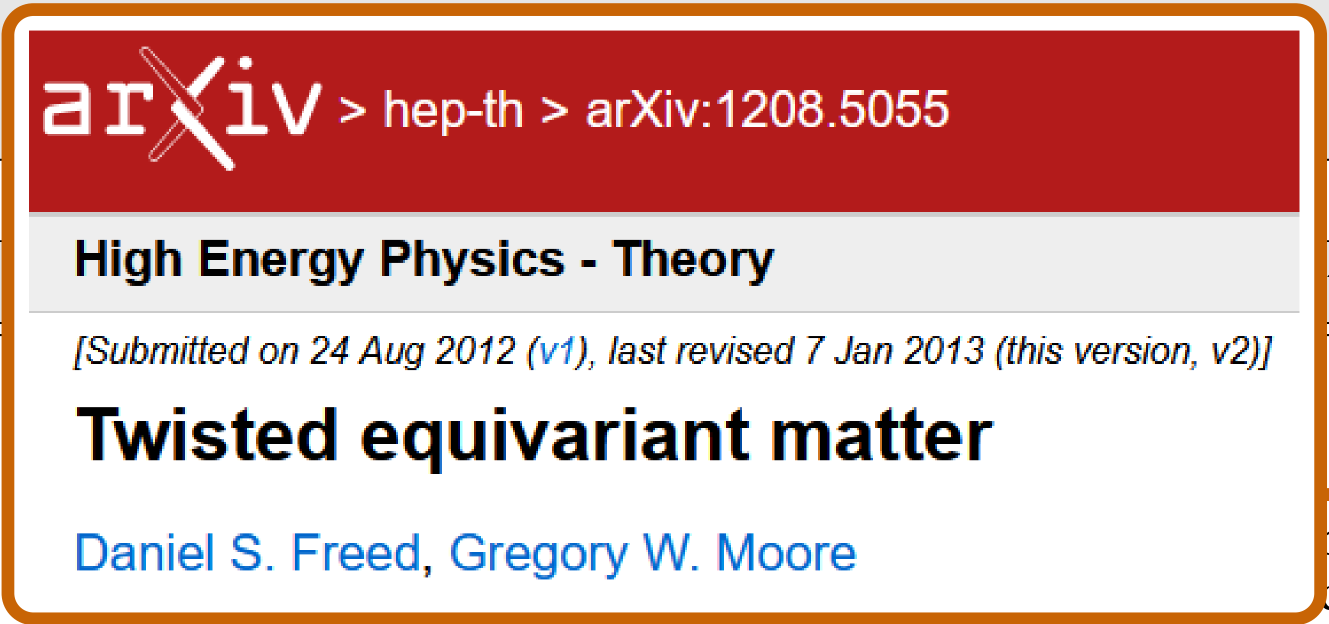

arXiv > hep-th > arXiv:1208.5055

High Energy Physics - Theory

[Submitted on 24 Aug 2012 (v1), last revised 7 Jan 2013 (this version, v2)]

Twisted equivariant matter

Daniel S. Freed, Gregory W. Moore

Topological Quantum

braiding of

d semi-metals

classified in TED K-theory of config. space:

arXiv > hep-th > arXiv:2206.13563

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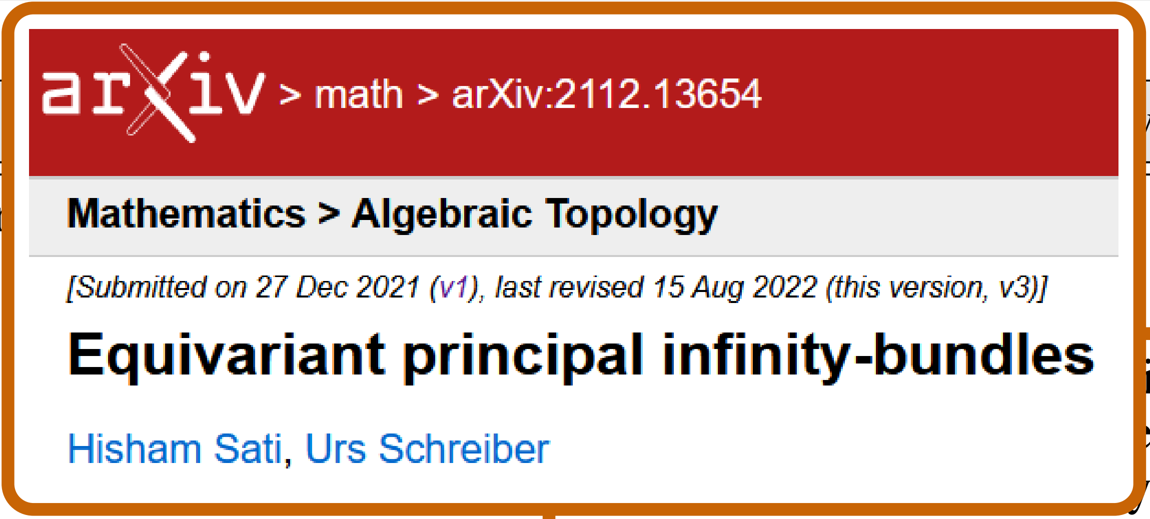
Topological Quantum Progr



arXiv > math > arXiv:2112.13654

Mathematics > Algebraic Topology

[Submitted on 27 Dec 2021 (v1), last revised 15 Aug 2022 (this version, v3)]

Equivariant principal infinity-bundles

Hisham Sati, Urs Schreiber

ic braiding of
red semi-metals
of config. space:

arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

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Hisham Sati, Urs Schreiber

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path $\longrightarrow$ **configuration space of distinct points** $\longrightarrow$ **unitary operators**

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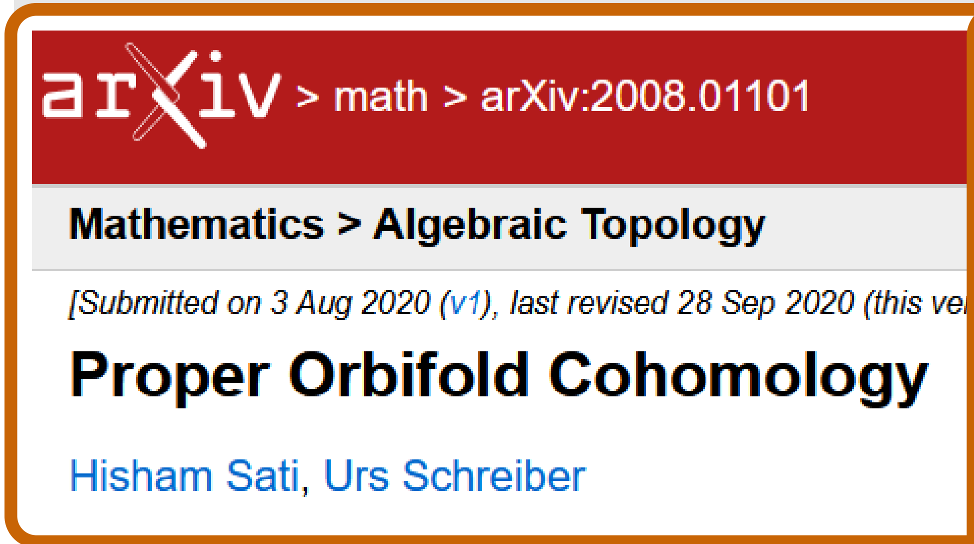


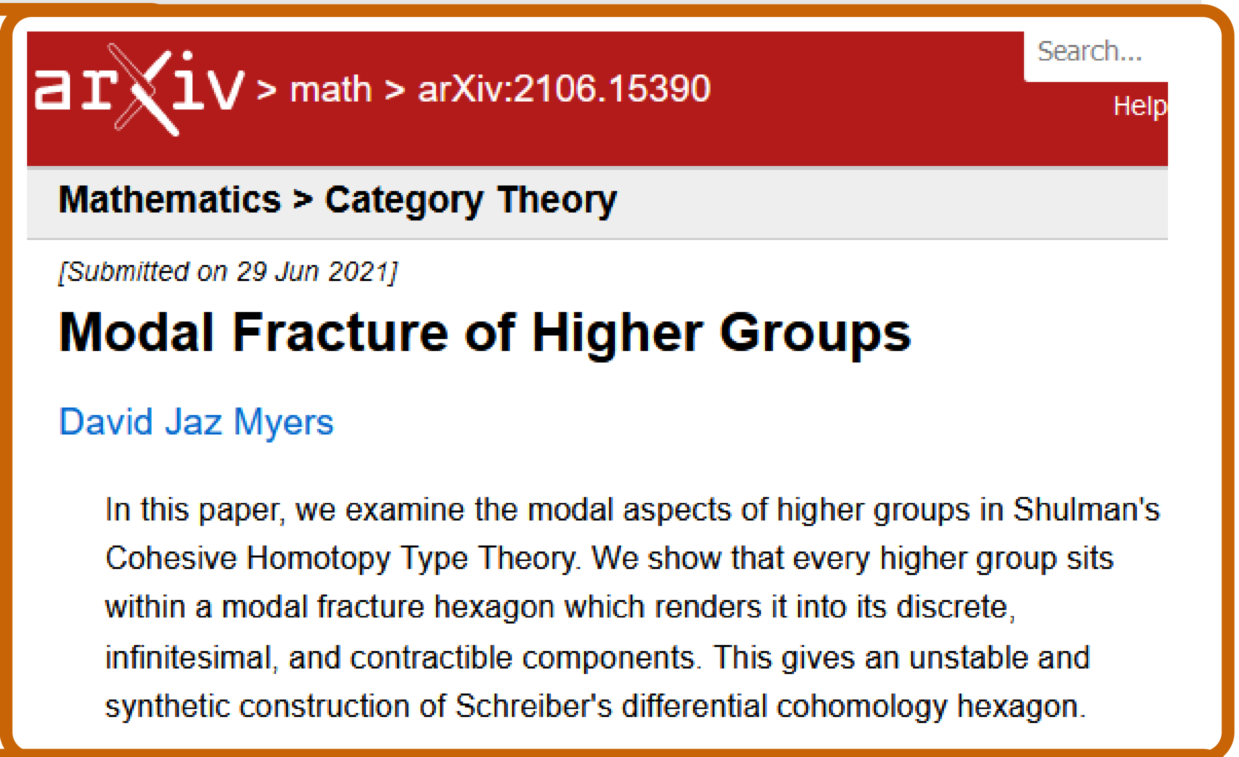
arXiv > math > arXiv:2008.01101

Mathematics > Algebraic Topology

[Submitted on 3 Aug 2020 (v1), last revised 28 Sep 2020 (this ve

Proper Orbifold Cohomology

Hisham Sati, Urs Schreiber



arXiv > math > arXiv:2106.15390

Search...

Help

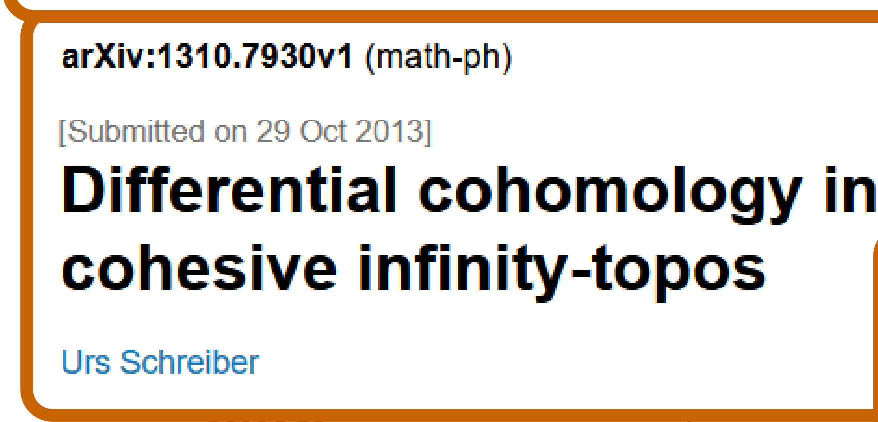
Mathematics > Category Theory

[Submitted on 29 Jun 2021]

Modal Fracture of Higher Groups

David Jaz Myers

In this paper, we examine the modal aspects of higher groups in Shulman's Cohesive Homotopy Type Theory. We show that every higher group sits within a modal fracture hexagon which renders it into its discrete, infinitesimal, and contractible components. This gives an unstable and synthetic construction of Schreiber's differential cohomology hexagon.

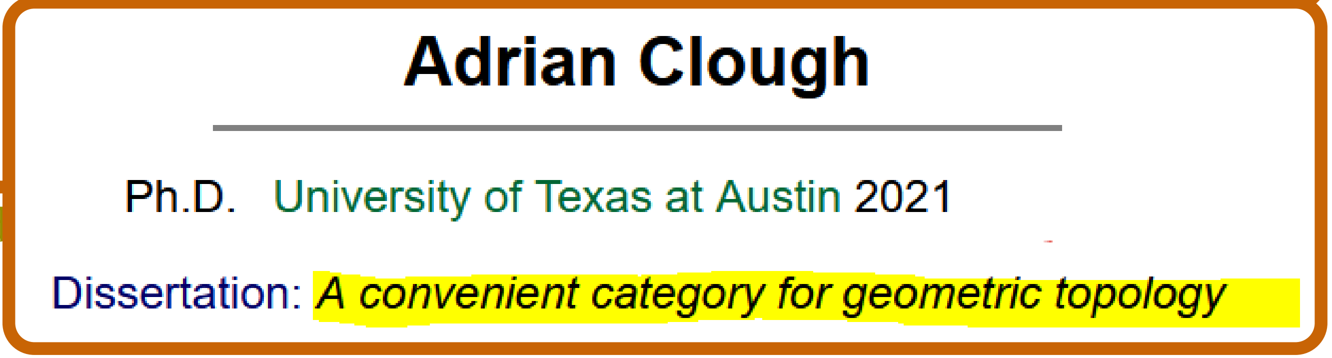


arXiv:1310.7930v1 (math-ph)

[Submitted on 29 Oct 2013]

Differential cohomology in a cohesive infinity-topos

Urs Schreiber



Adrian Clough

Ph.D. University of Texas at Austin 2021

Dissertation: A convenient category for geometric topology

path

topological quantum program

of d

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arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

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A				

To

arXiv > hep-th > arXiv:2203.11838

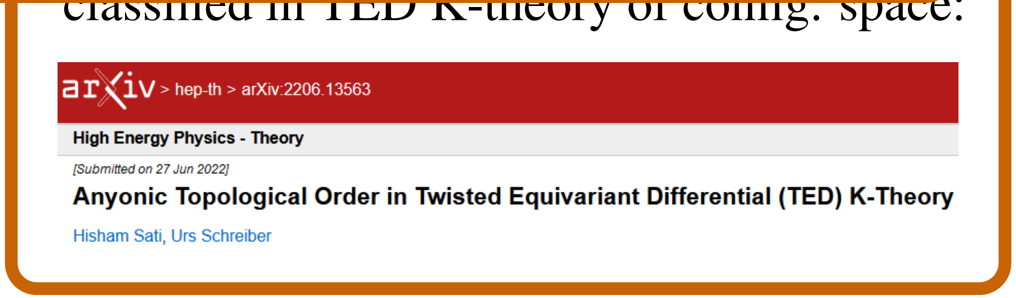
High Energy Physics - Theory

[Submitted on 22 Mar 2022]

Anyonic Defect Branes and Conformal Blocks in Twisted Equivariant Differential (TED) K-theory

Hisham Sati, Urs Schreiber

classified in TED K-theory of config. space:



arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

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Hisham Sati, Urs Schreiber

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path → **configuration space of distinct points** → **unitary operators**

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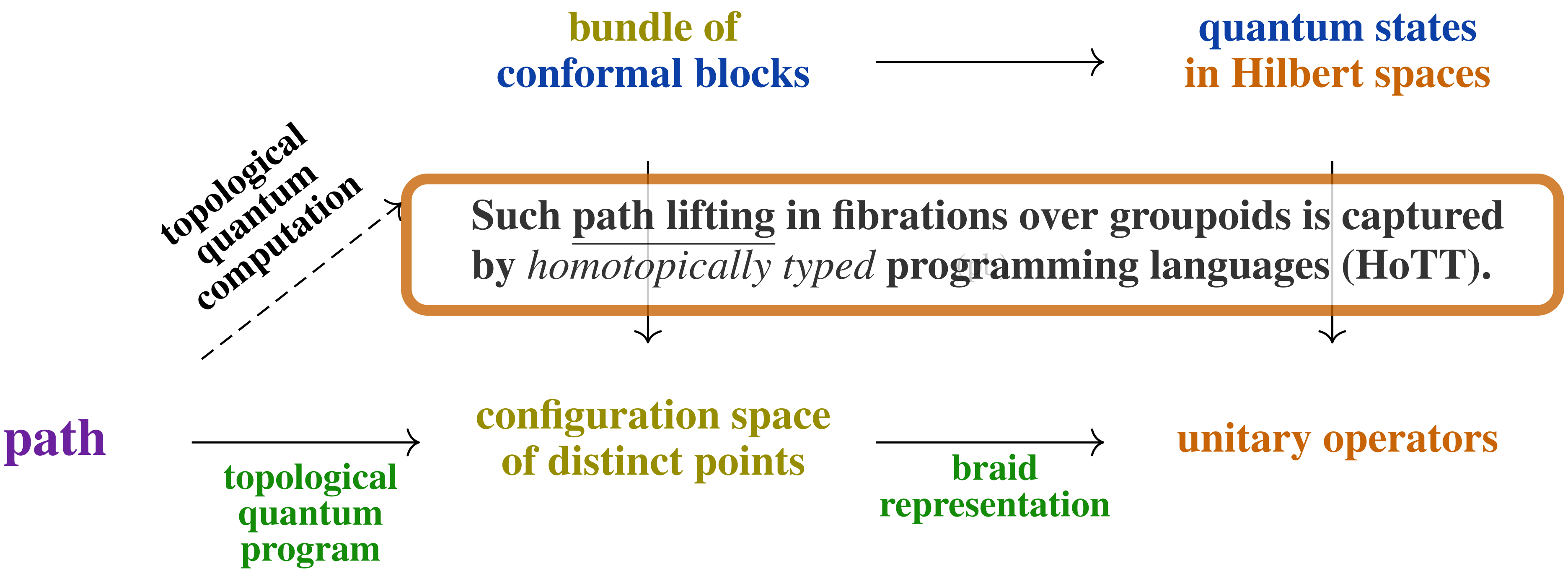
Topological Quantum Programming



bundle of conformal blocks → quantum states in Hilbert spaces

topological quantum computation

Such path lifting in fibrations over groupoids is captured by *homotopically typed* programming languages (HoTT).

path → configuration space of distinct points → unitary operators

topological quantum program

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Topological Quantum Programming		**Linear Homotopy Type Theory**

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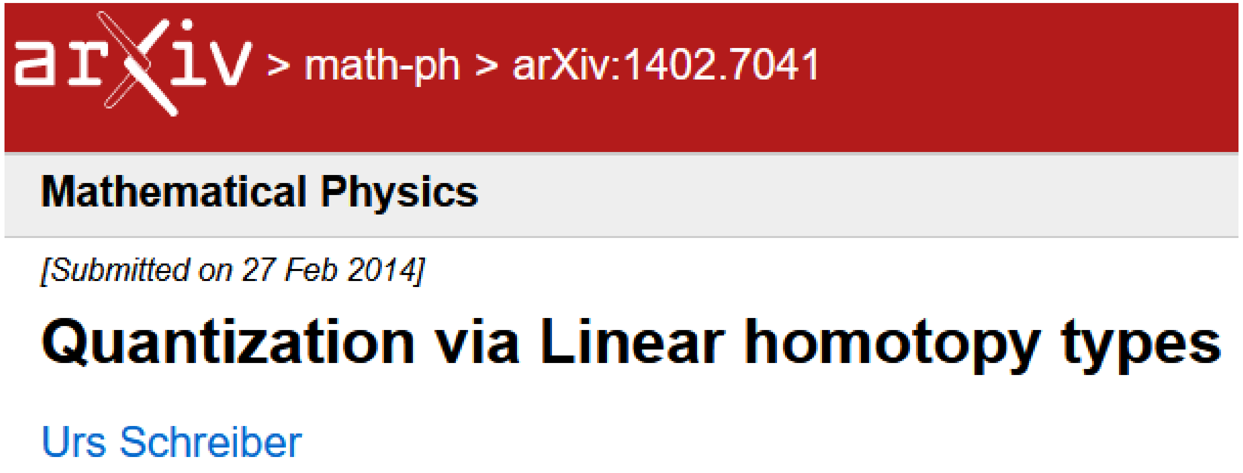


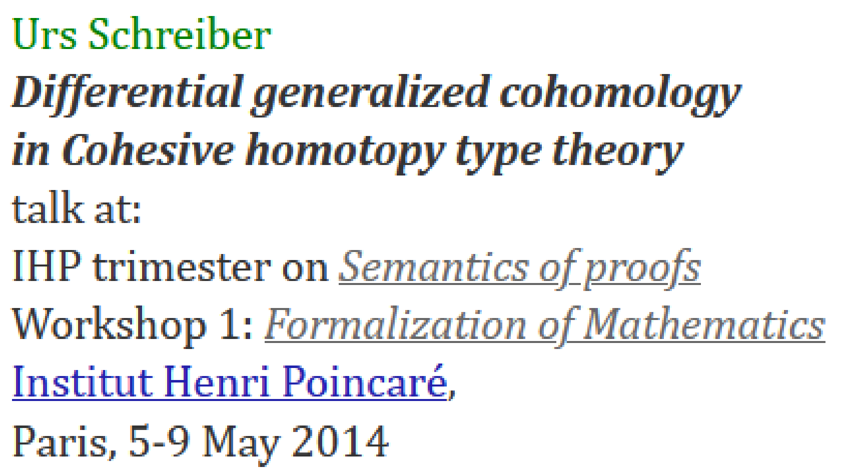
arXiv > math-ph > arXiv:1402.7041

Mathematical Physics

[Submitted on 27 Feb 2014]

Quantization via Linear homotopy types

Urs Schreiber



Urs Schreiber

Differential generalized cohomology in Cohesive homotopy type theory

talk at:

IHP trimester on *Semantics of proofs*

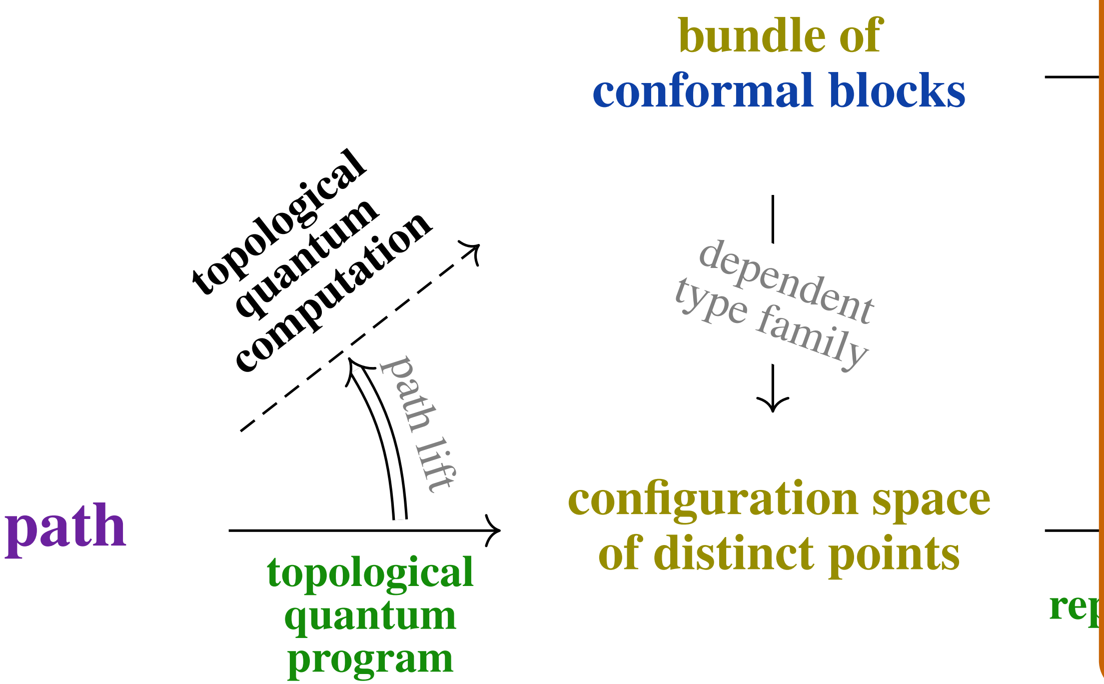
Workshop 1: *Formalization of Mathematics*

Institut Henri Poincaré,

Paris, 5-9 May 2014

Topological Quantum Programming

Linear Homotopy Type Theory



bundle of conformal blocks

dependent type family

configuration space of distinct points

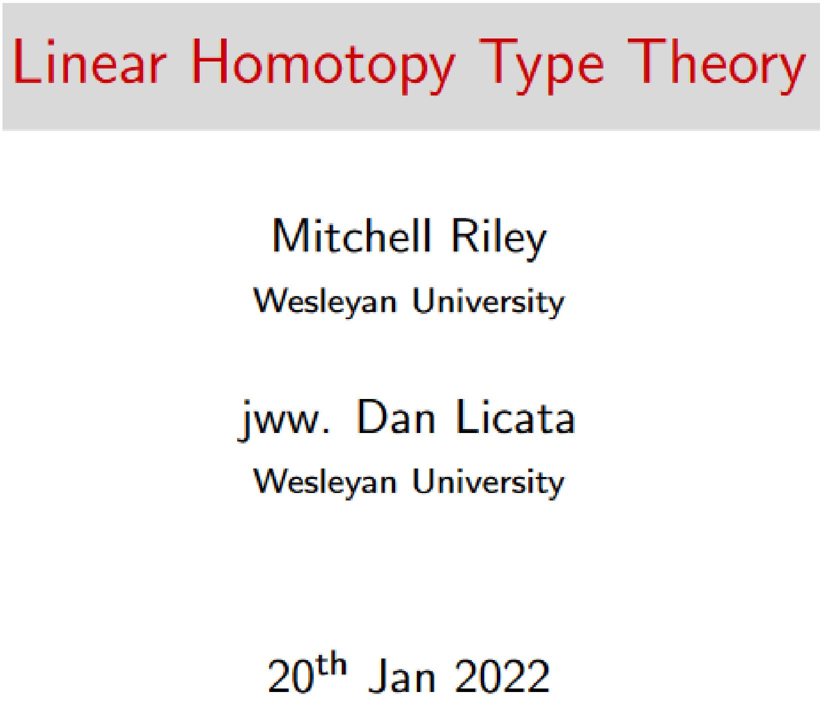
topological quantum computation

path lift

path

topological quantum program

rep



Linear Homotopy Type Theory

Mitchell Riley

Wesleyan University

jww. Dan Licata

Wesleyan University

20th Jan 2022

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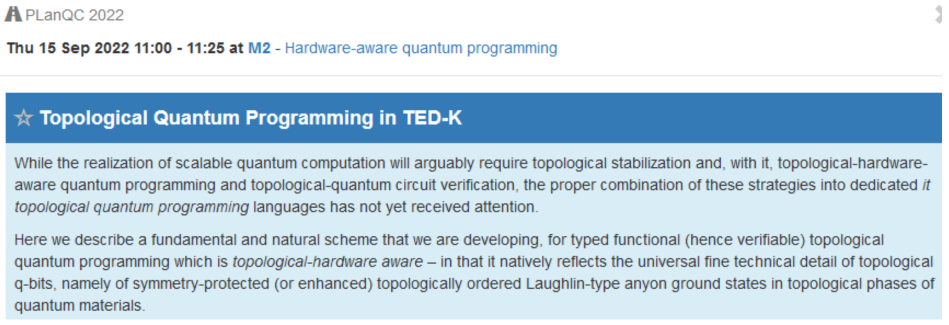
Topological Quantum Programming		**Linear Homotopy Type Theory**

Under this translation,
the fibration of conformal blocks,
has a slick construction in HoTT.



PLanQC 2022

Thu 15 Sep 2022 11:00 - 11:25 at M2 - Hardware-aware quantum programming

☆ Topological Quantum Programming in TED-K

While the realization of scalable quantum computation will arguably require topological stabilization and, with it, topological-hardware-aware quantum programming and topological-quantum circuit verification, the proper combination of these strategies into dedicated *it topological quantum programming* languages has not yet received attention.

Here we describe a fundamental and natural scheme that we are developing, for typed functional (hence verifiable) topological quantum programming which is *topological-hardware aware* – in that it natively reflects the universal fine technical detail of topological q-bits, namely of symmetry-protected (or enhanced) topologically ordered Laughlin-type anyon ground states in topological phases of quantum materials.

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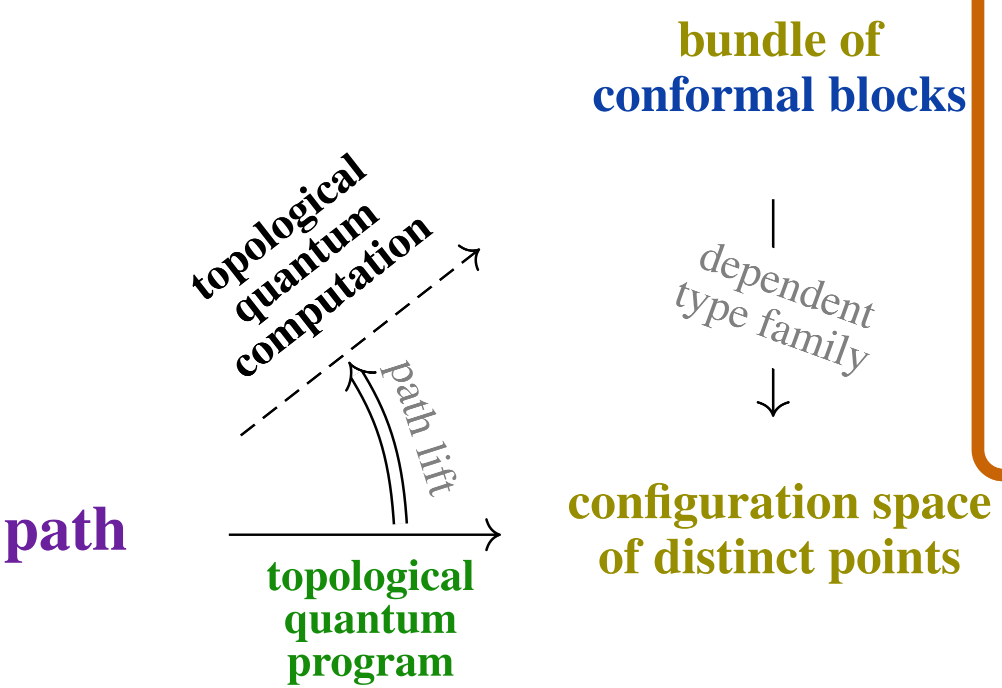
\arrow[Rightarrow, from=A, to=B, "\text{path lift}"']
\end{tikzcd}$$

⇒ **plan of attack**



braid representation

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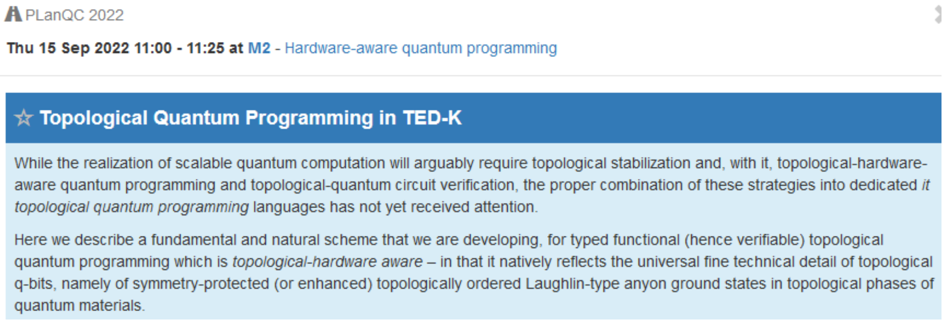
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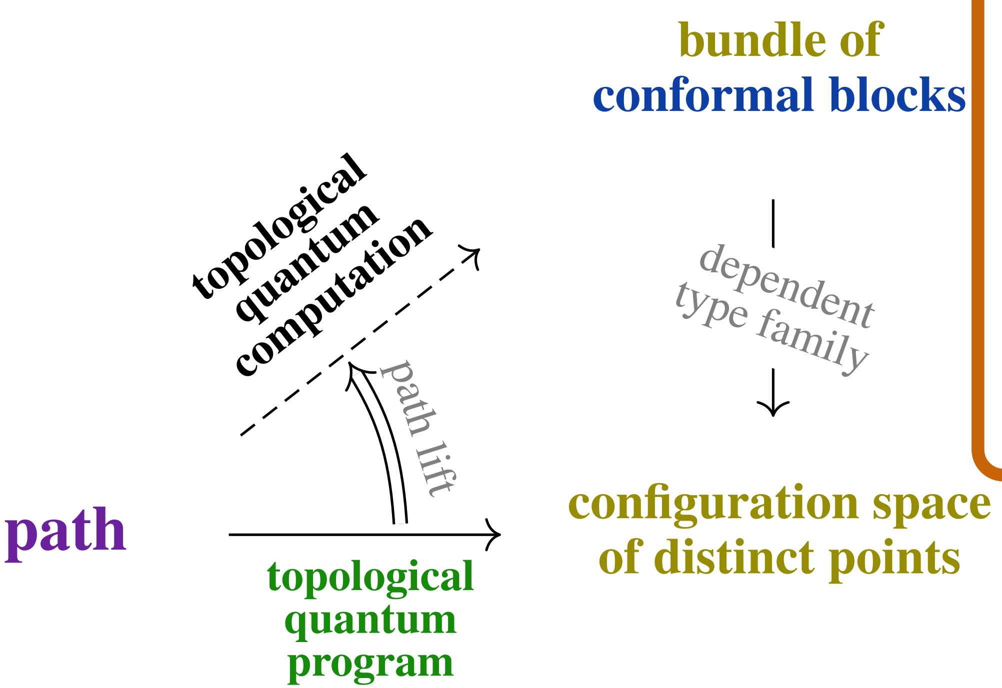
Programming platform:

Cohesive Homotopy Type Theory with dependent linear types

plan of attack



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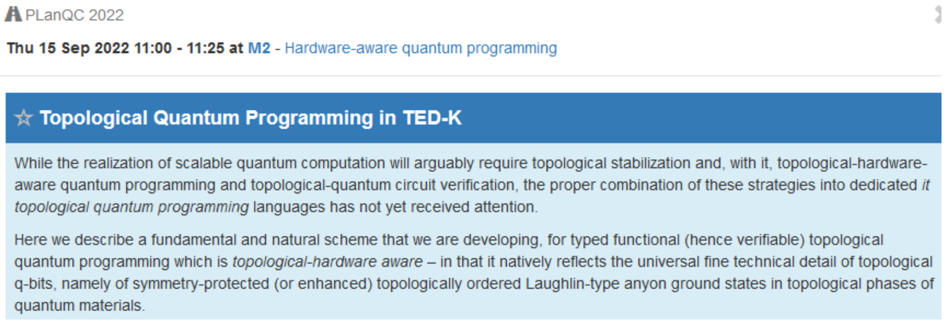

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☆ Topological Quantum Programming in TED-K

While the realization of scalable quantum computation will arguably require topological stabilization and, with it, topological-hardware-aware quantum programming and topological-quantum circuit verification, the proper combination of these strategies into dedicated *it topological quantum programming* languages has not yet received attention.

Here we describe a fundamental and natural scheme that we are developing, for typed functional (hence verifiable) topological quantum programming which is *topological-hardware aware* – in that it natively reflects the universal fine technical detail of topological q-bits, namely of symmetry-protected (or enhanced) topologically ordered Laughlin-type anyon ground states in topological phases of quantum materials.

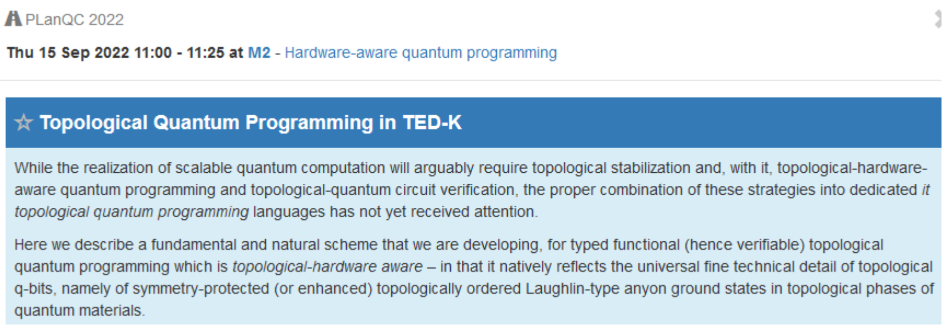
Programming platform: **Library/Module:**

$$\begin{tikzcd}
\boxed{\begin{array}{c}\text{Cohesive Homotopy}\\ \text{Type Theory with}\\ \text{dependent linear types}\end{array}} \arrow[r, "\text{implements}", "\underline{(1)}"'] & \boxed{\begin{array}{c}\text{TED-K-cohomology of}\\ \text{defect configurations in}\\ \text{crystallographic orbifolds}\end{array}}
\end{tikzcd}$$

plan of attack

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[d, "\text{dependent type family}"] & \\
\text{path} \arrow[ur, dashed, "\text{topological quantum computation}"] \arrow[r, "\text{topological quantum program}"'] \arrow[ur, phantom, "\Uparrow \text{ path lift}"' near end] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
\end{tikzcd}$$

The fibration of conformal blocks,
has a slick construction in HoTT.



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Programming platform: **Cohesive Homotopy** **Type Theory with dependent linear types**

implements → (1)

Library/Module: **TED-K-cohomology** **of defect configurations in crystallographic orbifolds**

emulates → (2)

Hardware platform: **Anyonic band nodes** **in topol. semimetals**

plan of attack

$$\begin{tikzcd} & \text{bundle of conformal blocks} \arrow[d, "\text{dependent type family}"] \\ \text{path} \arrow[r, "\text{topological quantum program}"', ""{name=P}] \arrow[ur, dashed, "\text{topological quantum computation}", ""'{name=L}] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators} \arrow[from=P, to=L, Rightarrow, "\text{path lift}"'] \end{tikzcd}$$

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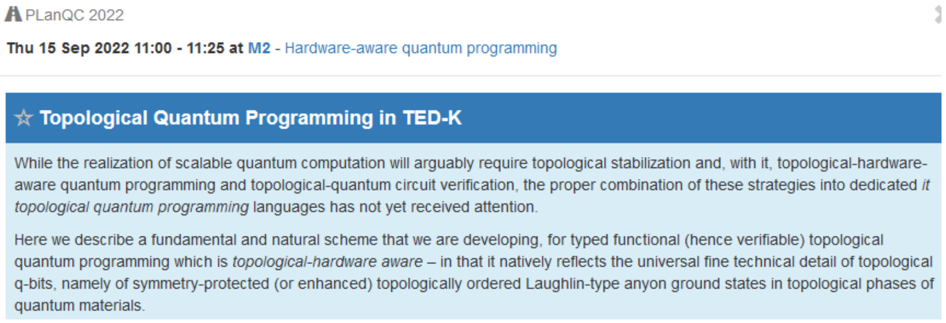

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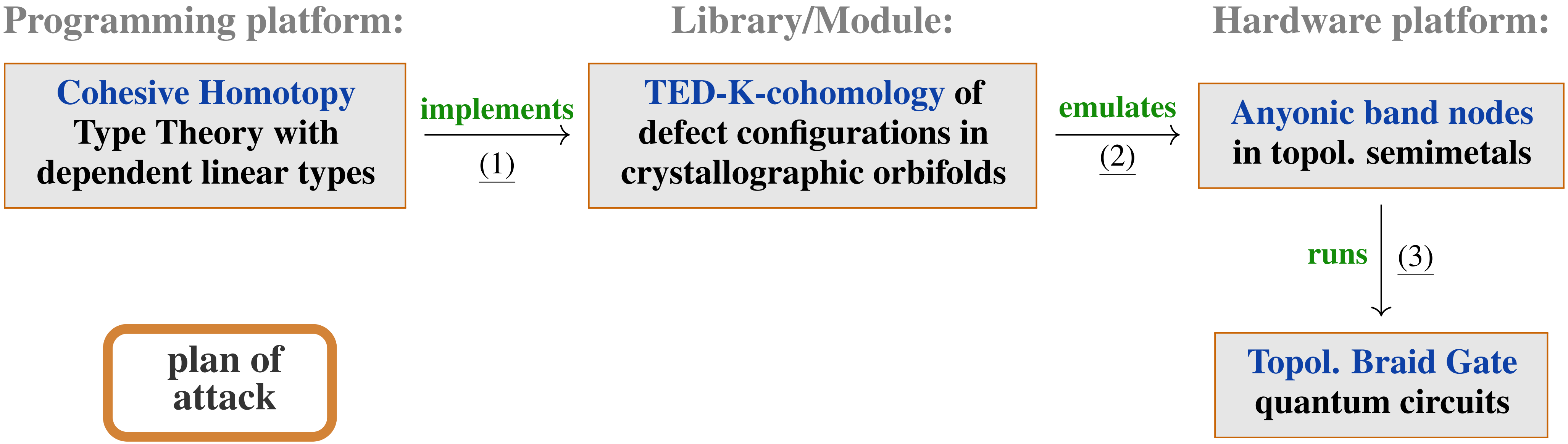


Programming platform:

Library/Module:

Hardware platform:

Cohesive Homotopy Type Theory with dependent linear types

implements (1)

TED-K-cohomology of defect configurations in crystallographic orbifolds

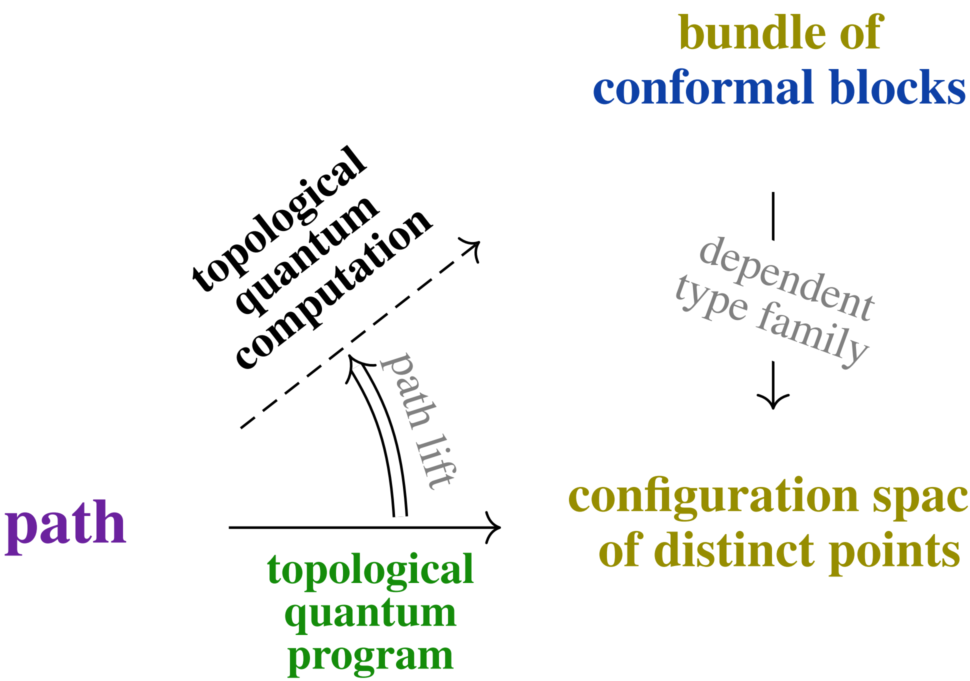
emulates (2)

Anyonic band nodes in topol. semimetals

runs (3)

Topol. Braid Gate quantum circuits

plan of attack



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dependent type family

topological quantum computation

path lift

path

topological quantum program

configuration space of distinct points

braid representation

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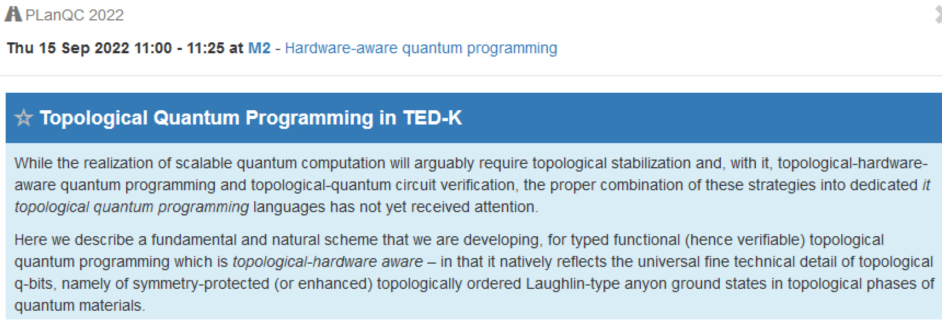

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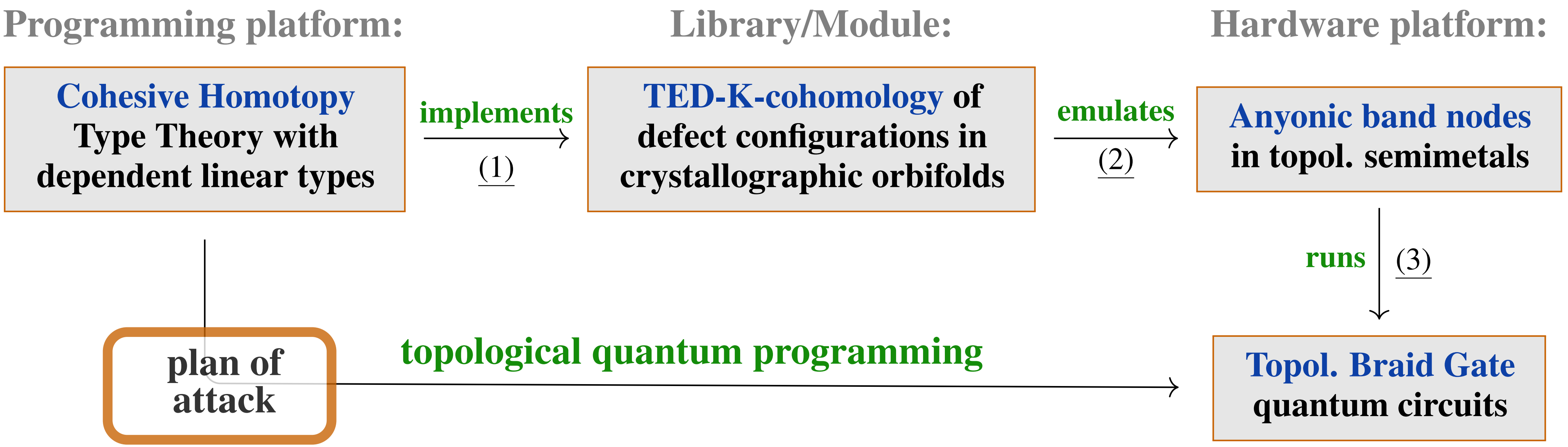

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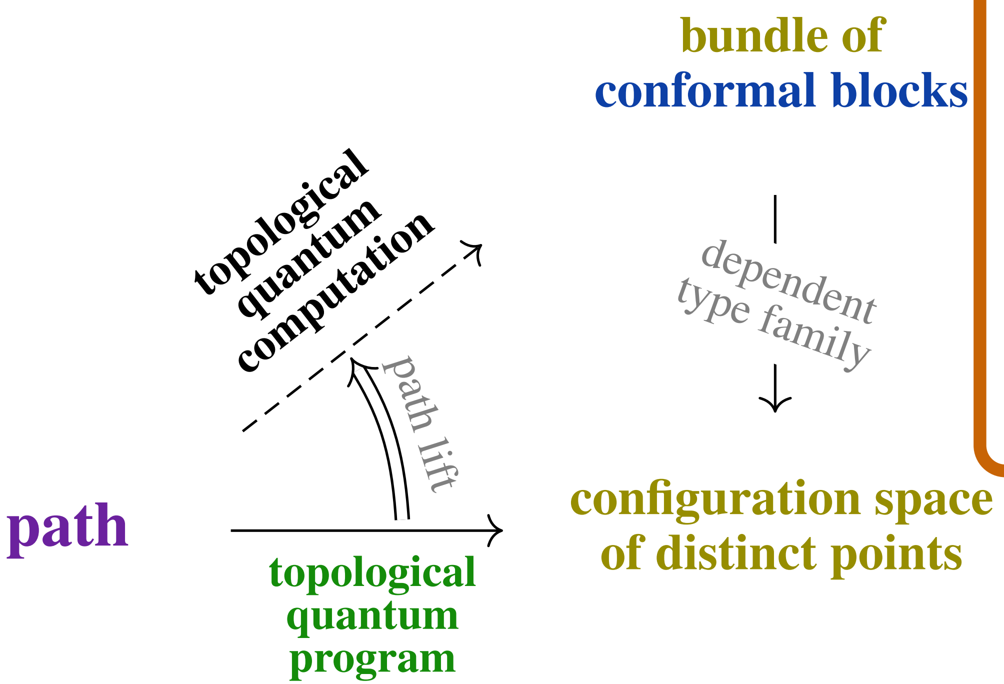

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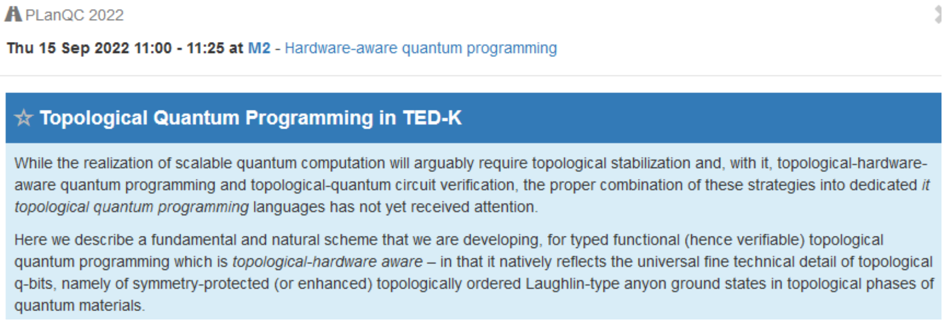

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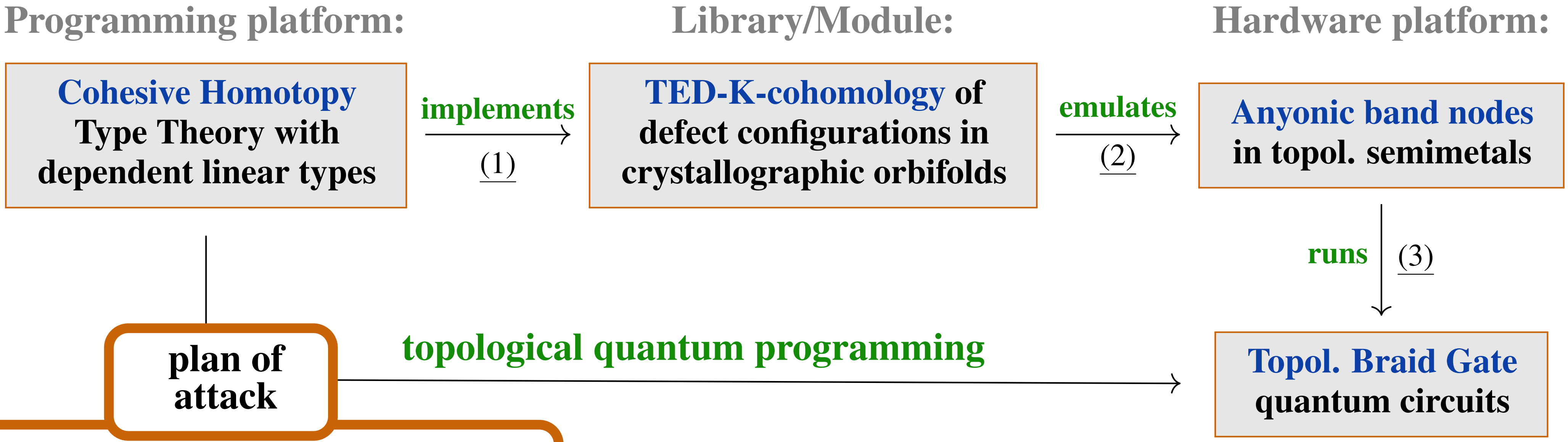

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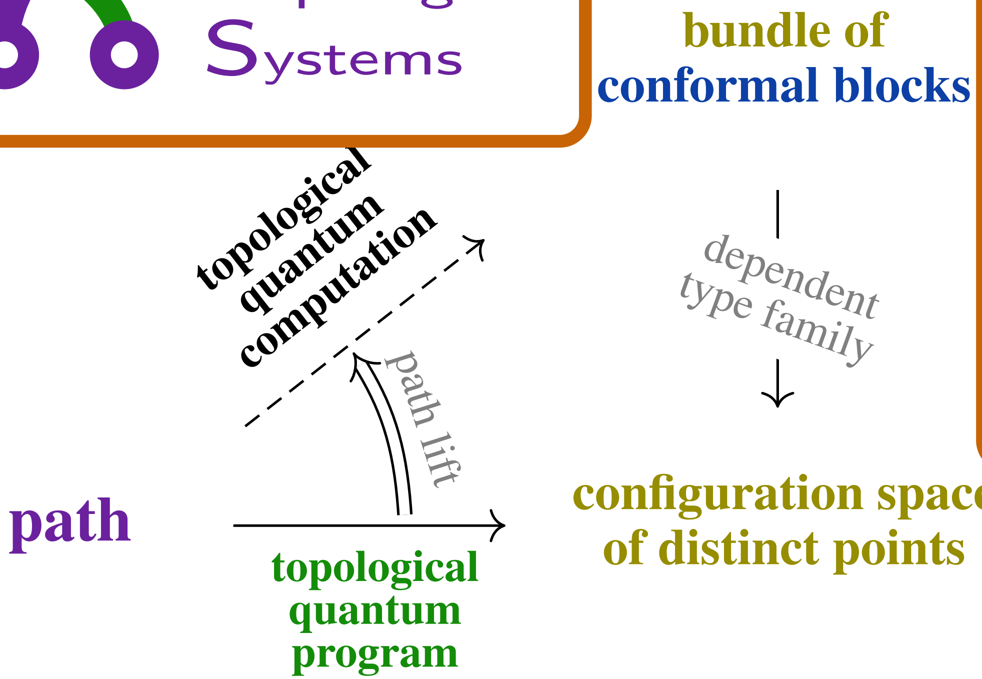
Center for Quantum & Topological Systems



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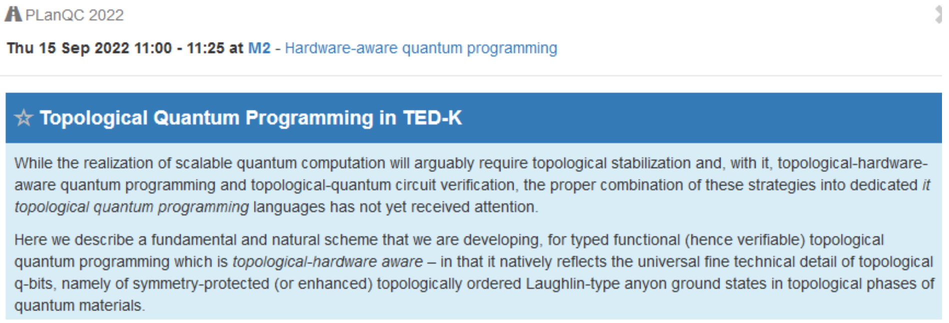

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