F-theory and 2d (0,2) Theories

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1601.02015 in collaboration with Timo Weigand

F-theory at (2,0)

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Setup

Setup: F-theory on elliptically fibered Calabi-Yau five-fold Y_5
$\Rightarrow$ 2d $N = (0,2)$ gauge theory, coupled to gravity.

Questions:

1. Why is this interesting?
2. What is the geometry-gauge theory dictionary?
3. Does this allow classification of 2d SCFTs?

Motivation

- ⋆ 2d $N = (0, 2)$ theories provide a rich and complex class of susy gauge theories. Despite long history, much remains to be understood.
- ⋆ Well-studied as heterotic worldsheet theories

  [Witten et al; Candelas, de la Ossa,....]

- ⋆ Recent resurgence in the context of M5-branes: In particular M5-branes on X_4 (embedded as co-associatives in G_2)

  [Gadde, Gukov, Putrov][Assel, SSN, Wong; to appear]

  Computation of elliptic genera, localization, etc have become available

  [Benini, Bobev][Benini, Eager, Hori, Tachikawa], [Closset, Sharpe...]

Goal: Develop a setup where 2d (0,2) theories can be constructed systematically and potentially comprehensively.

F-theory on elliptic Calabi-Yau five-folds provide such a framework

2d Vacua: Brief History

2d Superstring vacua: very sparce history.

- ⋆ Type II on CY4 and T^8/Γ [Gates, Gukov, Witten][Font]
- ⋆ 1998 and again since 2015: D1s probing CY4 singularities [Hanany, Uranga] [Franco, Kim, Seong, Yokoyama]
- ⋆ 1/2016: F-theory on elliptic CY5 `1601.02015` [SSN, Weigand]
- ⋆ 2/2016: heterotic/Type I/F-theory compactified to 2d `1602.04221` [Apruzzi, Heckman, Hassler, Melnikov]

Plan

I. 2d (0,2) Supersymmetry

II. F-theory on CY5: Gauge Theory

III. F-theory on CY5: Geometry

IV. F-theory on CY5: Effective Theory and Anomalies

V. F-theory on CY5: Spacetime-Worldsheet Corespondence

I. 2d (0,2) Supersymmetry

2d $(0,2)$ Supersymmetry

$\mathbb{R}^{1,1}$ with coordinates $y^{\pm} = y^0 \pm y^1$

$SO(1,1)_L \equiv U(1)_L$: ± 1 charge $\leftrightarrow$ $\pm$ 2d chirality

Negative chirality susy parameters: ϵ_- and $\bar{\epsilon}_-$.

Spectrum :

Vector Multiplet $(v_0 - v_1, \eta_-, \bar{\eta}_-; \mathfrak{D})$ and matter multiplets:

Multiplet	Content	SUSY
Chiral Φ	(φ, χ_+)	$\delta\varphi = -\sqrt{2}\,\epsilon_-\chi_+$
		$\delta\chi_+ = i\sqrt{2}(D_0 + D_1)\varphi\,\bar{\epsilon}_-$
Fermi P	ρ_-	$\delta\rho_- = \sqrt{2}\epsilon_-\,G - i\bar{\epsilon}_-\,E$

with $\bar{\mathcal{D}}_+\Phi = 0$ but $\bar{\mathcal{D}}_+ P = \sqrt{2}E$:

$$P = \rho_- - \sqrt{2}\theta^+ G - i\theta^+\bar{\theta}^+(D_0 + D_1)\rho_- - \sqrt{2}\bar{\theta}^+ E$$

$$\bar{P} = \bar{\rho}_- - \sqrt{2}\bar{\theta}^+ \bar{G} + i\theta^+\bar{\theta}^+(D_0 + D_1)\bar{\rho}_- - \sqrt{2}\theta^+ \bar{E}\,.$$

Interactions

Interacting theory with $i = 1, \cdots, \#\Phi$ Chirals and $a = 1, \cdots, \#P$ Fermis:

J-term (superpotentials) for $J^a = J^a(\Phi_i)$:

$$L^J = -\frac{1}{\sqrt{2}} \int d^2y \, d\theta^+ \; P_a J^a(\Phi_i)|_{\bar{\theta}^+=0} - \text{c.c.}$$
$$= -\int d^2y \left(G_a J^a + \rho_{-,a} \chi_{+,i} \frac{\partial J^a}{\partial \varphi_i} \right) - \text{c.c.}$$

E-term with $E_a = E_a(\Phi_i)$:

$$L^E = -\int d^2y \left(\bar{\rho}_{-,a} \chi_{+,i} \frac{\partial E_a}{\partial \varphi_i} + \rho_{-,a} \bar{\chi}_{+,i} \frac{\partial \bar{E}_a}{\partial \bar{\varphi}_i} \right) \subset -\frac{1}{2} \int d^2y d^2\theta \, P \bar{P} \, .$$

Supersymmetry:

$$\text{Tr} J^a E_a = 0 \, .$$

II. F-theory on CY5: Gauge Theory

F-theory on CY5

Elliptic CY5 with Kähler 4-fold base B_4 and (wtmlog) a section:

$$y^2 = x^3 + fx + g$$

- # Singular loci: $\{\Delta = 4f^3 + 27g^2 = 0\} \supset M_G$.

  7-branes wrap $M_G \longleftrightarrow$ gauge algebra $\mathfrak{g} \longleftrightarrow$ Singular fiber type

- # In gauge-theory limit: effective theory on 7-branes, i.e.
  8d SYM on $M_G \times \mathbb{R}^{1,1}$.

  1601.02015 [SSN, Weigand]

  1602.04221[Apruzzi, Heckman, Hassler, Melnikov]

- # M_G = Kähler 3-fold: $SO(6)_L \to U(3)_L \equiv U(1)_L \times SU(3)_L$
  Scalar supercharges on M_G: topological twist along M_G with $U(1)_R$:

$$J_{twist} = \frac{1}{2}\left(J_L + 3J_R\right) .$$

Two susy parameters, with $q_{twist} = 0$ and are $SO(1,1)_L$ left-chiral spinors:

$$\bar{\epsilon}_- = \mathbf{1}_{-1}\,, \qquad \epsilon_- = \mathbf{1}_{-1}$$

Twisted 8d SYM

Remaining symmetries after the partial twist:

$$SO(1,7)_L \times U(1)_R \;\rightarrow\; SU(3)_L \times SO(1,1)_L \times U(1)_{\text{twist}}$$

8d SYM spectrum: gauge fields $\mathbf{8}^{\mathbf{v}}_0$, fermions $\mathbf{8}^{\mathbf{c}}_{-1}$ and $\mathbf{8}^{\mathbf{s}}_{+1}$ and scalars $\mathbf{1}_{\pm 2}$.

After topological twisting: fields along M_G become forms:

$$U(1)_{\text{twist}} \text{ charge } q \geq 0 \; (q \leq 0)$$
$$\Updownarrow$$
$$\text{field is section of } \Omega^{(0,q)}(M_G) \; (\Omega^{(q,0)}(M_G)).$$

[$U(1)_L$ twisting corresponds to twisting with $K_{M_G} = \Omega^{(0,3)}$].

‘Bulk Spectrum’

$\pm$ = chirality in 2d and $L_{\mathbf{R}}$= line bundle breaking Ad$(G) \supset \mathbf{R} \oplus \overline{\mathbf{R}}$

Cohomology	Bosons	Fermions in **R**, $\bar{\mathbf{R}}$	Multiplet
$H^0_{\bar\partial}(M_G, L_{\mathbf{R}}) \oplus H^0_{\bar\partial}(M_G, L_{\mathbf{R}})^*$	$v_\mu, \mu = 0, 1$	$dblue\bar\eta_-, \eta_-$	Vector
$H^1_{\bar\partial}(M_G, L_{\mathbf{R}}) \oplus H^1_{\bar\partial}(M_G, L_{\mathbf{R}})^*$	$a_{\bar m}, \bar a_m$	$\psi_{+\bar m}, \bar\psi_{+m}$	Chiral and $\overline{\text{Chiral}}$
$H^2_{\bar\partial}(M_G, L_{\mathbf{R}}) \oplus H^2_{\bar\partial}(M_G, L_{\mathbf{R}})^*$	$-$	$\bar\rho_{-\bar m\bar n}, \rho_{-mn}$	$\overline{\text{Fermi}}$ and Fermi
$H^3_{\bar\partial}(M_G, L_{\mathbf{R}}) \oplus H^3_{\bar\partial}(M_G, L_{\mathbf{R}})^*$	$\bar\varphi_{\bar k\bar m\bar n}, \varphi_{kmn}$	$\bar\chi_{+\bar k\bar m\bar n}, \chi_{+kmn}$	$\overline{\text{Chiral}}$ and Chiral

Supersymmetry and Hitchin Equations

Dim redux and twist of 10d SYM supersymmetry resultsin $(0,2)$ in 2d:

- $\star$ Gaugino variation:

$$\mathfrak{D} = \frac{i}{2}\left(J \wedge J \wedge F_{M_G} + [\varphi, \overline{\varphi}]\right) .$$

- $\star$ Variation of Fermi

$$\delta\rho_- = \sqrt{2}\epsilon_-\, G - i\bar{\epsilon}_-\, E \quad \Rightarrow \quad \begin{cases} G_{mn} = \overline{F}_{mn} \\ E_{mn} = (\bar{\partial}_a^\dagger \varphi)_{mn} \end{cases}$$

- $\star$ BPS equations: Higgs bundle (a, φ) over M_G

$$D_+\varphi = D_+\bar{\varphi} = 0\,, \qquad F^{(0,2)} = F^{(2,0)} = 0\,, \qquad J \wedge J \wedge F + [\varphi, \bar{\varphi}] = 0$$

$\rightarrow$ cue Spectral covers, T-branes/Gluing data

‘Bulk’ Interactions

Interactions arise from overlaps of internal wave-functions:

Superpotential (J-term):

$$S^{(J)}_{\text{bulk}} = \mathbf{g}_{\alpha\beta\gamma} \int d^2y \; \rho^{\alpha}_{-} \, a^{\beta} \, \psi^{\gamma}_{+} + \text{c.c.}$$

with internal overlap

$$\mathbf{g}_{\alpha\beta\gamma} = \int_{M_G} \tilde{\rho}_{kmn\bar{n},\alpha} \wedge \hat{a}_{\bar{k},\beta} \wedge \hat{\psi}_{\bar{m},\gamma} \, , \qquad \tilde{\rho}_{kmn\bar{n},\alpha} = (\Omega \cdot \hat{\rho}_{\alpha})_{kmn\bar{n}}$$

E-term:

$$S^{(E)}_{\text{bulk}} = \mathbf{f}_{\alpha\mu\epsilon} \int d^2y \; \bar{\rho}^{\alpha}_{-} \left(\varphi^{\mu} \, \psi^{\epsilon}_{+} + \chi^{\mu}_{+} \, a^{\epsilon} \right) + \text{c.c.}$$

with internal overlap

$$\mathbf{f}_{\alpha\mu\epsilon} = \int_{M_G} \hat{\bar{\rho}}_{\bar{k}\bar{m},\alpha} \wedge \left(\hat{\varphi}_{kmn,\mu} \wedge \hat{\psi}_{\bar{n},\epsilon} \right)$$

Matter from Defects

Matter from interacting 7-branes or codim 2 singularity enhancements in the elliptic CY5 along $S_{\mathbf{R}}$ = matter surface.
Twisted SYM: $6d$ defect theory, with bulk compatible twist to 2d (0,2).

Spectrum:

Chirals: $\mathcal{S} = (\bar{S}, \bar{\sigma}_+)$ and $\mathcal{T} = (T, \tau_+)$
Fermi: $\bar{\mu}_-$

$$\begin{aligned}
\psi_+ \in H^1_{\bar\partial}(M_G, L_{\mathbf{R}}) \quad &\rightarrow \quad \tau_+ \in H^0_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes K^{1/2}_{S_{\mathbf{R}}}) \\
\bar\rho_- \in H^2_{\bar\partial}(M_G, L_{\mathbf{R}}) \quad &\rightarrow \quad \bar\mu_- \in H^1_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes K^{1/2}_{S_{\mathbf{R}}}) \\
\bar\chi_+ \in H^3_{\bar\partial}(M_G, L_{\mathbf{R}}) \quad &\rightarrow \quad \bar\sigma_+ \in H^2_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes K^{1/2}_{S_{\mathbf{R}}})\,,
\end{aligned}$$

$\rightarrow$ cue study of wave-function profiles.

Interactions

See also (for description in terms of W^{top}) [Apruzzi, Heckman, Hassler, Melnikov]

- ⋆ 'Bulk'-Matter-surface interactions: (codim 2)

$$J_{(\mu_-^\delta)} = -\mathbf{c}_{\delta\beta\epsilon}\mathcal{T}^\beta A^\epsilon$$

$$E^{(\rho_-^\alpha)} = -\mathbf{f}_{\alpha\mu\epsilon}\,\Phi^\mu A^\epsilon - \mathbf{b}_{\alpha\beta\gamma}\mathcal{T}^\beta\mathcal{S}^\gamma \quad E^{(\mu_-^\delta)} = -\mathbf{e}_{\delta\gamma\epsilon}\,\mathcal{S}^\gamma A^\epsilon$$

- ⋆ Cubic Matter-surface interactions for any $\mathcal{Z} = \mathcal{S}, \mathcal{T}$ and gauge invariant triplet of representations $\mathbf{R}_i$ (codim 3)

$$J_{\left(\mu_-^{\mathbf{R}_{b_1},\delta}\right)} = -\mathbf{h}_{\delta\epsilon\gamma}(\mathbf{R}_{b_1}\mathbf{R}_{b_2}\mathbf{R}_{b_3})\left(\mathcal{Z}_{b_2}^{\mathbf{R}_{b_2},\epsilon}\,\mathcal{Z}_{b_3}^{\mathbf{R}_{b_3},\gamma}\right)$$

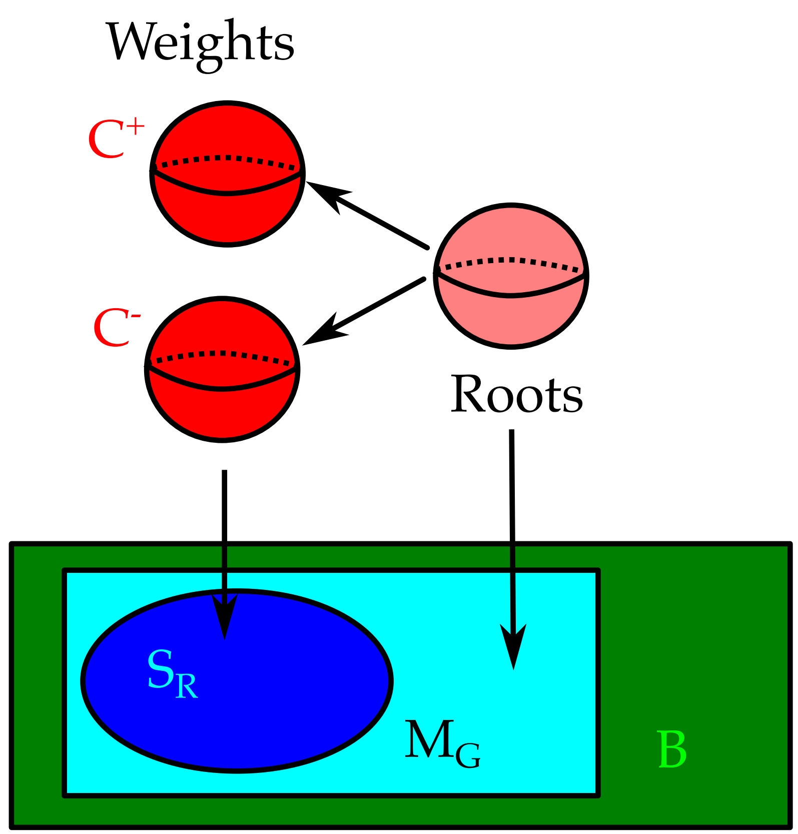
$$E^{\left(\mu_-^{\mathbf{R}_{a_1},\delta}\right)} = -\mathbf{d}_{\delta\epsilon\gamma}(\mathbf{R}_{a_1}\mathbf{R}_{a_2}\mathbf{R}_{a_3})\left(\mathcal{Z}_{a_2}^{\mathbf{R}_{a_2},\epsilon}\,\mathcal{Z}_{a_3}^{\mathbf{R}_{a_3},\gamma}\right)$$

- ⋆ Quartic Matter-surface interactions: (codim 4) $\rightarrow$ see example

III. F-theory on CY5: Geometry

F-theory and Singular Fibers

Numerous F-theory@20 Talks: Above codim 1: Singular fibers are trees of $\mathbb{P}^1$s associated to simple roots. Codim 2: these can split into weights:



How exactly this happens: see Box Graph paper

[Hayashi, Lawrie, Dave Morrison, SSN]

F-theory on elliptic CY5

Much as in higher-dimensions: Singular fibers above discriminant loci determine gauge algebra and higher codim give rise to matter (codim 2) and interactions (codim 3+).

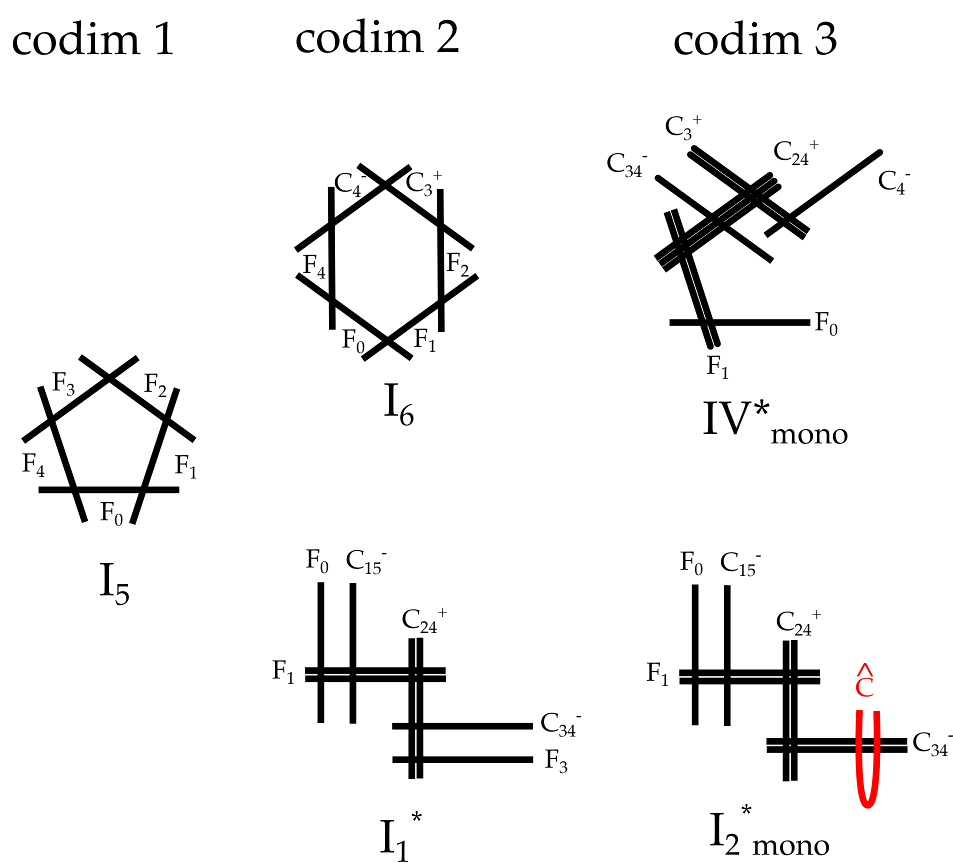
Codim	$\mathbb{P}^1$s in Fiber	Gauge Theory
1: M_G	Simple roots	Gauge algebra $\mathfrak{g}$
2: $S_{\mathbf{R}}$	Weights for Reps **R**	Matter in **R**
3: Σ	Splitting gauge invariantly	Cubic interactions
4: p	Further gauge invariant splitting	Quartic interactions

Note:

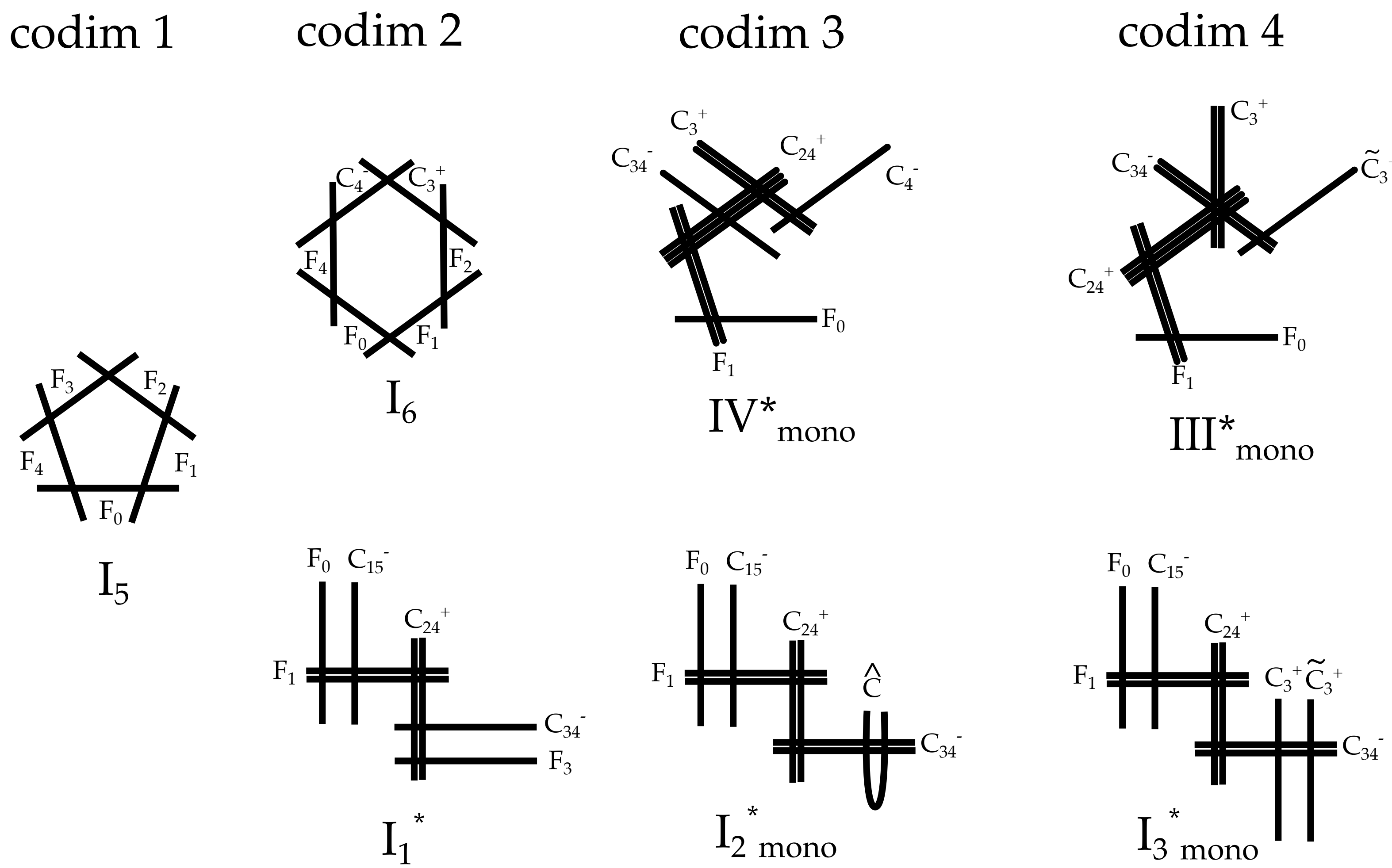
- Rational sections: $U(1)$ gauge factors $\Rightarrow$ engineer known and unknown (0,2) GLSM
- Non-abelian gauge groups occur quite naturally $\Rightarrow$ non-abelian generalizations easily accessible (e.g. GLSM into Grassmanians)

An Example

An old friend: $SU(5)$ with **10** and $\bar{\mathbf{5}}$ matter. Almost as old, but some interesting new effects in fiber: non-Kodaira I_n^* fibers from monodromy



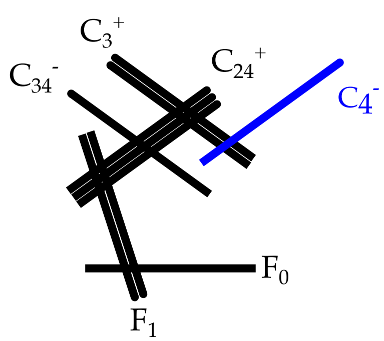
F_i= $\mathbb{P}^1$s associated to simple roots C = $\mathbb{P}^1$s associated to weights of **5** (C_i) and **10** (C_{ij}) with sign specifying whether $\pm$ the curve is effective.



codim 1
codim 2
codim 3
codim 4
F3 F2 F4 F1 F0
I5
C4- C3+ F4 F2 F0 F1
I6
C3+ C34- C24+ C4- F0 F1
IV*mono
C3+ C34- C̃3+ C24+ F0 F1
III*mono
F0 C15- C24+ F1 C34- F3
I1*
F0 C15- C24+ F1 Ĉ C34-
I2*mono
F0 C15- C24+ F1 C3+ C̃3+ C34-
I3*mono

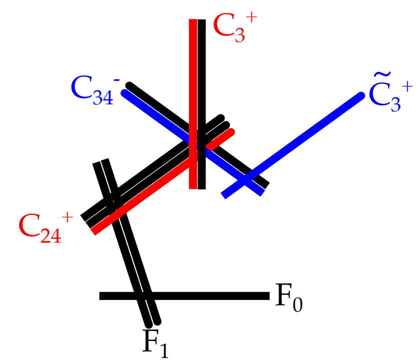
Quartic Couplings from Codim 4

codim 3



IV^*_{mono}

codim 4



III^*_{mono}

<u>Codim 3:</u>

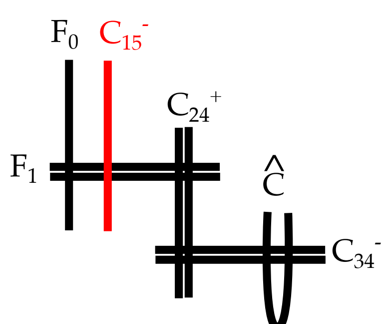
C_4^- participates in $\mathbf{10} \times \mathbf{10} \times \mathbf{5}$.

<u>Codim 4:</u>

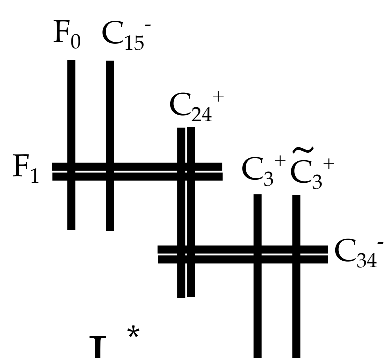
C_4^- splits furter into $C_{34}^- + \tilde{C}_3^+$

$\Rightarrow$ quartic coupling over codim 4 point

$$\mathbf{10} \times \mathbf{10} \times \mathbf{10} \times \bar{\mathbf{5}}$$



$I_2{}^*{}_{mono}$



$I_3{}^*{}_{mono}$

Likewise:

$$\bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \overline{\mathbf{10}}$$

IV. F-theory on CY5:
Effective Theory and Anomalies

LEEA by M/F duality

Comparison of the 1d Super-QM, which describes the effective theory of M-theory on resolved (not necessarily elliptic) CY5 with the circle-reduction of the 2d F-theory compactification:

M-theory on Y_5	$\xrightarrow{\text{Vol}(\mathbb{E}_\tau)\to 0}$	F-theory on Y_5
[Haupt, Lukas, Stelle] $\downarrow$		$\downarrow$
1d Super-Mechanics 2 supercharges	$\xrightarrow{R_A \sim \frac{1}{R_B} \to 0}$	2d $(0,2)$ Gauge Theory

G_4-flux

Fluxes are vital to generate chirality of the spectrum. Study via M/F [lessons from CY4: [Grimm, Hayashi]. Here: Dual M-theory was analyzed in [Haupt, Lukas, Stelle].

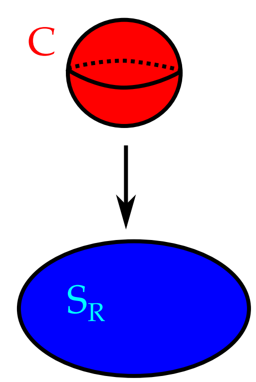
- ⋆ Flux quantization + susy: $G_4 + \frac{1}{2}c_2(Y_5) \in H^4(Y_5, \mathbb{Z}) \cap H^{(2,2)}(Y_5)$
- ⋆ Transversality constraints to ensure gauge fluxes:

$$\int_{Y_5} G_4 \wedge S_0 \wedge \omega_4 = 0 \quad \text{and} \quad \int_{Y_5} G_4 \wedge \omega_6 = 0\,, \qquad \forall\, \omega_4 \in H^4(B_4),\ \omega_6 \in H^6(B_4)$$

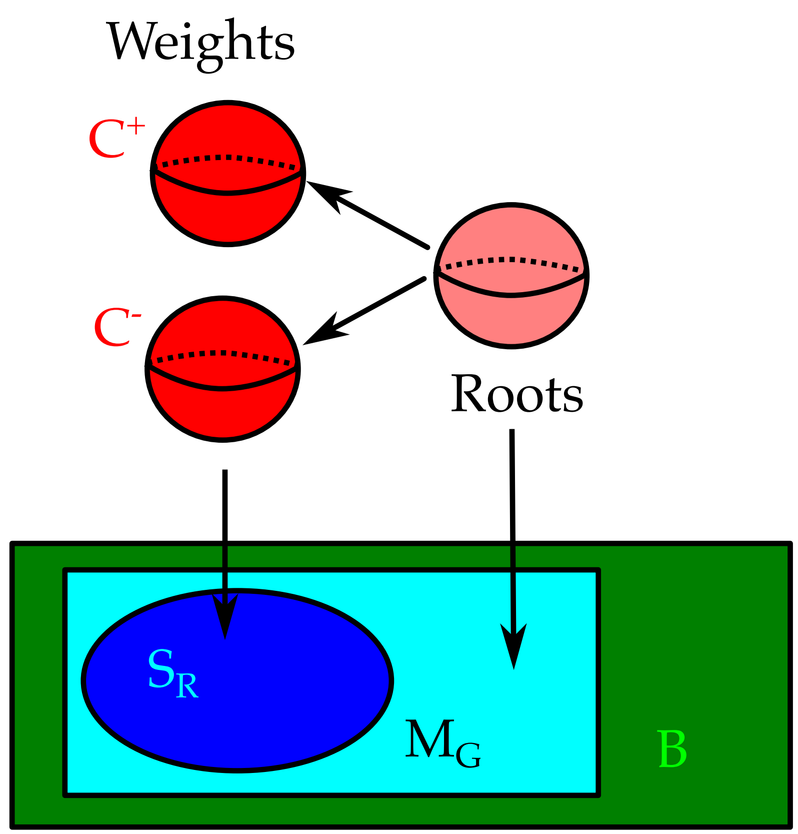
S_0= zero-section

Induced gauge flux: $\displaystyle\int_{C_\lambda} G_4 = c_1(L_{\mathbf{R}})$

Chirality contribution: $\displaystyle\chi(S_{\mathbf{R}}) = \frac{1}{2}\int_{S_{\mathbf{R}}} c_1^2(L_{\mathbf{R}})$



Direct relation of chirality to the intersections of G_4 and Cartans D_i:



E.g. in the situation $F_i \to C^+ + C^-$

$$\chi(S_{\mathbf{R}}) = \frac{1}{2}\int_{S_{\mathbf{R}}} c_1^2(L_{\mathbf{R}}) = -\frac{1}{2} G_4 \wedge G_4 \cdot_{Y_5} D_i$$

In general: using box graphs, can determine the chiralities in terms of these fiber intersections

$$D_i \cdot_{Y_5} \left(\frac{1}{24}[c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4\right)$$

$$= -\frac{1}{2}\sum_{\mathbf{R}} (n_{\mathbf{R}}^+ - n_{\mathbf{R}}^-) \left(\sum_{a=1}^{\dim(\mathbf{R})} \varepsilon(\lambda_a^{\mathbf{R}})\, \lambda_{ai}^{\mathbf{R}}\right)$$

$$= -\frac{1}{2}\sum_{\mathbf{R}} (n_{\mathbf{R}}^+ - n_{\mathbf{R}}^-) \left(\sum_{a=1}^{\dim(\mathbf{R})} D_i \cdot_{Y_5} C_{\lambda_a^{\mathbf{R}}}^{\varepsilon(\lambda_a^{\mathbf{R}})}\right) = (\star)$$

Will see: this follows from 1-loop CS terms.

Wrapped M2/D3-branes

Additional sectors of chiral matter: D3-branes wrapping curves C in the base B that intersect M_G: chiral 3-7 strings.

In M-theory: M2-brane states.

$$\text{Chiralities: } \#\text{ intersection points} = [M_G] \cdot_{B_4} [C^B_{\text{M2}}]$$

Key in anomaly cancellation.

First principle description in F-theory: from D3s wrapping curves ($\rightarrow$ in progress)

Anomalies and Tadpoles

Global consistency of the compactification: tadpole cancellation and anomaly cancellation. Again, consider M-theory effective action:

Two topological terms: $C_{\rm M2}$ = wrapped M2 curve class

$$S_{\rm M2} + S_{\rm curv} = -2\pi \int_{\mathbb{R}\times Y_5} C_3 \wedge \delta([C_{\rm M2}]) + 2\pi \int_{\mathbb{R}\times Y_5} C_3 \wedge \left(\frac{1}{24} c_4(Y_5) - \frac{1}{6} G_4 \wedge G_4 \right)$$

Via reduction of C_3 along $\omega_\alpha^{(1,1)}$ forms in Y_5: CS-terms:

$$S_{\rm top} = 2\pi \sum_\alpha \int_{\mathbb{R}} A_\alpha \wedge (k^\alpha_{\rm M2} + k^\alpha_{\rm curv}) \qquad \begin{cases} k^\alpha_{\rm M2} = -\int_{Y_5} \omega_\alpha \wedge \delta([C_{\rm M2}]) \\ k^\alpha_{\rm curv} = \int_{Y_5} \omega_\alpha \wedge \left(\frac{1}{24}[c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4 \right) \end{cases}$$

$$A_\alpha\text{-tadpole:} \qquad \delta([C_{\rm M2}]) = \frac{1}{24} c_4(Y_5) - \frac{1}{2} G_4 \wedge G_4$$

F-theory: 1-loop CS term

$k^i_{\rm curv} \equiv k^i_{1-{\rm loop}} = -\frac{1}{2} \sum_{\mathbf{R}} \left(n^+_{\mathbf{R}} - n^-_{\mathbf{R}} \right) \sum_{a=1}^{\dim(\mathbf{R})} q_{ai}\, {\rm sign}(m_0(\lambda^{\mathbf{R}}_a)) = (\star)$

Anomalies

Chiral fermions $\Rightarrow$ require gauge anomalies to cancel.
For non-ablian gauge anomaly:

- Bulk matter: $\mathcal{A}_{\text{bulk}}(\mathbf{R}) = -C(\mathbf{R})\chi(M_G, L_{\mathbf{R}})$
- Surface matter $\mathbf{R}$: $\mathcal{A}_{\text{surface}}(\mathbf{R}) = C(\mathbf{R})\chi(S_{\mathbf{R}}, L_{\text{R}})$
- 3-7 sector: $\mathcal{A}_{3-7} = -\,C(\mathbf{R})\int_{B_4}[M_G] \wedge [C^B_{\text{M2}}]$

$$\mathcal{A}_{\text{bulk}} + \mathcal{A}_{\text{surface}} + \mathcal{A}_{3-7} = 0$$

Note: tadpole cancellation implies anomaly cancellation via anomaly inflow (at least for perturbative vacua).

Abelian gauge anomalies: rich structure of GS/Stückelberg couplings.

V. F-theory on CY5: Spacetime-Worldsheet Corespondence

2d F-theory vacua = heterotic ws theories

Proposed correspondence:

A 2d $N = (0,2)$ F-theory compactification can be viewed as
(the UV completion of a) heterotic worldsheet theory

Example:

Heterotic on Quintic hypersurface in $\mathbb{P}^4$ + rk 3 vector bundle
$\leftrightarrow$ F on CY5 with rank 1 Mordell-Weil ($\rightarrow U(1)$) + G_4 flux.

In this case: phases of GLSM have interpretation in terms of topological transition in CY5: FI : $r \simeq G_4 \cdot S_1 \cdot J_B \cdot J_B$ ($S_1 =$ Shioda of the section)

$r \gg 0$: Non-linear sigma-model phase (no $U(1)$ gauge symmetry)
$r \ll 0$: Landau-Ginzburg phase ($\mathbb{Z}_5$)

$$
\begin{array}{ccccc}
\text{NLSM} - \text{phase} & & \text{GLSM} & & \text{LG} - \text{phase} \\
G = \emptyset & & G = U(1) & & G = \mathbb{Z}_5 \\
\tilde{Y}_5 & \xleftarrow[\text{transition}]{\text{conifold}} & Y_5 & \xrightarrow[\text{transition}]{\text{conifold}} & \hat{Y}_5 \\
\text{MW}(\tilde{Y}_5) = 0 & & \text{MW}(Y_5) = \mathbb{Z} & & \text{MW}(\hat{Y}_5) = 0 \\
\text{TS}(\tilde{Y}_5) = 0 & & \text{TS}(Y_5) = 0 & & \text{TS}(\hat{Y}_5) = \mathbb{Z}_5
\end{array}
$$

Remarks on 2d F-theory vacua

- The proposed correspondence can be useful in various ways:
  - Not necessarily critical string worldsheet theories, nevertheless F-theory provides framework to study them coupled also to gravity, see also [Apruzzi, Heckman, Hassler, Melnikov]
  - Generically, non-ablian gauge symmetries are present $\Rightarrow$ interesting models to study as GLSMs.
- Test whether a 2d F-theory vacuum flows to interesting SCFTs by computing elliptic genus [Benini, Eager, Hori,Tachikawa]
- Classification of 2d SCFTs $\rightarrow$ cue NHC

$\Rightarrow$ Lots of things to explore.

Epilogue

F-theory certainly has something going for itself when it comes to even dimensions:

$12d \leftarrow$ F-theory@20?

$10d \leftarrow$ [Morrison 2015]

$$\left.\begin{array}{l} 8d \\ 6d \\ 4d \end{array}\right\} \leftarrow \text{[Vafa], [Morrison Vafa]}$$

$2d \leftarrow$ F-theory@20

$0d \leftarrow ?$

Happy $n\times$20th Birthdays and many happy returns!