Non-Relativistic M2-Branes and AdS/CFT

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Introduction

Despite the efforts of everyone here M-theory is still very poorly understood.

AdS/CFT remains something of a miracle but offers a non-perturbative window into M-theory via 3D and 6D SCFT’s.

But 6D SCFT’s are poorly understood.

With M2-branes one had to sacrifice manifest R-symmetry and supersymmetry.

With M5-branes we need to sacrifice Lorentz invariance.

The last time I was hear I spoke about non-Lorentzian approaches to the M5-brane.

This time I'd like to revisit M2-branes in a non-relativistic limit and the corresponding AdS-duals: Interesting infinite dimensional symmetries.

Non-relativistic limits have been studied by a number of authors in recent years: [Gomis, Ooguri], [Gomis,Gomis,Kamimura], [Bagchi, Gopakumar], [Harmark, Orselli], Bergshoeff, Hartong, Obers, Oling,..., and no doubt see the talk by Blair

PART I: Non-relativistic limits of M2-brane Chern-Simons-Matter Theories

Consider a Chern-Simons matter theory with action of the form

$$S = -\frac{1}{c}\int \sqrt{-g}g^{\mu\nu}\mathrm{tr}(D_\mu \mathcal{Z}^M D_\nu \bar{\mathcal{Z}}^M) + V(\mathcal{Z}^M, \bar{\mathcal{Z}}^M) + S_{CS}$$

If the potential contains an explicit mass term

$$V(\mathcal{Z}^M, \bar{\mathcal{Z}}_M) = m^2c^2\mathcal{Z}^M\bar{\mathcal{Z}}_M + \hat{V}(\mathcal{Z}^M, \bar{\mathcal{Z}}_M)$$

then we can take a non-relativistic limit $c \to \infty$, $x^0 = ct$, $g = \eta$

$$\mathcal{Z}^M = e^{-imc^2t}\hat{\mathcal{Z}}^M$$

$$S = \int \mathrm{tr}(im\hat{\mathcal{Z}}^M D_t\hat{\bar{\mathcal{Z}}}_M - imD_t\hat{\mathcal{Z}}^M\hat{\bar{\mathcal{Z}}}_M - D_i\hat{\mathcal{Z}}^M D_i\hat{\bar{\mathcal{Z}}}_M) - \hat{V}(\hat{\mathcal{Z}}^M, \hat{\bar{\mathcal{Z}}}_M) + S_{CS}$$

Note that a divergence in the kinetic term is cancelled by a diverging term from the potential.

The Lorentz group is reduced to the Schrödinger group.

In a CFT we can't do this. But we can consider a similar limit

$$x^0 \to x^0 \qquad z \to \omega z \qquad z = x^1 + ix^2$$

(This is conformally equivalent to $x^0 \to cx^0$ with $c = \omega^{-1}$.)

In terms of the metric

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & \frac{\omega^2}{2} \\ 0 & \frac{\omega^2}{2} & 0 \end{pmatrix}$$

We note that

$$\sqrt{-g}g^{\mu\nu} = \omega^2 \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & \frac{2}{\omega^2} \\ 0 & \frac{2}{\omega^2} & 0 \end{pmatrix} = \begin{pmatrix} -\omega^2 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

Thus the limit $\omega \to 0$ is smooth and the action reduces to

$$S = -\int \; 2\mathrm{tr}(D\mathcal{Z}^M, \bar{D}\bar{\mathcal{Z}}_M) + 2\mathrm{tr}(\bar{D}\mathcal{Z}^M, D\bar{\mathcal{Z}}_M) + S_{CS}$$

The equations of motion are now

$$
\begin{aligned}
F_{z\bar{z}} &= 0 \\
F_{0z} &= \frac{2\pi}{k}\left(\mathcal{Z}^M D\bar{\mathcal{Z}}_M - D\mathcal{Z}^M\bar{\mathcal{Z}}_M\right) \\
(D\bar{D} + \bar{D}D)\mathcal{Z}^M &= 0
\end{aligned}
$$

So the spatial gauge field is flat. Solutions are

$$
\begin{aligned}
A_z &= 0 \\
A_0 &= \frac{2\pi}{k}\mathcal{Z}^M\bar{\mathcal{Z}}_M \\
\bar{\partial}\mathcal{Z}^M &= 0
\end{aligned}
$$

i.e. $\mathcal{Z}^M$ is holomorphic with any time dependence.

The action has an invariance under the infinite-dimensional set of diffeomorphisms.

But these aren't symmetries.

Symmetries are diffeomorphisms that preserve the metric, up to a conformal factor:

$$\frac{\partial x^\lambda}{\partial x'^\mu}\frac{\partial x^\rho}{\partial x'^\nu} g_{\lambda\rho}(x) = \Omega^2 g_{\mu\nu}(x) \ ,$$

For finite ω, $g \sim \eta$ and this reduces to the finite dimensional symmetry group $SO(2,3)$ of conformal transformations, including Lorentz transformations.

At $\omega = 0$ symmetries arise from

$$\sqrt{-g} g^{\mu\nu} \det\left(\frac{\partial x}{\partial x'}\right) \frac{\partial x'^{\lambda}}{\partial x^{\mu}} \frac{\partial x'^{\rho}}{\partial x^{\nu}} = \sqrt{-g}\Omega g^{\lambda\rho} = \Omega \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

We find z' is time-dependent holomorphic function of z and x'^0 is any function of x^0 with

$$\Omega = \frac{\partial x^0}{\partial x'^0}$$

This agrees with the solutions we found; namely the time dependence was arbitrary and the scalars were holomorphic functions.

But this is rather obscure point since we have removed all notion of time and dynamics.

We can find a more interesting situation if we pick one $\mathcal{Z}^M$, say $\mathcal{Z}^1$, and also rescale it:

$$\mathcal{Z}^1 \to \omega^{-1}\mathcal{Z}^1 \qquad \mathcal{Z}^A \to \mathcal{Z}^A \quad (A \neq 1)$$

This preserves the kinetic term for $\mathcal{Z}^1$ at $\omega = 0$.

But now the $D\mathcal{Z}^1\bar{D}\bar{\mathcal{Z}}^1 + \bar{D}\mathcal{Z}^1 D\bar{\mathcal{Z}}^1$ term diverges.

This can be cured by the following steps

- Note that

$$
\begin{aligned}
D\mathcal{Z}^1\bar{D}\bar{Z}^1 &= \partial(\mathcal{Z}^1\bar{D}\bar{\mathcal{Z}}_1) - \bar{\partial}(\mathcal{Z}^1 D\bar{\mathcal{Z}}_1) + \bar{D}\mathcal{Z}^1 D\bar{\mathcal{Z}}_1 - \mathcal{Z}^1[D,\bar{D}]\bar{\mathcal{Z}}_1 \\
&= \bar{D}\mathcal{Z}^1 D\bar{\mathcal{Z}}_1 + \partial(\mathcal{Z}^1\bar{D}\bar{\mathcal{Z}}_1) - \bar{\partial}(\mathcal{Z}^1 D\bar{\mathcal{Z}}_1) - i\mathcal{Z}^1 F_{z\bar{z}}(\bar{\mathcal{Z}}_1)
\end{aligned}
$$

- The red terms are a total derivative and are cancelled by including a background C-field:

$$
C = i\omega^{-2}dx^0 \wedge d\mathcal{Z}^1 \wedge d\bar{\mathcal{Z}}_1
$$

- The green term can be removed by shifting

$$
A_0 \to A_0 - \frac{2\pi}{k\omega^2}\mathcal{Z}^1\bar{\mathcal{Z}}_1
$$

- This introduces an ω^{-4} term in the kinetic term that needs to be cancelled by a $(\mathcal{Z}^1\bar{\mathcal{Z}}_1)^2\mathcal{Z}^A\bar{\mathcal{Z}}_A$ term in the potential

So now we just have a diverging term $-2\omega^{-2}\bar{D}\mathcal{Z}^1 D\bar{\mathcal{Z}}_1$.

- This is fixed (*c.f.* [Gomis, Ooguri]) by replacing

$$-\frac{2}{\omega^2}\int \bar{D}\mathcal{Z}^1 D\bar{\mathcal{Z}}_1 \to \int \bar{D}\mathcal{Z}^1\bar{H} + HD\bar{\mathcal{Z}}_1 + \frac{\omega^2}{4}H\bar{H}$$

- We can now set $\omega = 0$ and H becomes a Lagrange-multiplier imposing

$$\bar{D}\mathcal{Z}^1 = 0$$

Specifically, for M2-branes, the action becomes

$$
\begin{aligned}
S = \int & D_0 \mathcal{Z}^1 D_0 \bar{\mathcal{Z}}_1 + H D \bar{\mathcal{Z}}_1 + \bar{D} \mathcal{Z}^1 \bar{H} - 2 D \mathcal{Z}^A \bar{D} \bar{\mathcal{Z}}_A - 2 \bar{D} \mathcal{Z}^A D \bar{\mathcal{Z}}_A \\
& + \frac{2\pi i}{k} D_0 \mathcal{Z}^A [\bar{\mathcal{Z}}_1, \bar{\mathcal{Z}}_A; \mathcal{Z}^1] + \frac{2\pi i}{k} [\mathcal{Z}^1, \mathcal{Z}^A; \bar{\mathcal{Z}}_1] D_0 \bar{\mathcal{Z}}_A \\
& - \frac{4\pi^2}{3k^2} [\mathcal{Z}^A, \mathcal{Z}^1; \bar{\mathcal{Z}}_A][\bar{\mathcal{Z}}_B, \bar{\mathcal{Z}}_1; \mathcal{Z}^B] + \frac{16\pi^2}{3k^2} [\mathcal{Z}^A, \mathcal{Z}^1; \bar{\mathcal{Z}}_B][\bar{\mathcal{Z}}_A, \bar{\mathcal{Z}}_1, \mathcal{Z}^B] \\
& + \frac{8\pi^2}{3k^2} [\mathcal{Z}^A, \mathcal{Z}^B; \bar{\mathcal{Z}}_1][\bar{\mathcal{Z}}_A, \bar{\mathcal{Z}}_B; \mathcal{Z}^1] - \frac{4\pi^2}{3k^2} [\mathcal{Z}^1, \mathcal{Z}^A; \bar{\mathcal{Z}}_1][\bar{\mathcal{Z}}_B, \bar{\mathcal{Z}}_A; \mathcal{Z}^B] \\
& - \frac{4\pi^2}{3k^2} [\mathcal{Z}^B, \mathcal{Z}^A; \bar{\mathcal{Z}}_B][\bar{\mathcal{Z}}_1, \bar{\mathcal{Z}}_A; \mathcal{Z}^1] \\
& + \frac{ik}{2\pi} \left(A_0^L F_{z\bar{z}}^L + A_z^L F_{\bar{z}0}^L + A_{\bar{z}}^L F_{0z}^L + i A_0^L [A_z^L, A_{\bar{z}}^L] - L \to R \right)
\end{aligned}
$$

This was previously found in [Owen,NL],[Mouland, NL] where it was shown that it preserves all supersymmetries (with Fermions added).

What are the Bosonic symmetries?

Well it turns out that there is a symmetry for

$$x'^0 = x^0 + \epsilon F(x^0)$$

but now only if $F(x^0) = a + bx^0 + c(x^0)^2$. These generate the usual 1D conformal transformations of $SO(1,2)$

We still find symmetries

$$z' = z + \epsilon f(z, x^0)$$

for any holomorphic but time-dependent function f

Lastly there is a $U(1) \times SU(3)$ R-symmetry and a $\tilde{U}(1)$ 'Baryon' symmetry as is familiar for M2's.

What are the dynamics?

The equations of motion were obtained before [NL,Sacco] and analysed in [Kucharski, Owen, NL] (for $SU(2)\times SU(2)$):

H imposes the constraint

$$\bar{D}\mathcal{Z}^1 = 0$$

A_0 imposes a Gauss law constraint (for $D_0\mathcal{Z}^1 = 0$):

$$F^L_{z\bar{z}} = \frac{4\pi^2 i}{k^2}(\mathcal{Z}^A[\bar{\mathcal{Z}}_1,\bar{\mathcal{Z}}_A;\mathcal{Z}^1] - [\mathcal{Z}^1,\mathcal{Z}^A;\bar{\mathcal{Z}}_1]\bar{\mathcal{Z}}_A)$$

$$F^R_{z\bar{z}} = -\frac{4\pi^2 i}{k^2}([\bar{\mathcal{Z}}_1,\bar{\mathcal{Z}}_A;\mathcal{Z}^1]\mathcal{Z}^A - \bar{\mathcal{Z}}_A[\mathcal{Z}^1,\mathcal{Z}^A;\bar{\mathcal{Z}}_1])$$

Thus we find a (3-algebra) version of Hitchin's equations (when the $\mathcal{Z}^A$ are constant).

For $D_0\mathcal{Z}^1 \neq 0$ one can argue that the classical dynamics corresponds to motion on Hitchin Moduli space

What is the physical interpretation of this?

The M2-brane BPS equations are [Kim,Kim, Kwon,Nakajima]

$$
\begin{aligned}
\bar{D}\mathcal{Z}^1 &= 0 \ , \\
D\mathcal{Z}^A &= \bar{D}\mathcal{Z}^A = 0 \ , \\
[\mathcal{Z}^1, \mathcal{Z}^2; \bar{\mathcal{Z}}_2] &= [\mathcal{Z}^1, \mathcal{Z}^3; \bar{\mathcal{Z}}_3] = [\mathcal{Z}^1, \mathcal{Z}^4; \bar{\mathcal{Z}}_4] \ , \\
[\mathcal{Z}^1, \mathcal{Z}^A; \bar{\mathcal{Z}}_B] &= 0 \ (A \neq B) \ , \\
[\mathcal{Z}^A, \mathcal{Z}^B; \bar{\mathcal{Z}}_C] &= 0 \ , \\
D_0\mathcal{Z}^1 &= \frac{2\pi i}{k}[\mathcal{Z}^1, \mathcal{Z}^A; \bar{\mathcal{Z}}_A] \ , \\
D_0\mathcal{Z}^A &= \frac{2\pi i}{k}[\mathcal{Z}^1, \mathcal{Z}^A; \bar{\mathcal{Z}}_1] \ ,
\end{aligned}
$$

which we must supplement with the Gauss's law constraints

$$
\begin{aligned}
\frac{k}{2\pi}F^L_{z\bar{z}} &= \mathcal{Z}^M D_0\bar{\mathcal{Z}}_M - D_0\mathcal{Z}^M\bar{\mathcal{Z}}_M \\
\frac{k}{2\pi}F^R_{z\bar{z}} &= \bar{\mathcal{Z}}^M D_0\mathcal{Z}_M - D_0\bar{\mathcal{Z}}^M\mathcal{Z}_M
\end{aligned}
$$

The green BPS condition is not invariant under the the rescaling but we can make a shift

$$A_0^L = \mathcal{A}_0^L - \frac{2\pi}{k}\mathcal{Z}^1\bar{\mathcal{Z}}_1 \ ,$$
$$A_0^R = \mathcal{A}_0^R - \frac{2\pi}{k}\bar{\mathcal{Z}}_1\mathcal{Z}^1 \ ,$$

so that it becomes $\mathcal{D}_0\mathcal{Z}^A = 0$ then there is invariance under

$$\mathcal{Z}^1(t, \omega^{-1}z, \omega^{-1}\bar{z}) = \omega^{-1}\mathcal{Z}^1(t, z, \bar{z}) \ ,$$
$$\mathcal{Z}^A(t, \omega^{-1}z, \omega^{-1}\bar{z}) = \mathcal{Z}^A(t, z, \bar{z}) \ ,$$
$$\mathcal{A}_0(t, \omega^{-1}z, \omega^{-1}\bar{z}) = \mathcal{A}_0(t, z, \bar{z}) \ ,$$
$$A_z(t, \omega^{-1}z, \omega^{-1}\bar{z}) = \omega^{-1}A_z(t, z, \bar{z}) \ ,$$

Thus the limit corresponds to a non-relativistic expansion around BPS states.

In summary we have a 3D supersymmetric non-relativistic field theory with exotic 'symmetries'

- $SO(1,2)$ 1D conformal symmetry
- $U(1) \times SU(3)$ R-symmetry
- $\tilde{U}(1)$ Baryon symmetry
- arbitrary holomorhpic spatial transformations $z \to z + \epsilon f(z, x^0)$

The last ones act more like gauge symmetries. In particular their charges are boundary integrals (for $\partial_0 f = 0$):

$$Q[f] = \frac{i}{2} \oint f \operatorname{tr}(\mathcal{Z}^1 D_0 \bar{\mathcal{Z}}_1) d\bar{z} + c.c.$$

And there is a purely spatial conserved current $\bar{\partial} T = 0$:

$$T = \operatorname{tr}(\mathcal{Z}^1 D \bar{H} + 4 D \mathcal{Z}^A D \bar{\mathcal{Z}}_A)$$

It is instructive to see what happened to the charges in the latter case. Indeed even the total momentum, corresponding to $f = constant$ becomes a boundary term

In the original theory

$$\hat{P}_z = \mathrm{tr}\int D_0\hat{\mathcal{Z}}^M D\hat{\bar{\mathcal{Z}}}_M + D\hat{\mathcal{Z}}^M D_0\hat{\bar{\mathcal{Z}}}_M \ .$$

After rescaling $\hat{P}_z = \omega^{-1}P_z$ *etc.* and transforming we find

$$P_z = \mathrm{tr}\int \left[D\mathcal{Z}^1 D_0\bar{\mathcal{Z}}_1 + \frac{2\pi i}{k}\left([\mathcal{Z}^1,\mathcal{Z}^A;\bar{\mathcal{Z}}_1]D\bar{\mathcal{Z}}_A + D\mathcal{Z}^A[\bar{\mathcal{Z}}_1,\bar{\mathcal{Z}}_A;\mathcal{Z}^1]\right)\right] ,$$

Integrating by parts and using the constraints and equations of motion we simply find

$$P_z = i\mathrm{tr}\oint d\bar{z}\,\mathrm{tr}(\mathcal{Z}^1 D_0\bar{\mathcal{Z}}_1) \ .$$

A similar result occurs for rotations in the z-plane

PART II: Membrane-Newton-Cartan limit of $AdS_4 \times S^7$

It is well-known that the M2-branes have a dual description as a near horizon limit of

$$\hat{g} = \hat{\mathcal{H}}^{-\frac{2}{3}}(-dt^2 + dzd\bar{z}) + \hat{\mathcal{H}}^{\frac{1}{3}}(dud\bar{u} + dv^a dv^a)$$
$$\hat{C}_3 = \frac{i}{2}\hat{\mathcal{H}}^{-1}dt \wedge dz \wedge d\bar{z} + \hat{k}$$

where $d\hat{k} = 0$ and the function $\hat{\mathcal{H}}$ is

$$\hat{\mathcal{H}}(u^I, v^a) = 1 + \frac{\hat{R}^6}{(u\bar{u} + v^a v^a)^3} \ .$$

where $u \sim \mathcal{Z}^1$ parameterises $\mathbb{C}$ and $v^a \sim \mathcal{Z}^A$ parameterise $\mathbb{R}^6$

We need to consider the rescaling (this is conformally equivalent to the limit we considered above)

$$(t, z, u, v^a) \to (ct, c^{-\frac{1}{2}}z, cu, c^{-\frac{1}{2}}v^a)$$

that splits $\mathbb{R}^{11} \to \mathbb{R}^3 \oplus \mathbb{R}^8$ and take the limit $c \to \infty$ (with $\hat{R} = cR$)

Who is crazy enough to look at this weird limit? Happily these people: [Blair,Gallegos,Zinnato]

This is described by a Membrane-Newton-Cartan geometry

$$g = c^2 \tau_{\mu\nu} dx^\mu \otimes dx^\nu + \frac{1}{c} H_{\mu\nu} dx^\mu \otimes dx^\nu$$
$$g^{-1} = c H^{\mu\nu} \partial_\mu \otimes \partial_\nu + \frac{1}{c^2} \tau^{\mu\nu} \partial_\mu \otimes \partial_\nu$$

The key idea is that τ and H^{-1} exist and survive the limit $c \to \infty$.

Any subleading (in c) terms in τ and H can be neglected.

There are also local Galilean boosts that can be used to change the form of H.

We also need to decompose the C-field:

$$\hat{C}_3 = -c^3 dvol(\tau) + C_3 + c^{-3}\tilde{C}_3 + O(c^{-6}) \ .$$

The first term is the volume form along the "large" dimensions $(t, u, \bar{u})$

Note that the diverging volume form corresponds in the brane picture to a total derivative WZ-term:

$$S_{WZ} = \int *\hat{k} = \frac{i}{2}c^3 \int \mathrm{dtr}(dx^0 \wedge \mathcal{Z}^1 \wedge \bar{D}\bar{\mathcal{Z}}_1) + c.c$$

And indeed we used this to cancel a divergence above

$\tilde{C}_3$ plays the role of a Lagrange multiplier imposing a self-duality constraint on $F = dC_3$.

[Blair,Gallegos,Zinnato] give the resulting action obtained from eleven-dimensional supergravity in the limit $c \to \infty$.

In our case we find

$$\tau = -\frac{(u\bar{u})^2}{R^4} dt \otimes dt + \frac{R^2}{u\bar{u}} du \otimes d\bar{u} \; ,$$

$$H^{-1} = \frac{2R^4}{(u\bar{u})^2} \left(\partial \otimes \bar{\partial} + \bar{\partial} \otimes \partial\right) + \frac{u\bar{u}}{R^2} \left(\frac{\partial}{\partial v^a} \otimes \frac{\partial}{\partial v^a} \right) \; .$$

and

$$C_3 = \frac{i}{2} \left(\frac{(u\bar{u})^3}{R^6} \right) dt \wedge dz \wedge d\bar{z}$$

$$\tilde{C}_3 = \frac{i}{2} \left(\frac{3(u\bar{u})^2 v^a v^a}{R^6} \right) dt \wedge dz \wedge d\bar{z}$$

and the self-duality constraint on dC_3 is satisfied.

Geometrically this is $AdS_2 \times S^1 \times \mathbb{R}^2 \times \mathbb{R}^6$ (warped with flux)

Therefore we claim that the dual gravity theory is described by the [Blair,Gallegos,Zinnato] action about this background.

What About the Symmetries?

Since our QFT has the full supersymmetries (16+16 BLG or 12+12 ABJM) these should be present in the gravity dual

Recall the field theory had the symmetries:

- $SO(1,2)$
- $U(1) \times SU(3)$
- $\tilde{U}(1)$
- $z \to z(z, x^0)$

The first three all have associated Killing vectors in the dual geometry (as for M2's for generic k the translations in $\mathbb{R}^6$ are broken and the rotations reduced to $\tilde{U}(1) \times SU(3)$)

But there is a caveat:

It was argued in [Bagchi, Gopakumar] that in theories with AdS_2 duals the $SO(1,2)$ should enhance to a full affine current algebra due to asymptotic Killing vectors (*i.e.* diffeomorphisms generated by vector fields which are only Killing as you approach the boundary)

However for us such transformations induce extra terms in the C-field at the boundary that cannot be cancelled.

Thus we don't expect $SO(1,2)$ to enhance and indeed we don't see that in the gauge theory dual.

The transformations $z \to z + \epsilon f(z, x^0)$ are more interesting.

In the gravity dual we certainly have Killing vectors for translations and rotations in z.

However we don't see the spatial scale symmetry $z \to \lambda z$. Nor is there any indication of an infinite family of transformations.

Our claim is that most of these should be treated as gauge symmetries in the QFT and therefore not visible in the gravity dual: physical states are invariant.

Indeed the conserved charges are co-dimension two in the QFT and hence would be co-dimension three in the gravity dual.

So at most we expect the ‘global’ part of the holomorphic transformations are the $ISO(2)$ symmetries.

However it is possible that, in the $c \to \infty$ limit, even the $ISO(2)$ symmetries become trivial on the boundary and hence in the gauge theory.

Conclusions

In this talk I have discussed an amusing, but hopefully insightful, non-relativistic limit of M2-branes

- Novel Chern-Simons-Matter theory whose dynamics is given by motion on Hitchin's moduli space
- Infinite-dimensional spatial symmetry group
- Also gave the gravitational dual using the construction of [Blair,Gallegos,Zinnato]

We also matched the symmetries:

- 'predict' that the [Blair,Gallegos,Zinnato] action is supersymmetric
- spatial symmetries act trivially

Does AdS/CFT still work? Or have we broken it? If so how and why?

Can we reconstruct the finite c from the $c = \infty$ limit? *i.e.* recover Lorentz invariance in the gauge theory and bulk?

We would like to have a better understanding of the bulk geometry and its boundary

Is there is a non-trivial sector of the gauge theory where the $\mathcal{Z}^A$ are dynamical?

M2-branes also admit a string theory dual on $AdS_4 \times \mathbb{C}P^3$ [Arutyunov,Frolov],[Stefanski]. Can we make sense of our limit here?

THANK YOU

Some Old Slides

The $(2,0)$ superalgebra is [NL,Sacco][NL,Papageorgakis]

$$
\begin{aligned}
\delta X^i &= i\bar{\epsilon}\Gamma^i\Psi \\
\delta Y^\mu &= \frac{i}{2}\bar{\epsilon}\Gamma_{\lambda\rho}C^{\mu\lambda\rho}\Psi \\
\delta H_{\mu\nu\lambda} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi + i\bar{\epsilon}\Gamma^i\Gamma_{\mu\nu\lambda\rho}[Y^\rho, X^i, \Psi] \\
&\quad + \frac{i}{2}\bar{\epsilon}(\star C)_{\mu\nu\lambda}\Gamma^{ij}[X^i, X^j, \Psi] + \frac{3i}{4}\bar{\epsilon}\Gamma_{[\mu\nu|\rho\sigma}C^{\rho\sigma}{}_{\lambda]}\Gamma^{ij}[X^i, X^j, \Psi] \\
\delta A_\mu(\cdot) &= i\bar{\epsilon}\Gamma_{\mu\nu}[Y^\nu, \Psi, \ \cdot\ ] + \frac{i}{3!}\bar{\epsilon}C^{\nu\lambda\rho}\Gamma_{\mu\nu\lambda\rho}\Gamma^i[X^i, \Psi, \ \cdot\ ]\ , \\
\delta\Psi &= \Gamma^\mu\Gamma^i D_\mu X^i\epsilon + \frac{1}{2\cdot 3!}H_{\mu\nu\lambda}\Gamma^{\mu\nu\lambda}\epsilon - \frac{1}{2}\Gamma_\mu\Gamma^{ij}[Y^\mu, X^i, X^j]\epsilon \\
&\quad + \frac{1}{3!\cdot 3!}C_{\mu\nu\lambda}\Gamma^{\mu\nu\lambda}\Gamma^{ijk}[X^i, X^j, X^k]\epsilon
\end{aligned}
$$

$$
\Gamma_{012345}\epsilon = \epsilon \qquad \Gamma_{012345}\Psi = -\Psi
$$

X^I, Ψ and $H_{\mu\nu\lambda}$ are dynamical, A_μ and Y^μ are auxiliary but $C_{\mu\nu\lambda}$ is a background (abelian) 3-form

A standard (but trust me tedious) calculation shows that this system indeed closes on the following equations of motion

$$
\begin{aligned}
0 &= \Gamma^\rho D_\rho \Psi + \Gamma_\rho \Gamma^i [Y^\rho, X^i, \Psi] + \frac{i}{2 \cdot 3!} C^{\rho\sigma\tau} \Gamma_{\rho\sigma\tau} \Gamma^{ij} [X^i, X^j, \Psi] \\
0 &= D^2 X^i + [Y^\mu, X^j, [Y_\mu, X^j, X^i]] + \frac{1}{2 \cdot 3!} C^2 [X^j, X^k, [X^j, X^k, X^i]] \\
&\quad + fermions \\
0 &= D_{[\lambda} H_{\mu\nu\rho]} + \frac{1}{2} (\star C)_{[\mu\nu\lambda} [X^i, X^j, [Y_{\rho]}, X^i, X^j]] \\
&\quad + \frac{1}{4} \varepsilon_{\mu\nu\lambda\rho\sigma\tau} [Y^\sigma, X^i, D^\tau X^i] + fermions
\end{aligned}
$$

As well as constraints:

$$
\begin{aligned}
F_{\mu\nu}(\cdot) &= [Y^\lambda, H_{\mu\nu\lambda},\ \cdot\ ] - (\star C)_{\mu\nu\lambda}[X^i, D^\lambda X^i,\ \cdot\ ] + fermions \\
0 &= D_\mu Y^\nu - \frac{1}{2} H_{\mu\lambda\rho} C^{\nu\lambda\rho} \\
0 &= [Y^\mu, D_\mu(\cdot),\ \cdot'\ ] + \frac{1}{3}[D_\mu Y^\mu,\ \cdot\ ,\ \cdot'\ ] \\
0 &= C^{\mu\nu\lambda} D_\lambda(\cdot) - [Y^\mu, Y^\nu,\ \cdot\ ] \\
0 &= C \wedge Y \\
0 &= C_{\sigma[\mu\nu} C^\sigma{}_{\lambda]\rho}
\end{aligned}
$$

Somewhat unconventional (ugly? beautiful?).

There is a conserved supercurrent:

$$\begin{aligned}
S^\mu = & \, 2\pi i \langle D_\nu X^i, \Gamma^\nu \Gamma^\mu \Gamma^i \Psi \rangle + \frac{2\pi i}{4} \langle H_{\nu\lambda\rho}, \Gamma^{\nu\lambda\rho} \Gamma^\mu \Psi \rangle \\
& - \frac{2\pi i}{2} \langle [Y_\mu, X^i, X^j], \Gamma^\nu \Gamma^\mu \Gamma^{ij} \Psi \rangle \\
& + \frac{2\pi i}{3!^2} C_{\nu\lambda\rho} \langle [X^i, X^j, X^k], \Gamma^{\nu\lambda\rho} \Gamma^\mu \Gamma^{ijk} \Psi \rangle
\end{aligned}$$

and energy-momentum tensor :

$$\begin{aligned}
T_{\mu\nu} = & \, \frac{\pi}{2} \langle H_{\mu\lambda\rho}, H_\nu{}^{\lambda\rho} \rangle + 2\pi \langle D_\mu X^i, D_\nu X^i \rangle - \pi \eta_{\mu\nu} \langle D_\lambda X^i, D^\lambda X^i \rangle \\
& - \frac{\pi}{2} \eta_{\mu\nu} \langle [Y_\lambda, X^i, X^j], [Y^\lambda, X^i, X^j] \rangle \\
& + \frac{2\pi}{3!} (C_{\mu\lambda\rho} C_\nu{}^{\lambda\rho} - \frac{1}{6} \eta_{\mu\nu} C^2) \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle \\
& + \frac{\pi}{3!} C_{\mu\lambda\rho} (\star C)_\nu{}^{\lambda\rho} \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle + fermions
\end{aligned}$$

One can also compute the superalgebra and central charges.