Poisson structures from corners of field theories

based on joint work with

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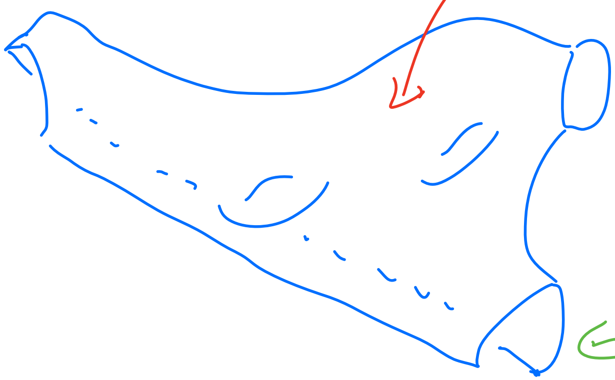
G. Canepa: focus on 3+1 gravity in coframe formulation

Field theory on manifolds with boundaries and corners



bulk: (BV) quantization
$\rightsquigarrow$ partition fn / state ψ

boundary: symplectic manifold M
$\rightsquigarrow$ its quantization $\mathcal{H} \ni \psi$

corners: Poisson manifold P and its quantization A

Σ boundary $\rightsquigarrow M_\Sigma$ | $\rightsquigarrow \mathcal{H}_\Sigma$

$\Sigma_1 \quad \Sigma_2$, C $\rightsquigarrow M_{\Sigma_1 \cup \Sigma_2} = \underline{M_1 \times_{P_c} M_2}$ | $\mathcal{H}_{\Sigma_1 \cup \Sigma_2} = \mathcal{H}_{\Sigma_1} \otimes_{A_c} \mathcal{H}_{\Sigma_2}$

$M_1 \to P_c \leftarrow M_2$

Part I: Some background & results

Affine Lie algebras

Let $(\mathfrak{g}, \langle , \rangle)$ be a f.d. quadratic Lie algebra (i.e. $\langle , \rangle$ invariant inner product)

- $\mathfrak{g}_{S^1} := \mathrm{Map}(S^1, \mathfrak{g})$ with pointwise Lie bracket $[f,g](x) = [f(x), g(x)]_{\mathfrak{g}}$

- $c(f,g) := \int_{S^1} \langle f, dg \rangle$ is a cocycle!

$f, g \in \mathfrak{g}_{S^1}$

$\Rightarrow \quad \hat{\mathfrak{g}} := \mathfrak{g}_{S^1} \oplus \mathbb{R}$ with $[f \oplus a, g \oplus b] := [f,g] \oplus c(f,g)$

is a Lie algebra

- $\hat{\mathfrak{g}}$ is called an affine Lie algebra
- Its representation theory is widely studied

- It is related to Chern-Simons theory and the Wess-Zumino-Witten model

- The Poisson algebra in the title is "$\hat{\mathfrak{g}}^*$"

viz., $\mathcal{F}^{\partial\partial} := \Omega^1(S^1) \otimes \mathfrak{g}^*$ with affine Poisson structure (defined on certain functionals, e.g., local ones)

- Associated to the corners of CS theory

2D generalization

Let $\mathfrak{g}$ be a f.d. Lie algebra

Γ a closed oriented surface

$\mathfrak{g}_\Gamma := \mathrm{Map}(\Gamma, \mathfrak{g})$ with pointwise Lie bracket

$\mathfrak{g}^*_\Gamma := \Omega^1(\Gamma) \otimes \mathfrak{g}^*$ with pointwise coadjoint action of $\mathfrak{g}$ on $\mathfrak{g}^*$

$\tilde{\mathfrak{g}}_\Gamma := \mathfrak{g}_\Gamma \oplus \mathfrak{g}^*_\Gamma$ with Lie bracket

$$[\theta \oplus \alpha, \varphi \oplus \beta]^{\sim} := [\theta, \varphi] \oplus (\mathrm{ad}^*_\theta \beta - \mathrm{ad}^*_\varphi \alpha)$$

Cocycle: $c(f\oplus\alpha, g\oplus\beta) = \int_\Gamma \big((\alpha, dg) - (\beta, df)\big)$

$(\ ,\ )$ canonical pairing of $\mathfrak{g}^*$ with $\mathfrak{g}$

$\Rightarrow \quad \hat{\mathfrak{g}}_\Gamma := \tilde{\mathfrak{g}}_\Gamma \oplus \mathbb{R}$ with

$[f\oplus\alpha\oplus a, g\oplus\beta\oplus b] := [f\oplus\alpha, g\oplus\beta]^{\sim} \oplus c(f\oplus\alpha, g\oplus\beta)$

is a Lie algebra

Generalizations

① $\mathfrak{g}$ - bundle over Γ $\rightsquigarrow$ $\mathfrak{g}_\Sigma$, $\mathfrak{g}^*_\Sigma$ sections

$$c\left(\theta\oplus\alpha, \phi\oplus\beta\right) = \int_\Gamma \left((\alpha, d_{A_0}\theta) - (\beta, d_{A_0}\phi) \right)$$

d_{A_0} covariant derivative w.r.t. some connection

② If $(\mathfrak{g}, \langle , \rangle)$ quadratic

$$c_\Lambda\left(\theta\oplus\alpha, \phi\oplus\beta\right) := c\left(\theta\oplus\alpha, \phi\oplus\beta\right) + \Lambda \int \langle \alpha, \beta \rangle$$

$\rightsquigarrow$ $\hat{\mathfrak{g}}^\Lambda_\Gamma$

- $\mathfrak{g}_\Gamma^{\Lambda}$ is related to 4D BF theory
  (with "cosmological" term for $\Lambda \neq 0$)
  It is worth being studied

- For 4D BF theory, one has to consider the Poisson manifold

$$\left(\mathfrak{g}_\Gamma^{\Lambda}\right)^* = \underbrace{\Omega^2(\Gamma)\otimes\mathfrak{g}^*}_{\ni B} \oplus \underbrace{\Omega^1(\Gamma)\otimes\mathfrak{g}}_{\ni A}$$

and the Poisson sub ...

$$= \{(A,B) : F_A + \Lambda B = 0\}$$

4D Gravity is related to the above:

- $\mathfrak{g} = so(3,1) \simeq \Lambda^2 \mathbb{R}^4$ (or $so(4)$ for Euclidean gravity)

- A further constraints

$$\mathrm{Pf}(B) = 0 \quad (\text{and } B \neq 0)$$

which defines a Poisson submanifold of P

Part II $\quad P_\infty$-structures from graded manifolds

A Poisson structure on a manifold M

is the same a function S of degree 2 on $T^*[1]M$

$C^\infty(T^*[1]M) := \Gamma(\Lambda TM)$

s.t. $\{S, S\} = 0$

functions of deg k is $\Gamma(\Lambda^k TM)$

In fact, $C^\infty(T^*[1]M) = \Gamma(\Lambda^\bullet TM)$

$\{\ ,\ \} = [\ ,\ ]_{SN}$

Generalization: allow M to be a graded manifold itself

$$\Rightarrow \quad S \rightsquigarrow \quad \pi = \pi_0 + \pi_1 + \pi_2 + \cdots, \qquad \pi_k \in \Gamma(\Lambda^k TM)$$

$$\deg \pi_k = 2-k$$

$$\{f_1, \dots, f_k\}_k := \pi_k(df_1, \dots, df_k)$$

L_∞-structure by multiderivations $=: P_\infty$

<u>Notation</u> We call such (M, S) a BF^2V manifold

Further generalization:

1) Replace $T^*[1]M$ by any graded manifold $\mathcal{M}$

with symplectic form of degree 1.

2) Write $\mathcal{M}$ as $T^*[1]M$ (choice of polarization)

Example $\mathfrak{g}$ finite dimensional Lie algebra

$\Rightarrow$ $\mathfrak{g}^*$ Poisson manifold w/ KKS Poisson structure

$\rightsquigarrow$ $\mathcal{M} = T^*[1]\mathfrak{g}^* = \mathfrak{g}^* \oplus \mathfrak{g}[1]$

$\pi = \pi_2 = \pi_{KKS} \rightsquigarrow S$

We can also write $\mathcal{M} = T^*[1]\mathfrak{g}[1]$

$S \rightsquigarrow \pi = \pi_1 = \delta_{CE}$ Chevalley-Eilenberg

$C^\infty(\mathfrak{g}[1]) = \Lambda^\cdot \mathfrak{g}^*$

A further generalization:

- A graded Poisson algebra with $S \in A_2$, $\{S, S\} = 0$ (e.g. $C^\infty(T^*[1]M)$)

- Splitting $A = \mathfrak{h} \oplus \mathfrak{g}$ (e.g. $\mathfrak{h} = C^\infty(M)$, $\mathfrak{g} = \Gamma(\Lambda^{>0} TM)$)

  $\mathfrak{h}, \mathfrak{g}$ Poisson subalgebras

  $\mathfrak{h}$ abelian

- Define desired brackets on $\mathfrak{h}$ [T. Voronov]

$$\{ \beta_1, \dots, \beta_k \}_k := P\{ \dots \{\{S, \beta_1\}, \beta_2\}, \dots, \beta_k\}$$

$\beta_i \in \mathfrak{h}$

$$P : A \to \mathfrak{h}$$

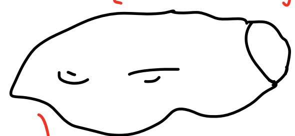
Claim ① (BV) Field theories yield a BF^2V structure on their codimension-2 corners

Polarization

Poisson$_\infty$-structure

② Boundary to corner

F_Σ F_Γ $\Gamma = \partial\Sigma$



P_∞-structure

symplectic structure

related by restriction of the fields to the corner

$F_\Sigma \to F_\Gamma$ "Poisson map"

- I will explain ①

This is pretty obvious in AKSZ theories

- Target $(\mathcal{Y},\ \omega_{\mathcal{Y}} = d\alpha_{\mathcal{Y}},\ S_{\mathcal{Y}})$ $\quad BF^nV$

symplectic of degree $n-1$ $\quad$ degree n $\quad \{S,S\}=0$

- Source $T^*[1]\Sigma$

k-manifold

Space of fields: $\mathrm{Map}(T^*[1]\Sigma,\ \mathcal{Y})$

with induced structure $\omega,\ S$: $\{S,S\}=0$ $\quad BF^{n-k}V$

$\deg = n-k-1$ $\quad \deg = n-k$

Take $k=n-2$

- The two examples I gave at the beginning are of this type
  - Chern-Simons
  - BF_4
- Interesting nonAKSZ example:

  4D Palatini-Cartan gravity

Example 1 $\quad BF_2$

- $\mathcal{Y} = T^*[1]\mathfrak{g}[1] = T^*[1]\mathfrak{g}^*$, $\quad \mathfrak{g}$ f.d. Lie algebra

$$n = 2$$

$$\dim \Sigma = 0 \rightsquigarrow \Sigma = \{pt\}$$

$$\mathrm{Map}(T[1]\Sigma, \mathcal{Y}) = \mathcal{Y}$$

This is the example we have considered before

Example 2 CS $\qquad n=3, \quad k=1$

$\mathcal{Y} = \mathfrak{g}[1]$, $(\mathfrak{g}, \langle , \rangle)$ quadratic Lie algebra
$\quad \ni a$

$$\omega_{\mathcal{Y}} = \frac{1}{2}\langle da, da\rangle = d\left(\frac{1}{2}\langle a, da\rangle\right)$$

$$S_{\mathcal{Y}} = \frac{1}{6}\langle a, [a,a]\rangle$$

$$\Sigma = S^1 \rightsquigarrow \mathrm{Map}(T[1]\Sigma, \mathfrak{g}[1]) = \underset{C}{\Omega^0(\Sigma)[1]\otimes\mathfrak{g}} \oplus \underset{A}{\Omega^1(\Sigma)\otimes\mathfrak{g}}$$

$$\omega = \int_{S^1} \langle \delta C, \delta A\rangle$$

$$S = \int_{S^1}\left(\frac{1}{2}\langle C, dC\rangle + \frac{1}{2}\langle A, [C,C]\rangle\right)$$

Choose the polarization:

$$\mathrm{Map}(T[1]\Sigma, \mathfrak{g}) = T^*[1]\,\Omega^1(\Sigma)\otimes\mathfrak{g}$$

" A: coordinate, $C \rightsquigarrow \frac{\delta}{\delta A}$ "

" $S \rightsquigarrow \pi_2 = \int_{S^1}\left(\langle \frac{\delta}{\delta A}, d\frac{\delta}{\delta A}\rangle + \frac{1}{2}\langle A, \frac{\delta}{\delta A}, \frac{\delta}{\delta A}\rangle\right)$ "

On linear functionals: $J_\beta := \int_{S^1}\langle \beta, A\rangle$, $\beta\in\Omega^0(S^1)\otimes\mathfrak{g} = \mathfrak{g}_{S^1}$

$$\{J_\beta, J_h\} = \underset{\substack{\shortparallel \\ \text{Lie bracket} \\ \text{on } \mathfrak{g}_{S^1}}}{J_{[\beta,h]}} \oplus \underbrace{\int_{S^1}\langle \beta, dh\rangle}_{\substack{\shortparallel \\ c(\beta,h)}}$$

$\rightsquigarrow$ affine Poisson structure on $\hat{\mathfrak{g}}_{S^1}$

Example 3 BF_4 $n=4,\ k=3,$

$\dim \Sigma = 2$

Fields

$\mathfrak{g}$-valued:		C	A	B^+	$\mathfrak{g}^*$-valued:	φ	γ	B
	form	0	1	2		0	1	2
	degree	1	0	-1		2	1	0

$$\omega = \int_\Sigma \delta B\, \delta C + \delta\gamma\, \delta A + \delta\varphi\, \delta B^+$$

(pairing is understood)

$$S = \int_\Sigma \frac{1}{2} B[C,C] + d_A C + \phi\left(F_A + [C,B^+]\right) + \Lambda\left(\frac{1}{2}\gamma\gamma + B\phi\right)$$

these terms require an invariant inner product

Polarization 1

$T^*[1]M, \quad M \ni (A, B, B^+)$

nonpositive degree

$S \rightsquigarrow \Pi = \Pi_1 + \Pi_2$

$$\Pi_1 = \int_\Sigma (F_A + \Lambda B) \frac{\delta}{\delta B^+}$$

$$\Pi_2 = \int_\Sigma \frac{1}{2} B \left[\frac{\delta}{\delta B}, \frac{\delta}{\delta B}\right] + \frac{\delta}{\delta A} d \frac{\delta}{\delta B} + A\left[\frac{\delta}{\delta A}, \frac{\delta}{\delta B}\right] + \Lambda \frac{\delta}{\delta A} \frac{\delta}{\delta A}$$

$$+ B^+\left[\frac{\delta}{\delta B^+}, \frac{\delta}{\delta B}\right]$$

- Π_2, forgetting B^+, yields the affine Poisson on $\left(\mathfrak{g}^{\Lambda}_{\Pi}\right)^*$
- B^+ and Π_1 corresponds to selecting the Poisson submanifold $\{F_A + \Lambda B = 0\}$

Polarization 2

$T^*[1]M$, $M \ni (\phi, \gamma, B)$, expand around a ref. connection A_0 (possibly trivial or flat)

$\int \rightsquigarrow \Pi = \Pi_0 + \Pi_1 + \Pi_2$

$$\Pi_0 = \int_\Sigma F_{A_0} + \Lambda\left(\frac{1}{2}\gamma\gamma + B\phi\right)$$

$$\Pi_1 = \int_\Sigma d_{A_0}\gamma \frac{\delta}{\delta B} + d_{A_0}\phi \frac{\delta}{\delta \gamma}$$

$$\Pi_2 = \int_\Sigma \frac{1}{2} B\left[\frac{\delta}{\delta B}, \frac{\delta}{\delta B}\right] + \gamma\left[\frac{\delta}{\delta \gamma}, \frac{\delta}{\delta B}\right] + \frac{1}{2}\phi\left[\frac{\delta}{\delta \gamma}, \frac{\delta}{\delta \gamma}\right] + \phi\left[\frac{\delta}{\delta \phi}, \frac{\delta}{\delta B}\right]$$

On linear functionals

$$J_\alpha := \int_\Sigma \alpha B, \quad M_\beta := \int_\Sigma \beta \psi, \quad K_\gamma := \int_\Sigma \gamma\phi$$

we get

$$\{,\}_0 = \int_\Sigma \phi F_{A_0} + \Lambda\left(\frac{1}{2}\psi\psi + B\phi\right)$$

$$\{J_\alpha\}_1 = M_{d_{A_0}\alpha}, \quad \{M_\beta\}_1 = K_{d_{A_0}\beta}, \quad \{K_\gamma\}_1 = 0$$

$$\{J_\alpha, J_{\tilde\alpha}\}_2 = J_{[\alpha,\tilde\alpha]}, \quad \{J_\alpha, M_\beta\}_2 = M_{[\alpha,\beta]},$$

$$\{J_\alpha, K_\gamma\}_2 = K_{[\alpha,\gamma]},$$

$$\{M_\beta, M_{\tilde\beta}\}_2 = K_{[\beta,\tilde\beta]}, \quad \text{otherwise zero}$$

Changing polarizations

We may realize $\mathcal{M} = T^*[1]M_1 = T^*[1]M_2$

Q: What is the relation between the P_∞-structures associated to M_1 and M_2?

<u>Conjecture</u> They are "Morita" equivalent

<u>Idea</u> BF^2V on $\mathcal{F}^\partial_\Sigma \rightsquigarrow$ consider $\mathcal{F}^\partial_{\Sigma \times I}$

GR in 4d (Palatini-Cartan) produces something related!

In the PC version the metric is replaced by a

cotrame e

- M 4-manifold that admits a Lorentzian structure

- $\begin{array}{c} V \\ \downarrow \\ M \end{array}$ a vector bundle isomorphic to TM endowed with a fiberwise Minkowski metric η

cotrame: $e : TM \xrightarrow{\simeq} V$ $\rightsquigarrow$ metric $g = e^* \eta = \eta(e,e)$

The theory is governed by the action functional

$$S_M[e,\omega] = \int_M \frac{1}{2} e e F_\omega + \frac{\Lambda}{24} eeee$$

Λ cosmological constant

Formally: $B := \frac{1}{2} ee \rightsquigarrow S^{BF}_M = \int BF_\omega + \frac{\Lambda}{6} BB$

On a 2d corner Σ one may define

$$J_\alpha := \int_\Sigma \frac{1}{2}\alpha\,(ee + \cdots) \qquad \alpha \in \Gamma(\Sigma, \Lambda^2 V|_\Sigma)$$

$$M_\beta := \int_\Sigma \beta\,(\iota_\xi e e + \cdots) \qquad \beta \in \Gamma(\Sigma, T^*\Sigma \otimes \Lambda^2 V|_\Sigma)$$

$$K_\gamma := \int_\Sigma \gamma\,(\iota_\xi e \iota_\xi e + \cdots) \qquad \gamma \in \Gamma(\Sigma, \Lambda^2 T^*\Sigma \otimes \Lambda^2 V|_\Sigma)$$

with ξ the ghost for diffeomorphisms

and the dots involve other fields

This produces the same brackets as for BF_4 in polarization 2

with $\mathfrak{g} = so(3,1)$

<u>Rem</u> The fiber of $\Lambda^2 V$ is isomorphic to $so(3,1)$

But one has to remember that e is nondegenerate

Setting $B := \frac{1}{2} ee + \cdots$, this can be imposed by

considering the Poisson submanifold

$$\left\{ B \in \Gamma(\Sigma, \Lambda^2 T^*\Sigma \otimes \Lambda^2 V|_\Sigma) \;\middle|\; B \neq 0 \text{ and } \underset{\underset{\Gamma(\Sigma, (\Lambda^2 T^*\Sigma)^{\otimes 2})}{\cap}}{Pf(B)} = 0 \right\}$$

GR is not an AKSZ theory

Q: How do we get this result?

A: There is a general procedure

BFV on the boundary

$\Downarrow$

BF^2V on the corner

1) Classically, a field theory produces

- a symplectic space of boundary fields
- a coisotropic submanifold thereof

Ex (Electromagnetism) Boundary fields: Electric field E
Vector potential A

$$\omega^\partial = \int \delta E \cdot \delta A \sqrt{\det g}$$

$$C = \{ \operatorname{div} E = 0 \}$$

Gauss law

General procedure [Kijowski–Tulczijew]

Local action functional S

δS yields

i) EL equations

ii) "Noether" 1-form $\tilde{\alpha}$

(e.g. $p\,dq$, $\int \vec{A}\cdot\delta\vec{E}\,\sqrt{\det q}$, ...)

boundary terms

Next a) define $\tilde{\omega}^\partial := \delta \tilde{\alpha}^\partial$

b) if needed, reduce by $\ker \omega^\partial$
(i.e. vector fields Y s.t. $\iota_Y \tilde{\omega}^\partial = 0$)

$\rightsquigarrow$ get ω^∂ on reduction

c) find the constraints: i.e., EL equations
w/ $\underline{\underline{\text{no}}}$ transversal components

Assume for simplicity constraints are first class

(φ^i) s.t. $\{\varphi^i, \varphi^j\} = f^{ij}_k \varphi^k$

some functions

Then [BFV]

i) Add ghosts c_i and antighosts b^i

gh# = $+1$ $\qquad$ -1

ii) $\omega^\partial \rightsquigarrow \omega^{BFV} = \omega^\partial + \sum_i \delta b^i \delta c_i$

iii) $S^\partial := \sum_i c_i \varphi^i + \cdots$ corrections in b

s.t. $\{ S^\partial, S^\partial \} = 0$ CME

<u>Thm</u> [BFV, Stasheff] 1) It is possible to find ...

s.t. CME satisfied

2) unique solution (up to)

3) in good cases: $H^0_Q \cong C^\infty(\underline{\Sigma})$ as Poisson algebra

$Q = \{ S^{BFV}, \cdot \}$ $\quad \Sigma = \{ \varphi^i = 0 \}$

2) Boundary to corner:

- Given BFV on Σ $(\partial\Sigma = \emptyset)$,

one can extend it to $\partial\Sigma \neq \emptyset$

- By a procedure similar to KT one gets

"BF^2V" data on $\mathcal{F}^{\partial\partial}$ = "fields on $\partial\Sigma$"

$\hookrightarrow$ $\omega^{\partial\partial}$, $S^{\partial\partial}$ $\quad$ gh $\omega^{\partial\partial} = 1$

gh $S^{\partial\partial} = 2$

s.t. • $\delta \omega^{\partial\partial} = 0$ (sometimes one can reduce by its kernel)

• $\exists\, Q^{\partial\partial}$: $\iota_{Q^{\partial\partial}} \omega^{\partial\partial} = \delta S^{\partial\partial}$

and $Q^{\partial\partial}$ is the projection of Q^{∂}

• some other conditions

Then $S^{\partial\partial}$ + polarization $\rightsquigarrow$ Poisson structure
(possibly up to homotopy)

- In AKPZ theories, this produces the same results as with the procedure discussed above

- For general relativity, this produces the result announced above

Thanks

Bonus material

Cofrane gravity in 3+1 dimensions

Bulk data:

- M 4-manifold that admits a Lorentzian structure

- $\begin{array}{c} V \\ \downarrow \\ M \end{array}$ a vector bundle isomorphic to TM endowed with a fiberwise Minkowski metric η

Fields

1) coframe: $e : TM \xrightarrow{\sim} V$

In particular, $e \in \Gamma(T^*M \otimes V)$

2) connection ω on the orthonormal frame bundle of V

The adjoint bundle may be identified with $\Lambda^2 V$ ("skew-symmetric matrices")

Locally, ω is a 1-form with values in $\Lambda^2 V$

Classical description of the boundary

- Fields: e, ω

- pre-symplectic form

$$\Omega = \delta \int \frac{1}{2} e e \delta\omega$$

kernel: $\omega \rightsquigarrow \omega + v$ with $ev = 0$

- constraints: $eF = 0, \; d_\omega e = 0$

Now refine the space of bulk fields

$$\mathcal{F}_M = \{(e,\omega) : \iota_\partial^* e \rightsquigarrow \text{space-like metric}\}$$

i.e. $\iota_\partial : \partial M \longrightarrow M$

$g^\partial := (\iota_\partial^* e, \iota_\partial^* e)$ tensor on ∂M

Condition g^∂ is positive definite

In this case, on ∂M, we can choose $e_n \in \Gamma(V)$ s.t. (e, e_n) basis of V

Moreover, ① $\underset{\text{bulk}}{d_\omega e = 0} \Rightarrow \begin{array}{l} e\, d_\omega e = 0 \\ e_n d_\omega e = e\sigma \\ \quad \text{for some } \sigma \end{array}$ on the boundary

② $$\left\{ (e,\omega) \text{ on } \partial M \text{ s.t. } \begin{array}{l} e\, d_\omega e = 0 \\ e_n d_\omega e = e\sigma \\ \quad \text{for some } \sigma \end{array} \right\} \simeq \left\{ (e,\omega) \text{ on } \partial M \right\} \Big/ \sim$$

symplectomorphism

$\omega \sim \omega + v$
$ev = 0$

$$\mathcal{F}^{\partial}_{\partial M} \qquad \omega^{\partial} = \int_{\partial M} e\,\delta e\,\delta\omega$$

Now on $\mathcal{F}^{\partial}_{\partial M}$ we still have to impose the constraints

$$\begin{cases} eF_\omega = 0 \\ ed_\omega e = 0 \end{cases}$$

They are 1st class, so the BFV construction is possible

From the BFV construction on $\Sigma = \partial M$

we then derive the BF^2V construction on $\Gamma = \partial \Sigma$

Einstein-Hilbert version

$$S = \int_M \sqrt{g}\, R + \Lambda \sqrt{g}$$

ADM decomposition near ∂M (with assumption: $g_{\partial M}$ nondegenerate)

yields $\mathcal{F}^\partial_{EH}$, ω^∂_{EH}, $\underbrace{\mathcal{C}_{EH}}_{\substack{\| \\ \{\text{constrained fields}\}}}$ momentum and Hamiltonian constraints

- $\underline{\mathcal{C}_{EH}} \simeq \widetilde{\mathcal{C}_{PC}}$ ← Palatini–Cartan
- $\mathcal{C}_{EH}$ may be described in terms of BFV

But $BFV \rightsquigarrow BF^2V$ is a mess in this case

because the constraints contain second derivatives