Center for
Quantum&
Topological
Systems

Initial Researchers’ Meeting

NYU Abu Dhabi, 13-14 Sept 2022

Urs Schreiber: **“Motivation, Strategy & Technology”**

slides and further pointers at: ncatlab.org/nlab/show/CQTS#InitialResearcherMeeting-Schreiber

(1) –

(2) –

(3) –

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(1) – The Problem:

Practical Foundations of
Topological Quantum Computation

(2) –

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Motivation/Claim: What
Atomic/Nuclear Physics was to math of 1st 1/2 of 20th century,
Quantum Computation is to Cohesive Linear Homotopy Theory.

Topological Quantum Computation

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Cohesive Linear Homotopy for
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More indication of **technical details**.

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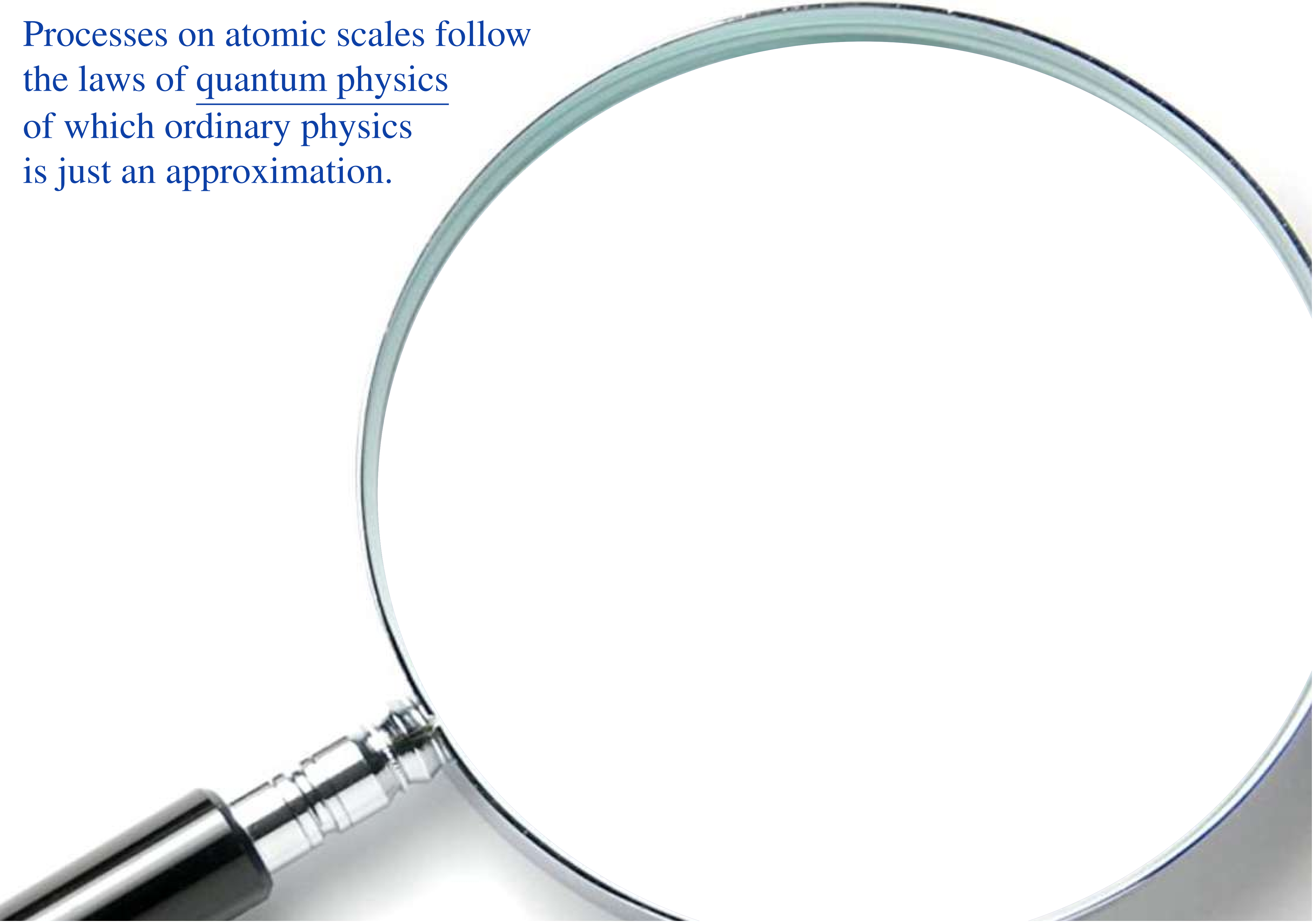

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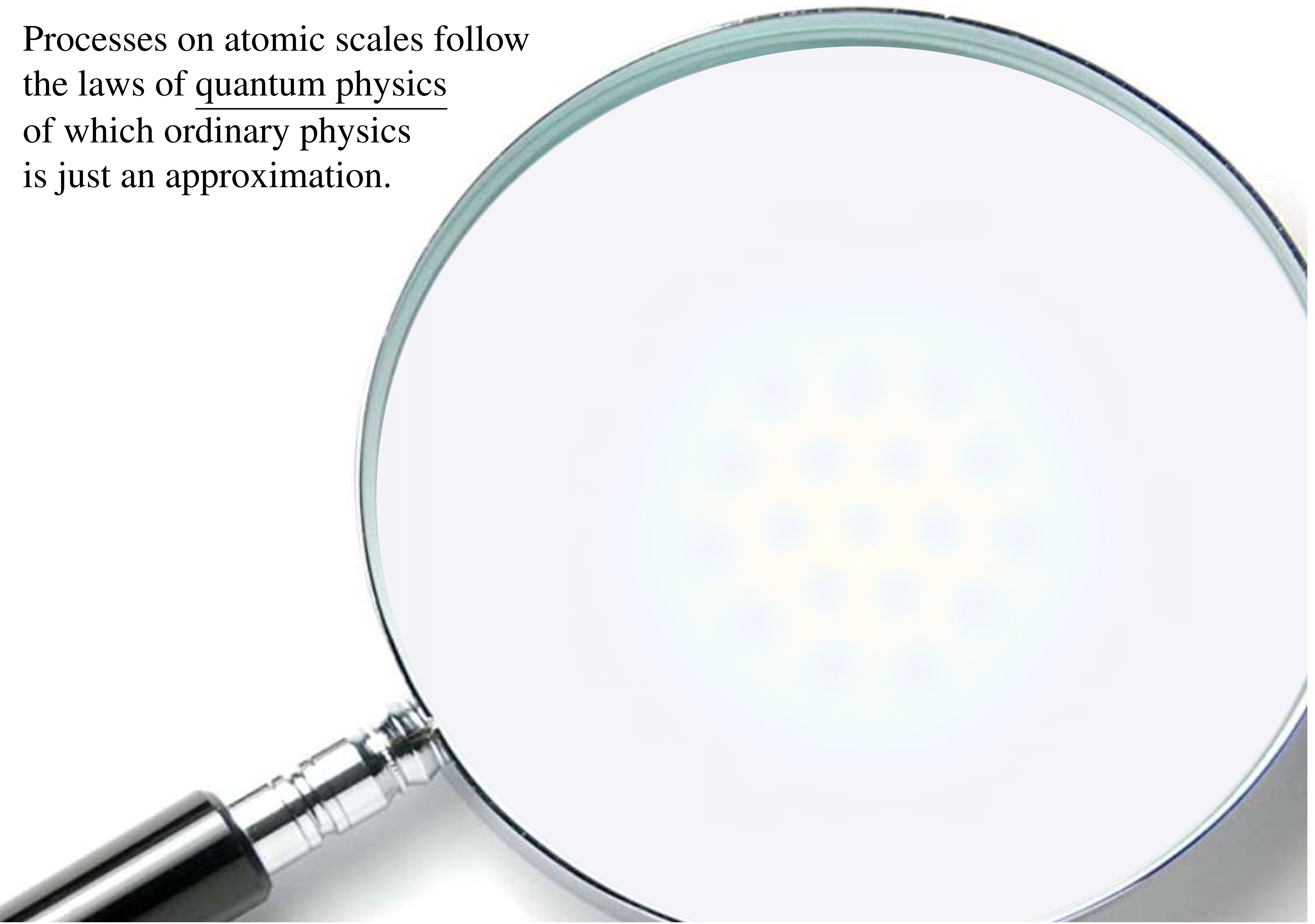

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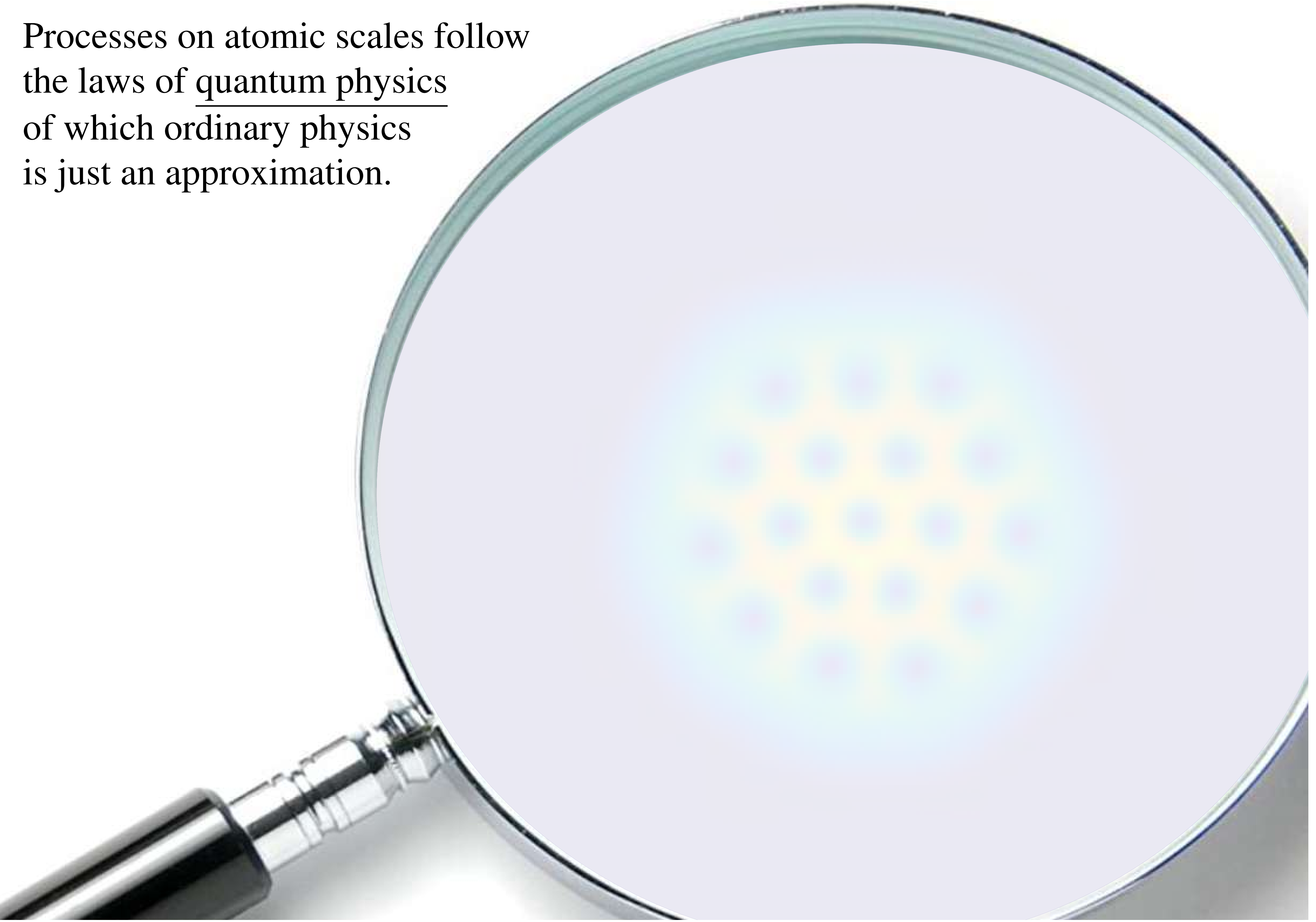

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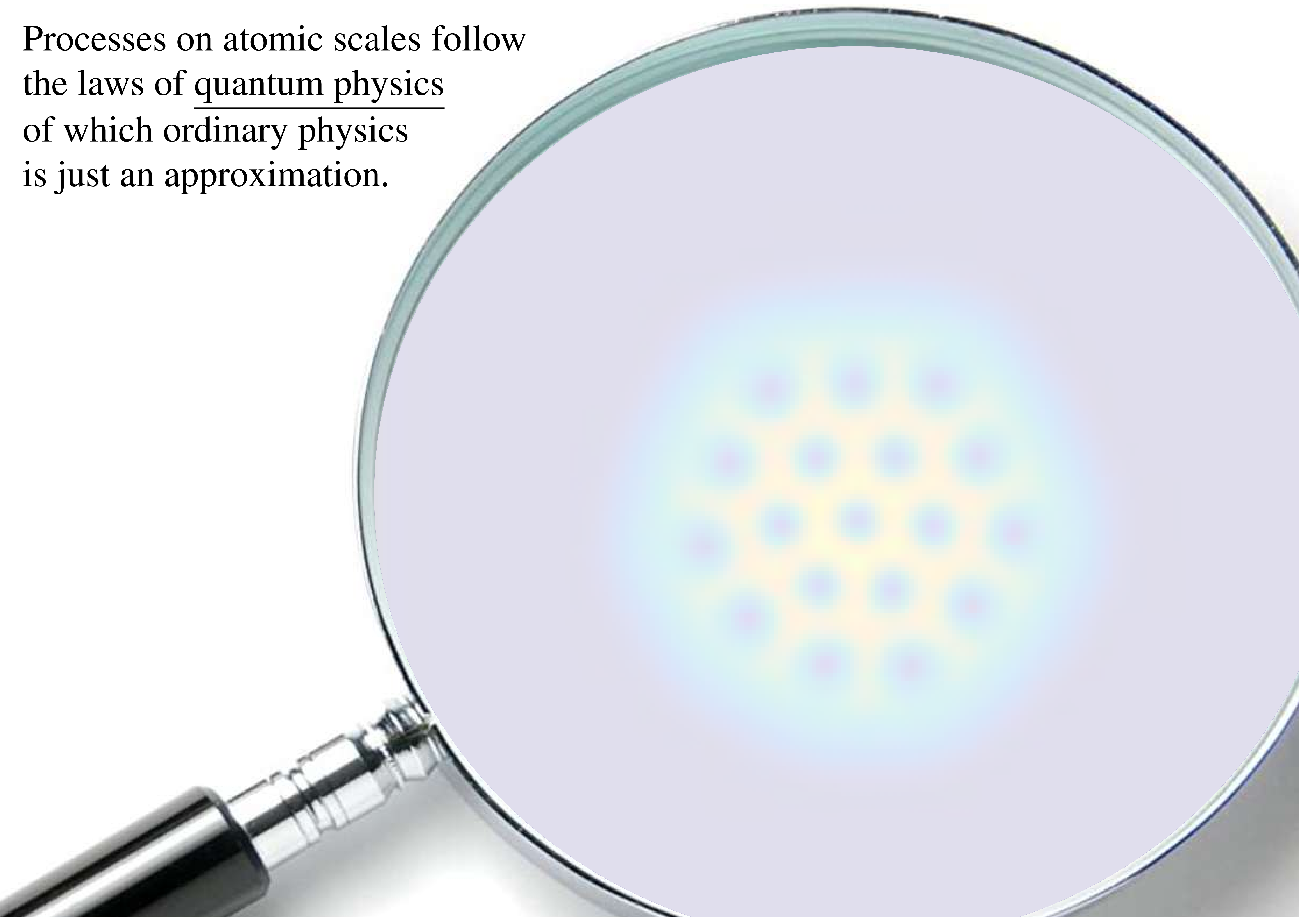

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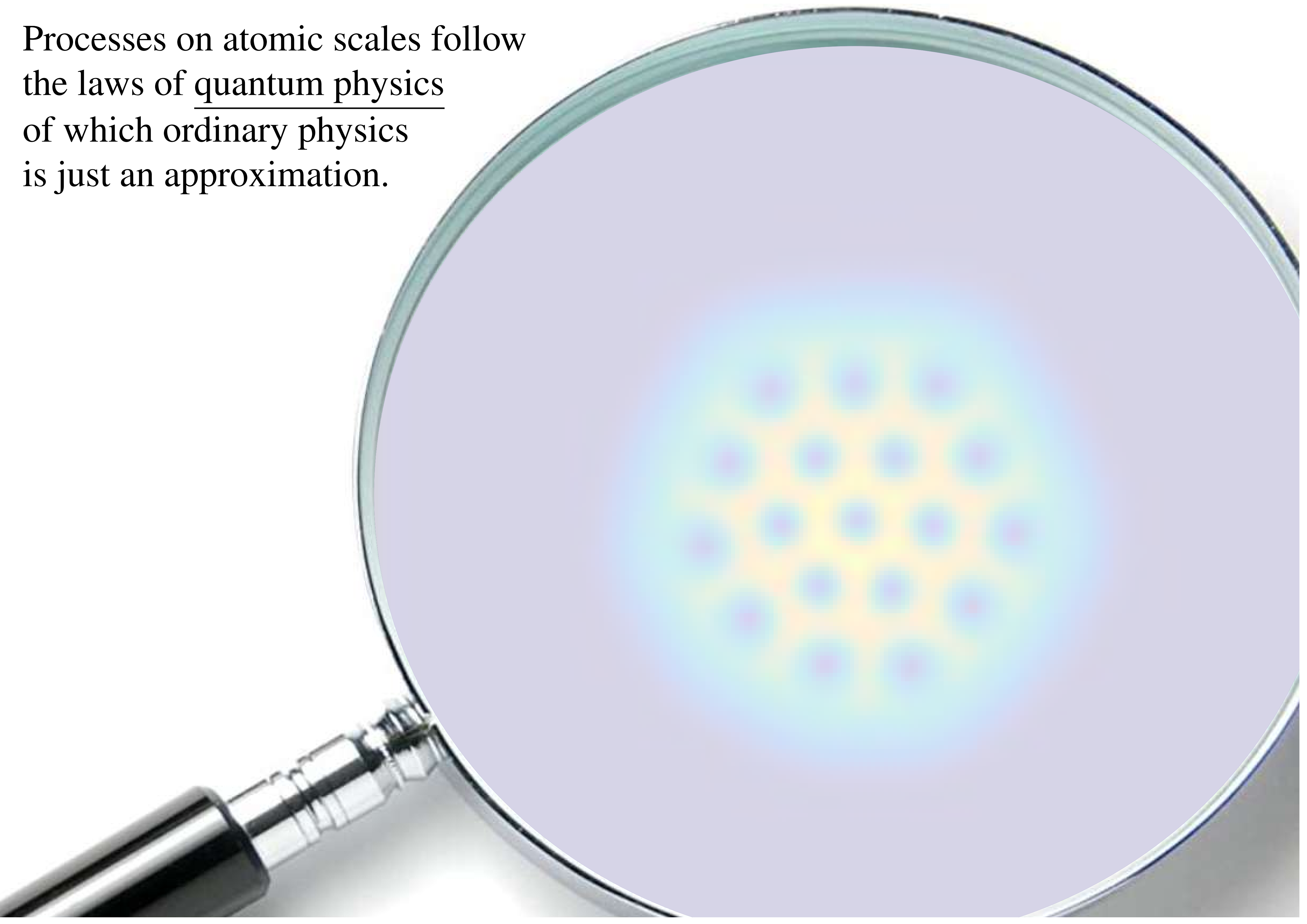

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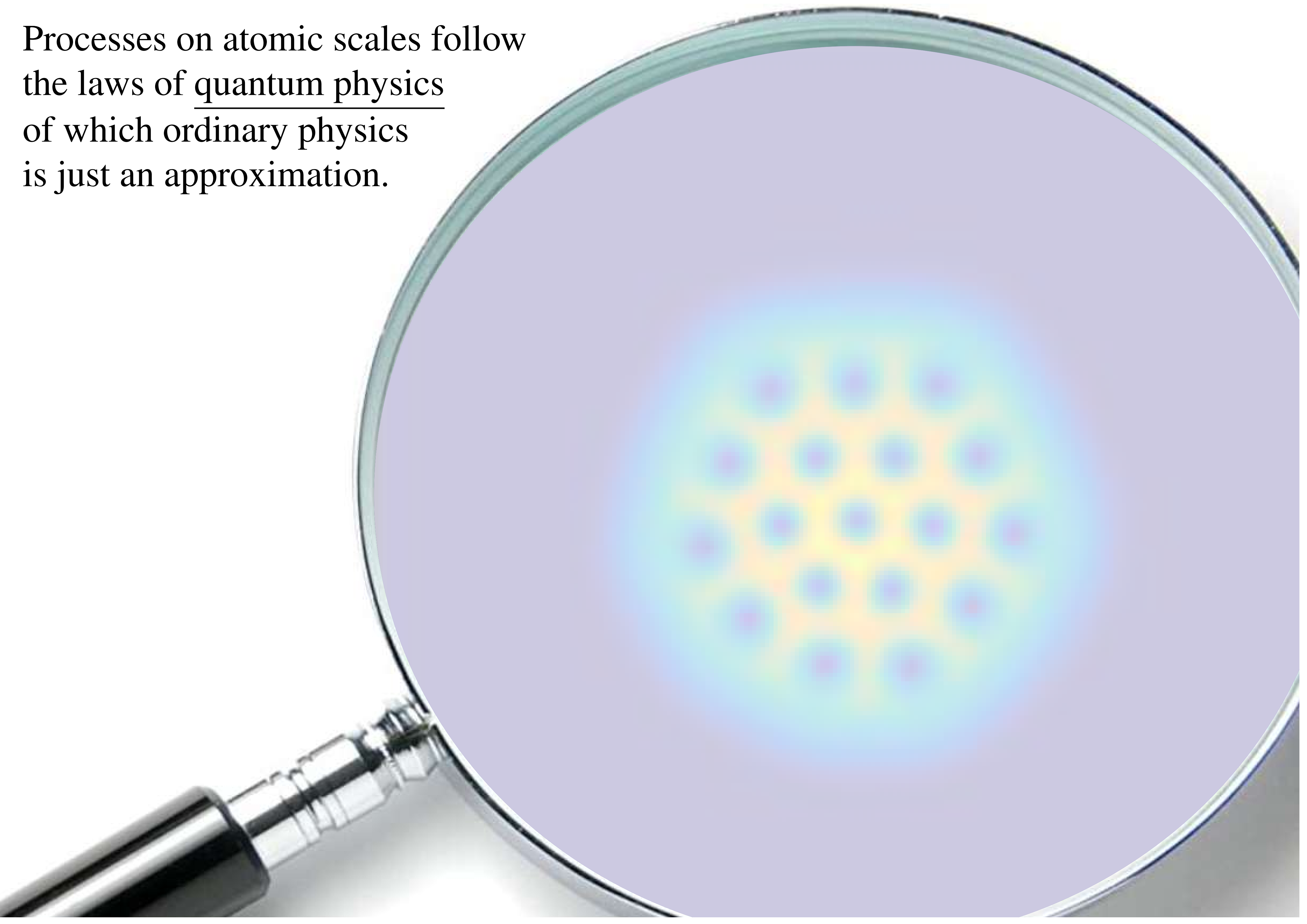

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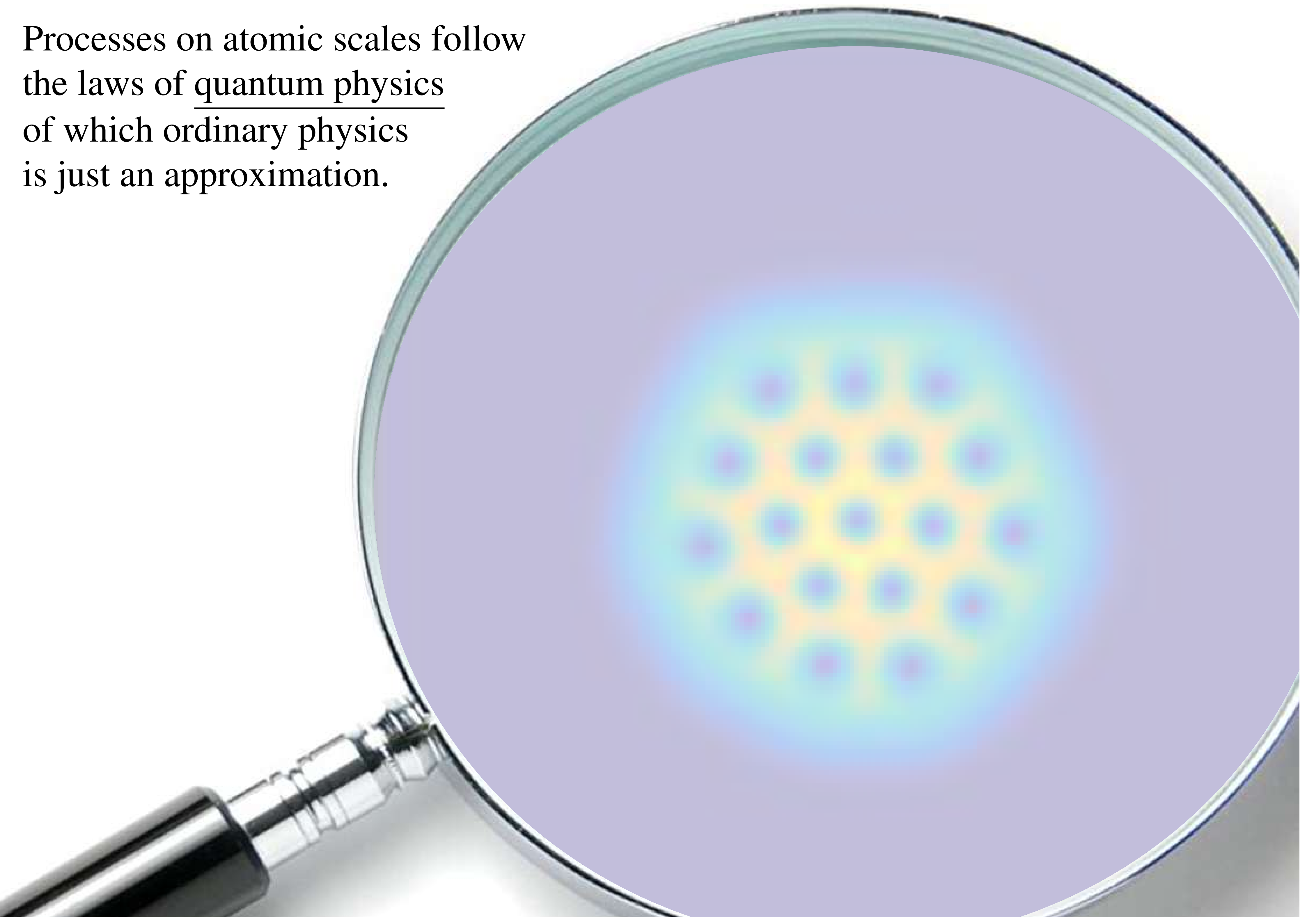

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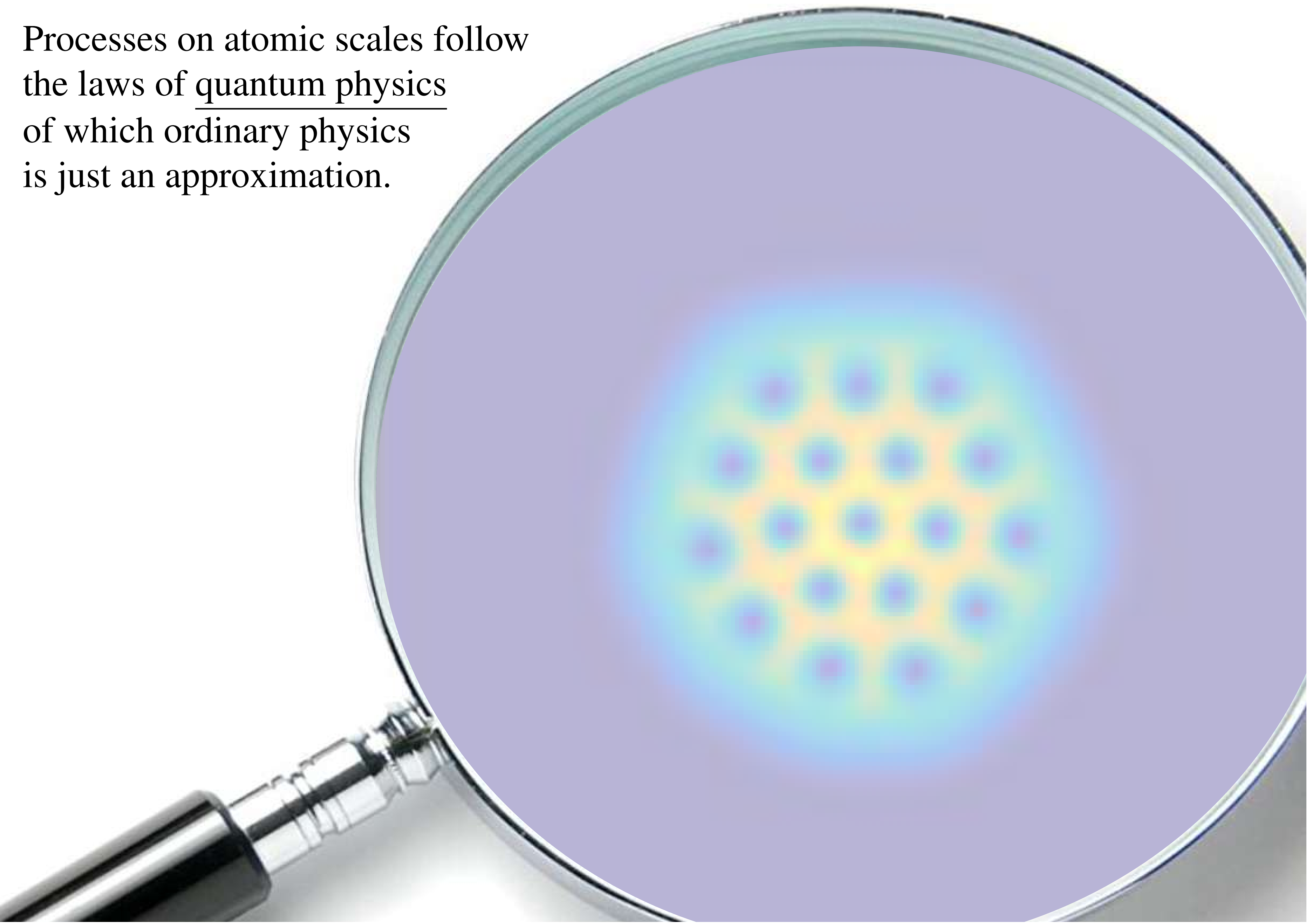

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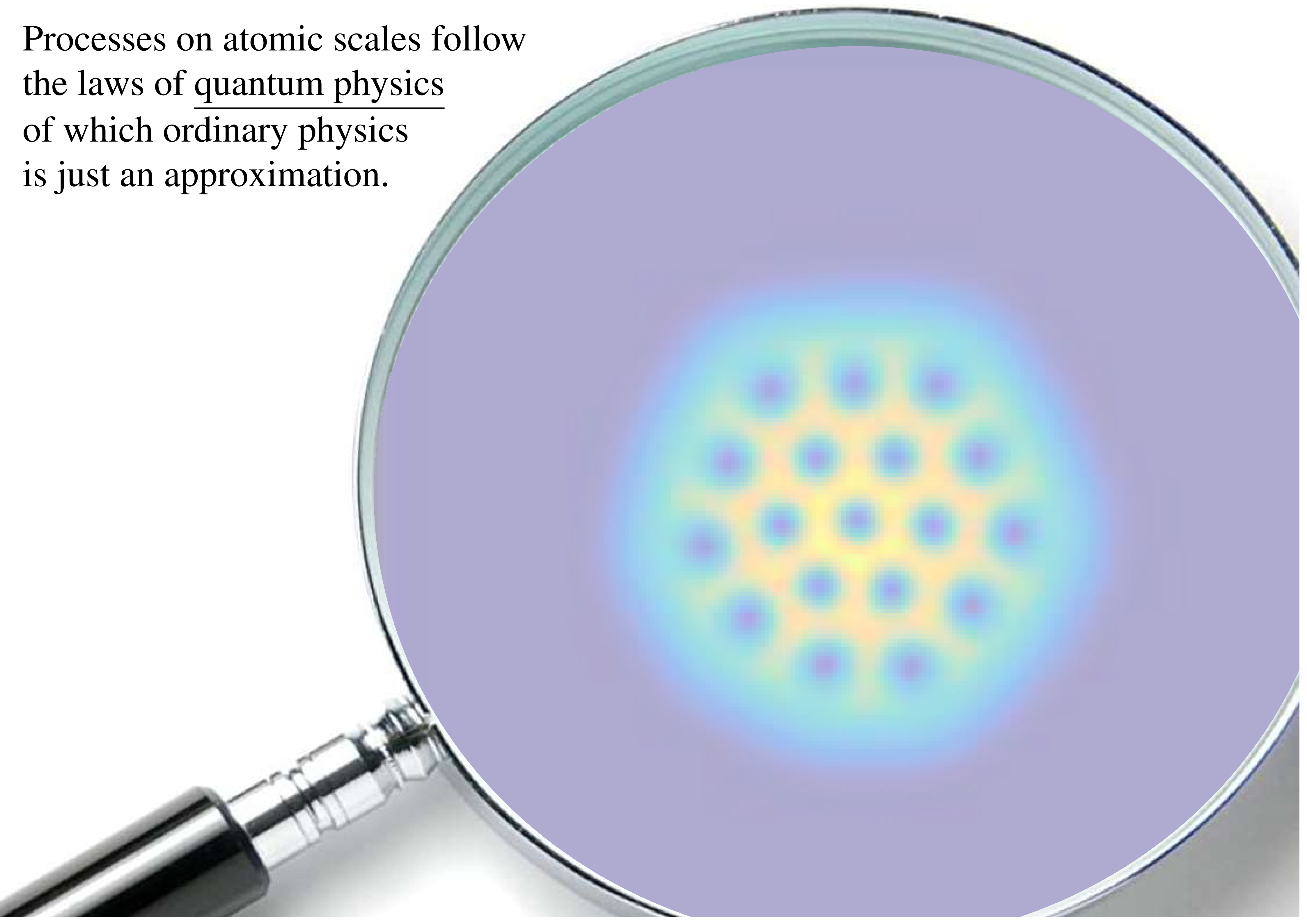

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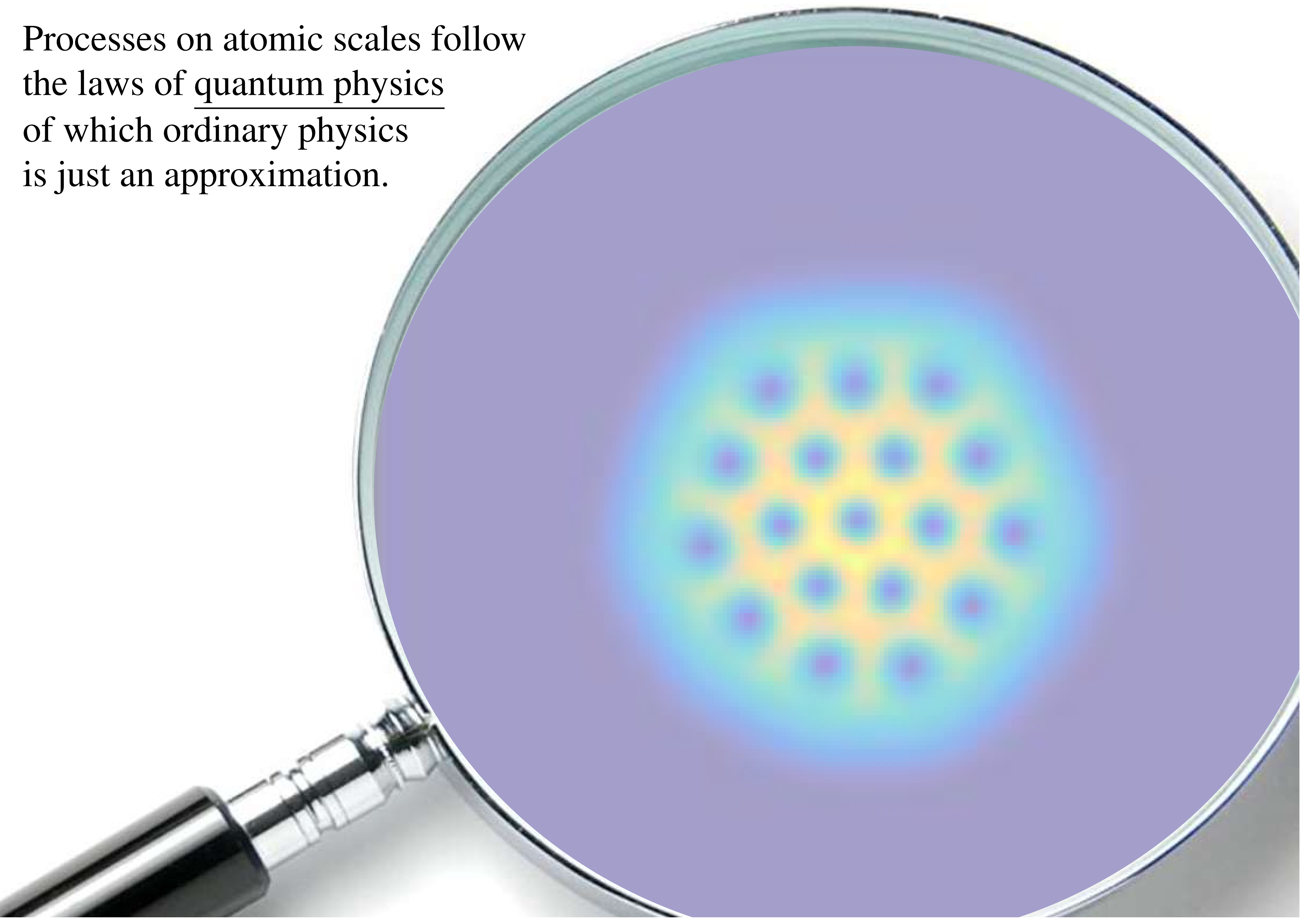

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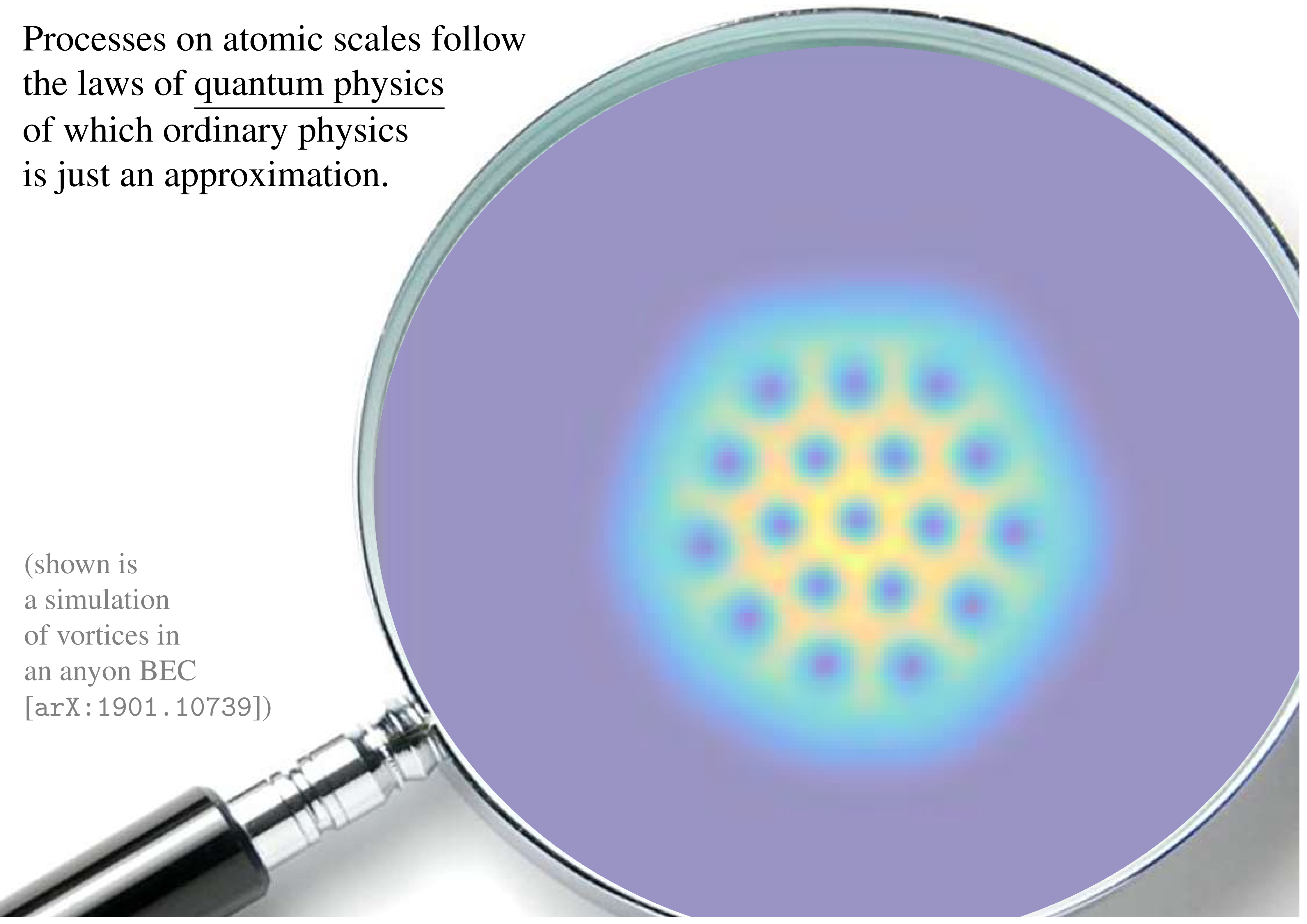

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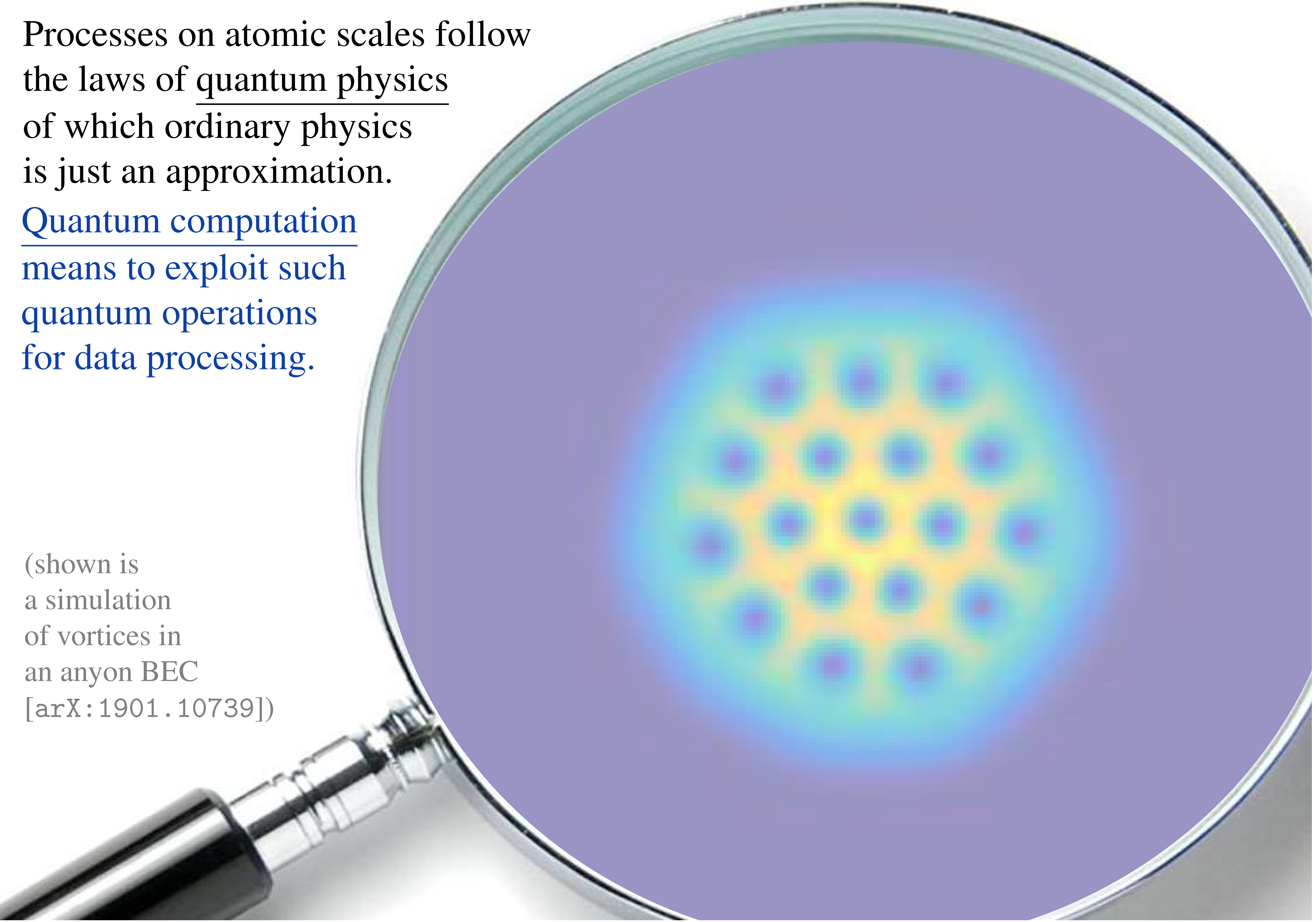

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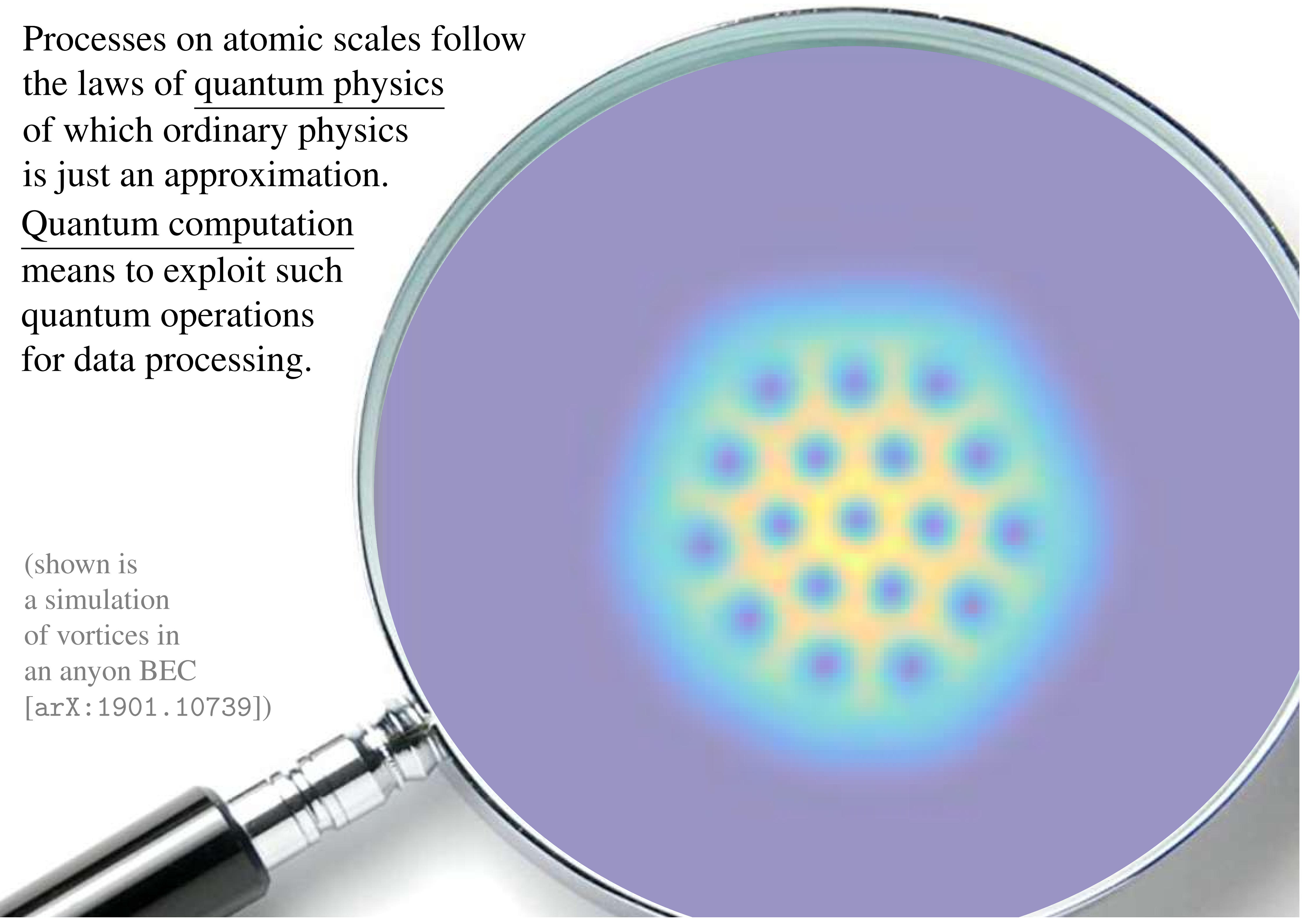

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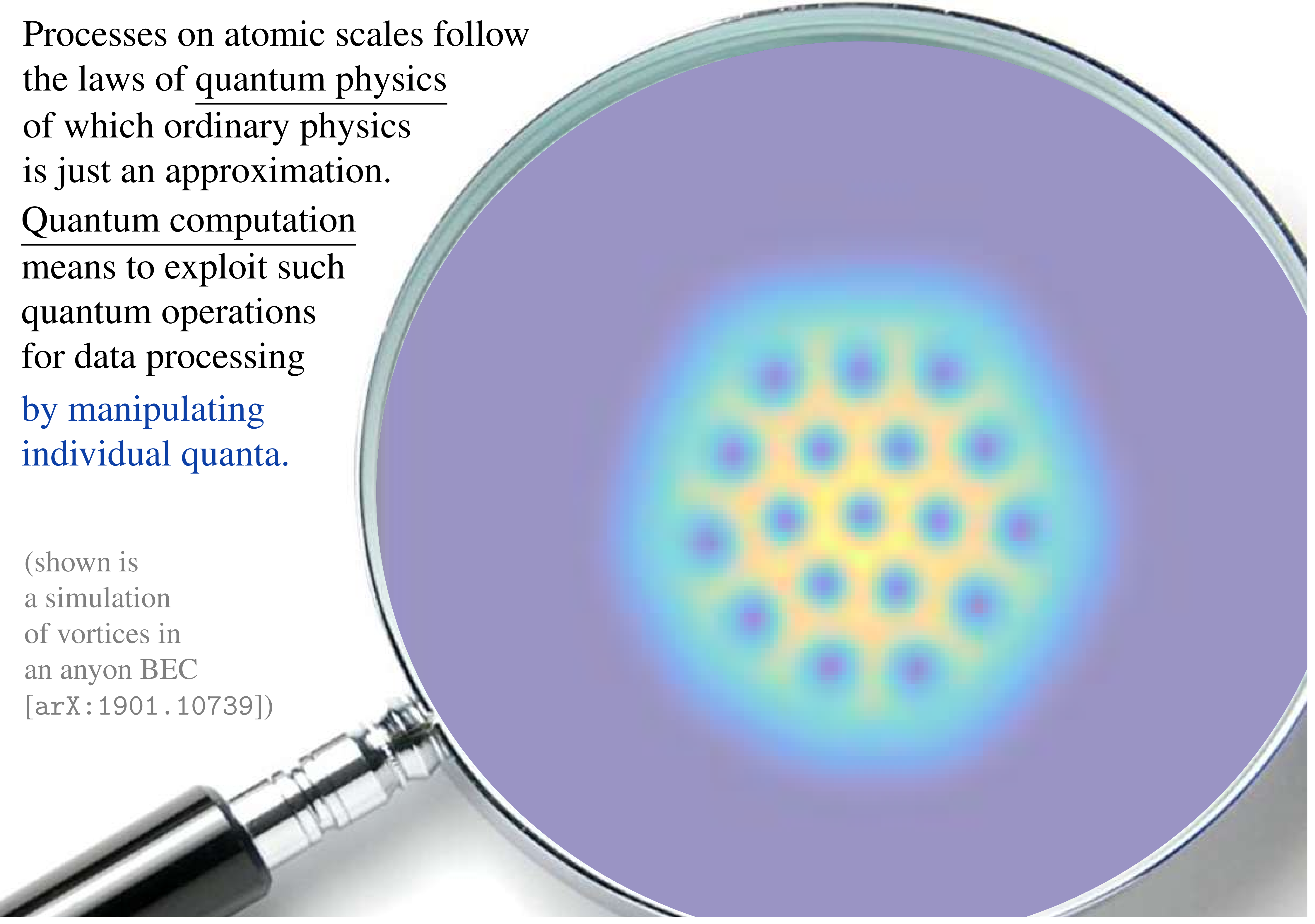

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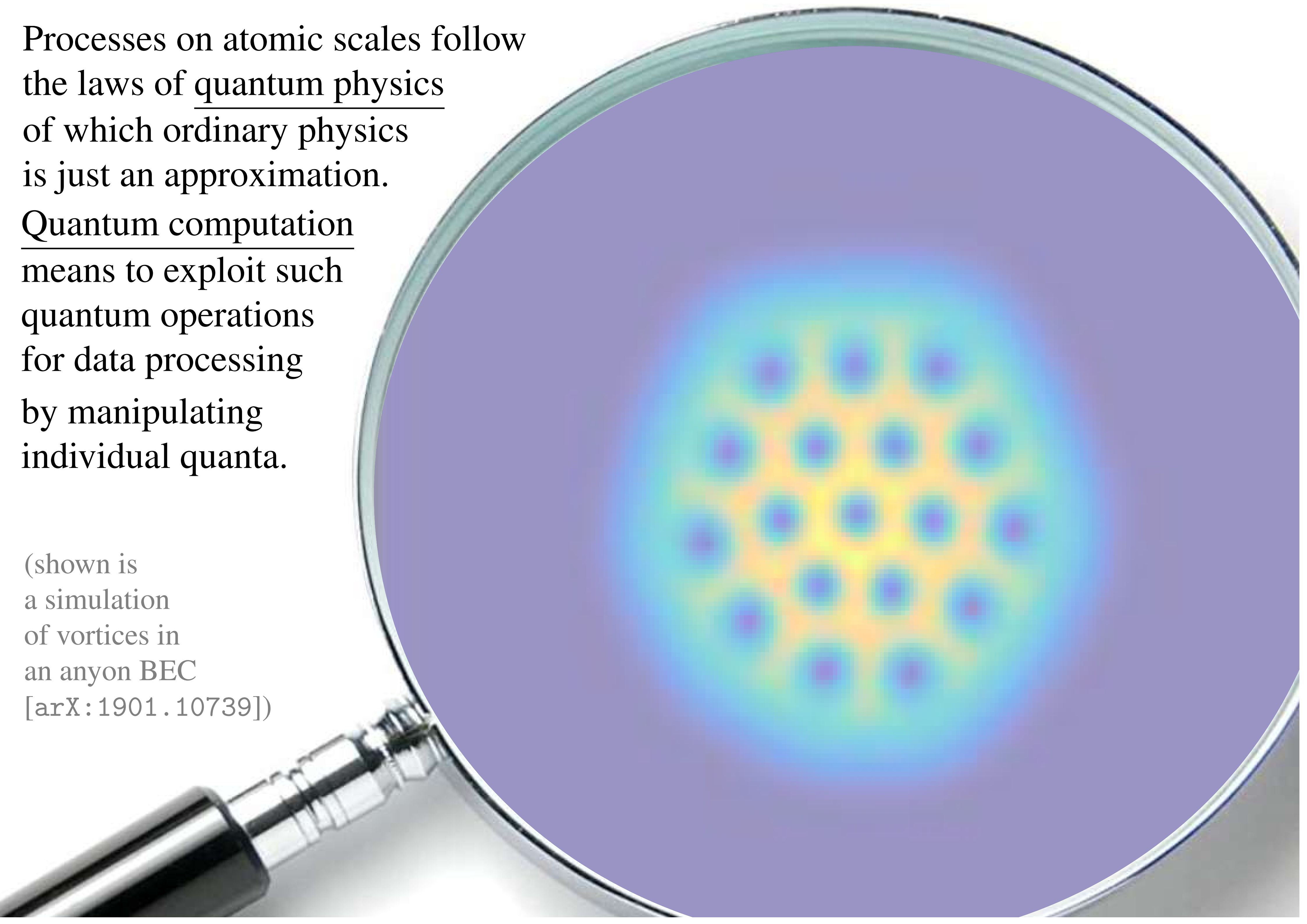

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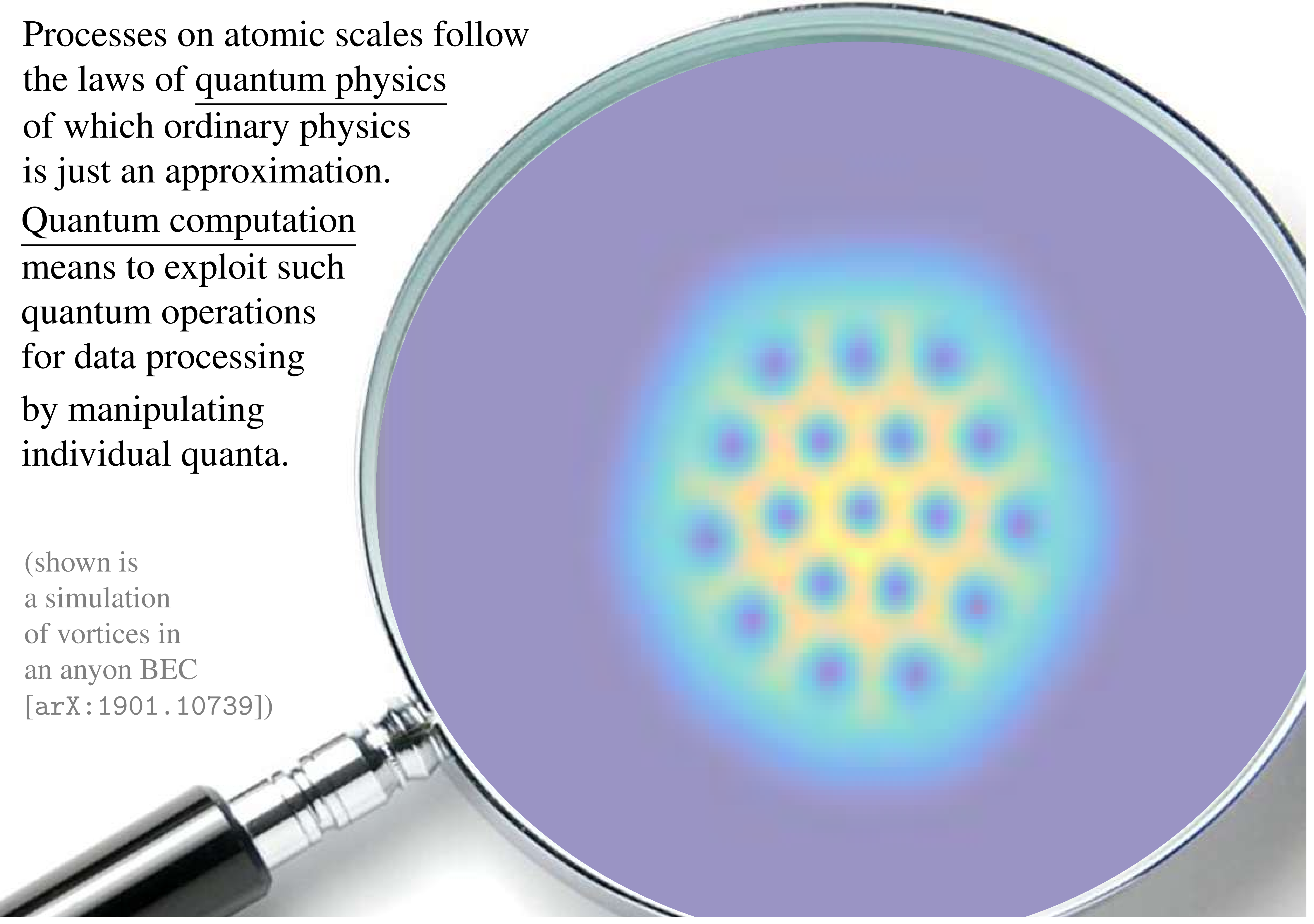

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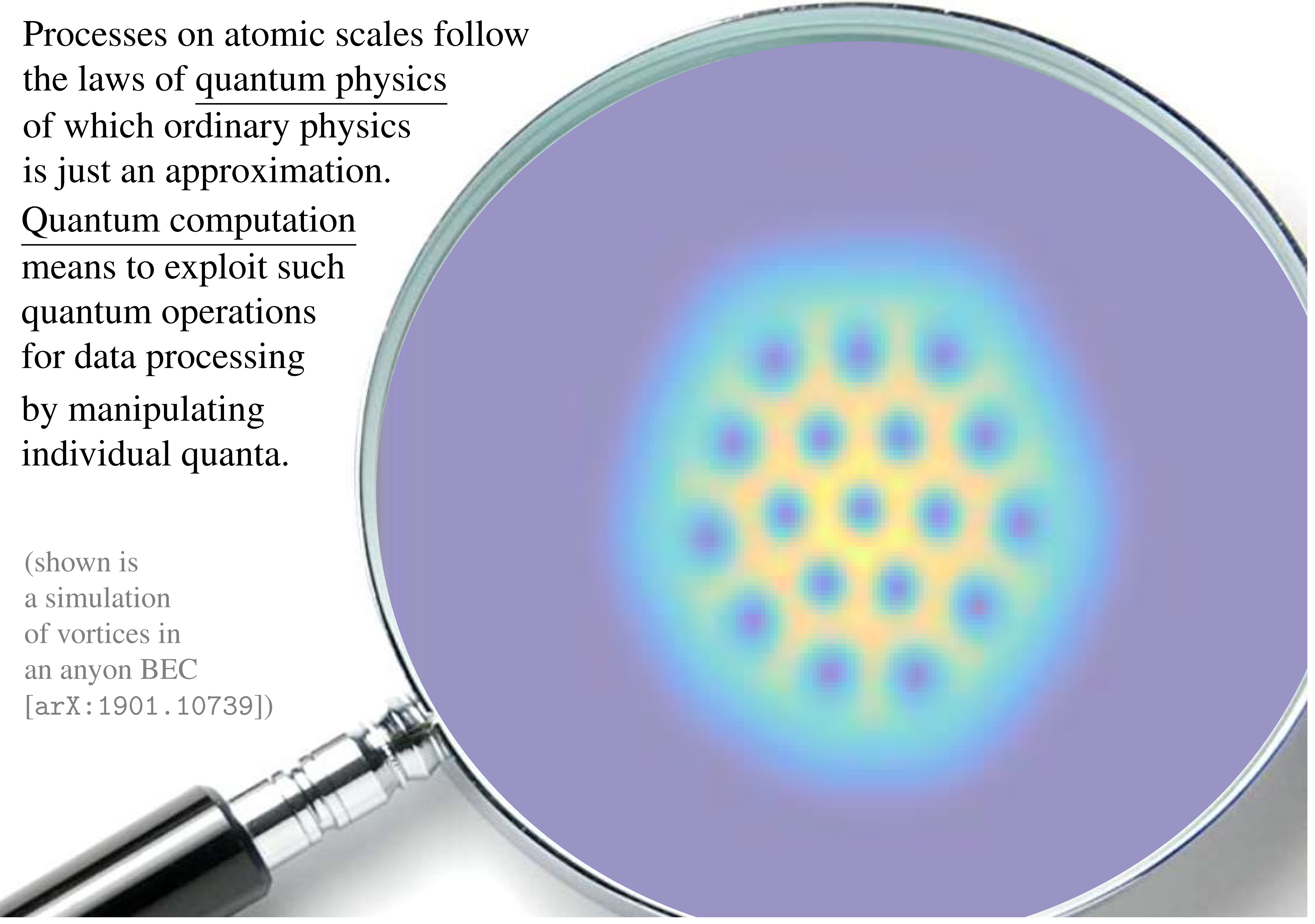

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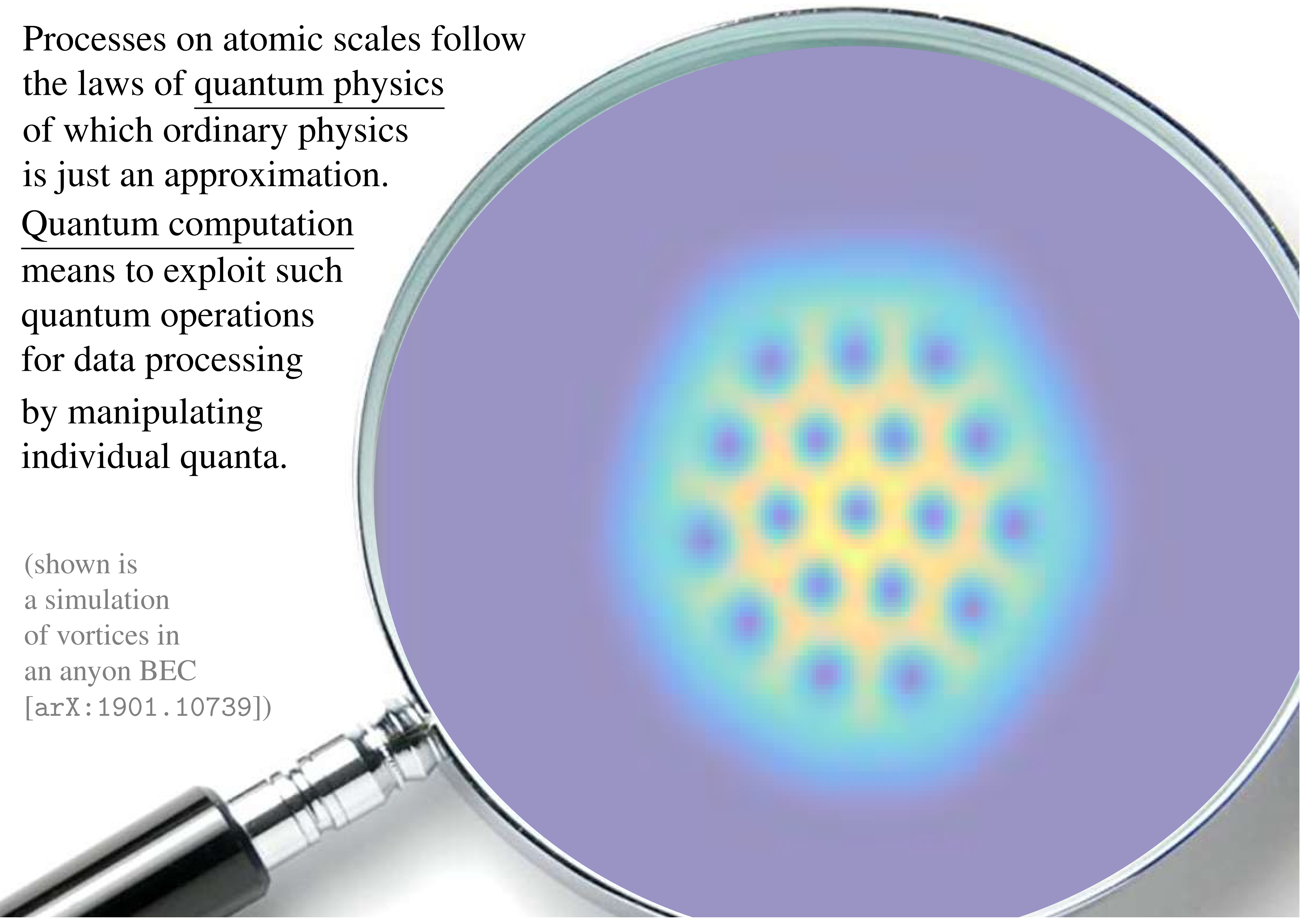

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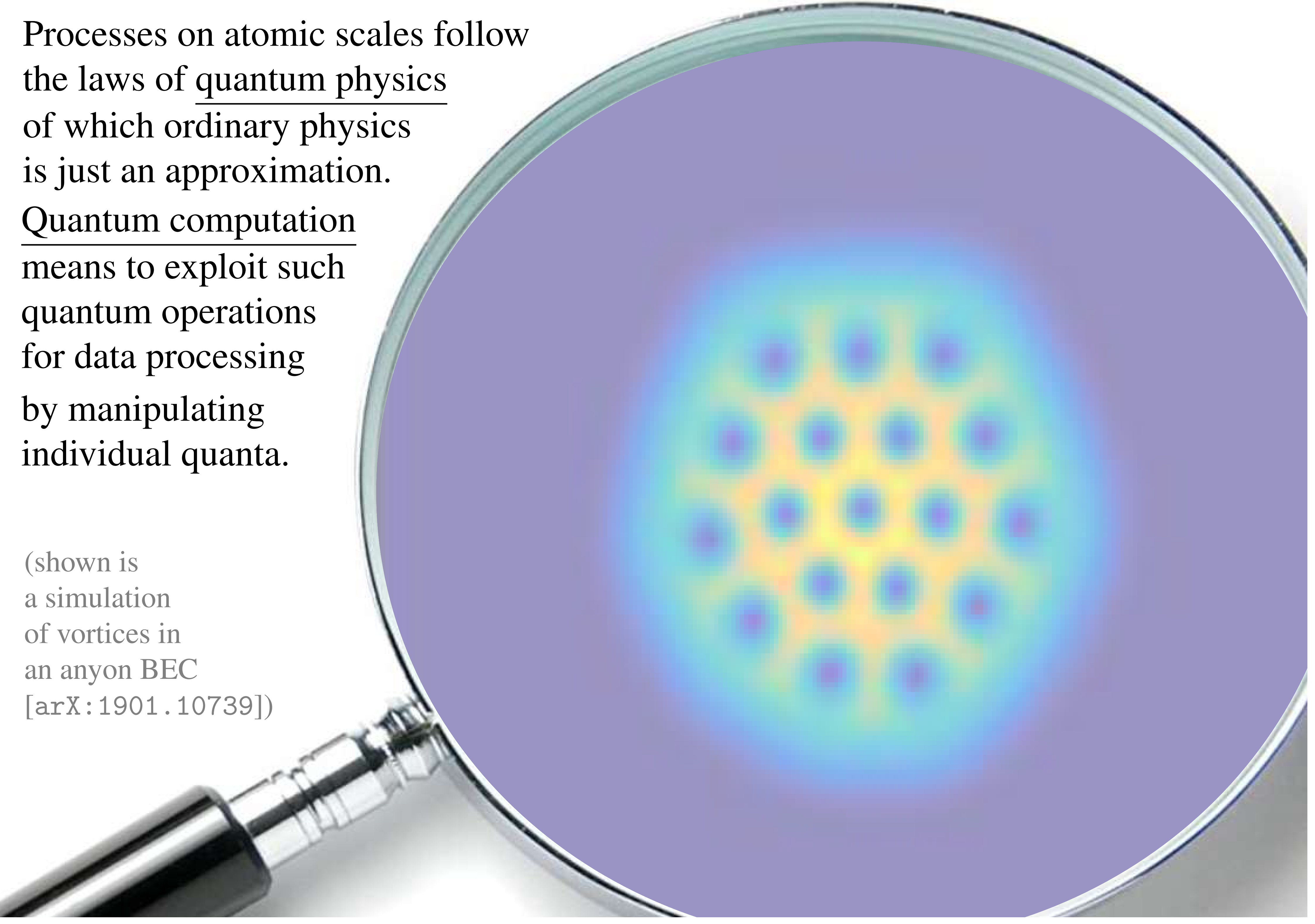

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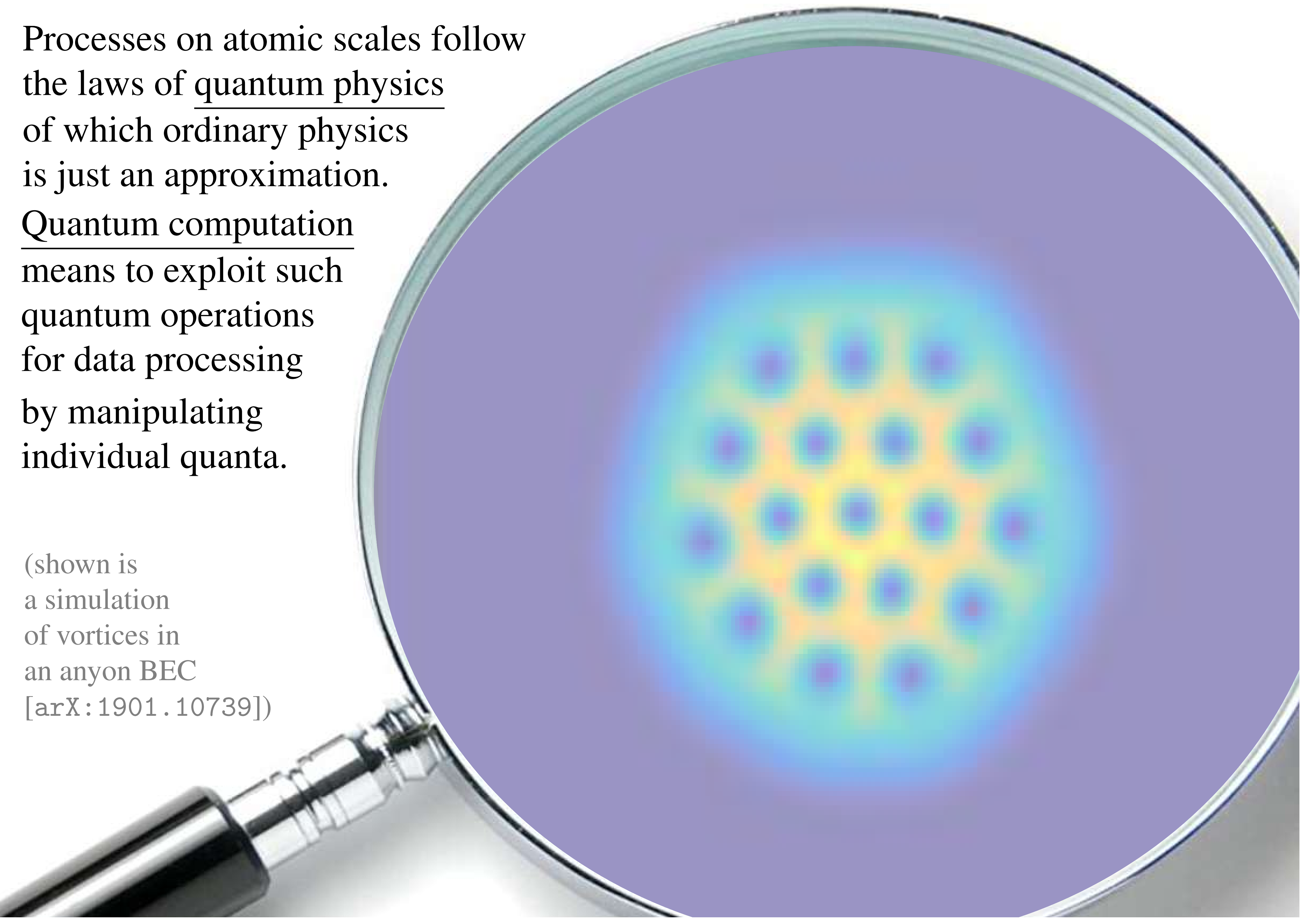

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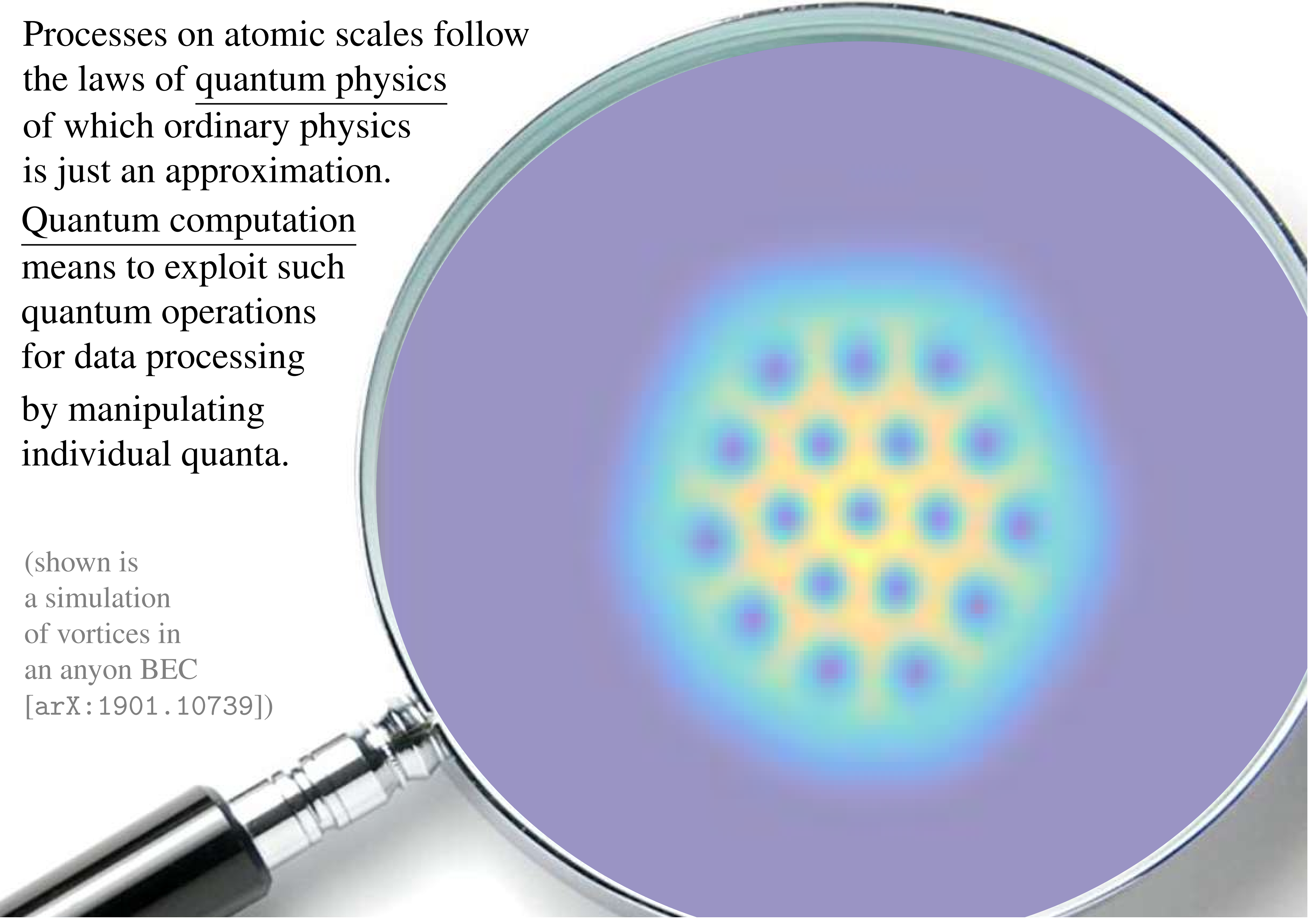

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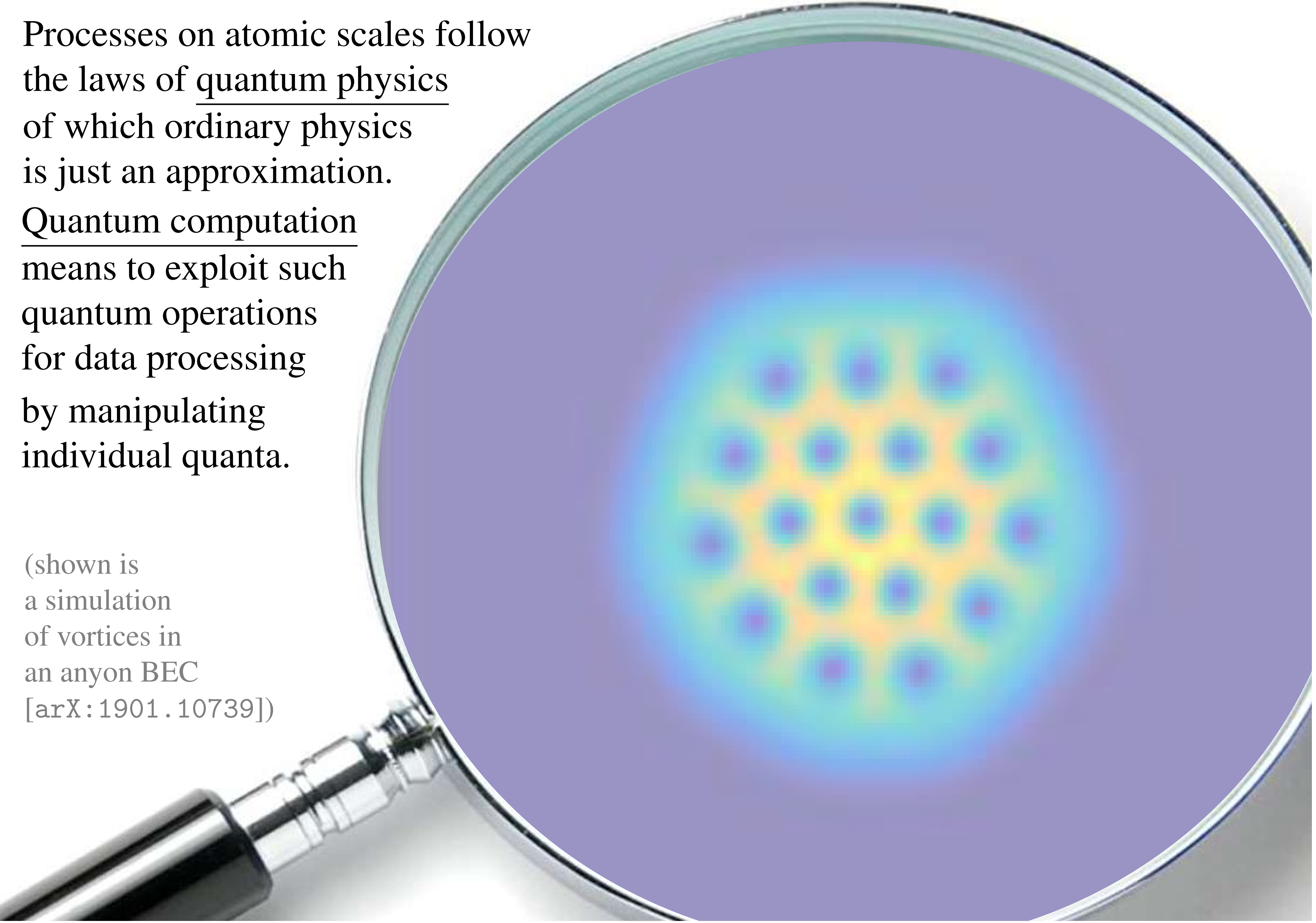

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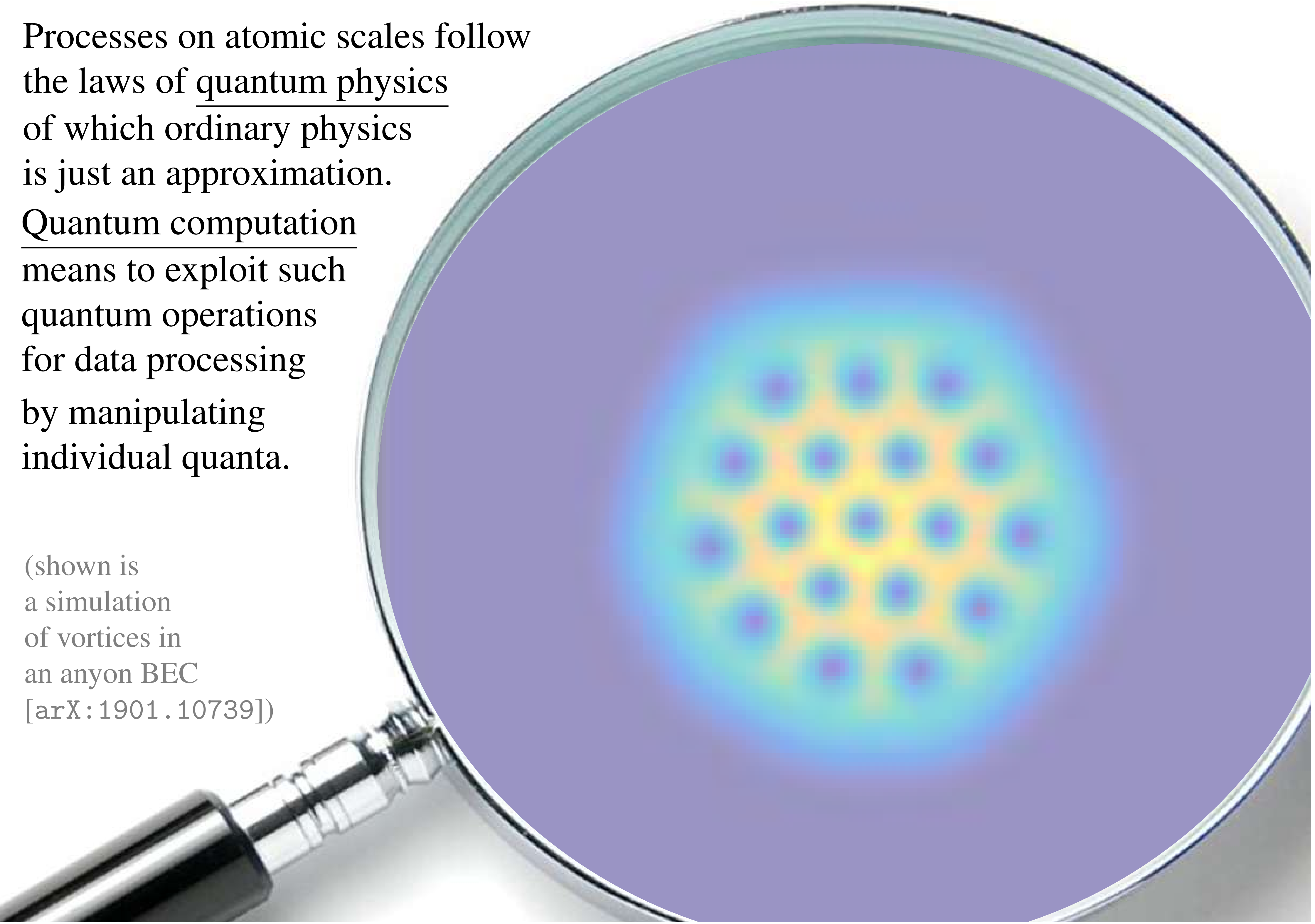

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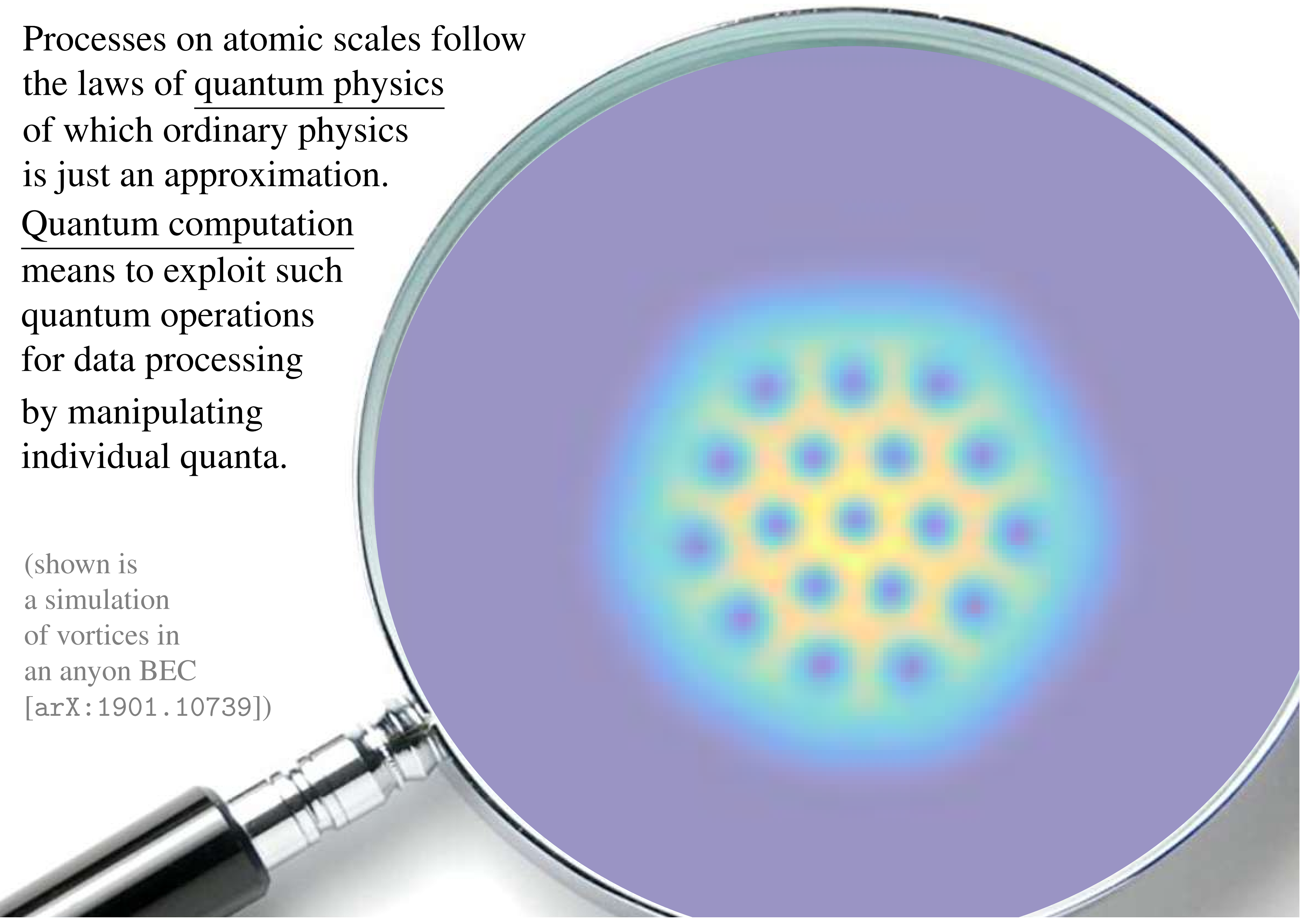

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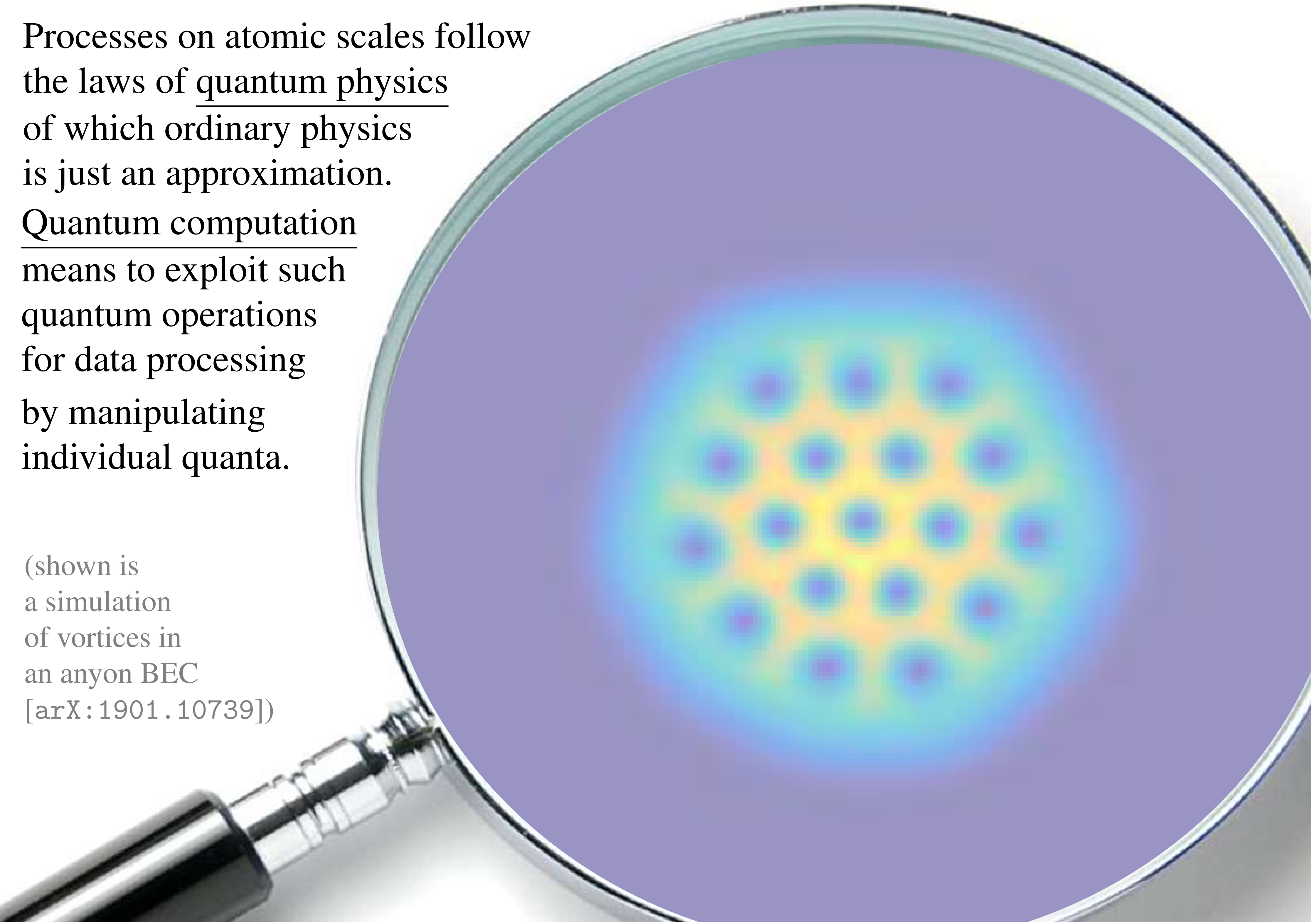

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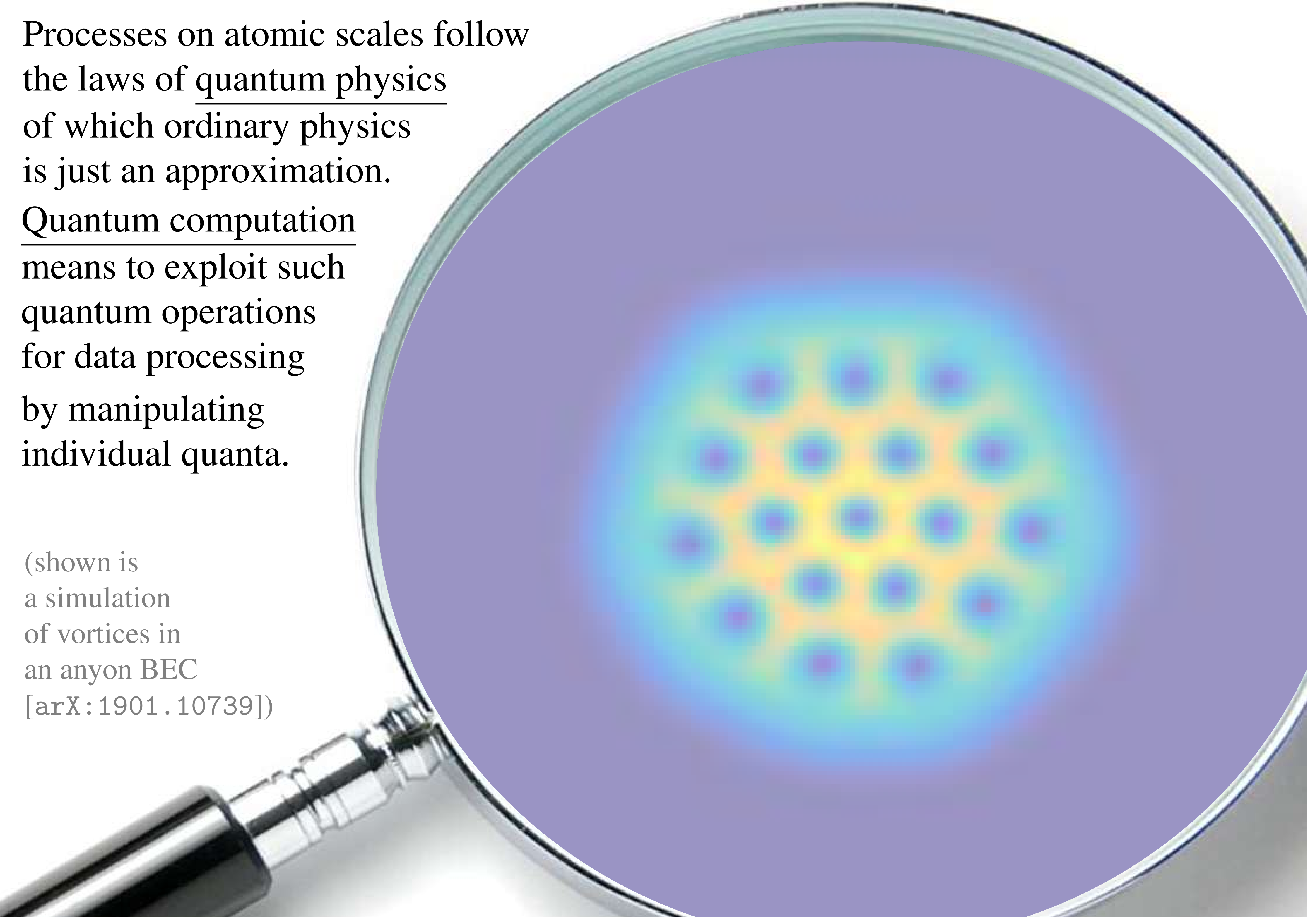

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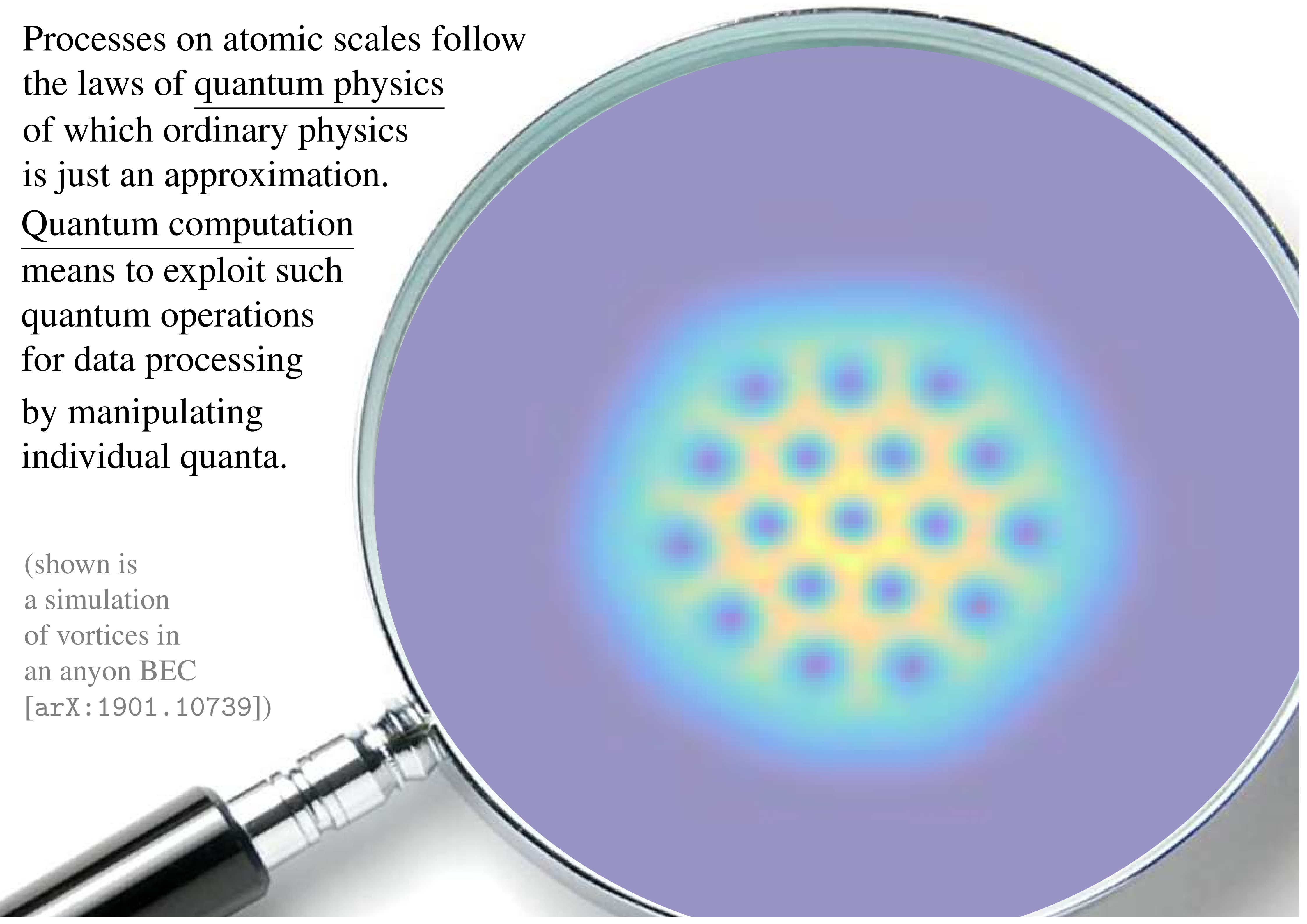

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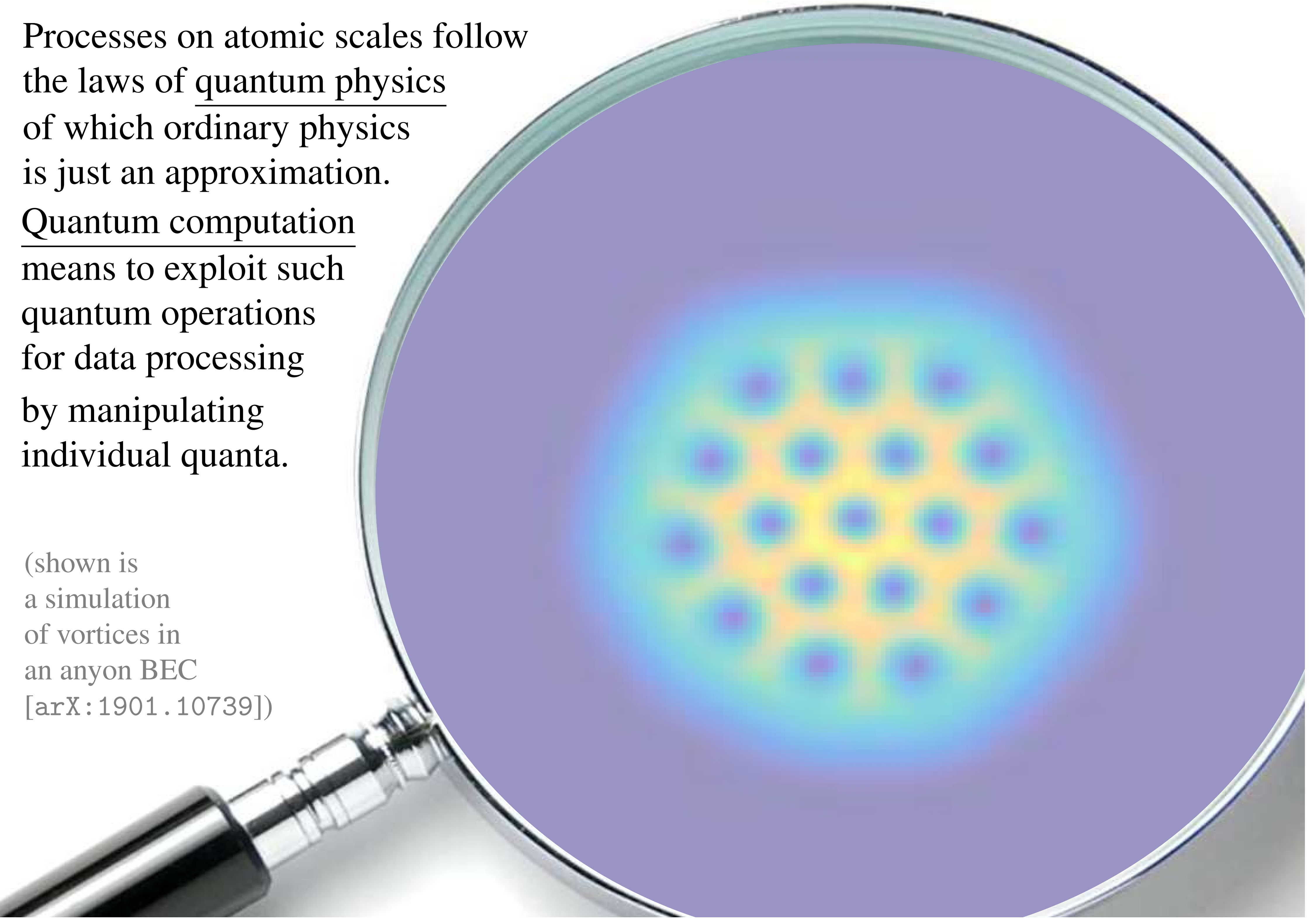

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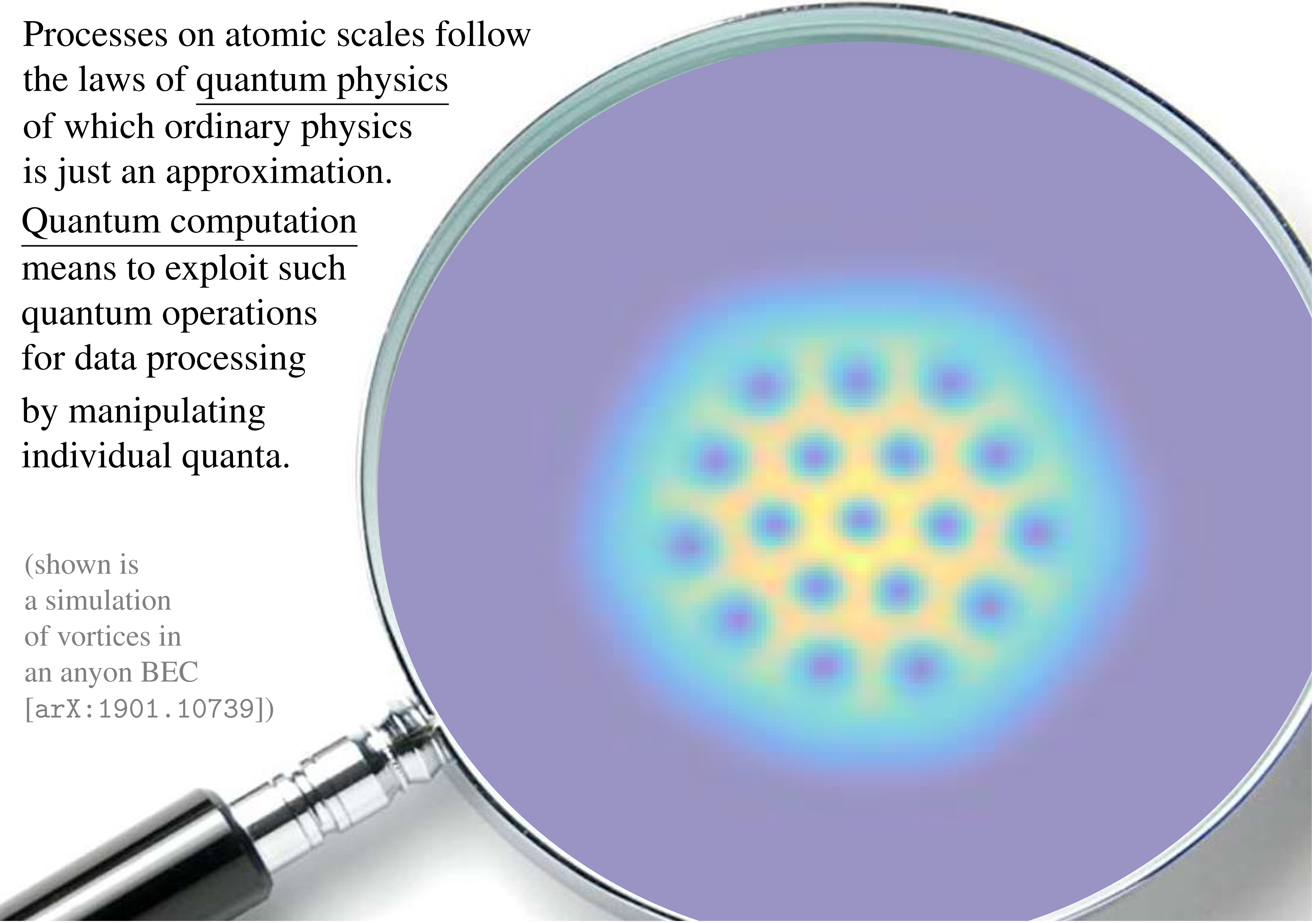

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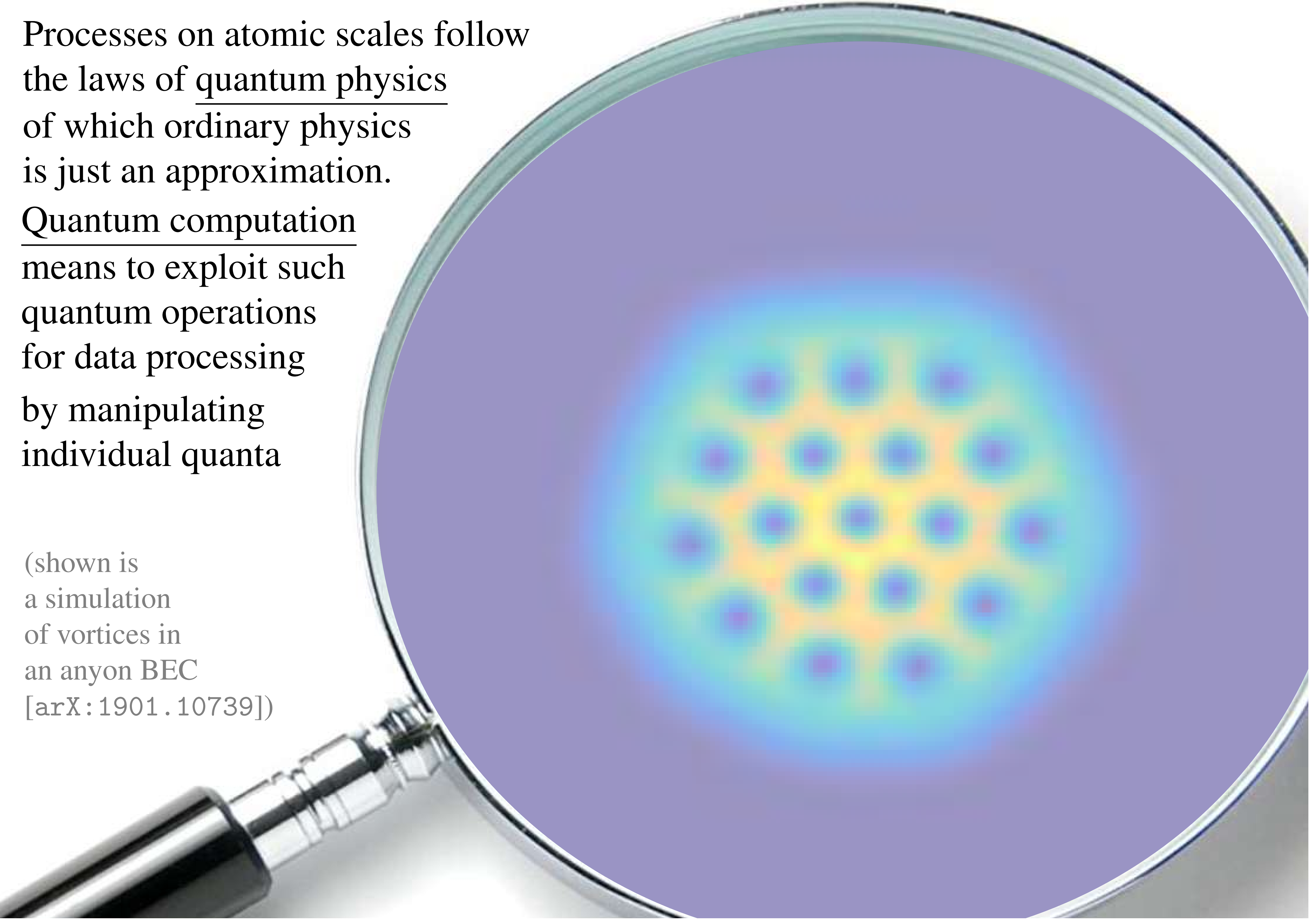

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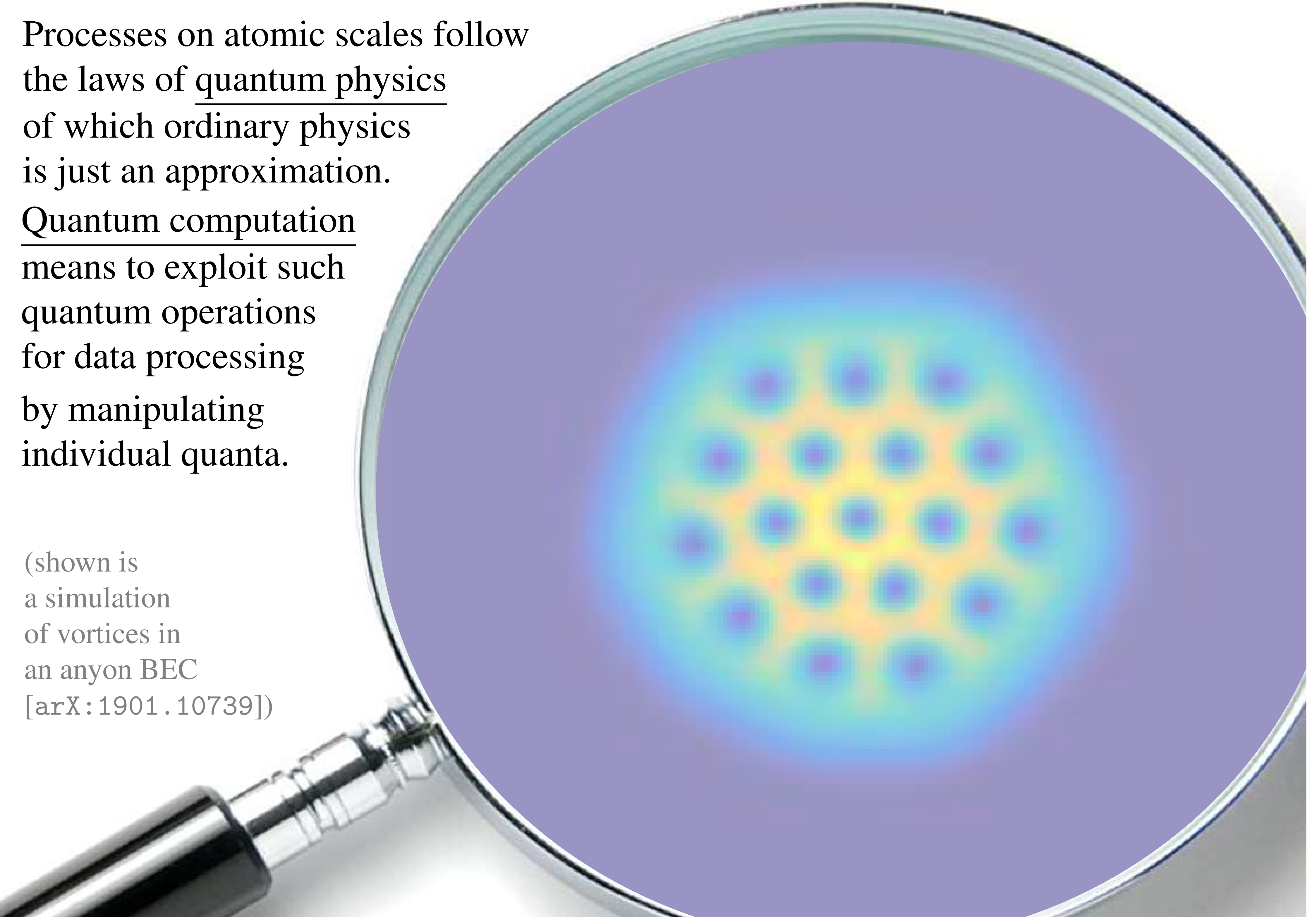

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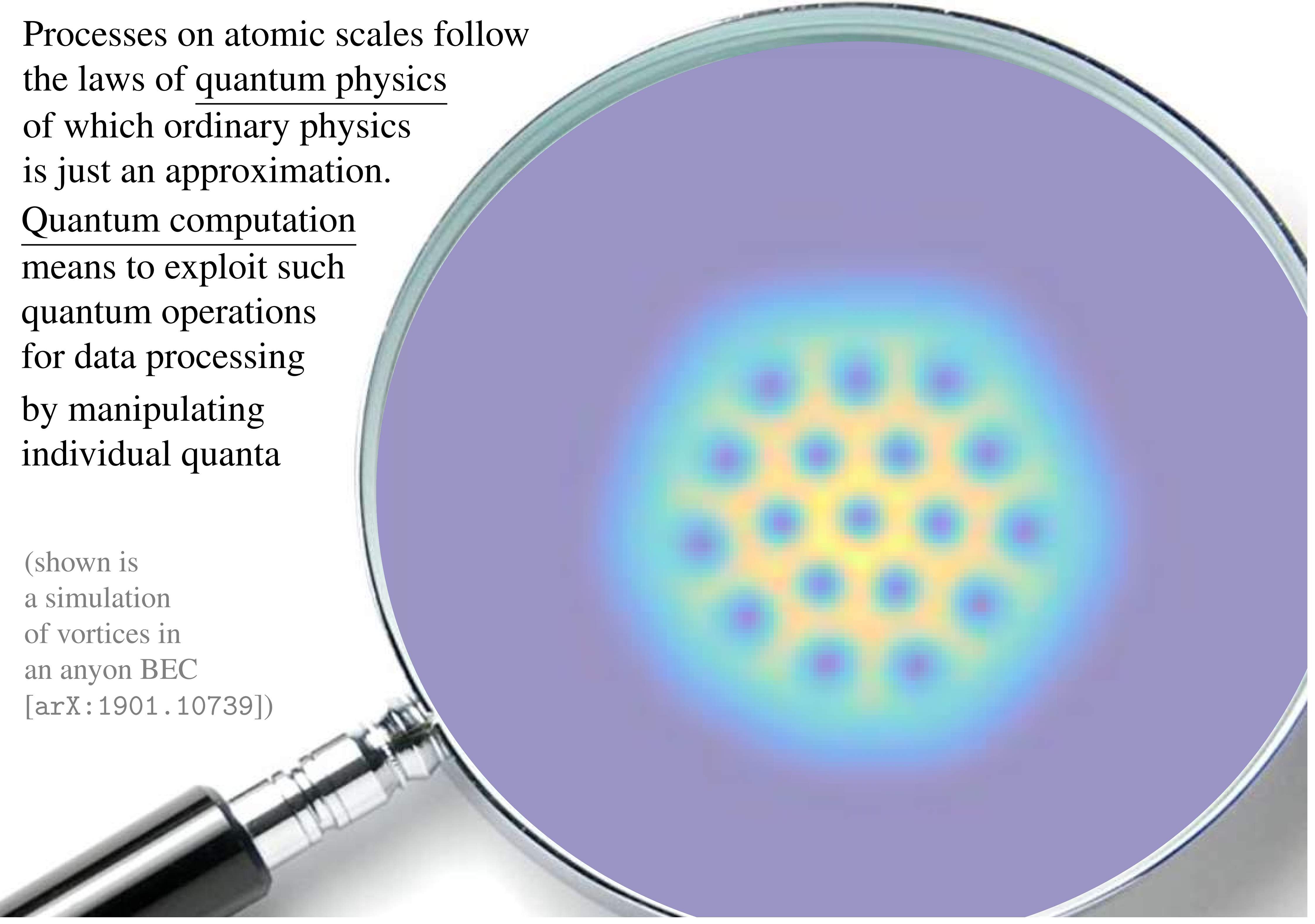

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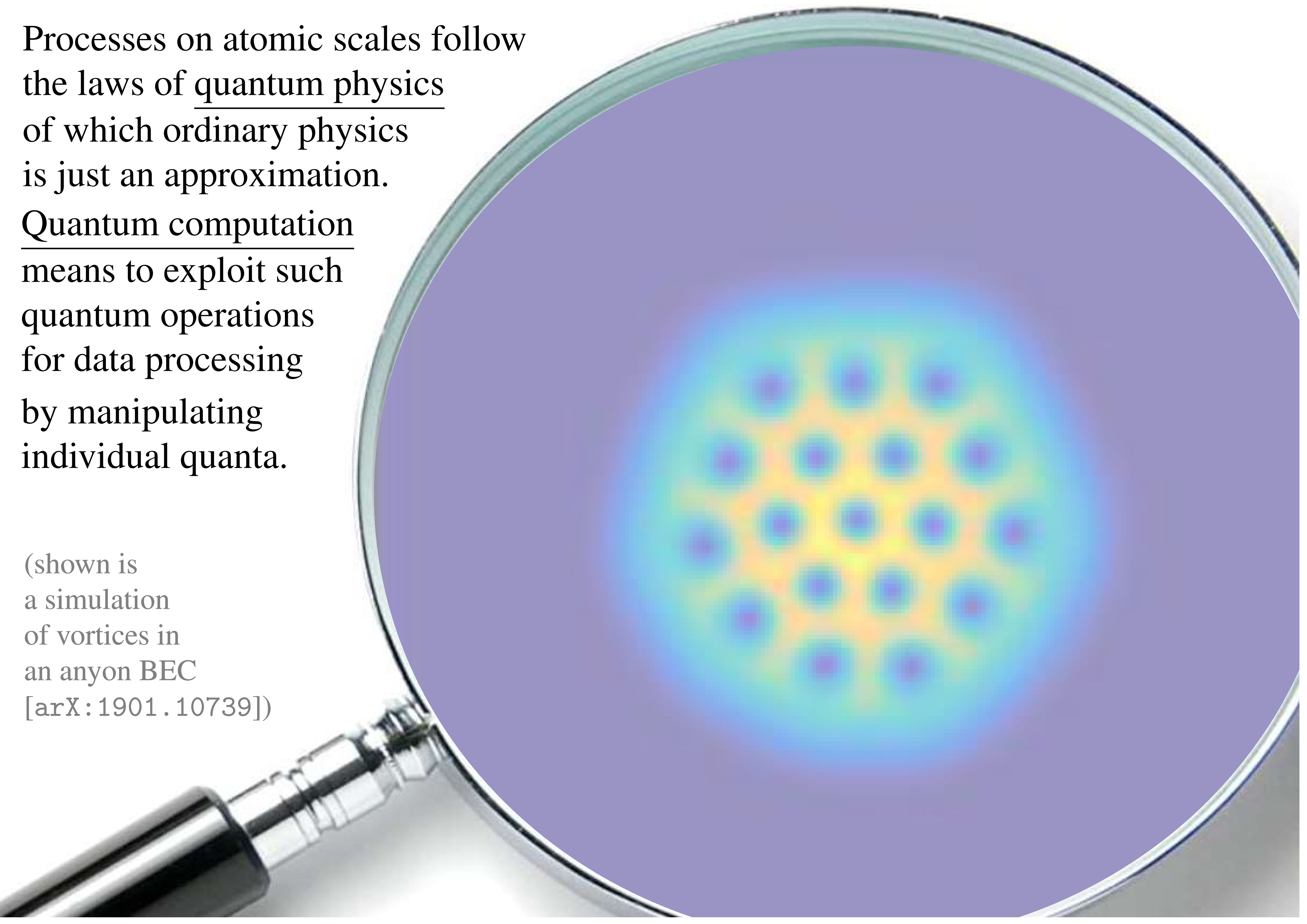

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“ because nature isn’t classical, dammit,
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Quantum factoring would break existing encryption
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But

Quantum Instability:

so far
these demonstrations are contrived
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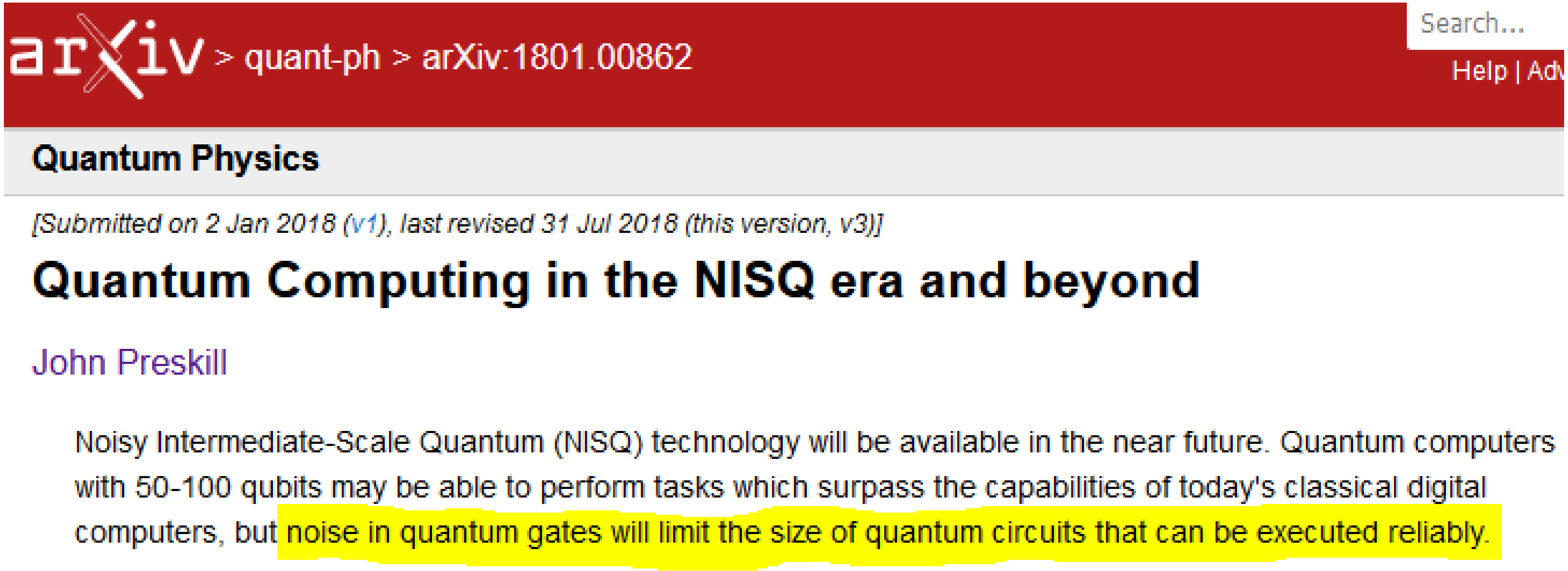

Quantum Physics

[Submitted on 2 Jan 2018 (v1), last revised 31 Jul 2018 (this version, v3)]

Quantum Computing in the NISQ era and beyond

John Preskill

Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably.

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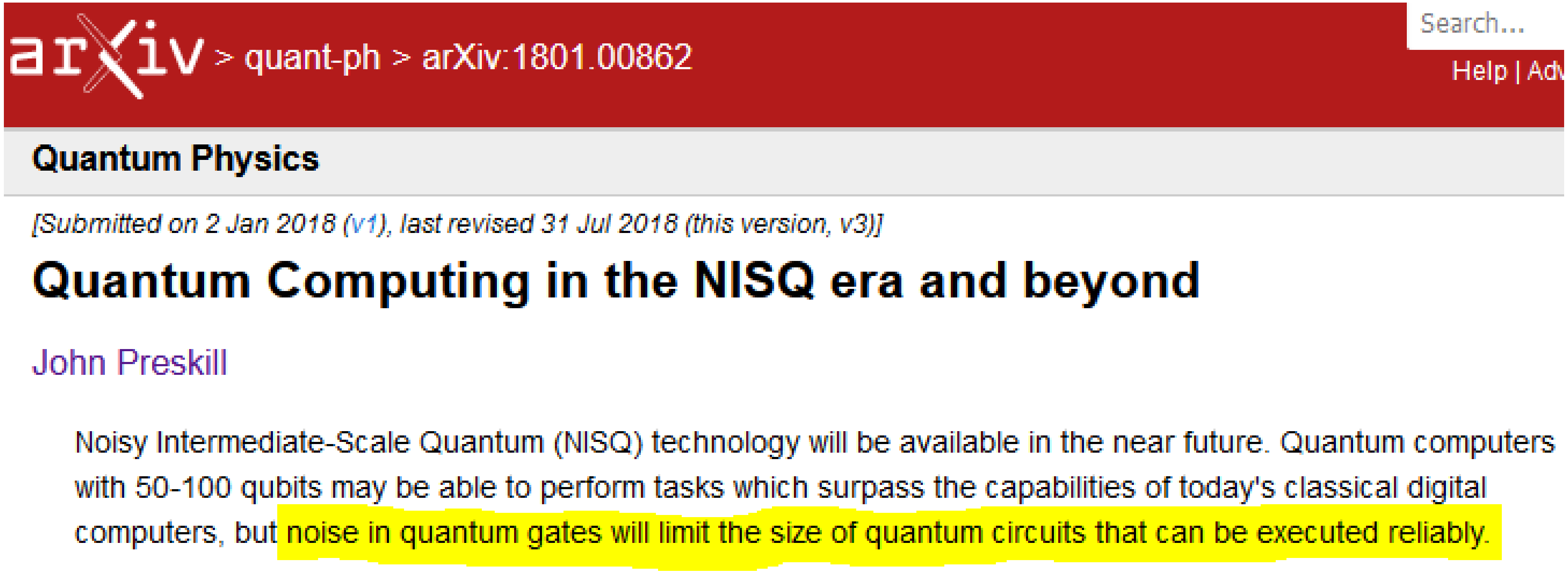

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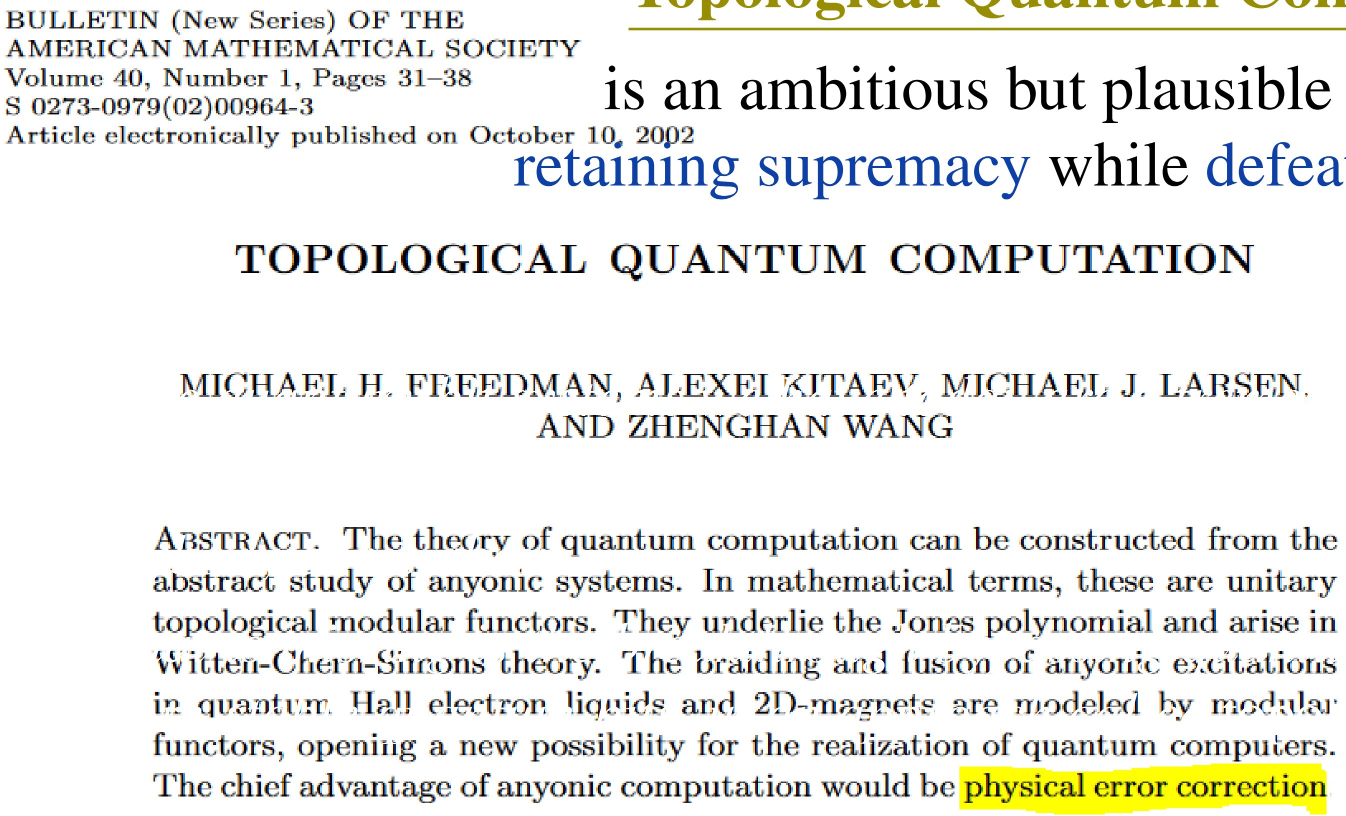

BULLETIN (New Series) OF THE
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Volume 40, Number 1, Pages 31–38
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Article electronically published on October 10, 2002

TOPOLOGICAL QUANTUM COMPUTATION

MICHAEL H. FREEDMAN, ALEXEI KITAEV, MICHAEL J. LARSEN,
AND ZHENGHAN WANG

Abstract. The theory of quantum computation can be constructed from the abstract study of anyonic systems. In mathematical terms, these are unitary topological modular functors. They underlie the Jones polynomial and arise in Witten-Chern-Simons theory. The braiding and fusion of anyonic excitations in quantum Hall electron liquids and 2D-magnets are modeled by modular functors, opening a new possibility for the realization of quantum computers. The chief advantage of anyonic computation would be physical error correction

Quantum Supremacy:

Quantum Computation is expected to
be enormously more powerful
than Classical Digital Computation
(for special but crucial applications).

Quantum Instability:

But Quantum Computation is prone to
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Topological Quantum Computation:

is an ambitious but plausible strategy for
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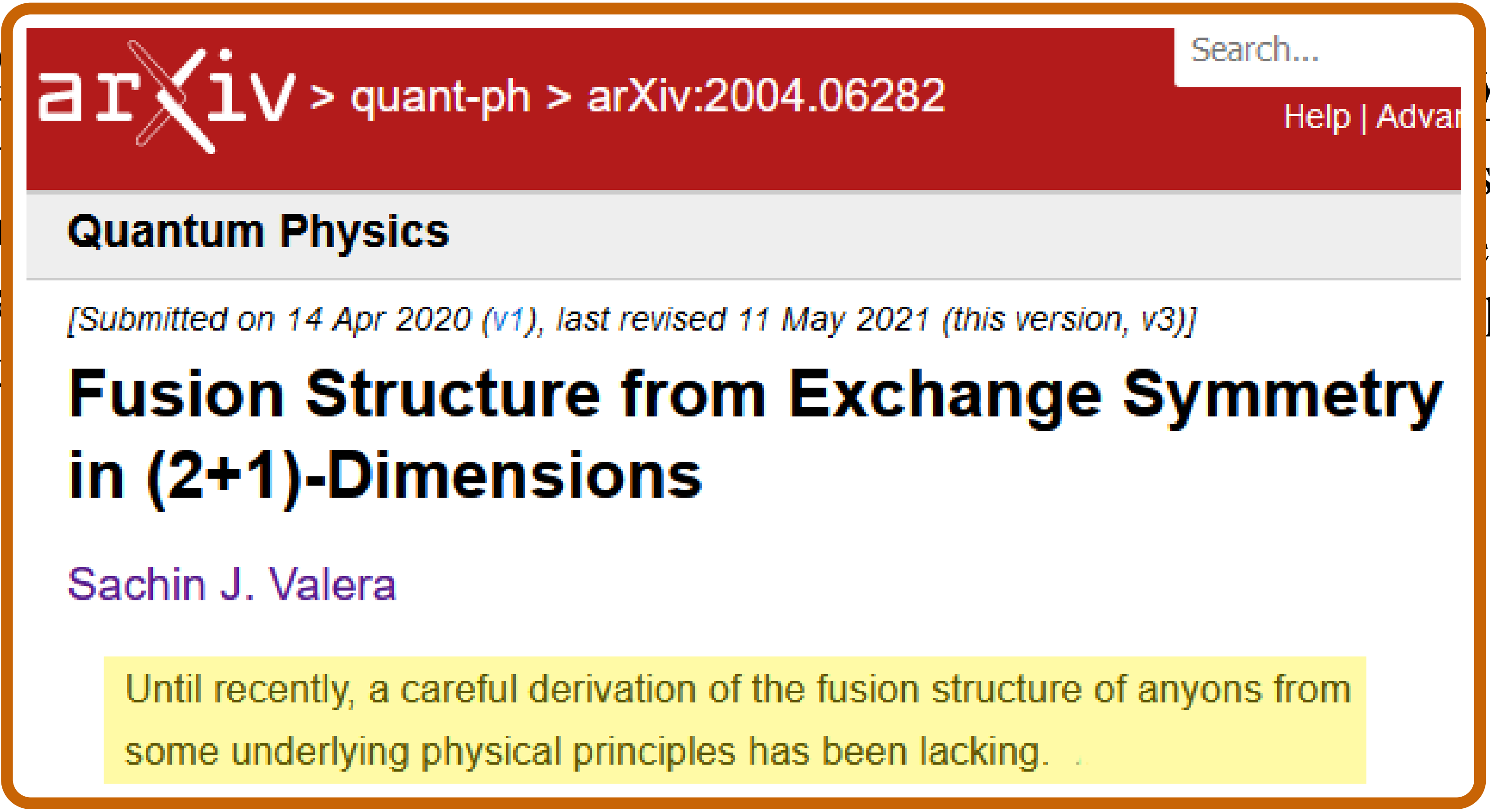


arXiv > quant-ph > arXiv:2004.06282

Quantum Physics

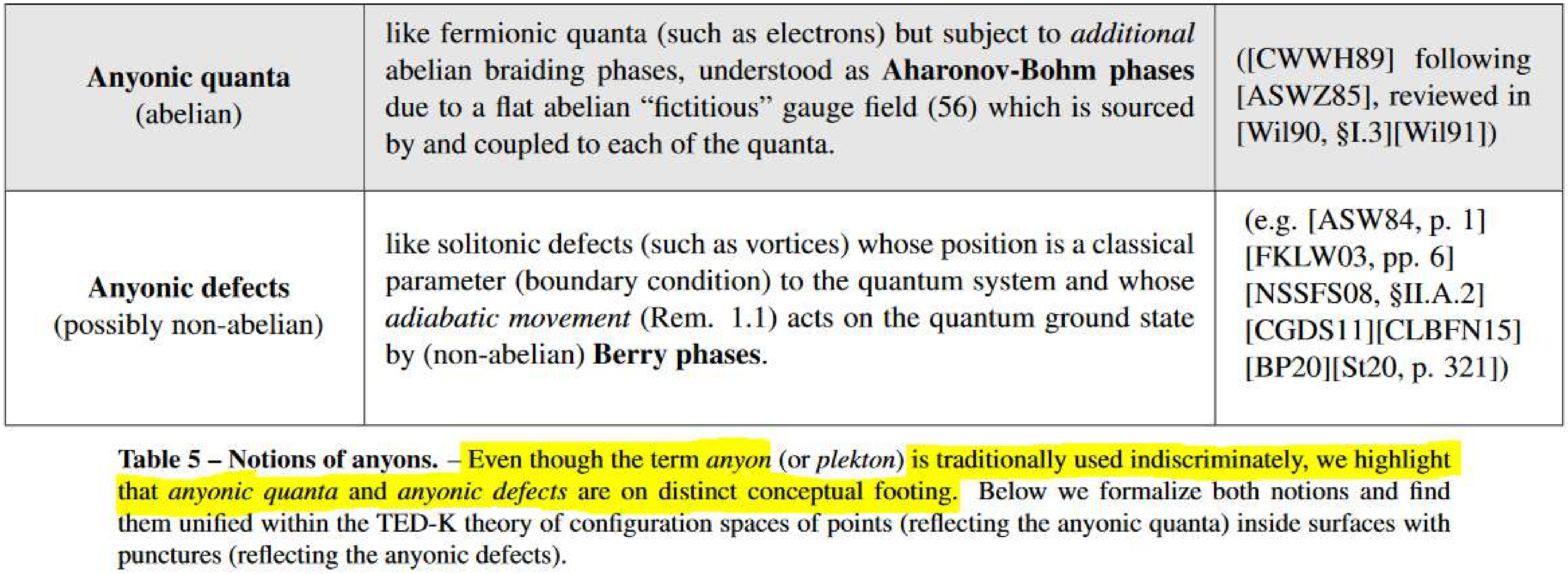
[Submitted on 14 Apr 2020 (v1), last revised 11 May 2021 (this version, v3)]

Fusion Structure from Exchange Symmetry in (2+1)-Dimensions

Sachin J. Valera

Until recently, a careful derivation of the fusion structure of anyons from some underlying physical principles has been lacking.

But theoretical foundations had remained shaky



Anyonic quanta (abelian)	like fermionic quanta (such as electrons) but subject to *additional* abelian braiding phases, understood as **Aharonov-Bohm phases** due to a flat abelian "fictitious" gauge field (56) which is sourced by and coupled to each of the quanta.	([CWWH89] following [ASWZ85], reviewed in [Wil90, §I.3][Wil91])
Anyonic defects (possibly non-abelian)	like solitonic defects (such as vortices) whose position is a classical parameter (boundary condition) to the quantum system and whose *adiabatic movement* (Rem. 1.1) acts on the quantum ground state by (non-abelian) **Berry phases**.	(e.g. [ASW84, p. 1] [FKLW03, pp. 6] [NSSFS08, §II.A.2] [CGDS11][CLBFN15] [BP20][St20, p. 321])

Table 5 – Notions of anyons. – Even though the term *anyon* (or *plekton*) is traditionally used indiscriminately, we highlight that *anyonic quanta* and *anyonic defects* are on distinct conceptual footing. Below we formalize both notions and find them unified within the TED-K theory of configuration spaces of points (reflecting the anyonic quanta) inside surfaces with punctures (reflecting the anyonic defects).

But theoretical foundations had remained shaky

arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

Anyonic Topological Order in Twisted Equivariant Differential (TED) K-Theory

Hisham Sati, Urs Schreiber

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10 maart 2021 - 20:21 door Tomas van Dijk @tomasvd

Majorana: not fraud, but confirmation bias

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retractionwatch.com/2022/04/24/authors-retract-second-

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but first: *What is the idea?* ⟶

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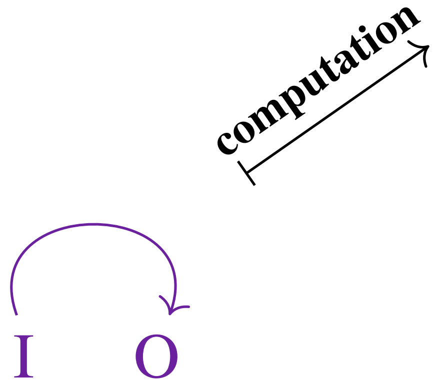
cf. [van Leeuwen & Wiedermann (2017)]

[Sati & Schreiber, PlanQC **2022** 33 (2022)]



To compute is to **execute**

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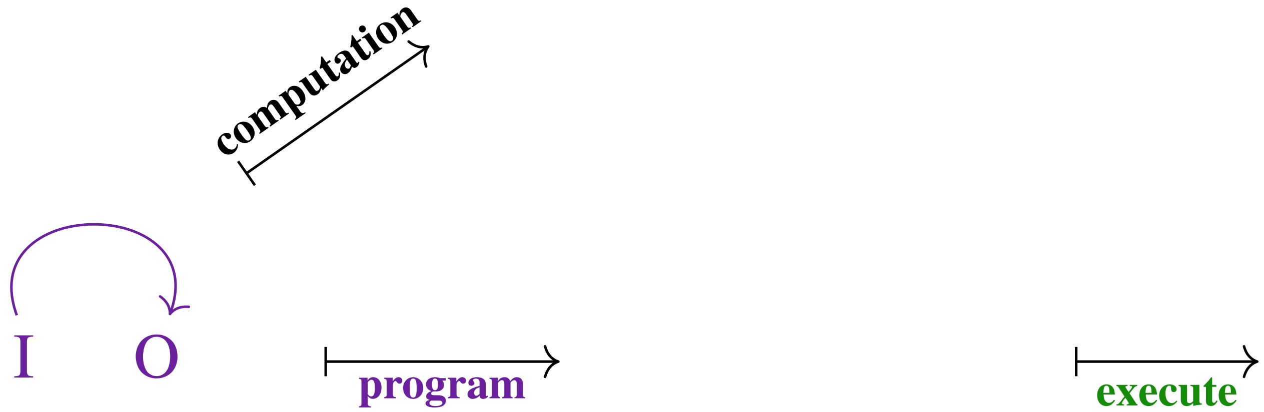

computation

I O

program

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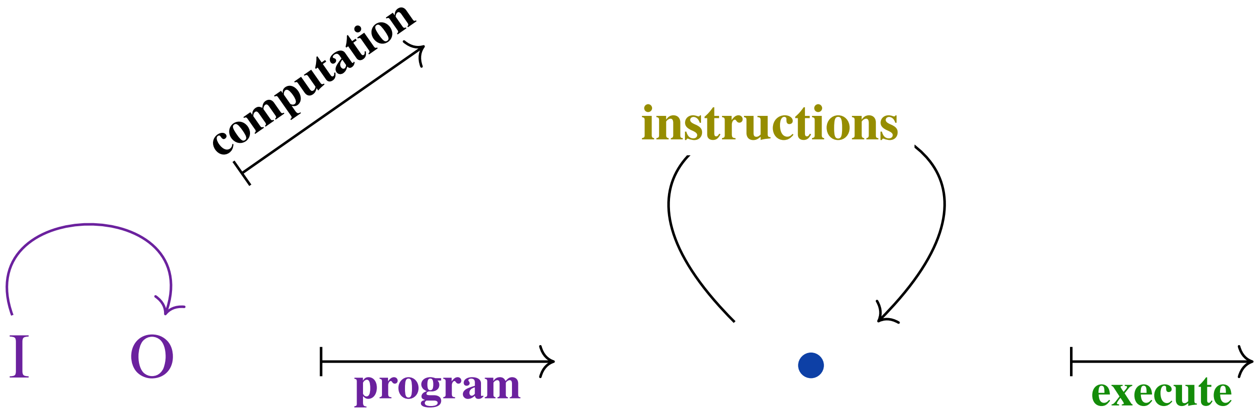

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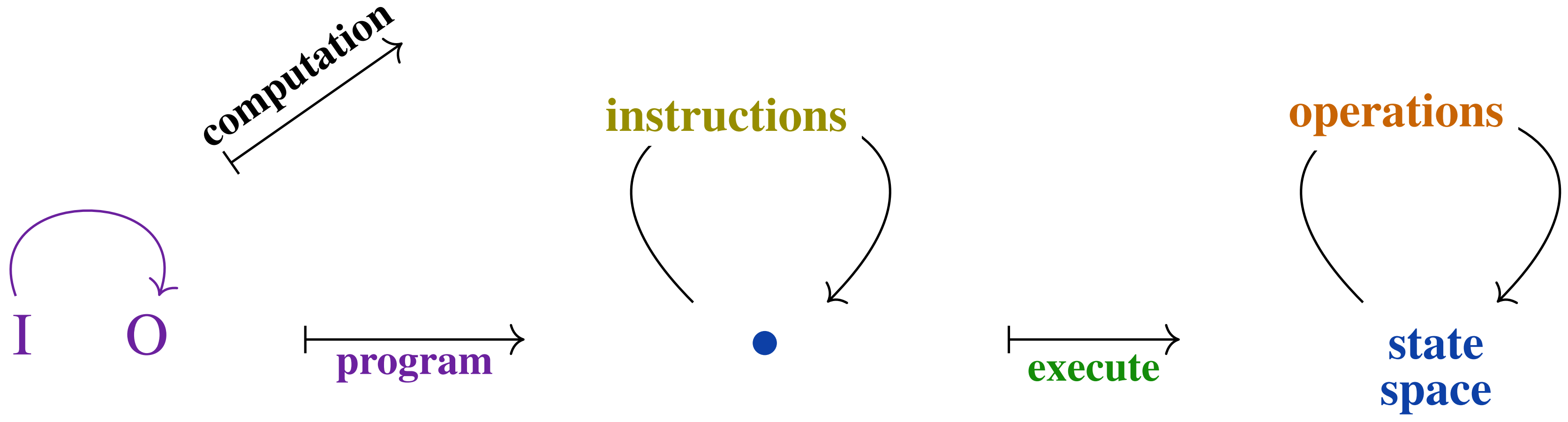

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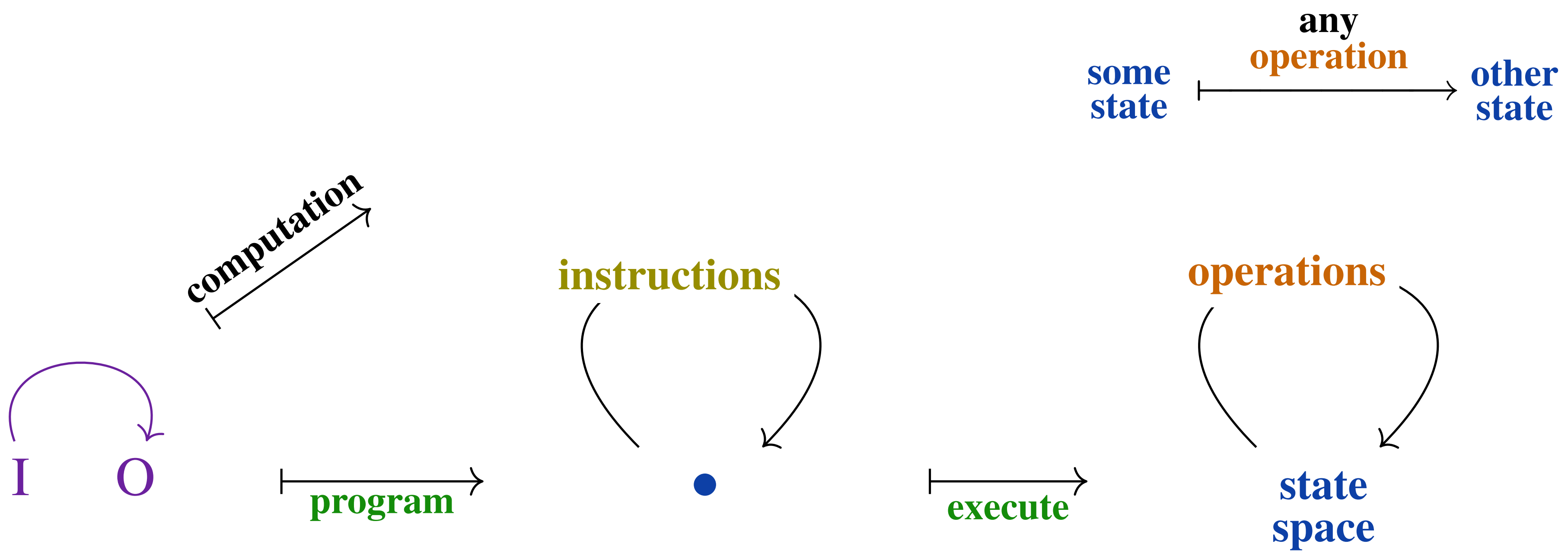

any
operation

some state ↦ **other state**

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To compute is to **execute**
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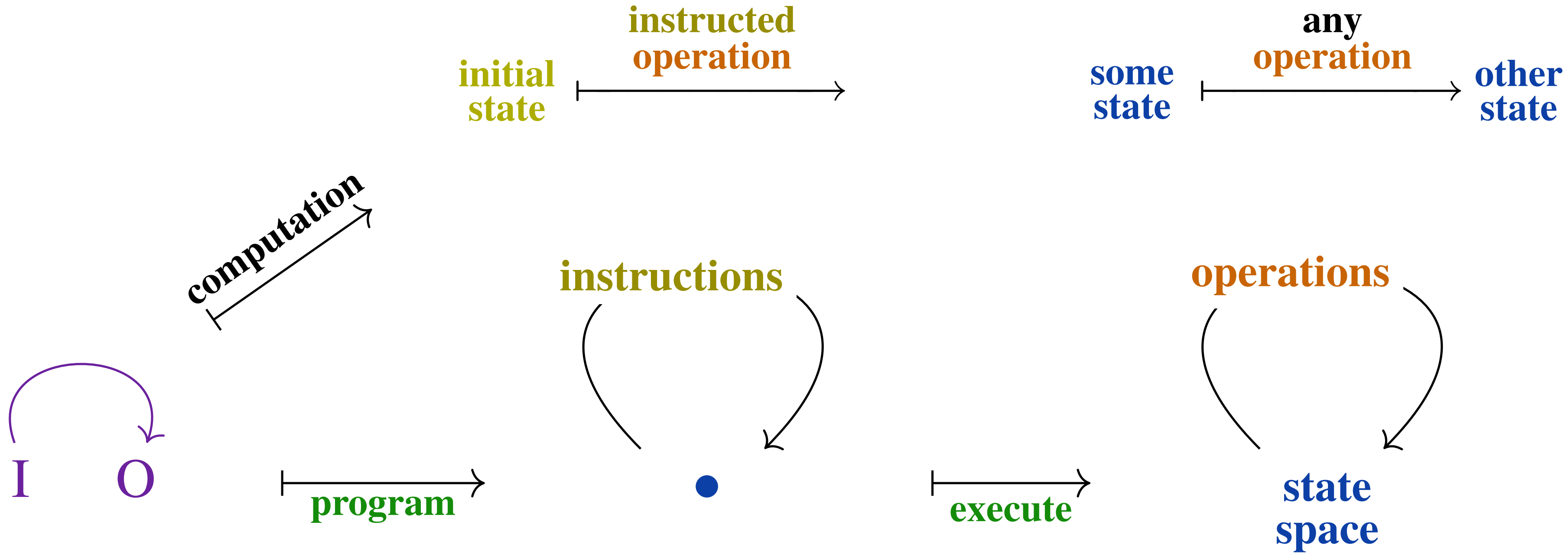

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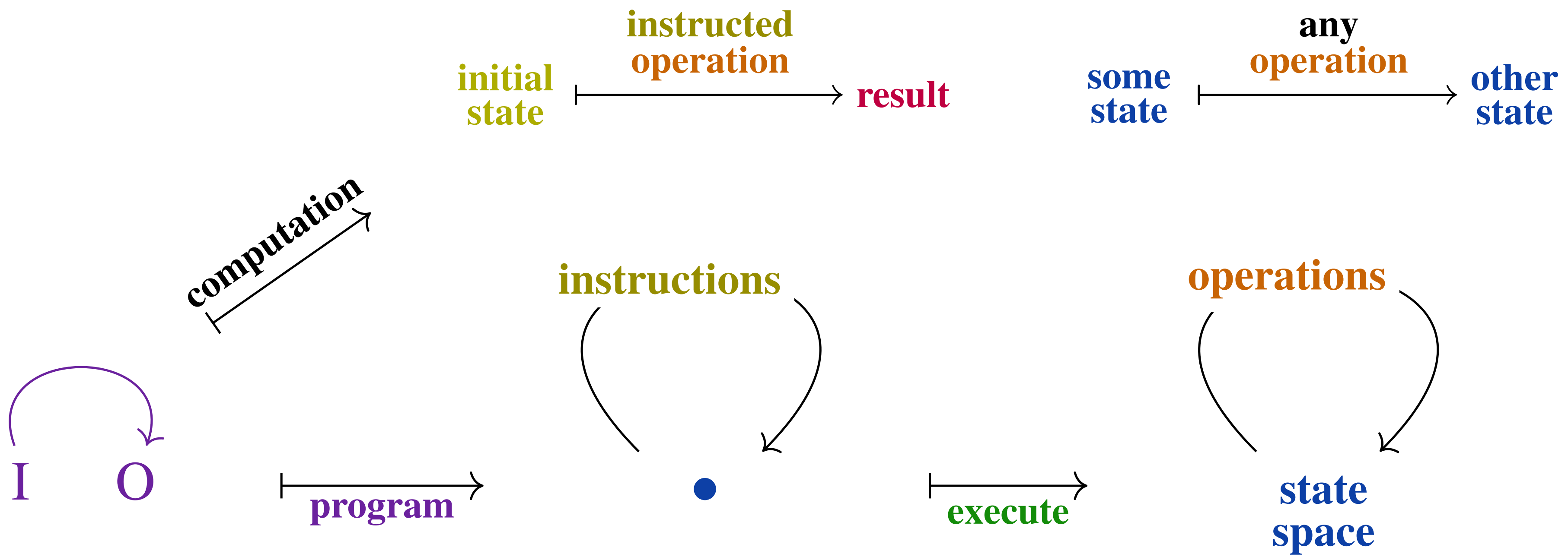

initial state ↦ **instructed operation** ↦ **result**

some state ↦ **any operation** ↦ **other state**

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 & \textbf{instructed operations} \arrow[r] \arrow[d] \arrow[dr, phantom, "\text{(pb)}"] & \textbf{operations on states} \arrow[d] \\
\textbf{path} \arrow[ur, dashed, "\textbf{computation on initial state}"] \arrow[r, "\textbf{program}"'] & \textbf{instructions} \arrow[r, "\textbf{execution}"'] & \textbf{operations}
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[Sati & Schreiber, PlanQC **2022** 33 (2022)]

Aside: Formalization by path lifting in Homotopy Type Theory:

$$\begin{tikzcd}
& \textbf{instructed operations} \arrow[r] \arrow[d, "\text{dependent type family}"] \arrow[dr, phantom, "\text{(pb)}"] & \textbf{operations on states} \arrow[d, "\text{univalent type universe}"] \\
\textbf{path} \arrow[r, "\textbf{program}"', ""{name=P}] \arrow[ur, dashed, "\textbf{computation on initial state}", ""'{name=C}] & \textbf{instructions} \arrow[r, "\textbf{execution}"'] & \textbf{operations}
\arrow[Rightarrow, from=P, to=C, "\text{path lift}"']
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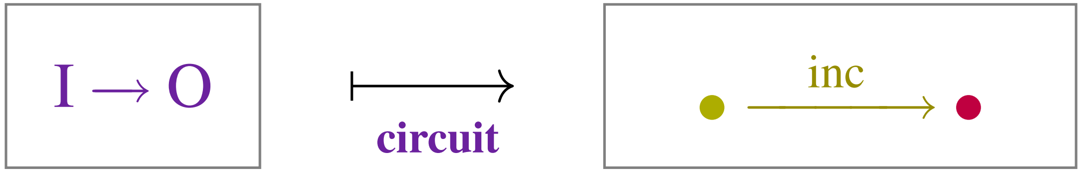
Example: Classical computation resulting in cyclic permutation of 3 numbers:



$$\boxed{I \to O} \;\; \xmapsto[\textbf{circuit}]{} \;\; \boxed{\bullet \xrightarrow{\text{inc}} \bullet}$$

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$$\boxed{\mathrm{I} \to \mathrm{O}} \xmapsto[\textbf{program}]{\qquad} \boxed{\bullet \xrightarrow{\ \mathrm{inc}\ } \bullet} \xmapsto[\textbf{execution}]{\qquad} \boxed{\mathbb{F}_3^3 \xrightarrow{(\bullet + 1)\ \mathrm{mod}\ 3} \mathbb{F}_3^3}$$

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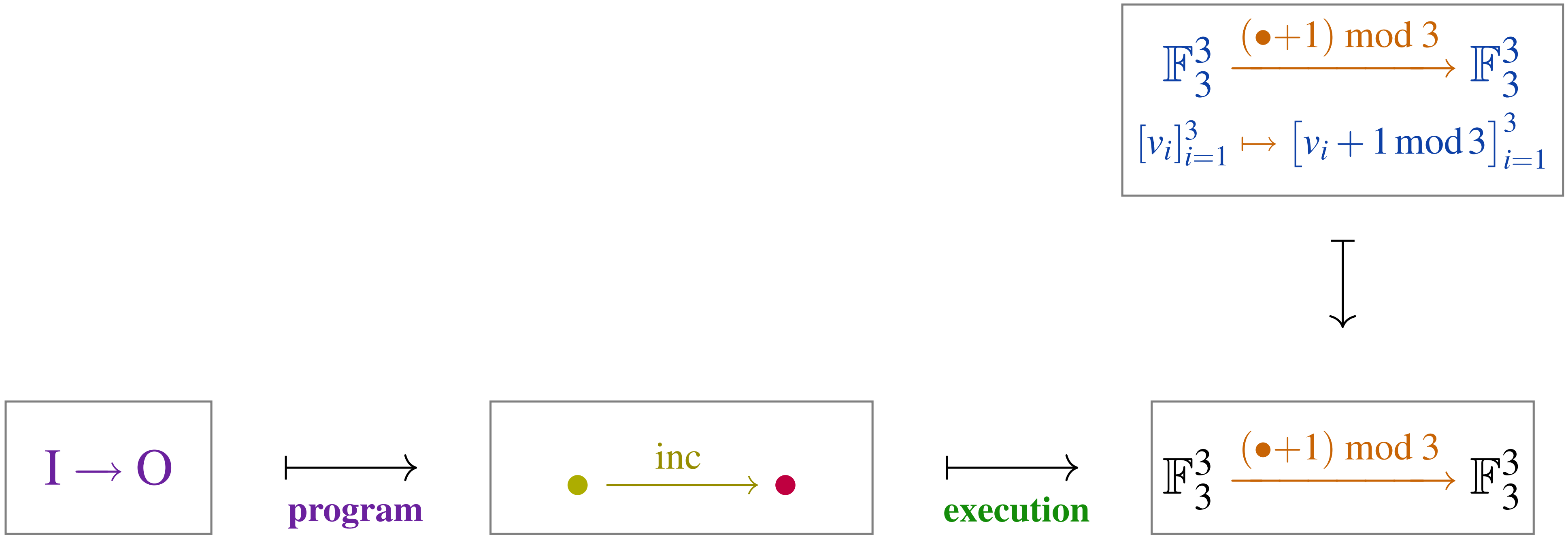

$$\begin{array}{ccc} \mathbb{F}_3^3 & \xrightarrow{(\bullet+1)\bmod 3} & \mathbb{F}_3^3 \\ [v_i]_{i=1}^3 & \mapsto & \big[v_i + 1 \bmod 3\big]_{i=1}^3 \end{array}$$

$$\downarrow$$

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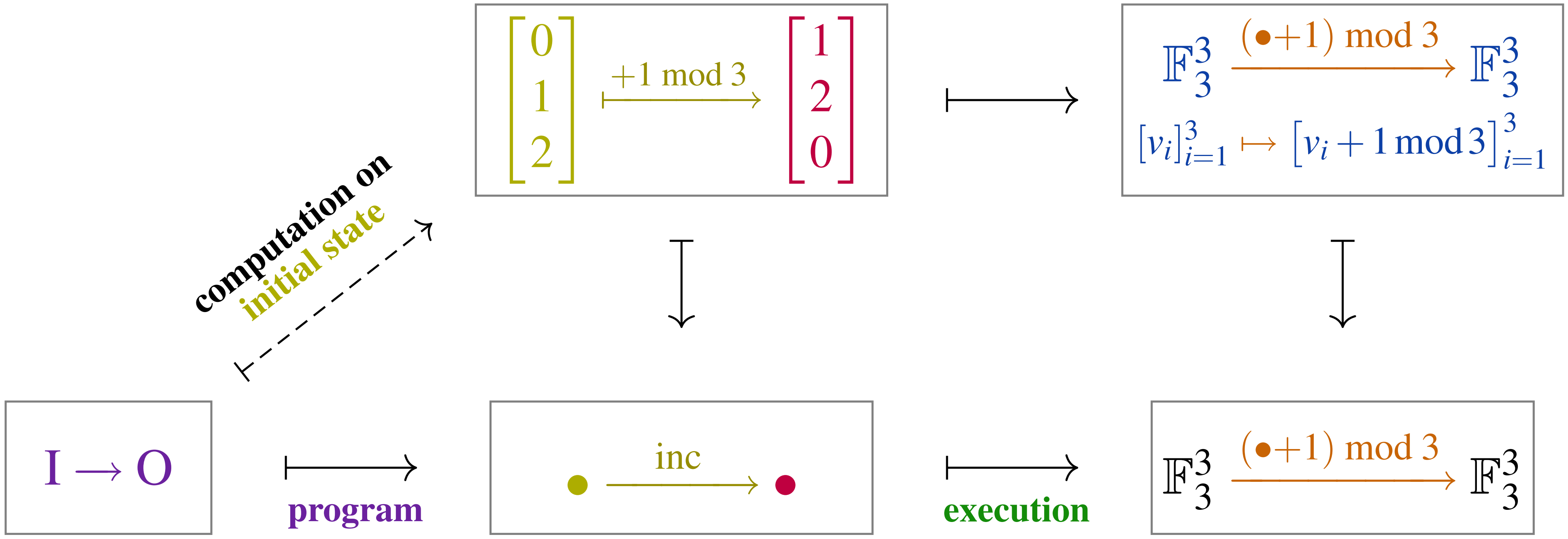

$$\begin{bmatrix}0\\1\\2\end{bmatrix} \xmapsto{+1 \bmod 3} \begin{bmatrix}1\\2\\0\end{bmatrix} \longmapsto \begin{array}{c} \mathbb{F}_3^3 \xrightarrow{(\bullet+1) \bmod 3} \mathbb{F}_3^3 \\ [v_i]_{i=1}^3 \mapsto [v_i + 1 \bmod 3]_{i=1}^3 \end{array}$$

computation on **initial state**

$$I \to O \quad \xmapsto[\textbf{program}]{} \quad \bullet \xrightarrow{\text{inc}} \bullet \quad \xmapsto[\textbf{execution}]{} \quad \mathbb{F}_3^3 \xrightarrow{(\bullet+1) \bmod 3} \mathbb{F}_3^3$$

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e.g. mechanical gates:

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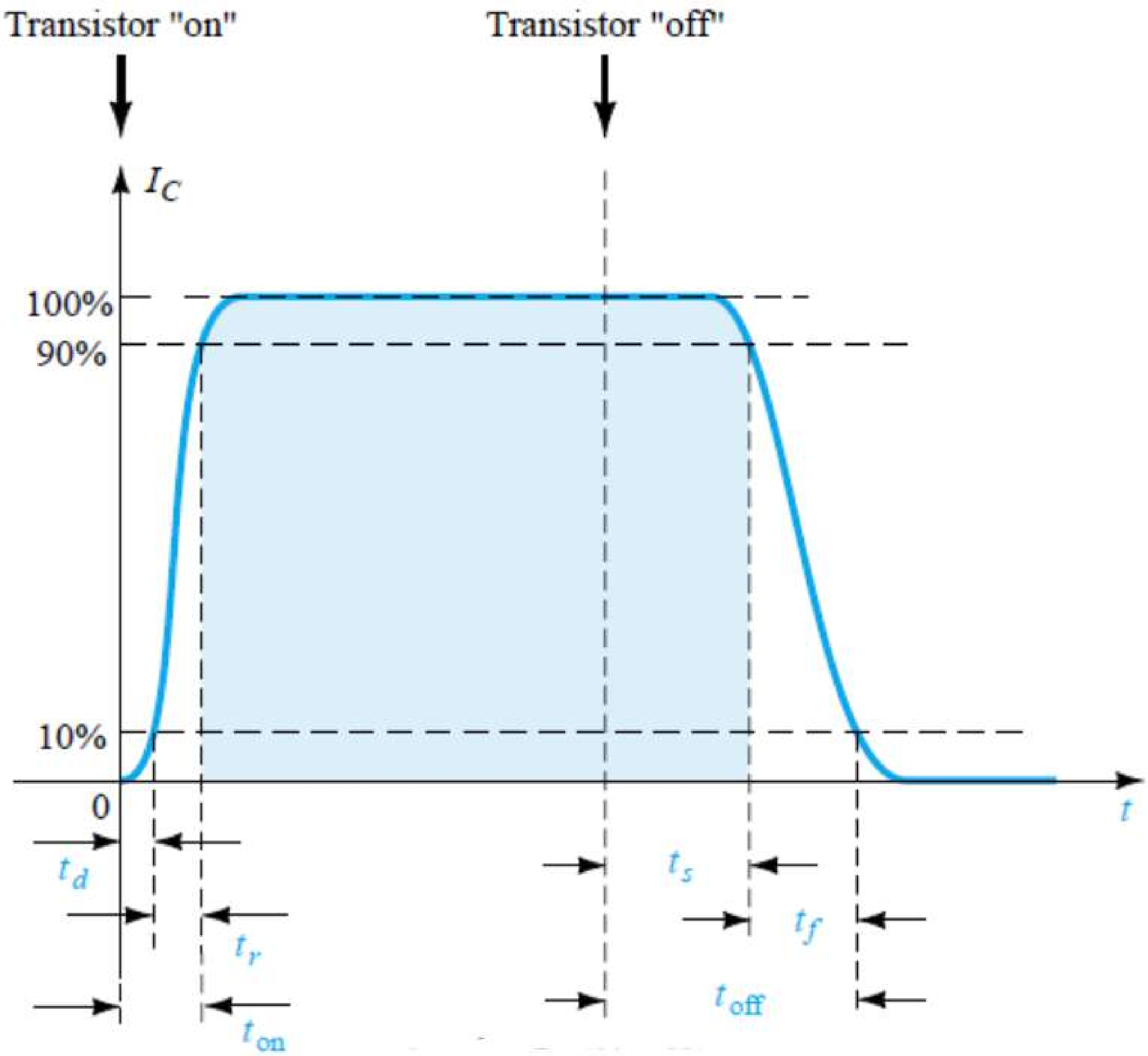

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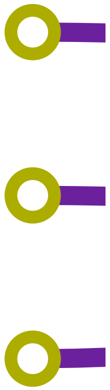

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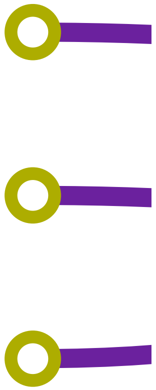

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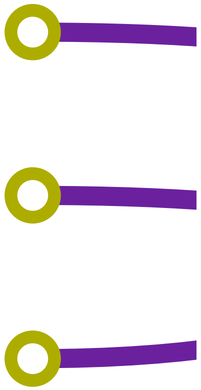

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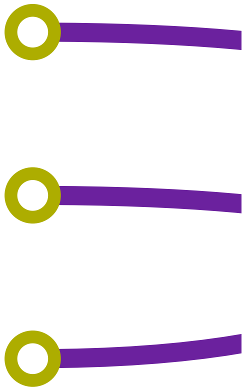

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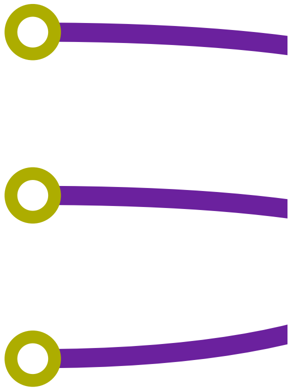

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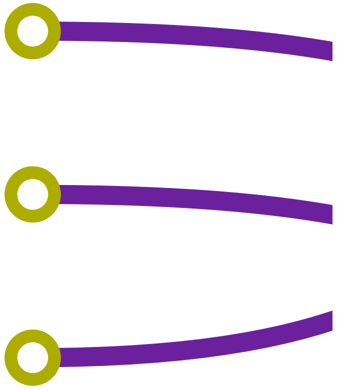

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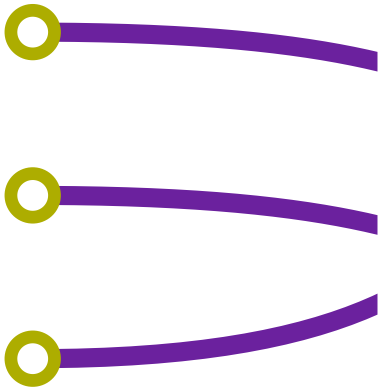

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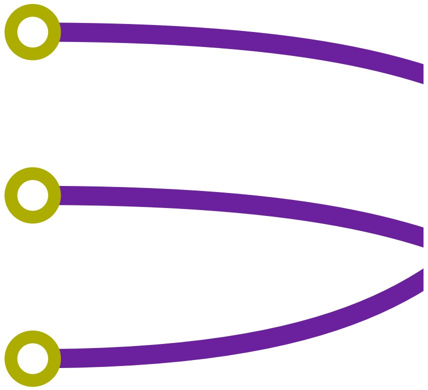

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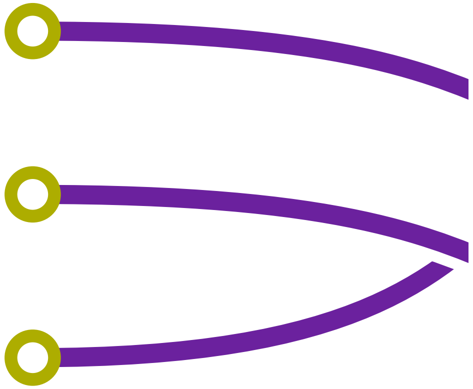

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result should be stable
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Coarse-grain state space into a bit lattice.

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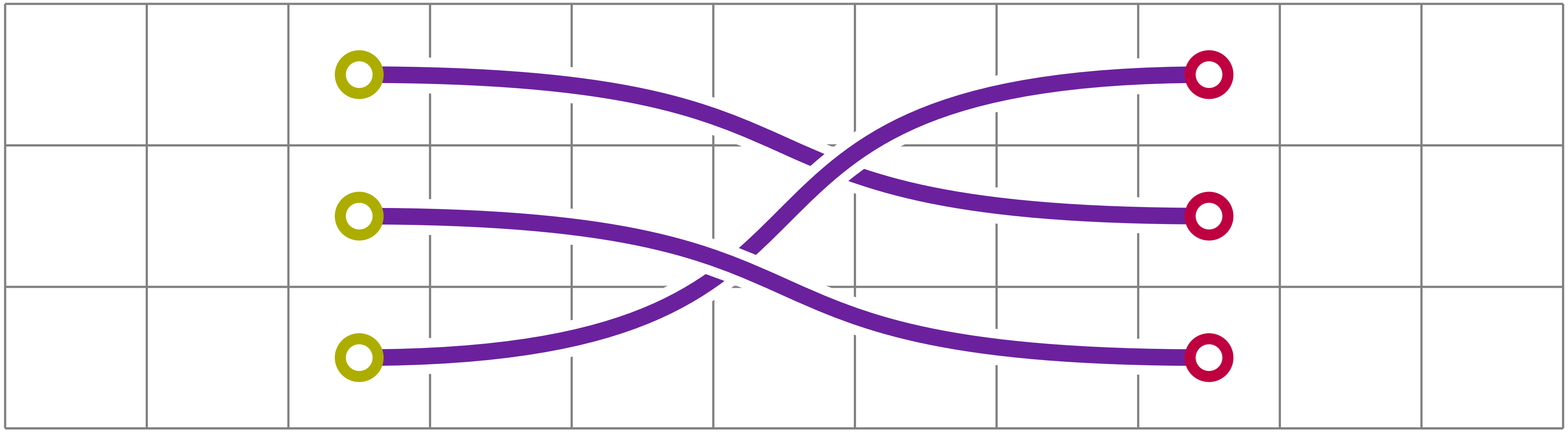

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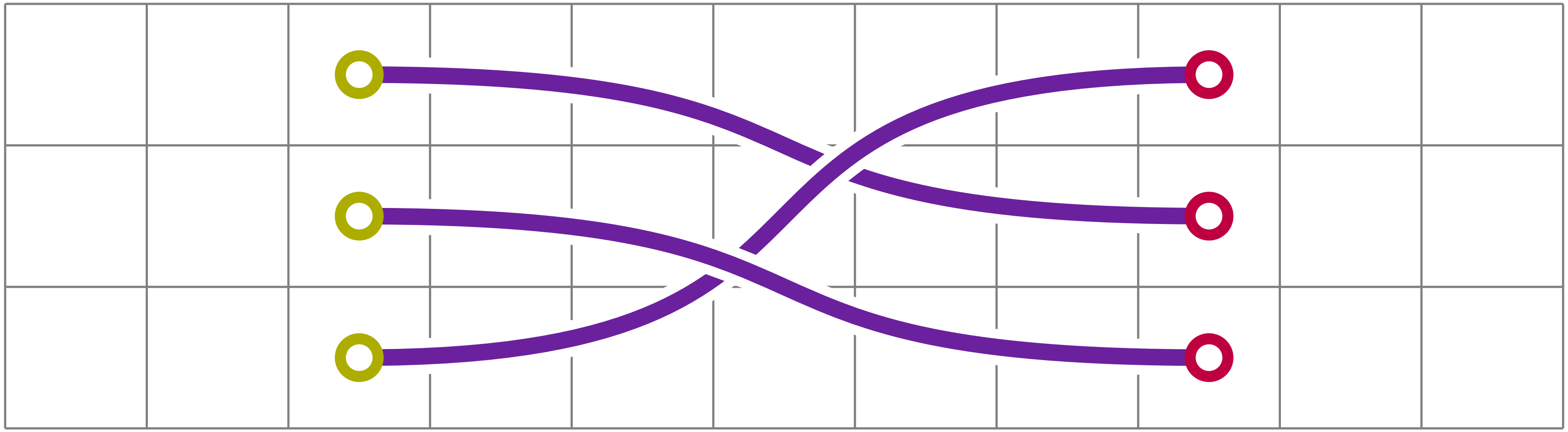

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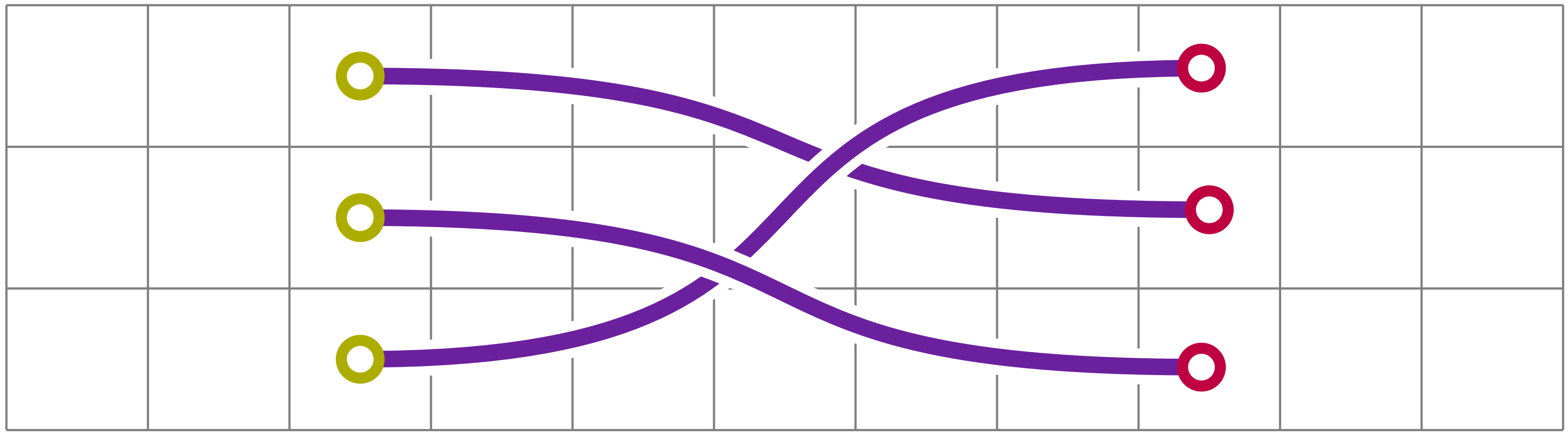

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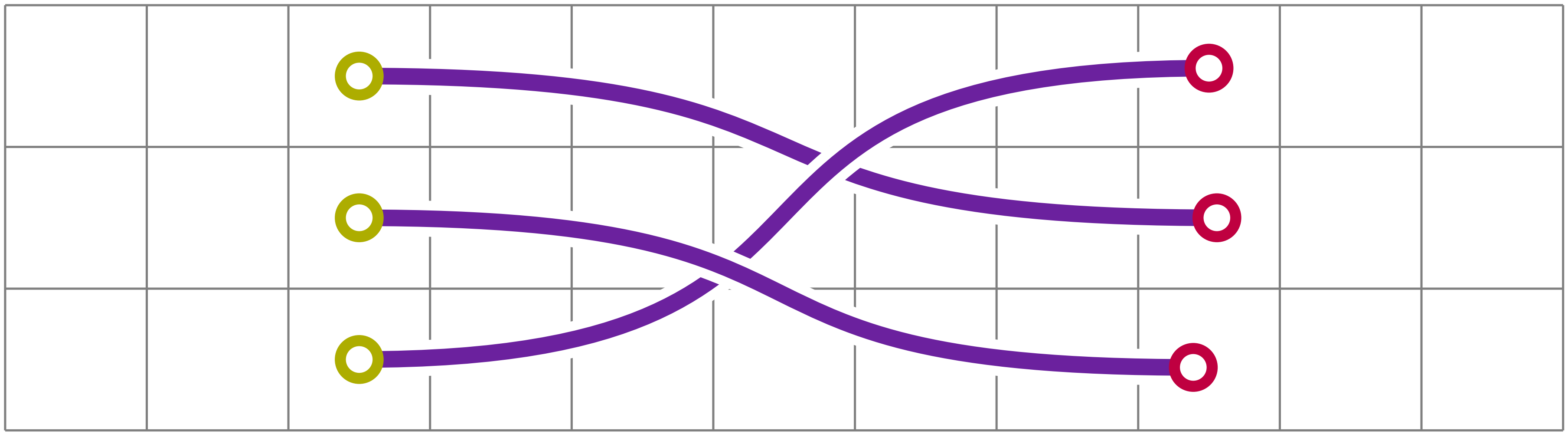

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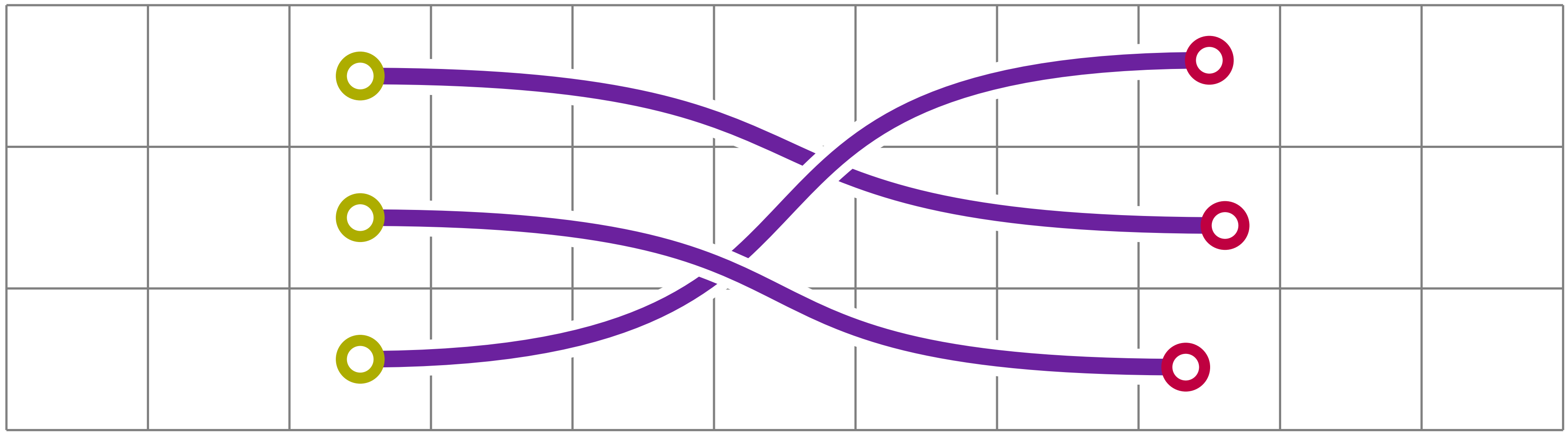

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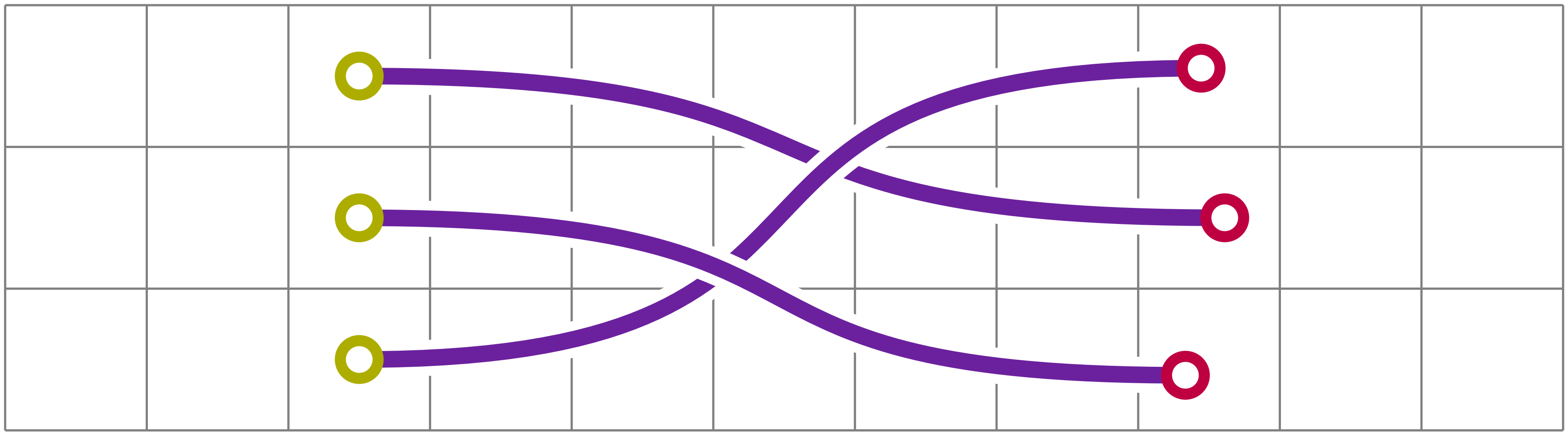

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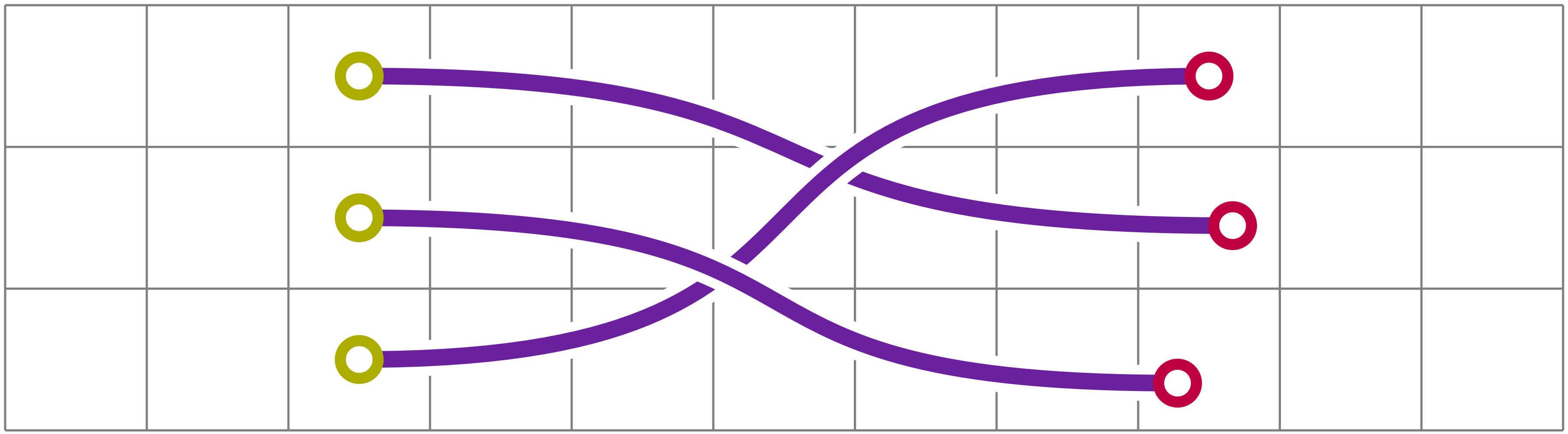

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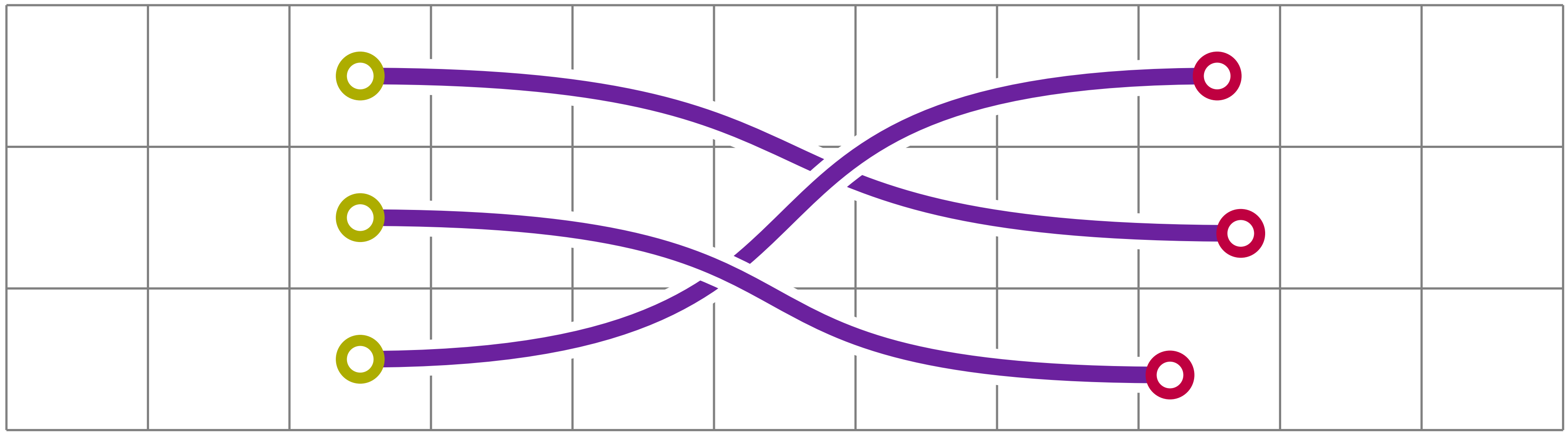

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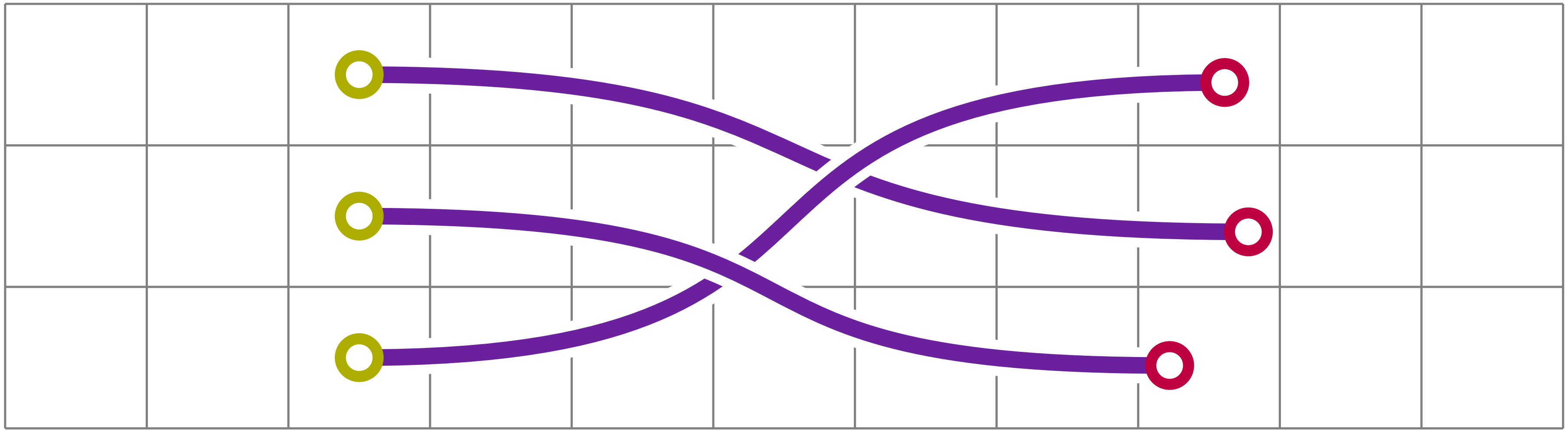

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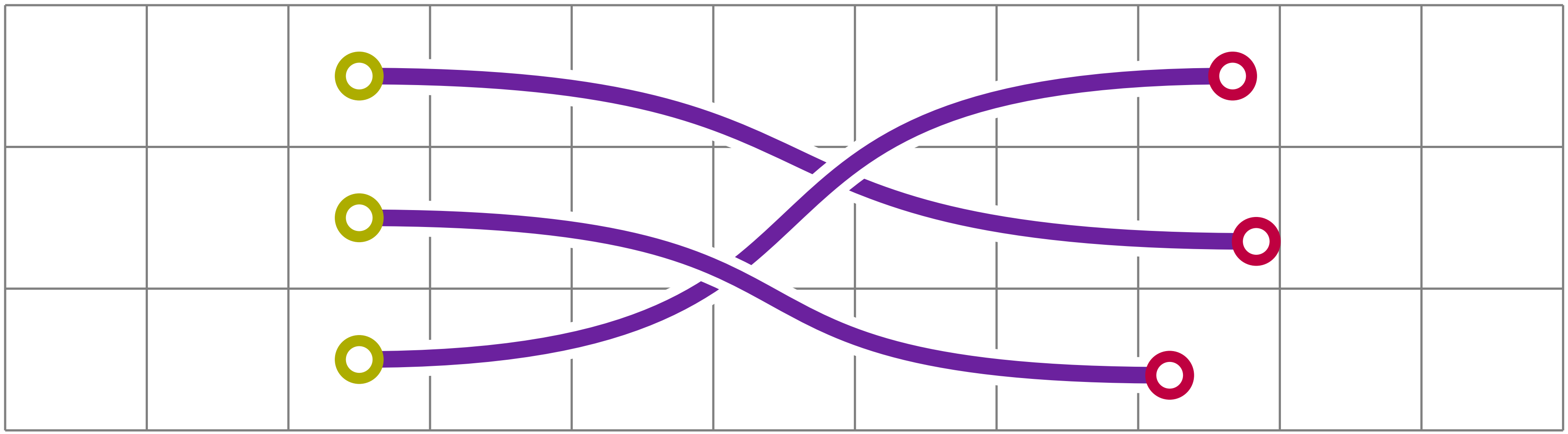

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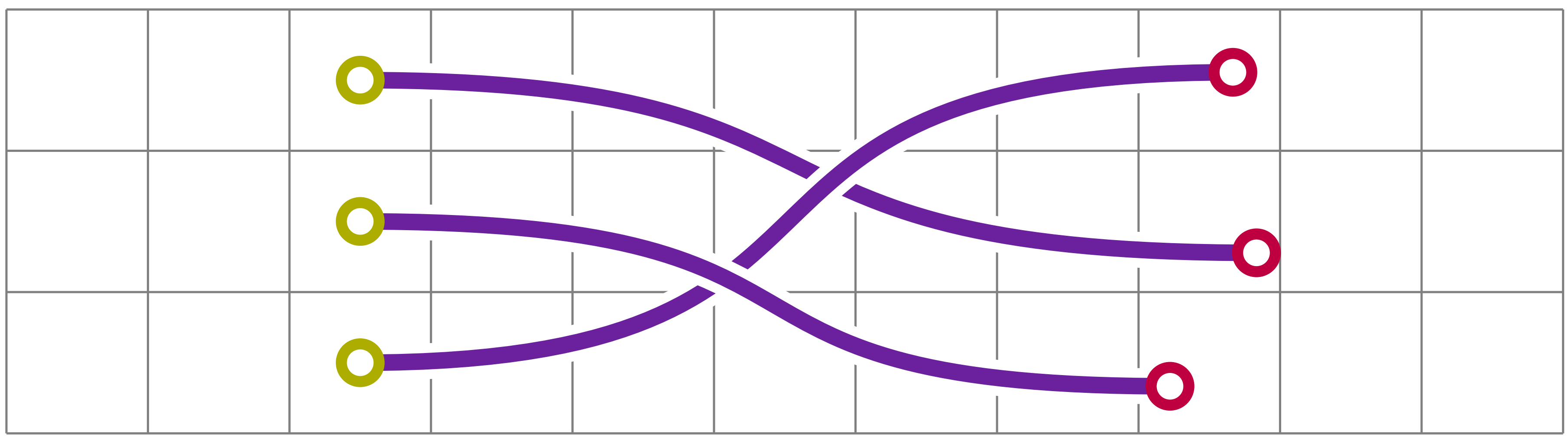

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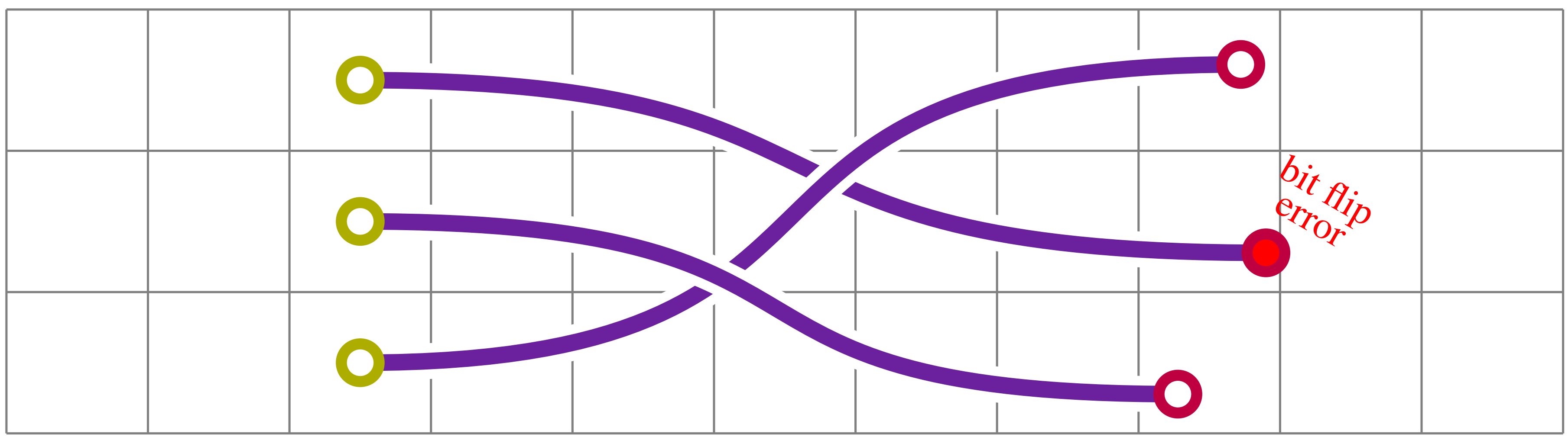

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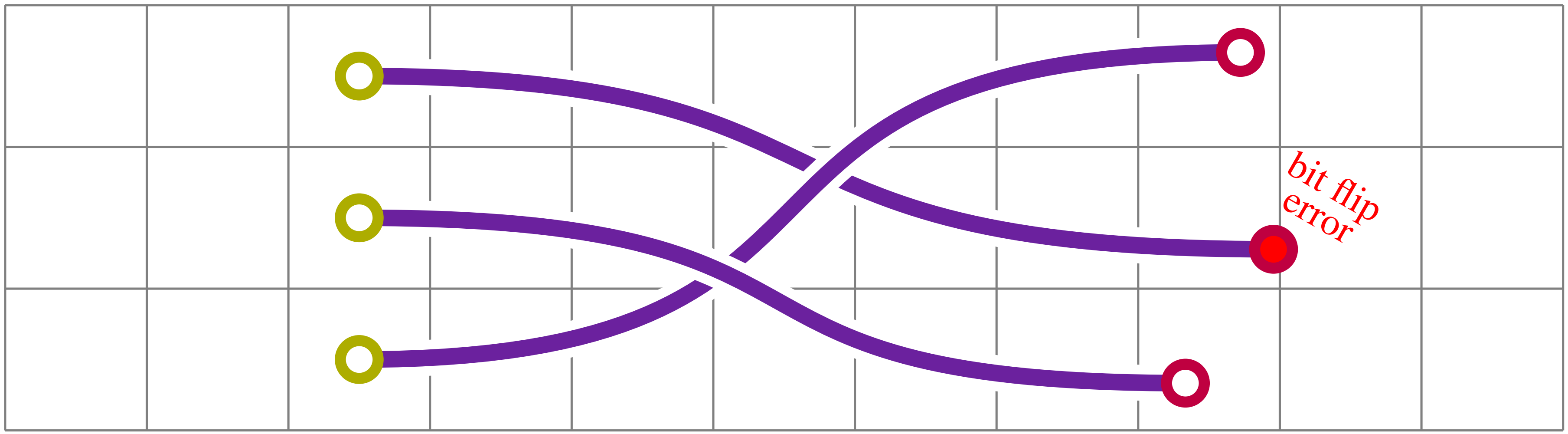

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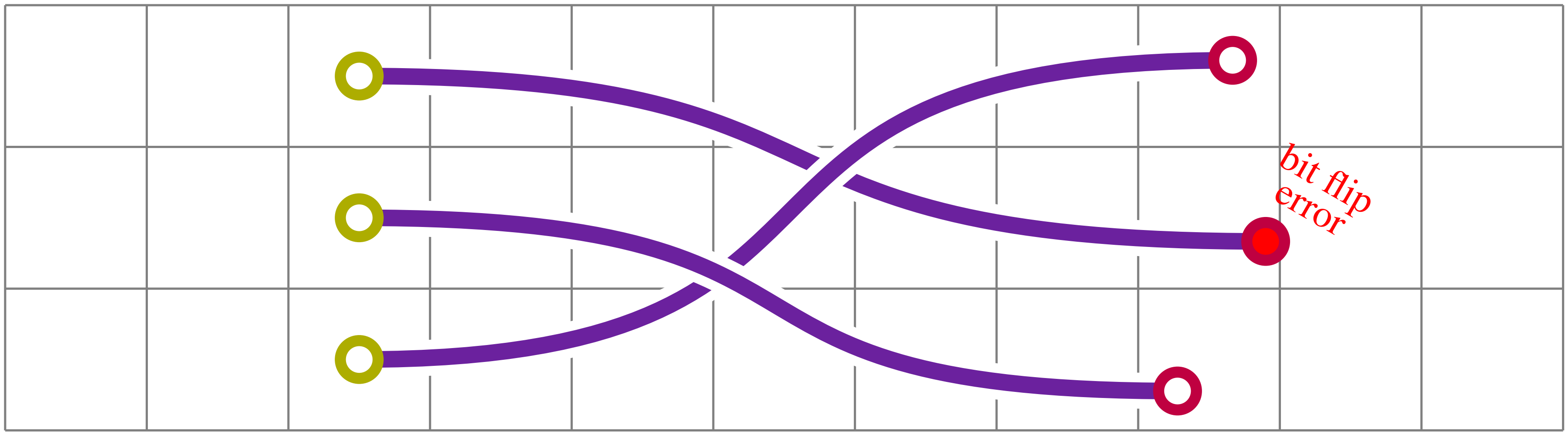

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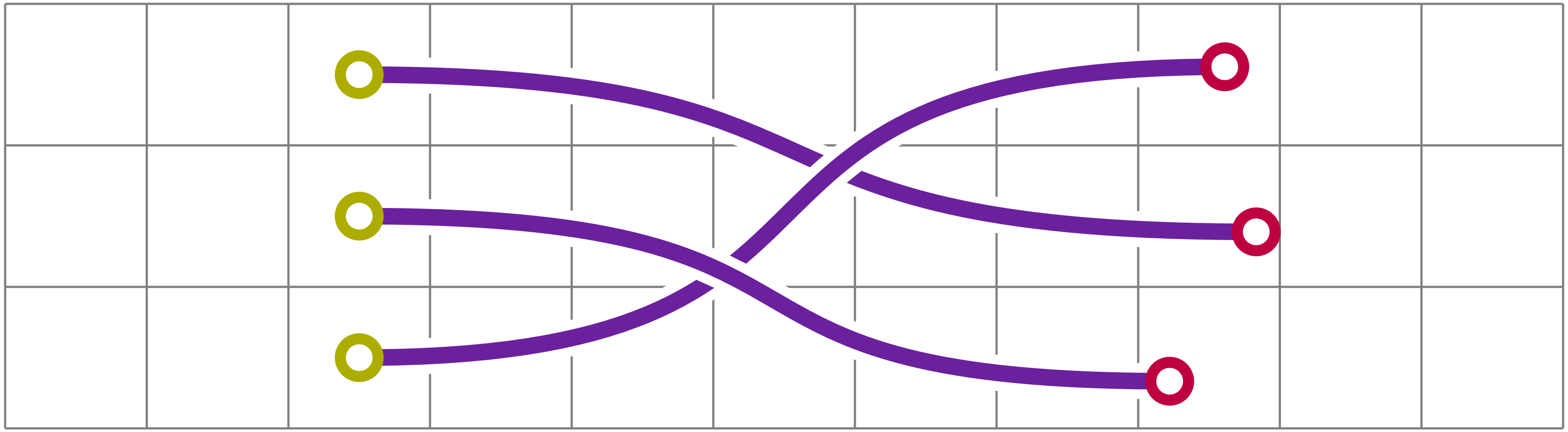

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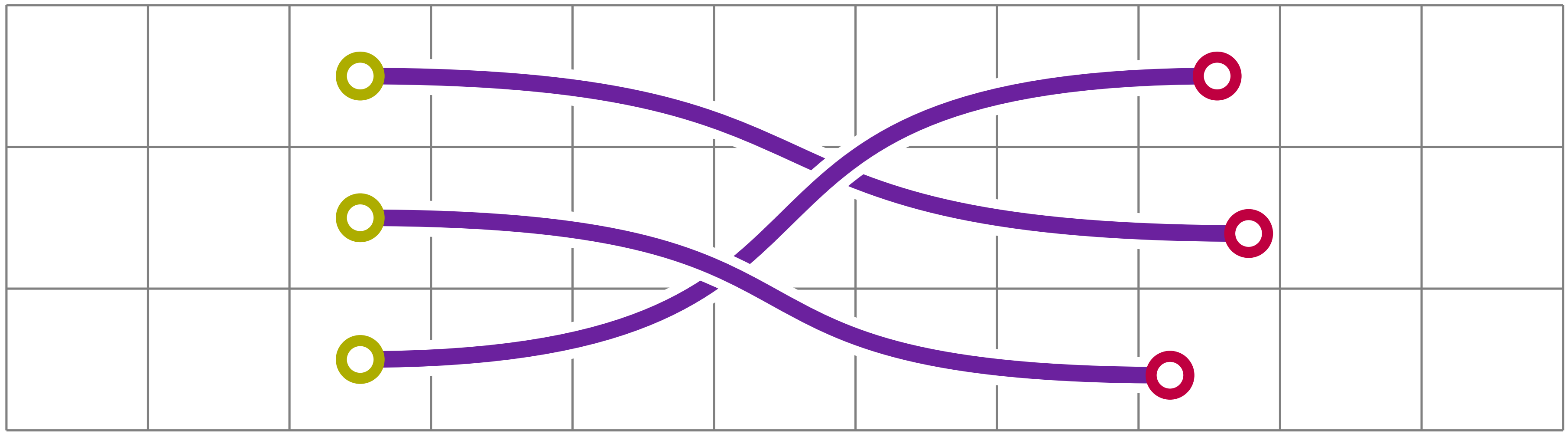

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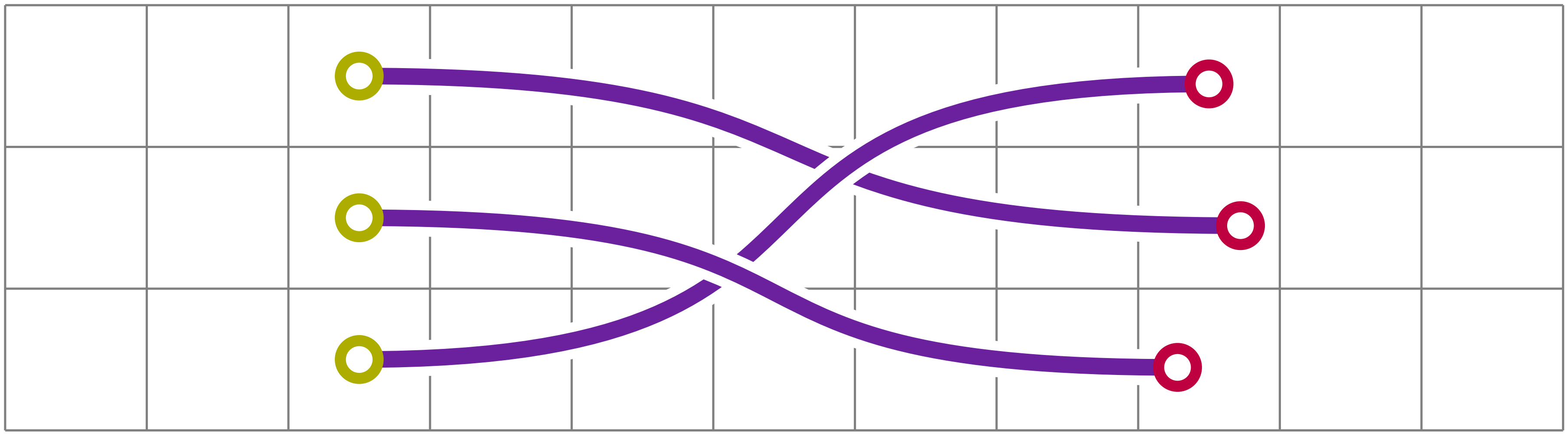

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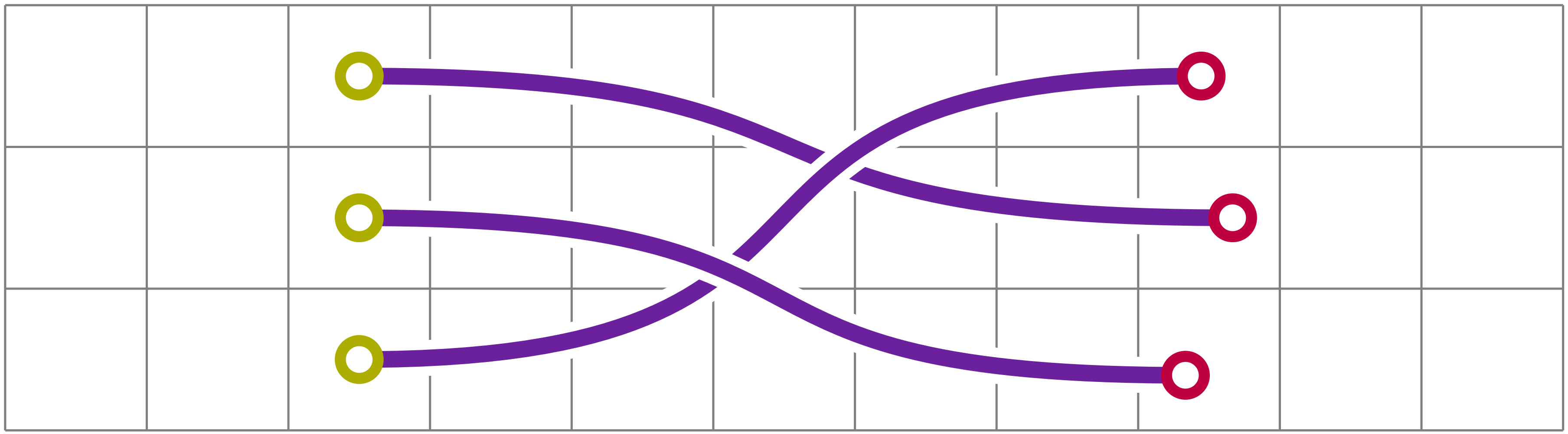

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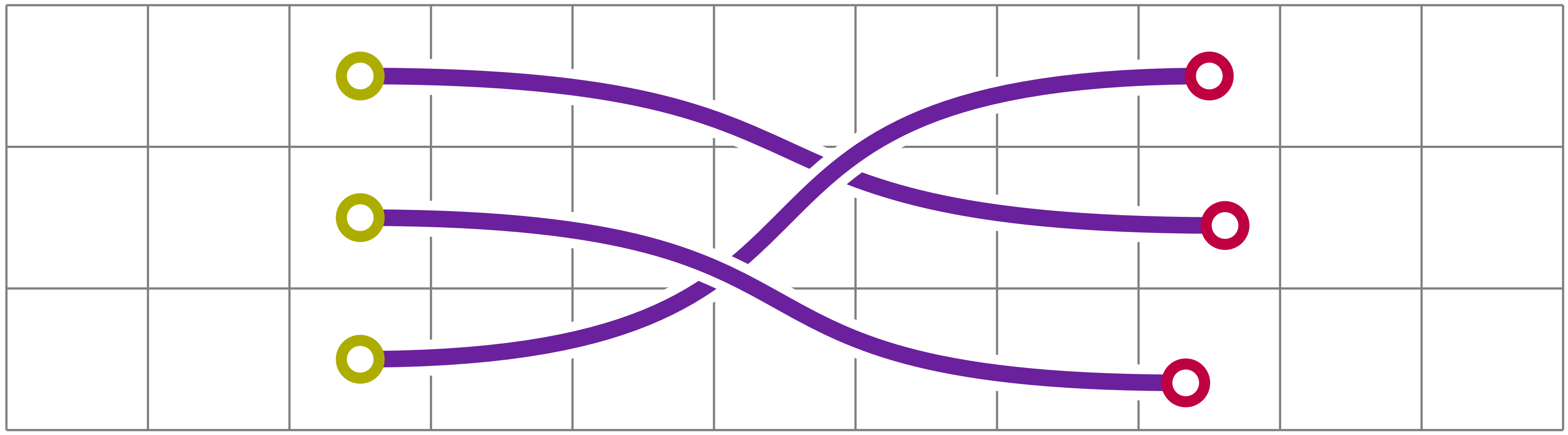

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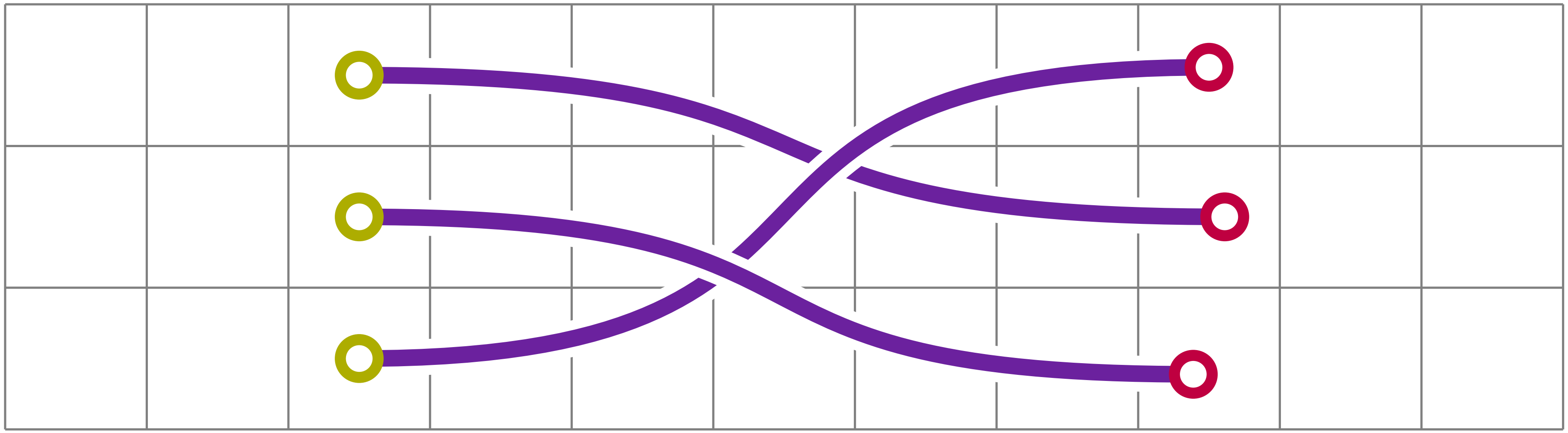

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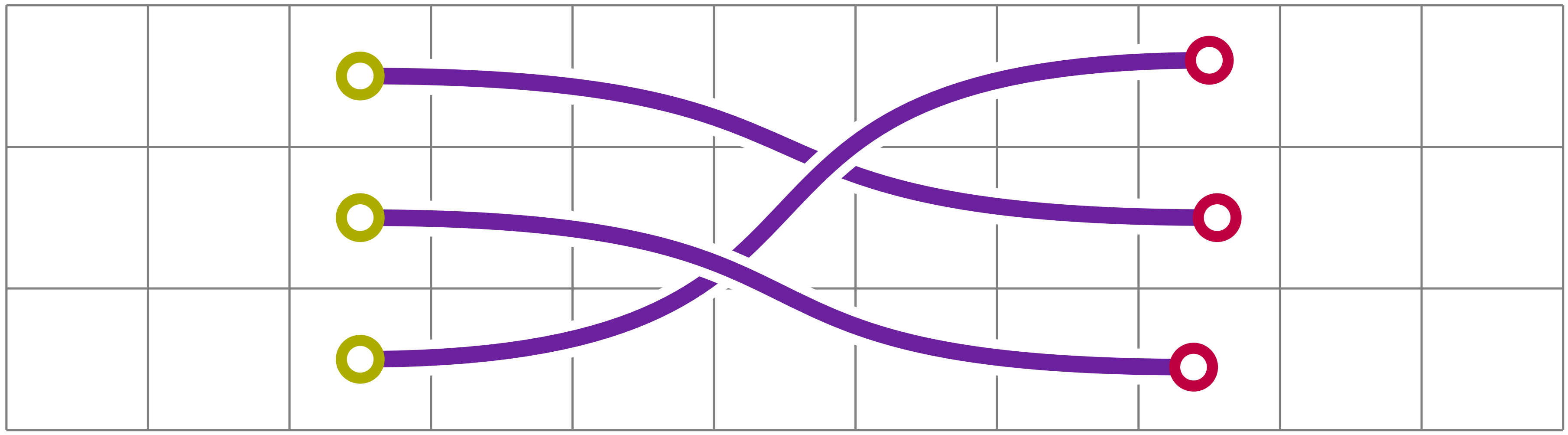

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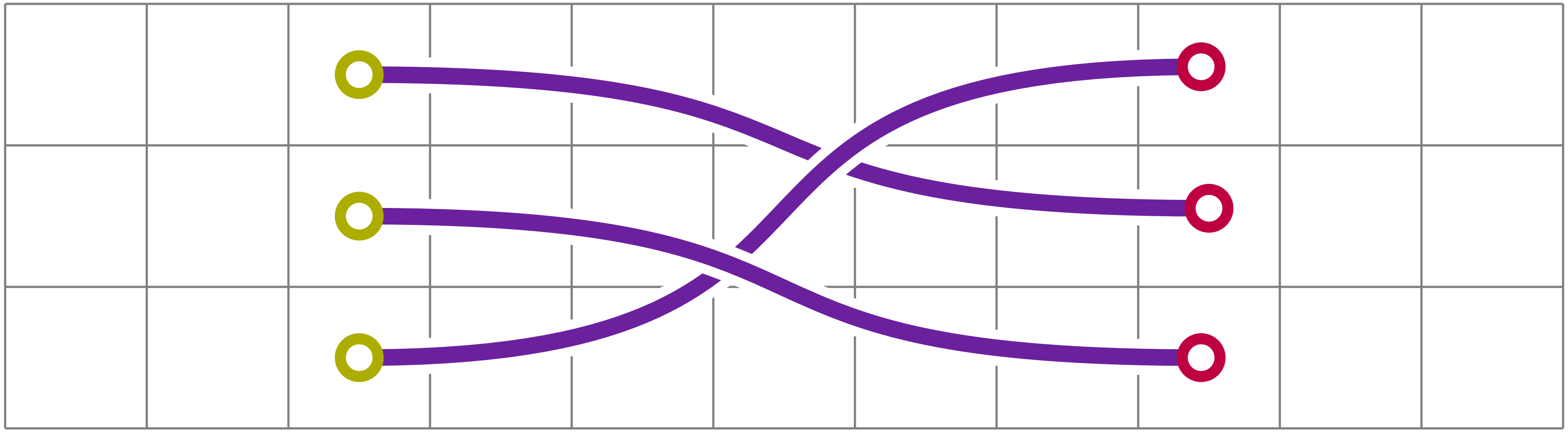

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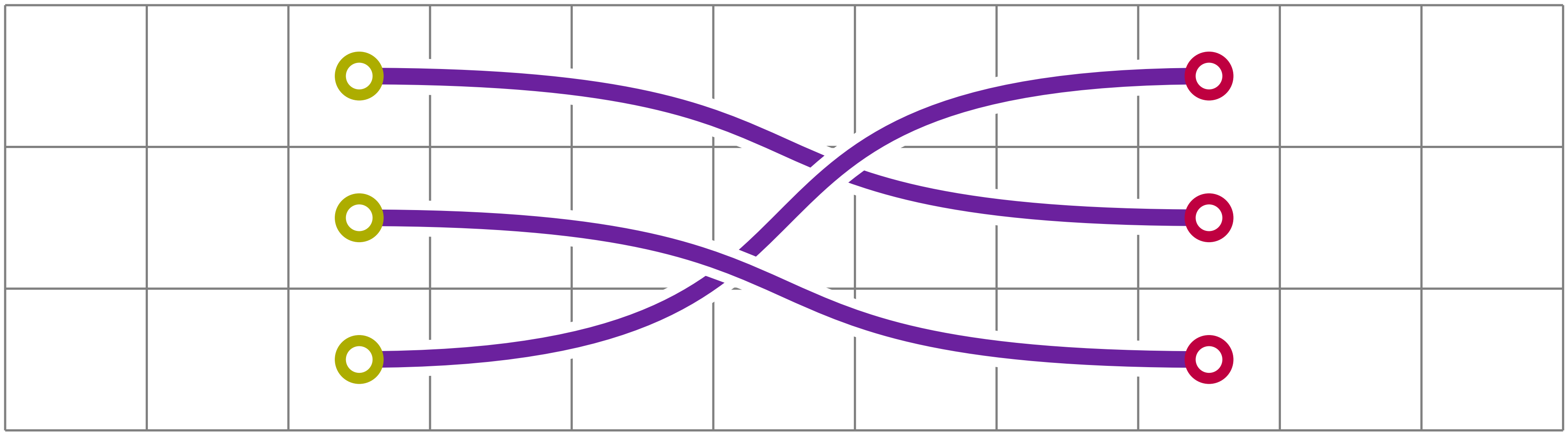

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Strategy of **Topological Quantum Computation:**
Use topologically invariant quantum processes.

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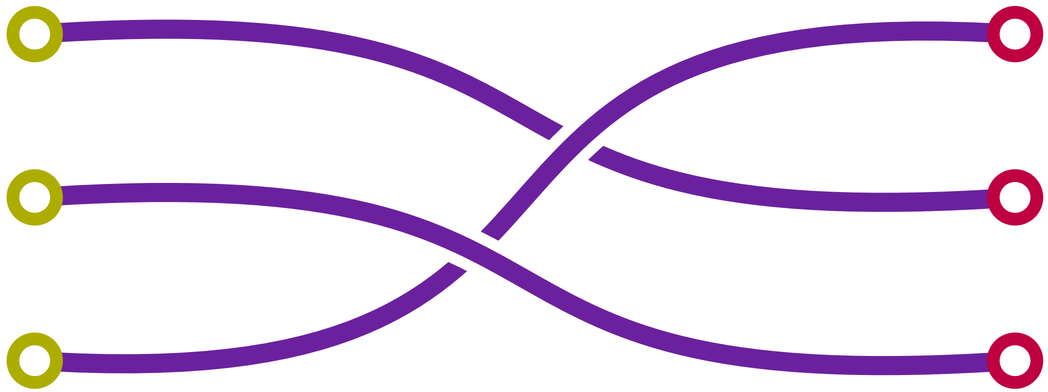

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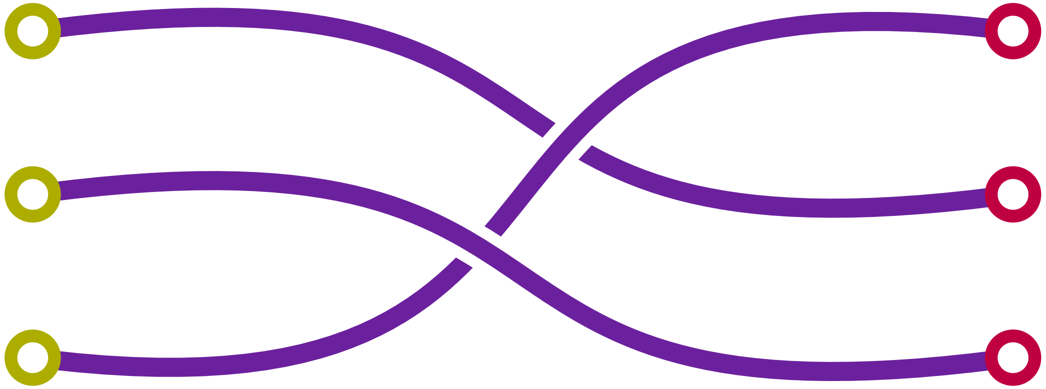

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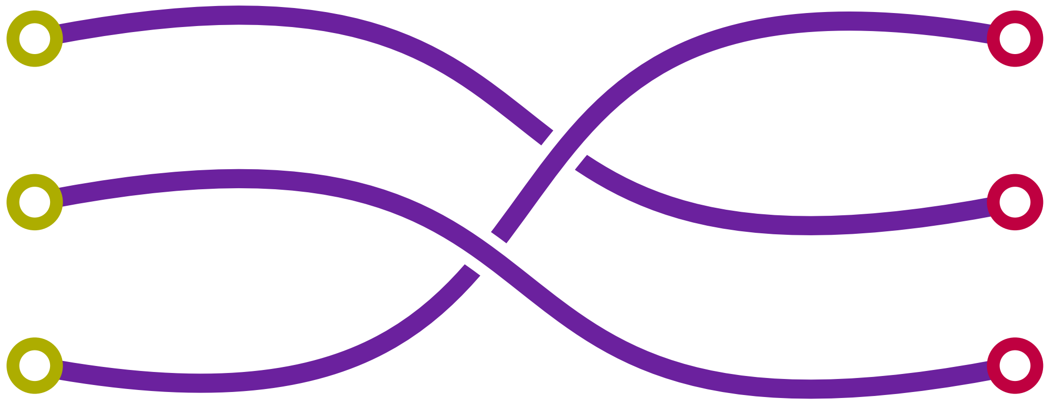

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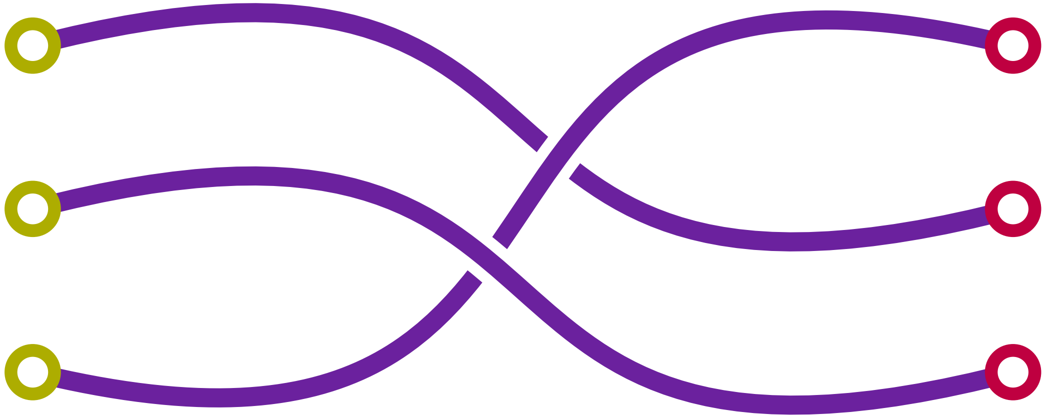

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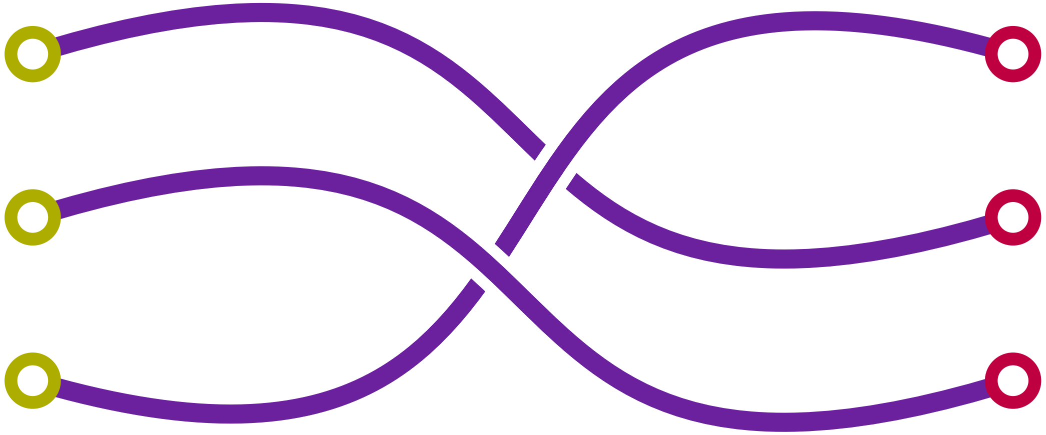

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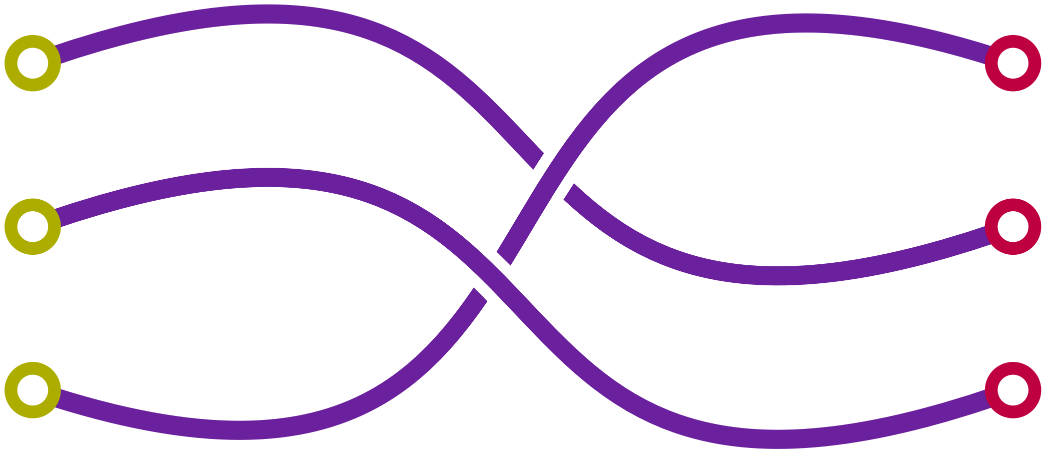

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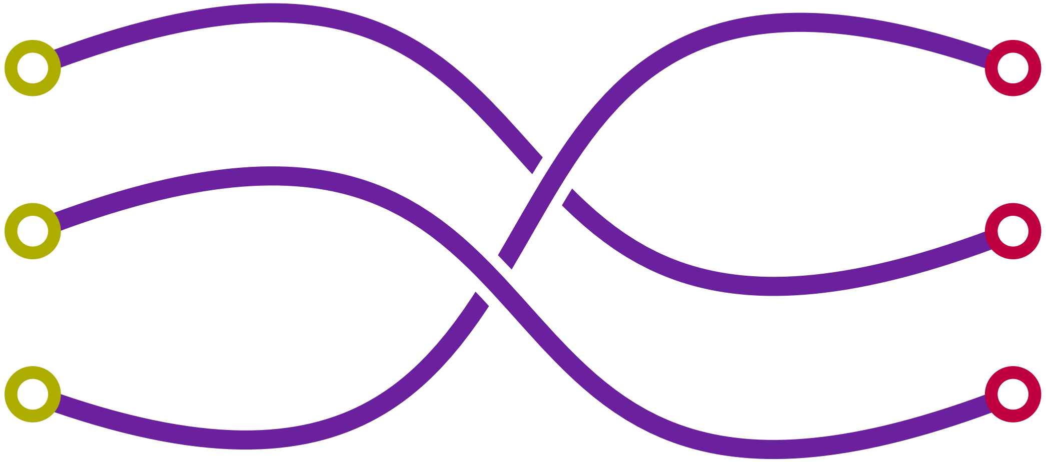

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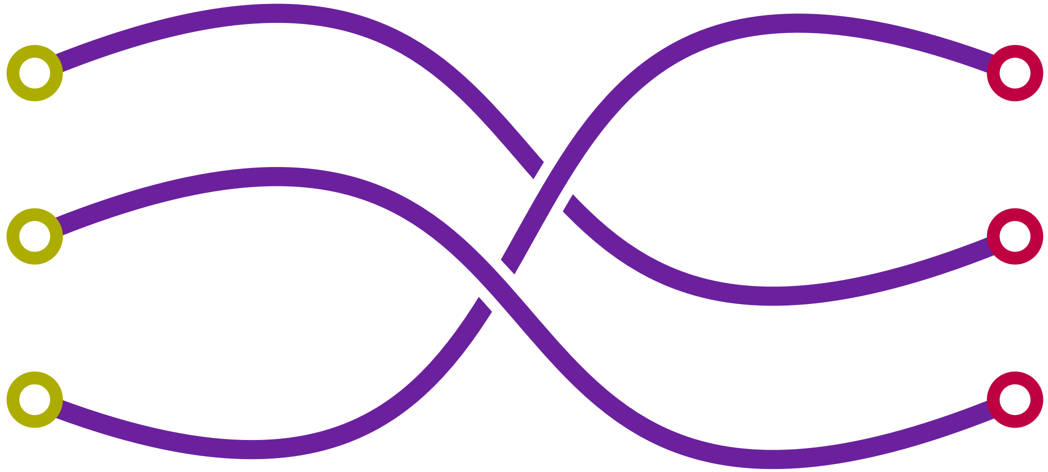

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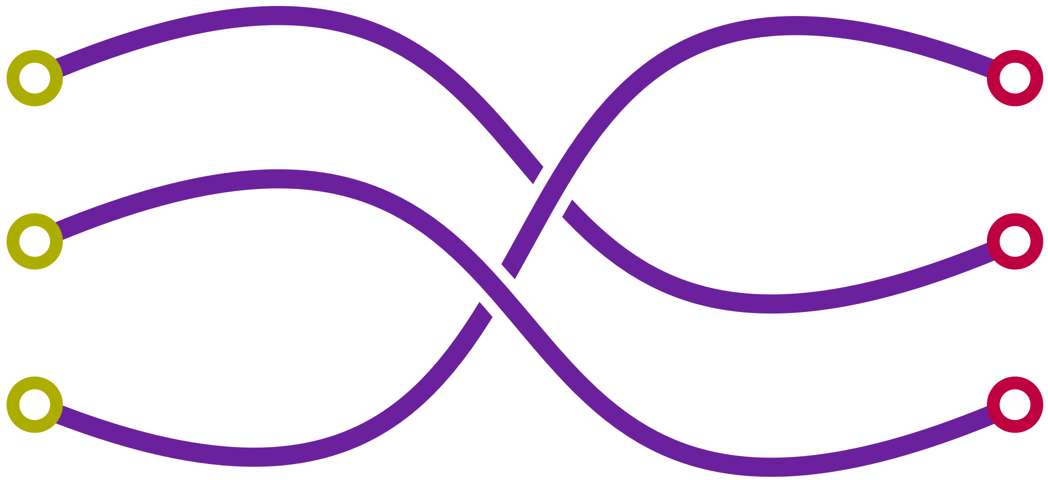

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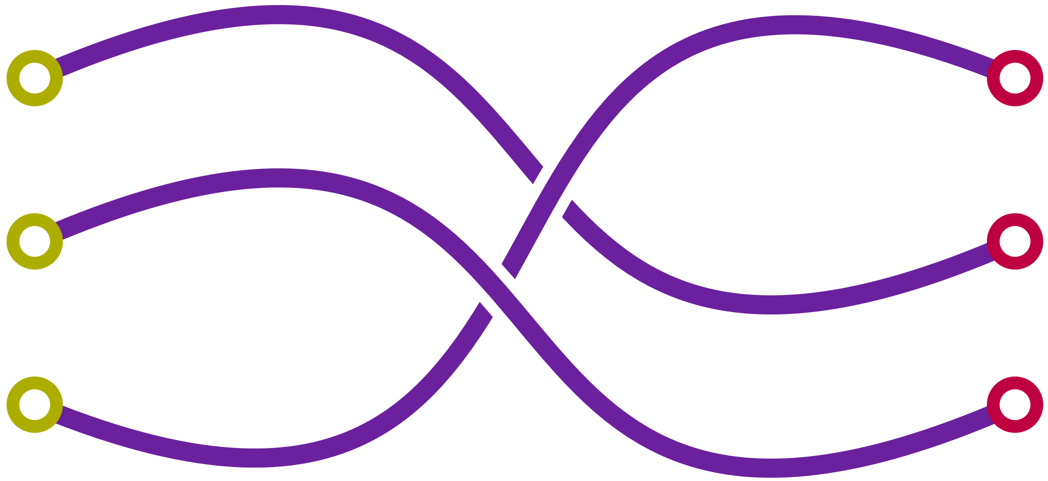

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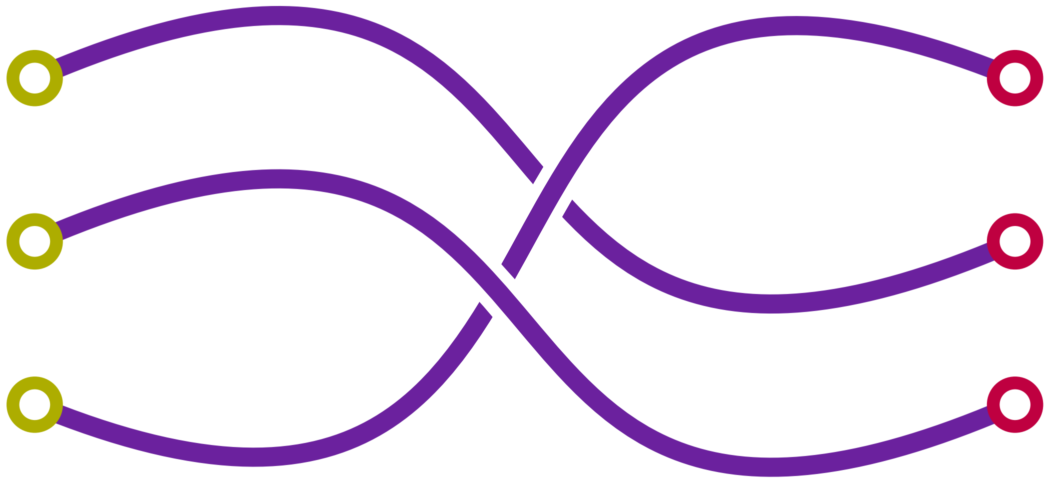

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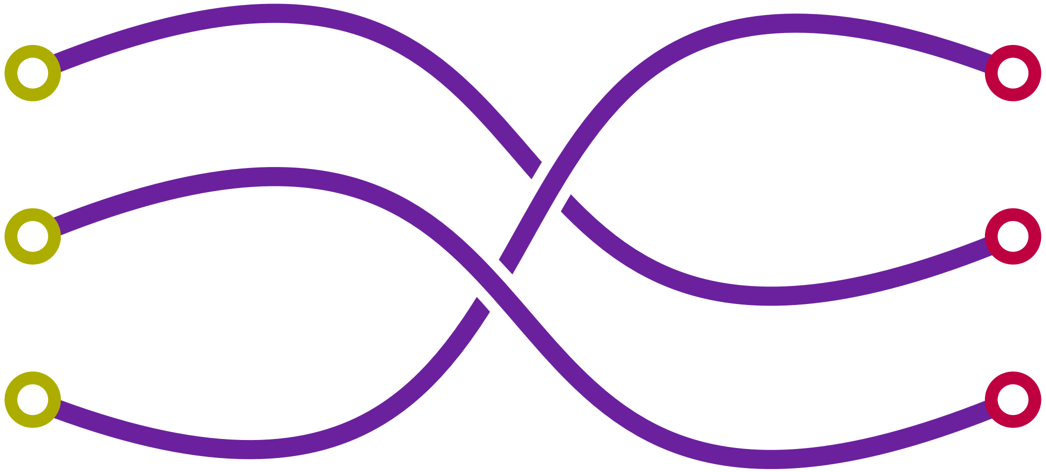

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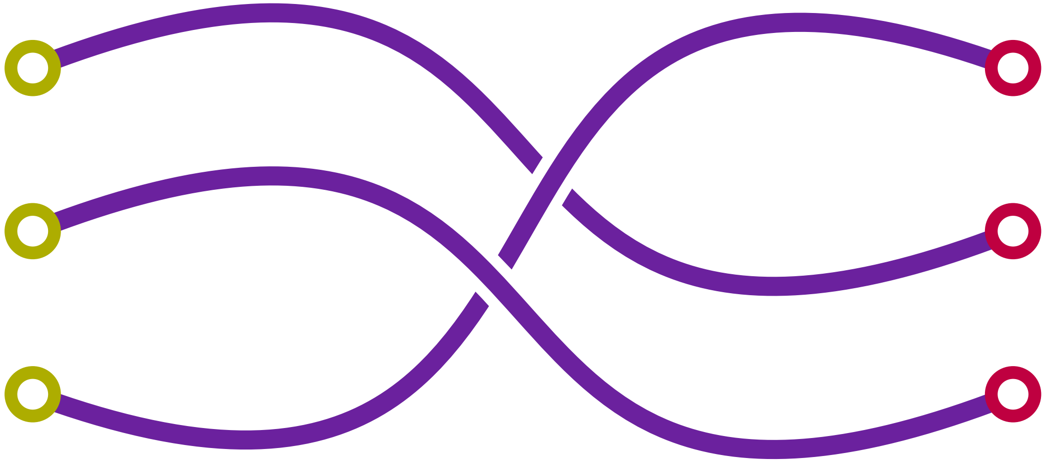

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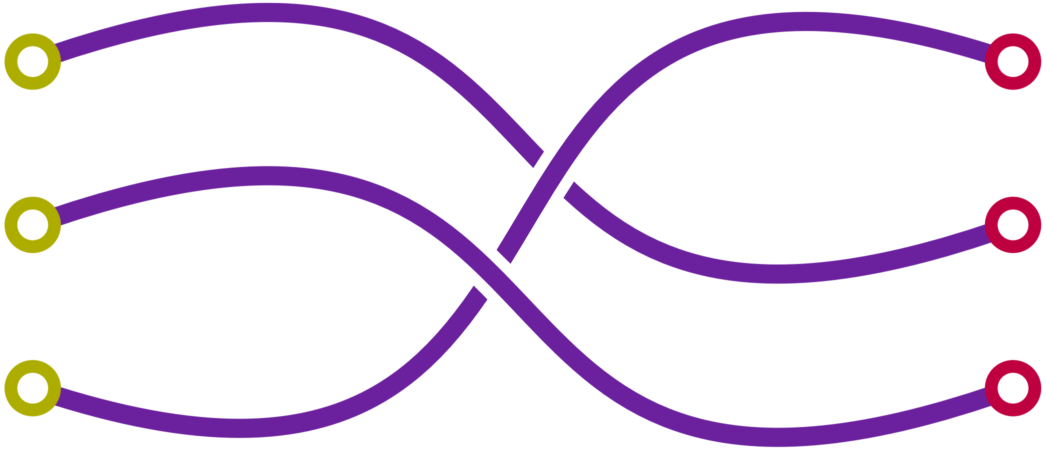

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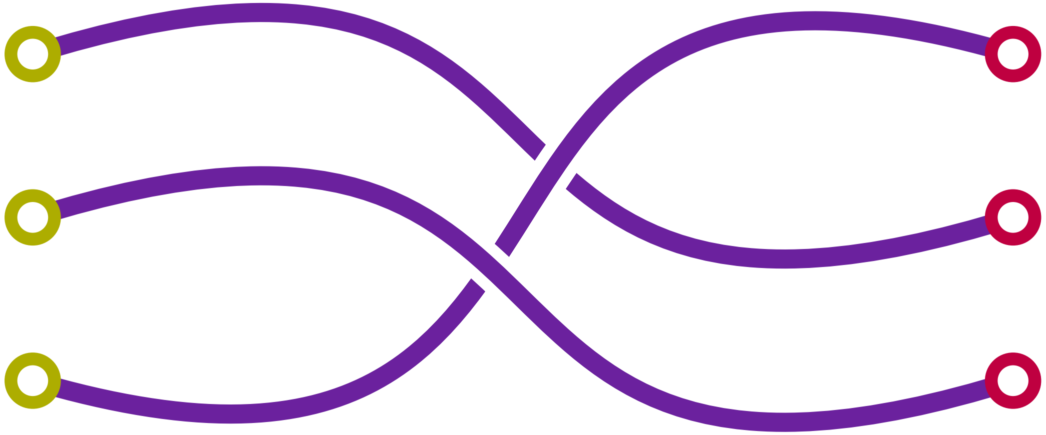

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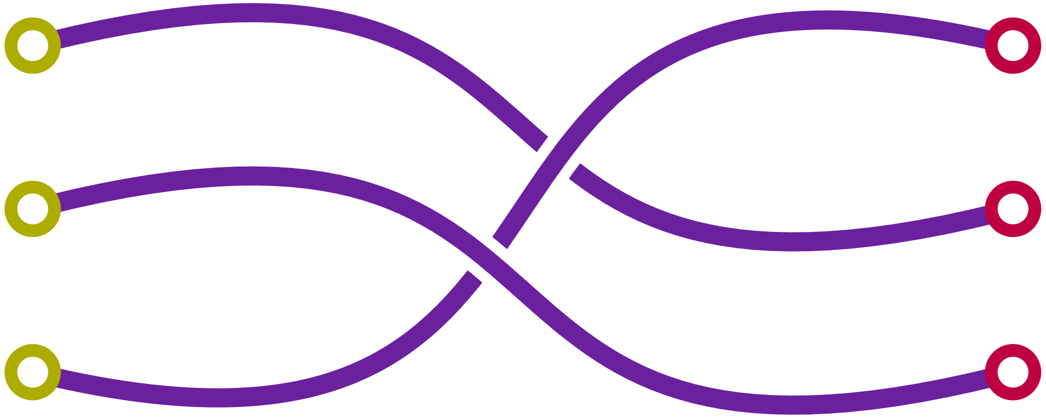

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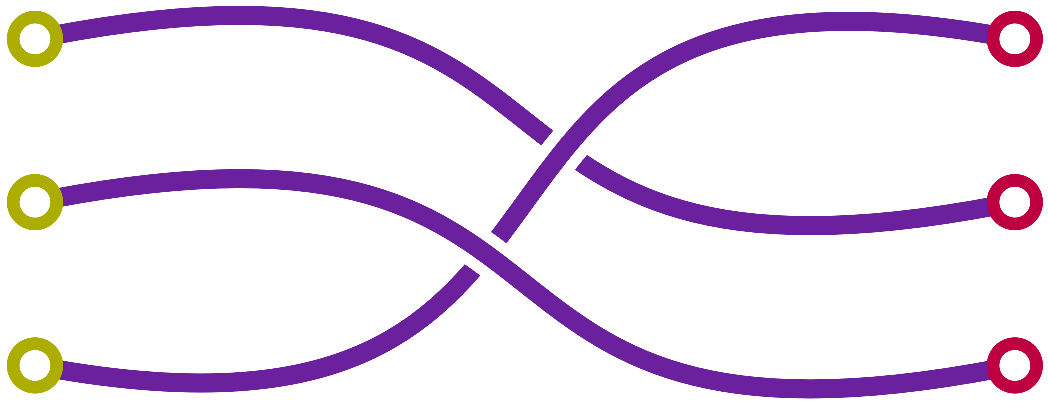

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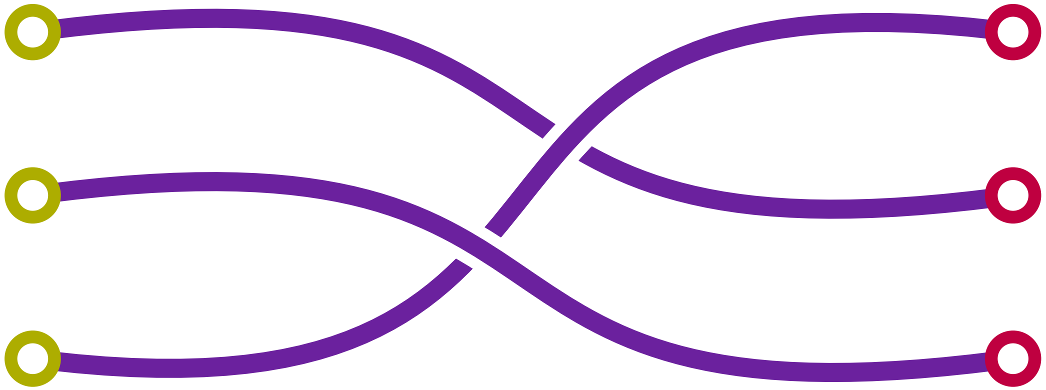

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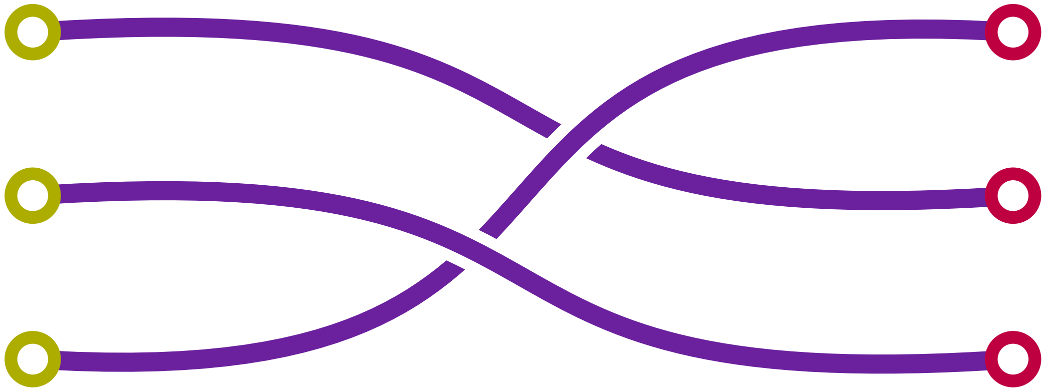

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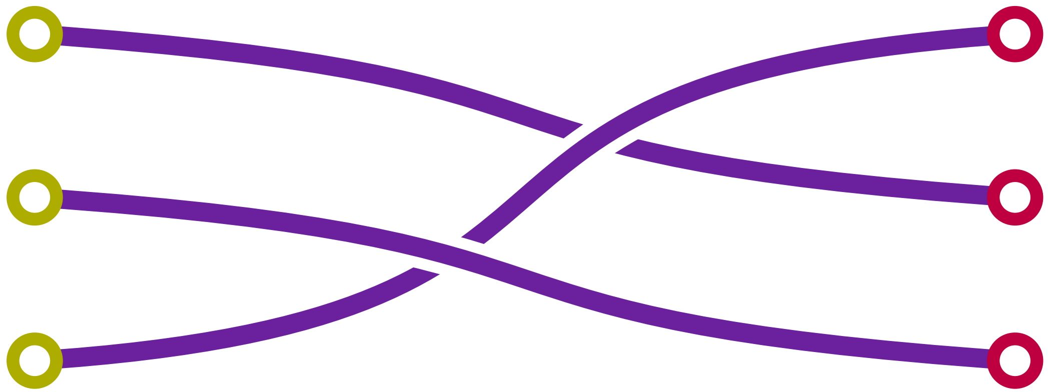

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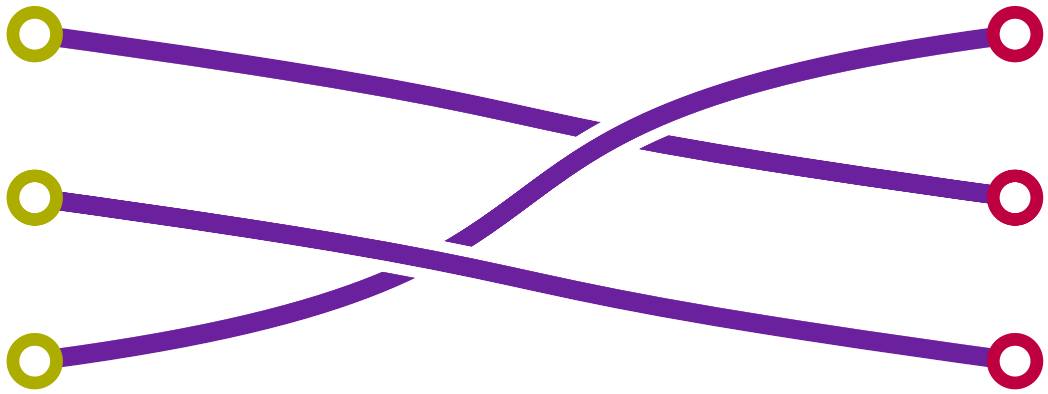

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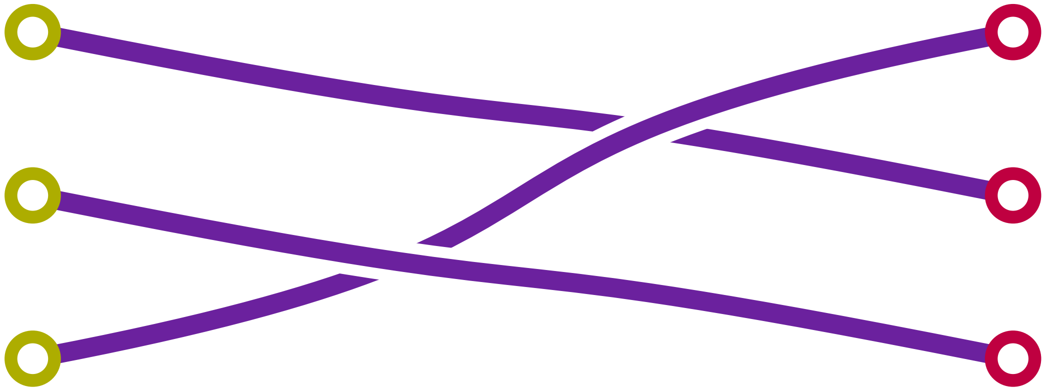

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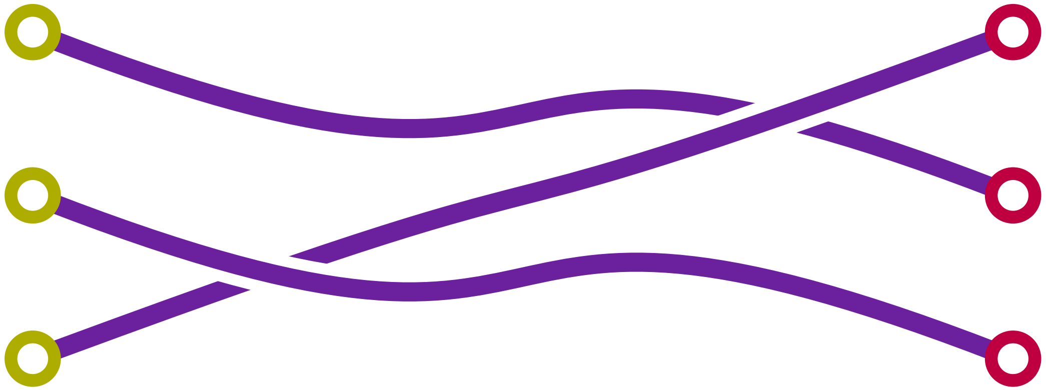

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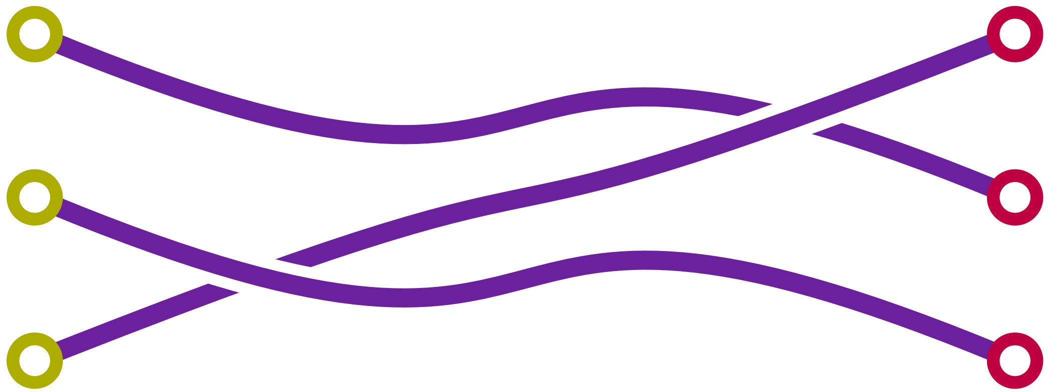

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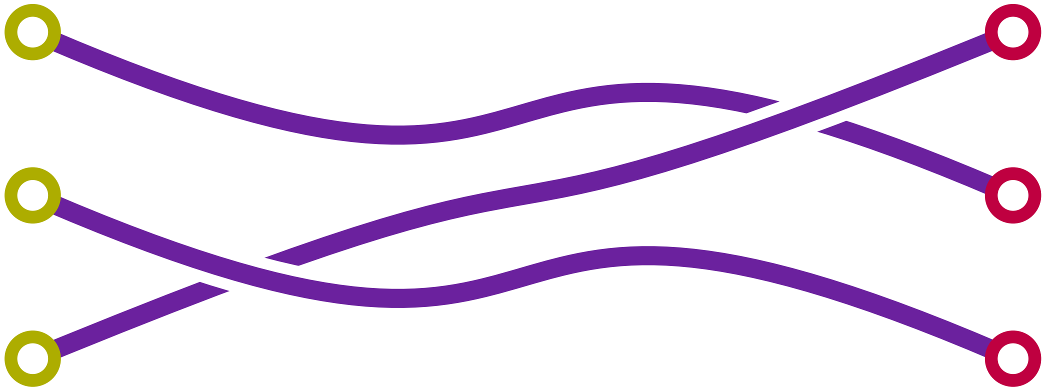

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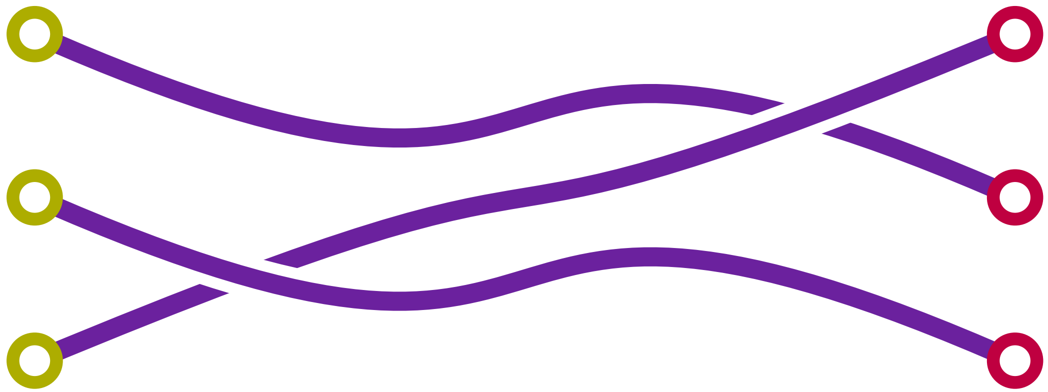

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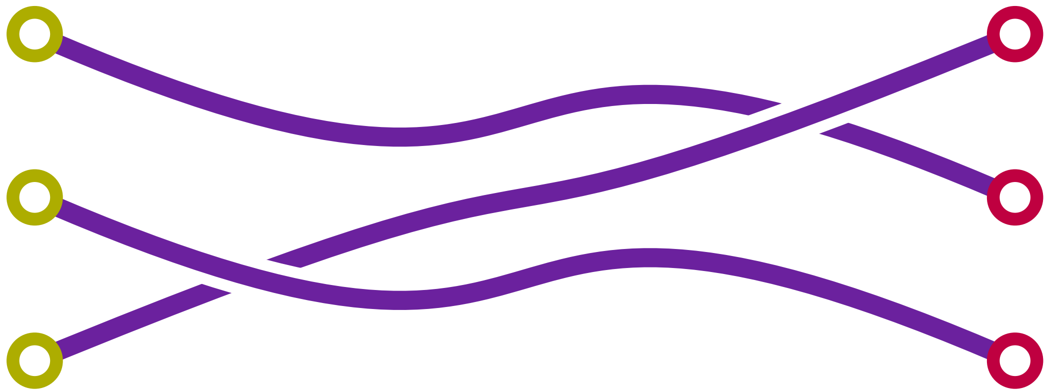

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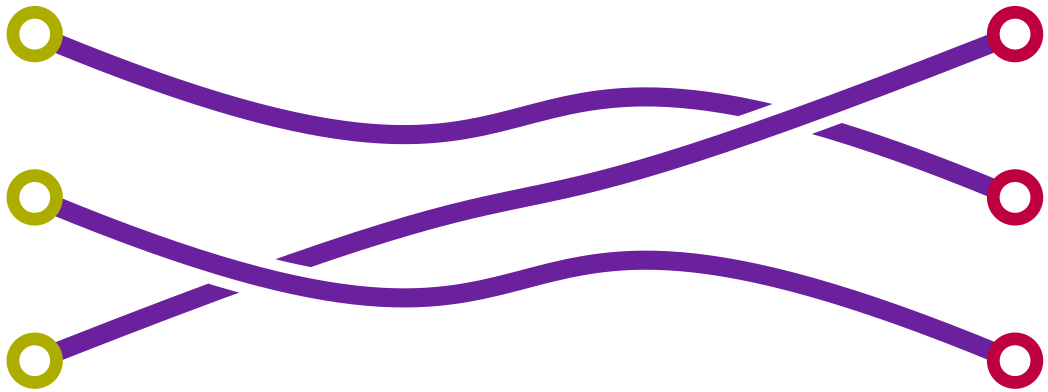

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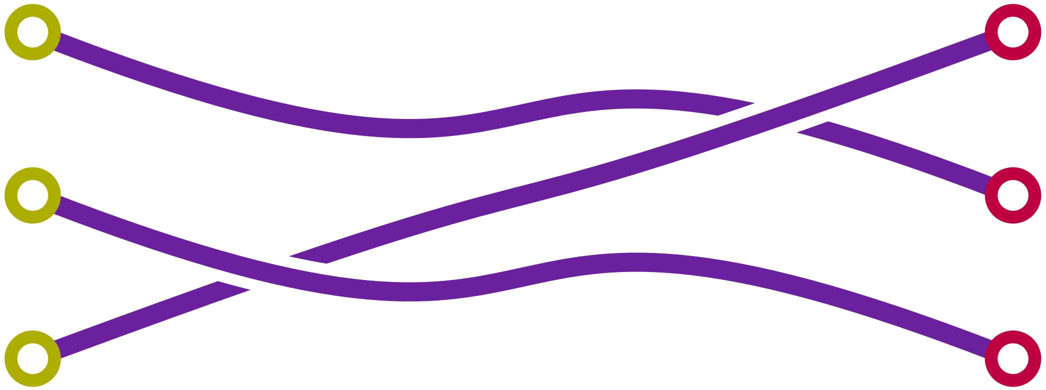

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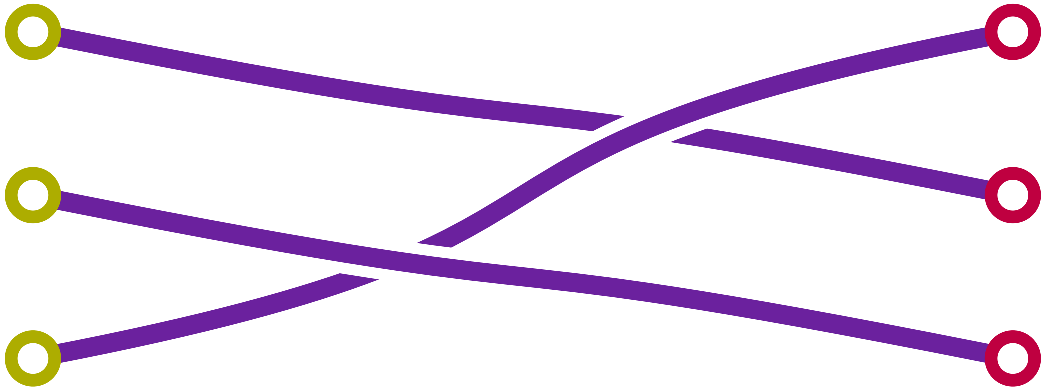

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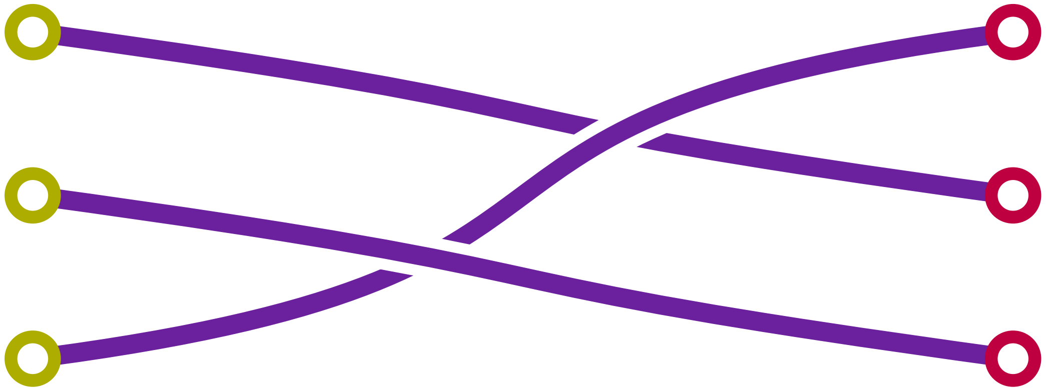

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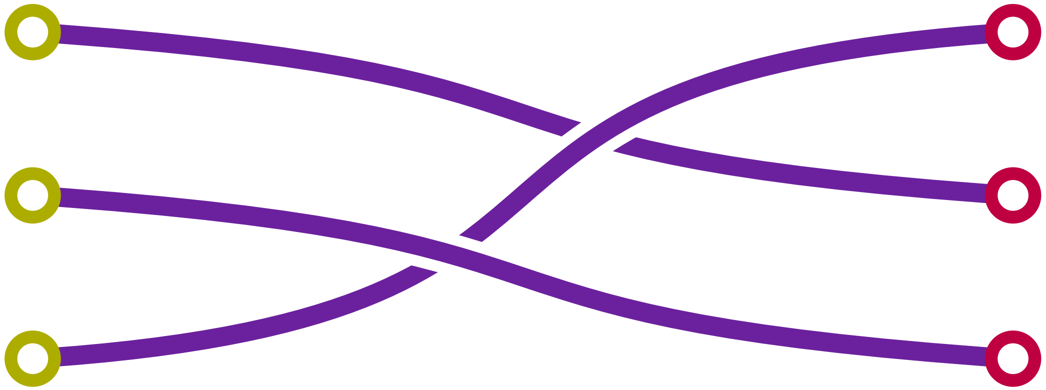

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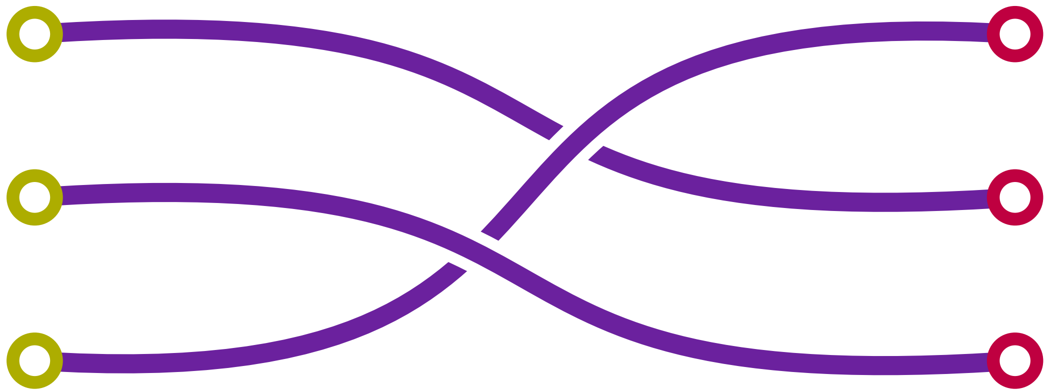

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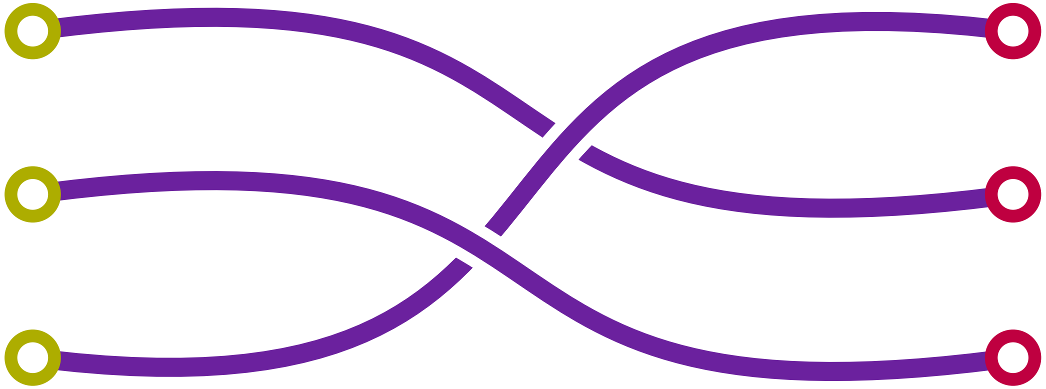

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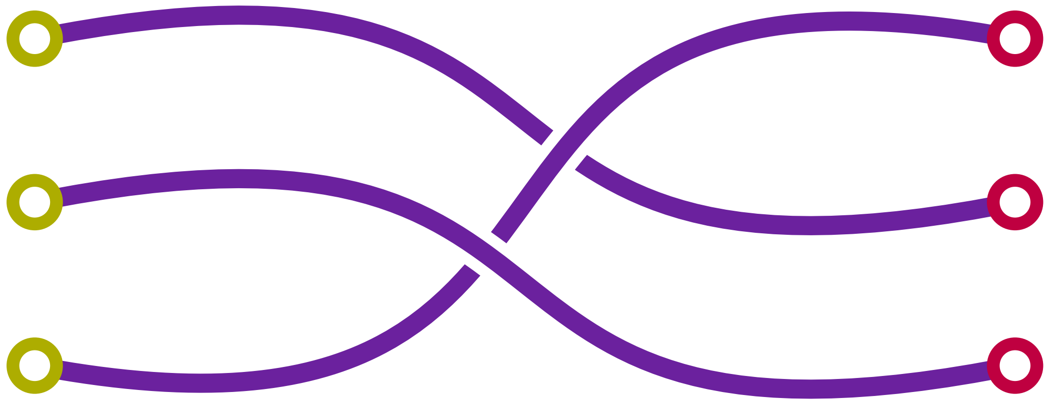

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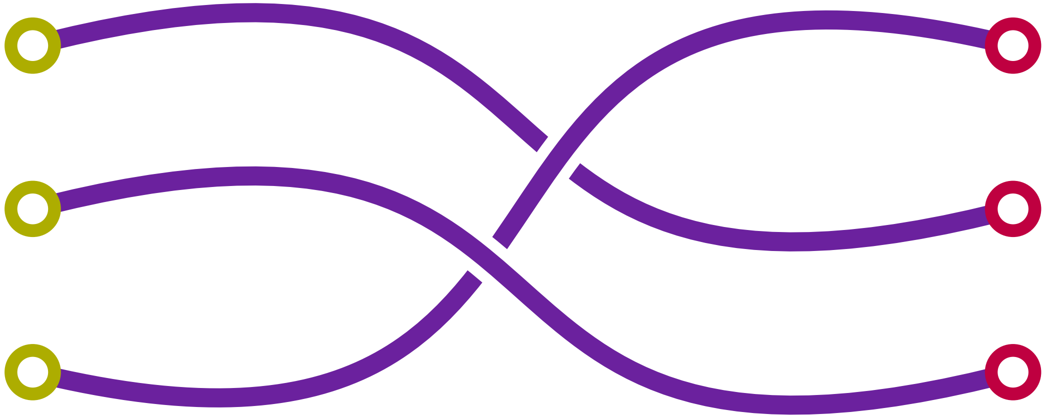

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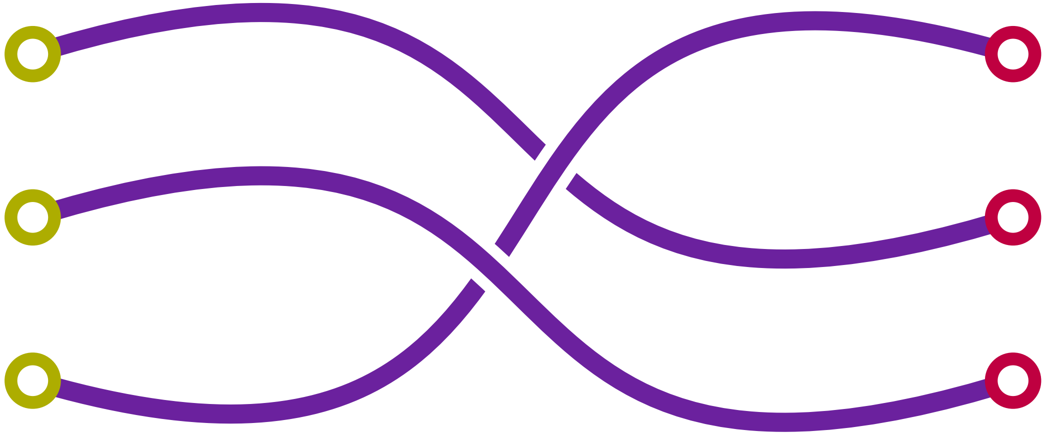

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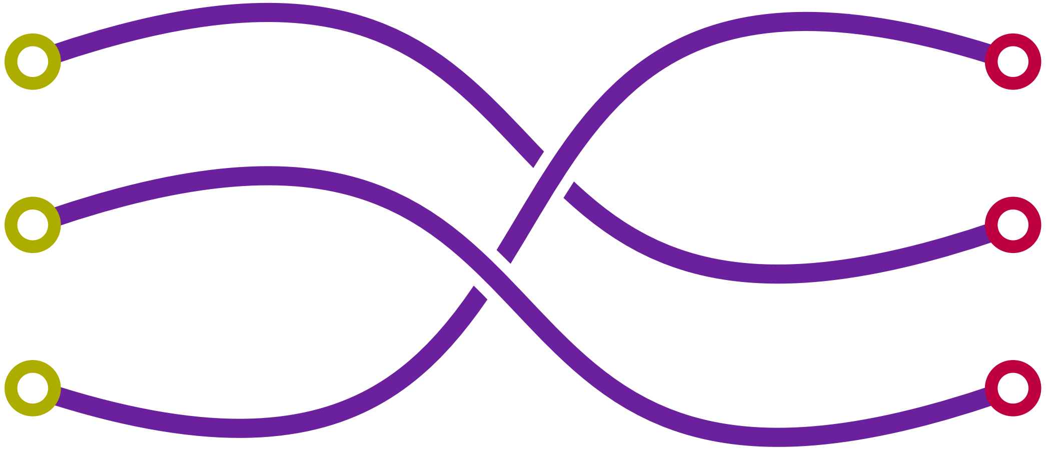

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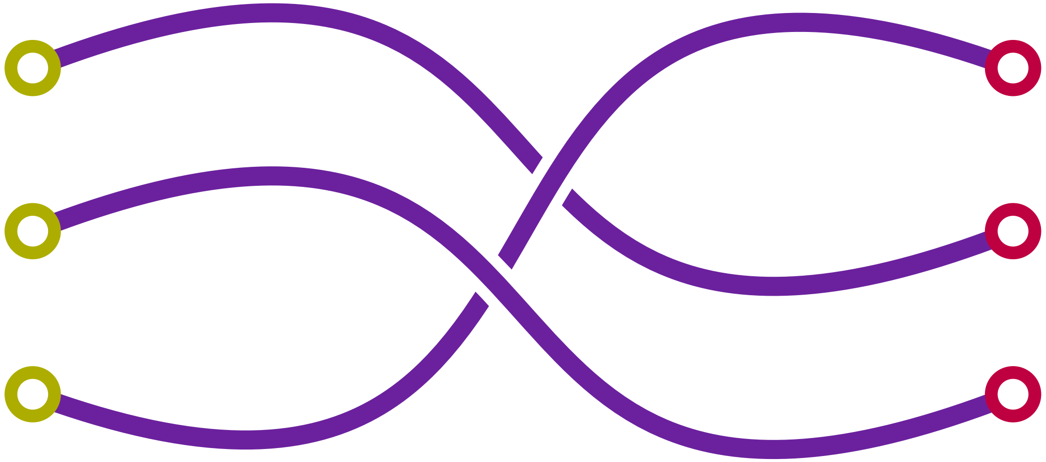

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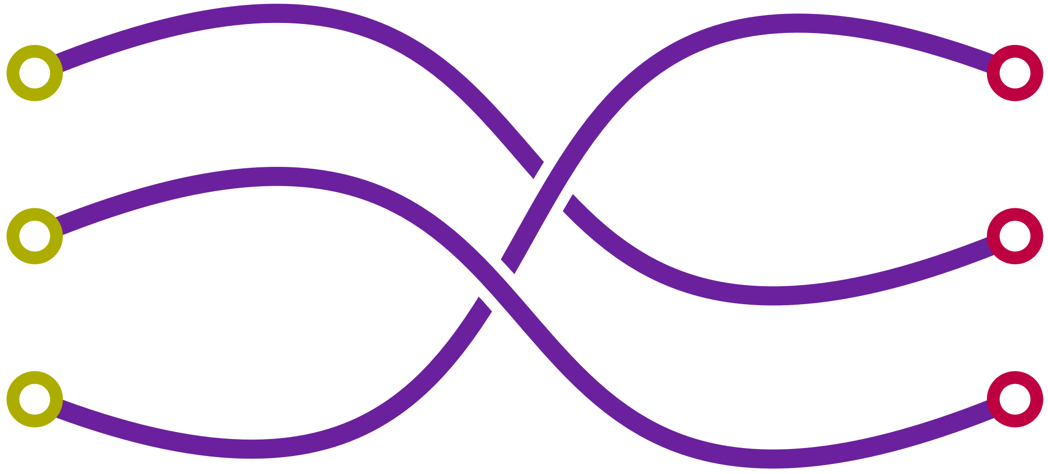

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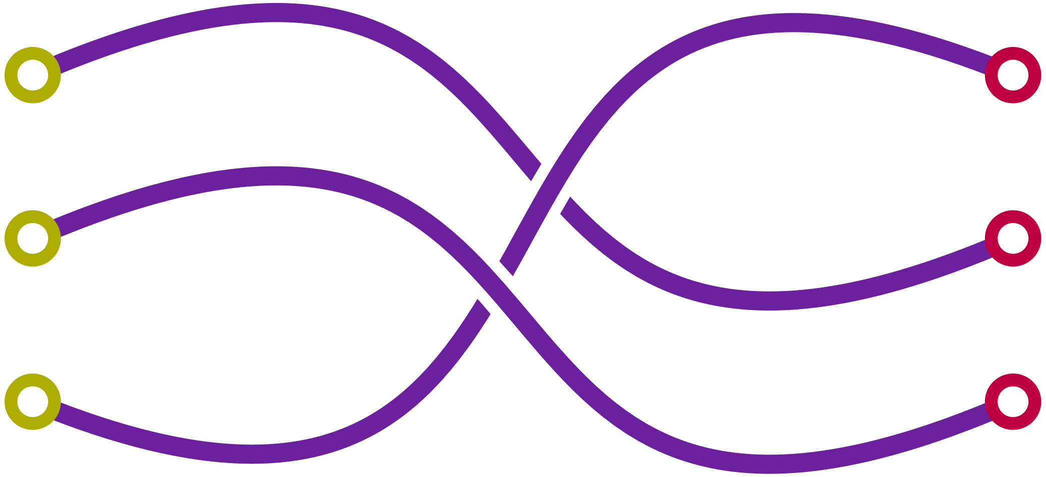

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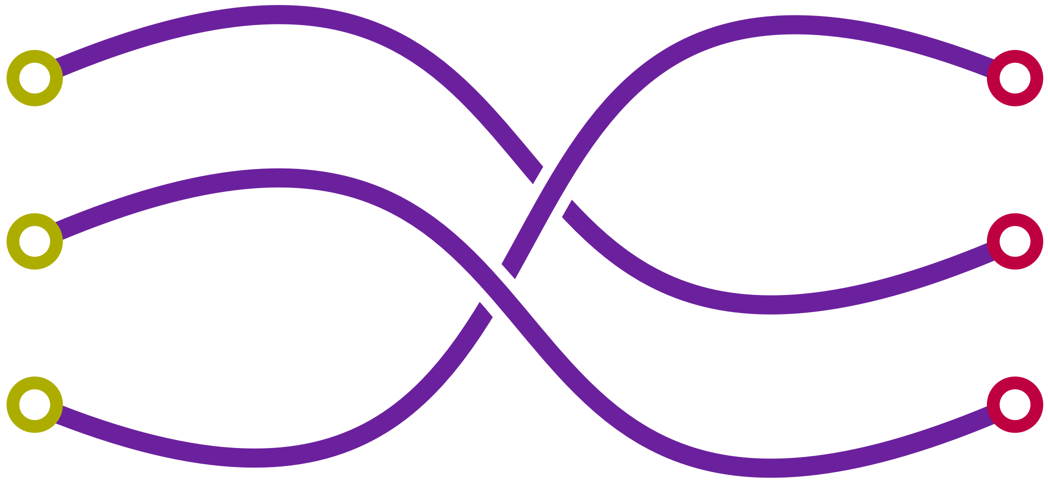

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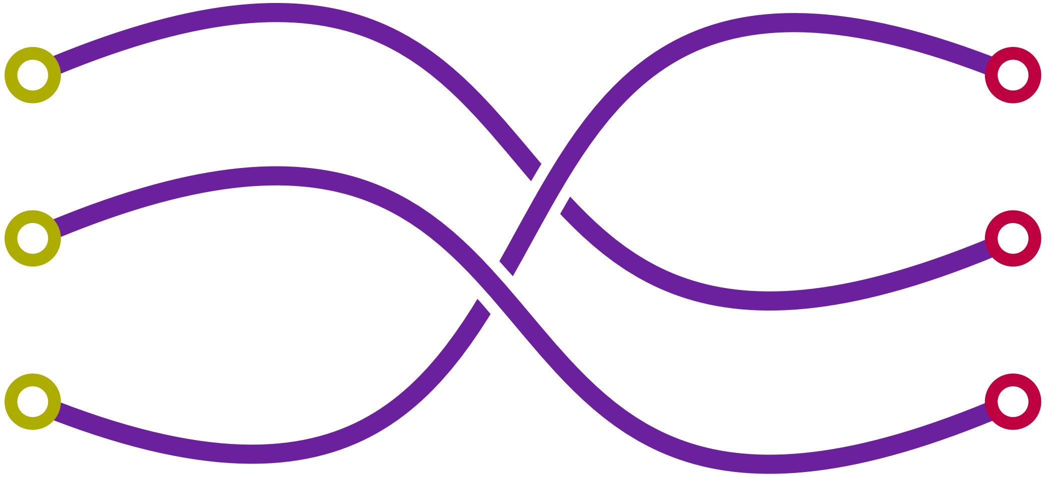

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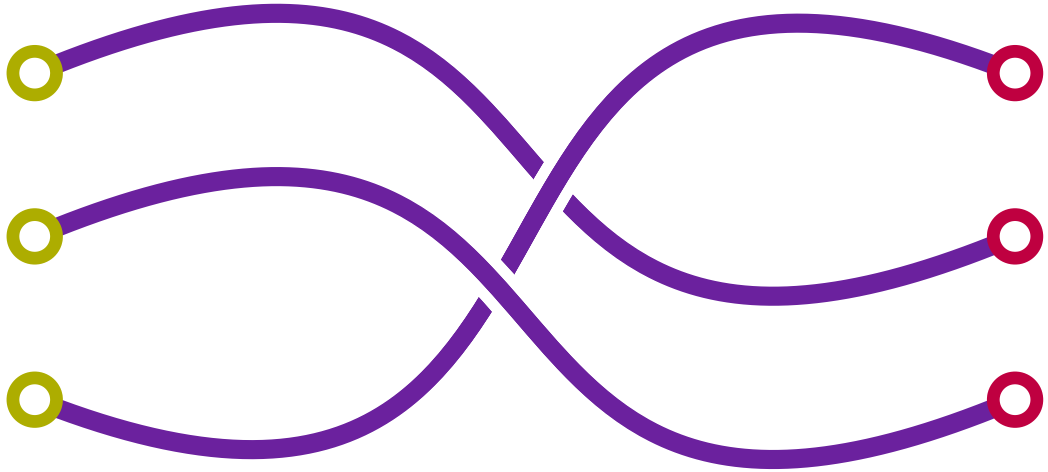

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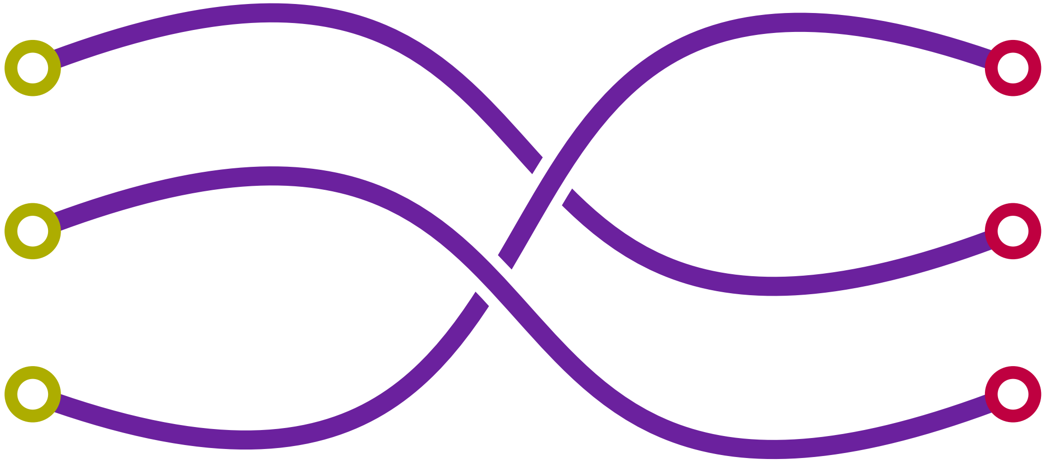

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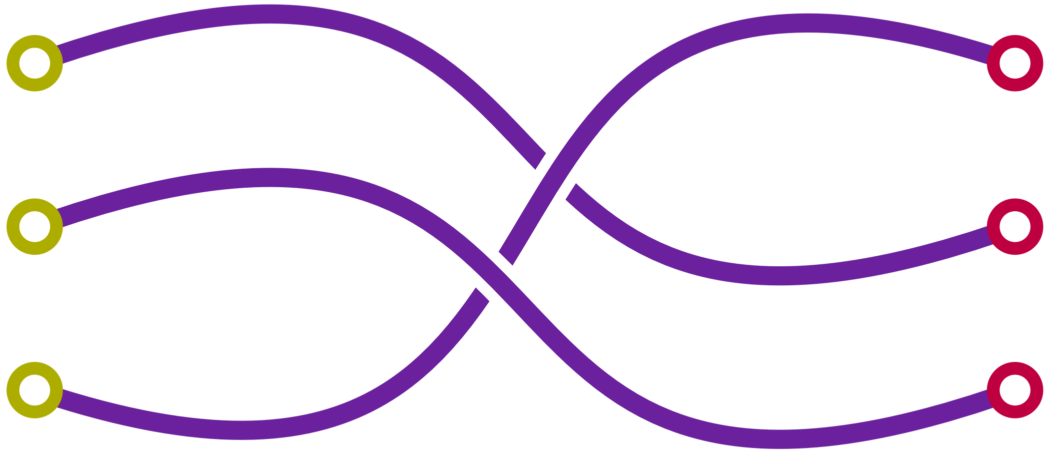

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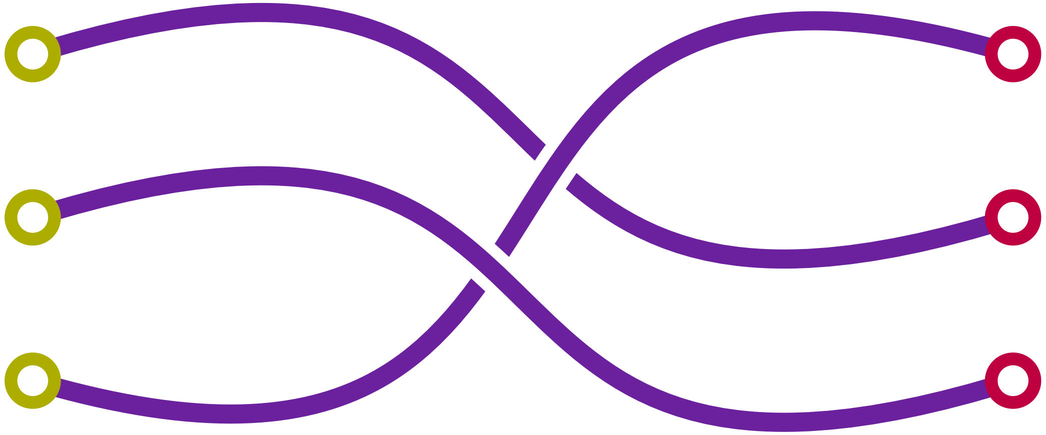

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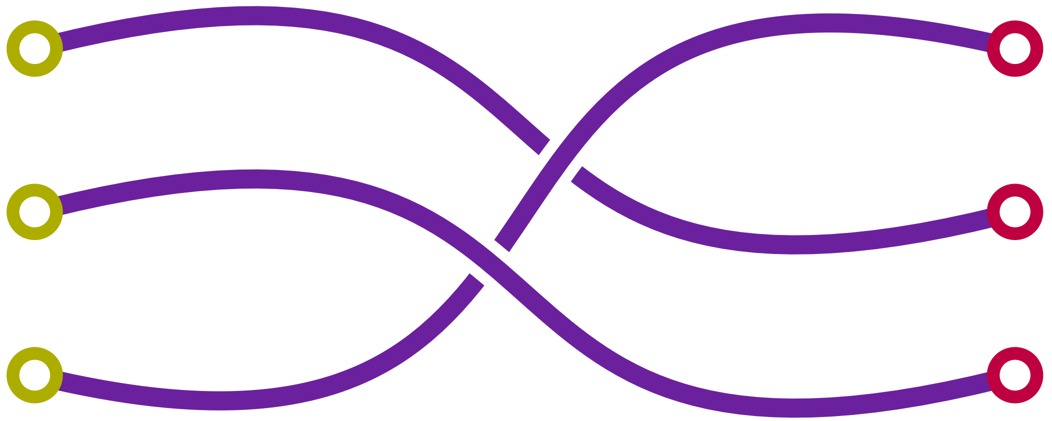

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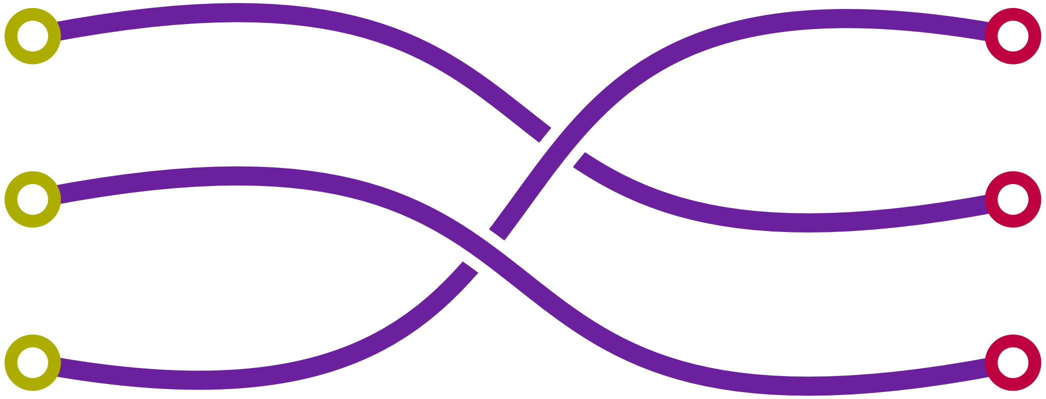

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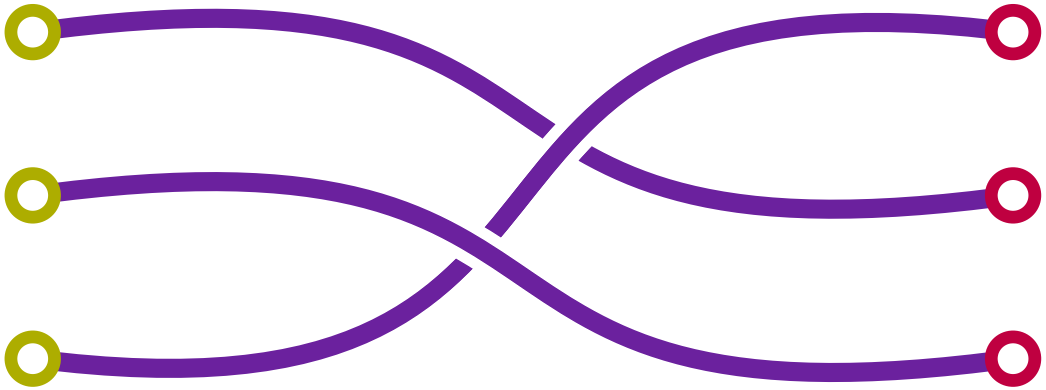

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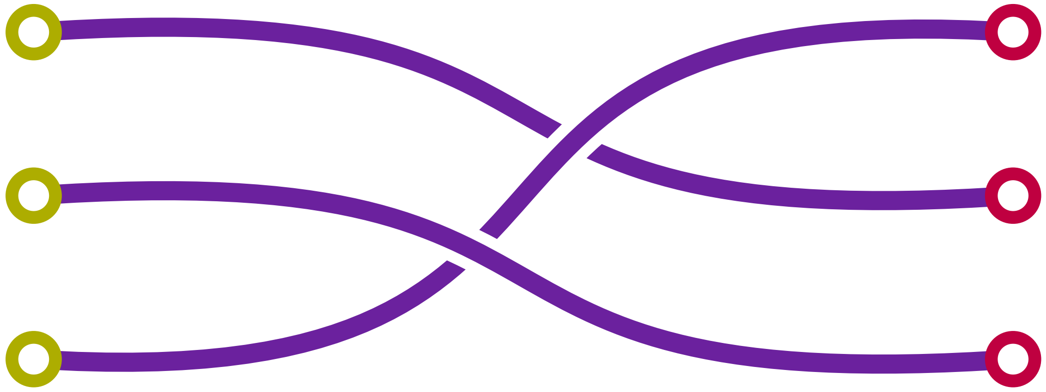

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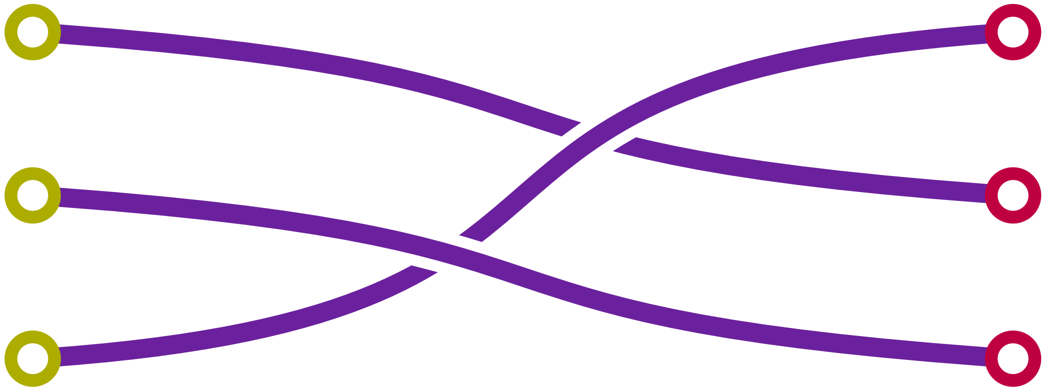

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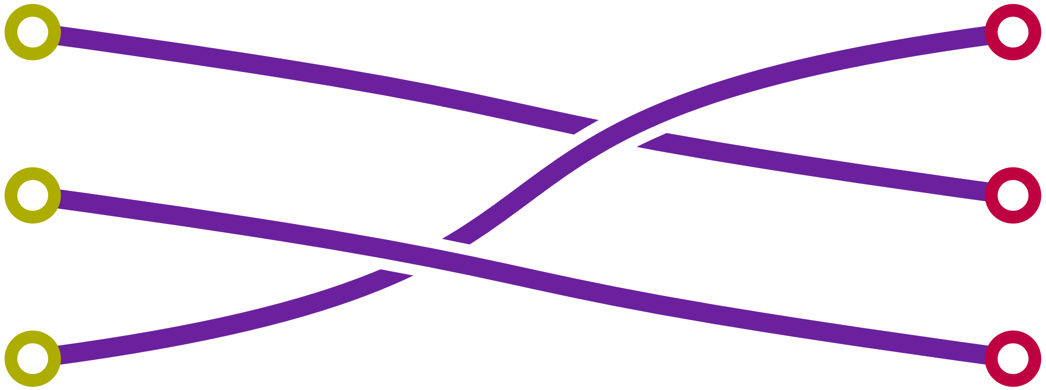

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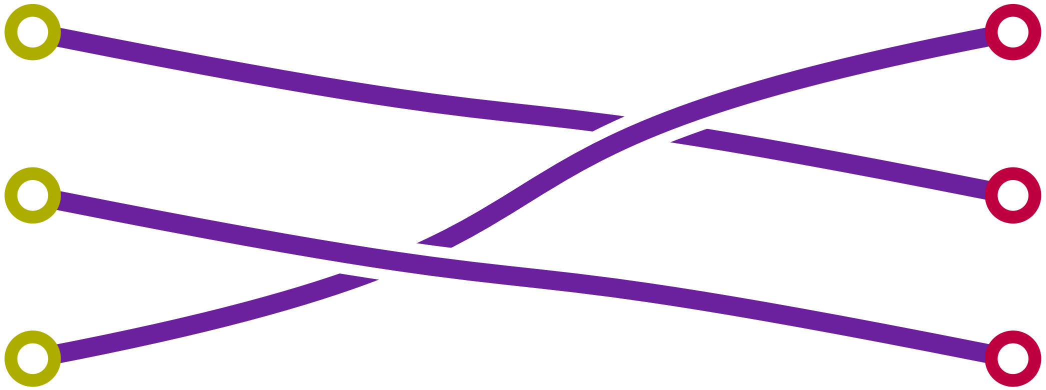

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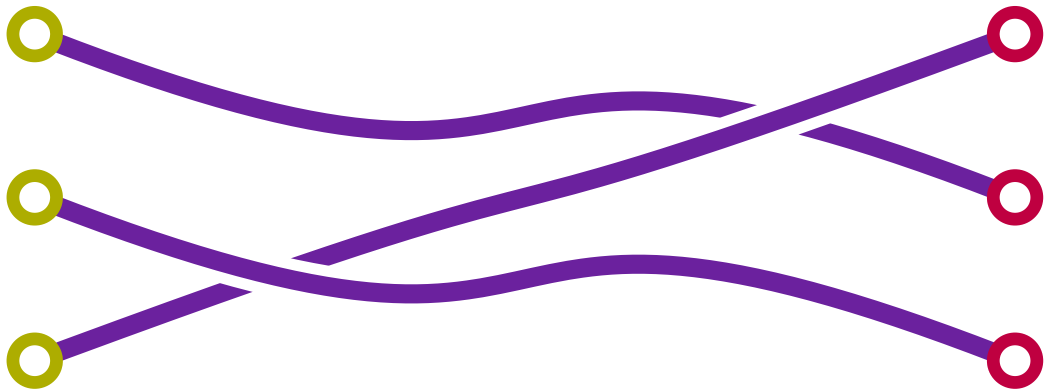

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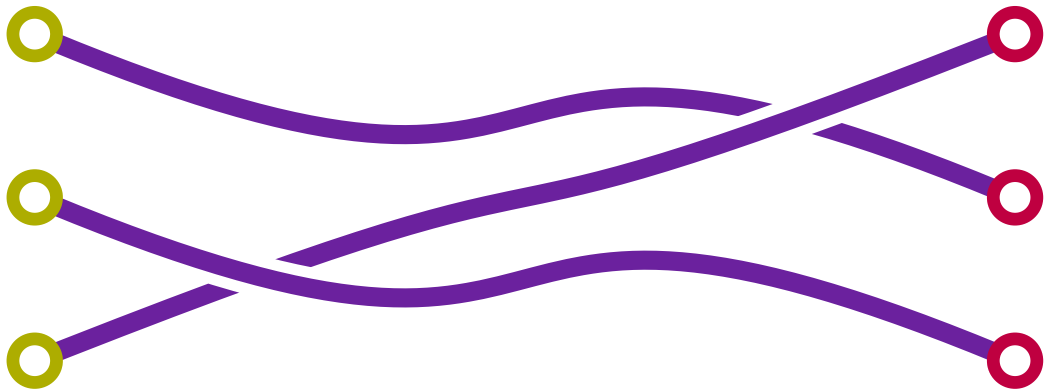

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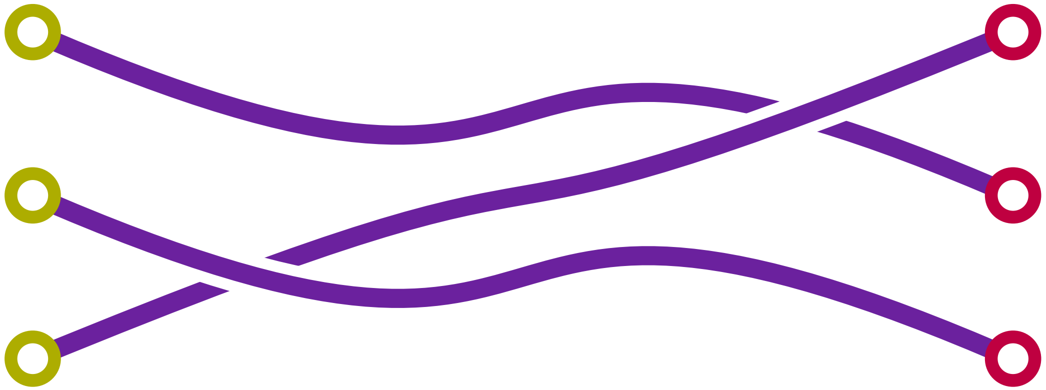

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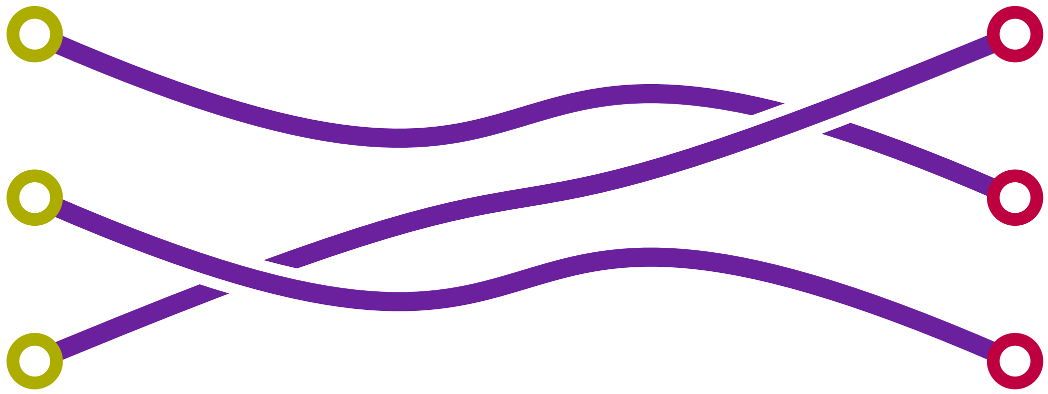

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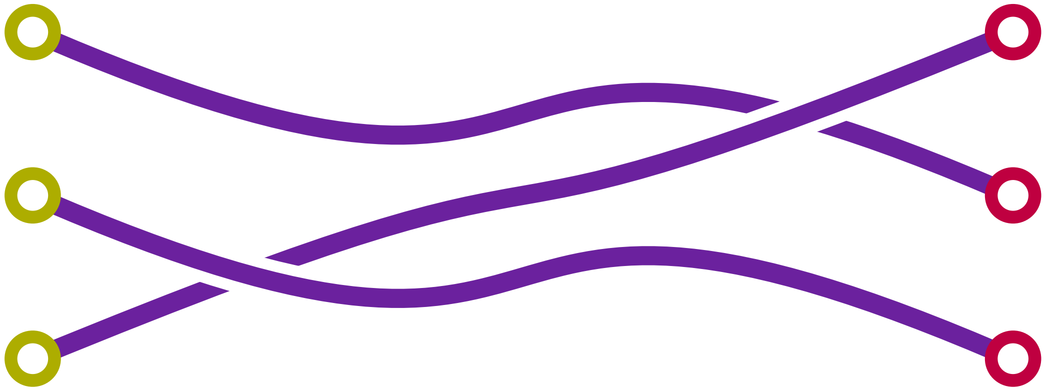

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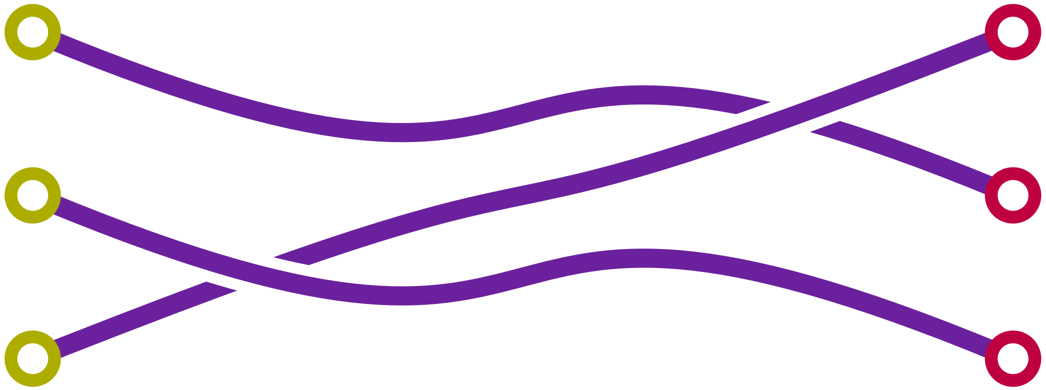

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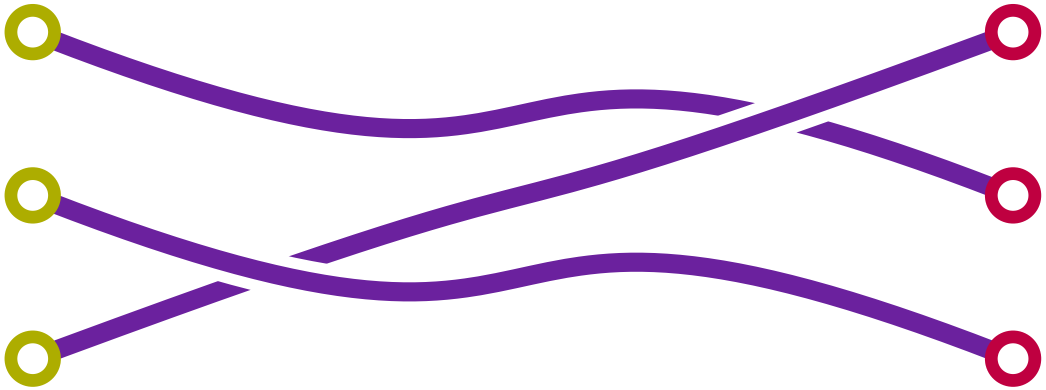

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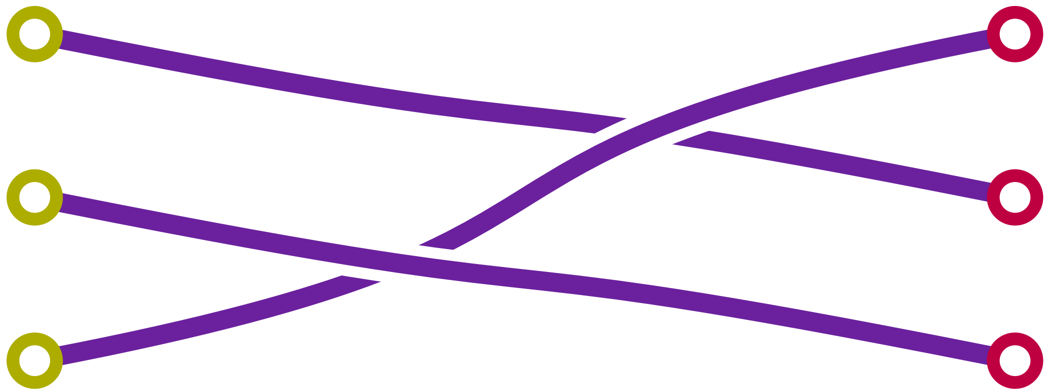

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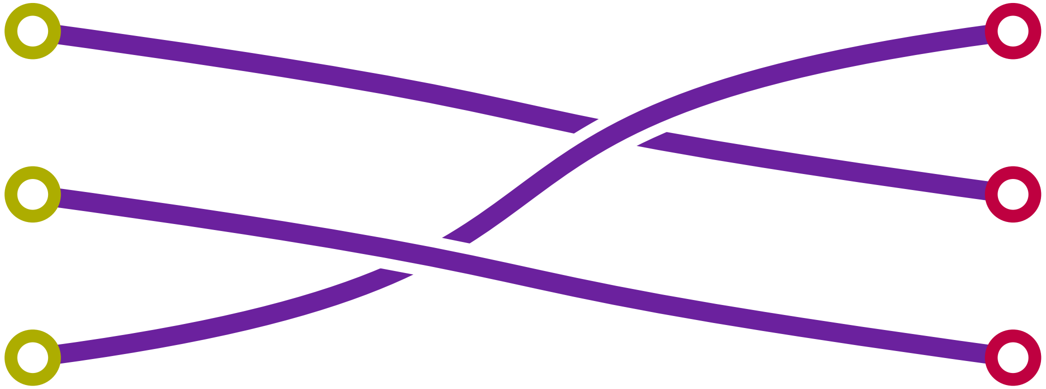

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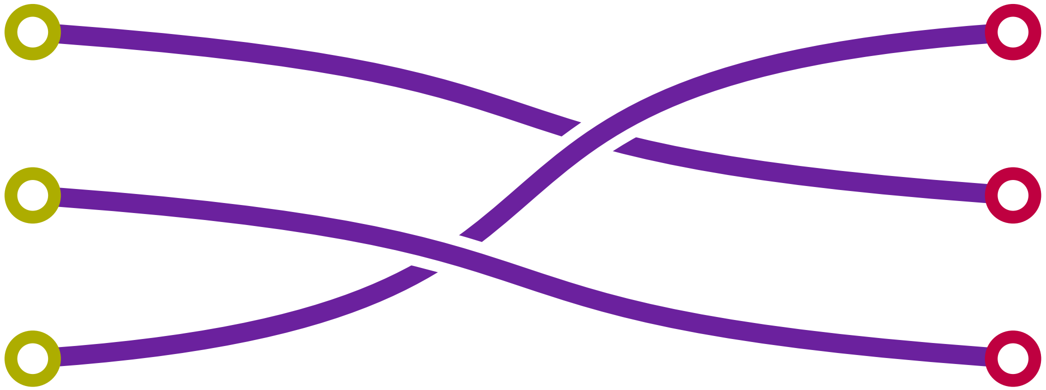

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But (why and where) do such processes even exist?

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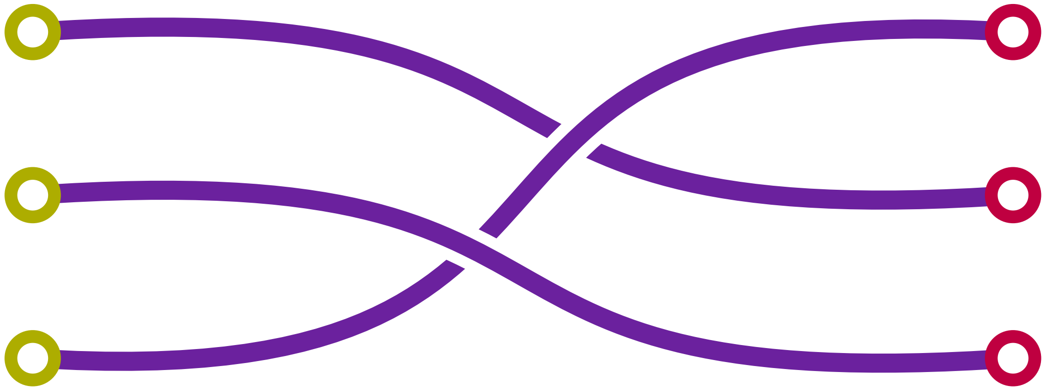

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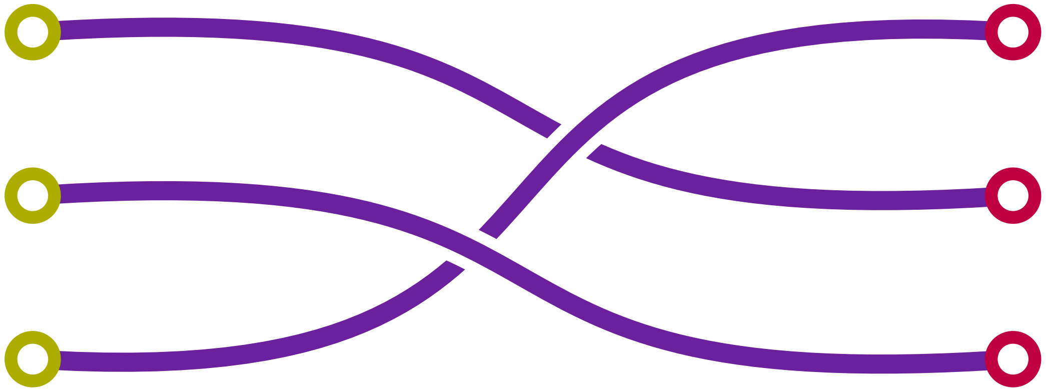

But (why and where) do such processes even exist?

On atomic scales, particles are waves; whose energy is *quantized*.

ground state: $E = 0$

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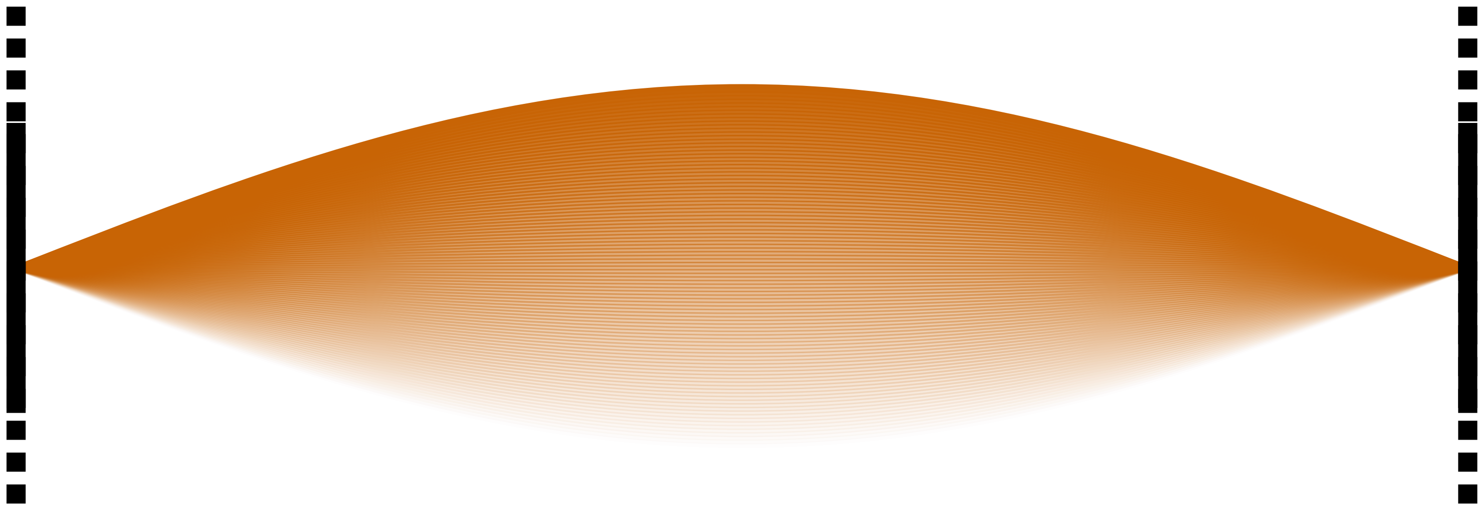

first excited state: $E = \hbar\omega$

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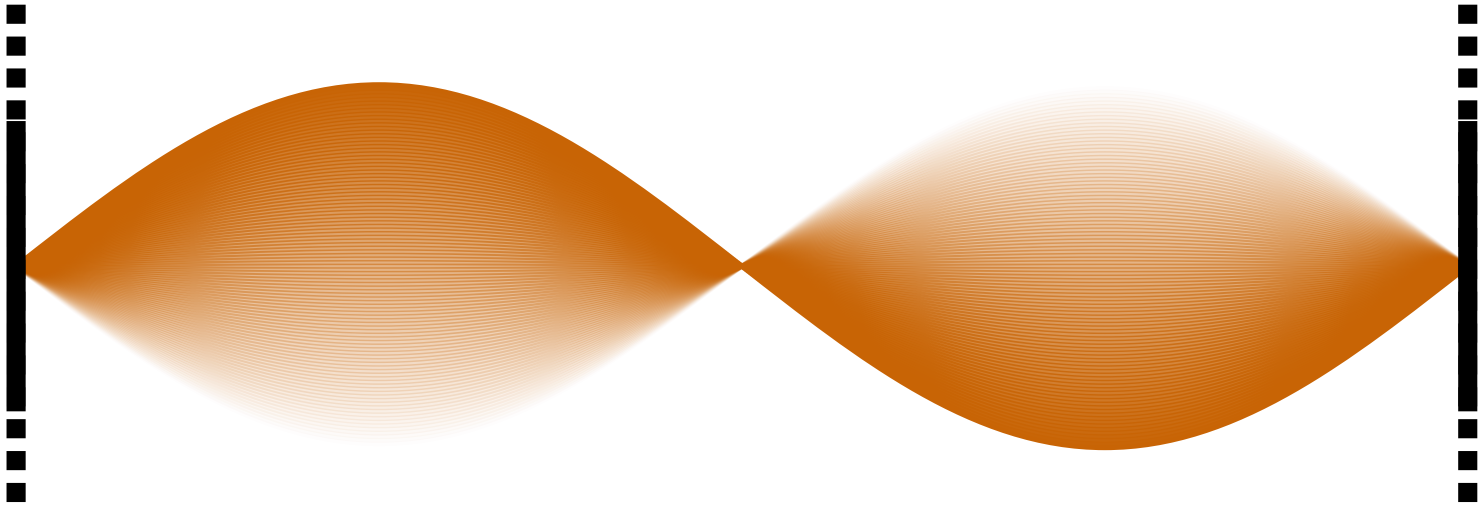

second excited state: $E = 2\hbar\omega$

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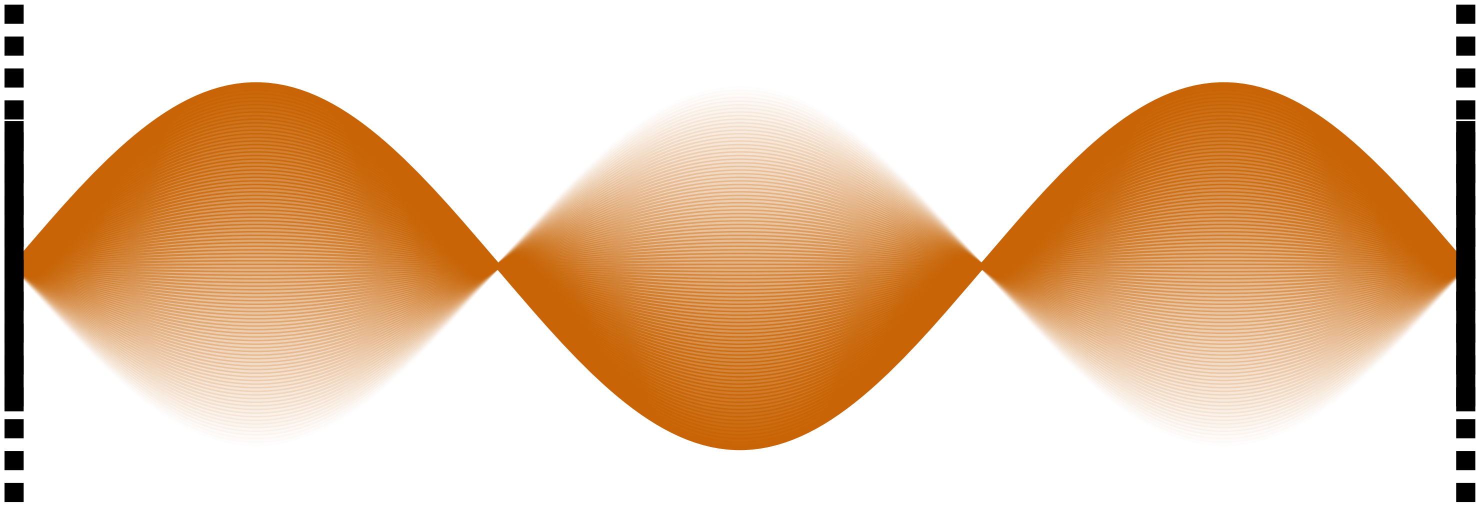

third excited state: $E = 3\hbar\omega$

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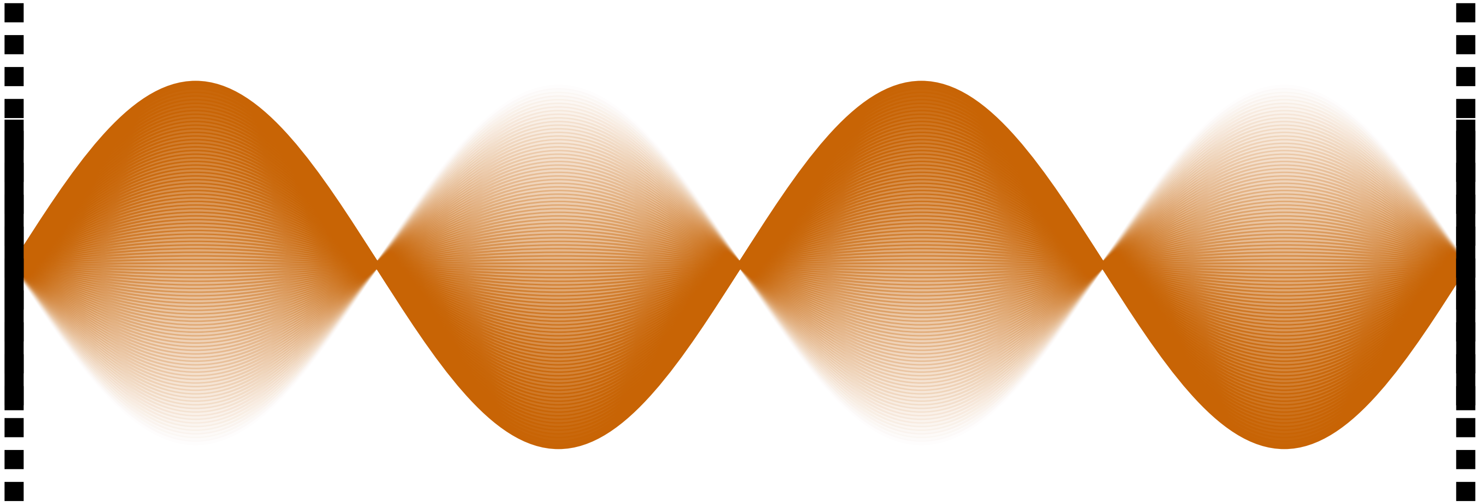

fourth excited state: $E = 4\hbar\omega$

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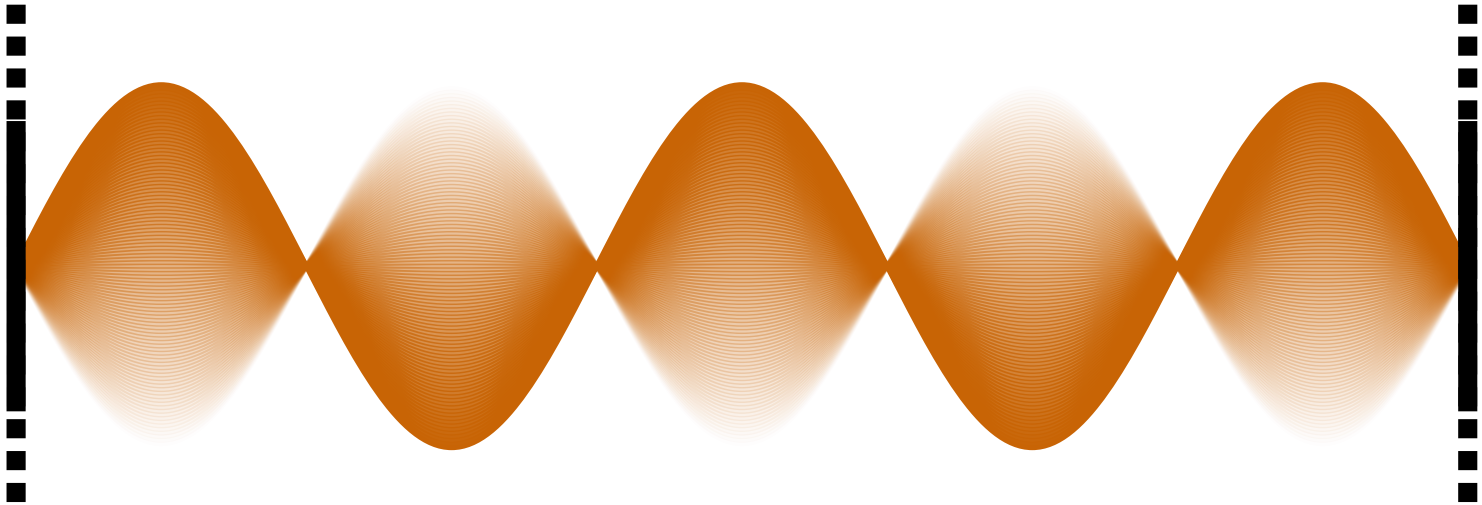
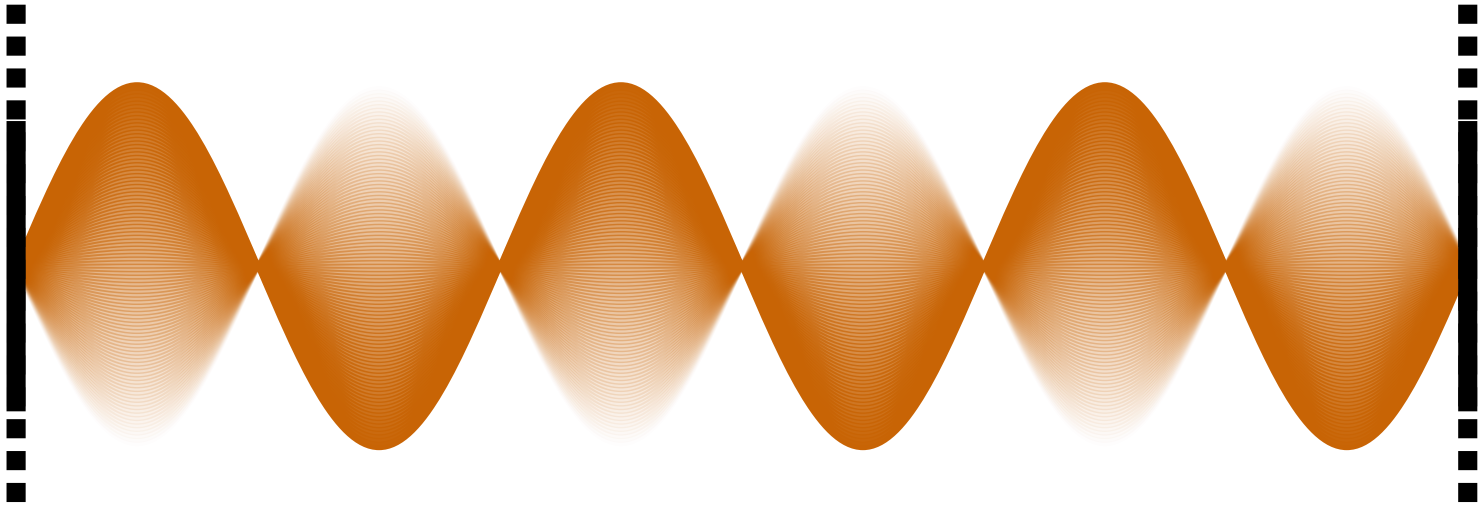


fifth excited state: $E = 5\hbar\omega$

But (why and where) do such processes even exist?

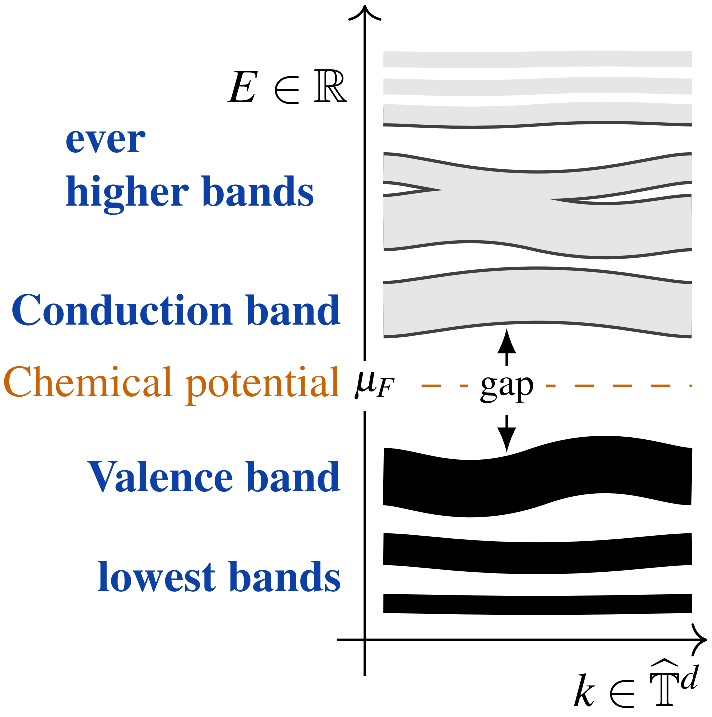
On atomic scales, particles are waves; whose energy is *quantized*.



sixth excited state: $E = 6\hbar\omega$

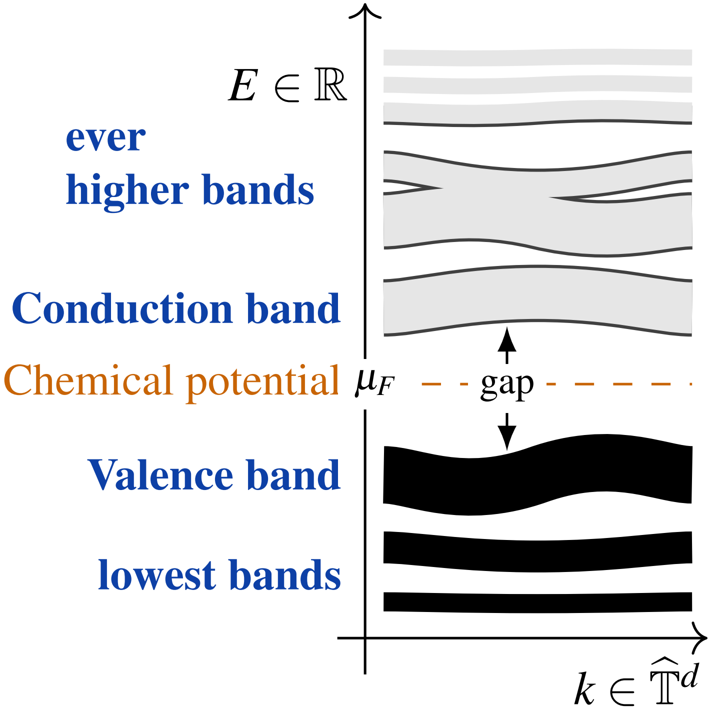
But (why and where) do such processes even exist?

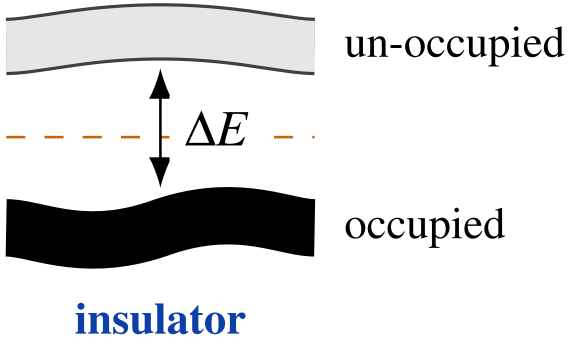
As very many particles come together in a crystal
their excitation energies accumulate in “bands”
but energy gaps *may* remain.



But (why and where) do such processes even exist?

If the ground state remains separated by an energy gap ΔE then it is *completely* undisturbed by disturbances $< \Delta E$.





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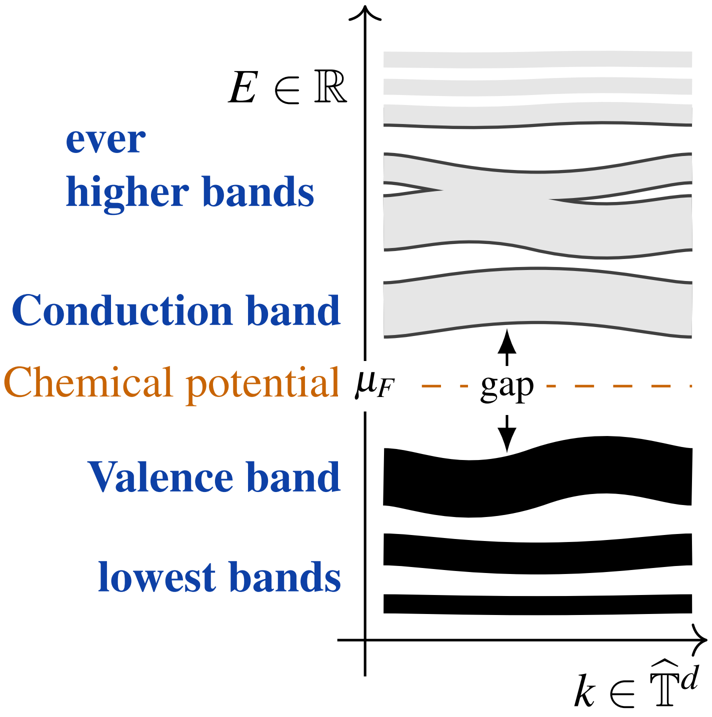

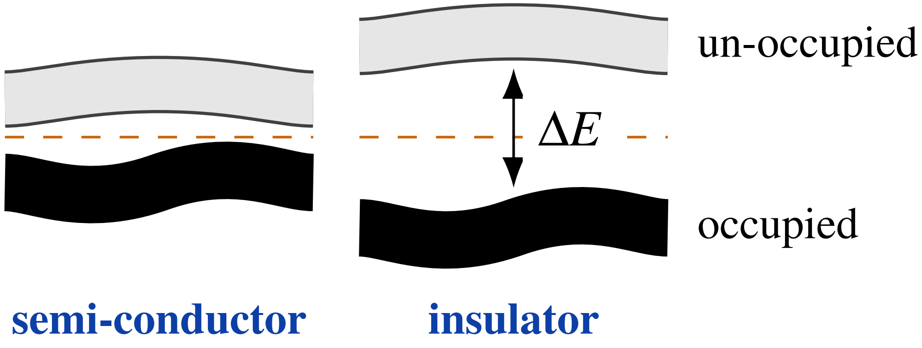

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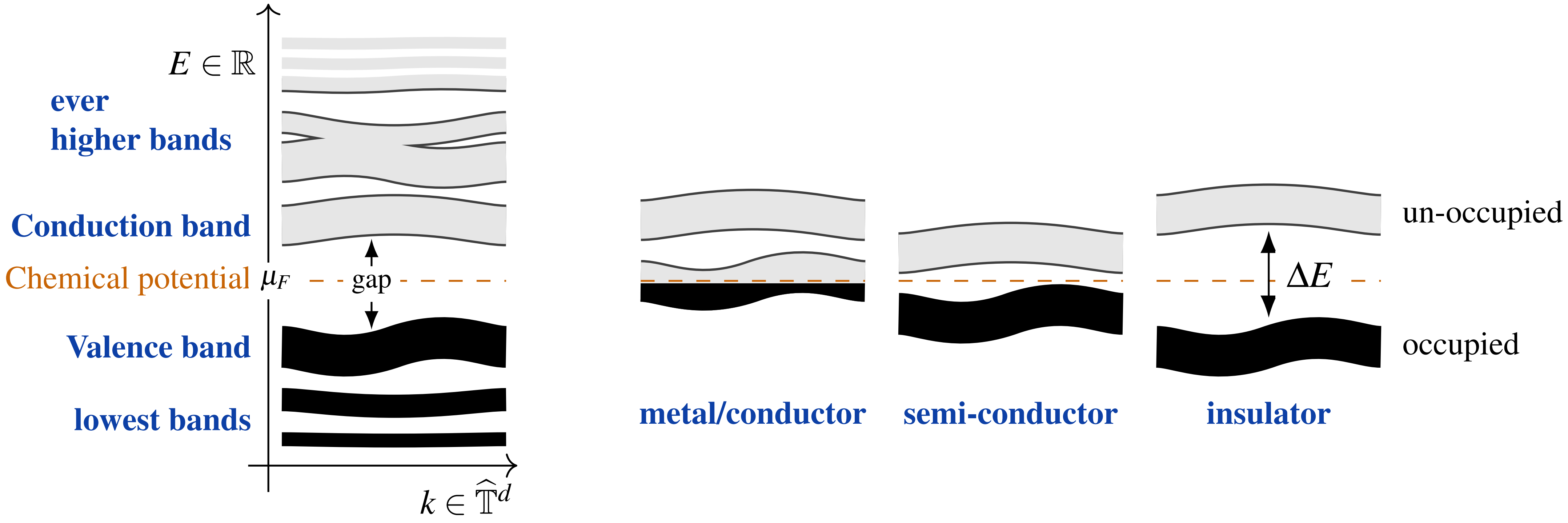
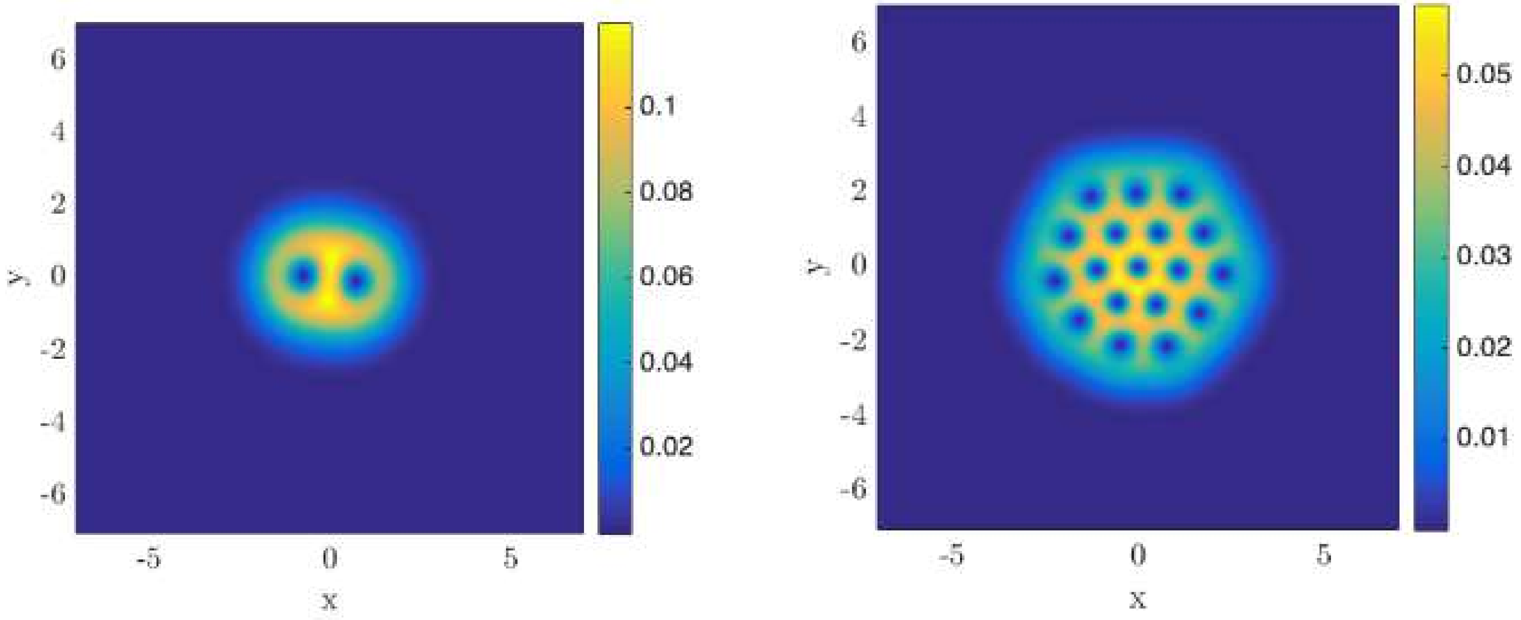


But (why and where) do such processes even exist?

So if such a gapped ground state depends on position of *point defects*, then their adiabatic movement is a topological quantum process.



(numerical simulation from `arXiv:1901.10739`)

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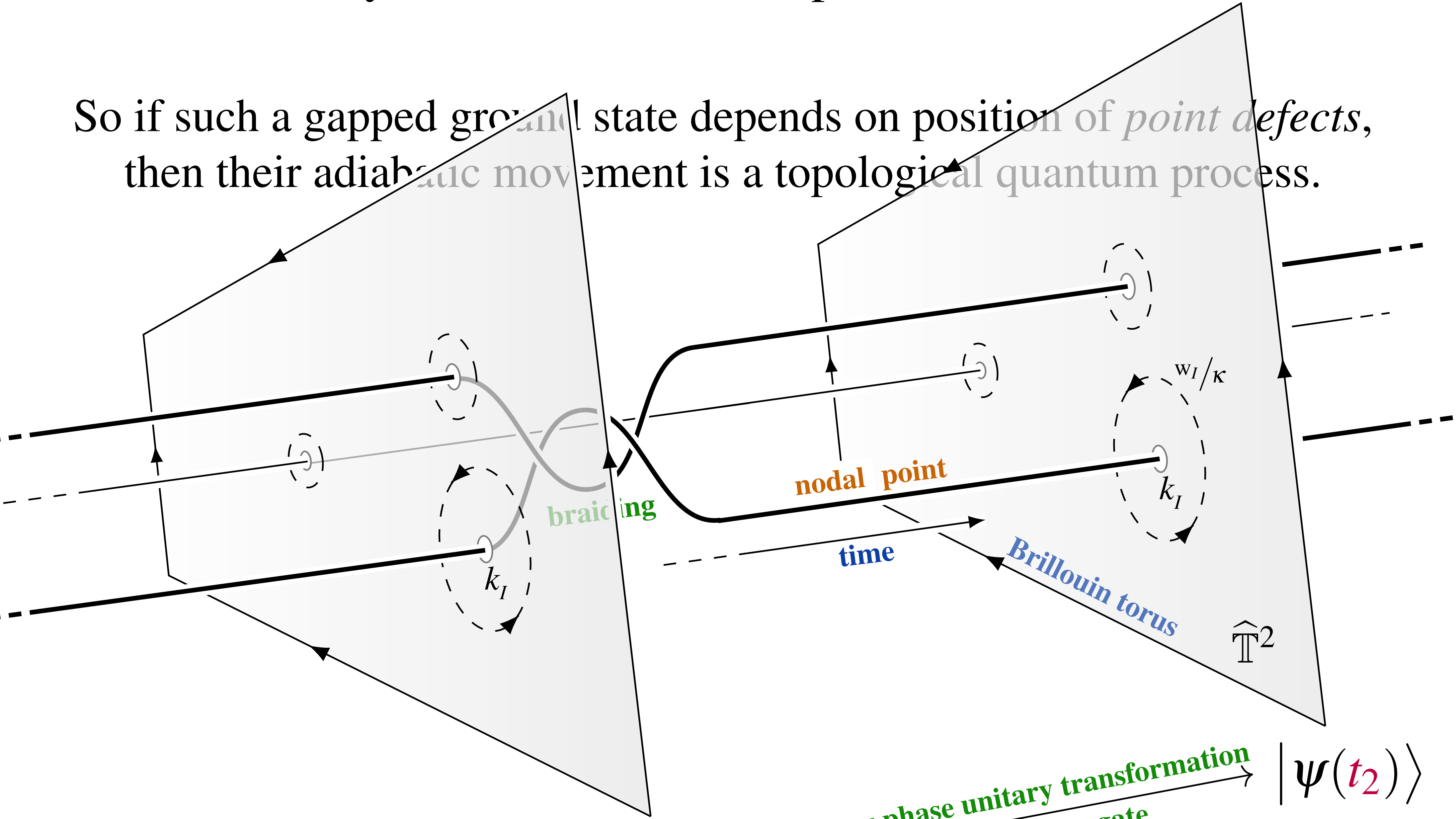

some ground state for fixed defect positions $k_1, k_2, \cdots$ **at time** t_1

$\left|\psi(t_1)\right\rangle$

Berry phase unitary transformation

= adiabatic quantum gate

$\left|\psi(t_2)\right\rangle$

another ground state fo fixed defect positions $k_1, k_2, \cdots$ **at time** t_2

$\overbrace{\textbf{To compute}}^{\boxed{\text{reliably}}}$ is

the case of

Topological Quantum Computation

[Sati & Schreiber, PlanQC **2022** 33 (2022)]

$\overbrace{\textbf{To compute}}^{\boxed{\text{reliably}}}$ is to **execute**

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Topological Quantum Computation

[Sati & Schreiber, PlanQC **2022** 33 (2022)]

$\boxed{I \longrightarrow O}$

topological quantum computation

$\longmapsto$

braid representation

reliably

To compute is to **execute** sequences of **instructions**

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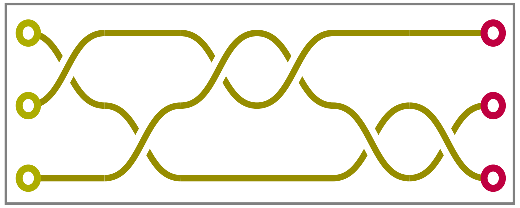
Topological Quantum Computation

[Sati & Schreiber, PlanQC **2022** 33 (2022)]

topological quantum computation

$I \longrightarrow O$

topological quantum circuit



braid representation

$\overbrace{\textbf{To compute}}^{\boxed{\text{reliably}}}$ is to **execute**
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as composable **operations**

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Topological Quantum Computation
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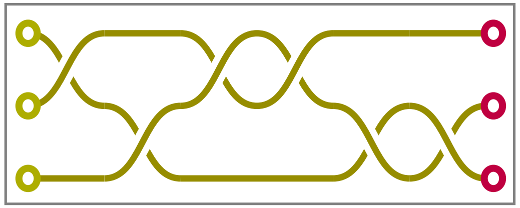
topological quantum computation

$\mathrm{I} \longrightarrow \mathrm{O}$

$\longmapsto$ **topological quantum circuit**



$\longmapsto$ **braid representation**

$$\mathcal{H}_3 \xrightarrow{U} \mathcal{H}_3$$

$\overbrace{\textbf{To compute}}^{\boxed{\text{reliably}}}$ is to **execute** sequences of **instructions** as composable **operations** on a chosen **state space**,

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Topological Quantum Computation

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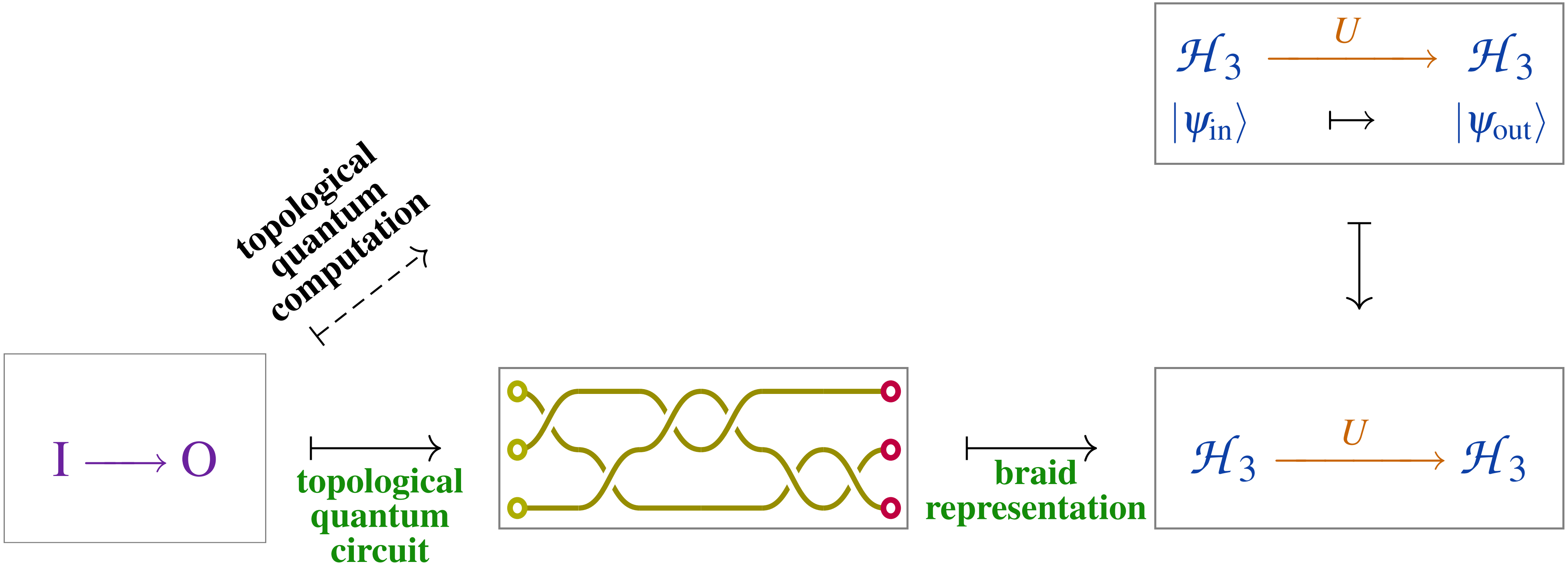

$$\begin{tikzcd} \mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3 \\ \lvert\psi_{\mathrm{in}}\rangle \arrow[r, mapsto] & \lvert\psi_{\mathrm{out}}\rangle \end{tikzcd}$$

topological quantum computation

$\mathrm{I} \longrightarrow \mathrm{O}$

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$\overbrace{\textbf{To compute}}^{\boxed{\text{reliably}}}$ is to **execute**
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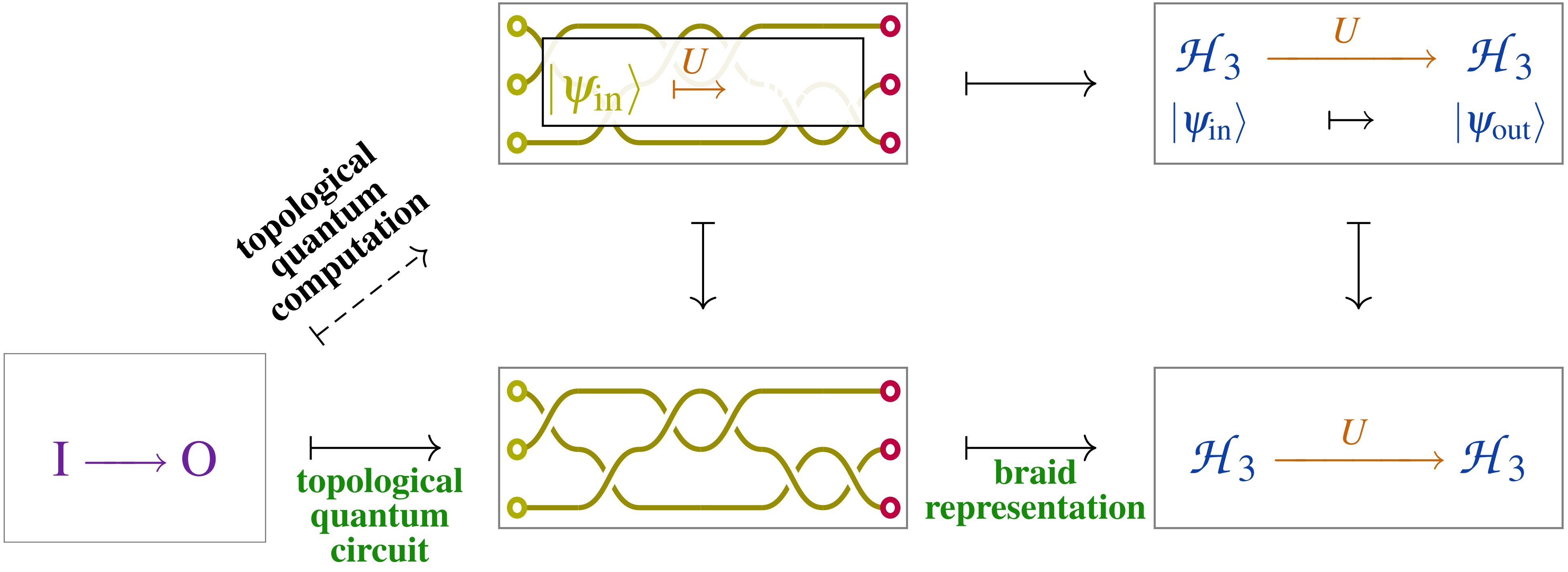

$|\psi_{\mathrm{in}}\rangle \overset{U}{\longmapsto}$

$\mathcal{H}_3 \xrightarrow{U} \mathcal{H}_3$

$|\psi_{\mathrm{in}}\rangle \mapsto |\psi_{\mathrm{out}}\rangle$

topological quantum computation

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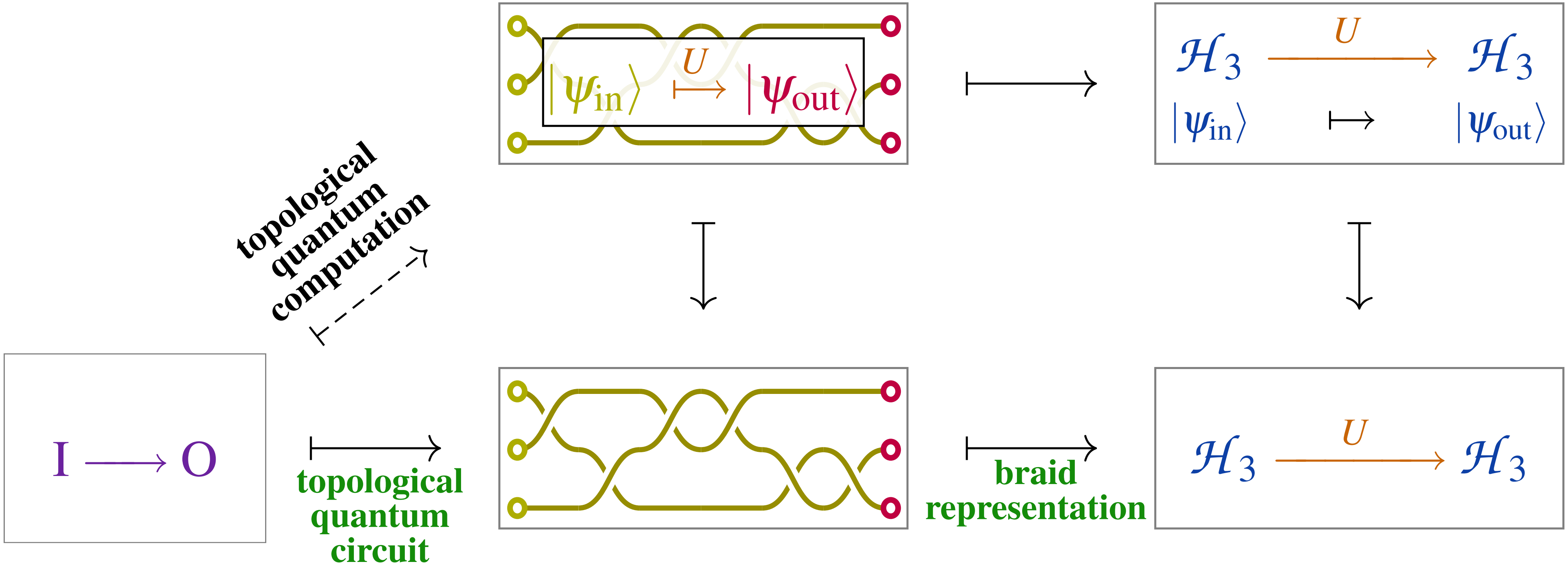

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$$\begin{tikzcd}
 & \textbf{bundle of conformal blocks} \arrow[r] \arrow[d] \arrow[dr, phantom, "\text{(pb)}"] & \textbf{quantum states in Hilbert spaces} \arrow[d] \\
\textbf{path} \arrow[ur, dashed, "\textbf{topological quantum computation}"] \arrow[r, "\textbf{topological quantum program}"'] & \textbf{configuration space of distinct points} \arrow[r, mapsto, "\textbf{braid representation}"'] & \textbf{unitary operators}
\end{tikzcd}$$

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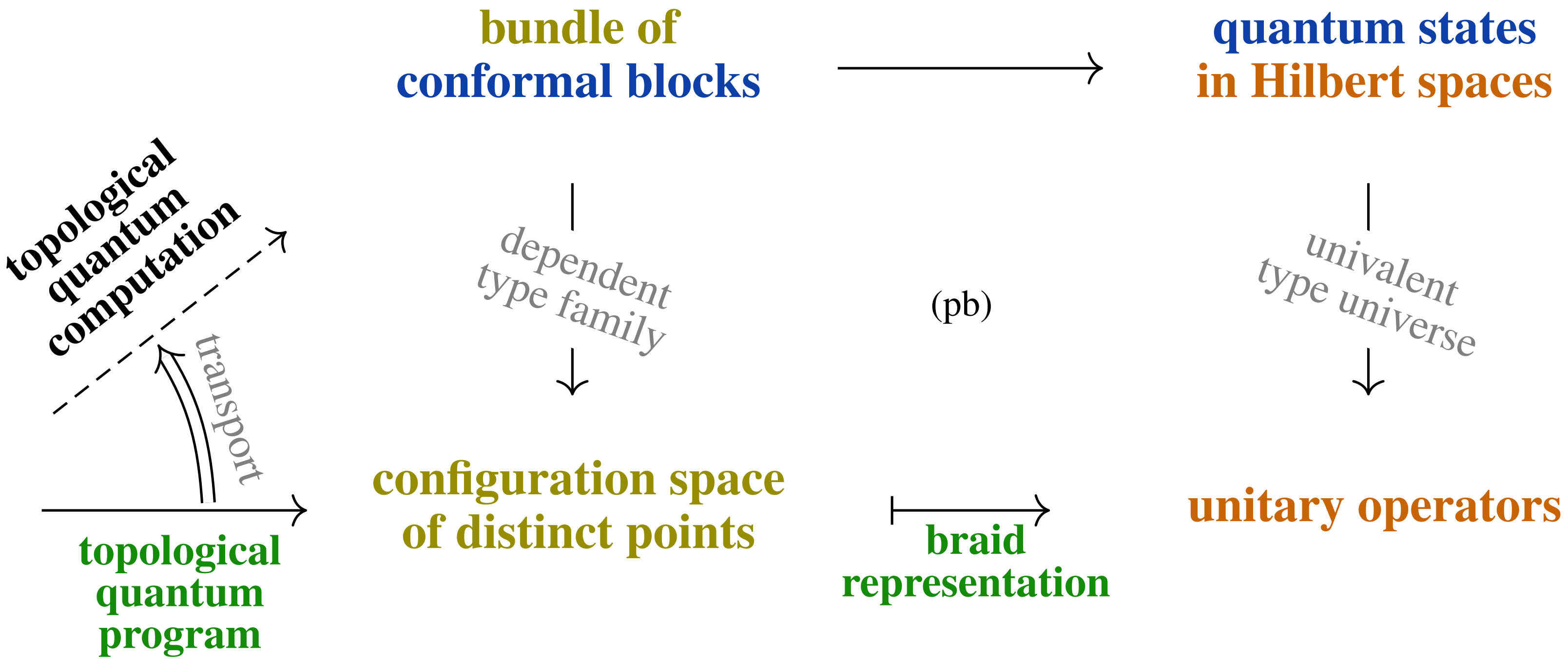
[Sati & Schreiber, PlanQC **2022** 33 (2022)]

Claim: This has natural construction in Homotopy Type Theory:


bundle of conformal blocks

quantum states in Hilbert spaces

topological quantum computation

dependent type family

(pb)

univalent type universe

transport

path

topological quantum program

configuration space of distinct points

braid representation

unitary operators

Quantum materials with these properties are called topological phases of matter

Quantum materials with these properties are called topological phases of matter

Nobel Lecture, Aula Magna, Stockholm University, December 8, 2016

Topological Quantum Matter

F. Duncan M. Haldane
Princeton University

- The TKNN formula (on behalf of David Thouless)
- The Chern Insulator and the birth of “topological insulators”

Quantum materials with these properties are called topological phases of matter

Quantum materials with these properties are called topological phases of matter exhibiting topological order.

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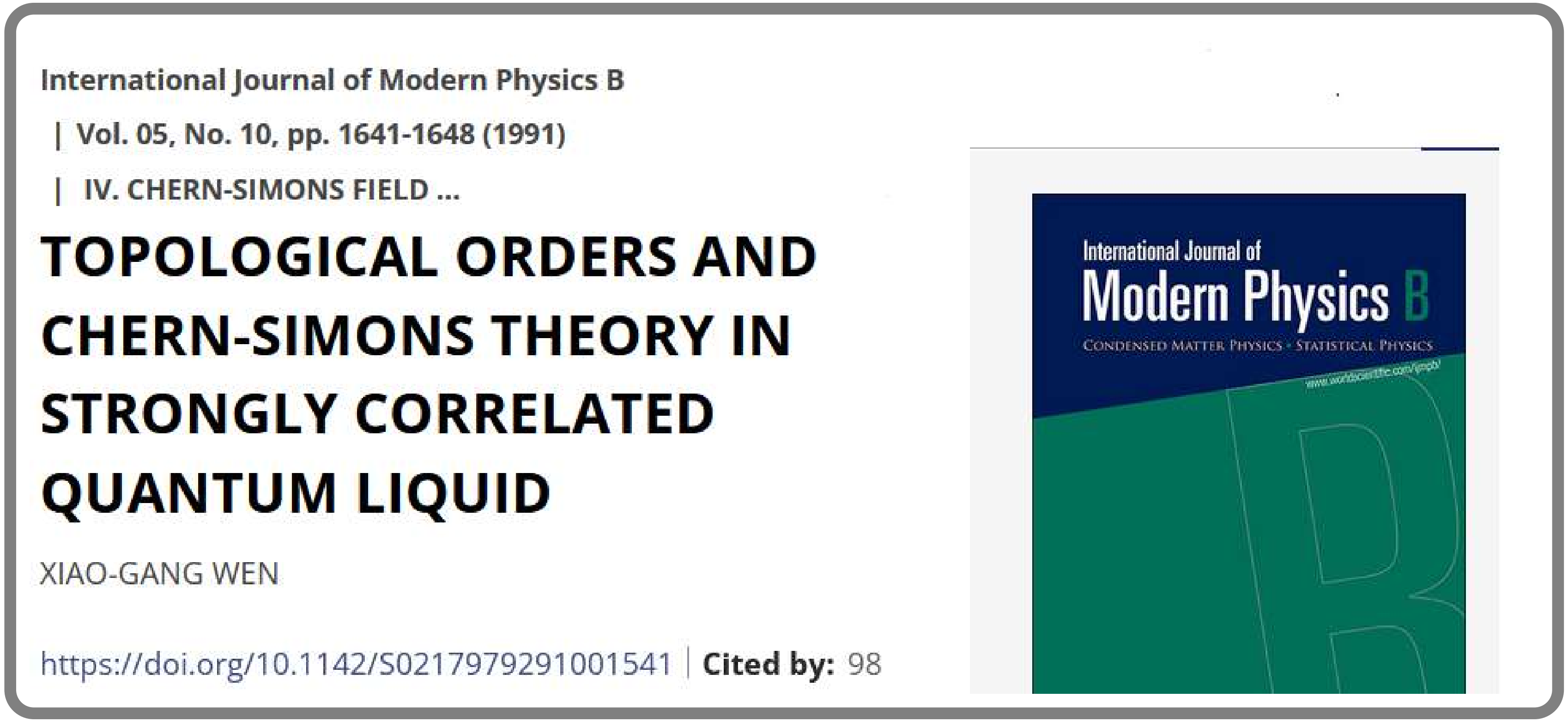

International Journal of Modern Physics B

| Vol. 05, No. 10, pp. 1641-1648 (1991)

| IV. CHERN-SIMONS FIELD ...

TOPOLOGICAL ORDERS AND CHERN-SIMONS THEORY IN STRONGLY CORRELATED QUANTUM LIQUID

XIAO-GANG WEN

https://doi.org/10.1142/S0217979291001541 | **Cited by:** 98

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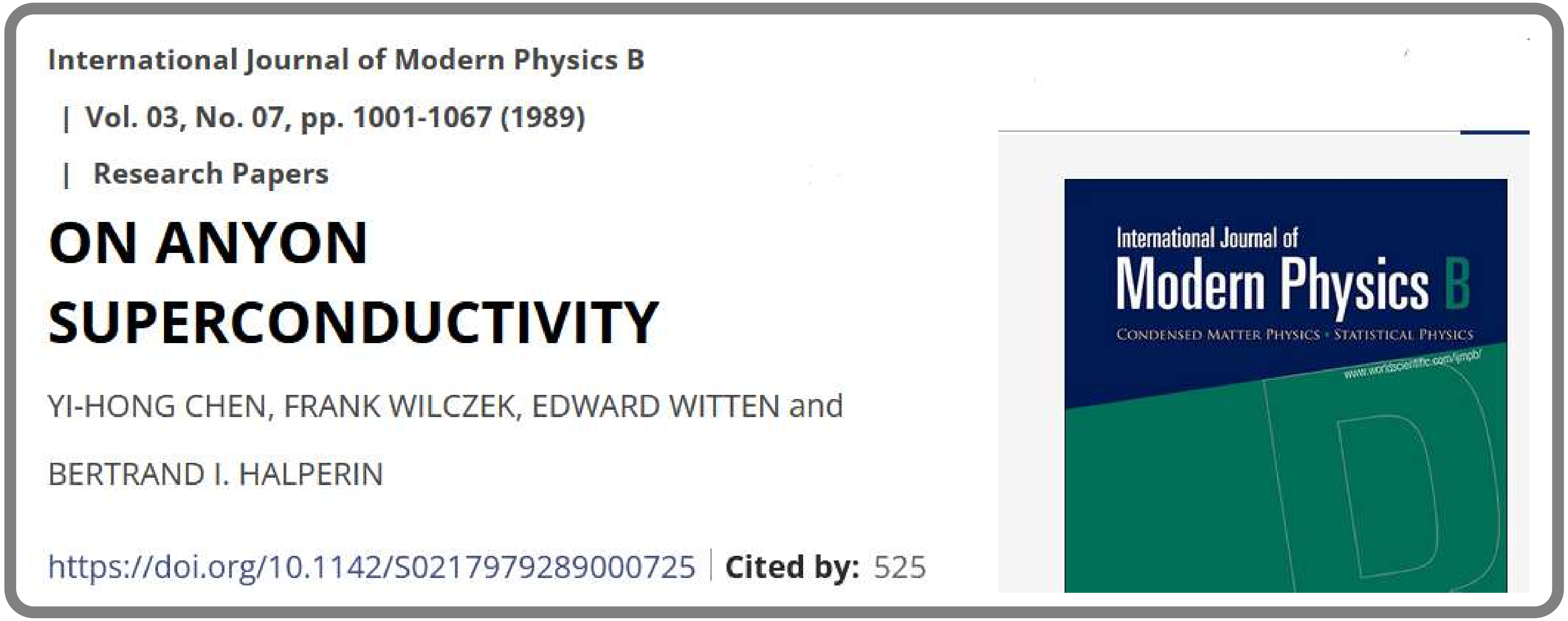

International Journal of Modern Physics B

| Vol. 03, No. 07, pp. 1001-1067 (1989)

| Research Papers

ON ANYON SUPERCONDUCTIVITY

YI-HONG CHEN, FRANK WILCZEK, EDWARD WITTEN and BERTRAND I. HALPERIN

https://doi.org/10.1142/S0217979289000725 | **Cited by:** 525

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promising! – needs more investigation

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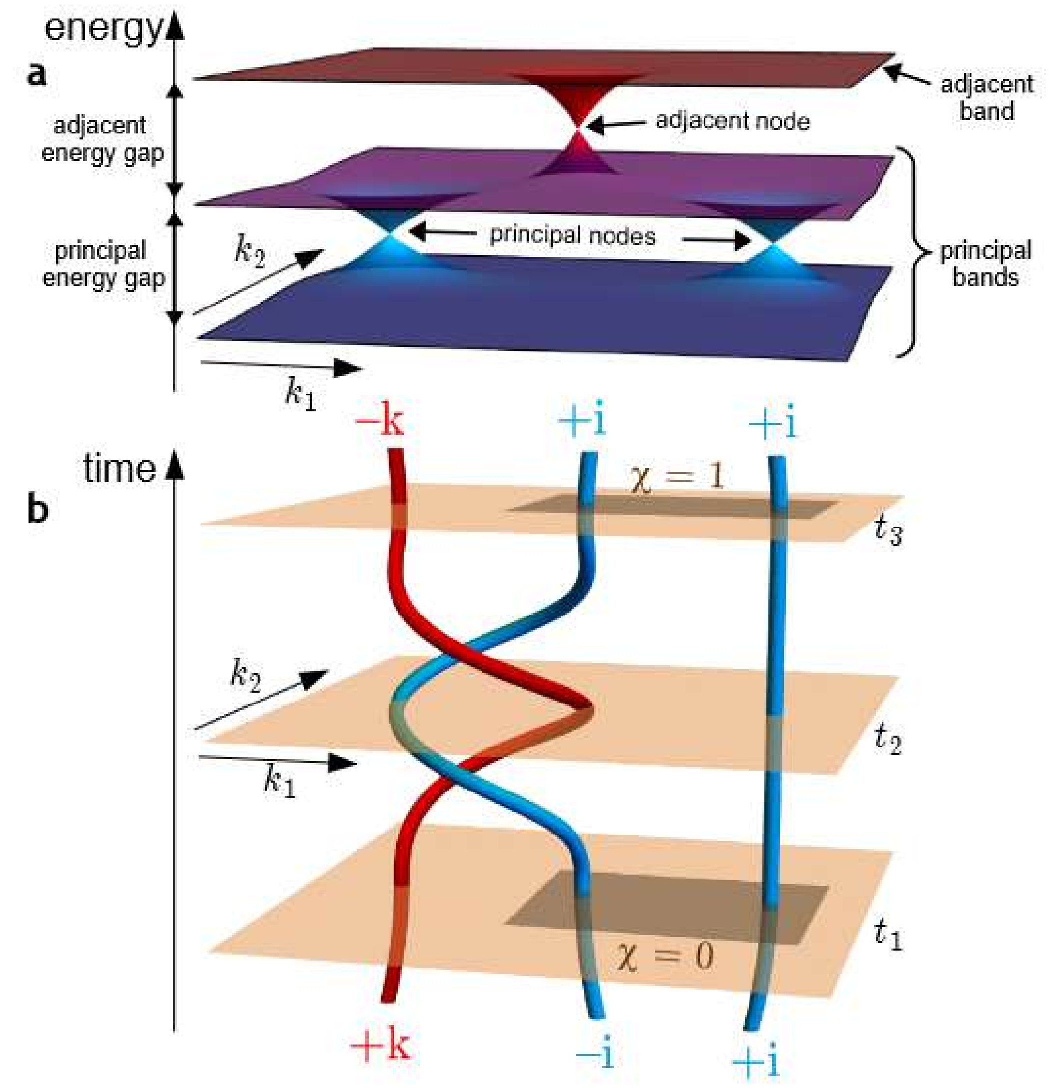

FIG. 1. **Reciprocal braiding of band nodes.**

Bzdušek et al., Nature Phys. **16** (2020)

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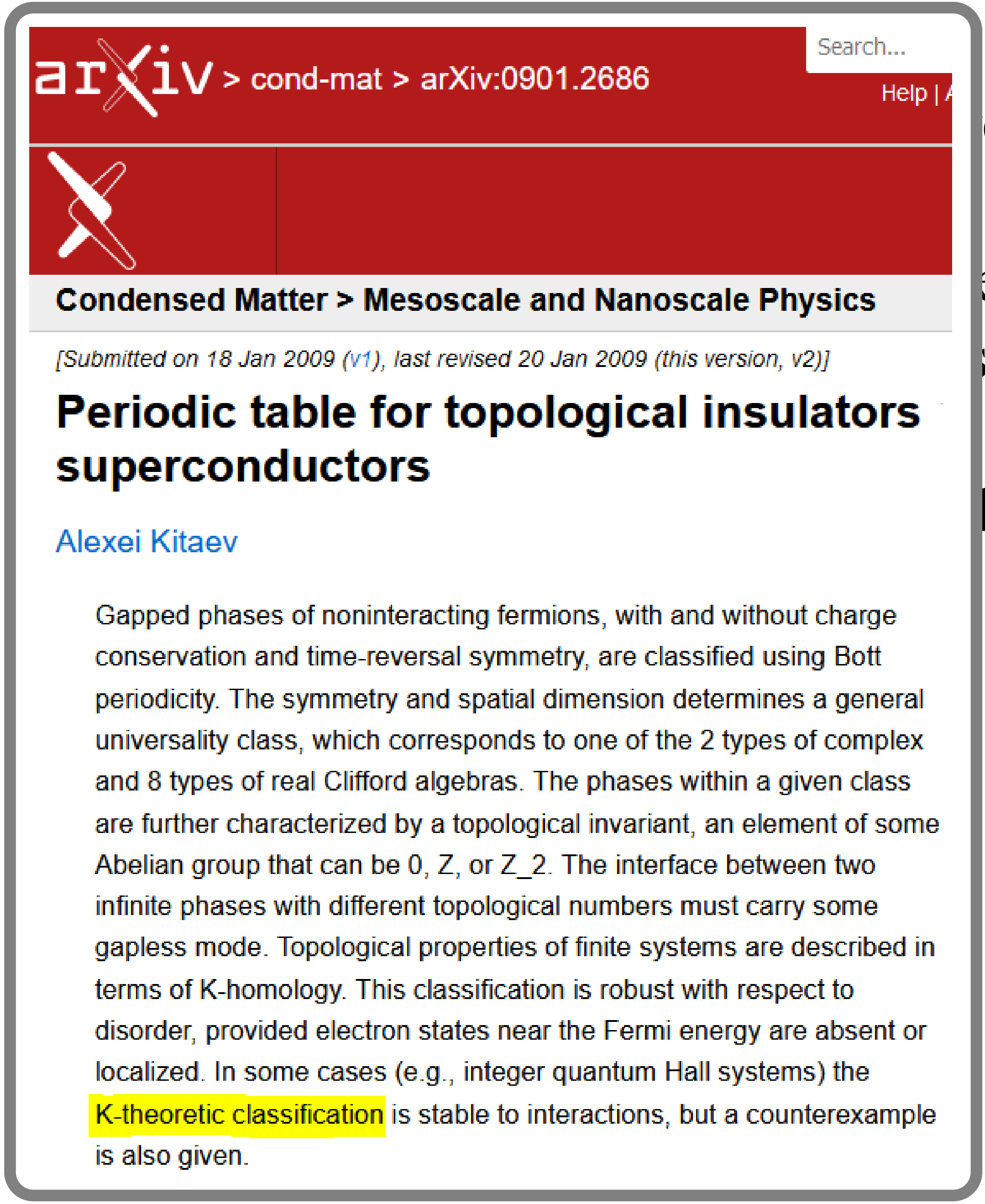

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 18 Jan 2009 (v1), last revised 20 Jan 2009 (this version, v2)]

Periodic table for topological insulators superconductors

Alexei Kitaev

Gapped phases of noninteracting fermions, with and without charge conservation and time-reversal symmetry, are classified using Bott periodicity. The symmetry and spatial dimension determines a general universality class, which corresponds to one of the 2 types of complex and 8 types of real Clifford algebras. The phases within a given class are further characterized by a topological invariant, an element of some Abelian group that can be 0, Z, or Z_2. The interface between two infinite phases with different topological numbers must carry some gapless mode. Topological properties of finite systems are described in terms of K-homology. This classification is robust with respect to disorder, provided electron states near the Fermi energy are absent or localized. In some cases (e.g., integer quantum Hall systems) the K-theoretic classification is stable to interactions, but a counterexample is also given.

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(1) – The Problem:

Practical Foundations of
Topological Quantum Computation

(2) – The Strategy:

Cohesive Linear Homotopy for
Holographic Condensed Matter Theory

(3) – The Technology:

TED K-Cohomology of
Cohomotopy Moduli Spaces

there is a curious dictionary

Condensed/Quantum Matter	Alg. Topology/Geom. Homotopy

there is a curious dictionary

Condensed/Quantum Matter	AdS/CMT ⟷	**String/M-Theory**	flux, charge quantization ⟷	**Alg. Topology/Geom. Homotopy**

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$\mathcal{H}$

Hilbert space of quantum states

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\mathcal{H}_1 \arrow[r, bend left, "U"] & \mathcal{H}_2
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Hilbert space of quantum states

unitary evolution

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P_1

external classical parameters at time t_1

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Hilbert space of quantum states at parameter value P_1

$$\begin{tikzcd}
P_1 \arrow[r, squiggly, "p_{12}"] & P_2
\end{tikzcd}$$

adiabatic flow of parameters

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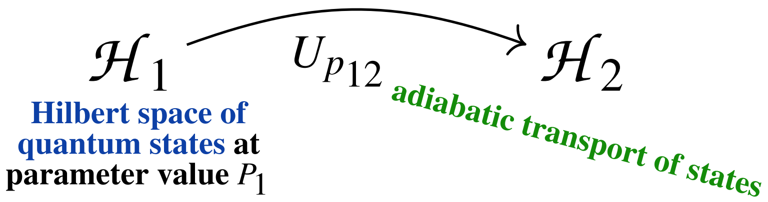

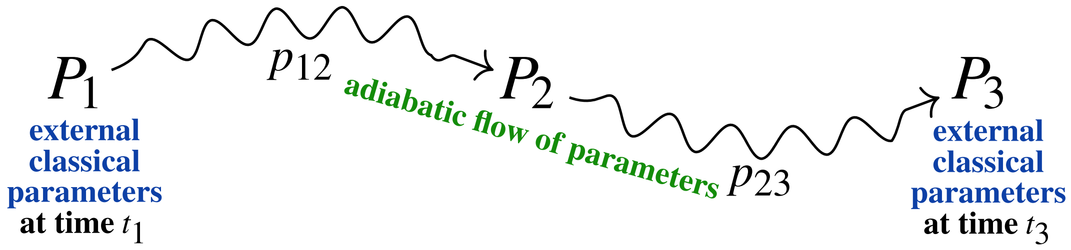

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Hilbert space of quantum states at parameter value P_1

adiabatic transport of states

Hilbert space of quantum states at parameter value P_3

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parameter groupoid

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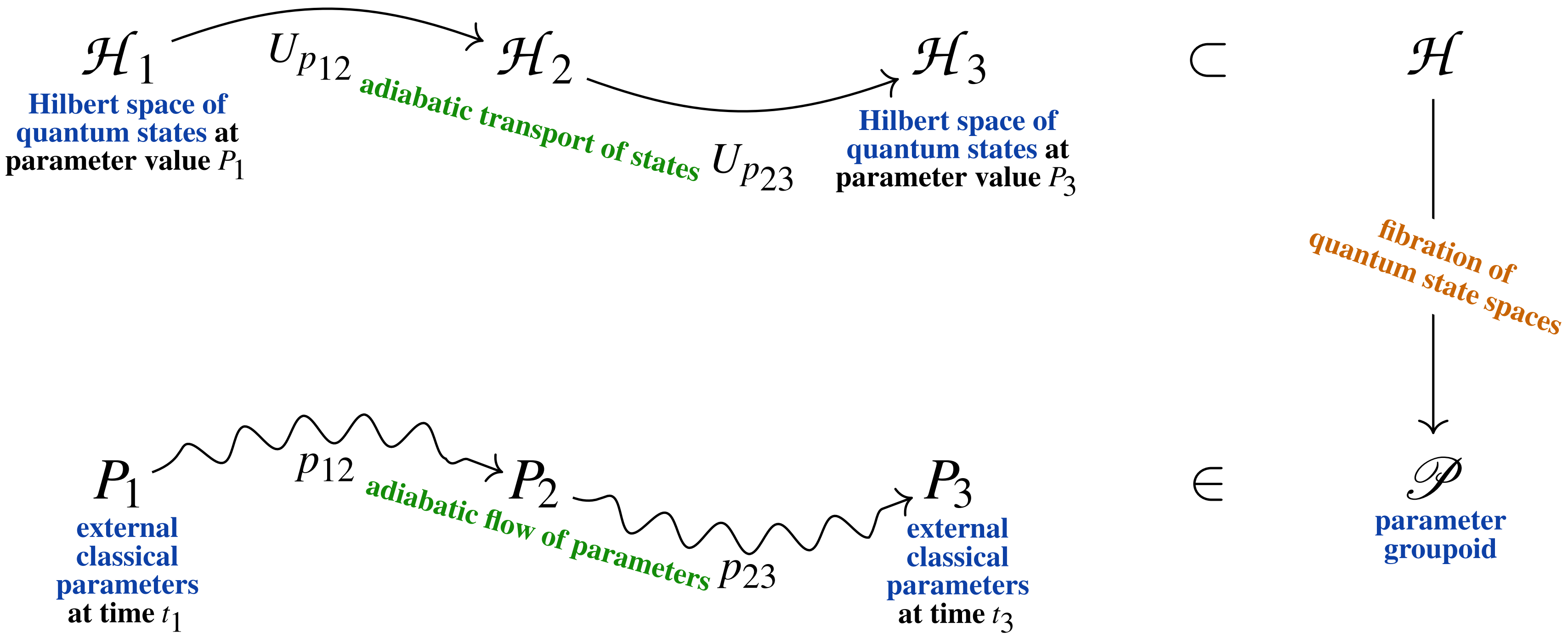

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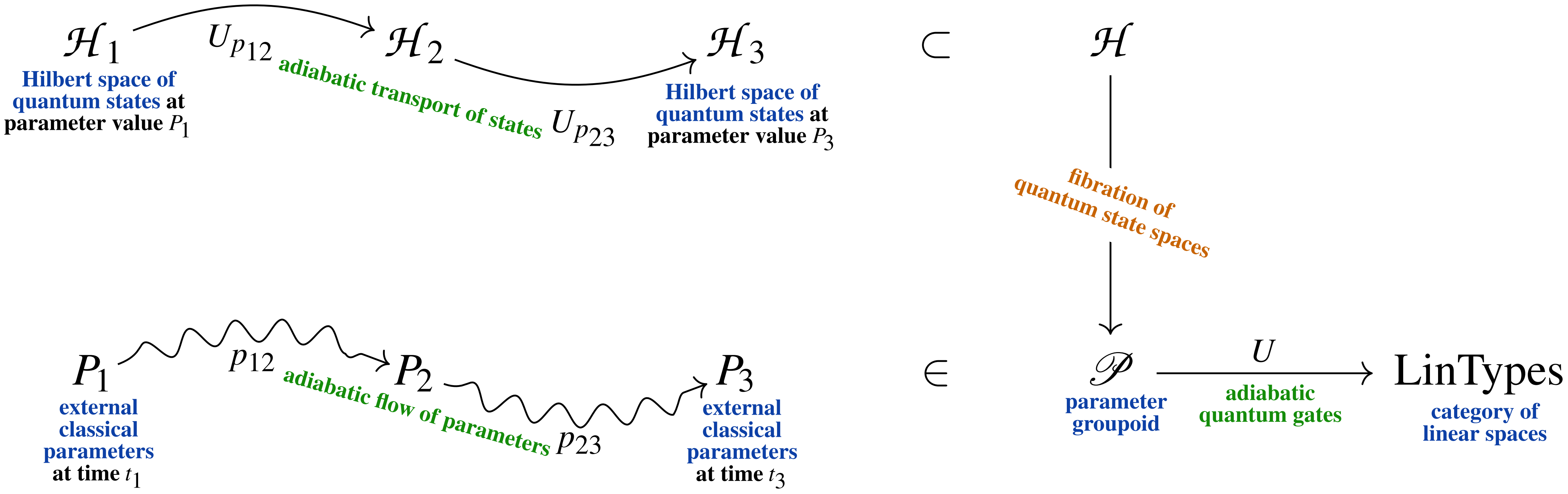

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$$\begin{tikzcd}
\underset{\substack{\textbf{Hilbert space of}\\ \textbf{quantum states at}\\ \textbf{parameter value } P_1}}{\mathcal{H}_1} \arrow[r, bend left, "U_{p_{12}}"] & \mathcal{H}_2 \arrow[r, bend right, "U_{p_{23}}", "\textbf{adiabatic transport of states}"'] & \underset{\substack{\textbf{Hilbert space of}\\ \textbf{quantum states at}\\ \textbf{parameter value } P_3}}{\mathcal{H}_3} & \subset & \mathcal{H} \arrow[r] \arrow[d, "\substack{\textbf{fibration of}\\ \textbf{quantum state spaces}}"'] \arrow[dr, phantom, "\text{(pb)}"] & \widehat{\mathrm{LinTypes}} \arrow[d, "\substack{\textbf{univalent}\\ \textbf{fibration of}\\ \textbf{linear spaces}}"] \\
\underset{\substack{\textbf{external}\\ \textbf{classical}\\ \textbf{parameters}\\ \textbf{at time } t_1}}{P_1} \arrow[r, squiggly, "p_{12}"] & P_2 \arrow[r, squiggly, "p_{23}", "\textbf{adiabatic flow of parameters}"'] & \underset{\substack{\textbf{external}\\ \textbf{classical}\\ \textbf{parameters}\\ \textbf{at time } t_3}}{P_3} & \in & \underset{\substack{\textbf{parameter}\\ \textbf{groupoid}}}{\mathscr{P}} \arrow[r, "U", "\substack{\textbf{adiabatic}\\ \textbf{quantum gates}}"'] & \underset{\substack{\textbf{category of}\\ \textbf{linear spaces}}}{\mathrm{LinTypes}}
\end{tikzcd}$$

there is a curious dictionary

Condensed/Quantum Matter	$\xleftrightarrow{\text{AdS/CMT}}$	**String/M-Theory**	$\xleftrightarrow[\text{quantization}]{\text{flux, charge}}$	**Alg. Topology/Geom. Homotopy**
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Topological Quantum Programming

Example. For TQC one takes:

parameters = sets of distinct points in plane

parameter paths = braids of their worldlines

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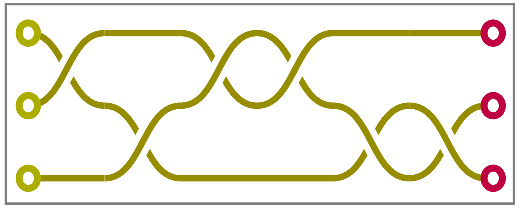

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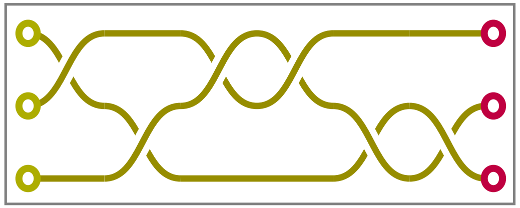

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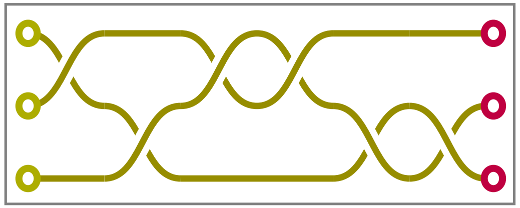
Topological Quantum Programming

Adiabatic transprt along such parameters is a unitary *braid representation*



braid

$\xmapsto{\textbf{braid representation}}$

$$\begin{tikzcd}\mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3\end{tikzcd}$$

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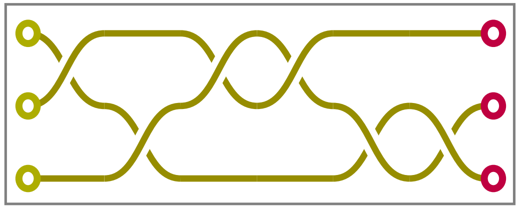

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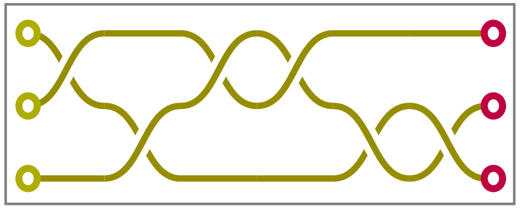
$$\begin{tikzcd}
\mathcal{H}_3 \arrow[r, "U"] & \mathcal{H}_3 \\
\lvert\psi_{\mathrm{in}}\rangle \arrow[u, phantom, sloped, "\in"] \arrow[r, mapsto] & \lvert\psi_{\mathrm{out}}\rangle \arrow[u, phantom, sloped, "\in"]
\end{tikzcd}$$

$$\big\downarrow$$



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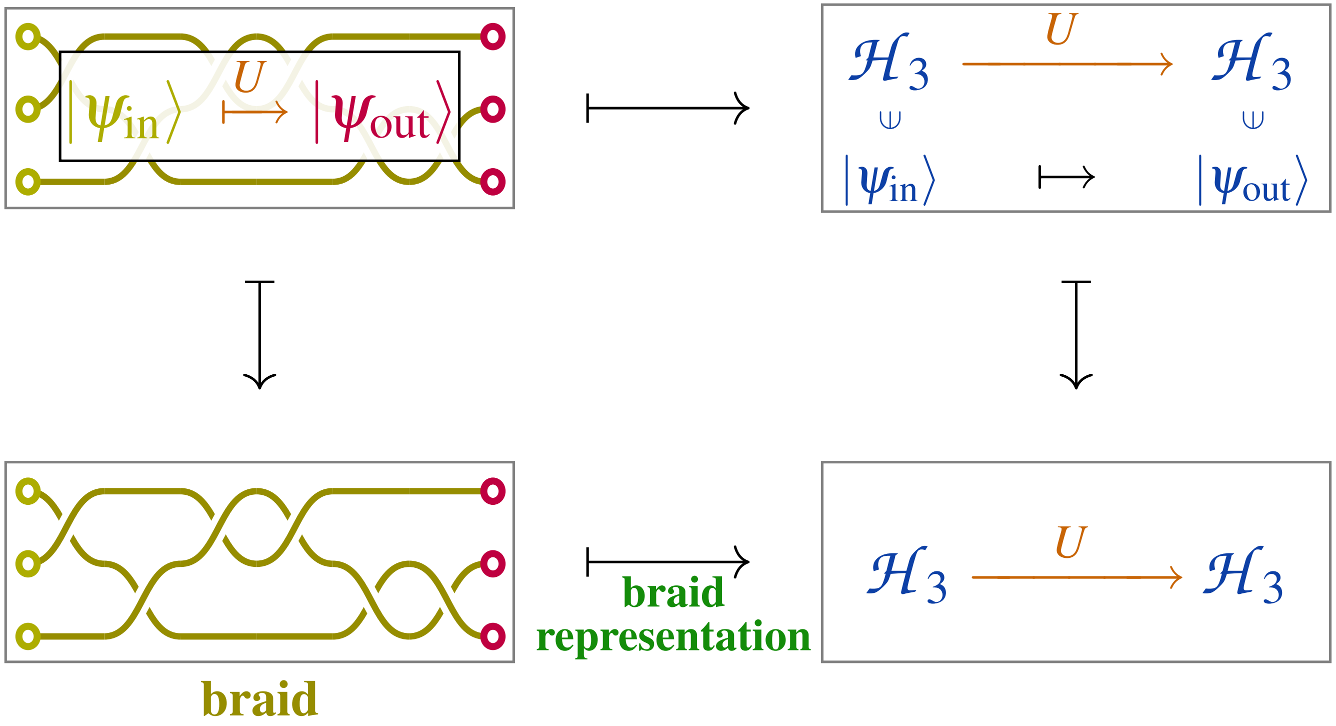

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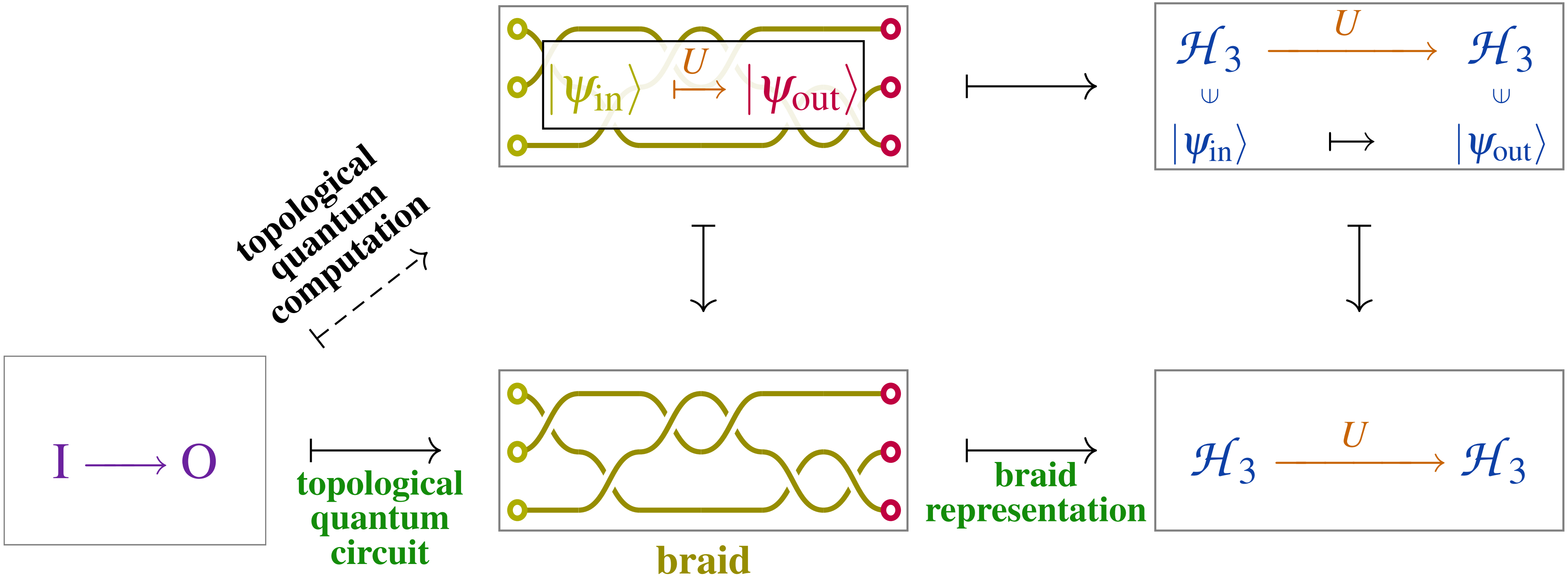
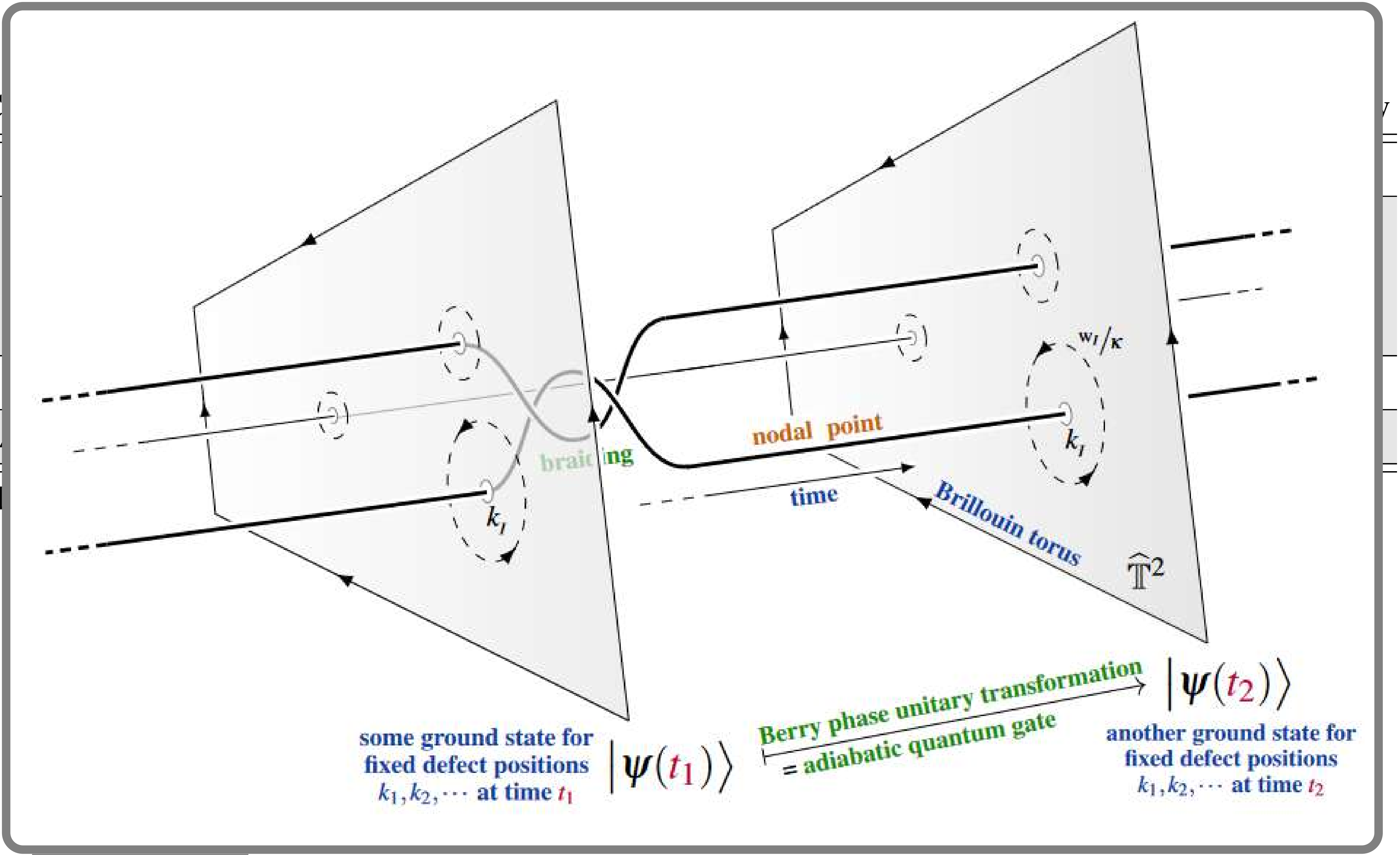
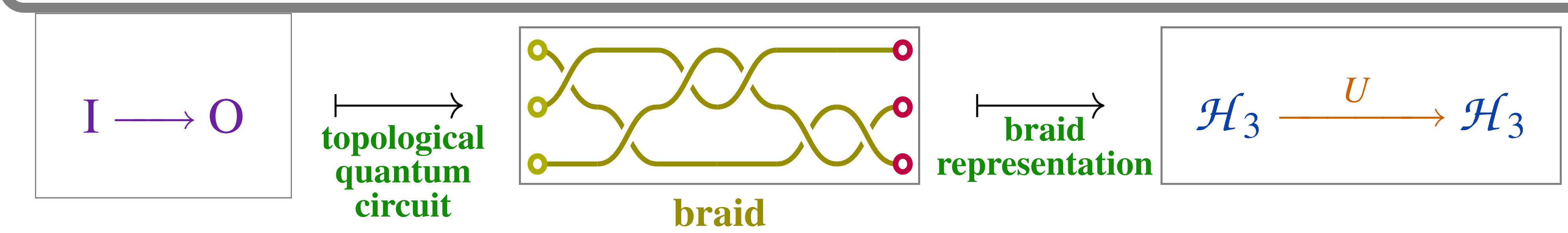




w_1/κ
k_1
k_1
braiding
nodal point
time
Brillouin torus
$\widehat{\mathbb{T}}^2$
some ground state for fixed defect positions $k_1, k_2, \cdots$ at time t_1
$|\psi(t_1)\rangle$
Berry phase unitary transformation
= adiabatic quantum gate
$|\psi(t_2)\rangle$
another ground state for fixed defect positions $k_1, k_2, \cdots$ at time t_2



$\mathrm{I} \longrightarrow \mathrm{O}$
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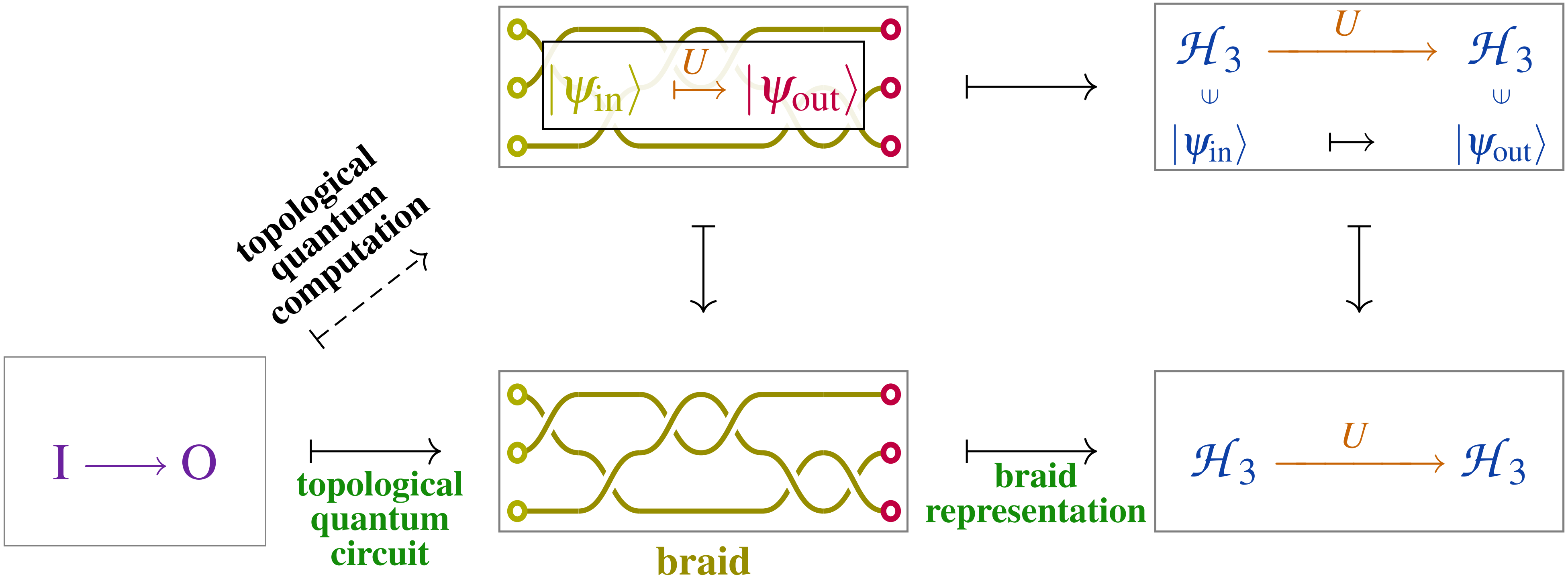

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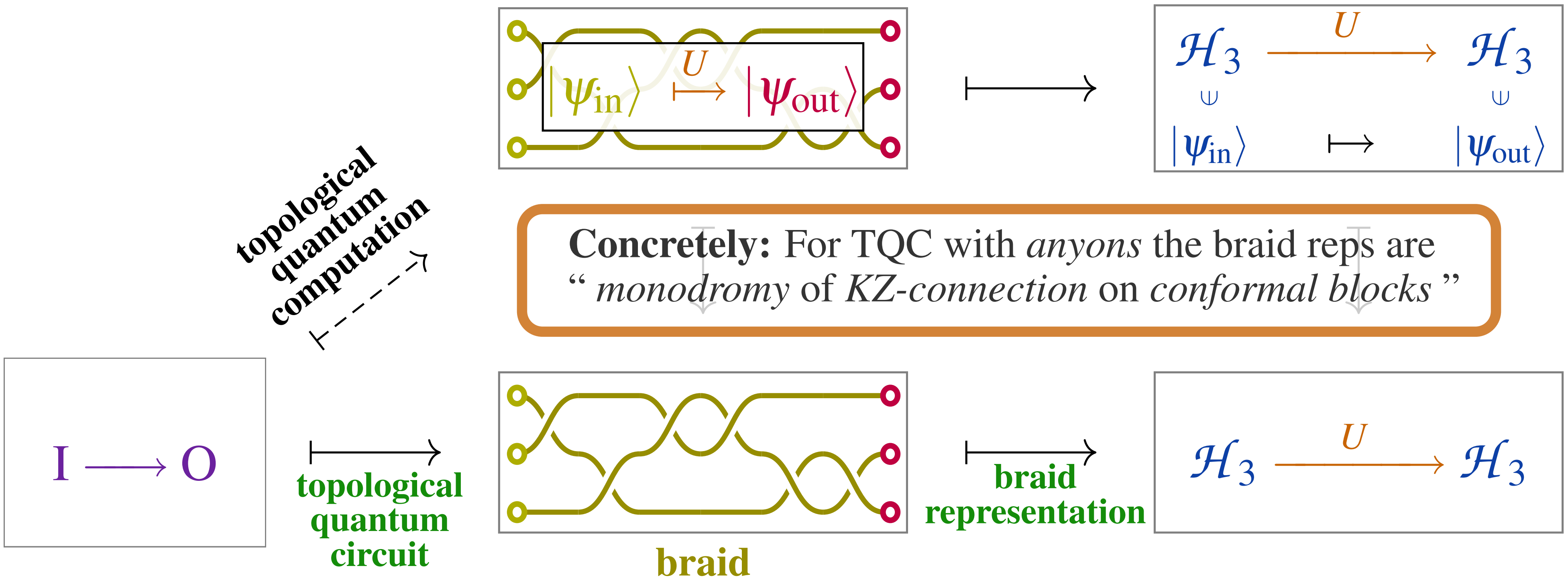

Concretely: For TQC with *anyons* the braid reps are "*monodromy* of *KZ-connection* on *conformal blocks*"

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Topological Quantum Programming

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[r] \arrow[d] & \text{quantum states in Hilbert spaces} \arrow[d] \\
\text{path} \arrow[ur, dashed, "\text{topological quantum computation}"] \arrow[r, "\text{topological quantum program}"'] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
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This describes adiabatic braiding of *band nodes* of topol. ordered semi-metals classified in TED K-theory of config. space:

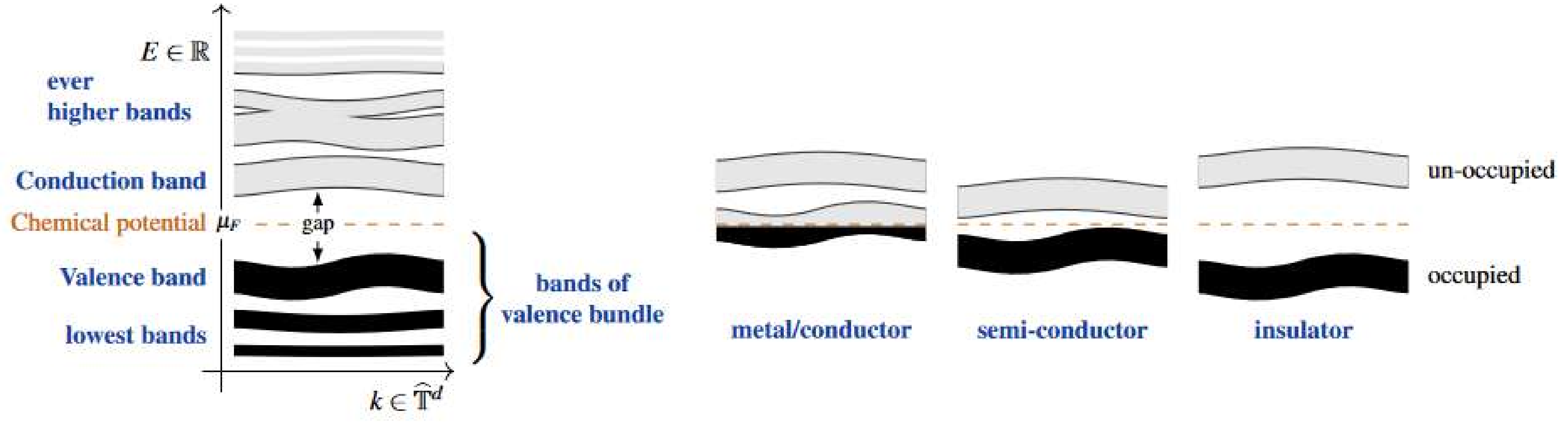
arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

Anyonic Topological Order in Twisted Equivariant Differential (TED) K-Theory

Hisham Sati, Urs Schreiber



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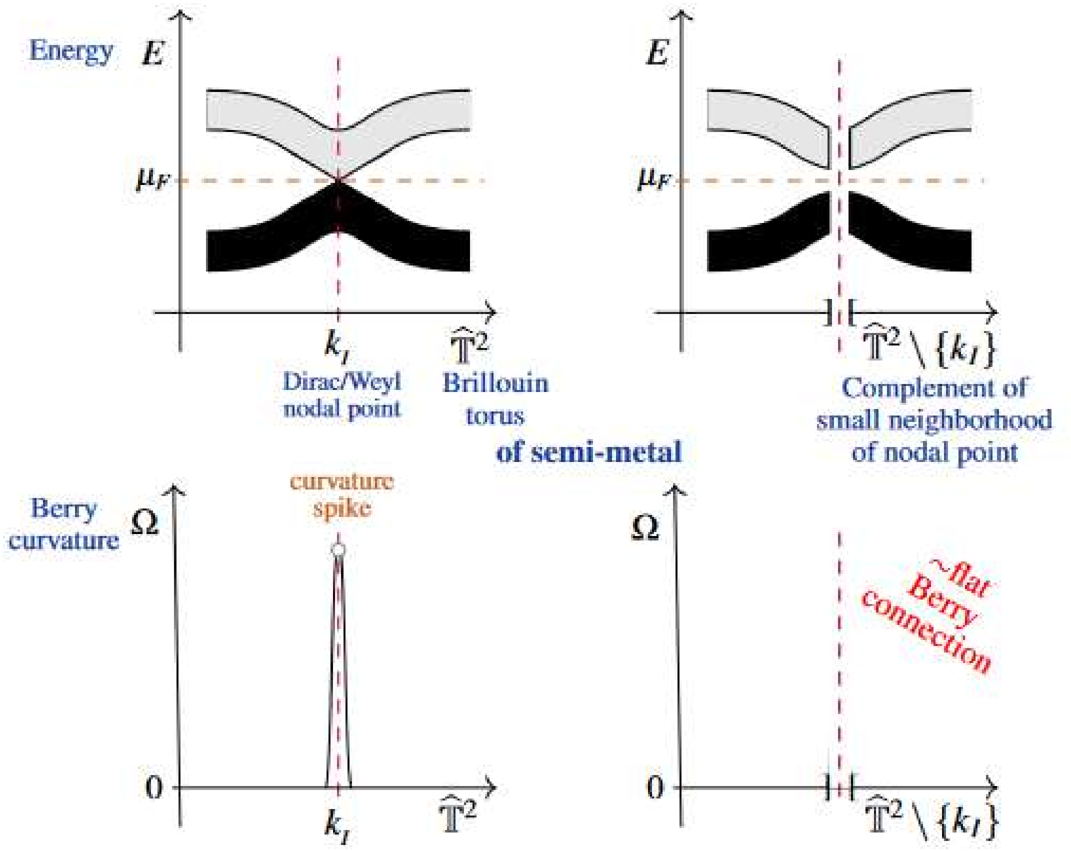

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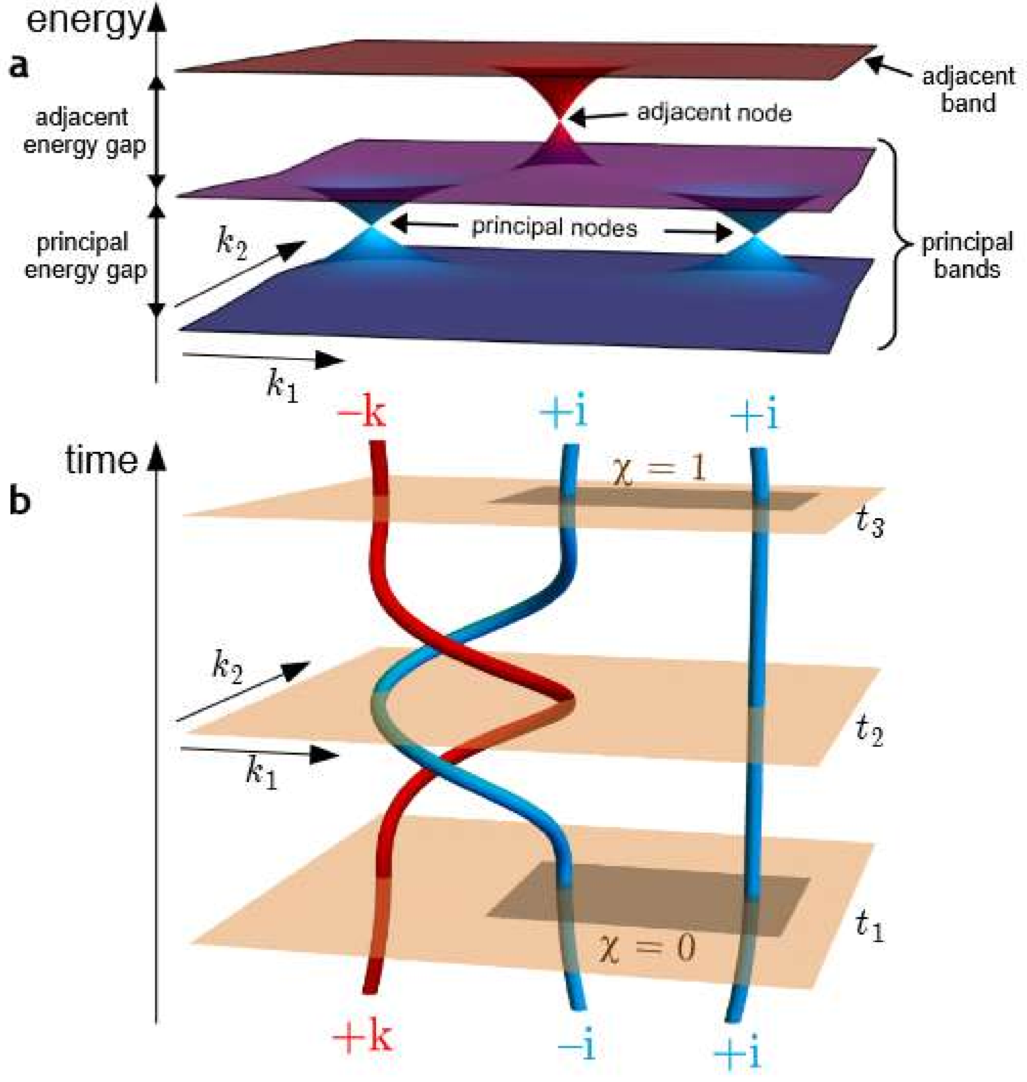
arXiv > cond-mat > arXiv:1907.10611

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 24 Jul 2019 (v1), last revised 7 Mar 2021 (this version, v4)]

Non-Abelian reciprocal braiding of Weyl points and its manifestation in ZrTe

Adrien Bouhon, QuanSheng Wu, Robert-Jan Slager, Hongming Weng, Oleg V. Yazyev, Tomáš Bzdušek



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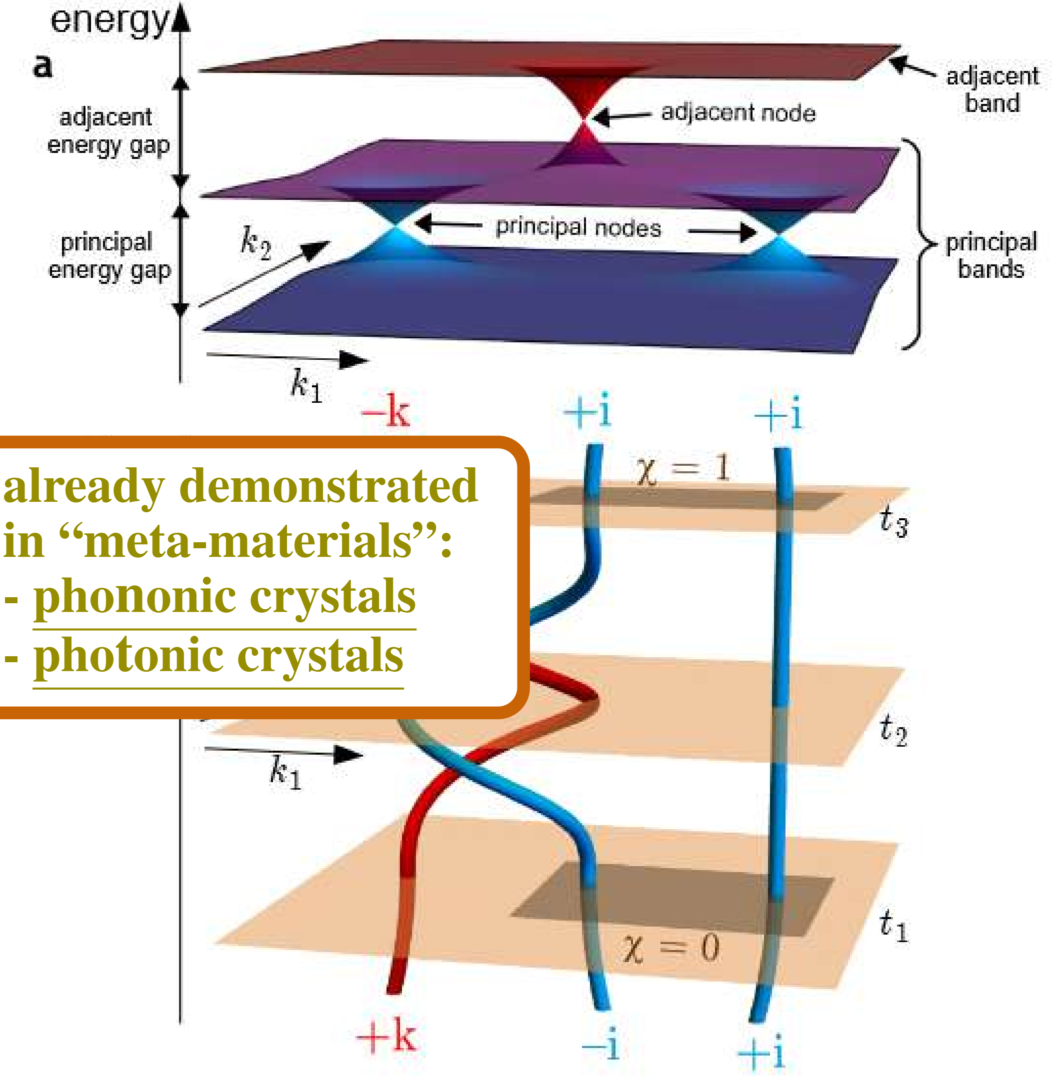

FIG. 1. **Reciprocal braiding of band nodes.**

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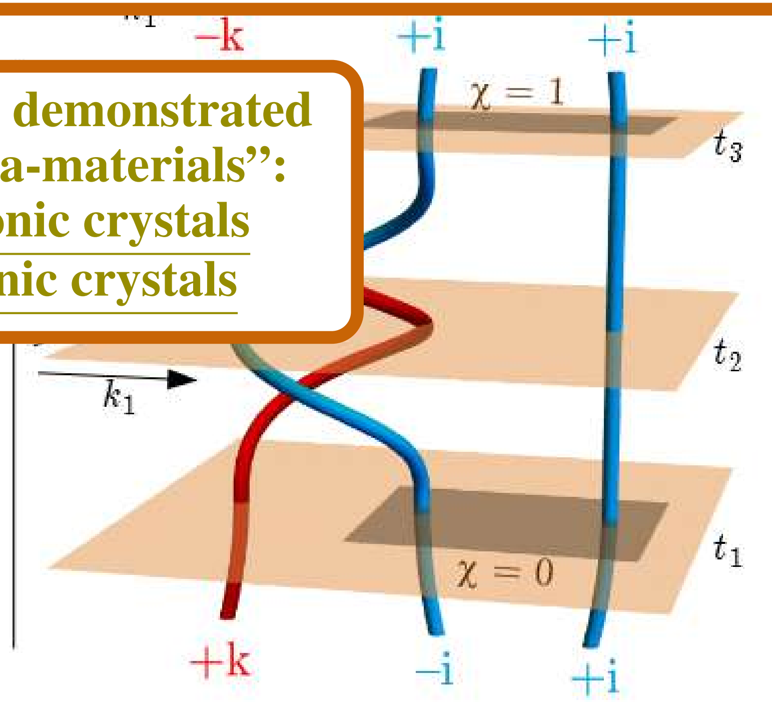

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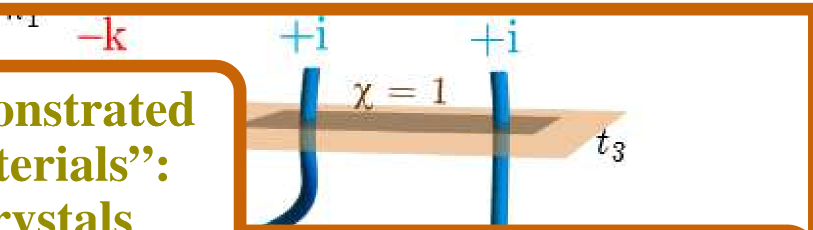

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…hed by De Gruyter February 2, 2022

Nodal lines in momentum space: topological invariants and recent realizations in photonic and other

Haedong Park, Wenlong Gao, Xiao Zhang and Sang Soon Oh

From the journal Nanophotonics

https://doi.org/10.1515/nanoph-2021-0692

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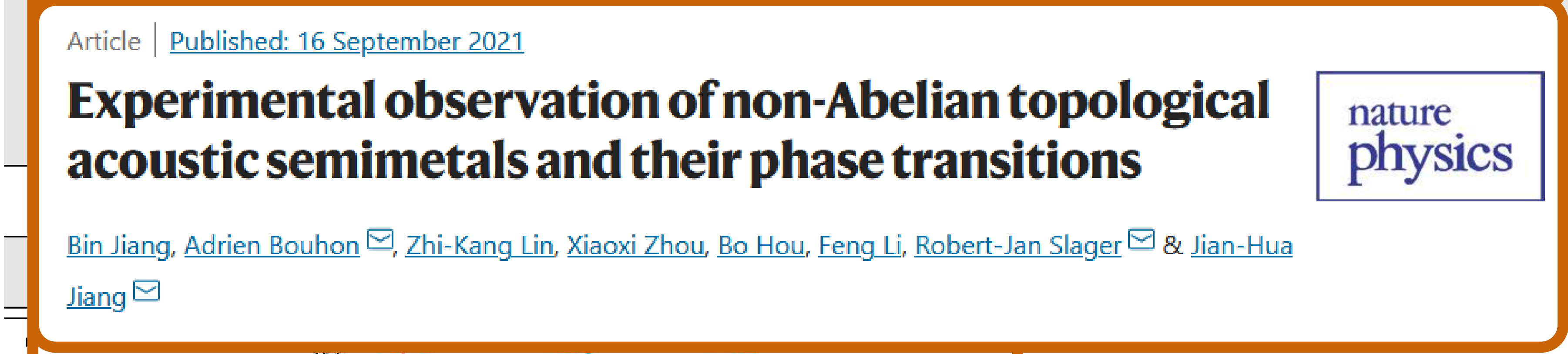
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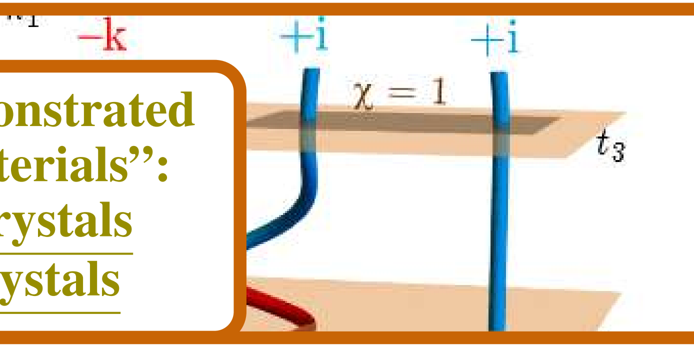

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Article | Published: 28 March 2022

Non-Abelian braiding on photonic chips

Xu-Lin Zhang, Feng Yu, Ze-Guo Chen, Zhen-Nan Tian, Qi-Dai Chen, Hon… Ma

Nature Photonics **16**, 390–395 (2022) | Cite this article

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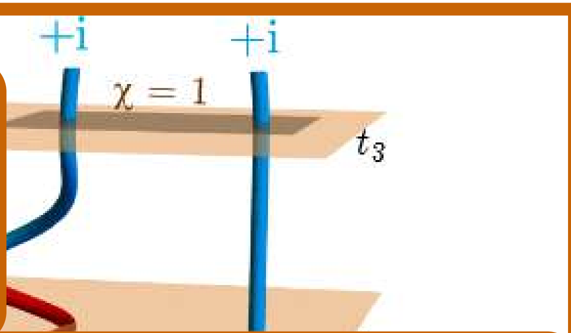

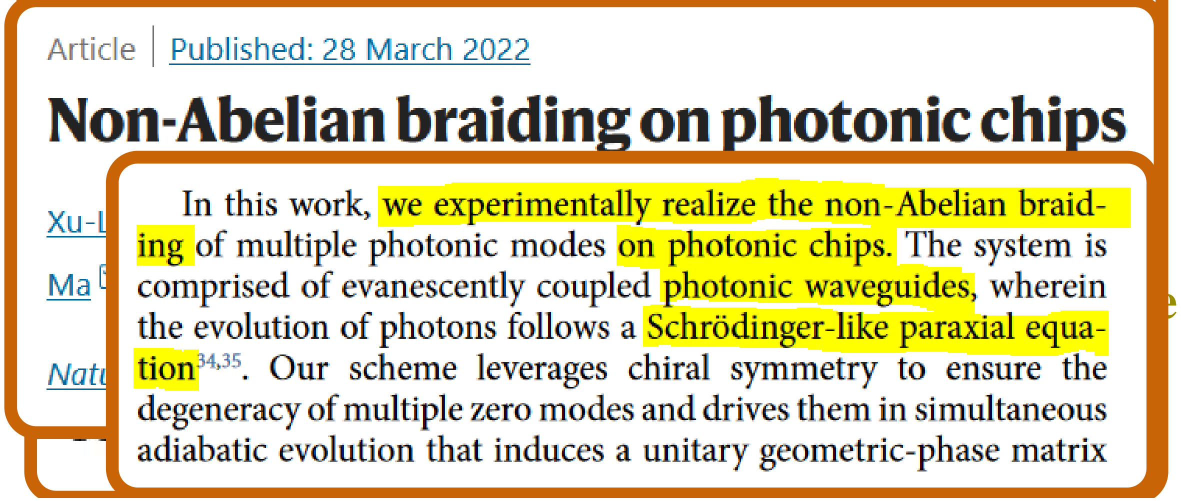

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Non-Abelian braiding on photonic chips

Xu-L
Ma
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In this work, we experimentally realize the non-Abelian braiding of multiple photonic modes on photonic chips. The system is comprised of evanescently coupled photonic waveguides, wherein the evolution of photons follows a Schrödinger-like paraxial equation[34,35]. Our scheme leverages chiral symmetry to ensure the degeneracy of multiple zero modes and drives them in simultaneous adiabatic evolution that induces a unitary geometric-phase matrix

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flux, charge
quantization

ry ⟷ **Alg. Topology/Geom. Homotopy**

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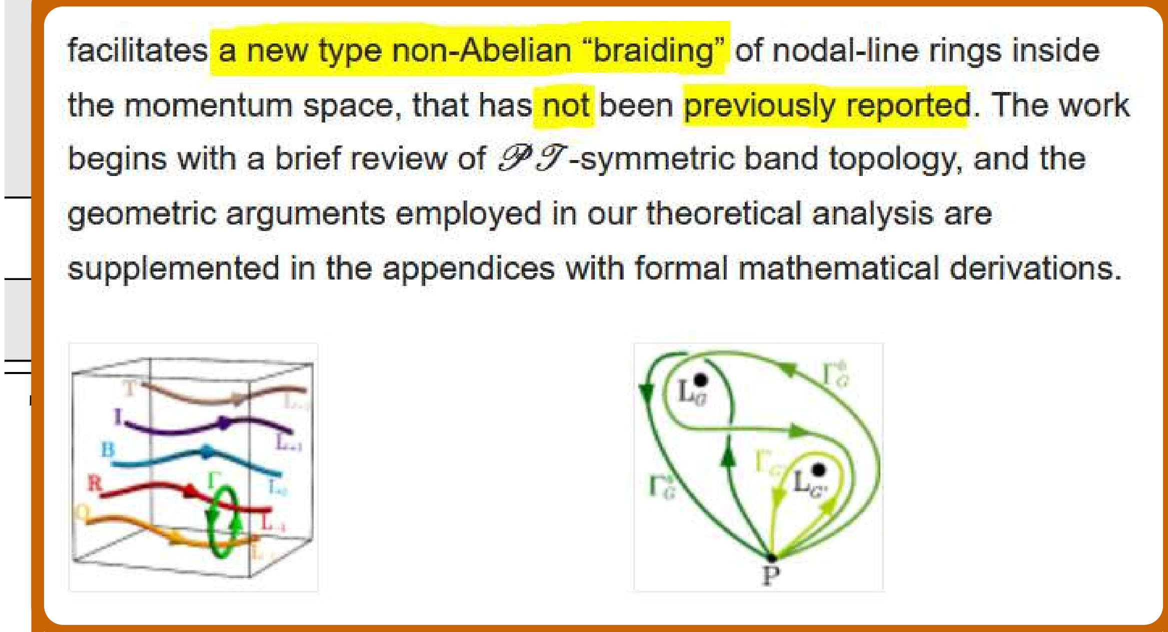
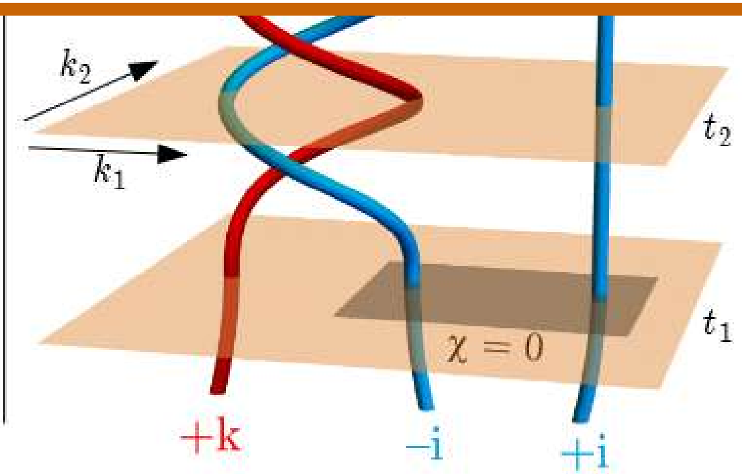
Fibrations of vector spaces

Non-Abelian topology of nodal-line rings in $\mathscr{PT}$-symmetric systems

Apoorv Tiwari and Tomáš Bzdušek
Phys. Rev. B **101**, 195130 – Published 18 May 2020



facilitates a new type non-Abelian "braiding" of nodal-line rings inside the momentum space, that has not been previously reported. The work begins with a brief review of $\mathscr{PT}$-symmetric band topology, and the geometric arguments employed in our theoretical analysis are supplemented in the appendices with formal mathematical derivations.



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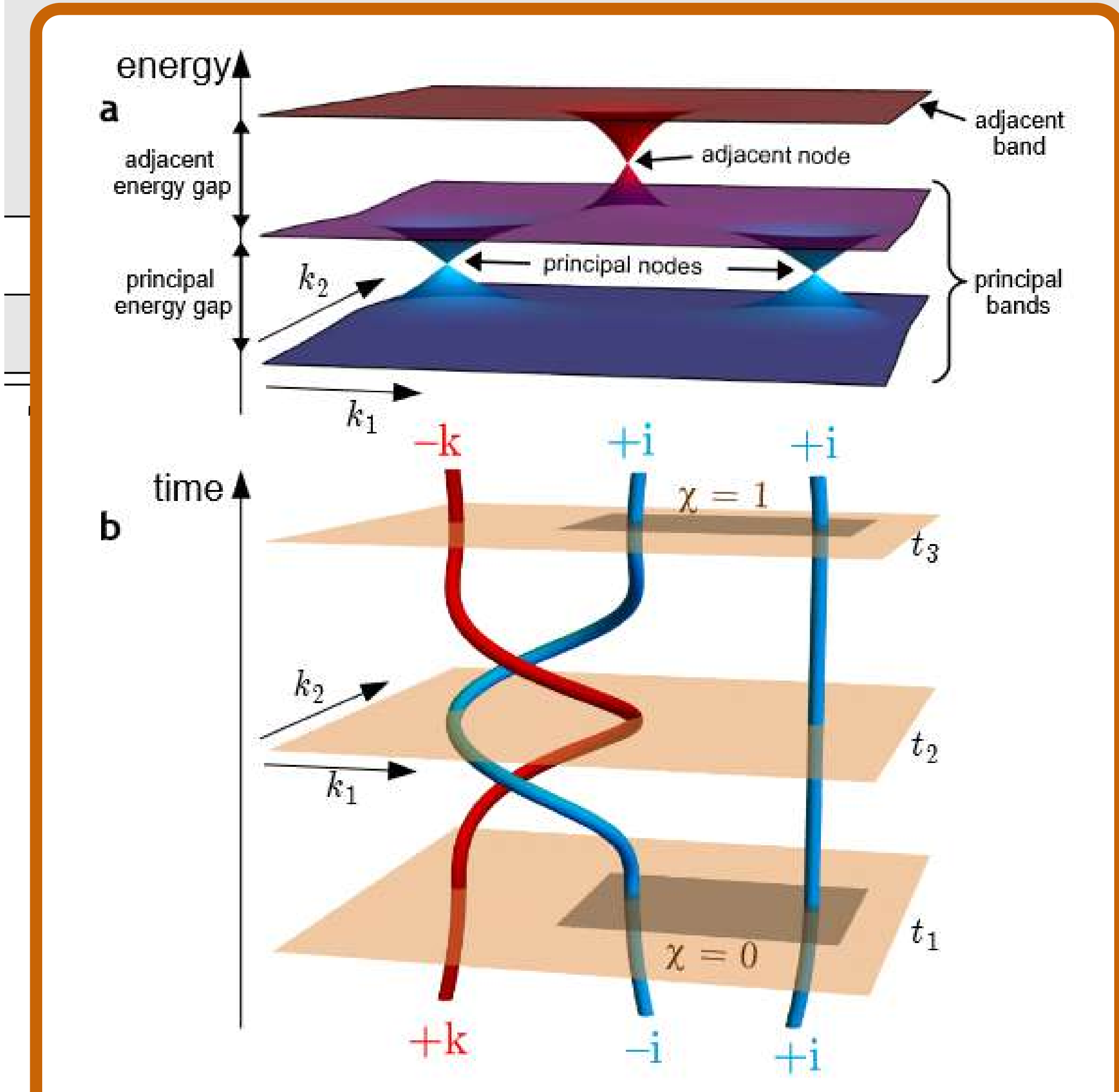

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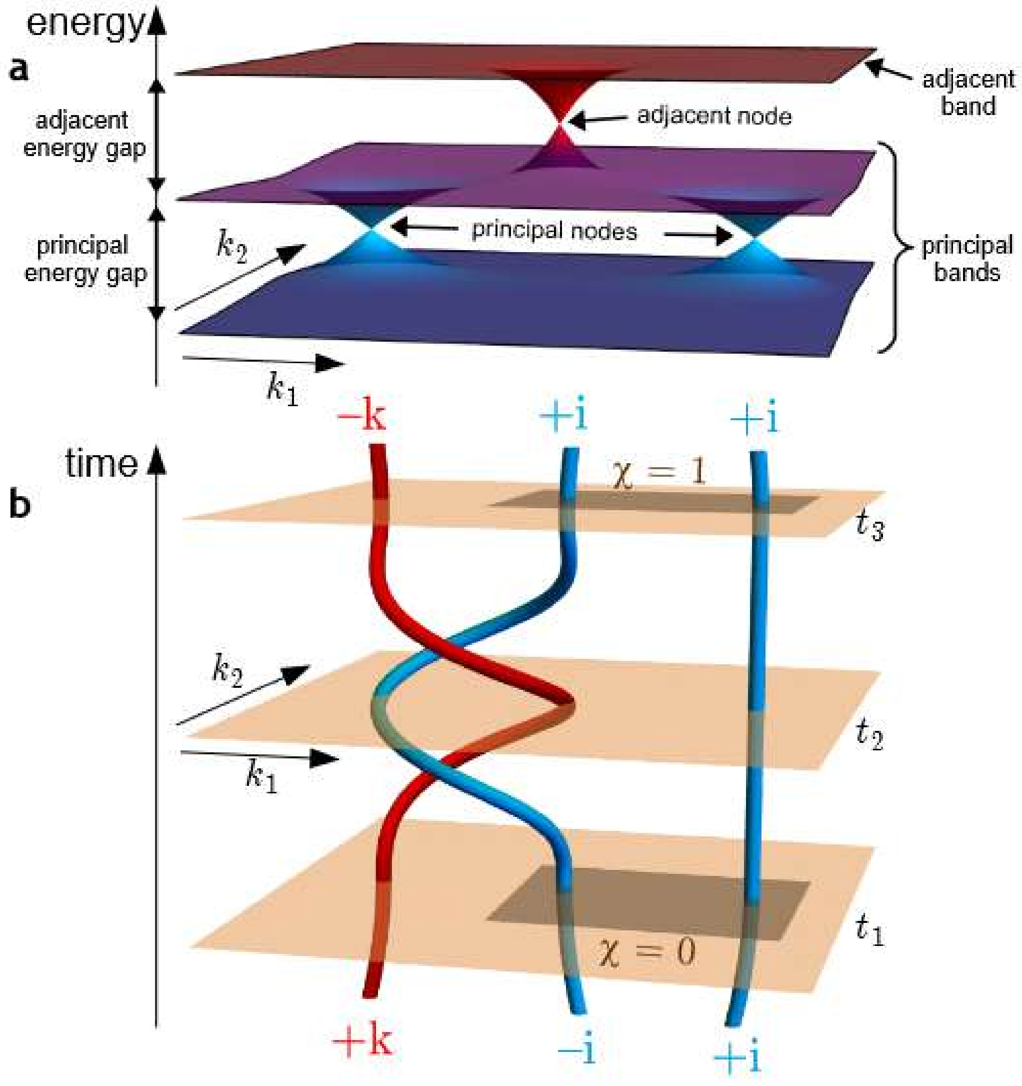
To do: Scrutinize further evidence that/when/how such band nodes indeed qualify as anyons in momentum space.



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$\xrightarrow{}$ **braid representation** **unitary operators**

there is a curious dictionary

flux, charge

Condense

To do: Scrutinize further evidence that/when/how such band nodes indeed qualify as anyons in momentum space.

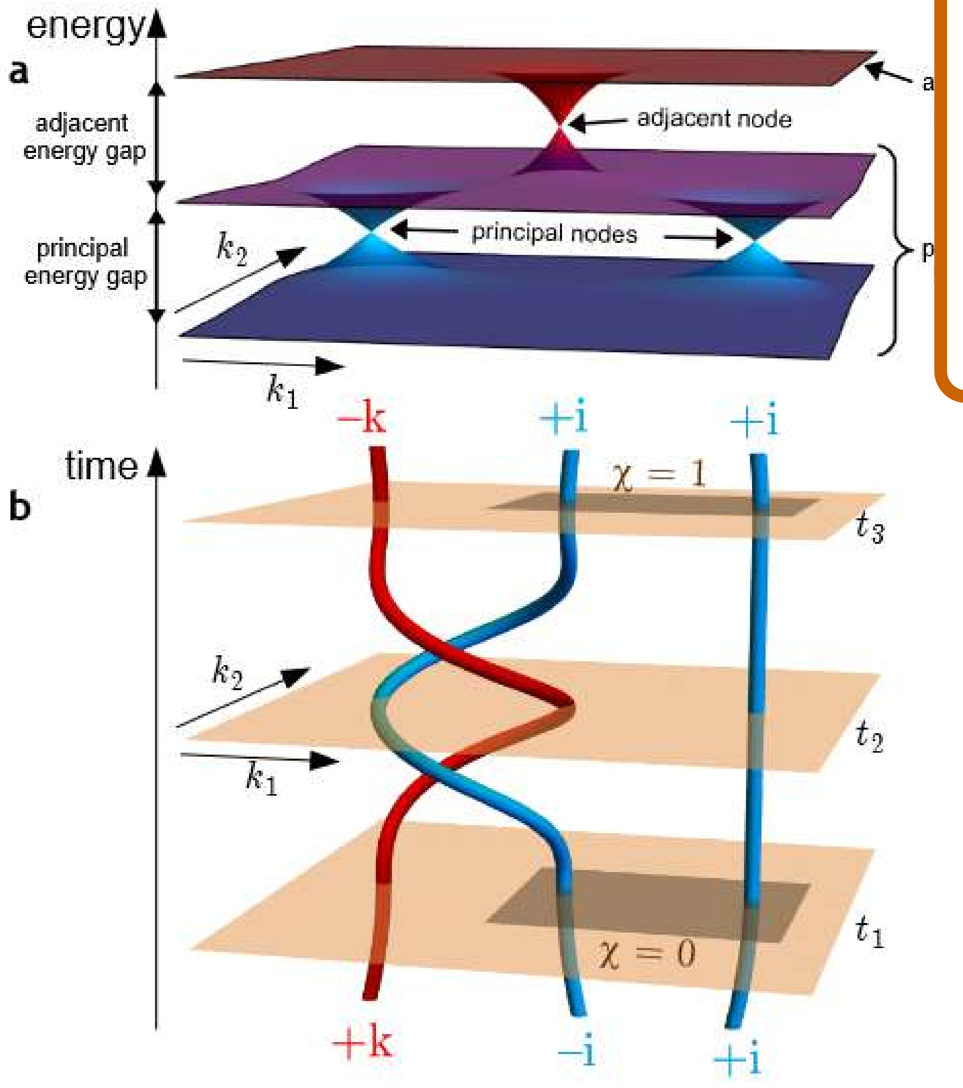
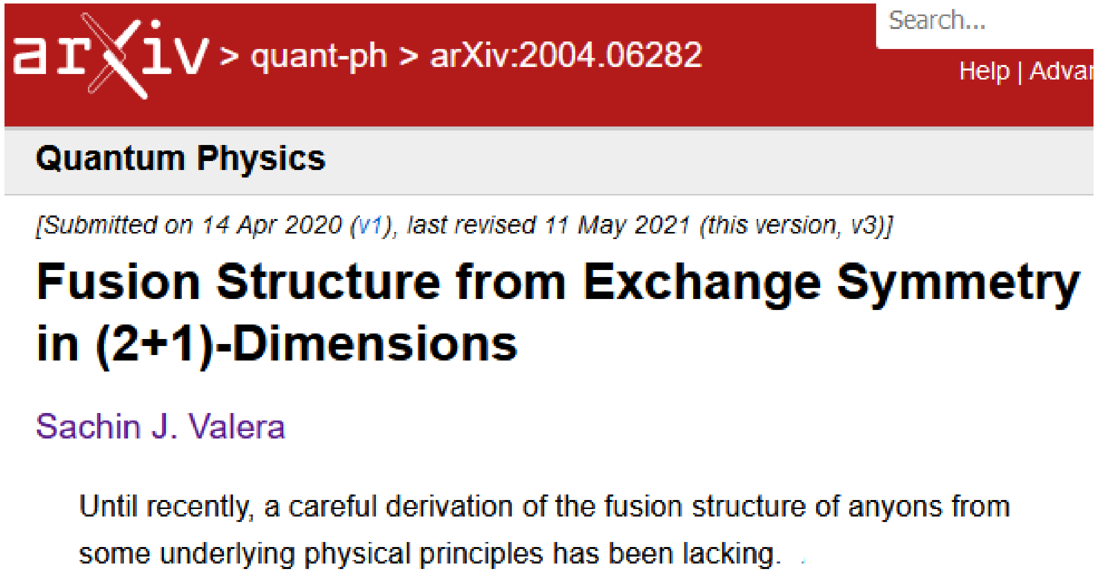


FIG. 1. **Reciprocal braiding of band nodes.**



Quantum Physics

[Submitted on 14 Apr 2020 (v1), last revised 11 May 2021 (this version, v3)]

Fusion Structure from Exchange Symmetry in (2+1)-Dimensions

Sachin J. Valera

Until recently, a careful derivation of the fusion structure of anyons from some underlying physical principles has been lacking.

This describes adiabatic braiding of *band nodes* of topol. ordered semi-metals classified in TED K-theory of config. space:

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

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Condensed/Quantum Matter $\xleftrightarrow{\text{AdS/CMT}}$ **String/M-Theory** $\xleftrightarrow{\text{flux, charge quantization}}$ **Alg. Topology/Geom. Homotopy**

Adiabatic transport of states	Moduli monodromy	Fibrations of vector spaces

Topological Quantum Programming

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[d] & \\
\text{path} \arrow[ur, dashed, "\text{topological quantum computation}"] \arrow[r, "\text{topological quantum program}"'] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
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This describes adiabatic braiding of *band nodes* of topol. ordered semi-metals classified in TED K-theory of config. space:

arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

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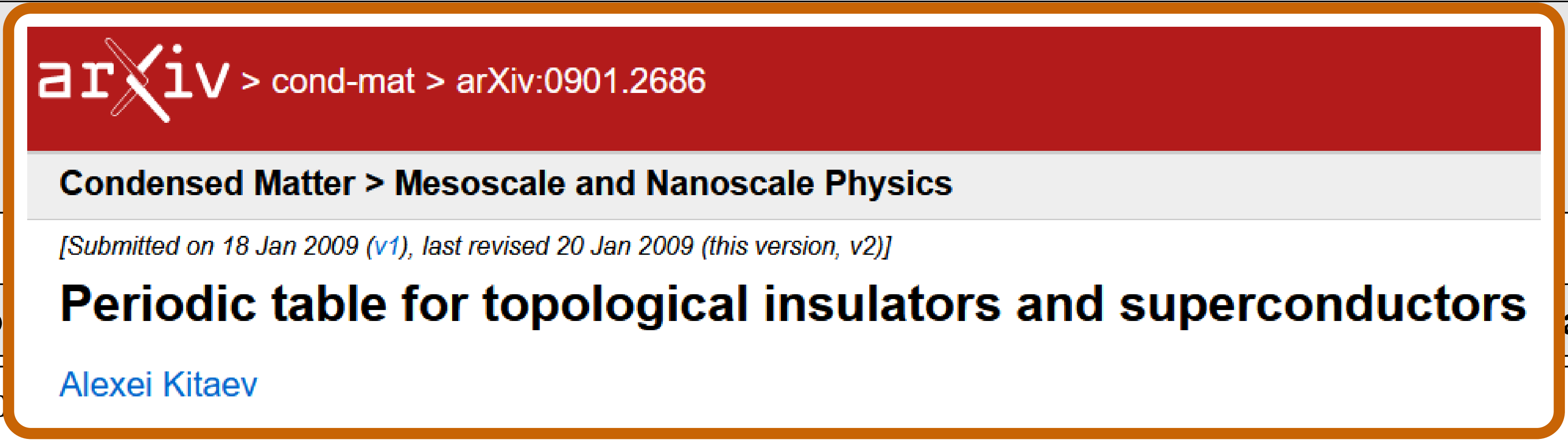
Adiab				aces
Topolo				



arXiv > cond-mat > arXiv:0901.2686

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 18 Jan 2009 (v1), last revised 20 Jan 2009 (this version, v2)]

Periodic table for topological insulators and superconductors

Alexei Kitaev

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[d] & \\
\text{path} \arrow[r, "\text{topological quantum program}"'] \arrow[ur, dashed, "\text{topological quantum computation}"] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
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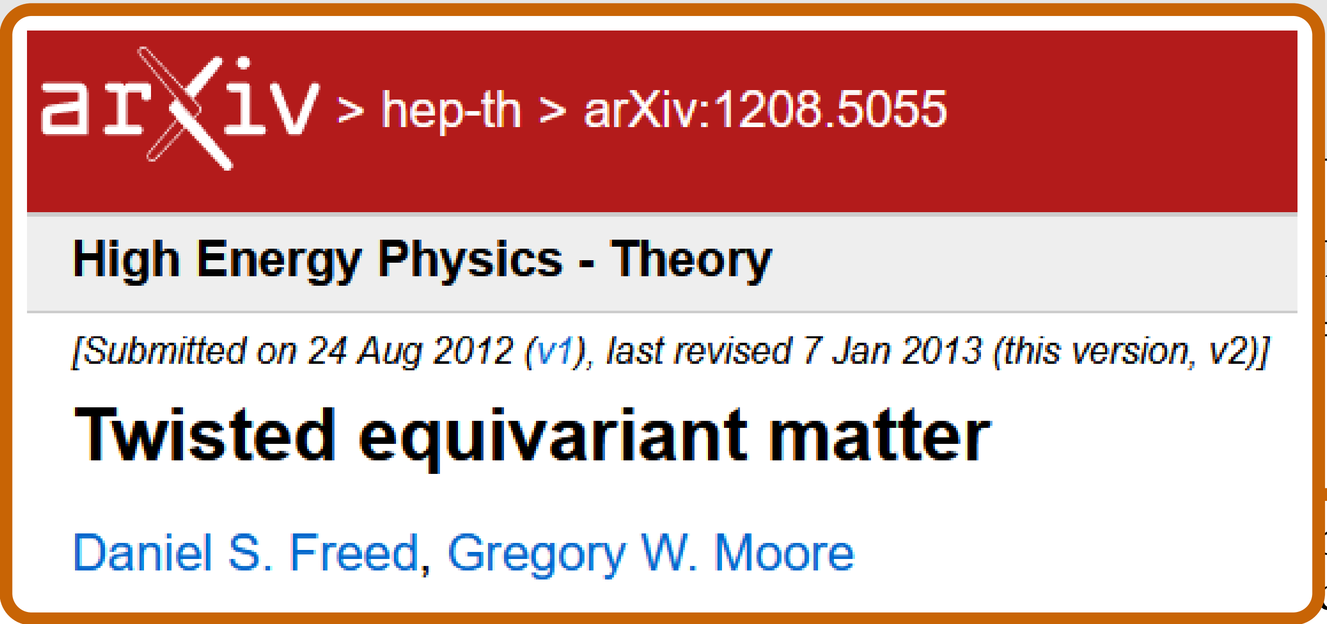
Topological Quantum				



arXiv > hep-th > arXiv:1208.5055

High Energy Physics - Theory

[Submitted on 24 Aug 2012 (v1), last revised 7 Jan 2013 (this version, v2)]

Twisted equivariant matter

Daniel S. Freed, Gregory W. Moore

braiding of
d semi-metals
classified in TED K-theory of config. space:

arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

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path ⟶ **configuration space of distinct points** ⟶ **unitary operators**

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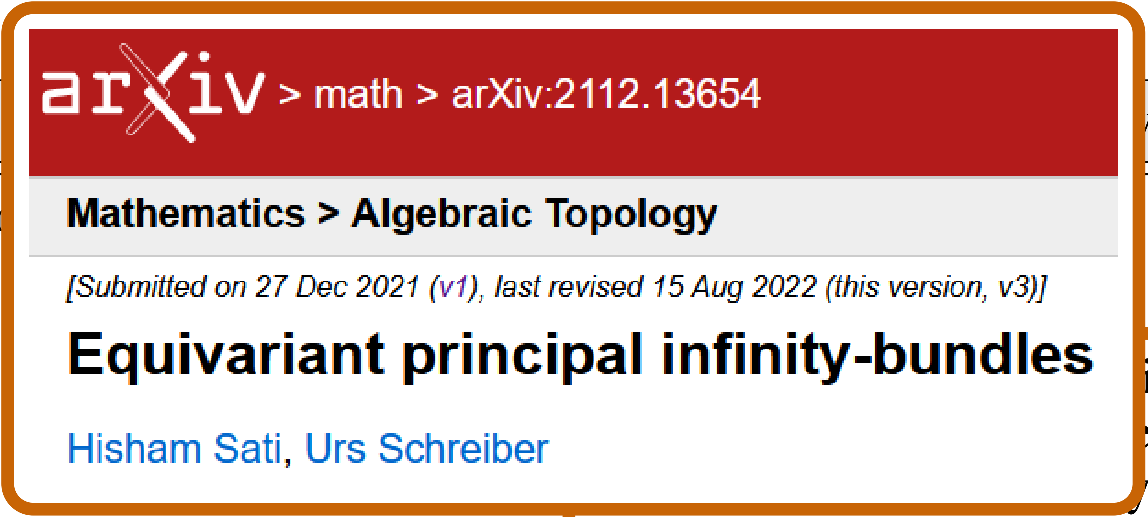
Adiabatic transport of states … vector spaces

Topological Quantum Progr



arXiv > math > arXiv:2112.13654

Mathematics > Algebraic Topology

[Submitted on 27 Dec 2021 (v1), last revised 15 Aug 2022 (this version, v3)]

Equivariant principal infinity-bundles

Hisham Sati, Urs Schreiber

ic braiding of
ered semi-metals
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arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

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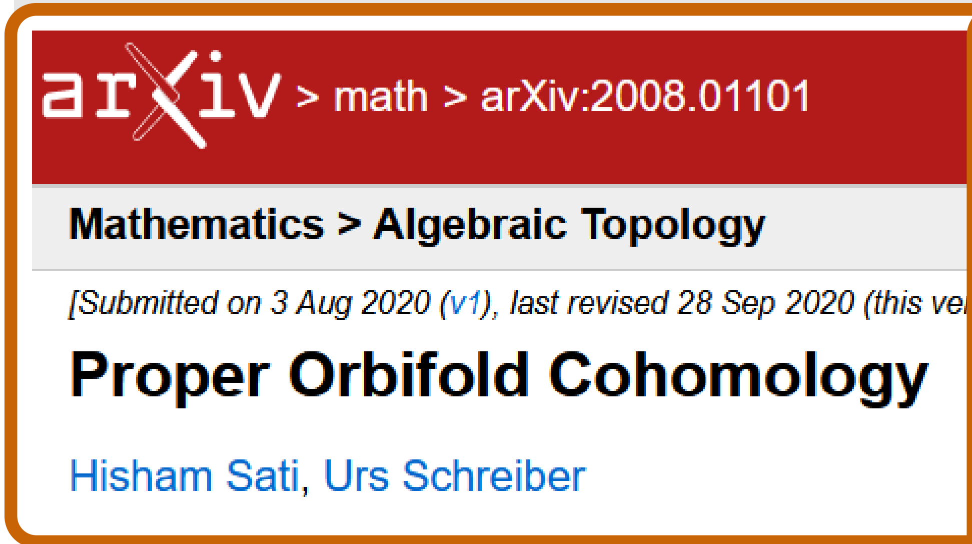


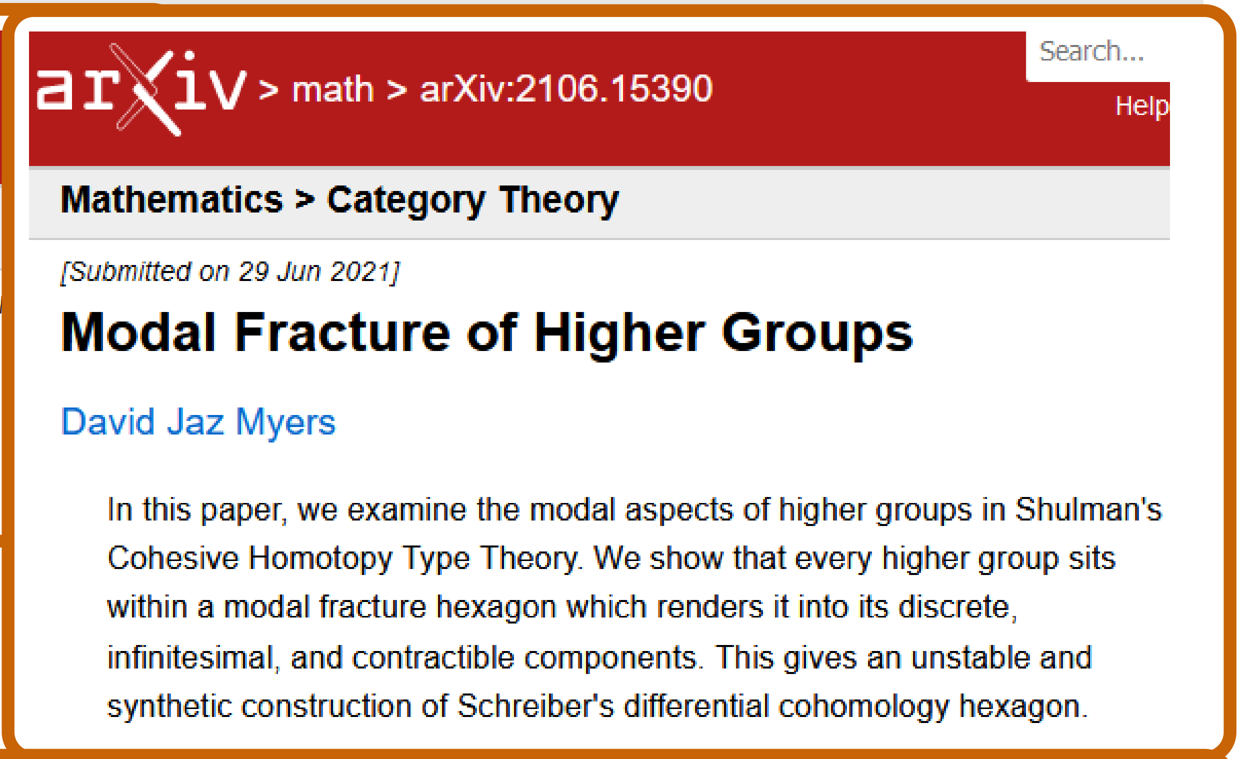
arXiv > math > arXiv:2008.01101

Mathematics > Algebraic Topology

[Submitted on 3 Aug 2020 (v1), last revised 28 Sep 2020 (this ve

Proper Orbifold Cohomology

Hisham Sati, Urs Schreiber



arXiv > math > arXiv:2106.15390

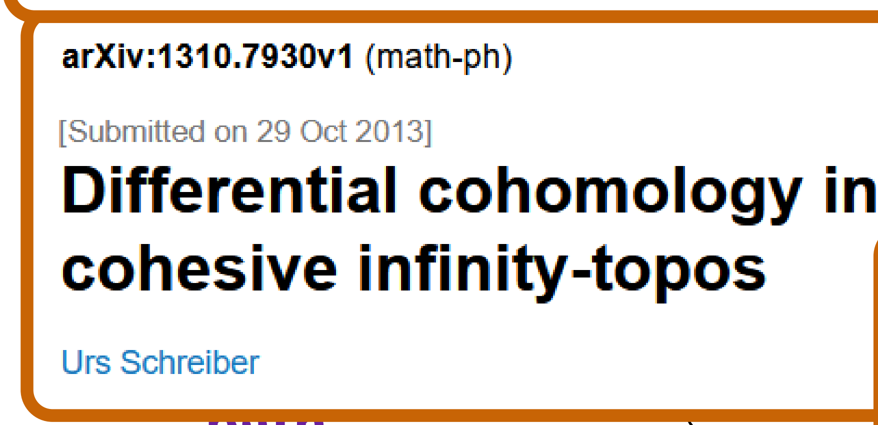
Mathematics > Category Theory

[Submitted on 29 Jun 2021]

Modal Fracture of Higher Groups

David Jaz Myers

In this paper, we examine the modal aspects of higher groups in Shulman's Cohesive Homotopy Type Theory. We show that every higher group sits within a modal fracture hexagon which renders it into its discrete, infinitesimal, and contractible components. This gives an unstable and synthetic construction of Schreiber's differential cohomology hexagon.

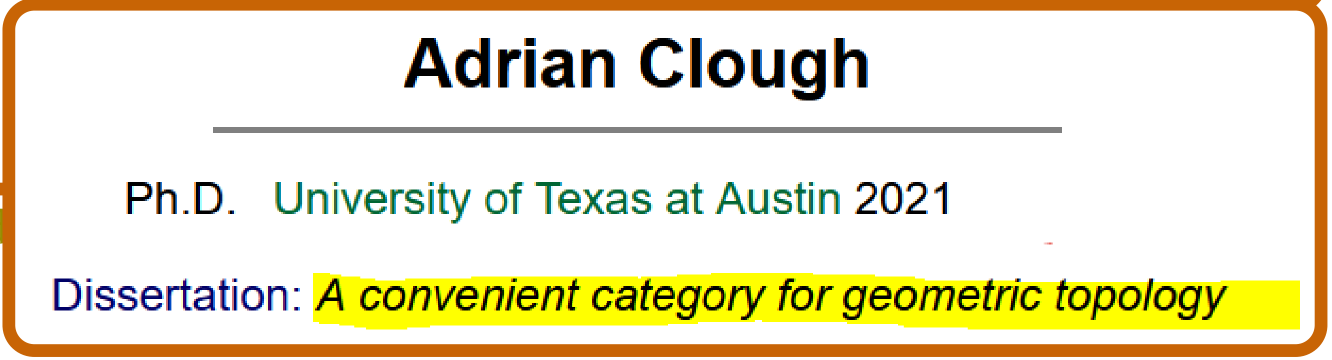


arXiv:1310.7930v1 (math-ph)

[Submitted on 29 Oct 2013]

Differential cohomology in a cohesive infinity-topos

Urs Schreiber



Adrian Clough

Ph.D. University of Texas at Austin 2021

Dissertation: *A convenient category for geometric topology*

path

topological quantum program

of d

there is a curious dictionary

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arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

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topological order		higher gauge field		twisted-
(anyonic) interactions		(defect) M-branes		Co-Bordism/-Homotopy
A				

To

arXiv > hep-th > arXiv:2203.11838

High Energy Physics - Theory

[Submitted on 22 Mar 2022]

Anyonic Defect Branes and Conformal Blocks in Twisted Equivariant Differential (TED) K-theory

Hisham Sati, Urs Schreiber

classified in TED K-theory of config. space:

arXiv > hep-th > arXiv:2206.13563

High Energy Physics - Theory

[Submitted on 27 Jun 2022]

Anyonic Topological Order in Twisted Equivariant Differential (TED) K-Theory

Hisham Sati, Urs Schreiber

topological quantum computation

path $\xrightarrow{\text{topological quantum program}}$ **configuration space of distinct points** $\xrightarrow{\text{braid representation}}$ **unitary operators**

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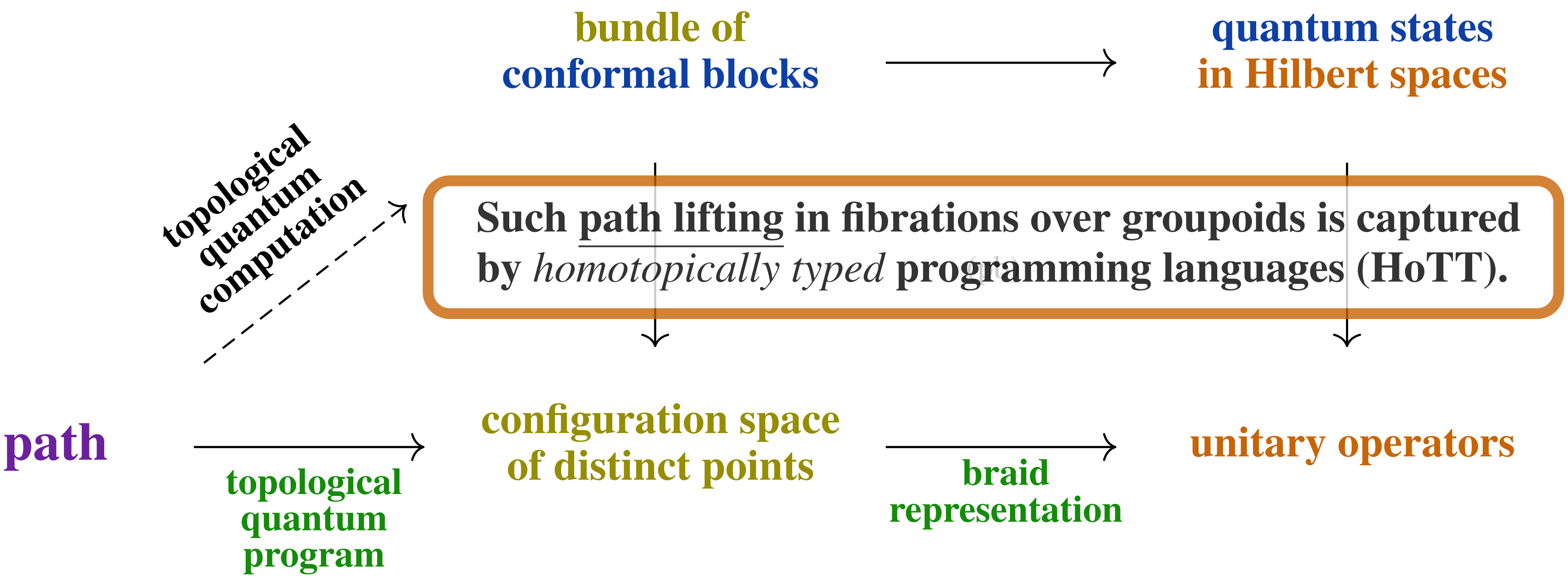

bundle of conformal blocks → **quantum states in Hilbert spaces**

topological quantum computation

path → **configuration space of distinct points** → **unitary operators**

topological quantum program

braid representation

Such path lifting in fibrations over groupoids is captured by *homotopically typed* programming languages (HoTT).

there is a curious dictionary

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Topological Quantum Programming				**Linear Homotopy Type Theory**

$$\begin{tikzcd}
 & \text{bundle of conformal blocks} \arrow[r] \arrow[d, "\text{dependent type family}"] \arrow[dr, phantom, "\text{(pb)}"] & \text{quantum states in Hilbert spaces} \arrow[d, "\text{univalent type universe}"] \\
\text{path} \arrow[r, "\text{topological quantum program}"', ""{name=D}] \arrow[ur, dashed, "\text{topological quantum computation}", ""{name=U}] & \text{configuration space of distinct points} \arrow[r, "\text{braid representation}"'] & \text{unitary operators}
\arrow[Rightarrow, from=D, to=U, "\text{path lift}"']
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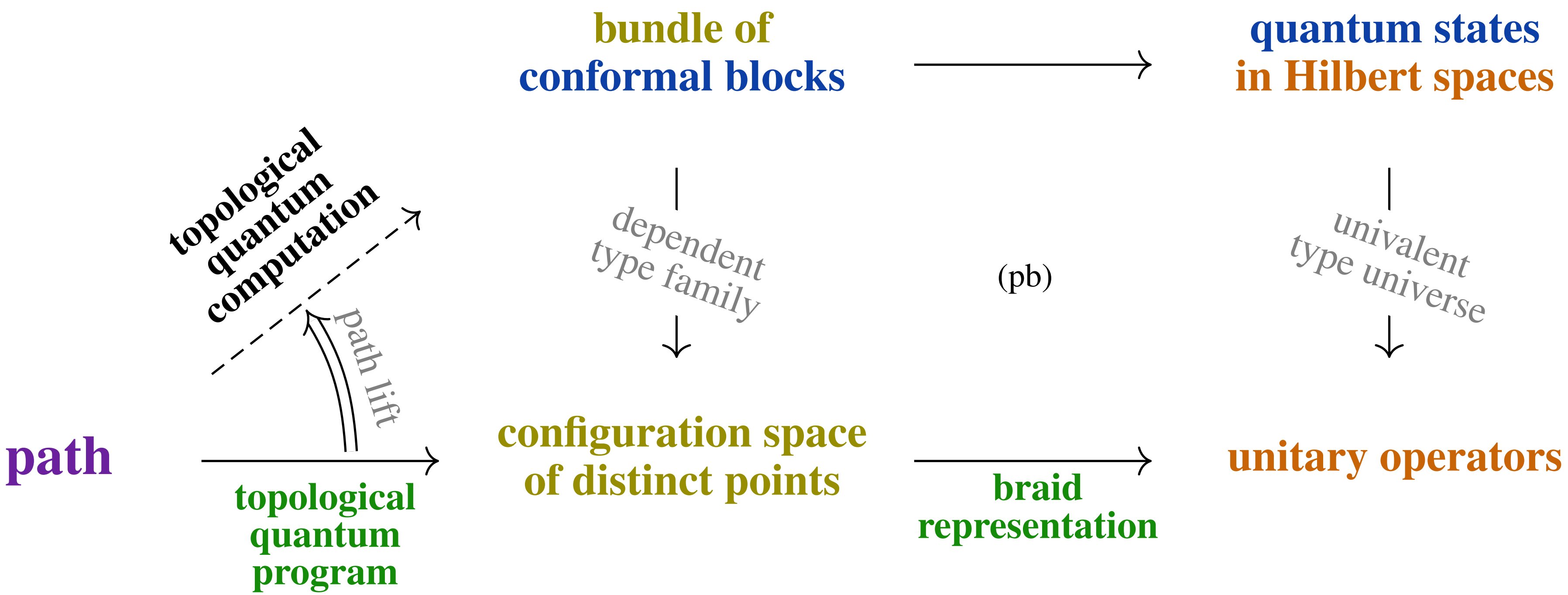

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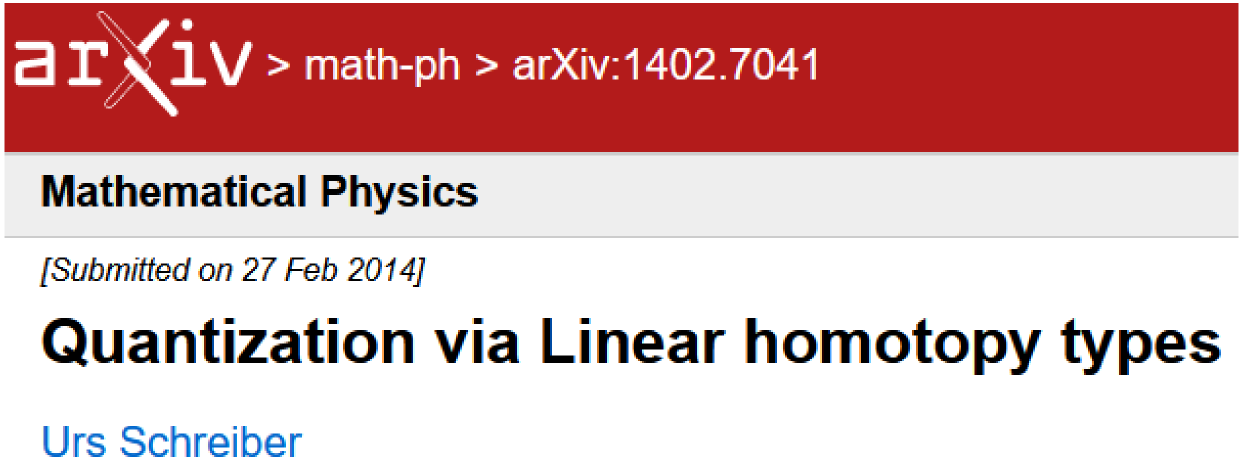


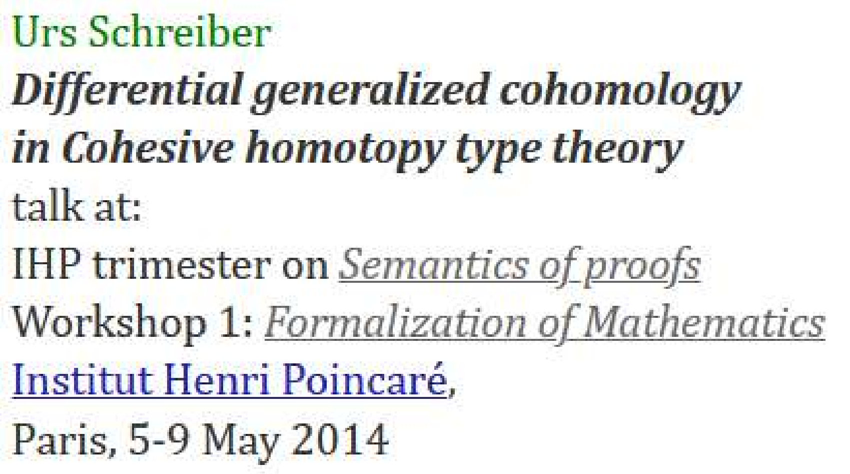
arXiv > math-ph > arXiv:1402.7041

Mathematical Physics

[Submitted on 27 Feb 2014]

Quantization via Linear homotopy types

Urs Schreiber



Urs Schreiber

Differential generalized cohomology in Cohesive homotopy type theory

talk at:

IHP trimester on *Semantics of proofs*

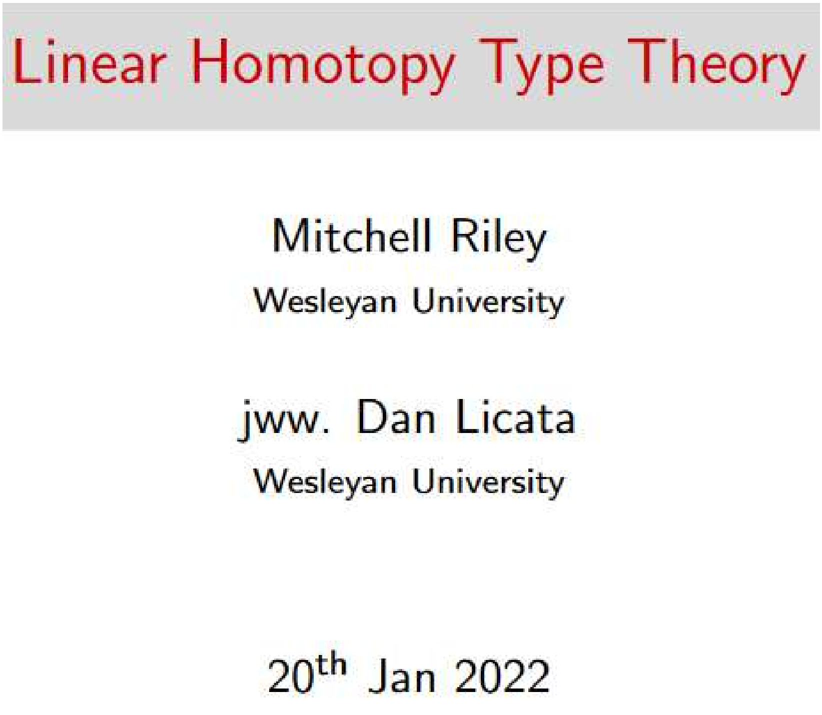
Workshop 1: *Formalization of Mathematics*

Institut Henri Poincaré,

Paris, 5-9 May 2014

Topological Quantum Programming

Linear Homotopy Type Theory



Linear Homotopy Type Theory

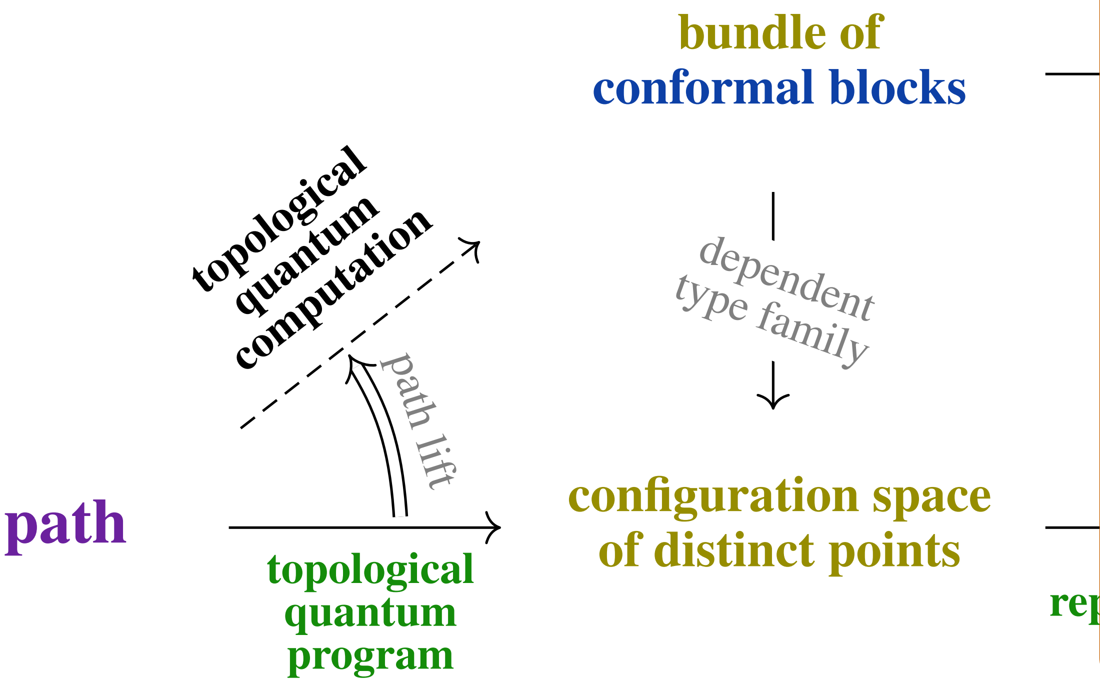
Mitchell Riley

Wesleyan University

jww. Dan Licata

Wesleyan University

20th Jan 2022



bundle of conformal blocks

dependent type family

configuration space of distinct points

path

topological quantum program

path lift

topological quantum computation

rep

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Berry phases	gauge field	cohesive differential-
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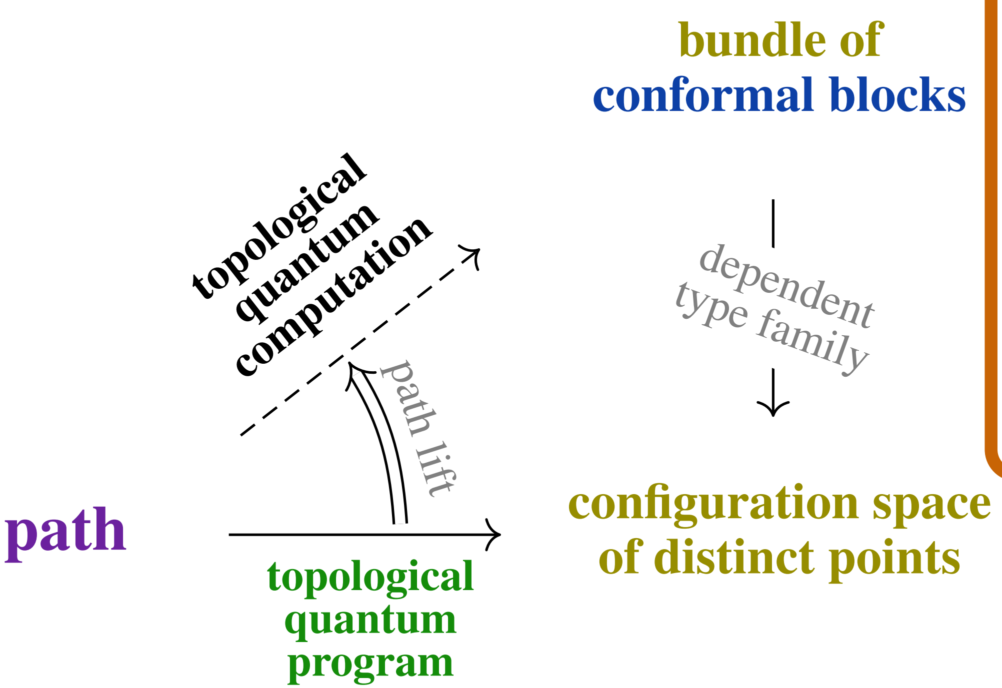
(anyonic) interactions	(defect) M-branes	Co-Bordism/-Homotopy
Adiabatic transport of states	Moduli monodromy	Fibrations of mapping spectra

Topological Quantum Programming **Linear Homotopy Type Theory**



Under this translation,
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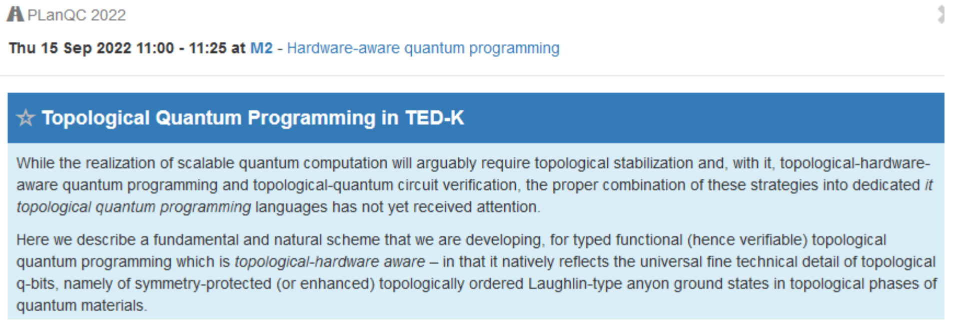

Thu 15 Sep 2022 11:00 - 11:25 at M2 - Hardware-aware quantum programming

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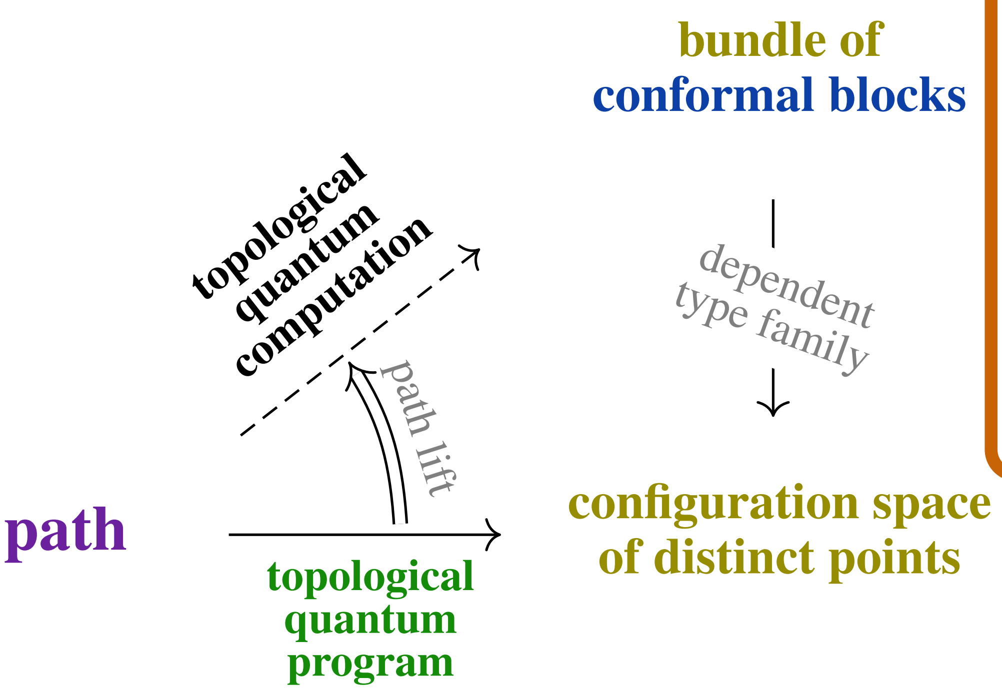
⇒ **plan of attack**



braid representation → **unitary operators**

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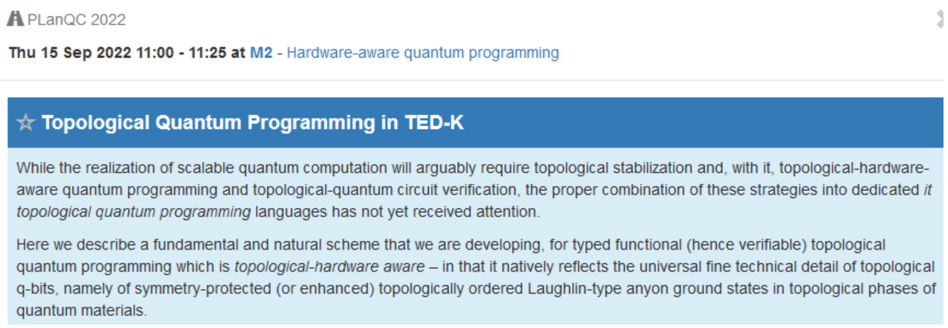
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Programming platform:

Cohesive Homotopy
Type Theory with
dependent linear types

plan of
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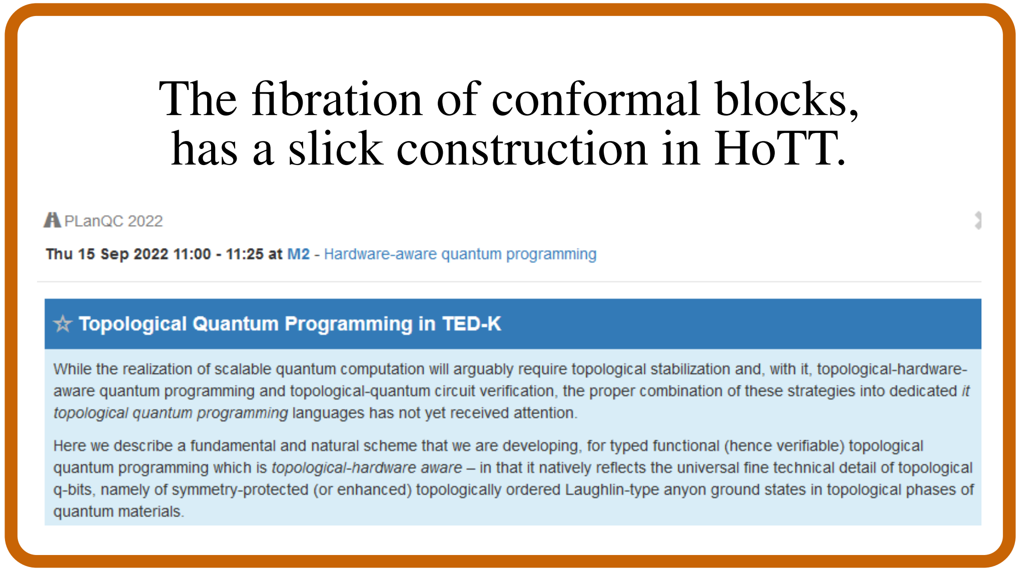

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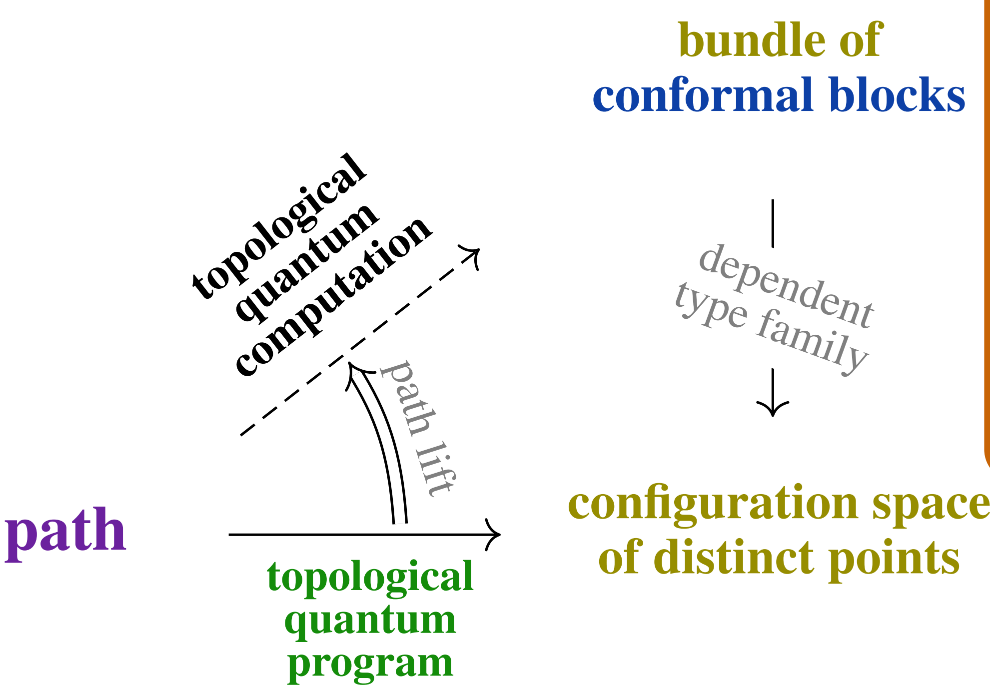

topological quantum computation

path lift

bundle of conformal blocks

dependent type family

path

topological quantum program

configuration space of distinct points

braid representation

unitary operators

Programming platform:

Cohesive Homotopy Type Theory with dependent linear types

implements

(1)

Library/Module:

TED-K-cohomology of defect configurations in crystallographic orbifolds

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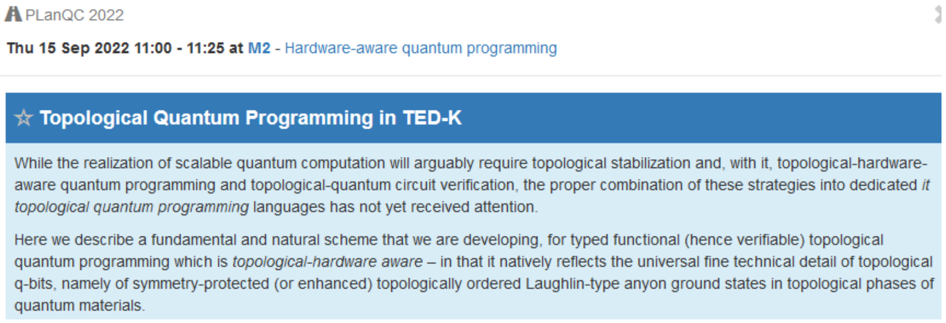

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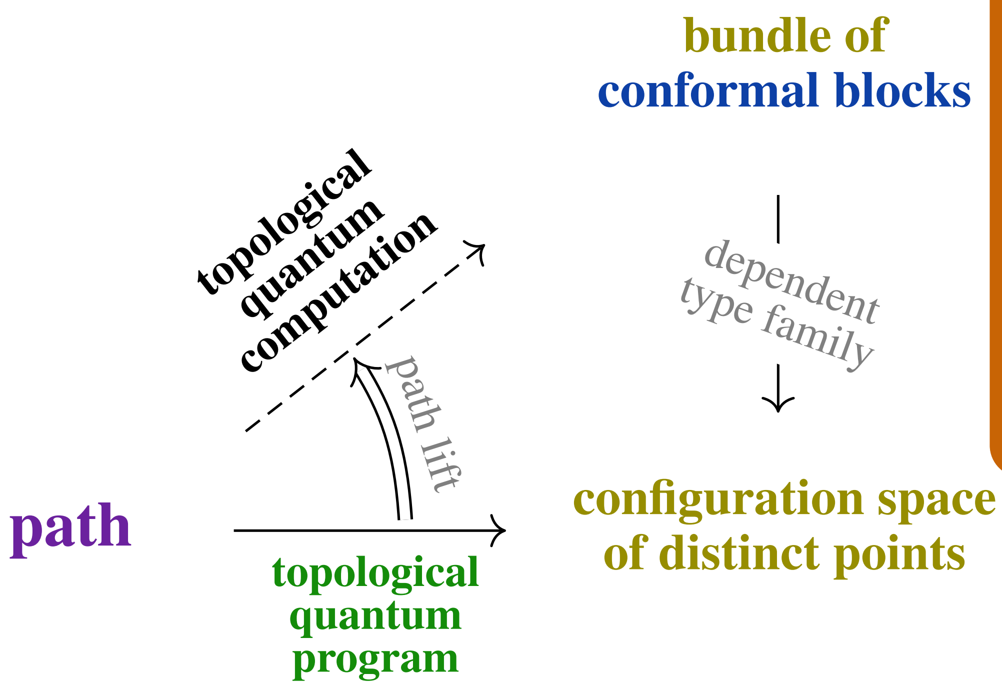

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Anyonic band nodes in topol. semimetals

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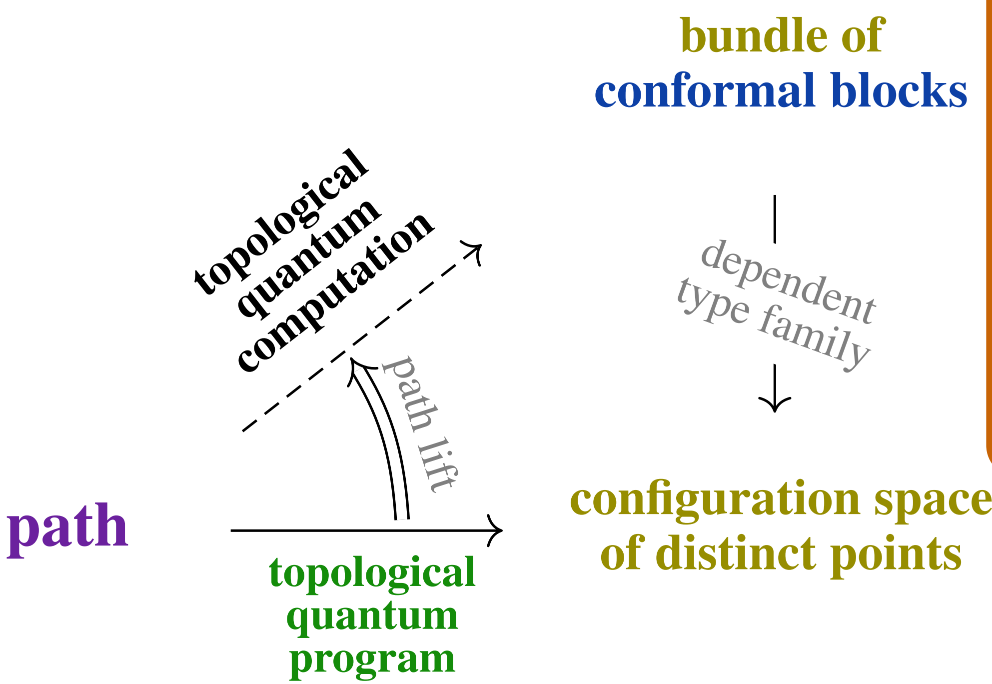

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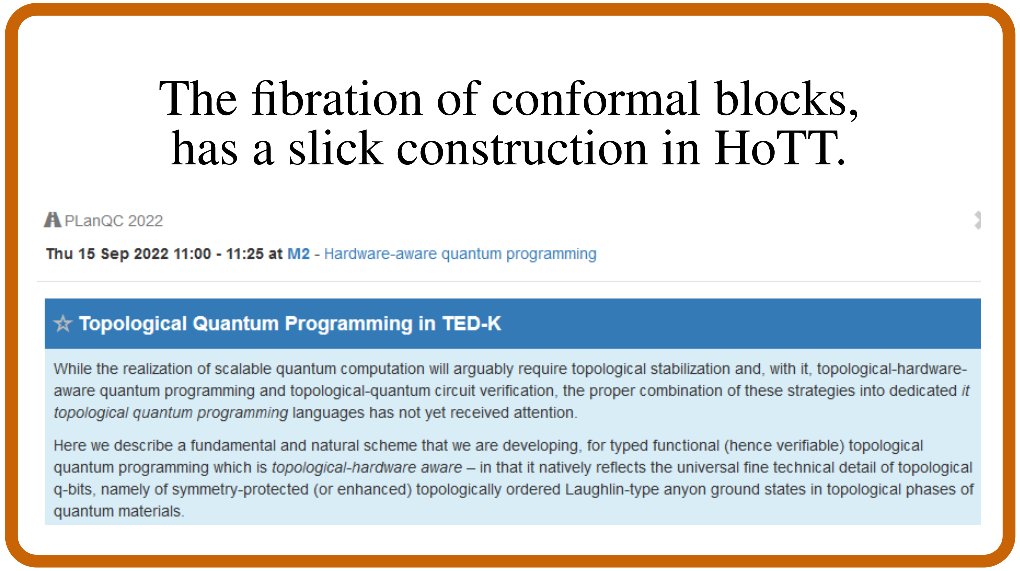
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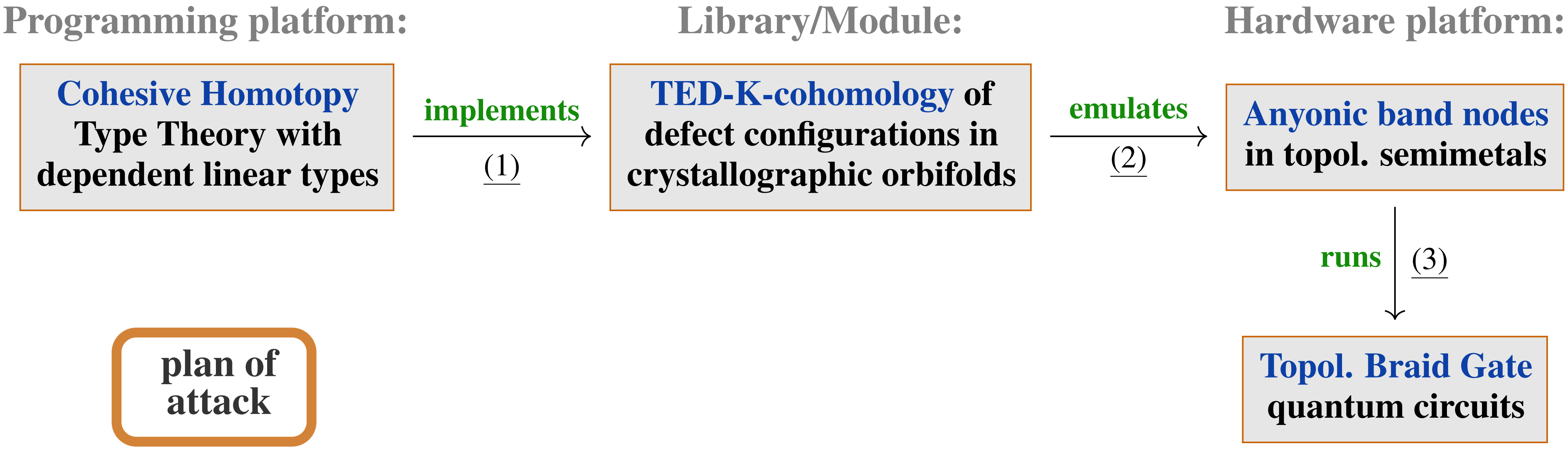

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Topol. Braid Gate quantum circuits

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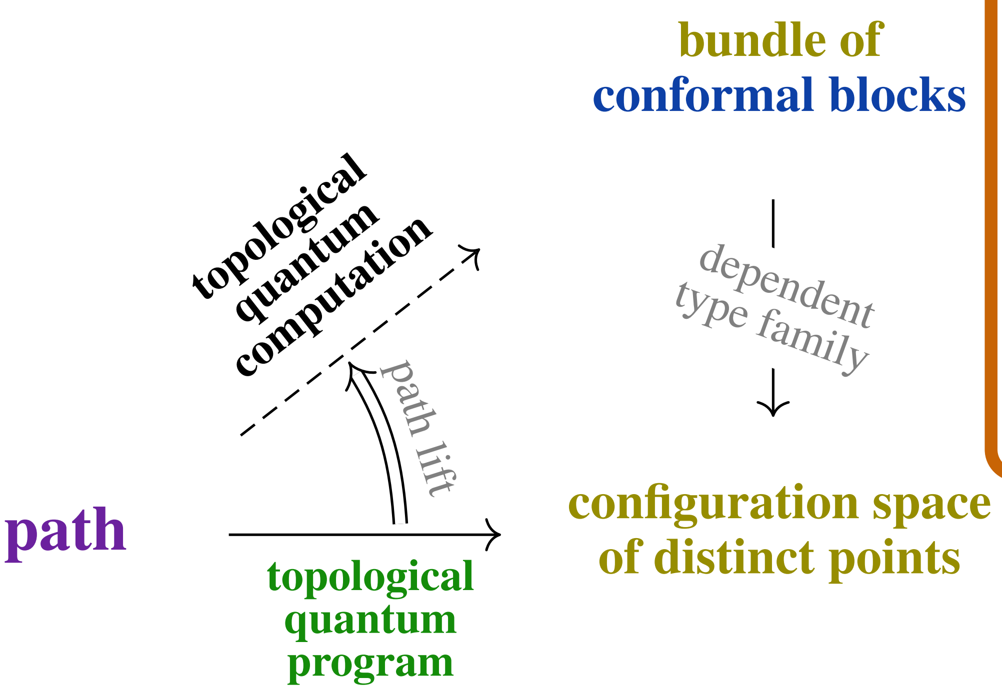

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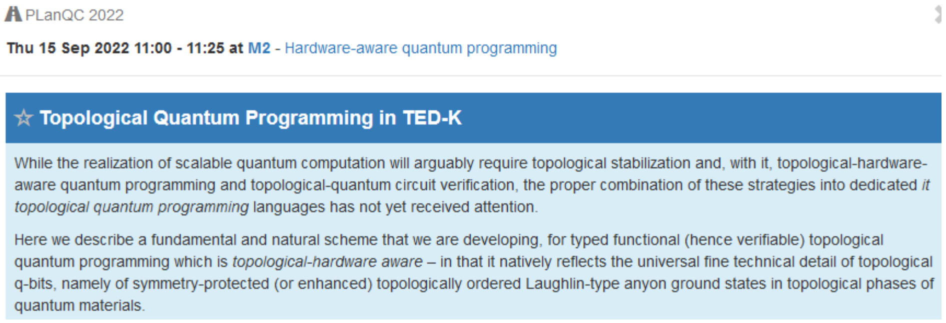

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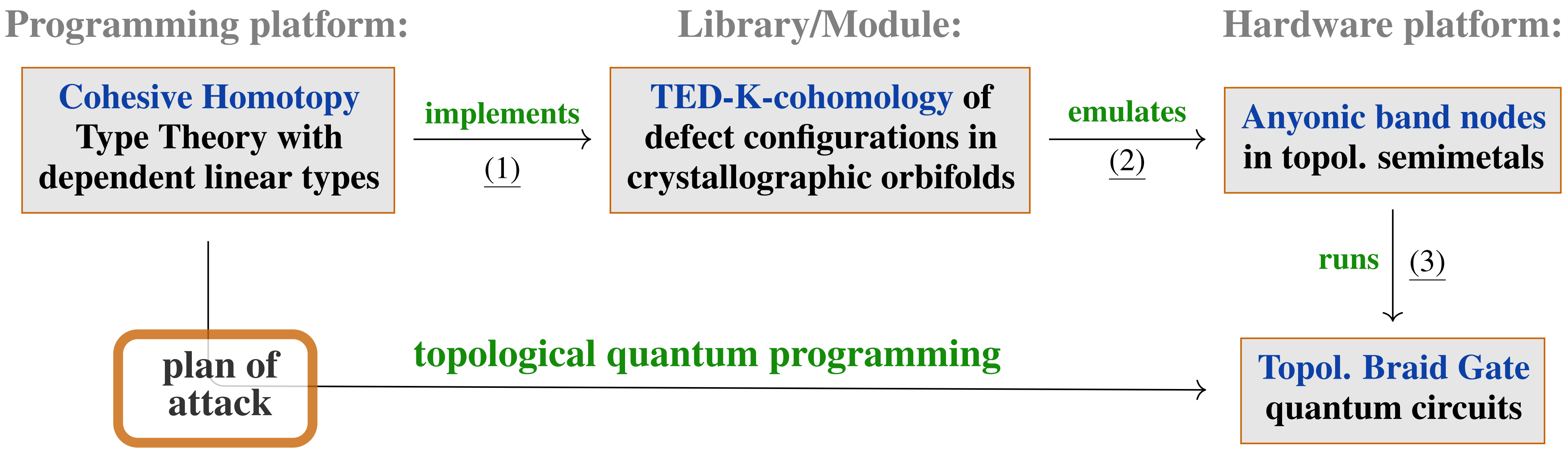

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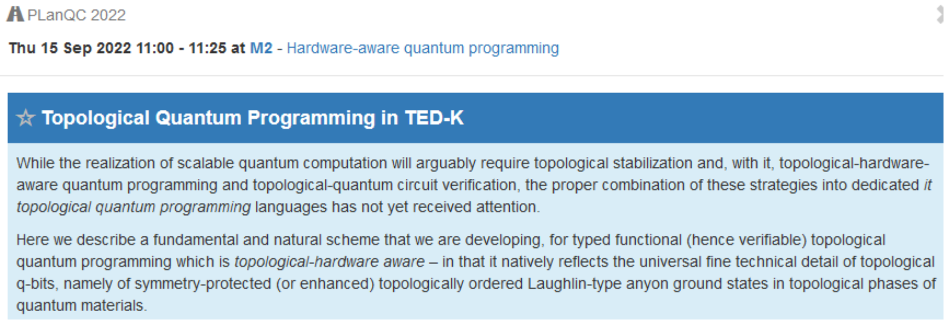

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(1) – The Problem:

Practical Foundations of
Topological Quantum Computation

(2) – The Strategy:

Cohesive Linear Homotopy for
Holographic Condensed Matter Theory

(3) – The Technology:

TED K-Cohomology of
Cohomotopy Moduli Spaces

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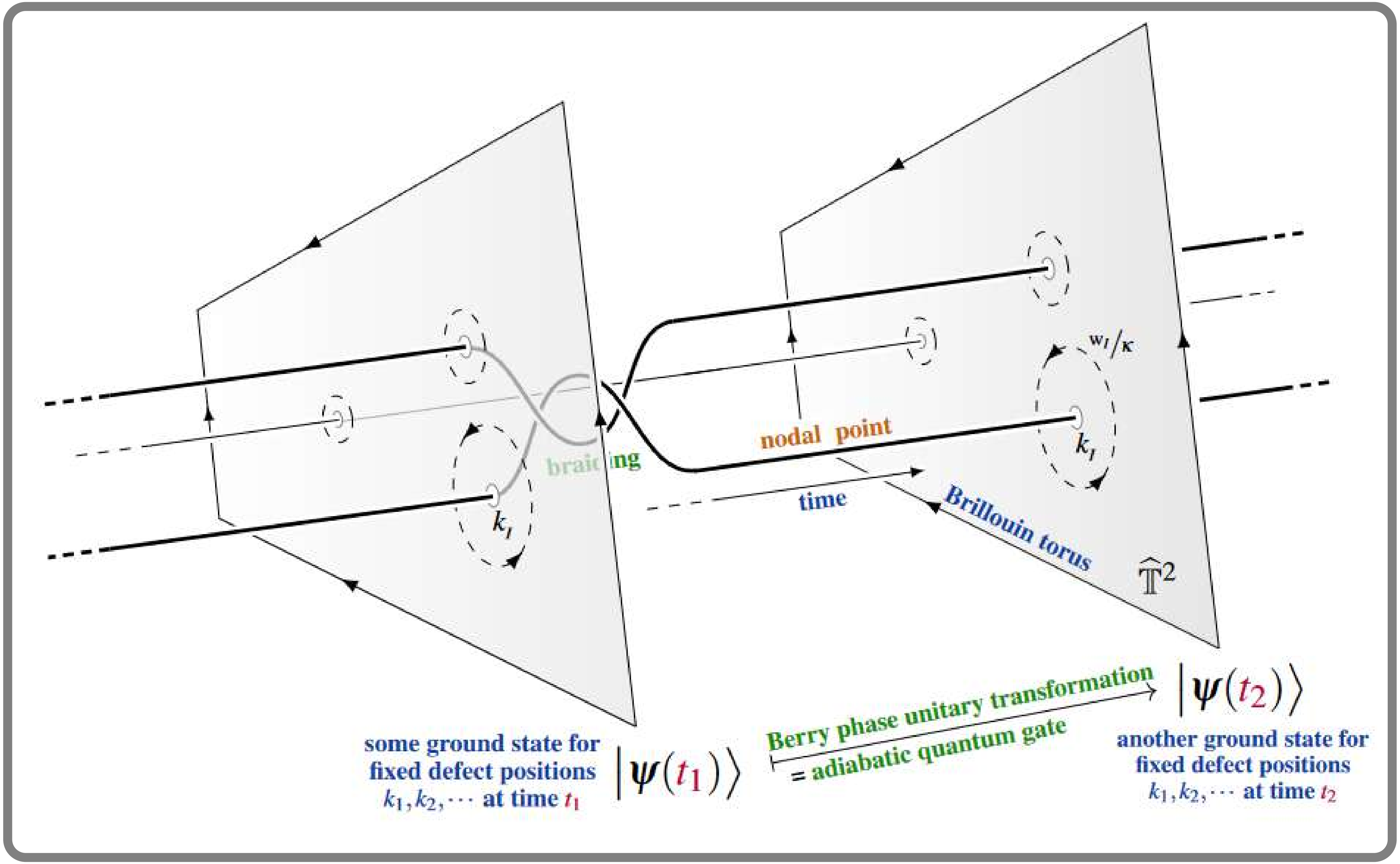

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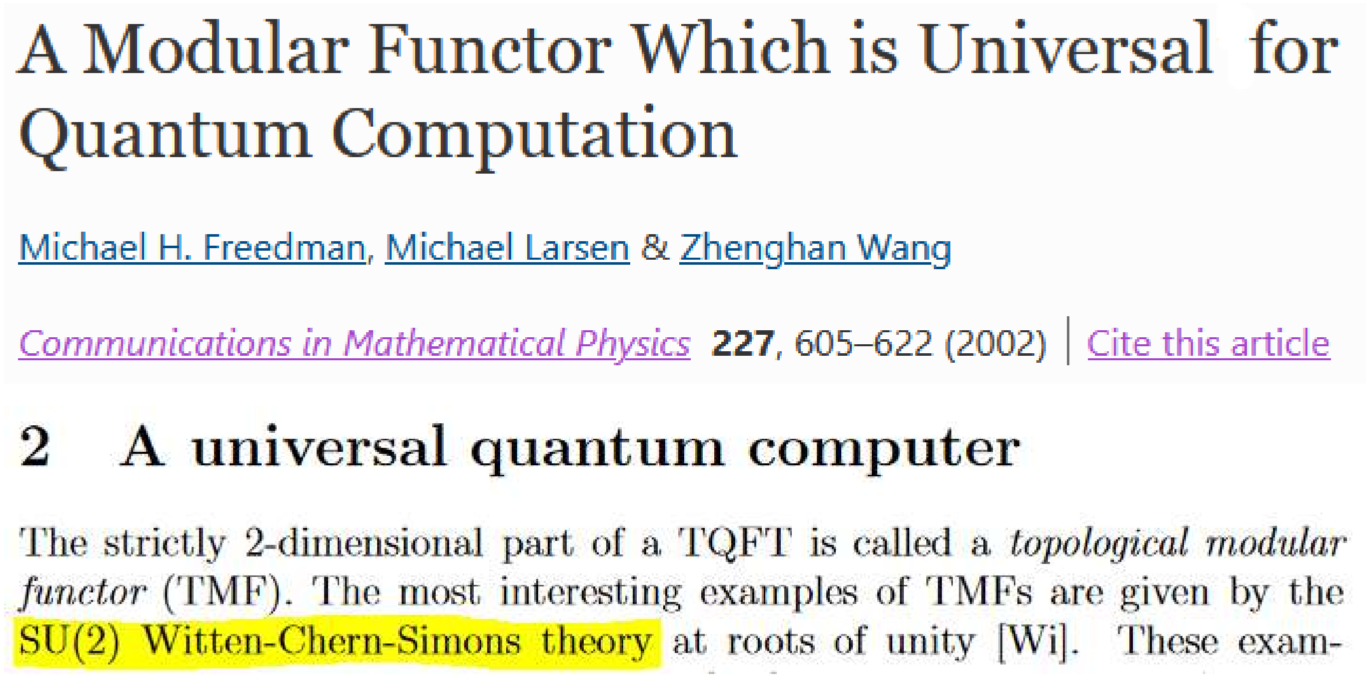

A Modular Functor Which is Universal for Quantum Computation

Michael H. Freedman, Michael Larsen & Zhenghan Wang

Communications in Mathematical Physics **227**, 605–622 (2002) | Cite this article

2 A universal quantum computer

The strictly 2-dimensional part of a TQFT is called a *topological modular functor* (TMF). The most interesting examples of TMFs are given by the SU(2) Witten-Chern-Simons theory at roots of unity [Wi]. These exam-

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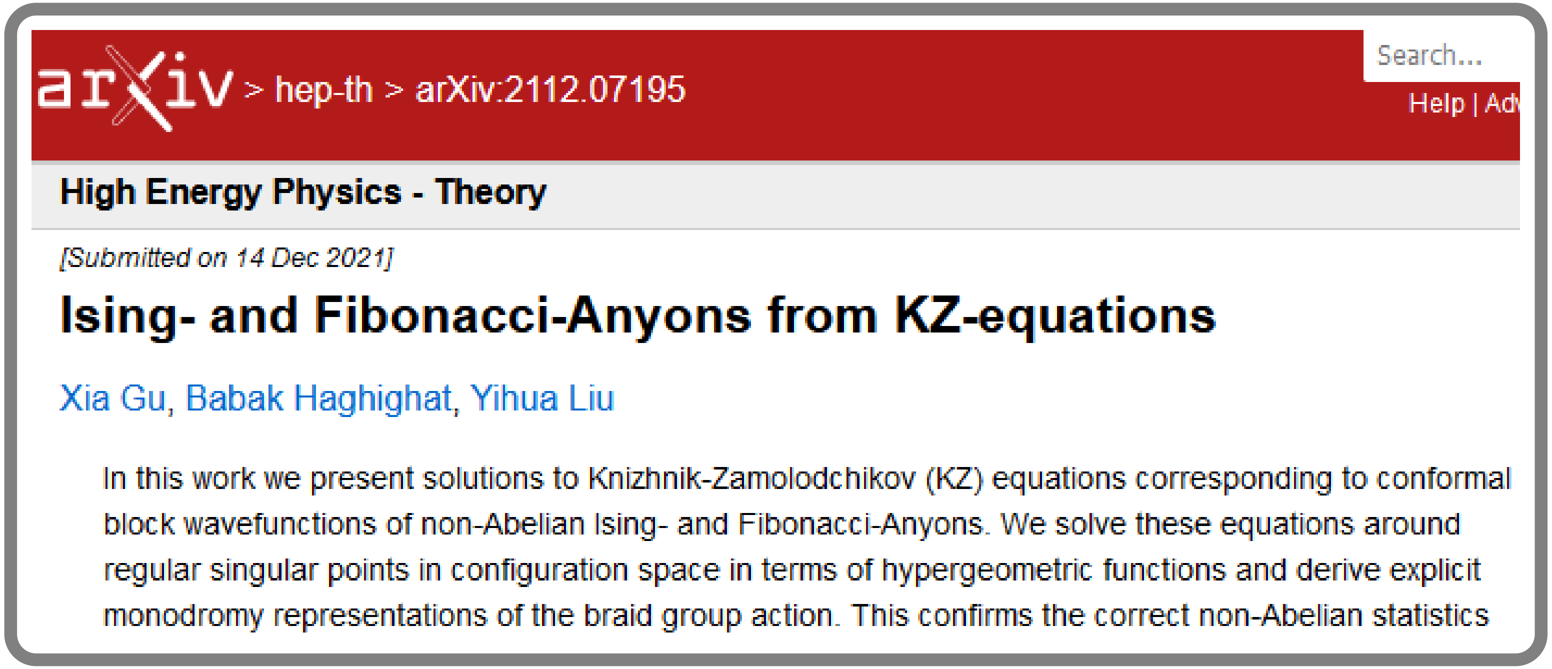

High Energy Physics - Theory

[Submitted on 14 Dec 2021]

Ising- and Fibonacci-Anyons from KZ-equations

Xia Gu, Babak Haghighat, Yihua Liu

In this work we present solutions to Knizhnik-Zamolodchikov (KZ) equations corresponding to conformal block wavefunctions of non-Abelian Ising- and Fibonacci-Anyons. We solve these equations around regular singular points in configuration space in terms of hypergeometric functions and derive explicit monodromy representations of the braid group action. This confirms the correct non-Abelian statistics

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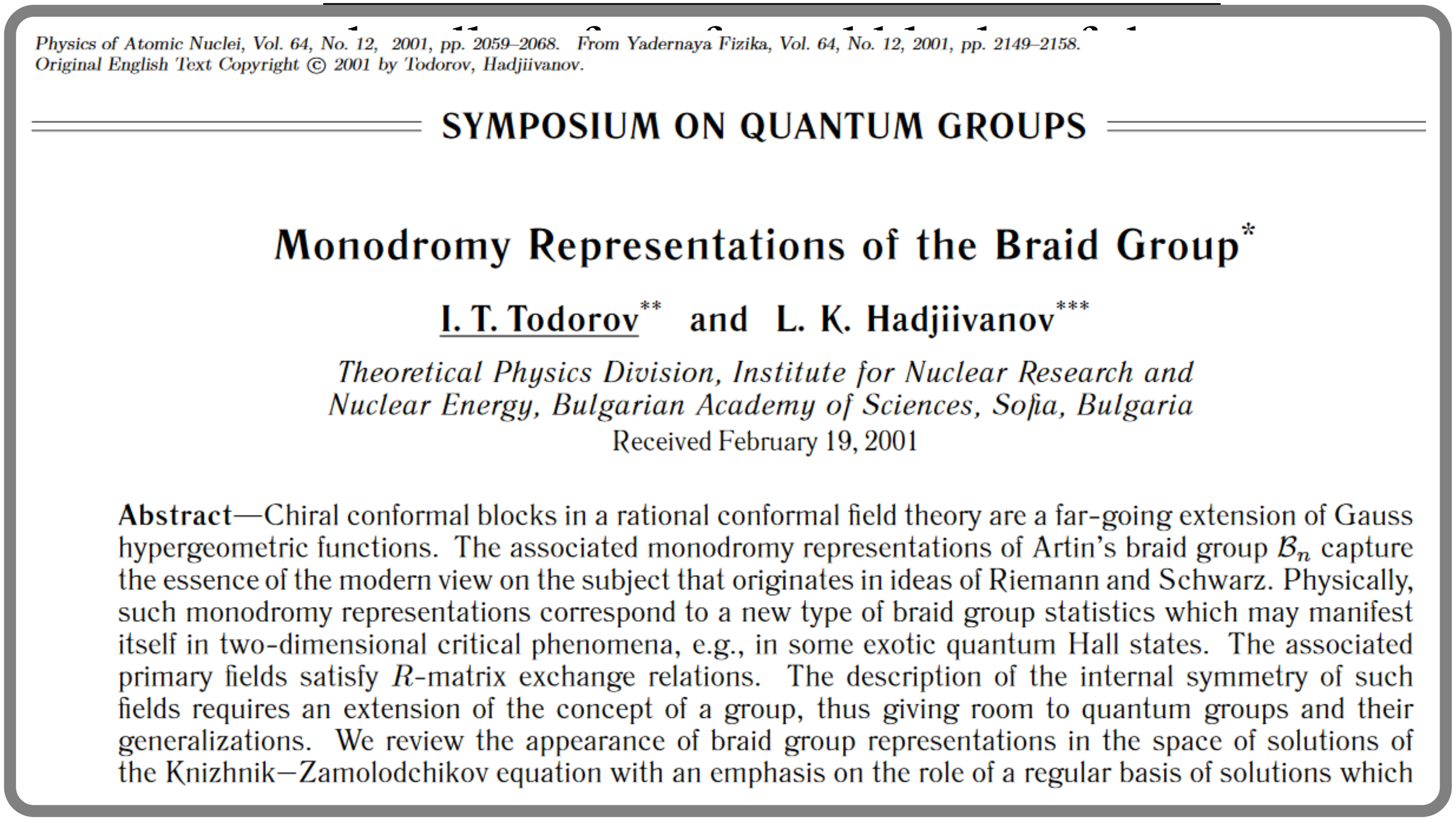

Physics of Atomic Nuclei, Vol. 64, No. 12, 2001, pp. 2059–2068. From Yadernaya Fizika, Vol. 64, No. 12, 2001, pp. 2149–2158.
Original English Text Copyright © 2001 by Todorov, Hadjiivanov.

SYMPOSIUM ON QUANTUM GROUPS

Monodromy Representations of the Braid Group*

I. T. Todorov** and **L. K. Hadjiivanov*****

*Theoretical Physics Division, Institute for Nuclear Research and
Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria*

Received February 19, 2001

Abstract—Chiral conformal blocks in a rational conformal field theory are a far-going extension of Gauss hypergeometric functions. The associated monodromy representations of Artin's braid group $\mathcal{B}_n$ capture the essence of the modern view on the subject that originates in ideas of Riemann and Schwarz. Physically, such monodromy representations correspond to a new type of braid group statistics which may manifest itself in two-dimensional critical phenomena, e.g., in some exotic quantum Hall states. The associated primary fields satisfy R-matrix exchange relations. The description of the internal symmetry of such fields requires an extension of the concept of a group, thus giving room to quantum groups and their generalizations. We review the appearance of braid group representations in the space of solutions of the Knizhnik–Zamolodchikov equation with an emphasis on the role of a regular basis of solutions which

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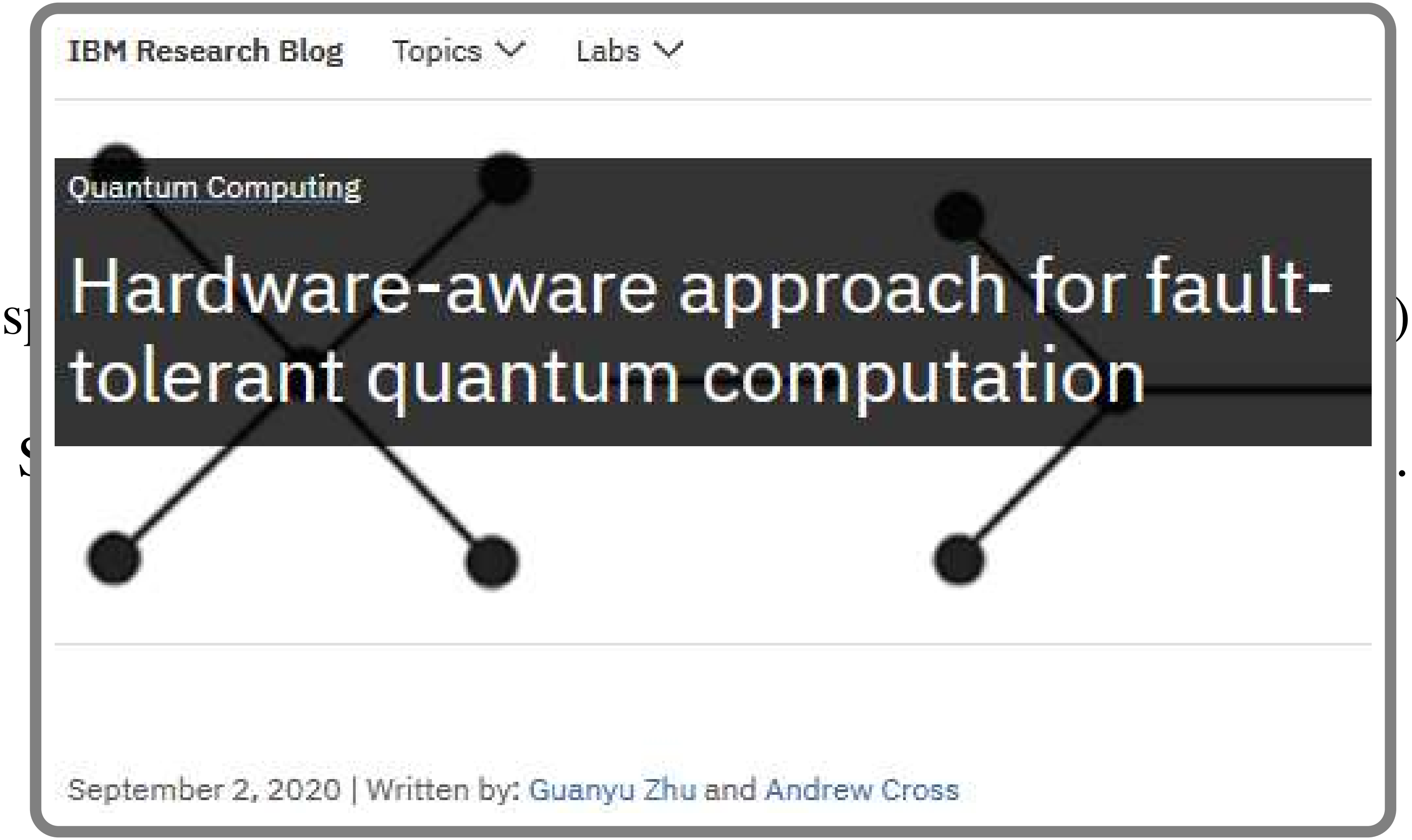

IBM Research Blog Topics Labs

Quantum Computing

Hardware-aware approach for fault-tolerant quantum computation

September 2, 2020 | Written by: Guanyu Zhu and Andrew Cross

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Programming languages suited for describing *bundles* are *dependently typed*

classifying map

$$X \xrightarrow{\vdash P} \text{Types}$$

system of X-dependent types

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$$\begin{tikzcd} & \widehat{\mathrm{Types}} \arrow[d, "\text{univalent type bundle}"] \\ X \arrow[r, "\overset{\text{classifying map}}{\vdash P}"] & \mathrm{Types} \end{tikzcd}$$

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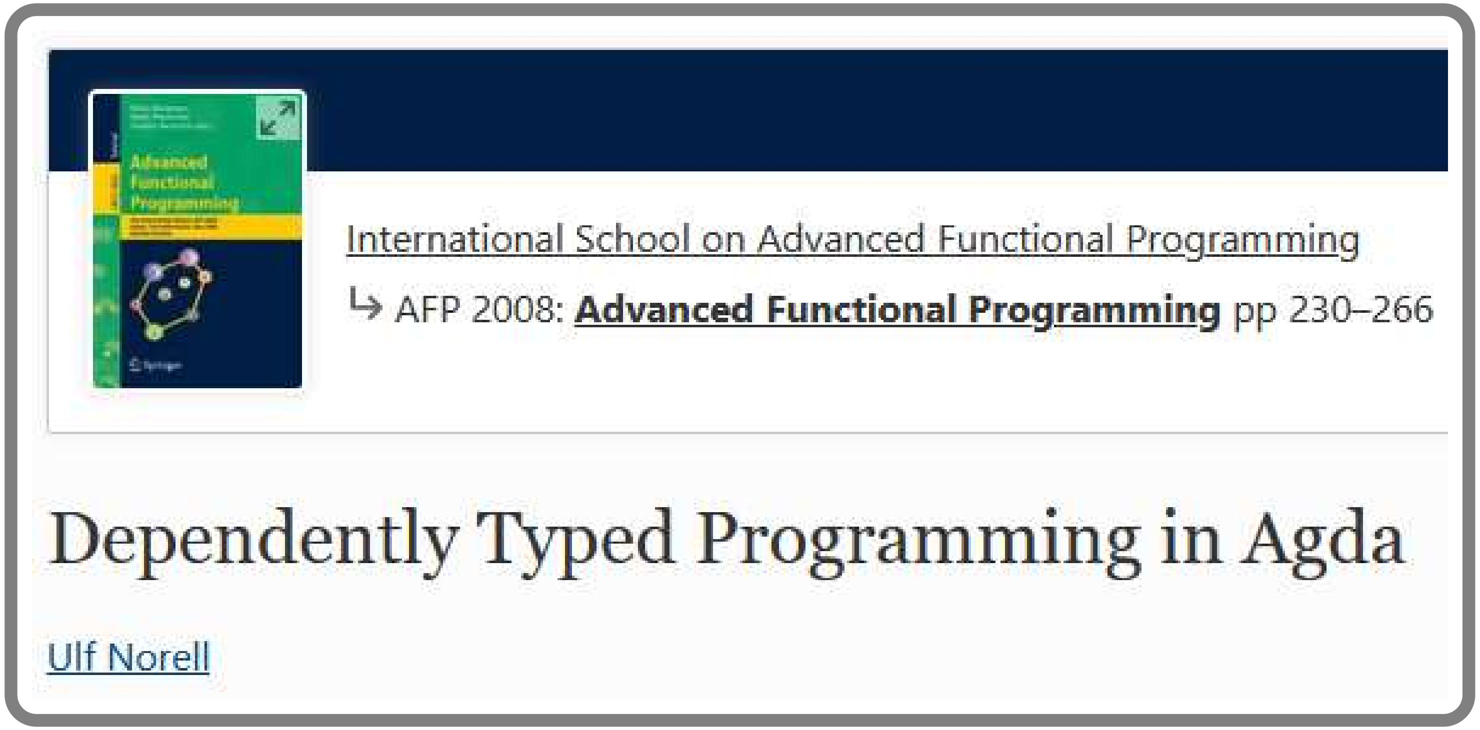

International School on Advanced Functional Programming

↳ AFP 2008: **Advanced Functional Programming** pp 230–266

Dependently Typed Programming in Agda

Ulf Norell

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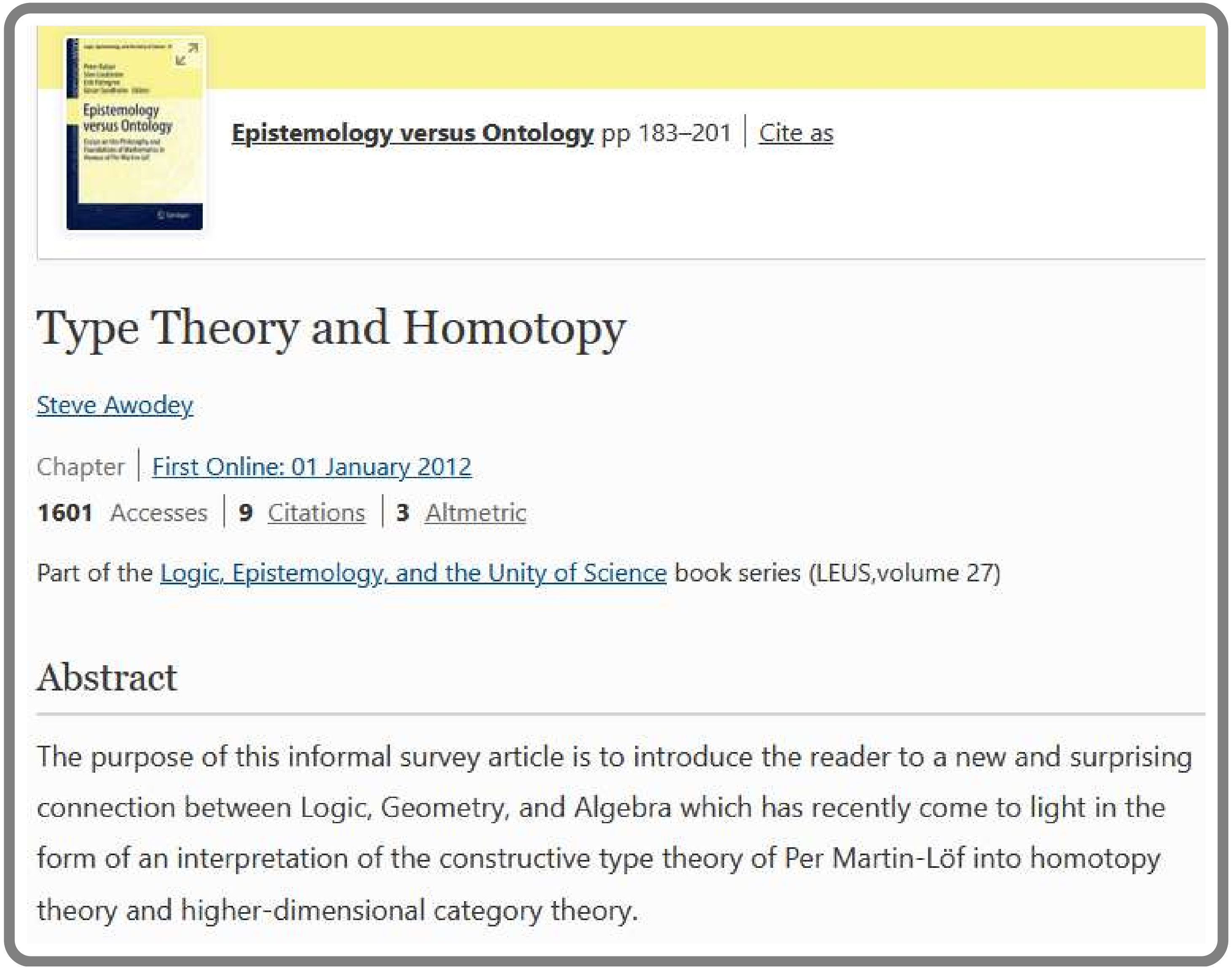

Epistemology versus Ontology pp 183–201 | Cite as

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Chapter | First Online: 01 January 2012

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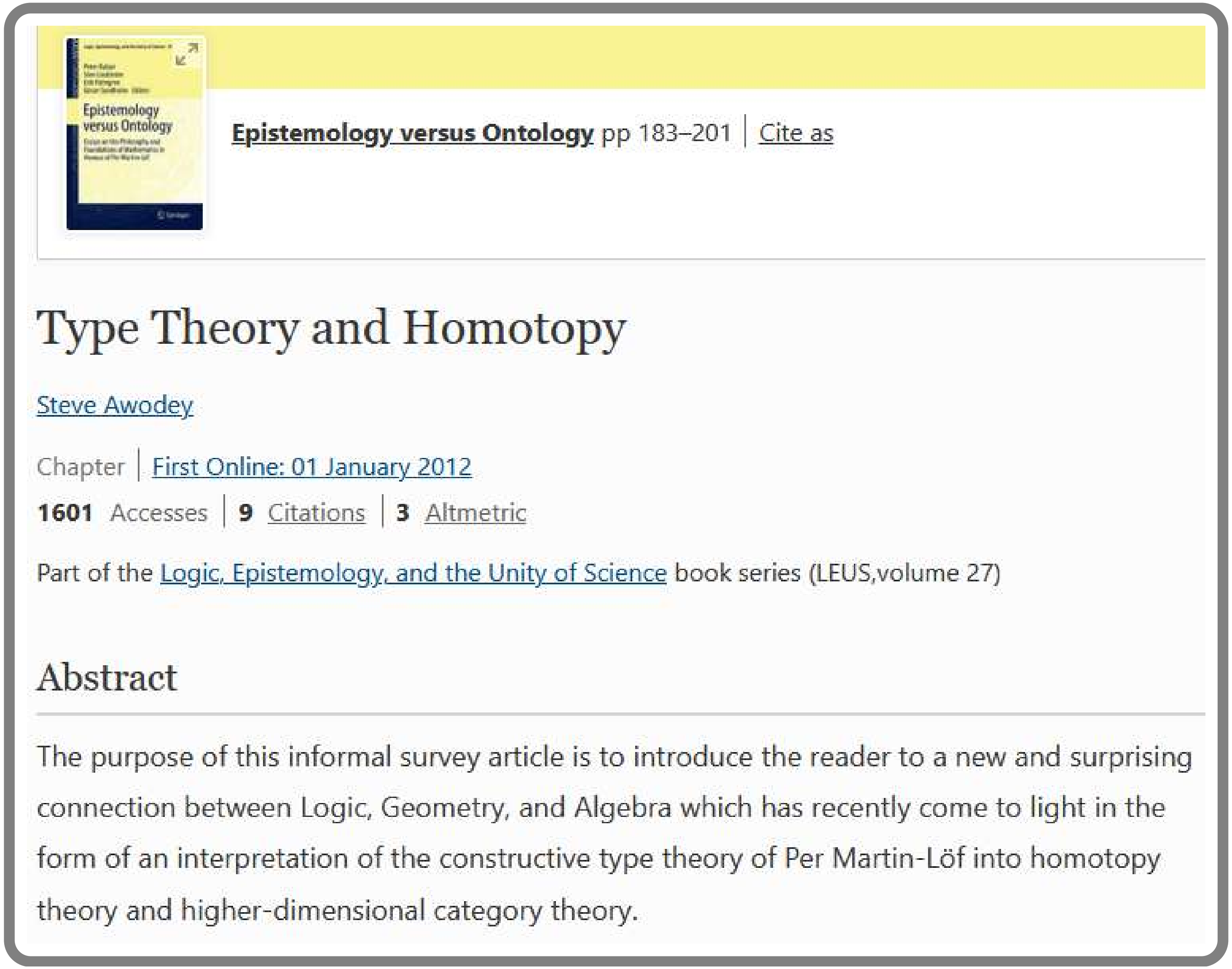

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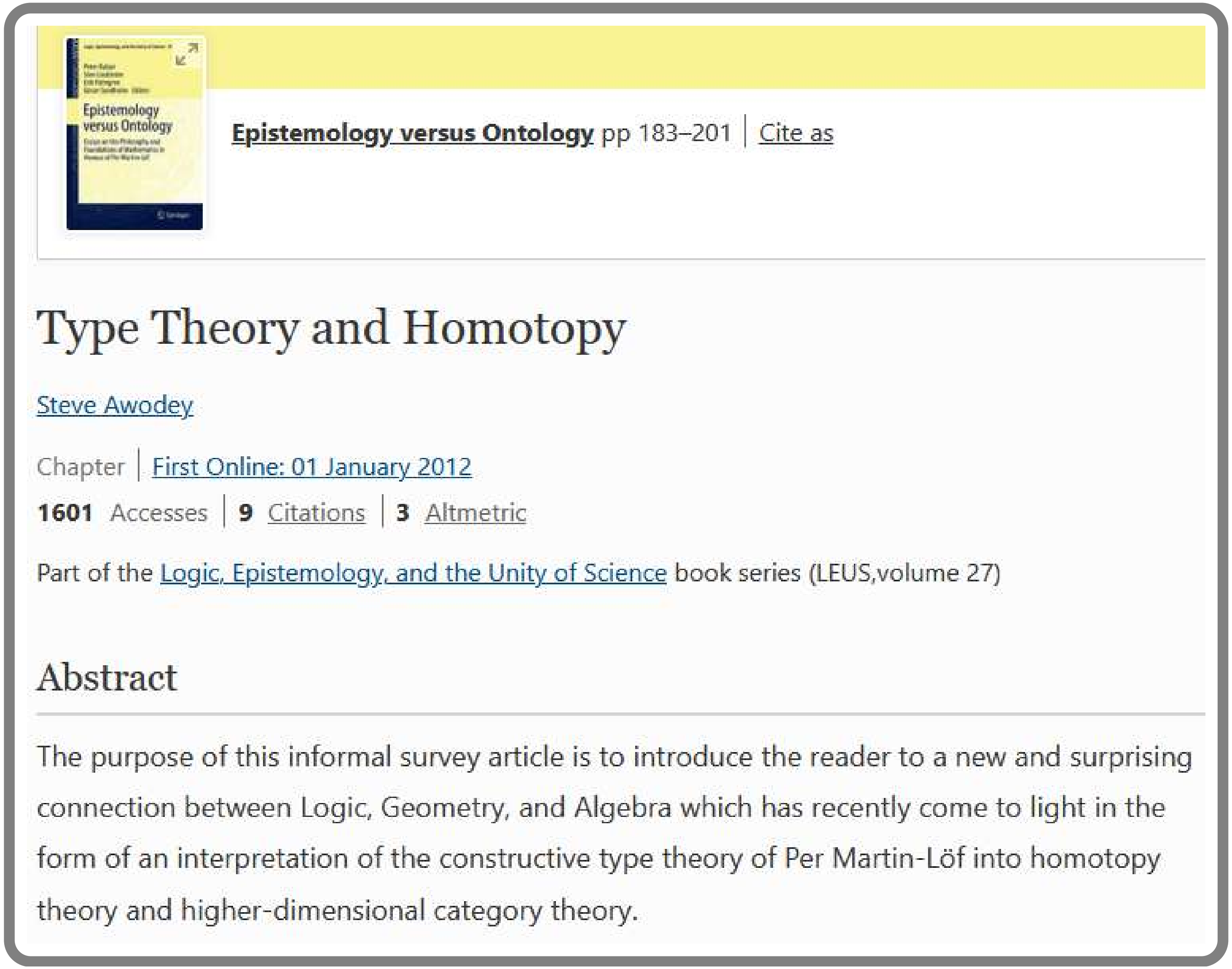

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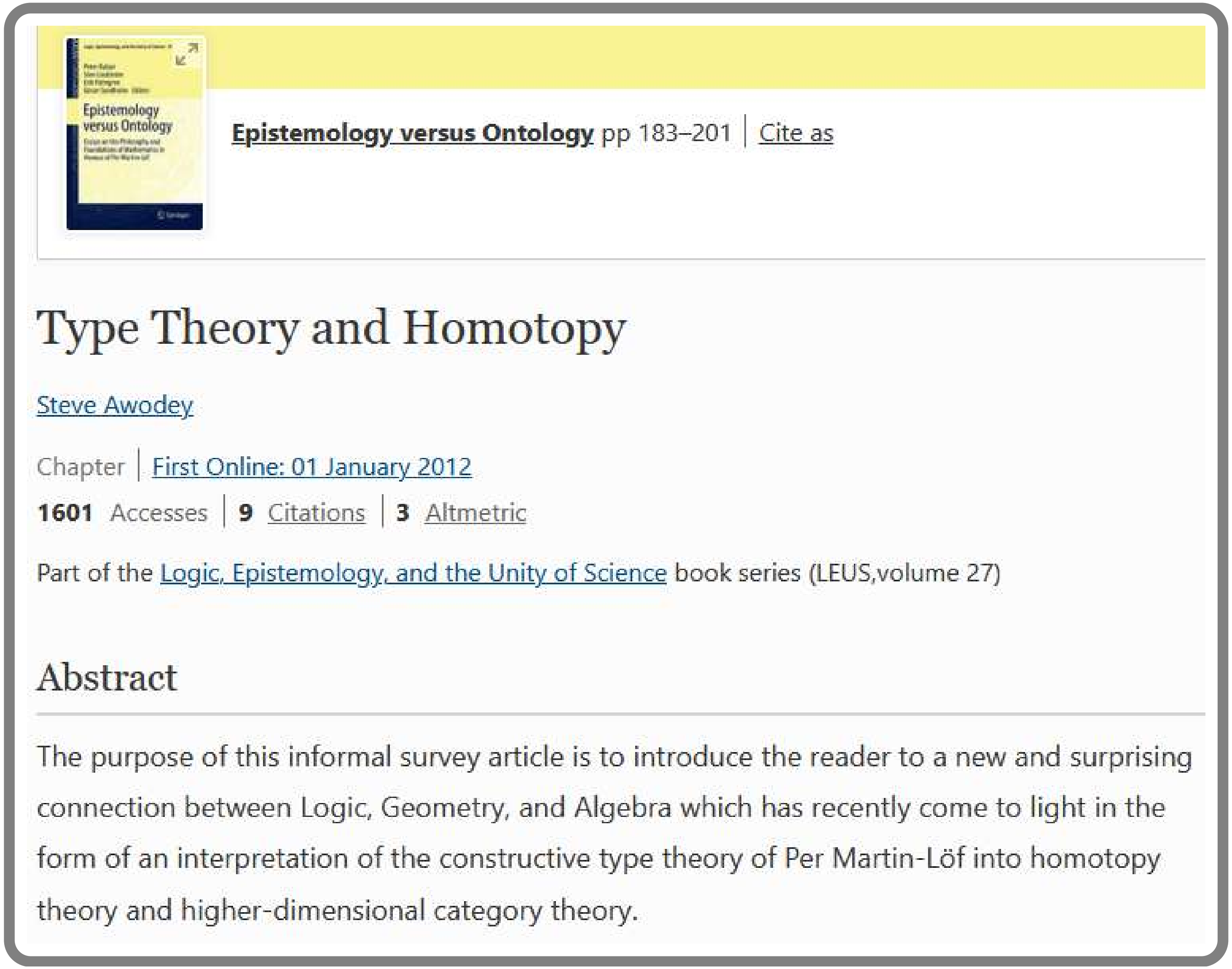

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Twenty-Five Years of Constructive Type Theory

GIOVANNI SAMBIN and JAN SMITH

CHAPTER

6 The groupoid interpretation of type theory

Martin Hofmann, Thomas Streicher

https://doi.org/10.1093/oso/9780198501275.003.0008 Pages 83–112

Published: October 1998

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Homotopy theoretic models of identity types

Published online by Cambridge University Press: **01 January 2009**

STEVE AWODEY and MICHAEL A. WARREN

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$$\mathbf{B}G = \left\{ \begin{tikzcd} & * \arrow[dr, rightsquigarrow, "g_2"] & \\ * \arrow[ur, rightsquigarrow, "g_1"] \arrow[rr, rightsquigarrow, "g_1 \cdot g_2"] & & * \end{tikzcd} \;\middle|\; g_i \in G \right\}$$

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$$\begin{tikzcd} & P(y) \arrow[dr, "\mathrm{tr}(\gamma_2)"] & \\ P(x) \arrow[ur, "\mathrm{tr}(\gamma_1)"] \arrow[rr, "\mathrm{tr}(\gamma_3)"] & & P(z) \end{tikzcd}$$

$X :\; \mathrm{Types}$

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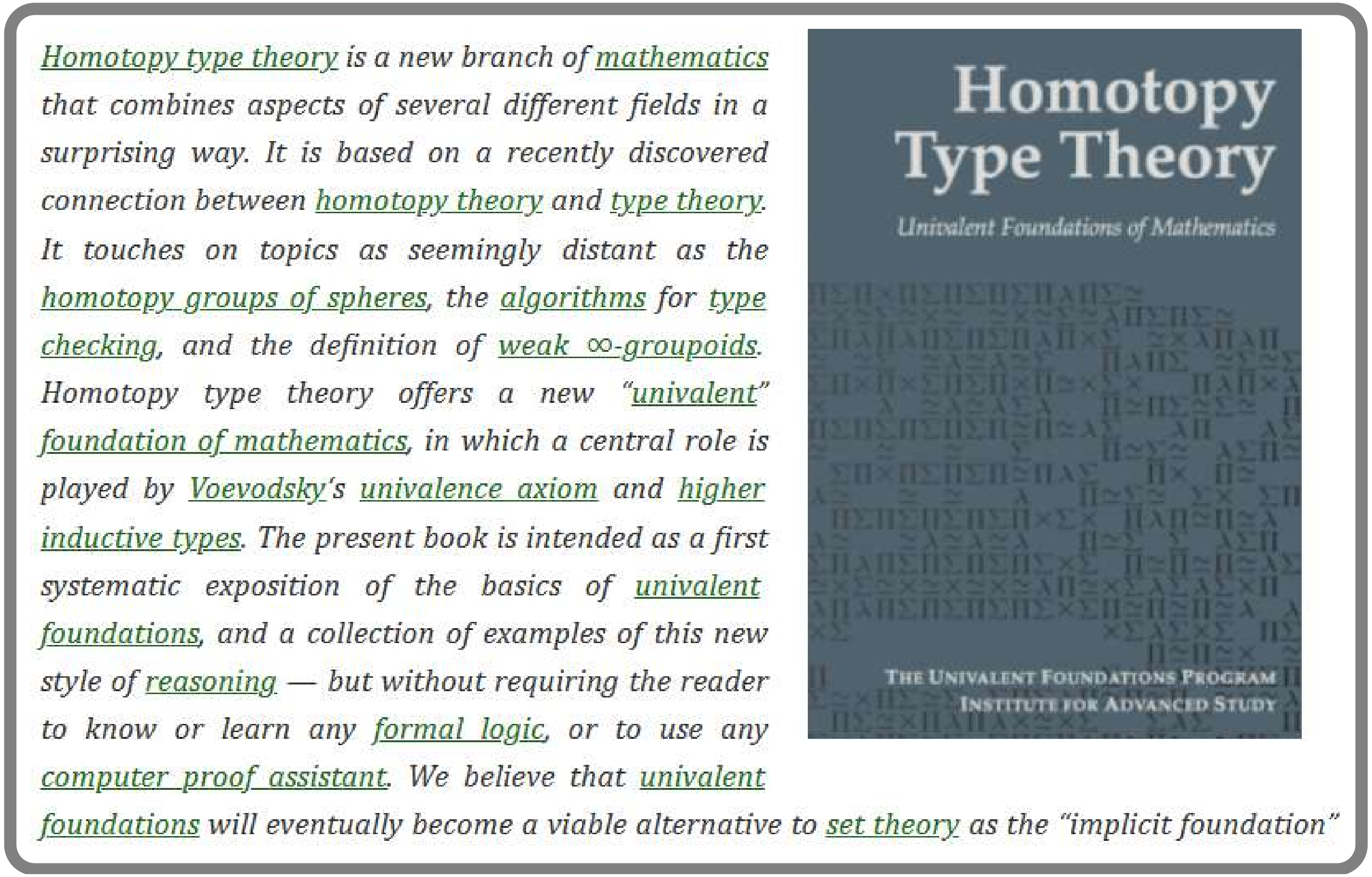

Homotopy type theory is a new branch of mathematics that combines aspects of several different fields in a surprising way. It is based on a recently discovered connection between homotopy theory and type theory. It touches on topics as seemingly distant as the homotopy groups of spheres, the algorithms for type checking, and the definition of weak ∞-groupoids. Homotopy type theory offers a new "univalent" foundation of mathematics, in which a central role is played by Voevodsky's univalence axiom and higher inductive types. The present book is intended as a first systematic exposition of the basics of univalent foundations, and a collection of examples of this new style of reasoning — but without requiring the reader to know or learn any formal logic, or to use any computer proof assistant. We believe that univalent foundations will eventually become a viable alternative to set theory as the "implicit foundation"

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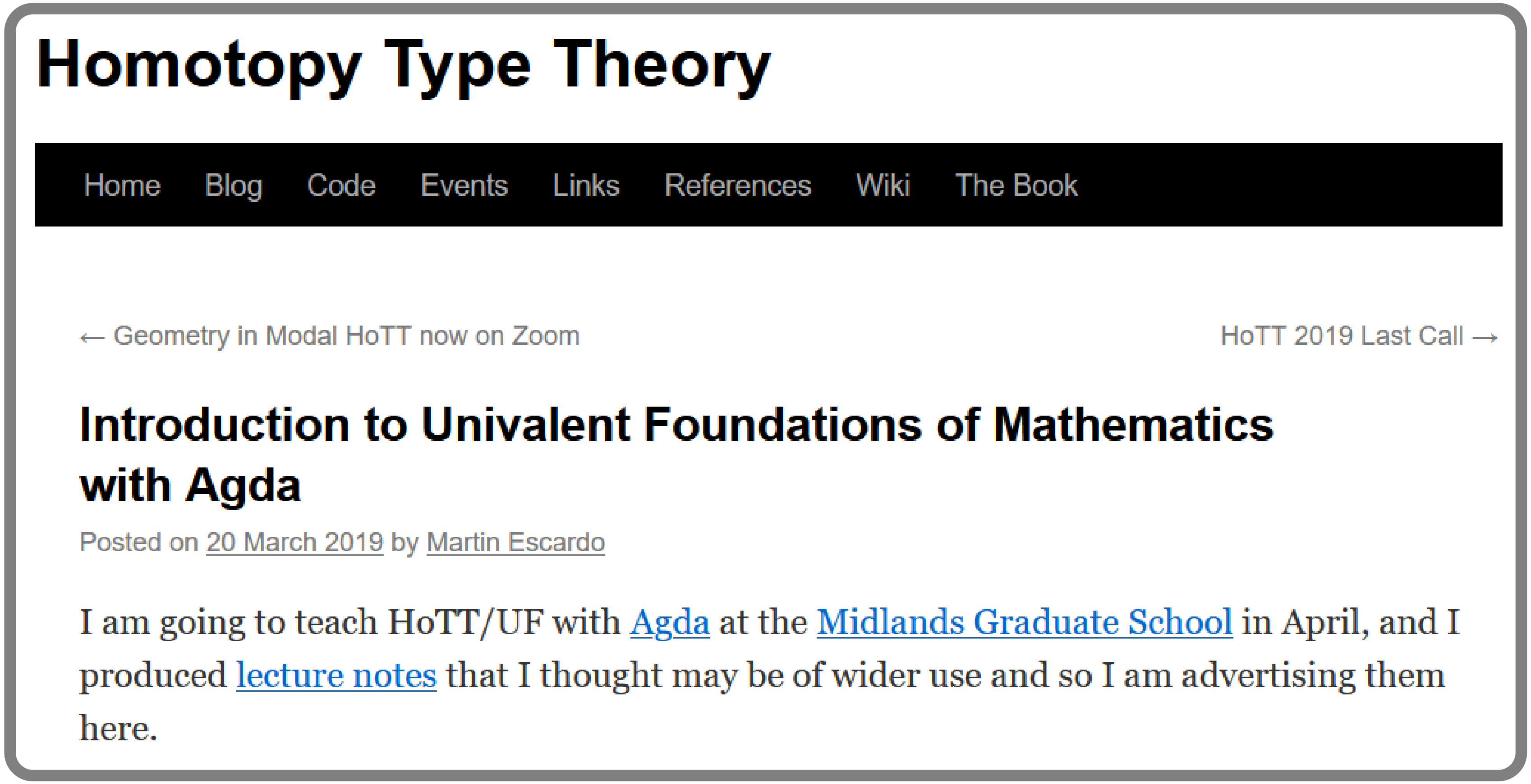

Homotopy Type Theory

Introduction to Univalent Foundations of Mathematics with Agda

Posted on 20 March 2019 by Martin Escardo

I am going to teach HoTT/UF with Agda at the Midlands Graduate School in April, and I produced lecture notes that I thought may be of wider use and so I am advertising them here.

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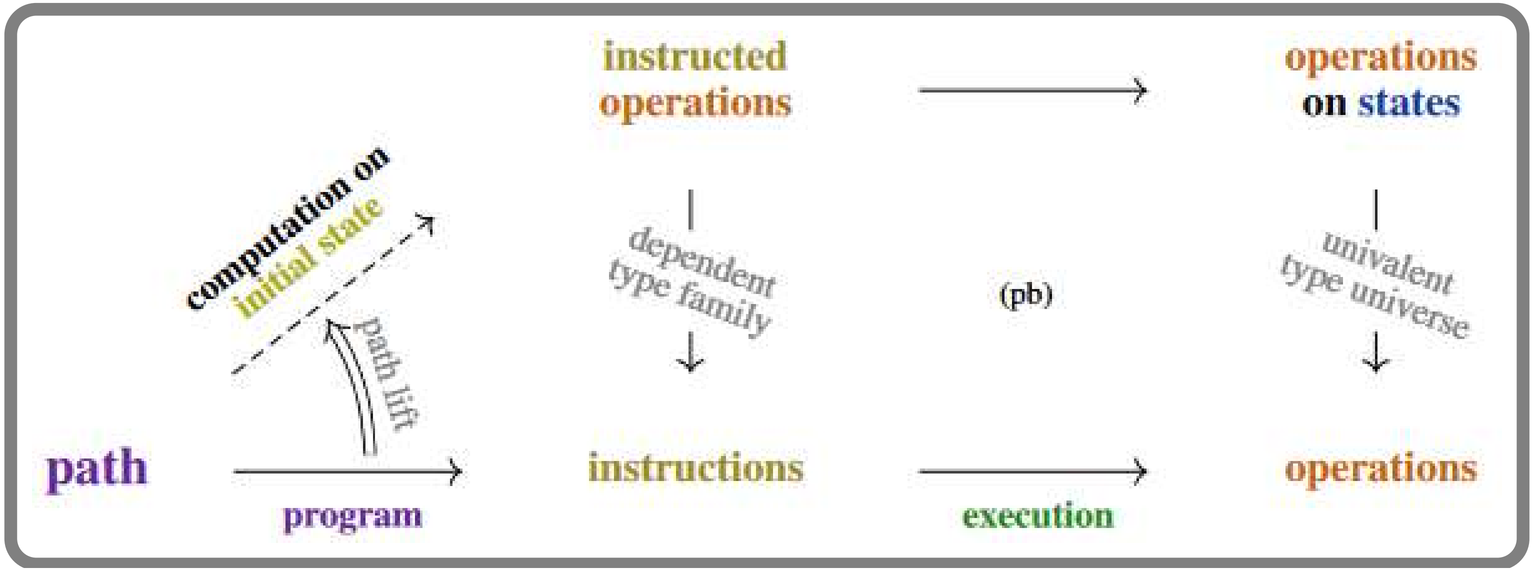

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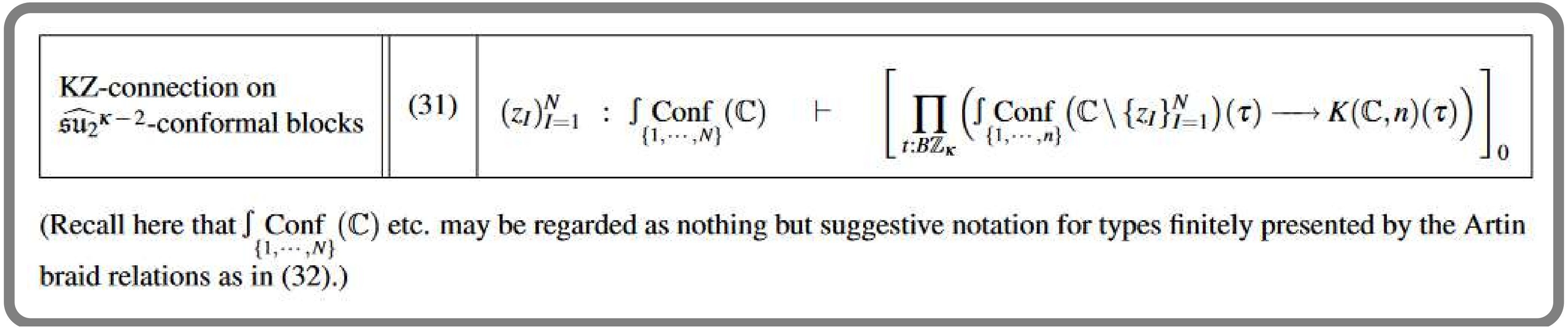

KZ-connection on $\widehat{\mathfrak{su}_2}^{\kappa-2}$-conformal blocks	(31)	$(z_I)_{I=1}^N \;:\; \int \underset{\{1,\cdots,N\}}{\text{Conf}}(\mathbb{C}) \quad \vdash \quad \left[\prod_{t:B\mathbb{Z}_\kappa} \left(\int \underset{\{1,\cdots,n\}}{\text{Conf}} (\mathbb{C} \setminus \{z_I\}_{I=1}^N)(\tau) \longrightarrow K(\mathbb{C},n)(\tau) \right) \right]_0$

(Recall here that $\int \underset{\{1,\cdots,N\}}{\text{Conf}}(\mathbb{C})$ etc. may be regarded as nothing but suggestive notation for types finitely presented by the Artin braid relations as in (32).)

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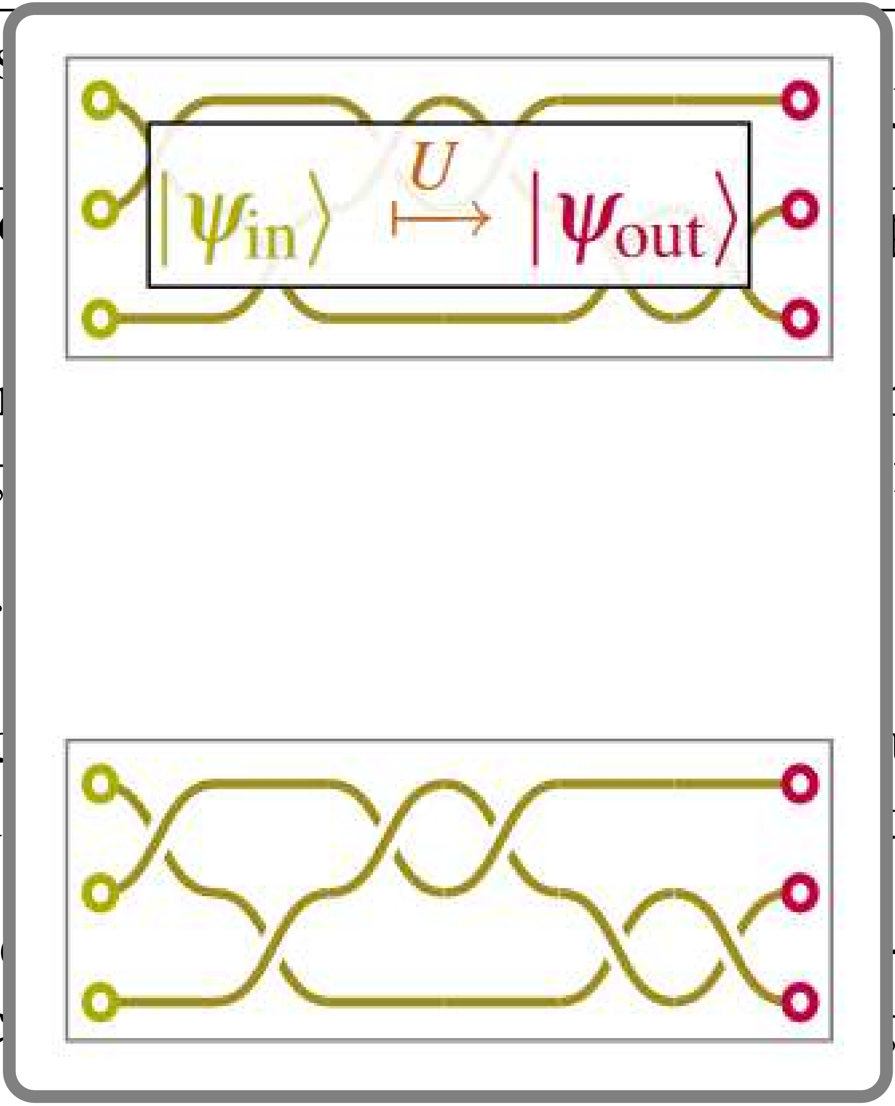

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⇒ natural & powerful topological-hardware-aware Q-pogramming paradigm

In HoTT, data types come with paths between their terms

$$\boxed{\begin{array}{rl} X & \in \text{Types} \\ x, y & \in X \end{array} \quad \vdash \quad \text{Paths}_X(x,y) = \left\{ x \overset{\gamma}{\rightsquigarrow} y \right\} \in \text{Types}}$$

akin to continuous paths in topological spaces.

Such HoTT programming languages turn out to be remarkably fundamental, arguably serving as a new foundation for mathematics.

We now first offer the following observations:

(1.) Reversible circuit execution – such as in quantum computation – is described by *path lifting* in dependent homotopy type families.

(2.) The dependent homotopy type family of $\mathfrak{su}(2)$-conformal blocks has a slick construction in HoTT programming languages;

(3.) its path lifting operation is exactly anyonic braid gate execution.

⇒ natural & powerful topological-hardware-aware Q-pogramming paradigm

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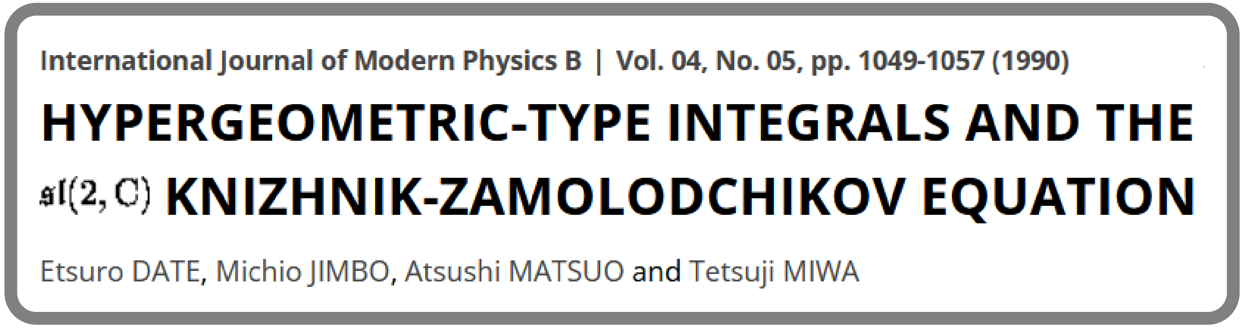

International Journal of Modern Physics B | Vol. 04, No. 05, pp. 1049-1057 (1990)

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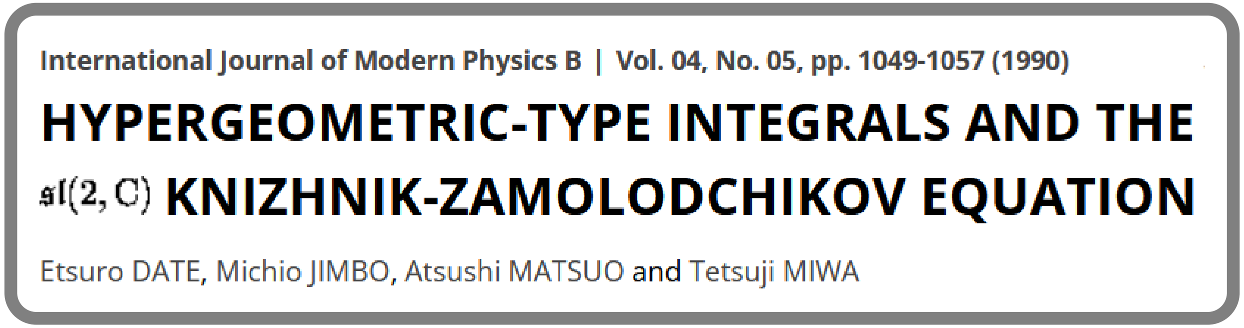

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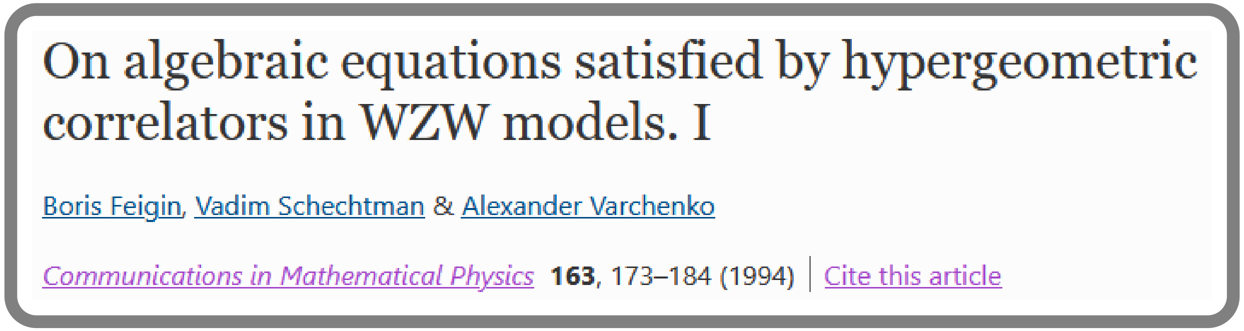

On algebraic equations satisfied by hypergeometric correlators in WZW models. I

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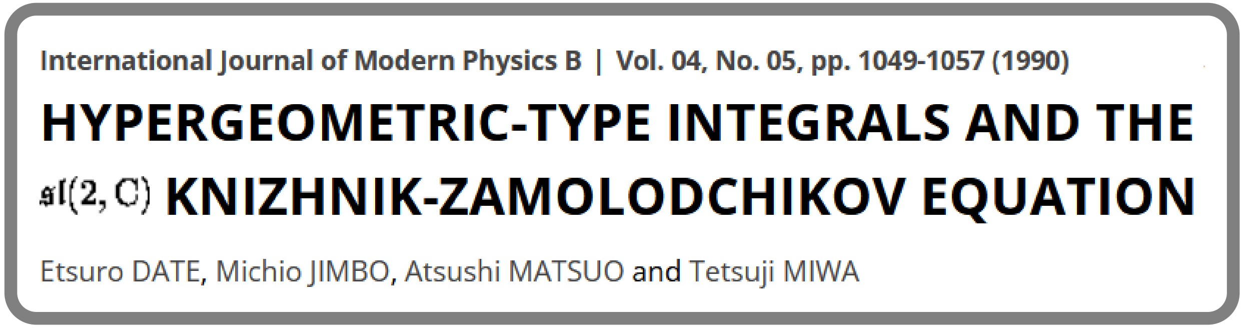

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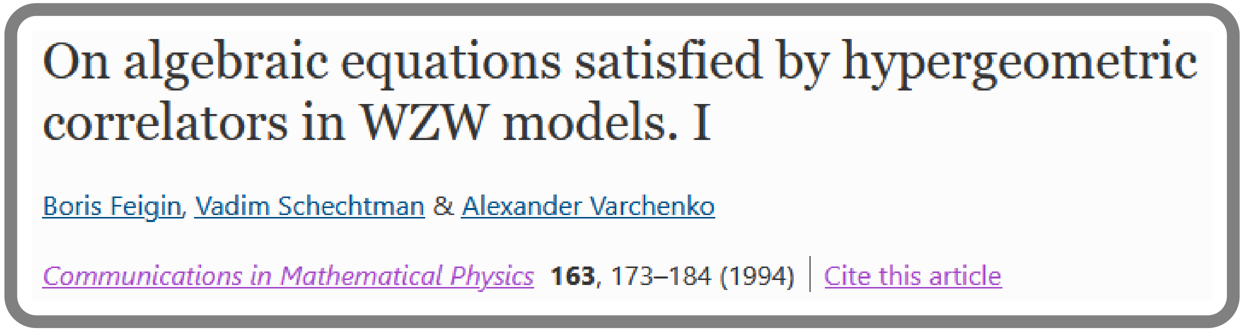

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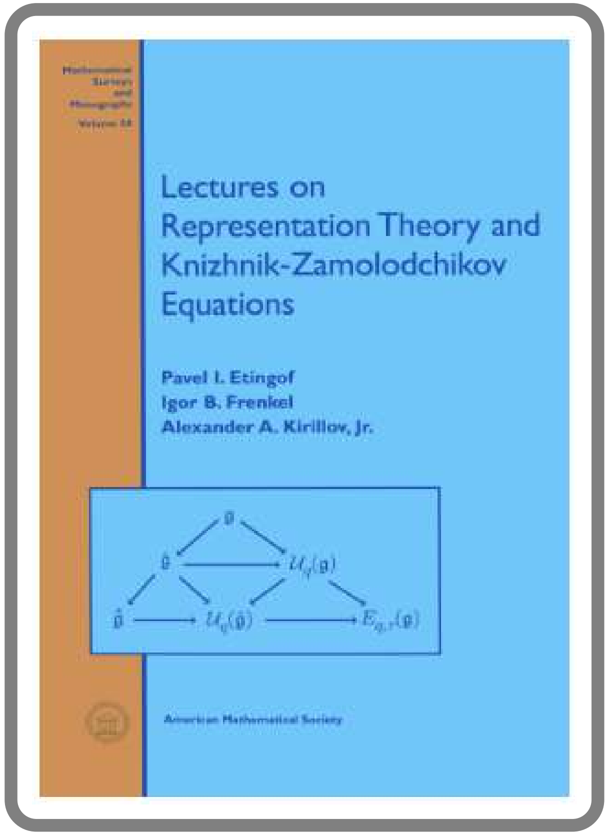

Lectures on
Representation Theory and
Knizhnik-Zamolodchikov
Equations

Pavel I. Etingof
Igor B. Frenkel
Alexander A. Kirillov, Jr.

American Mathematical Society

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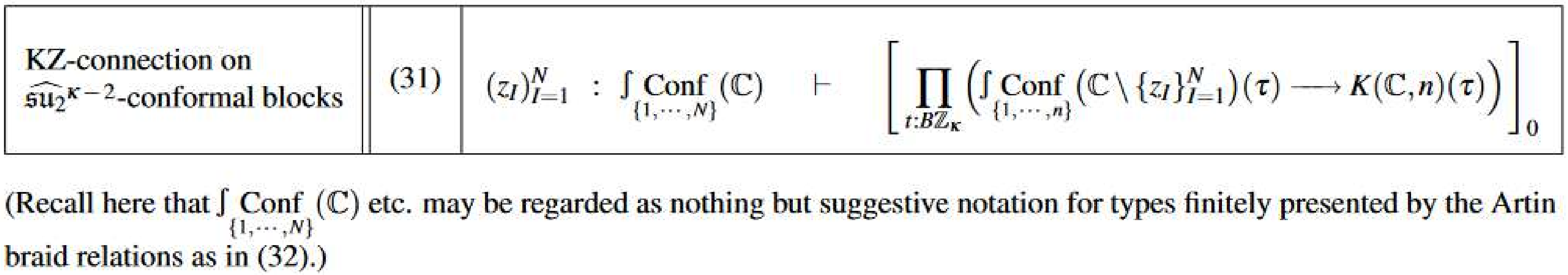

KZ-connection on $\widehat{\mathfrak{su}}_2{}^{\kappa-2}$-conformal blocks	(31)	$(z_I)_{I=1}^N \;:\; \int \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C}) \quad \vdash \quad \left[\prod_{t:B\mathbb{Z}_\kappa} \left(\int \underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathbb{C}\setminus\{z_I\}_{I=1}^N\big)(\tau) \longrightarrow K(\mathbb{C},n)(\tau)\right)\right]_0$

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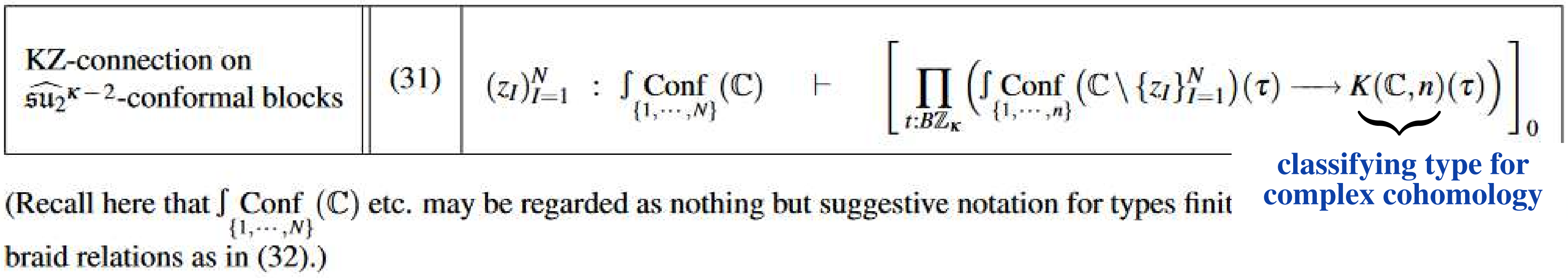

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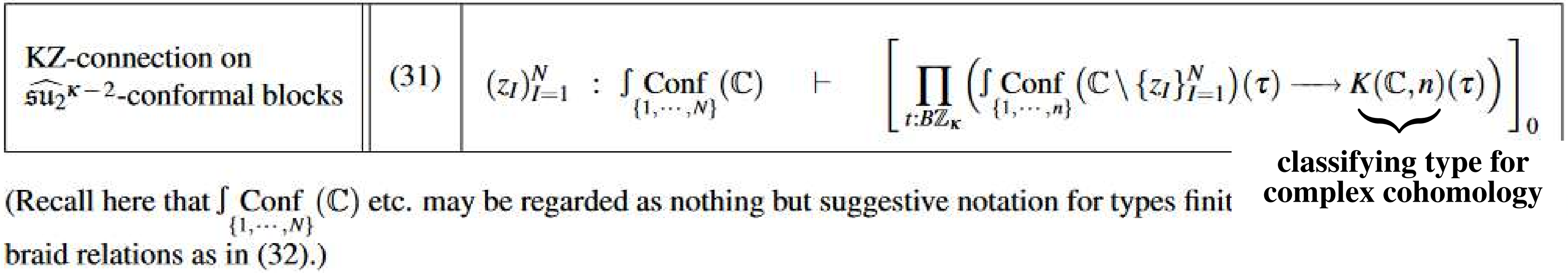

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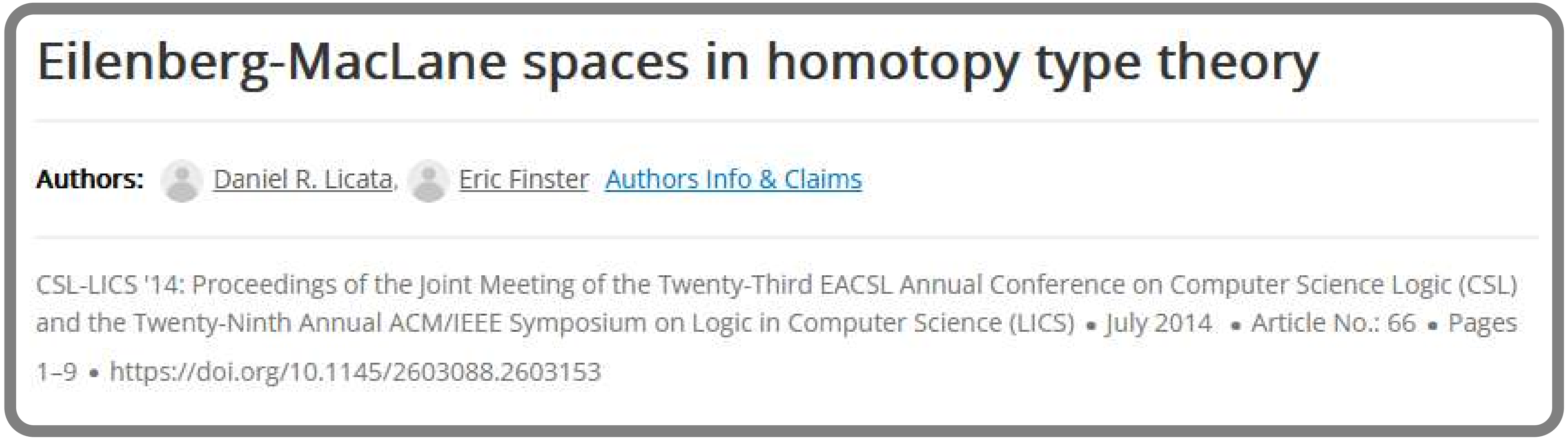

Eilenberg-MacLane spaces in homotopy type theory

Authors: Daniel R. Licata, Eric Finster Authors Info & Claims

CSL-LICS '14: Proceedings of the Joint Meeting of the Twenty-Third EACSL Annual Conference on Computer Science Logic (CSL) and the Twenty-Ninth Annual ACM/IEEE Symposium on Logic in Computer Science (LICS) • July 2014 • Article No.: 66 • Pages 1–9 • https://doi.org/10.1145/2603088.2603153

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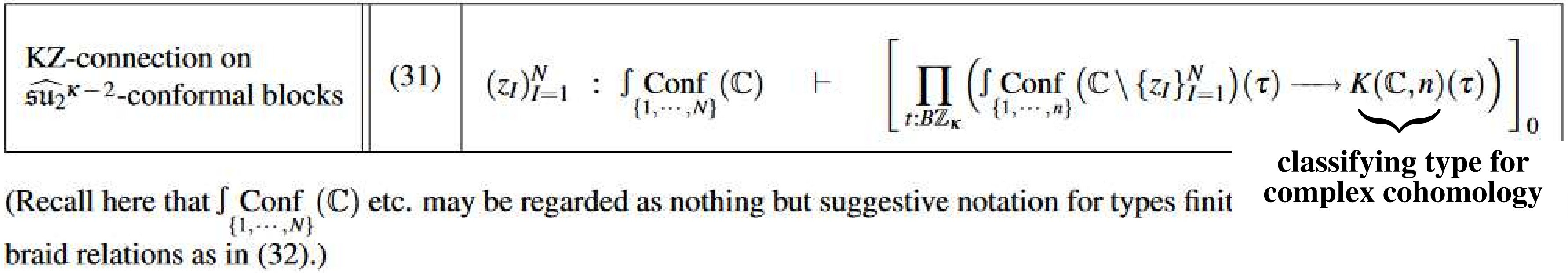

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classifying type for
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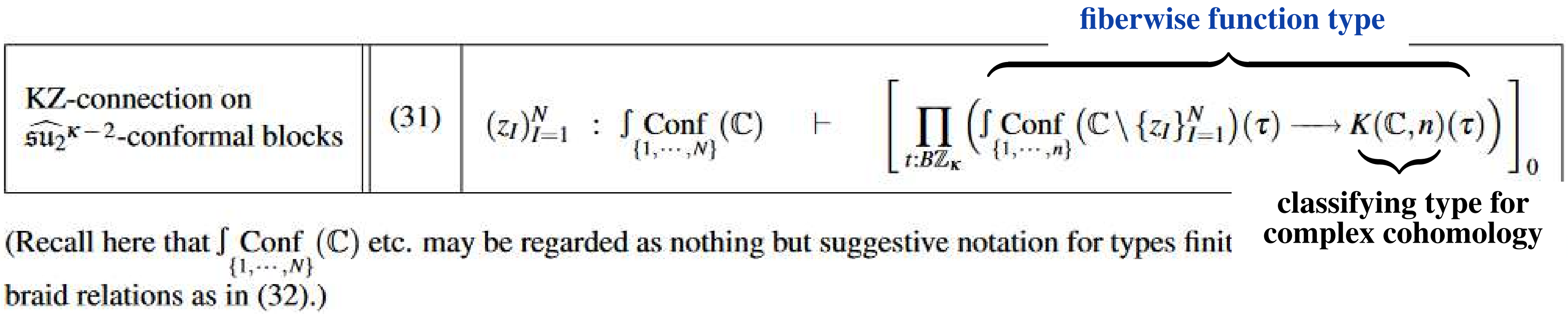

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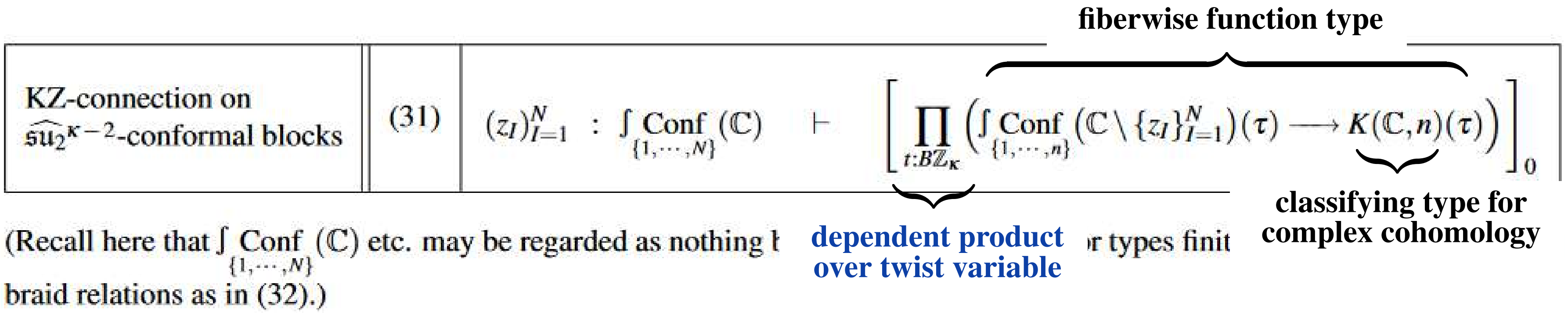

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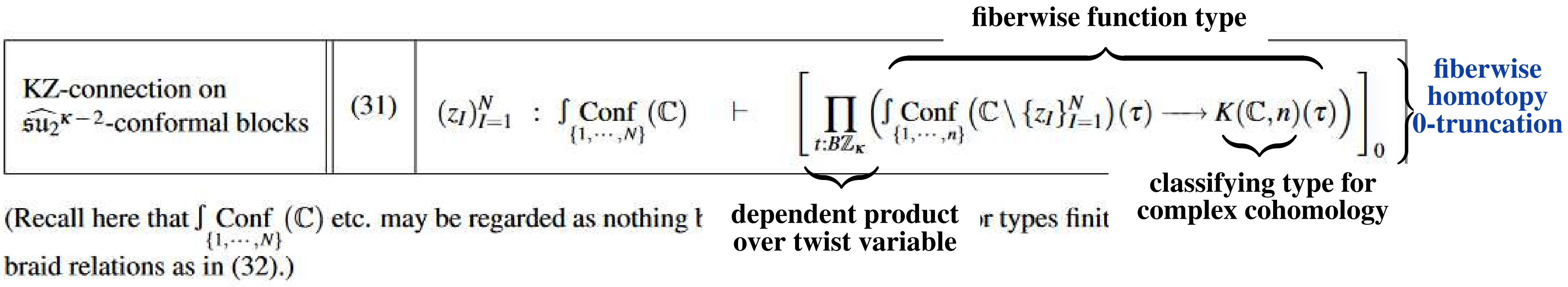

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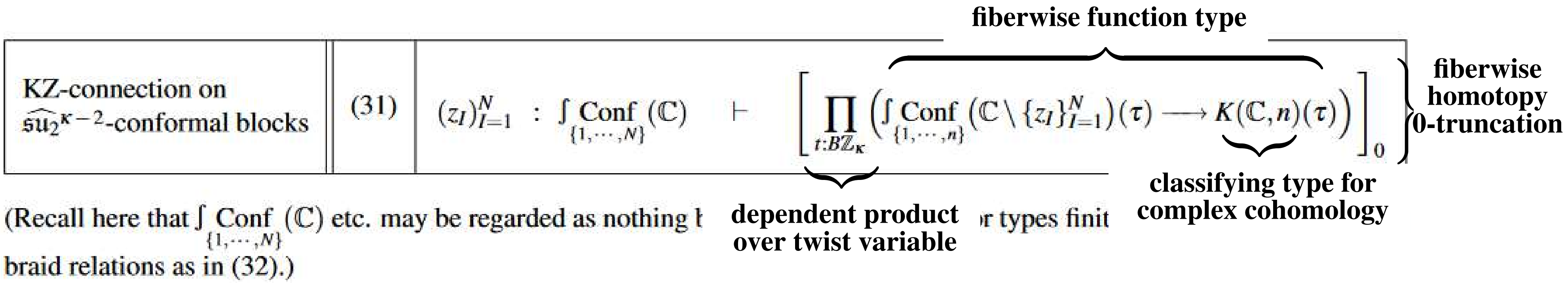

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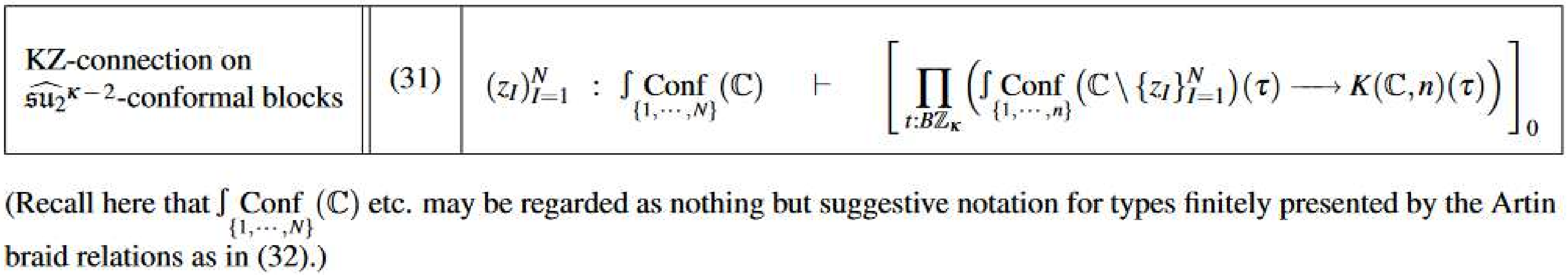

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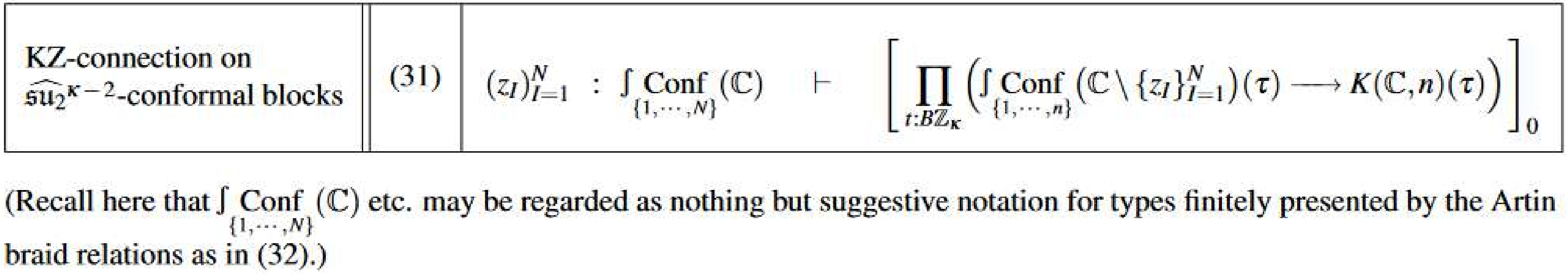

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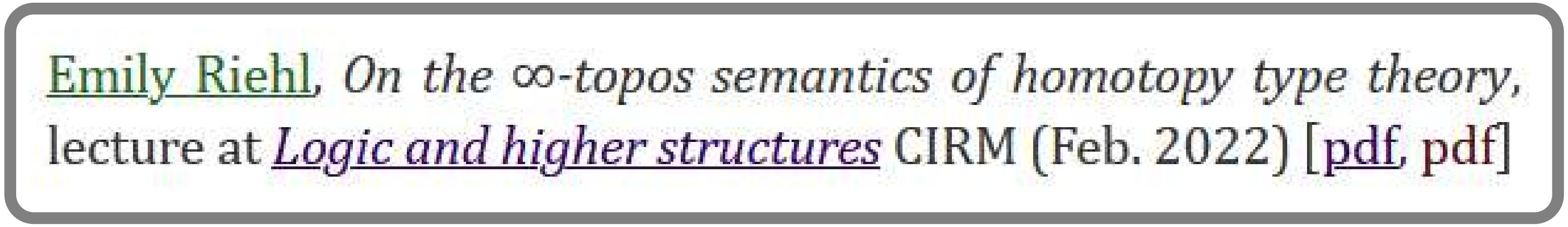

Emily Riehl, *On the ∞-topos semantics of homotopy type theory*,
lecture at *Logic and higher structures* CIRM (Feb. 2022) [pdf, pdf]

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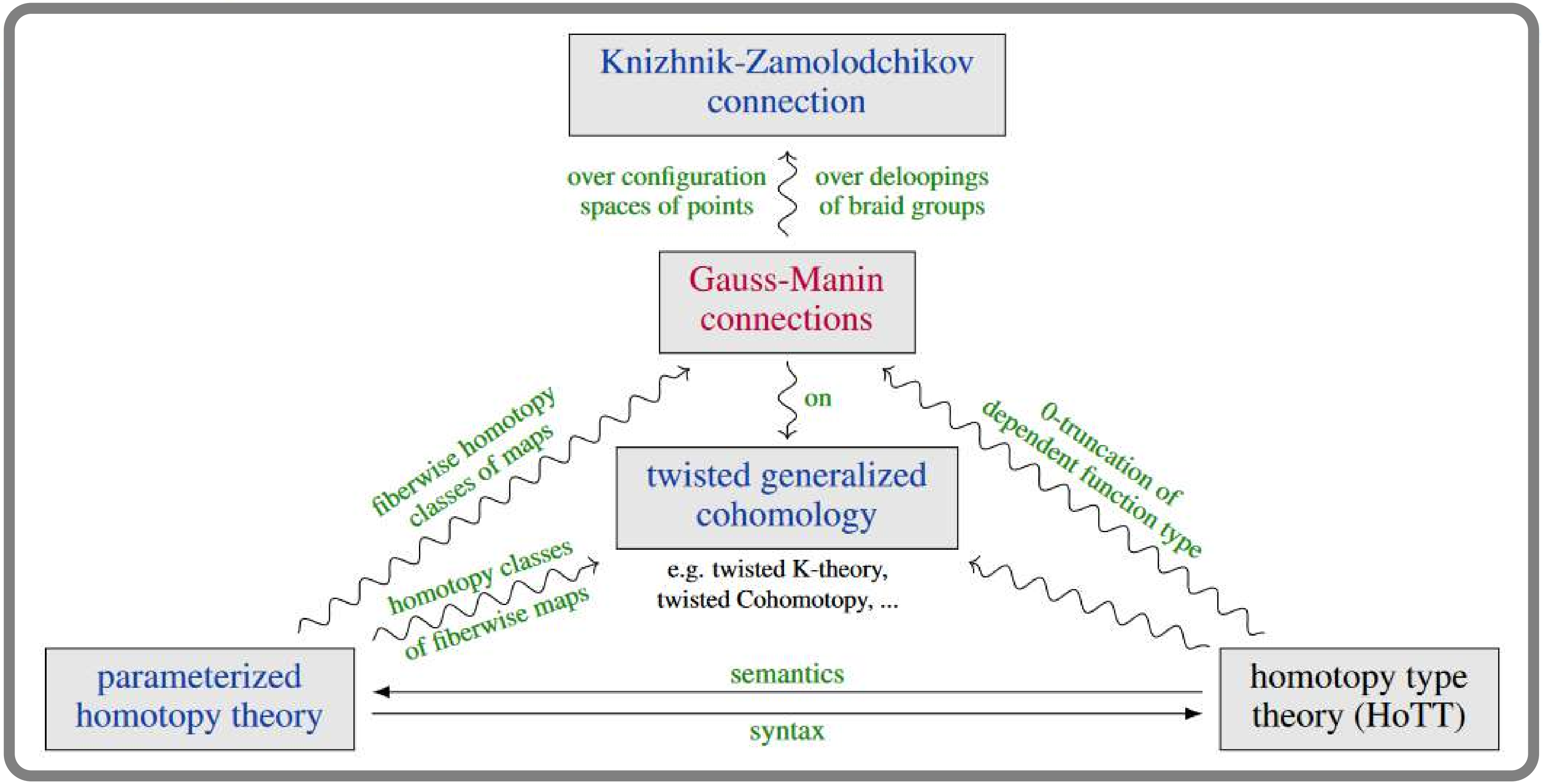

Knizhnik-Zamolodchikov connection

over configuration spaces of points

over deloopings of braid groups

Gauss-Manin connections

on

twisted generalized cohomology

e.g. twisted K-theory, twisted Cohomotopy, ...

fiberwise homotopy classes of maps

homotopy classes of fiberwise maps

0-truncation of dependent function type

parameterized homotopy theory

semantics

syntax

homotopy type theory (HoTT)

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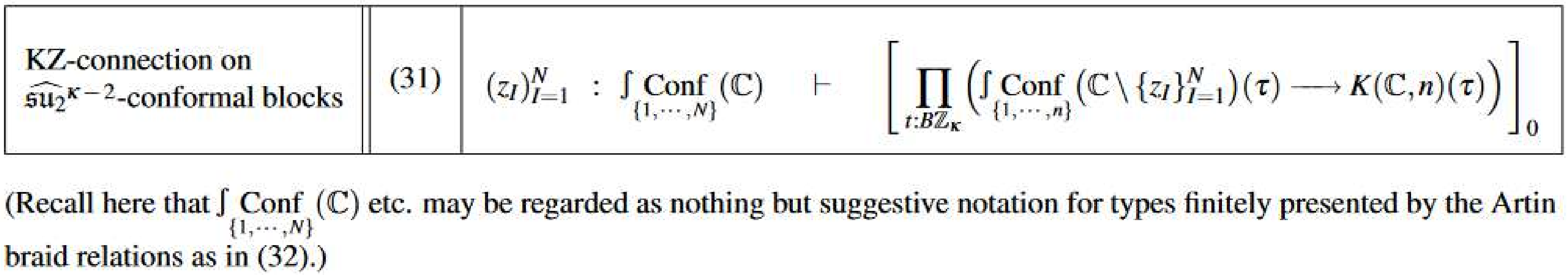

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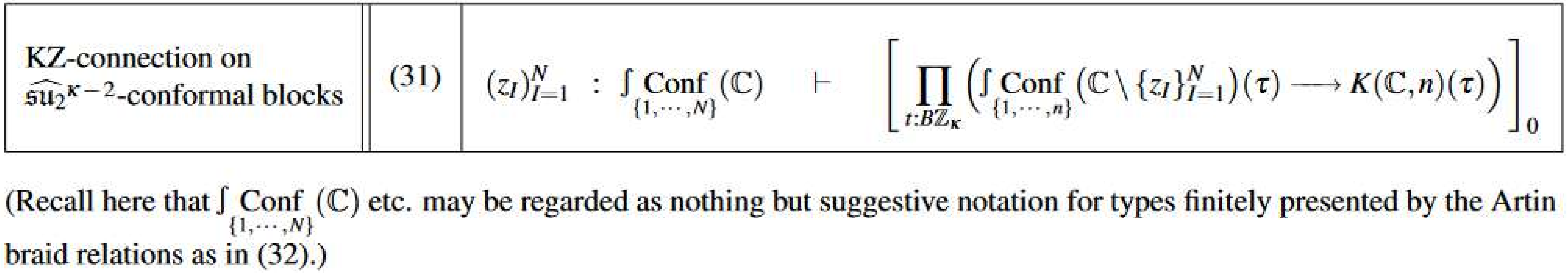

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Claim: Its transport operation is the monodromy braid representation

$P : \; X \longrightarrow \mathrm{Types}$

$$\begin{tikzcd}
 & P(y) \arrow[dr, "\mathrm{tr}(\gamma_2)"] & \\
P(x) \arrow[ur, "\mathrm{tr}(\gamma_1)"] \arrow[rr, "\mathrm{tr}(\gamma_3)"] & & P(z)
\end{tikzcd}$$

$X : \; \mathrm{Types}$

$$\begin{tikzcd}
 & y \arrow[dr, squiggly, "\gamma_2"] & \\
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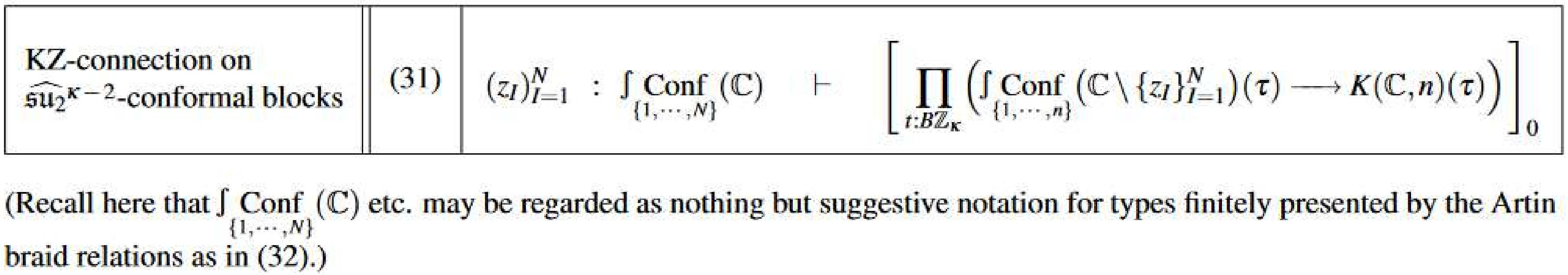

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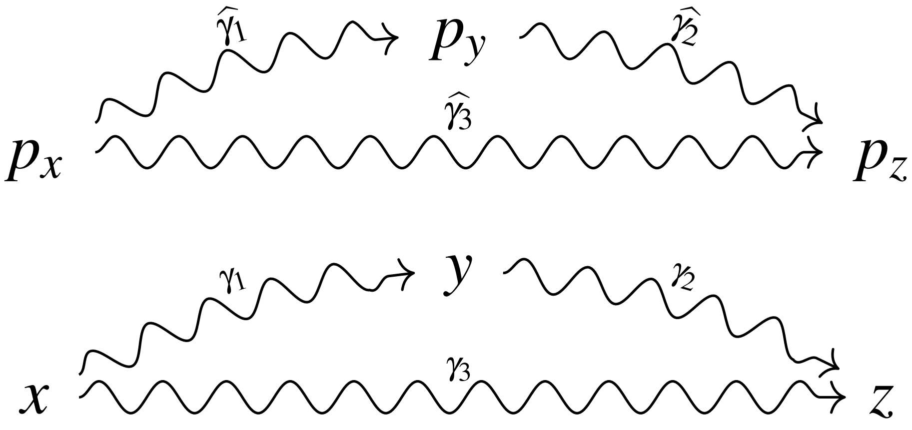

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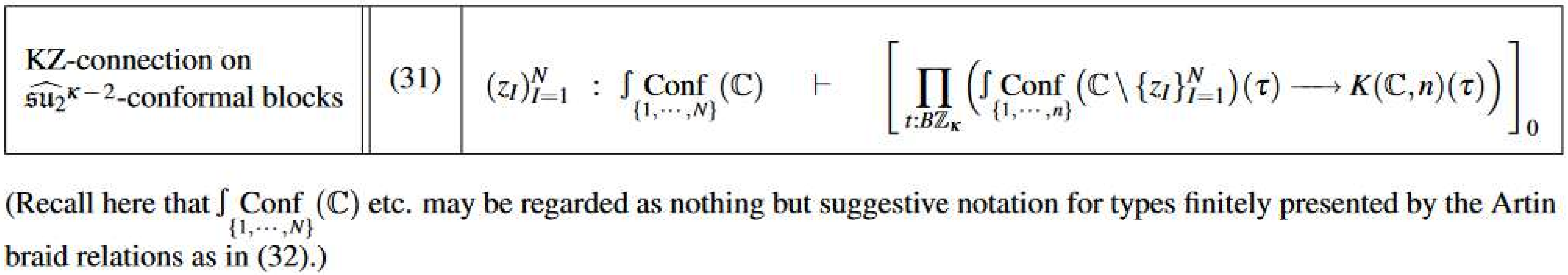

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Hence coding this type family into a HoTT language like `Agda`
gives a Topological Quantum Programming Language

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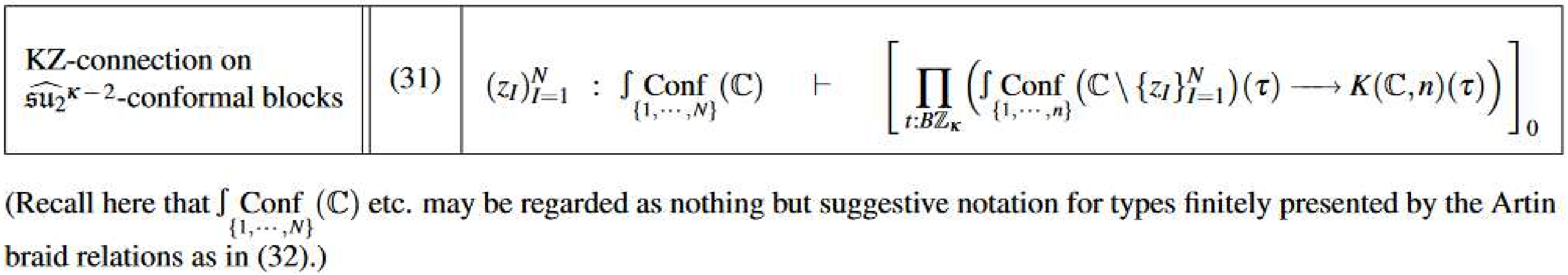

KZ-connection on $\widehat{\mathfrak{su}_2}^{\kappa-2}$-conformal blocks	(31)	$(z_I)_{I=1}^N \;:\; \int \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C}) \quad \vdash \quad \left[\prod_{t:B\mathbb{Z}_\kappa} \left(\int \underset{\{1,\cdots,n\}}{\mathrm{Conf}} \left(\mathbb{C} \setminus \{z_I\}_{I=1}^N\right)(\tau) \longrightarrow K(\mathbb{C},n)(\tau) \right) \right]_0$

(Recall here that $\int \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{C})$ etc. may be regarded as nothing but suggestive notation for types finitely presented by the Artin braid relations as in (32).)

Claim: Its transport operation is the monodromy braid representation
and its path lifting is execution of $\mathfrak{su}(2)$-anyon braid gates.

Hence coding this type family into a HoTT language like `Agda`
gives a Topological Quantum Programming Language
which is fully aware of topological anyon braid quantum gates.

Namely, bundles of $\mathfrak{su}(2)$-conformal blocks
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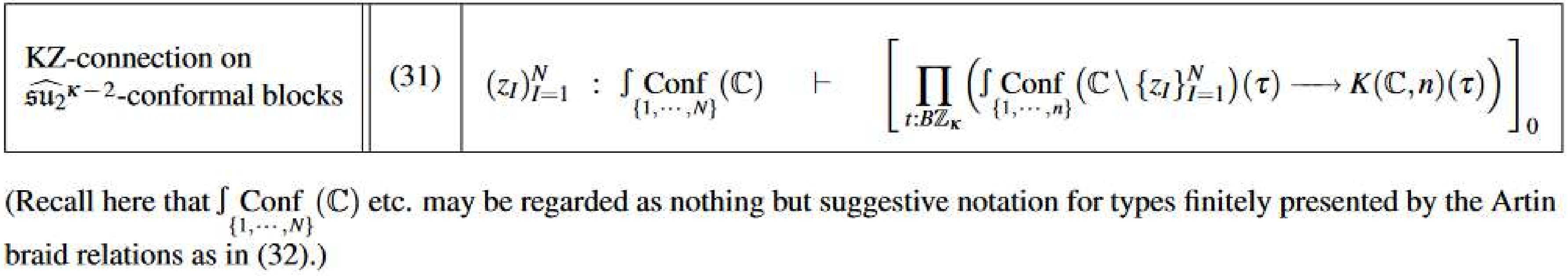

KZ-connection on $\widehat{\mathfrak{su}}_2{}^{\kappa-2}$-conformal blocks	(31)	$(z_I)_{I=1}^N \;:\; \int_{\{1,\cdots,N\}} \mathrm{Conf}\,(\mathbb{C}) \quad \vdash \quad \left[\prod_{t:B\mathbb{Z}_\kappa} \left(\int_{\{1,\cdots,n\}} \mathrm{Conf}\,(\mathbb{C}\setminus\{z_I\}_{I=1}^N)(\tau) \longrightarrow K(\mathbb{C},n)(\tau)\right)\right]_0$

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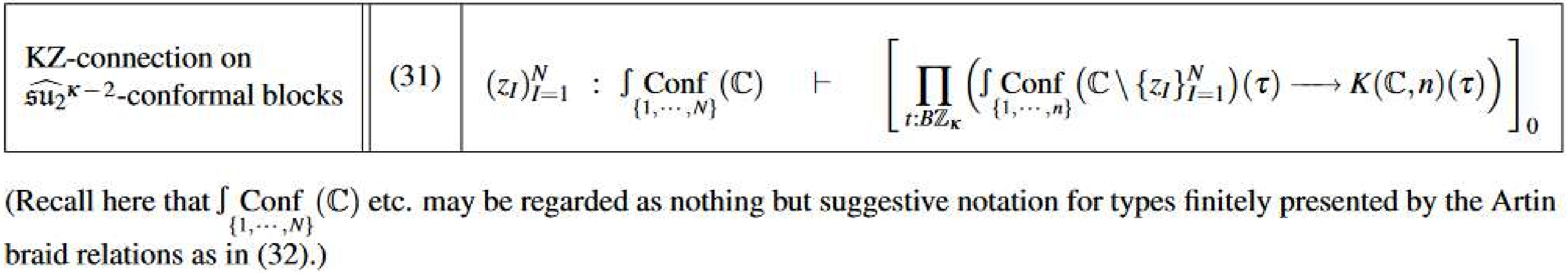

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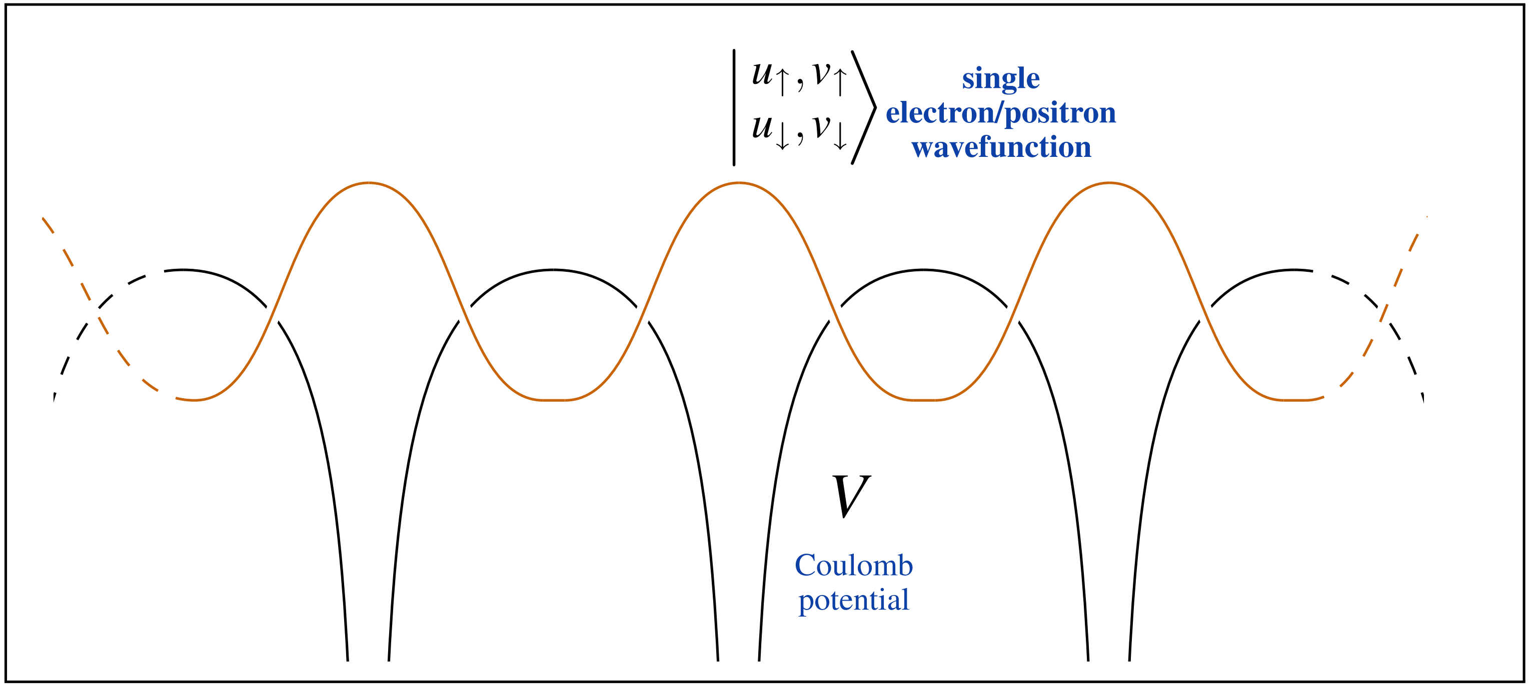
In fact, yet more fine-detail of TQC hardware is naturally HoTT-codeable ⟶

Vacua of electron/positron field in Coulomb background.

Fact ([KS77, CHO82]). The vacua of the free Dirac quantum field in a classical Coulomb background...



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Fact ([KS77, CHO82]). The vacua of the free Dirac quantum field in a classical Coulomb background are characterized by Fredholm operators...

$$\begin{tikzcd}
\textbf{finite-dimensional kernel} & & & \textbf{finite-dimensional cokernel} \\
\ker(F) \arrow[r, hook] & \mathcal{H} \arrow[r, "\overset{\textbf{Fredholm operator}}{F}", "\text{bounded linear}"'] & \mathcal{H} \arrow[r, two heads] & \mathrm{coker}(F) \\
\overbrace{\psi \in \mathcal{H} \,\big\vert\, \forall_{\phi} \, \langle \phi | F | \psi \rangle = 0} & & & \overbrace{\psi \in \mathcal{H} \,\big\vert\, \forall_{\phi} \, \langle \psi | F | \phi \rangle = 0}
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on the single-electron/positron Hilbert space:

single electron Hilbert space

electron states in dressed vacuum

$$\begin{tikzcd} \ker(F) \arrow[r, hook] & \mathcal{H} \arrow[ddrr, "F"] & & \mathcal{H} & \\ & \oplus & & \oplus & \\ & \mathcal{H} \arrow[uurr, "F^*"] & & \mathcal{H} \arrow[r, two heads] & \operatorname{coker}(F) \end{tikzcd}$$

dressed vacuum Fredholm operator

positron states in dressed vacuum

single positron Hilbert space

total charge in dressed vacuum

number of electrons in dressed vacuum state

number of positrons in dressed vacuum state

$$\begin{aligned} \operatorname{ind}(F) & = \dim\big(\ker(F)\big) - \dim\big(\operatorname{coker}(F)\big) \\ & = \dim\big(\operatorname{coker}(F^*)\big) - \dim\big(\ker(F^*)\big) \end{aligned}$$

Quantum symmetries.

On these dressed vacua of electron/positron states
the following *CPT-twisted projective group*

even projective unitary group

$$\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \Big(\overset{\textbf{grading involution}}{\underbrace{\mathbb{Z}_2}_{\{\mathrm{e},P\}}} \times \overset{\textbf{complex conjugation}}{\underbrace{\mathbb{Z}_2}_{\{\mathrm{e},T\}}} \Big)$$

group of quantum symmetries

$$C := PT\,, \quad P\cdot\big[U_+,U_-\big] \;:=\; \big[U_-,U_+\big]\cdot P\,, \qquad T\cdot\big[U_+,U_-\big] \;:=\; \big[\overline{U}_+,\overline{U}_-\big]\cdot T$$

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naturally acts by conjugation:

$$\begin{array}{rcccl}
[U_+,U_-] & : & F & \longmapsto & U_+^{-1}\circ F\circ U_- \\
C\cdot[U_+,U_-] & : & F & \longmapsto & U_-^{-1}\circ F^t\circ U_+ \\
P\cdot[U_+,U_-] & : & F & \longmapsto & U_-^{-1}\circ F^*\circ U_+ \\
T\cdot[U_+,U_-] & : & F & \longmapsto & U_+^{-1}\circ \overline{F}\circ U_-
\end{array}$$

Twisted equivariant KR-theory – As a single diagram of smooth groupoids.

Homotopy classes of quantum-symmetry equivariant families
of such self-adjoint odd Fredholm operators
constitute *twisted equivariant* KR-*cohomology*:

$$
\mathrm{KR}^{\tau}_{G}(\mathrm{X}) \;:=\; \left\{
\begin{tikzcd}
& \overset{\text{space of self-adjoint odd Fredholm operators}}{\mathrm{Fred}^0_{\mathbb{C}}} /\!\!/ \left(\overset{\text{group of quantum symmetries}}{\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, P\} \times \{\mathrm{e}, T\}} \right) \arrow[d] \\
\overset{\text{orbi-orientifold}}{\mathrm{X} /\!\!/ G} \arrow[ur, dashed, "\text{equivariant family of Fredholm operators}", "\text{cocycle in TE-K-theory}"'] \arrow[r, "\text{twist } \tau"'] & {}
\end{tikzcd}
\right\}_{\big/ \sim_{\mathrm{htpy}}}
$$

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Free topological phases of matter.

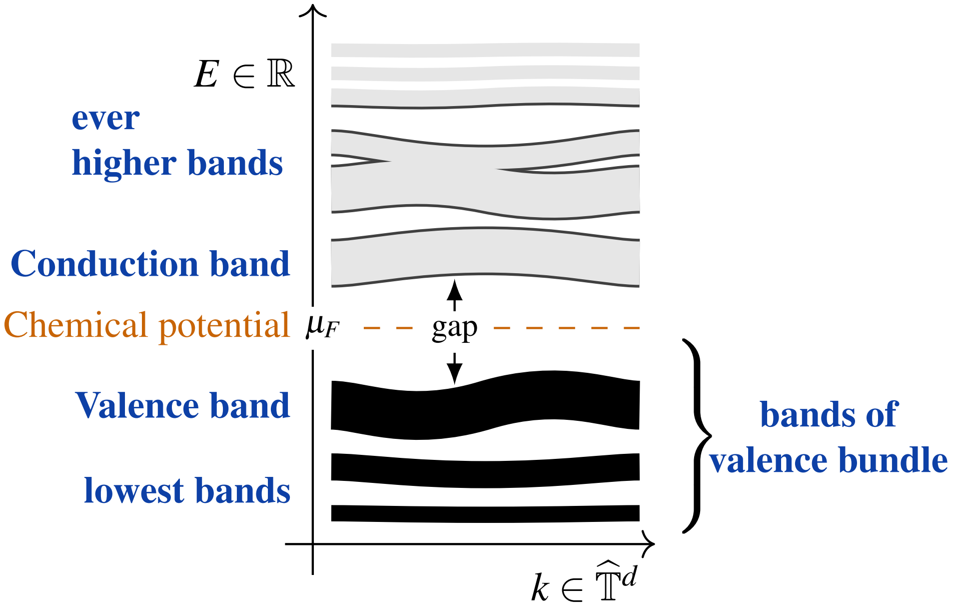
⇒ Idea: *Single*-particle valence bundle of electrons in crystalline insulator
classified by topological K-theory of Brillouin torus
equivariant wrt quantum symmetries [Kitaev 09] [?]

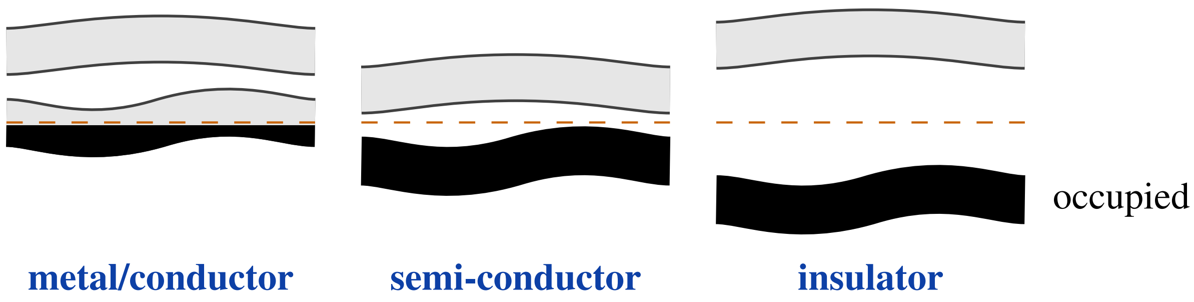
Single particle valence bundle

$$\begin{tikzcd} \mathcal{V} \arrow[d] & = \left\{ k \in \widehat{\mathbb{T}}^d, \, |\psi\rangle \in \mathcal{H} \oplus \mathcal{H} \;\middle|\; \big|\langle \psi | H_k | \psi \rangle\big| \leq \mu_F \right\} \subset \mathcal{B} \\ \widehat{\mathbb{T}}^d & \end{tikzcd}$$

Bundle of all relativistic Bloch states

Brillouin torus of momenta in crystal





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Bundle of all relativistic Bloch states

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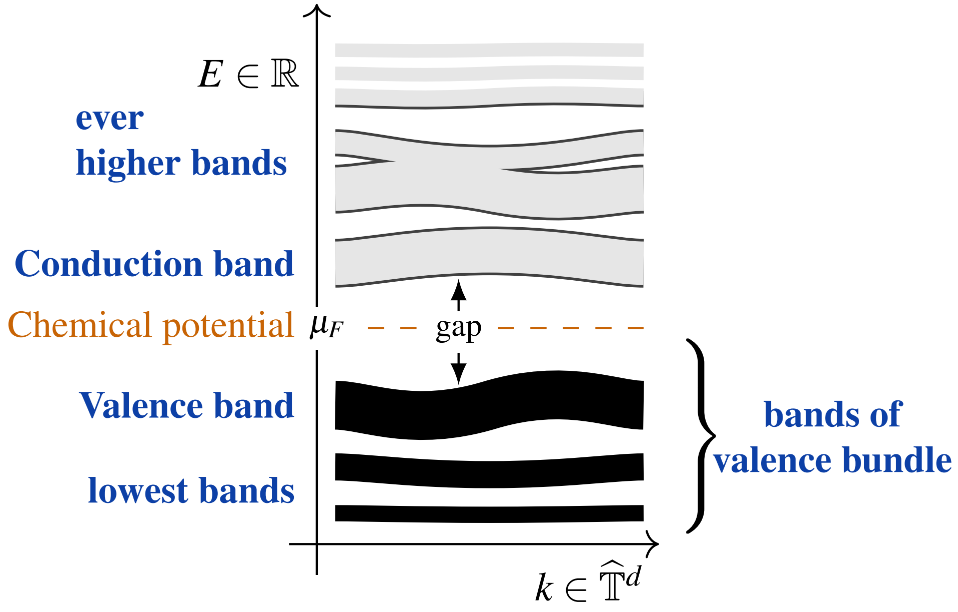
Brillouin torus of momenta in crystal

→ *n*-particle story



$E \in \mathbb{R}$

ever higher bands

Conduction band

Chemical potential μ_F

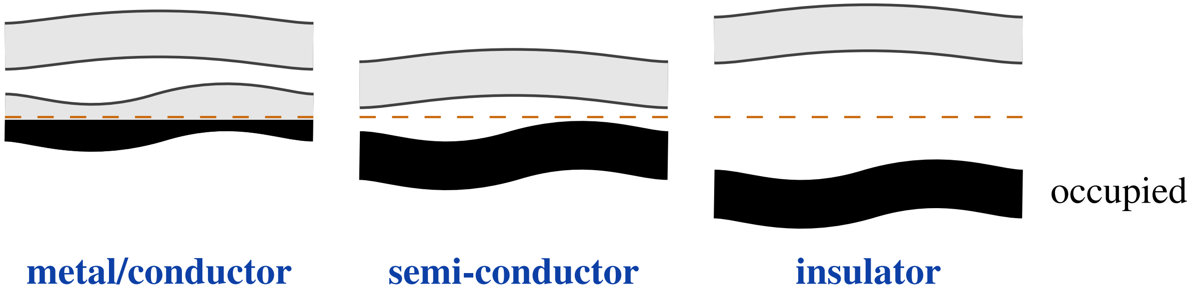
gap

Valence band

lowest bands

bands of valence bundle

$k \in \widehat{\mathbb{T}}^d$



occupied

metal/conductor

semi-conductor

insulator

CPT Quantum symmetries.

$$\begin{tikzcd}
\mathbf{B}\big(\{\mathrm{e}, T\}\big) \arrow[rr, "\substack{\text{pure quantum T-symmetry} \\ T \longmapsto \widehat{T}}"] \arrow[dr, hook] & & \mathbf{B}\left(\dfrac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, T\} \right) \arrow[r] \arrow[dl] & \mathbf{B}\big(\mathbf{B}\mathrm{U}(1) \rtimes \{\mathrm{e}, T\}\big) \\
& \mathbf{B}\big(\{\mathrm{e}, P\} \times \{\mathrm{e}, T\}\big) & &
\end{tikzcd}$$

Here is how to compute the possible quantum T-symmetries...

CPT Quantum symmetries.

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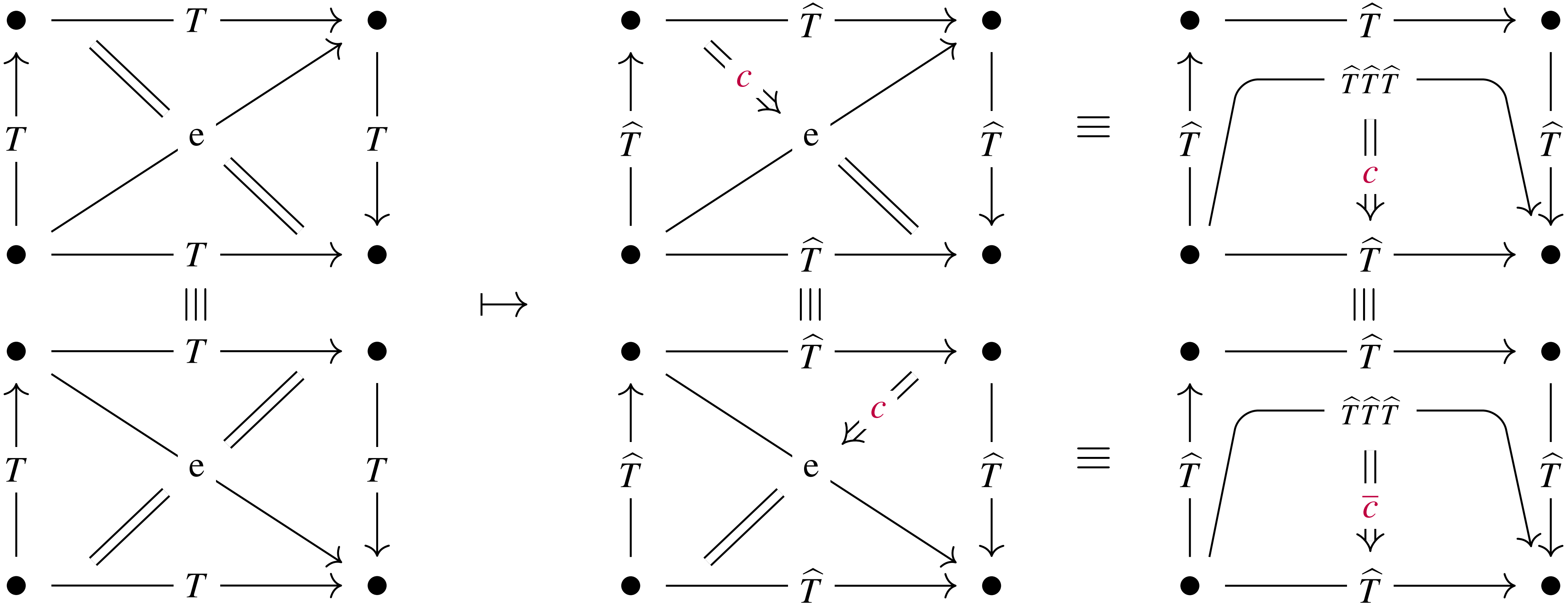

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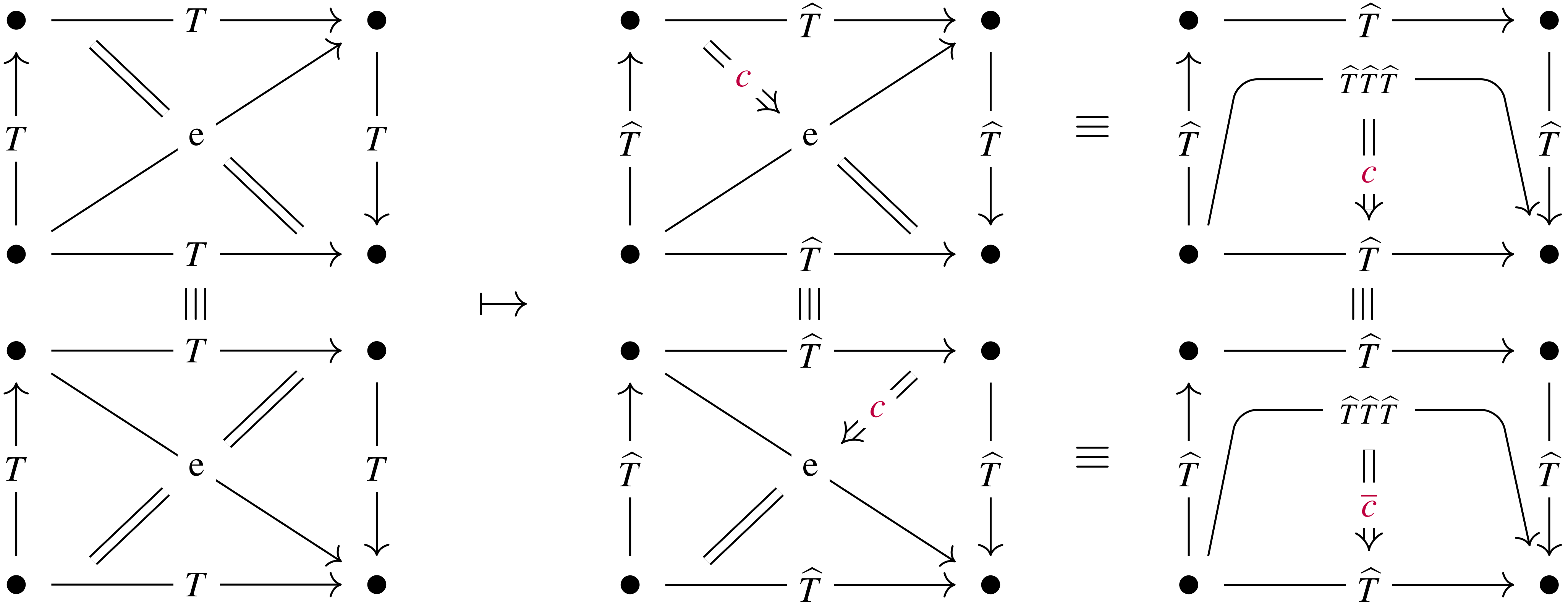

So $\bar{c} = c$ and hence there are two choices for quantum T-symmetry, up to homotopy:

$$\widehat{T}^2 = \pm 1 \quad \text{and similarly} \quad \widehat{C}^2 = \pm 1\,.$$

Example – Orientifold KR-theory

Let I be Inversion action on 2-torus $\widehat{\mathbb{T}}^2 \simeq \mathbb{R}^2/\mathbb{Z}^2$ and trivial action on observables

$$\begin{tikzcd} \mathbb{T}^2 \arrow[r, "I"] & \mathbb{T}^2 \\ k \arrow[r, mapsto] & -k\,, \end{tikzcd} \qquad \begin{tikzcd} \mathrm{Fred}^0_{\mathbb{C}} \arrow[r, "I"] & \mathrm{Fred}^0_{\mathbb{C}} \\ F \arrow[r, mapsto] & F\,. \end{tikzcd}$$

If T acts as I on $\mathbb{T}^2$, then $\mathrm{KR}^{\widehat{T}^2 = +1}$ is *Atiyah's Real K-theory* aka *orienti-fold* K-theory:

$$\mathrm{KR}\Big(\widehat{\mathbb{T}}^{0,2}\Big) \;\simeq\; \left\{ \begin{tikzcd} & & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[d] \\ \mathbb{T}^2 /\!\!/ \{\mathrm{e}, I\} \arrow[urr, dashed] \arrow[rr, "\widehat{T}^2 = +1"] \arrow[dr, "I \mapsto T"', "\text{inversion of space}"] & & \mathbf{B}\big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[dl, "\text{complex conj. of observables}"'] \\ & \mathbf{B}\{\mathrm{e}, T\} \arrow[u, phantom, "\text{combined with}"] & \end{tikzcd} \right\} \Big/ \sim_{\mathrm{htpy}}$$

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But what happens on I-fixed loci i.e. on "orientifolds" ? ⟶

CPT Quantum symmetries – 10 global choices.

(following [?, Prop. 6.4])

Equivariance group	$G =$	$\{\mathrm{e}\}$	$\{\mathrm{e}, P\}$	$\{\mathrm{e}, T\}$		$\{\mathrm{e}, C\}$		$\{\mathrm{e}, T\} \times \{\mathrm{e}, C\}$			
Realization as quantum symmetry $\tau :$	$\widehat{T}^2 =$			$+1$	-1			$+1$	-1	-1	$+1$
	$\widehat{C}^2 =$					$+1$	-1	$+1$	$+1$	-1	-1
Maximal induced Clifford action anticommuting with all G-invariant odd Fredholm operators	$E_{-3} =$								$\mathrm{i}\widehat{T}\widehat{C}\beta$		
	$E_{-2} =$					$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$		
	$E_{-1} =$		$\widehat{P}\beta$			$\widehat{C}\beta$		$\widehat{C}\beta$	$\widehat{C}\beta$		
	$E_{+0} =$	β	β	β	$\begin{pmatrix} \beta & 0 \\ 0 & -\beta \end{pmatrix}$	β	β	β	β	β	β
	$E_{+1} =$				$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$		$\widehat{C}\beta$			$\widehat{C}\beta$	$\widehat{C}\beta$
	$E_{+2} =$				$\begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$		$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$	
	$E_{+3} =$				$\begin{pmatrix} 0 & -\widehat{T} \\ \widehat{T} & 0 \end{pmatrix}$					$\mathrm{i}\widehat{T}\widehat{C}\beta$	
	$E_{+4} =$				$\begin{pmatrix} 0 & \mathrm{i}\widehat{T} \\ \mathrm{i}\widehat{T} & 0 \end{pmatrix}$						
τ-twisted G-equivariant KR-theory of fixed loci	$\mathrm{KR}^{\tau} =$	KU^0	KU^1	KO^0	KO^4	KO^2	KO^6	KO^1	KO^3	KO^5	KO^7

$$\left\{\begin{array}{lll|cc|ll} \text{bounded opers.} & \widehat{F} : \mathcal{H}^2 \xrightarrow[\mathbb{K}-\text{linear}]{\text{bounded}} \mathcal{H}^2 & & & & \text{bounded oper.} & E_0, \cdots, E_p : \mathcal{H}^2 \xrightarrow[\mathbb{K}-\text{linear}]{\text{bounded}} \mathcal{H}^2 \\ \text{self-adjoint} & \widehat{F}^* = \widehat{F} := F + F^* & & \begin{array}{c}\text{graded comm.}\\ E_i \circ \widehat{F} = -\widehat{F} \circ E_i\end{array} & \text{with} & \text{(anti-)self-adjoint} & (E_i)^* = \mathrm{sgn}_i \cdot E_i \\ \text{Fredholm} & \dim\big(\ker(\widehat{F})\big) < \infty & & & & \text{Clifford gen.} & E_i \circ E_j + E_j \circ E_i = 2\mathrm{sgn}_i \cdot \delta_{ij} \end{array}\right\}$$

$$=: \mathrm{Fred}_{\mathbb{C}}^{-p}$$

$$[?]: \;\; \Big\{ X \xrightarrow[\text{cnts}]{} \mathrm{Fred}^p_{\mathbb{K}} \Big\} \big/_{\sim \text{htpy}} \;=\; \left\{ \begin{array}{lcl} \mathrm{KU}^p(X) = \mathrm{KU}^{p+2}(X) & \vert & \mathbb{K} = \mathbb{C} \\ \mathrm{KO}^p(X) = \mathrm{KO}^{p+8}(X) & \vert & \mathbb{K} = \mathbb{R} \end{array} \right.$$

Maximal induced Clifford action anticommuting with all G-invariant odd Fredholm operators	$E_{-3} =$								$\mathrm{i}\widehat{T}\widehat{C}\beta$		
	$E_{-2} =$					$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$		
	$E_{-1} =$		$\widehat{P}\beta$			$\widehat{C}\beta$		$\widehat{C}\beta$	$\widehat{C}\beta$		
	$E_{+0} =$	β	β	β	$\begin{pmatrix} \beta & 0 \\ 0 & -\beta \end{pmatrix}$	β	β	β	β	β	β
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	$E_{+2} =$				$\begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$		$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$	
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τ-twisted G-equivariant KR-theory of fixed loci	$\mathrm{KR}^\tau =$	KU^0	KU^1	KO^0	KO^4	KO^2	KO^6	KO^1	KO^3	KO^5	KO^7

Example – TI-equivariant KR-theory is KO^0-theory.

The combination $T \cdot I$ acts trivially on the domain space and by complex conjugation on observables.

Hence $(T \cdot I)$-equivariant $(\widehat{T}^2 = +1)$-twisted KR-theory is KO^0-theory:

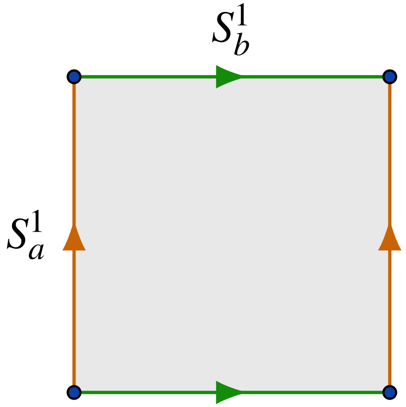
$$\mathrm{KO}^0(X) \;\simeq\; \left\{ \begin{tikzcd} & & \mathrm{Fred}^0_{\mathbb{C}} \sslash \big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[d] \\ X \times * \sslash \{\mathrm{e}, TI\} \arrow[urr, dashed] \arrow[rr, "\widehat{T}^2 = +1"] \arrow[dr, "TI \mapsto T"', "\text{no action on space}"] & \text{combined with} & \mathbf{B}\big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \arrow[dl, "\text{complex conj. of observables}"'] \\ & \mathbf{B}\{\mathrm{e}, T\} & \end{tikzcd} \right\} \Big/ \sim_{\mathrm{htpy}}$$

$n =$	0	1	2	3	4	5	6	7	8	9	$\cdots$
$\mathrm{KO}^0(S^n_*) =$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\cdots$

Example – TI-equivariant KR-theory of punctured torus.

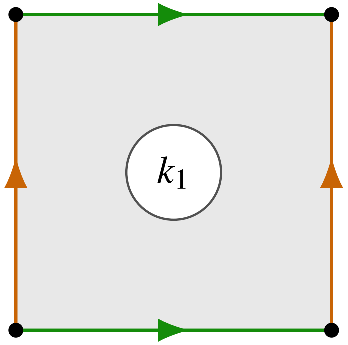
So the TI-equivariant $(\widehat{T}^2 = +1)$-twisted KR-theory of the N-punctured torus is

$$\begin{aligned}
&\mathrm{KR}^{(\widehat{T}^2 = +1)}\big(\widehat{\mathbb{T}}^2 \setminus \{k_1, \cdots, k_N\}\big) \\
&\simeq\ \mathrm{KO}^0\big(\widehat{\mathbb{T}}^2 \setminus \{k_1, \cdots, k_N\}\big) \\
&\simeq\ \mathrm{KO}^0\Big(\bigvee\nolimits_{1+N} S^1_*\Big) \qquad (N \geq 1) \\
&\simeq\ \bigoplus_{1+N} \mathbb{Z}_2
\end{aligned}$$



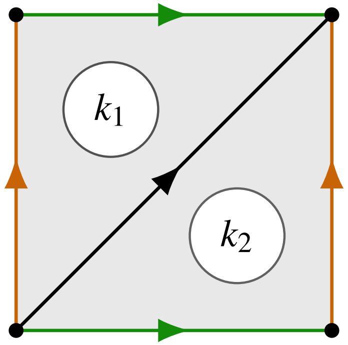
$\widehat{\mathbb{T}}^2$

$\underset{\text{stbl}}{\simeq} S^1_a \vee S^1_b \vee S^2_{\text{bulk}}$



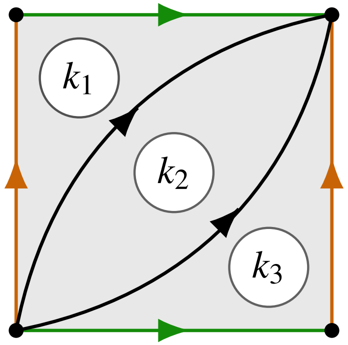
$\widehat{\mathbb{T}}^2 \setminus \{k_1\}$

$\underset{\text{htpy}}{\simeq} S^1_a \vee S^1_b$



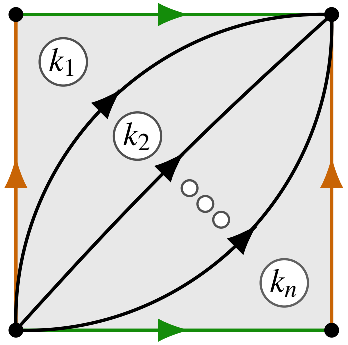
$\widehat{\mathbb{T}}^2 \setminus \{k_1, k_2\}$

$\underset{\text{htpy}}{\simeq} S^1_a \vee S^1_b \vee S^1$



$\widehat{\mathbb{T}}^2 \setminus \{k_1, k_2, k_3\}$

$\underset{\text{htpy}}{\simeq} S^1_a \vee S^1_b \vee S^1 \vee S^1$



$\widehat{\mathbb{T}}^2 \setminus \{k_1, \cdots, k_n\}$

$\underset{\text{htpy}}{\simeq} \bigvee_{1+n} S^1$

The B-field twist.

Besides these CPT-quantum symmetries,
K-theory generically admits the famous *twisting by a B-field*:

The homotopy fiber sequence of 2-stacks discussed before

$$\mathbf{B}\mathrm{U}(\mathcal{H}) \longrightarrow \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big) \xrightarrow[]{\overset{\textbf{universal Dixmier-Douady class}}{\mathrm{DD}}} \mathbf{B}^2\mathrm{U}(1)$$

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induces a surjection of equivalence classes of equivariant higher bundles

equivariant projective bundles **equivariant bundle gerbes**

$$\pi_0 \, \mathrm{Maps}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big)\Big) \xrightarrow{\;\mathrm{DD}_*\;} \!\!\!\!\twoheadrightarrow \;\; \pi_0 \, \mathrm{Maps}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}^2\mathrm{U}(1)\Big)$$

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$$\underbrace{\pi_0\,\mathrm{Maps}\Big(\widehat{X /\!\!/ G}, \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big)\Big)}^{\text{equivariant projective bundles}} \xrightarrow{\mathrm{DD}_*} \!\!\!\!\rightarrow \overbrace{\pi_0\,\mathrm{Maps}\Big(\widehat{X /\!\!/ G}, \mathbf{B}^2\mathrm{U}(1)\Big)}^{\text{equivariant bundle gerbes}}$$

which has a natural section:

$$\underbrace{\overbrace{\pi_0\mathrm{Maps}\big(\widehat{X /\!\!/ G}, \mathbf{B}^2\mathrm{U}(1)\big)}^{\text{“stable twists”}}}_{\text{equivariant bundle gerbes}} \hookrightarrow \underbrace{\pi_0\,\mathrm{Maps}\left(\widehat{X /\!\!/ G}, \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \big(\{\mathrm{e}, C\} \times \{\mathrm{e}, P\}\big)\right)\right)}_{\text{full quantum-symmetry twists}}.$$

The B-field twist – Inner local systems.

On fixed loci (orbi-singularities)

$$\mathrm{X}/\!\!/G \;\simeq\; \mathrm{X} \times * /\!\!/ G \;=\; \mathrm{X} \times \mathbf{B}G$$

the B-field twist induces *secondary* twists by “inner local systems”:

stable twists over fixed locus

$$\mathrm{Maps}\big(\mathrm{X} \times * /\!\!/ G,\, \mathbf{B}^2\mathrm{U}(1)\big) \simeq \mathrm{Maps}\big(\mathrm{X} \times \mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1)\big)$$

$$\simeq \mathrm{Maps}\big(\mathrm{X},\, \mathrm{Maps}(\mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1))\big)$$

The B-field twist – Inner local systems.

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stable twists over fixed locus

$$\begin{aligned} \mathrm{Maps}\big(\mathrm{X} \times * /\!\!/ G, \mathbf{B}^2\mathrm{U}(1)\big) &\simeq \mathrm{Maps}\big(\mathrm{X} \times \mathbf{B}G, \mathbf{B}^2\mathrm{U}(1)\big) \\ &\simeq \mathrm{Maps}\big(\mathrm{X}, \mathrm{Maps}(\mathbf{B}G, \mathbf{B}^2\mathrm{U}(1))\big) \\ &\simeq \mathrm{Maps}\big(\mathrm{X}, \mathbf{B}G^* \times \mathbf{B}^2\mathrm{U}(1)\big) \end{aligned}$$

Here we are assuming $G \underset{\mathrm{fin}}{\subset} \mathrm{SU}(2)$ so that $H^2_{\mathrm{Grp}}\big(G, \mathrm{U}(1)\big) = 0$.

And $G^* := \mathrm{Hom}\big(G, \mathrm{U}(1)\big)$ denotes the Pontrjagin-dual group.

The B-field twist – Inner local systems.

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\mathrm{Maps}\big(\mathrm{X} \times \ast /\!\!/ G,\, \mathbf{B}^2\mathrm{U}(1)\big) &\simeq \mathrm{Maps}\big(\mathrm{X} \times \mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1)\big) \\
&\simeq \mathrm{Maps}\big(\mathrm{X},\, \mathrm{Maps}(\mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1))\big) \\
&\simeq \mathrm{Maps}\big(\mathrm{X},\, \mathbf{B}G^\ast \times \mathbf{B}^2\mathrm{U}(1)\big) \\
&\simeq \mathrm{Maps}\big(\mathrm{X},\, \mathbf{B}G^\ast\big) \;\times\; \mathrm{Maps}\big(\mathrm{X},\, \mathbf{B}^2\mathrm{U}(1)\big)
\end{aligned}$$

inner local systems **bundle gerbes**

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The B-field twist – Inner local systems – The diagrammatics.

Hence the

inner local system-twisted KU-*cohomology*
of a G-orbi-singularity of shape X

arises as follows:

$$\mathrm{KU}_G^{n+[\omega_1]}(X) \;=\; \left\{ \begin{tikzcd} & \mathrm{Fred}^n_{\mathbb{C}} /\!\!/ \mathrm{PU}(\mathcal{H}) \arrow[d] \\ X \times * /\!\!/ G \arrow[ur, dashed, "\text{cocycle}"] \arrow[r, "\tau", "\text{inner local system twist}"'] & \mathbf{B}\mathrm{PU}(\mathcal{H}) \end{tikzcd} \right\} \Big/ \sim_{\mathrm{htpy}}$$

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The B-field twist – Inner local systems – The proof.

For the proof we consider the following diagram [**?**, Ex. 4.1.56][**?**, §3]:

$$\begin{tikzcd}
\mathbf{B}G \arrow[r, bend left, "\mathrm{stbl}_0"{name=U}] \arrow[r, bend right, "\mathrm{stbl}_0"'{name=D}] & \mathbf{B}\mathrm{PU}(\mathcal{H}) \arrow[from=U, to=D, Rightarrow, "\rho \in G^*"]
\end{tikzcd}$$

$$\begin{tikzcd}
\bullet \arrow[d, "g"] & & \underset{\substack{[\rho_i] \in \\ \mathrm{Irr}(G)}}{\bigoplus} \rho_i \otimes \ell^2(\mathbb{C}) \arrow[r, "v \mapsto 1_\rho \otimes v"] \arrow[d, "\underset{[\rho_i]}{\oplus}(\rho_i(g)\otimes \mathrm{id})"] & \underset{\substack{[\rho_i] \in \\ \mathrm{Irr}(G)}}{\bigoplus} \rho_i \otimes \ell^2(\mathbb{C}) \arrow[d, "\underset{[\rho_i]}{\oplus}(\rho_i(g)\otimes \mathrm{id})"] \\
\bullet \arrow[r, phantom, "\longmapsto"] & & \underset{\substack{[\rho_i] \in \\ \mathrm{Irr}(G)}}{\bigoplus} \rho_i \otimes \ell^2(\mathbb{C}) \arrow[r, "v \mapsto 1_\rho \otimes v"'] \arrow[ur, Rightarrow, "\rho(g)(1_\rho)"] & \underset{\substack{[\rho_i] \in \\ \mathrm{Irr}(G)}}{\bigoplus} \rho_i \otimes \ell^2(\mathbb{C})
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\mathbf{B}G \arrow[r, bend left, "\mathrm{stbl}_0"] \arrow[r, bend right, "\mathrm{stbl}_0"'] \arrow[r, phantom, "\Downarrow \rho \in G^*"] & \mathbf{B}\mathrm{PU}(\mathcal{H})
\end{tikzcd}$$

stable G-representation

$$\begin{tikzcd}
\bigoplus_i \rho_i \otimes \ell^2(\mathbb{C}) \arrow[rr, "v \mapsto 1_\rho \otimes v"] \arrow[dr, "F", "\text{Fredholm operator}"'] \arrow[dd, "\bigoplus_i \rho_i \otimes \ell^2(\mathbb{C})" description] & & \bigoplus_i \rho_i \otimes \ell^2(\mathbb{C}) \arrow[dr, "{[\rho] \cdot F}"] & \\
& \bigoplus_i \rho_i \otimes \ell^2(\mathbb{C}) \arrow[rr, "v \mapsto 1_\rho \otimes v", "\text{tensoring with unit of group character}"'] \arrow[dd, "\bigoplus_s \rho_i(g) \otimes \mathrm{id}" description] & & \bigoplus_i \rho_i \otimes \ell^2(\mathbb{C}) \arrow[dd, "\bigoplus_s \rho_i(g) \otimes \mathrm{id}" description] \\
\bigoplus_i \rho_i \otimes \ell^2(\mathbb{C}) \arrow[dr, "F"'] & & & \\
& \bigoplus_i \rho_i \otimes \ell^2(\mathbb{C}) \arrow[rr, "v \mapsto 1_\rho \otimes v"'] & & \bigoplus_i \rho_i \otimes \ell^2(\mathbb{C})
\end{tikzcd}$$

action of group character on equivariant Fredholm operator

equivariance of Fredholm operator

projective intertwining action
of group character

The B-field twist – Inner local systems – Chern character.

One aspect of these twistings becomes transparent under the *Chern character*:

$$\underset{\text{complex K-theory}}{\mathrm{KU}^0(\mathrm{X})} \xrightarrow[\text{Chern character}]{} \mathrm{KU}^0(\mathrm{X};\mathbb{C}) \;\simeq\; \overbrace{\bigoplus_{d\in\mathbb{N}} H^{2d}\Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathrm{X};\mathbb{C}\big),\mathrm{d}\Big)}^{\text{periodic de Rham cohomology}}$$

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For twist by B-field $\widehat{B}_2$ there is a closed differential 3-form H_3 such that:

$$\overset{\text{plain B-field -twisted K-theory}}{\mathrm{KU}^{n+\widehat{B}_2}(\mathrm{X})} \xrightarrow[\textbf{Chern character}]{\textbf{twisted}} \mathrm{KU}^{\widehat{B}_2}(\mathrm{X};\mathbb{C}) \;\simeq\; \overset{\text{3-twisted periodic de Rham cohomology}}{\bigoplus_{d\in\mathbb{Z}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathrm{X};\mathbb{C}\big),\mathrm{d}+H_3\wedge\Big)}$$

The B-field twist – Inner local systems – Chern character.

One aspect of these twistings becomes transparent under the Chern character:

complex K-theory

periodic de Rham cohomology

$$\mathrm{KU}^0(\mathrm{X}) \xrightarrow[\text{Chern character}]{} \mathrm{KU}^0(\mathrm{X};\mathbb{C}) \simeq \bigoplus_{d\in\mathbb{N}} H^{2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}),\mathrm{d}\Big)$$

For twist by B-field $\widehat{B}_2$ there is a closed differential 3-form H_3 such that:

plain B-field
-twisted K-theory

3-twisted periodic de Rham cohomology

$$\mathrm{KU}^{n+\widehat{B}_2}(\mathrm{X}) \xrightarrow[\text{Chern character}]{\text{twisted}} \mathrm{KU}^{\widehat{B}_2}(\mathrm{X};\mathbb{C}) \simeq \bigoplus_{d\in\mathbb{Z}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}),\mathrm{d}+H_3\wedge\Big)$$

For twist by inner C_κ-local system, there is closed 1-form ω_1 with holon. in $C_\kappa \subset \mathrm{U}(1)$ such that:

inner local system
-twisted K-theory

1-twisted periodic de Rham cohomology

$$\mathrm{KU}^{n+[\omega_1]}_{C_\kappa}(\mathrm{X}) \xrightarrow[\text{Chern character}]{\text{twisted equivariant}} \bigoplus_{\substack{d\in\mathbb{Z}\\ 1\leq r\leq\kappa}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}),\mathrm{d}+r\cdot\omega_1\wedge\Big)$$

of A-type singularity

The B-field twist – Inner local systems – Chern character.

One aspect of these twistings becomes transparent under the Chern character:

This is the hidden 1-twisting in TED-K – that we will next relate to anyons. ⟶

inner local system
-twisted K-theory

1-twisted periodic de Rham cohomology

$$\mathrm{KU}^{n+[\omega_1]}_{C_\kappa}(\mathrm{X}) \xrightarrow[\text{Chern character}]{\text{twisted equivariant}} \bigoplus_{\substack{d\in\mathbb{Z}\\ 1\le r\le\kappa}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}), \mathrm{d} + r\cdot\omega_1\wedge\Big)$$

of A-type singularity

TED-Cohomological incarnation of Conformal blocks.

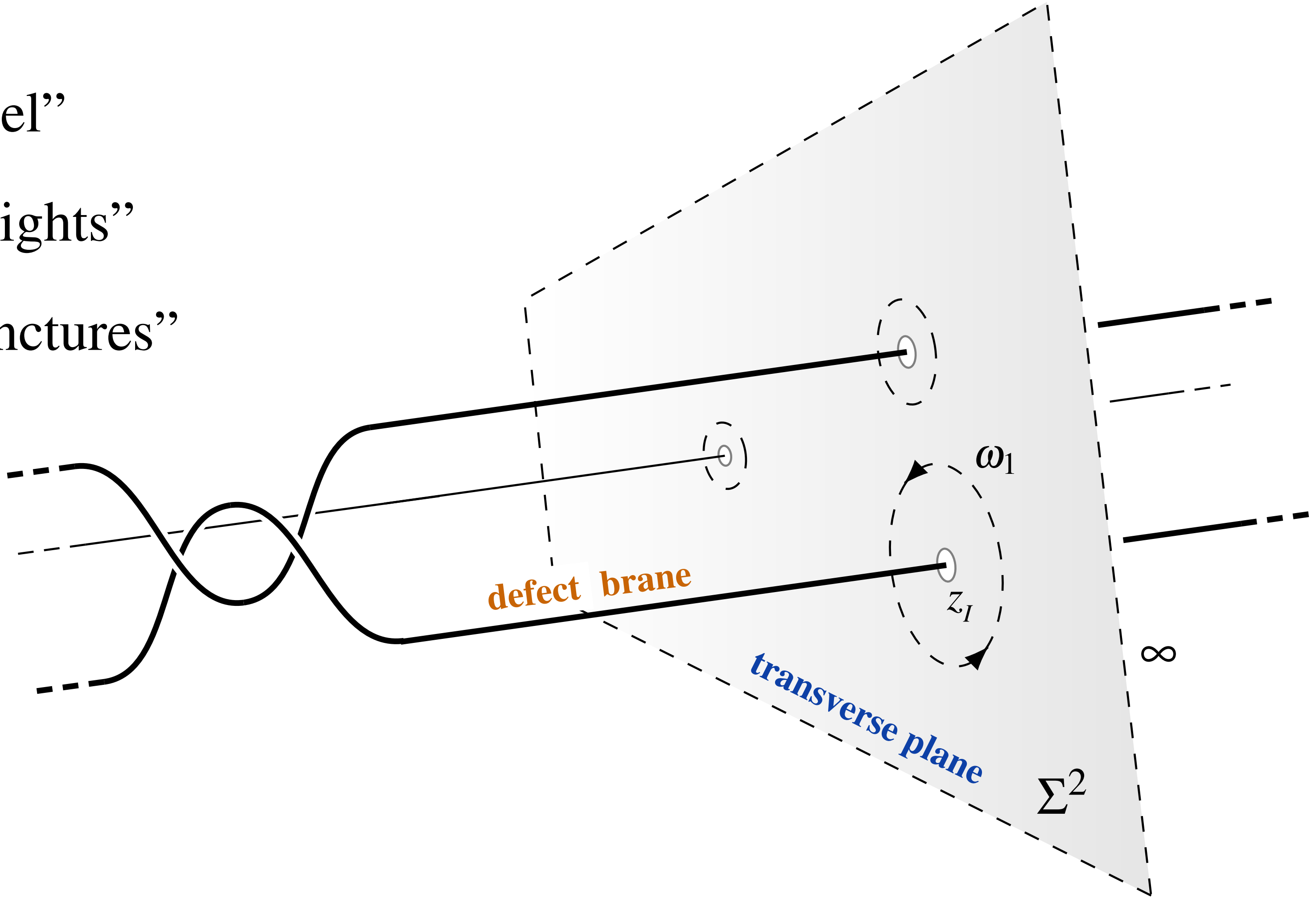
Consider

$\kappa \;:=\; k+2$ “level”

$\mathrm{w}_I \;\in\; \{0,\cdots,k\}$ “weights”

$z_I \;\in\; \{z_1,\cdots,z_N\}$ “punctures”

$\omega_1 \;:=\; \sum_I -\frac{\mathrm{w}_I}{\kappa}\frac{\mathrm{d}z}{z-z_I}$



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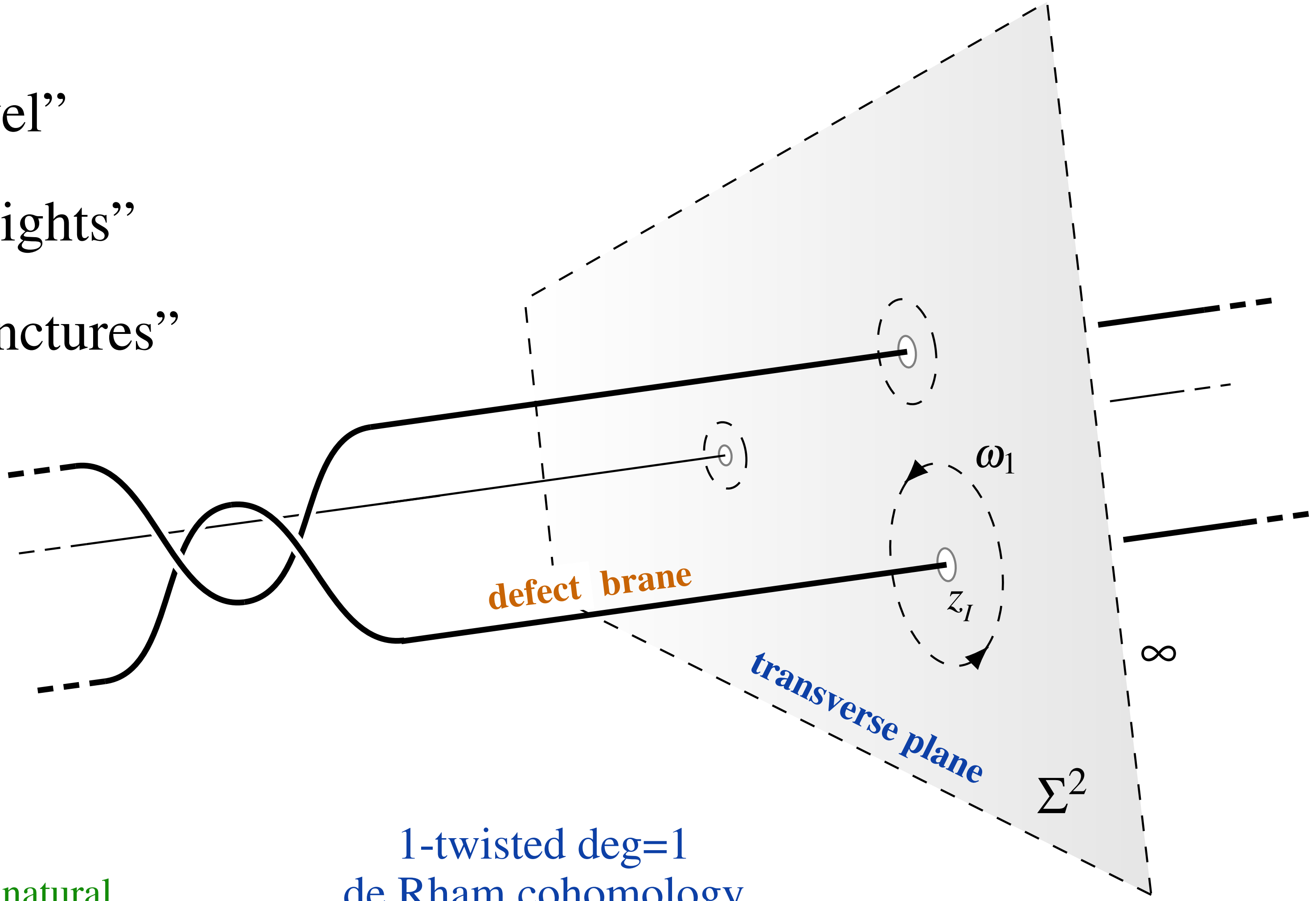

$\mathfrak{su}(2)$-affine deg=1
conformal blocks

natural
inclusion

1-twisted deg=1
de Rham cohomology

$$\mathrm{CnfBlck}^1_{\widehat{\mathfrak{sl}_2}^k}(\vec{\mathrm{w}},\vec{z}) \xhookrightarrow{\qquad\qquad} H^1\Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{C}\setminus\{\vec{z}\}\big),\, \mathrm{d}+\omega_1\wedge\Big)$$

FSV92, Cor. 3.4.2

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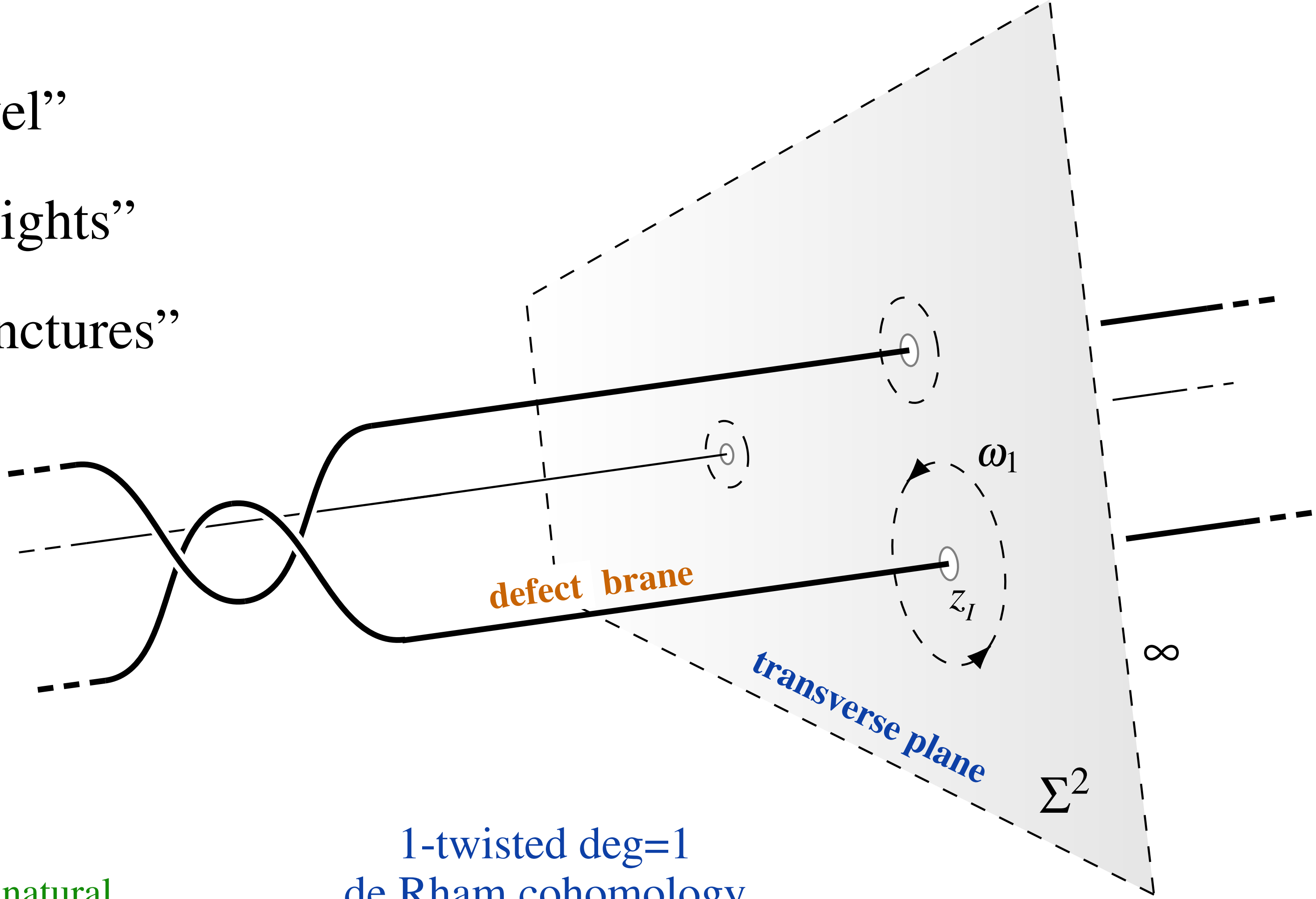

$\mathfrak{su}(2)$-affine deg=1
conformal blocks

1-twisted deg=1
de Rham cohomology

$$\mathrm{CnfBlck}^1_{\widehat{\mathfrak{sl}_2}^k}(\vec{\mathrm{w}},\vec{z}) \xhookrightarrow{\text{natural inclusion}} H^1\Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{C}\setminus\{\vec{z}\}\big),\, \mathrm{d}+\omega_1\wedge\Big)$$

FSV92, Cor. 3.4.2

$$\xhookrightarrow{\text{natural inclusion}} \mathrm{KU}^{1+\omega_1}\Big(\big(\mathbb{C}\setminus\{\vec{z}\}\big)\times *\!/\!\!/C_\kappa;\,\mathbb{C}\Big)$$

SS22a, Prop. 2.16

inner local system-twisted deg=1
K-theory of $\mathbb{A}_{\kappa-1}$-singularity

(as explained above)

Account for interactions by passage to configuration space.

Interacting *n*-electron wavefunctions are functions on the space of *n* points in Bri-torus

Account for interactions by passage to configuration space.

Interacting n-electron wavefunctions are functions on the space of n points in Bri-torus

Pauli exclusion ⇒ these span vector bundle away from the locus of coinciding points:

Account for interactions by passage to configuration space.

Interacting n-electron wavefunctions are functions on the space of n points in Bri-torus
Pauli exclusion ⇒ these span vector bundle away from the locus of coinciding points:

Slater determinants of Bloch states

Slater-Bloch valence bundle of interacting n-electron states

$$\begin{tikzcd} \mathscr{V}_n \arrow[d] & \subset \coprod_{(k^1,\cdots,k^n)} \mathrm{Span}\Big\{ \Psi_{i_1,\cdots,i_n}\Big(\big(k^1,s^1\big),\cdots,\big(k^n,s^n\big)\Big)\Big\}_{\substack{(i_1,\cdots,i_n)\\(s^1,\cdots,s^n)}} \\ \underset{\{1,\cdots,n\}}{\mathrm{Conf}}\Big(\widehat{\mathbb{T}}^d \setminus \{k_1,\cdots,k_N\}\Big) & = \Big\{(k^1,\cdots,k^n)\in\big(\widehat{\mathbb{T}}^d\big)^n \,\Big\vert\, \underset{i\neq j}{\forall}\, k^i \neq k^j \;\;\text{and}\;\; \underset{i,I}{\forall}\, k^i\neq k_I\Big\}. \end{tikzcd}$$

configuration space of n “probe” points

in complement of N “nodal” points inside the Brillouin torus

Pauli exclusion

nodal singularities

Account for interactions by passage to configuration space.

Interacting n-electron wavefunctions are functions on the space of n points in Bri-torus
Pauli exclusion ⇒ these span vector bundle away from the locus of coinciding points:

Slater determinants of Bloch states

Slater-Bloch valence bundle of interacting n-electron states

$$\begin{tikzcd}
\mathscr{V}_n \arrow[r, phantom, "\subset"] \arrow[d] & \displaystyle\coprod_{(k^1,\cdots,k^n)} \mathrm{Span}\Big\{ \Psi_{i_1,\cdots,i_n}\Big(\big(k^1,s^1\big),\cdots,\big(k^n,s^n\big)\Big) \Big\}_{\substack{(i_1,\cdots,i_n)\\(s^1,\cdots,s^n)}} \\
\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\Big(\underset{\text{in complement of } N \text{ “nodal” points inside the Brillouin torus}}{\widehat{\mathbb{T}}^d \setminus \{k_1,\cdots,k_N\}}\Big) = \Big\{ (k^1,\cdots,k^n) \in \big(\widehat{\mathbb{T}}^d\big)^n \,\Big\vert\, \underset{i\neq j}{\forall}\; \underset{\text{Pauli exclusion}}{k^i \neq k^j} \;\text{ and }\; \underset{i,I}{\forall}\; \underset{\text{nodal singularities}}{k^i \neq k_I} \Big\}. &
\end{tikzcd}$$

configuration space of n “probe” points

This locus is known as the **configuration space of n points**. (see e.g. SS22, §2.2)

TED-Cohomological incarnation of Conformal blocks.

So consider, more generally, *configuration spaces of points*

$$\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathrm{X}\big) \;:=\; \Big\{ z^1, \cdots, z^n \in \mathrm{X} \,\Big|\, \underset{i<j}{\forall}\; z^i \neq z^j \Big\}.$$

with $\omega_1 := \sum_{1 \leq i \leq n} \sum_{I} -\frac{\mathrm{w}_I}{\kappa} \frac{\mathrm{d}z}{z - z_I} + \sum_{1 \leq i < j \leq n} \frac{2}{\kappa} \frac{\mathrm{d}z}{z^i - z^j}$ on $\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathbb{C} \setminus \{\vec{z}\}\big)$

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Then:

$$\overset{\substack{\mathfrak{su}(2)\text{-affine deg}=n\\ \text{conformal blocks}}}{\mathrm{CnfBlck}^{n}_{\widehat{\mathfrak{sl}_2}^{k}}\big(\vec{\mathrm{w}},\vec{z}\big)} \hookrightarrow \overset{\substack{\text{1-twisted deg}=n\text{ de Rham cohomology}\\ \text{of configuration space of } n \text{ points}}}{H^n\Big(\Omega^\bullet_{\mathrm{dR}}\Big(\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathbb{C}\setminus\{\vec{z}\}\big)\Big),\, \mathrm{d}+\omega_1\wedge\Big)}$$

FSV92, Cor. 3.4.2

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$\mathfrak{su}(2)$-affine deg=n
conformal blocks

1-twisted deg=n de Rham cohomology
of configuration space of n points

$$\mathrm{CnfBlck}^n_{\widehat{\mathfrak{sl}_2}^k}(\vec{\mathrm{w}}, \vec{z}) \hookrightarrow H^n\Big(\Omega^\bullet_{\mathrm{dR}}\Big(\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathbb{C} \setminus \{\vec{z}\}\big)\Big), \mathrm{d} + \omega_1 \wedge\Big)$$

FSV92, Cor. 3.4.2

$$\hookrightarrow \mathrm{KU}^{n+\omega_1}\Big(\Big(\underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathbb{C} \setminus \{\vec{z}\}\big)\Big) \times * /\!\!/ C_\kappa; \mathbb{C}\Big)$$

[?, Thm. 2.18]

inner local system-twisted deg=n K-theory
of configurations in $\mathbb{A}_{\kappa-1}$-singularity

TED-Cohomological incarnation of Conformal blocks.

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[?, Thm. 2.18]

inner local system-twisted deg=n K-theory of configurations in $\mathbb{A}_{\kappa-1}$-singularity

The previous statement is subsumed since $\underset{\{1\}}{\mathrm{Conf}}(\mathrm{X}) = \mathrm{X}$.

Conclusion.

The commonly expected $\widehat{\mathfrak{su}_2}^k$-charges of anyons and defect branes *are* reflected in the TED-K-theory of *configuration spaces of points* in 2-dimensional transverse spaces *inside* $\mathbb{A}_{k+1}$*-orbi-singularities*.

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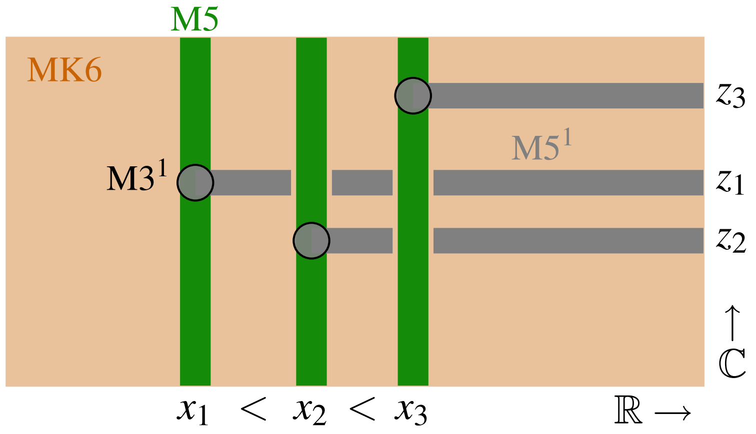
$$\overset{\substack{\text{Configuration space of}\\ \text{ordered points in the plane}}}{\coprod_n \underset{\{1,\cdots,n\}}{\mathrm{Conf}}(\mathbb{C})} \;\simeq\; \overbrace{\coprod_n \mathrm{Conf}_n\big(\mathbb{C};\,\mathbb{R}_{\mathrm{cpt}}\big)}^{\substack{\text{3-Cohomotopy cocycle space}\\ \text{for codim=1 branes}\\ \mathrm{Map}^*(\mathbb{R}_+\wedge\mathbb{C}_{\mathrm{cpt}},\,S^3)\,\simeq}} \underset{\substack{\coprod_n \mathrm{Conf}_n\big(*;\,(\mathbb{R}\times\mathbb{C})_{\mathrm{cpt}}\big)\\ \text{Fiber product of respective configuration spaces}\\ \text{(of un-ordered points escaping to transverse infinity)}\\ \text{reflecting the brane intersections}}}{\times} \overbrace{\coprod_n \mathrm{Conf}_n\big(\mathbb{R};\,\mathbb{C}_{\mathrm{cpt}}\big)}^{\substack{\text{3-Cohomotopy cocycle space}\\ \text{for codim-2 branes}\\ \mathrm{Map}^*(\mathbb{R}_{\mathrm{cpt}}\wedge\mathbb{C}_+,\,S^3)\,\simeq}}$$

e.g.: $\underset{\{1,\cdots,3\}}{\mathrm{Conf}}(\mathbb{C}) \simeq \Bigg\{$



$\Bigg\}$

The moduli space of flat M3-branes according to <u>*Hypothesis H*</u> is the configuration space of ordered points in their transverse plane.

Physics underlying Topological Quantum Theory controlling Computation

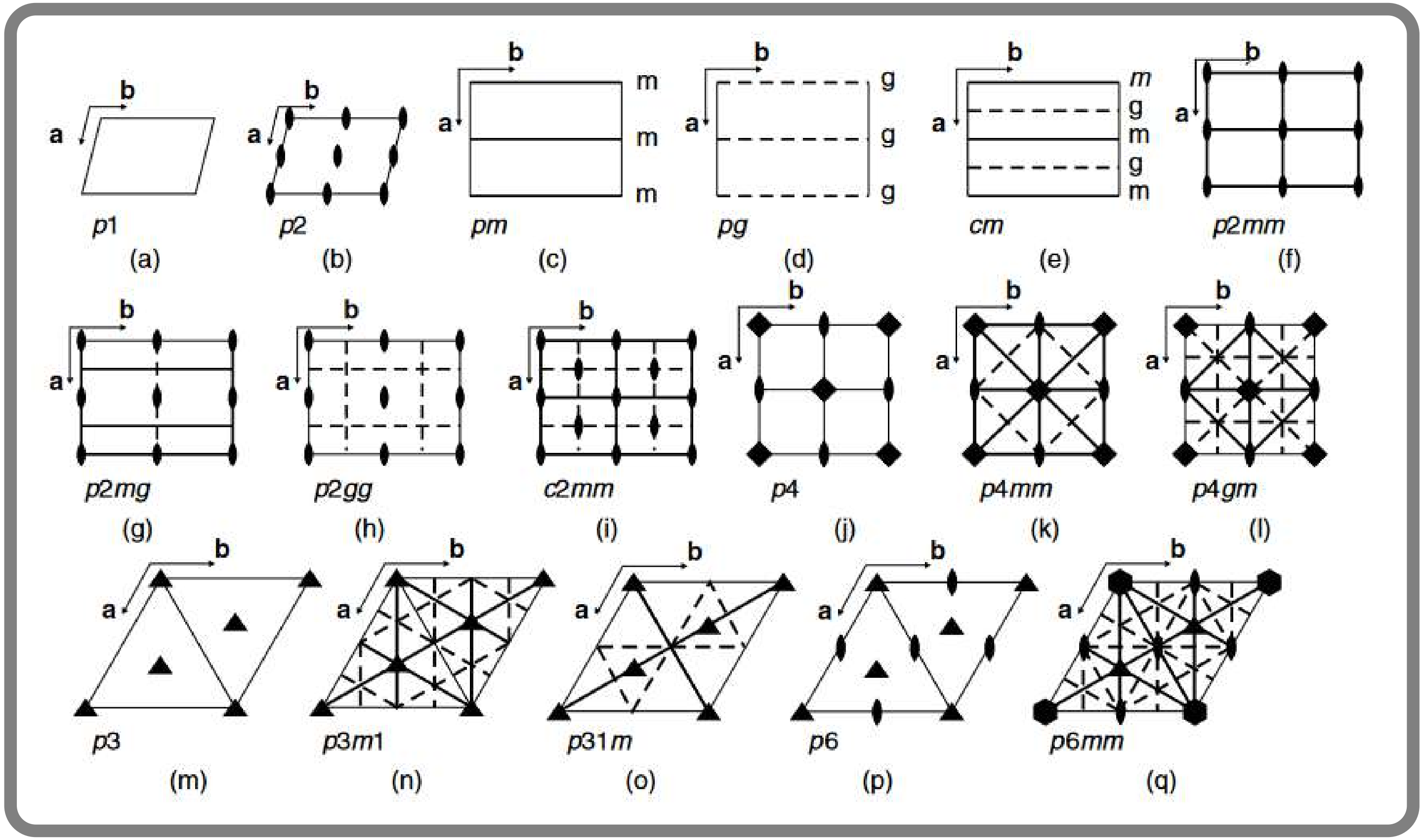
In summary,
we arrive at
the following
picture.

Physics Theory
underlying controlling
Topological Quantum Computation

electron states ↔ Brillouin torus

[Brillouin (1930)]

Physics underlying Topological Quantum Computation
Theory controlling Topological Quantum Computation



electron states ↔ Brillouin torus

[Brillouin (1930)]

Physics underlying Topological Quantum Theory controlling Computation

electron states ⟷ Brillouin torus

[Brillouin (1930)]

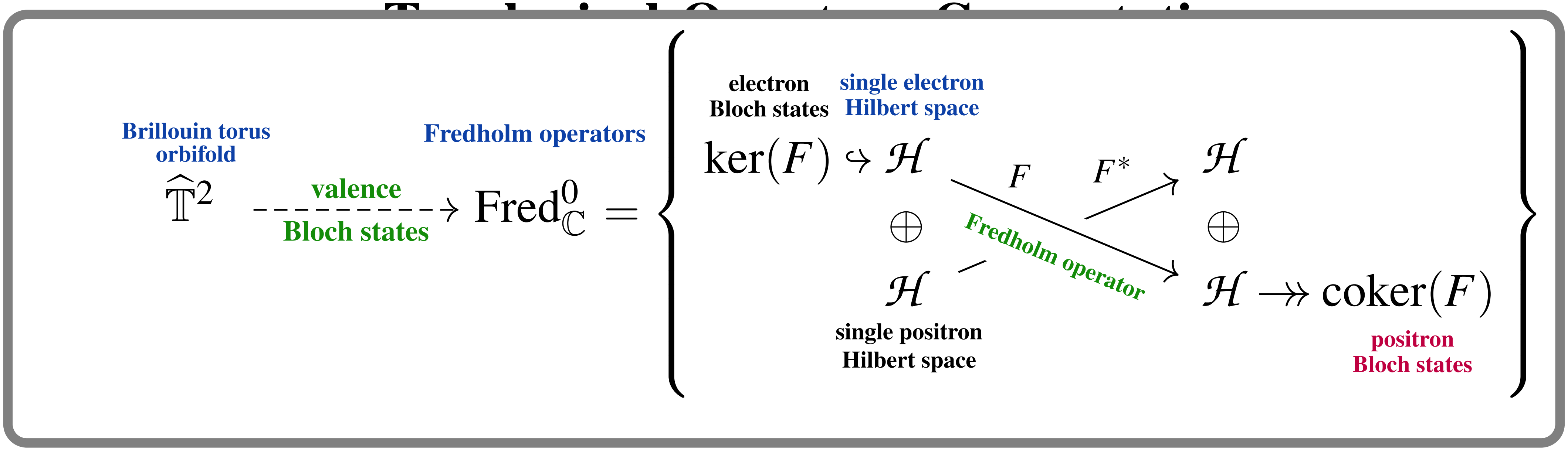
Physics **Theory**
underlying **controlling**
Topological Quantum Computation

topological phases		topological
(deformation classes)	↔	K-theory of
electron states	↔	Brillouin torus

[Kitaev (2009)]

Physics underlying

Theory controlling



Brillouin torus orbifold

valence Bloch states

Fredholm operators

electron Bloch states

single electron Hilbert space

Fredholm operator

single positron Hilbert space

positron Bloch states

$$\begin{tikzcd}[ampersand replacement=\&] \widehat{\mathbb{T}}^2 \arrow[r, dashed] \& \mathrm{Fred}^0_{\mathbb{C}} = \left\{ \begin{tikzcd} \ker(F) \arrow[r, hook] \& \mathcal{H} \arrow[drr, "F"] \& \& \mathcal{H} \\ \& \oplus \& \& \oplus \\ \& \mathcal{H} \arrow[urr, "F^*"'] \& \& \mathcal{H} \arrow[r, two heads] \& \mathrm{coker}(F) \end{tikzcd} \right\} \end{tikzcd}$$

topological phases (deformation classes) ↔ topological K-theory of

electron states ↔ Brillouin torus

[Kitaev (2009)]

Physics underlying Topological Quantum Computation
Theory controlling Topological Quantum Computation

topological phases (deformation classes) ↔ topological K-theory of

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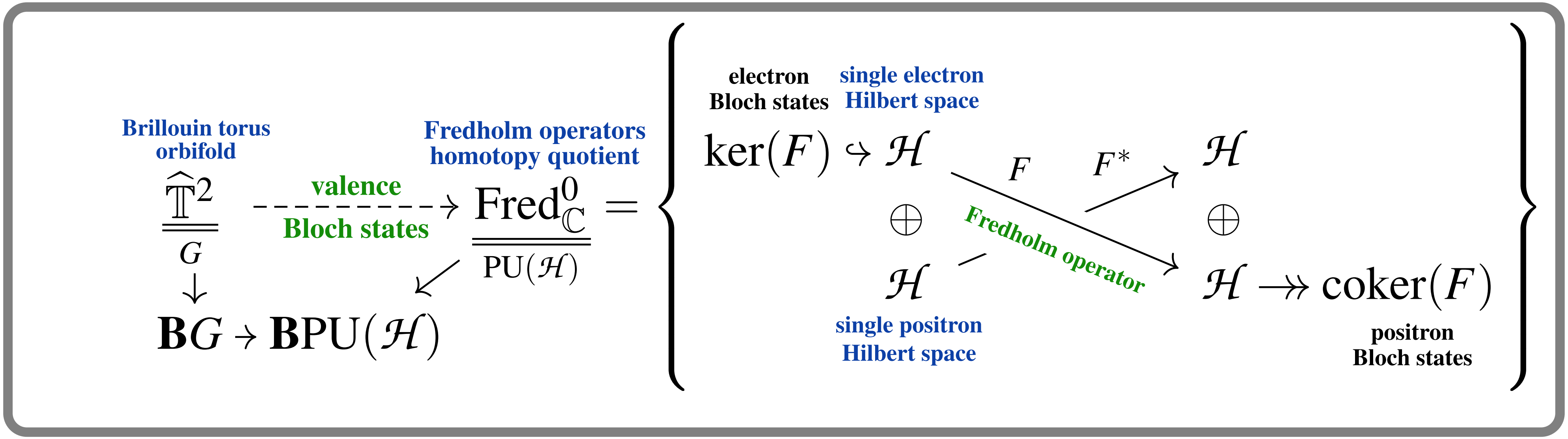
Physics Theory
underlying controlling
Topological Quantum Computation

quantum symmetries	↔	equivariant
topological phases	↔	topological
deformation classes	↔	K-theory of
electron states	↔	Brillouin torus

[Freed & Moore (2013)]

Physics Theory
underlying controlling
Topological Quantum Computation

quantum symmetries ↔ equivariant



[Freed & Moore (2013)]

Physics		**Theory**
underlying		**controlling**
Topological	**Quantum**	**Computation**

quantum symmetries	↔	equivariant
topological phases	↔	topological
deformation classes	↔	K-theory of
electron states	↔	Brillouin torus

[Freed & Moore (2013)]

Physics Theory
underlying controlling
Topological Quantum Computation

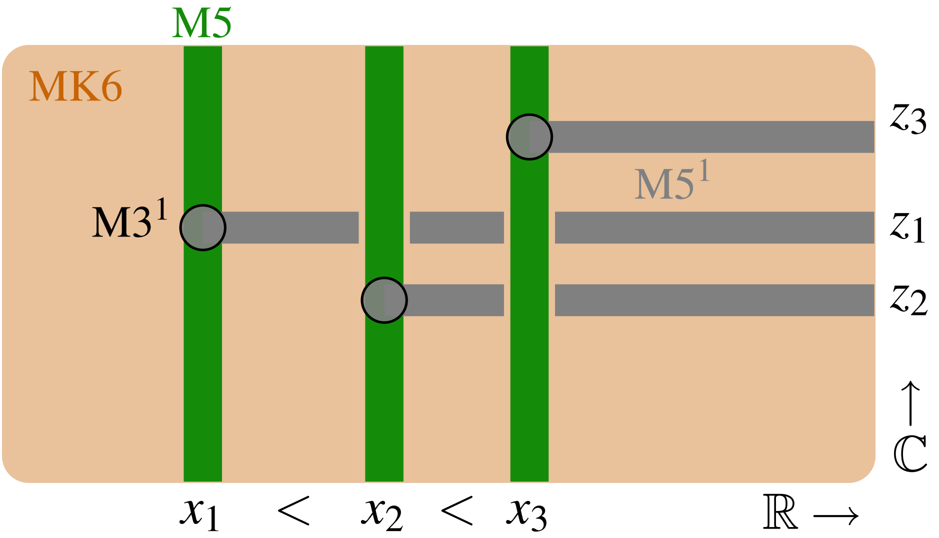
quantum symmetries ↔ equivariant

topological phases ↔ topological
deformation classes ↔ K-theory of
strongly interacting ↔ configurations in
electron states ↔ Brillouin torus

[Sati & Schreiber (2022a) (2022b)]

Physics Theory
underlying controlling
Topological Quantum Computation

$$\underset{\{1,\cdots,3\}}{\mathrm{Conf}}(\mathbb{C}) \;\simeq\; \left\{ \quad \right\}$$



strongly interacting ↔ configurations in

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[Sati & Schreiber (2022a) (2022b)]

Physics Theory
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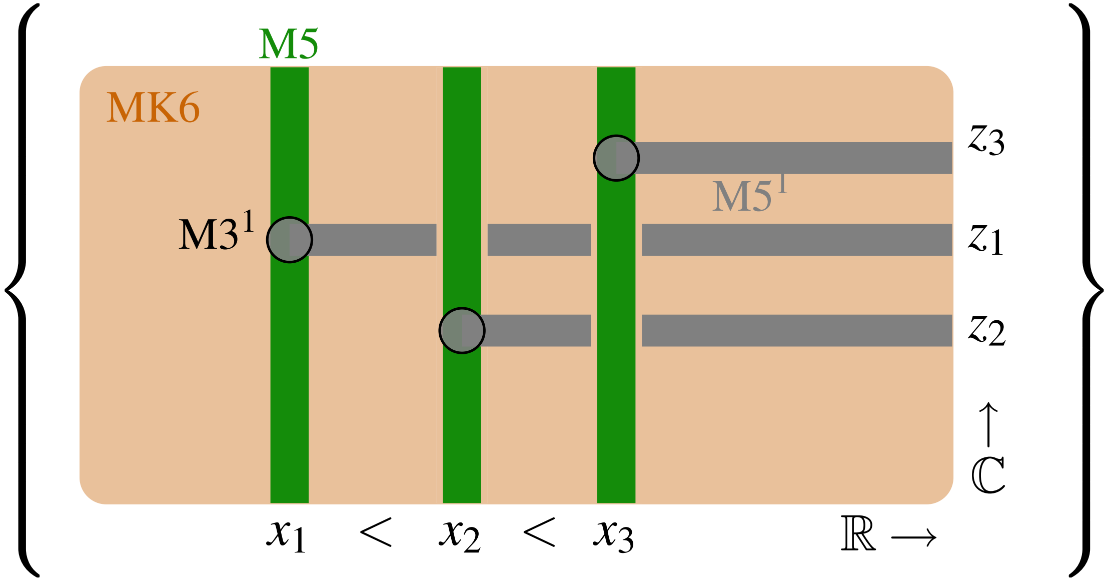

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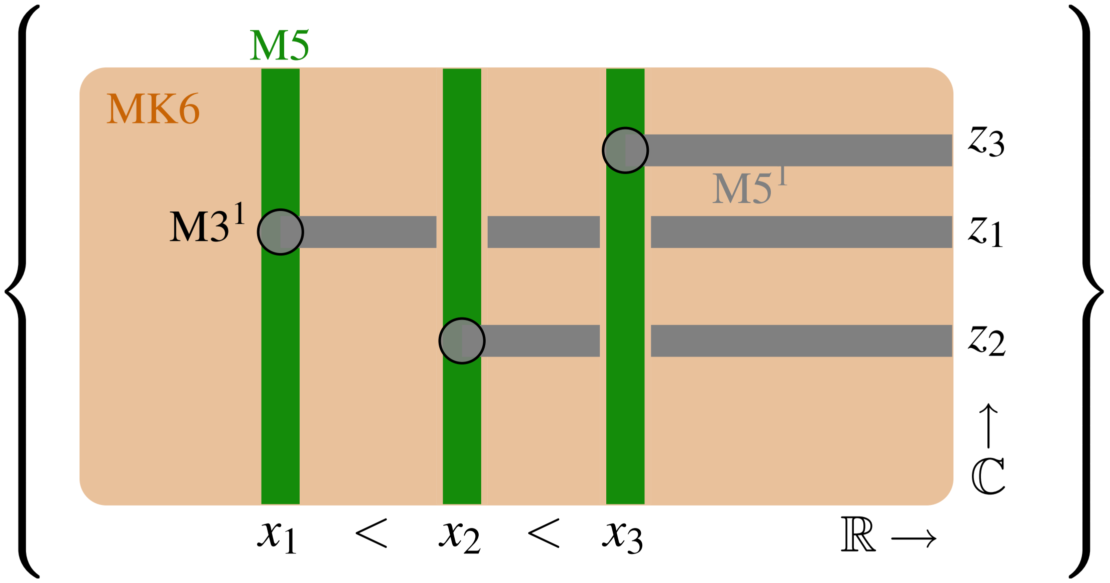

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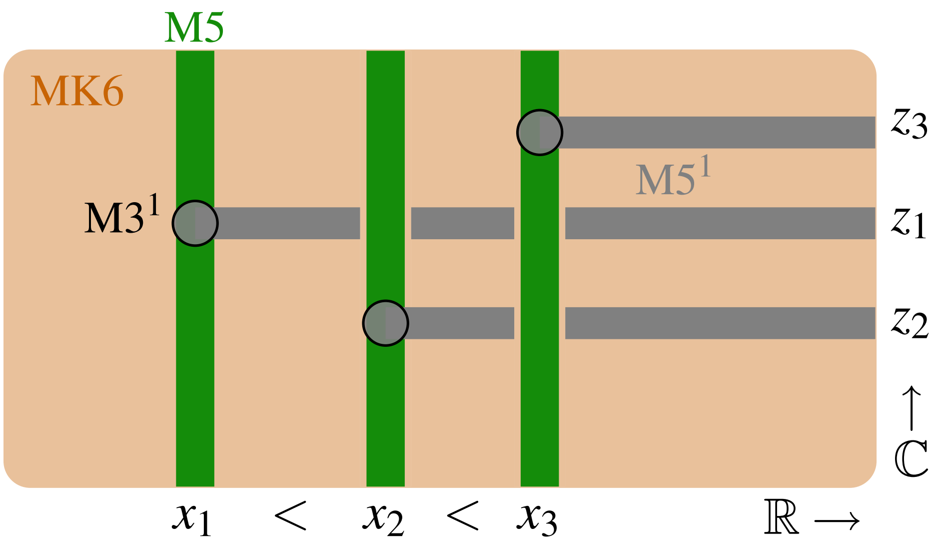

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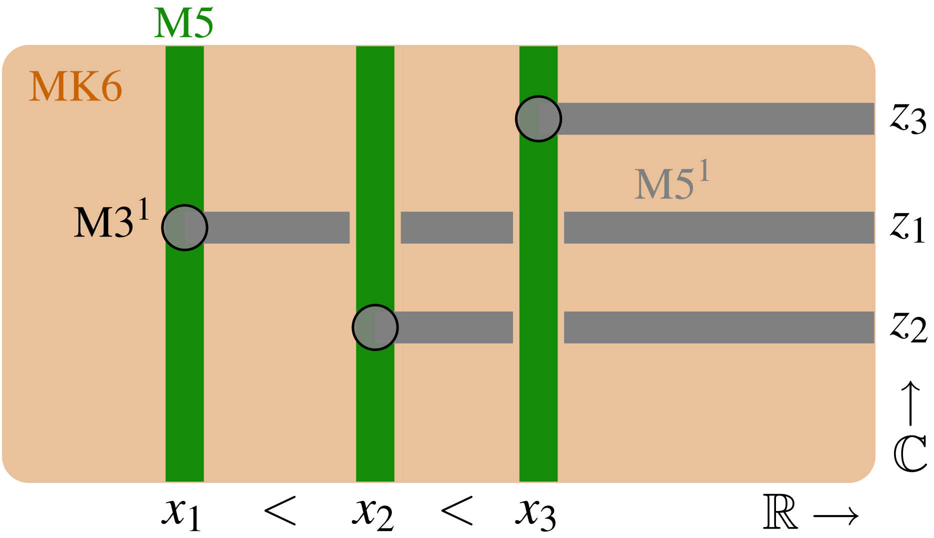

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Physics Theory

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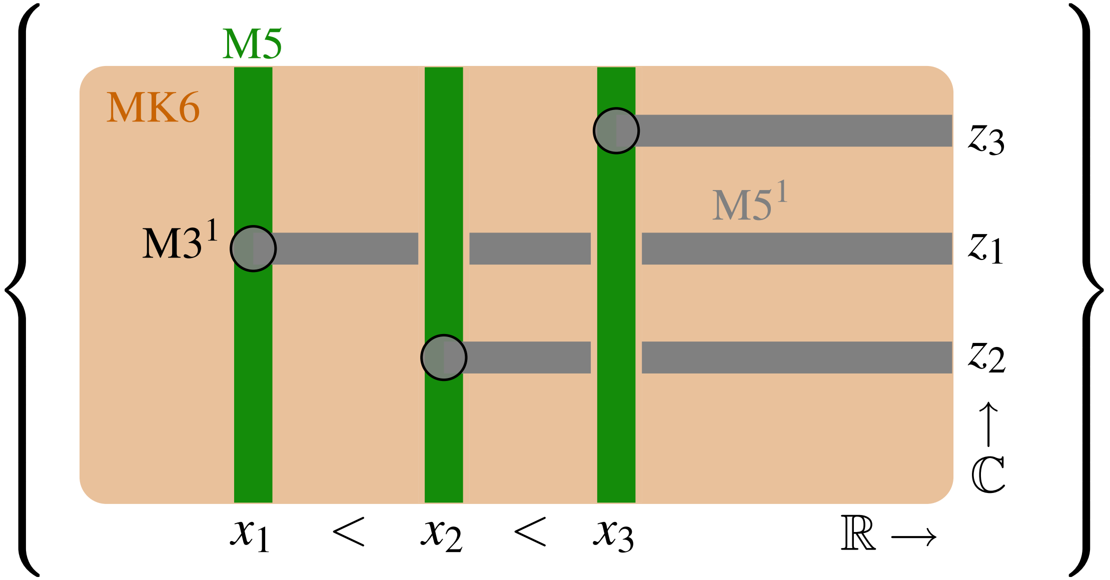

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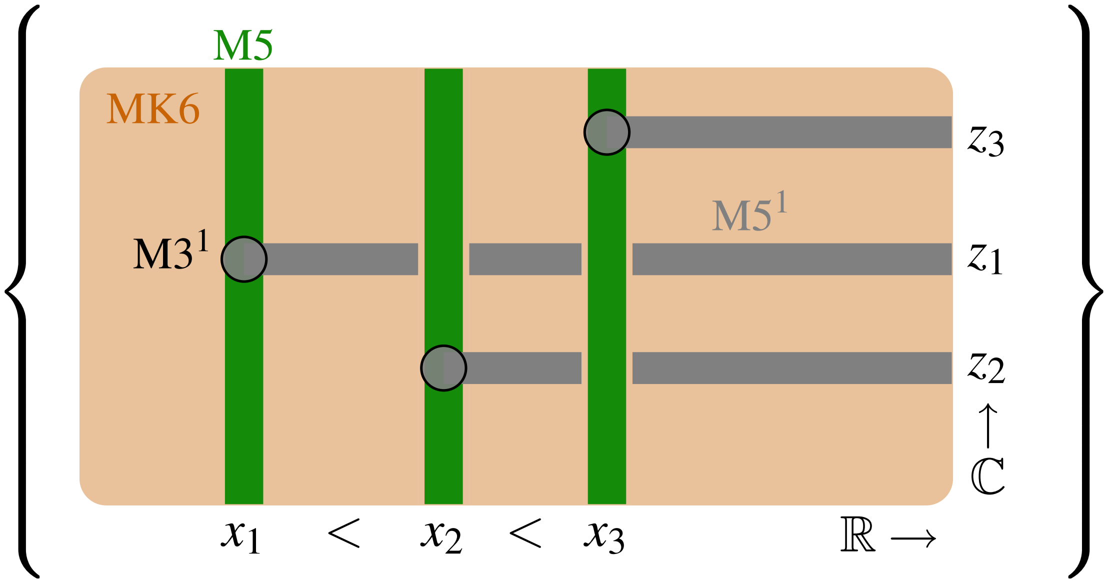

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[Sati & Schreiber (2022a) (2022b)]

Physics Theory
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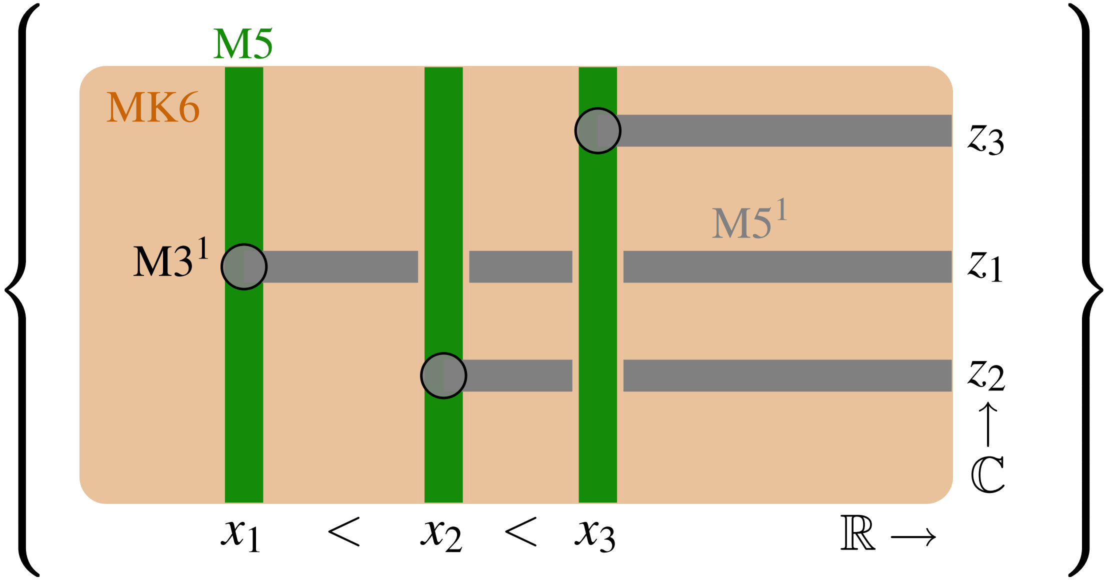

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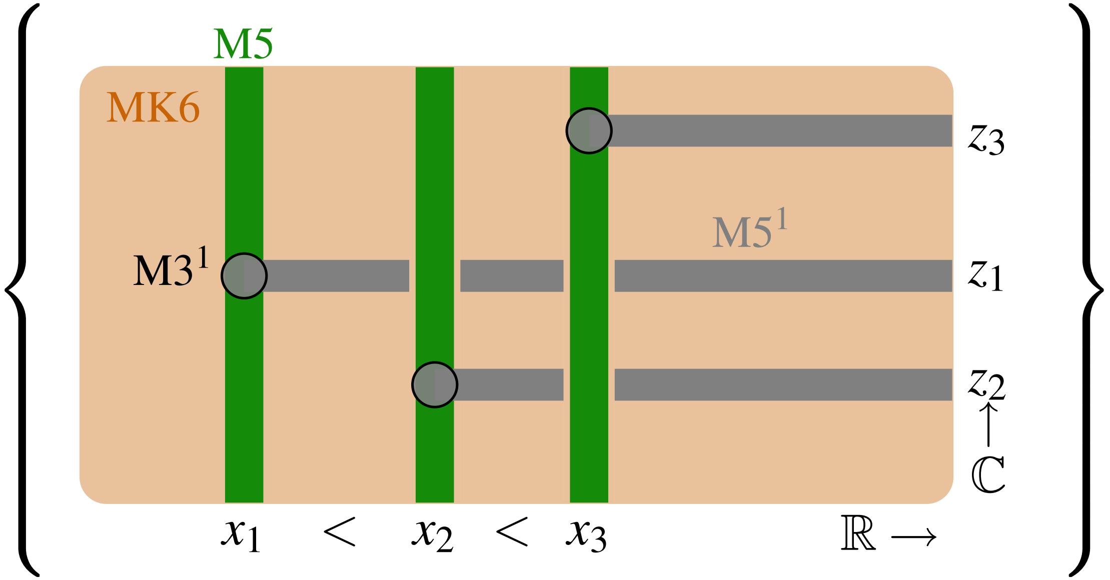

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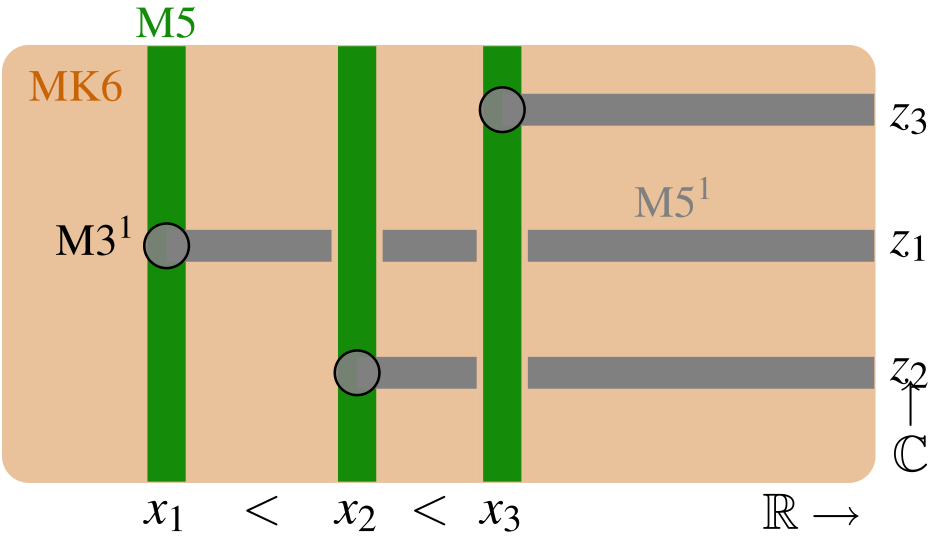

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Physics Theory
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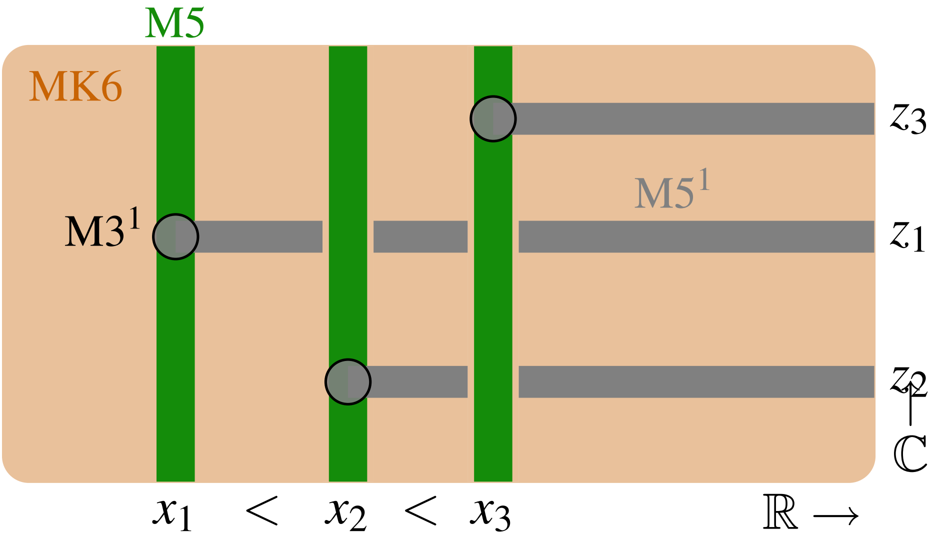

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Physics Theory
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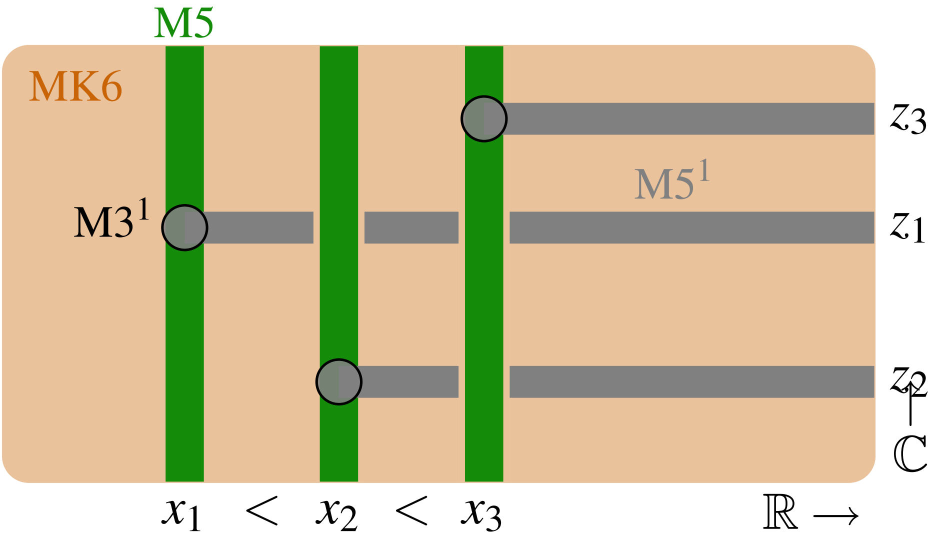


strongly interacting ↔ configurations in

electron states ↔ Brillouin torus

[Sati & Schreiber (2022a) (2022b)]

Physics Theory
underlying controlling
Topological Quantum Computation

quantum symmetries ↔ equivariant

topological phases ↔ topological
deformation classes ↔ K-theory of
strongly interacting ↔ configurations in
electron states ↔ Brillouin torus

[Sati & Schreiber (2022a) (2022b)]

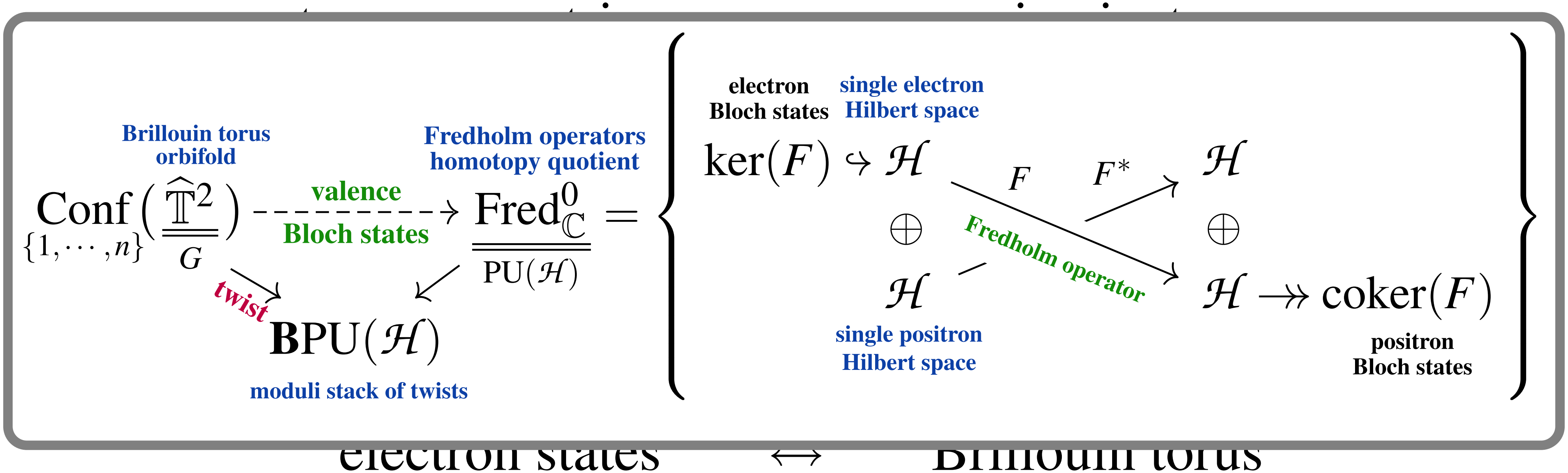
Physics Theory
underlying controlling
Topological Quantum Computation

anyon species	↔	twisted
quantum symmetries	↔	equivariant
anyon wavefunctions	↔	differential
topological phases	↔	topological
deformation classes	↔	K-theory of
strongly interacting	↔	configurations in
electron states	↔	Brillouin torus

[Sati & Schreiber (2022b)]

Physics underlying Topological Quantum Computation
Theory controlling Topological Quantum Computation

anyon species ↔ twisted



$$\begin{tikzcd}
\mathrm{Conf}_{\{1,\cdots,n\}}\Big(\widehat{\mathbb{T}}^2 \sslash G\Big) \arrow[rr, dashed, "\text{valence}", "\text{Bloch states}"'] \arrow[dr, "\text{twist}"'] & & \mathrm{Fred}^0_{\mathbb{C}} \sslash \mathrm{PU}(\mathcal{H}) \arrow[dl] & = \\
& \mathbf{B}\mathrm{PU}(\mathcal{H}) & &
\end{tikzcd}$$

$$\left\{\begin{tikzcd}
\ker(F) \arrow[r, hook] & \mathcal{H} \arrow[drr, "F", "\text{Fredholm operator}"'] & & \mathcal{H} \\
& \oplus & & \oplus \\
& \mathcal{H} \arrow[urr, "F^*"] & & \mathcal{H} \arrow[r, two heads] & \mathrm{coker}(F)
\end{tikzcd}\right\}$$

electron states ↔ Brillouin torus

[Sati & Schreiber (2022b)]

Physics Theory
underlying controlling
Topological Quantum Computation

anyon species ↔ twisted
quantum symmetries ↔ equivariant
anyon wavefunctions ↔ differential
topological phases ↔ topological
deformation classes ↔ K-theory of
strongly interacting ↔ configurations in
electron states ↔ Brillouin torus

[Sati & Schreiber (2022b)]

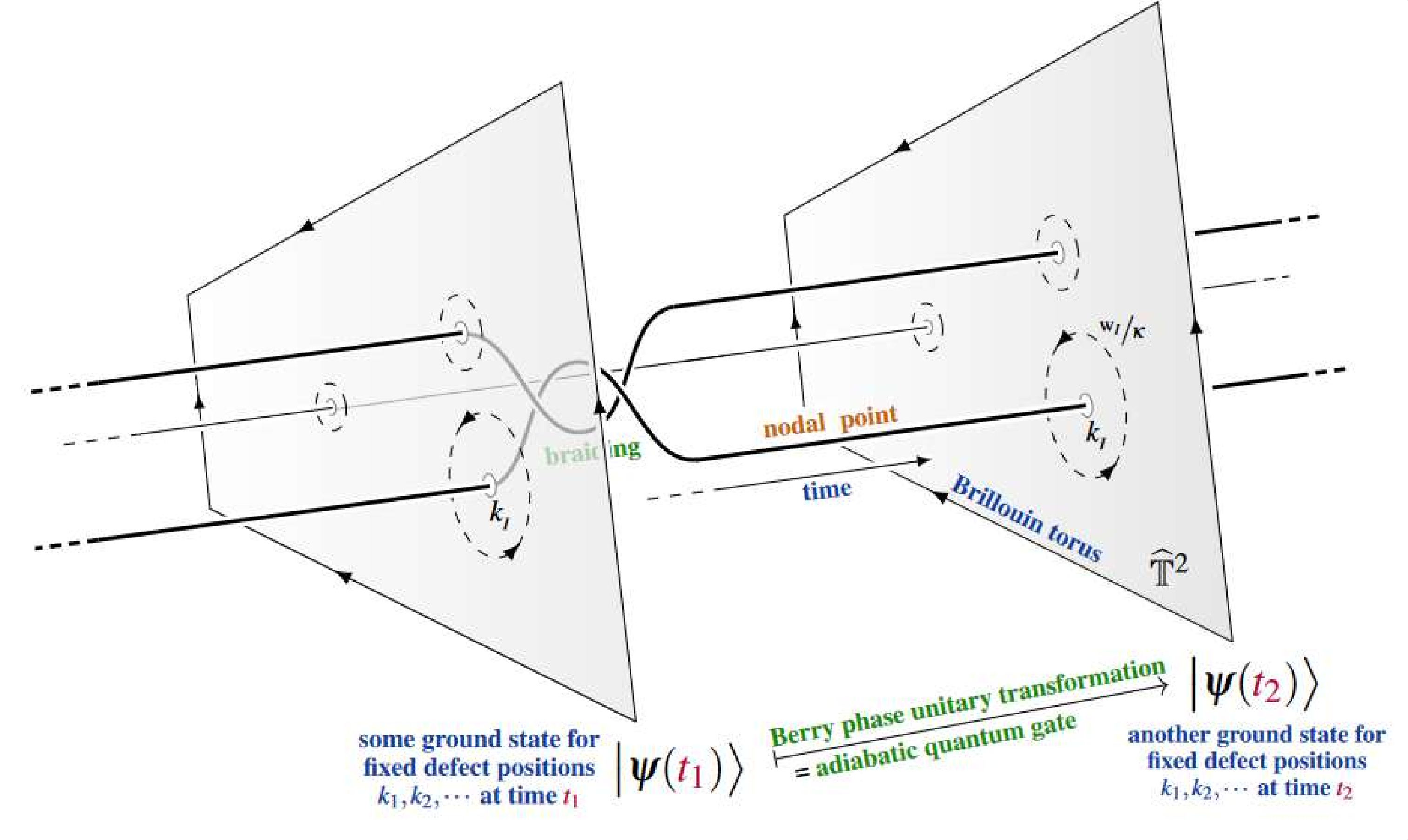
Physics Theory
underlying controlling
Topological Quantum Computation

anyon braiding	↔	GM-connection on
anyon species	↔	twisted
quantum symmetries	↔	equivariant
anyon wavefunctions	↔	differential
topological phases	↔	topological
deformation classes	↔	K-theory of
strongly interacting	↔	configurations in
electron states	↔	Brillouin torus

[Sati & Schreiber, PlanQC **2022** 33 (2022)]

Physics Theory
underlying controlling
Topological Quantum Computation

anyon braiding ↔ GM-connection on



Physics Theory
underlying controlling
Topological Quantum Computation

anyon braiding ↔ GM-connection on
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[Sati & Schreiber, PlanQC **2022** 33 (2022)]

Physics **Theory**
underlying **controlling**
Topological Quantum Computation

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electron states	↔	Brillouin torus

[Kitaev (2003)] [Freedman, Kitaev, Larsen & Wang (2003)]

Physics		**Theory**
underlying		**controlling**
Topological	**Quantum**	**Computation**
anyon braiding	↔	GM-connection on
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[Sati & Schreiber (2022a) (2022b) (2022c)]