Topological quantum states for quantum computing and metrology

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Quantum teleportation of Majorana Zero Modes

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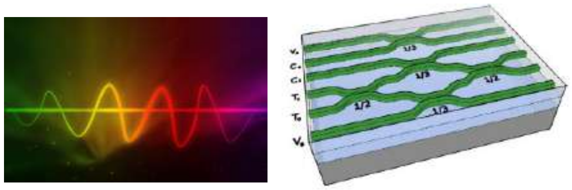
Huang, Narozniak, TB et al. Phys. Rev. Lett. **126**, 090502 (2021)

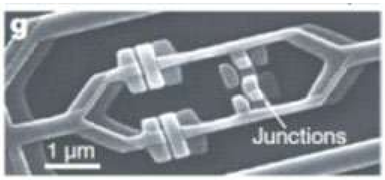
Quantum computing

Currently there are a handful of platforms for building quantum computers

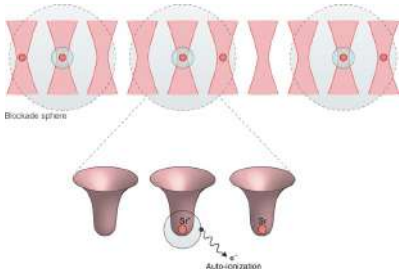
Optics



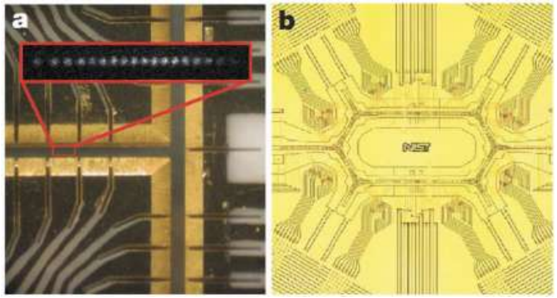
Superconducting qubits



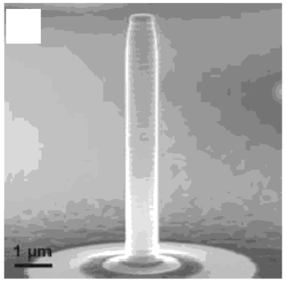
Cold atoms



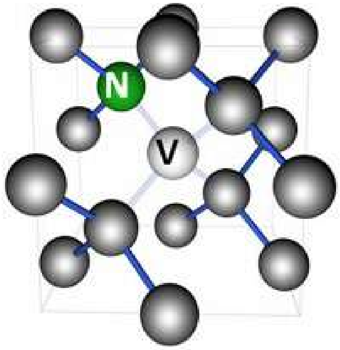
Ion traps



Quantum dots (semiconductors)

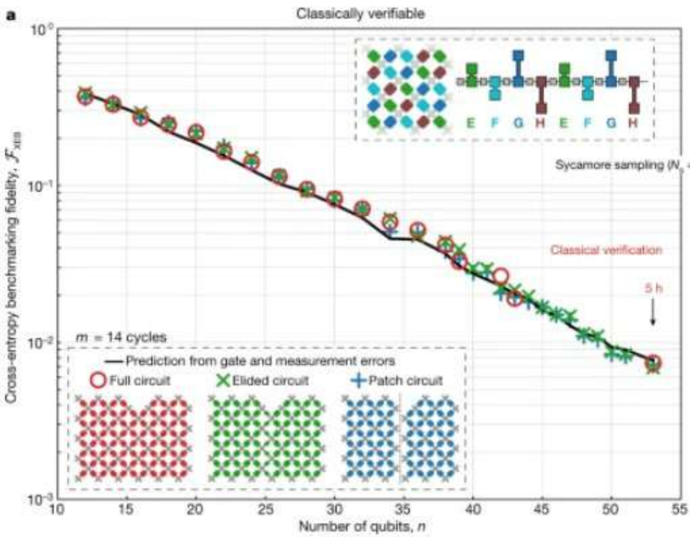


N-V centers (diamond)

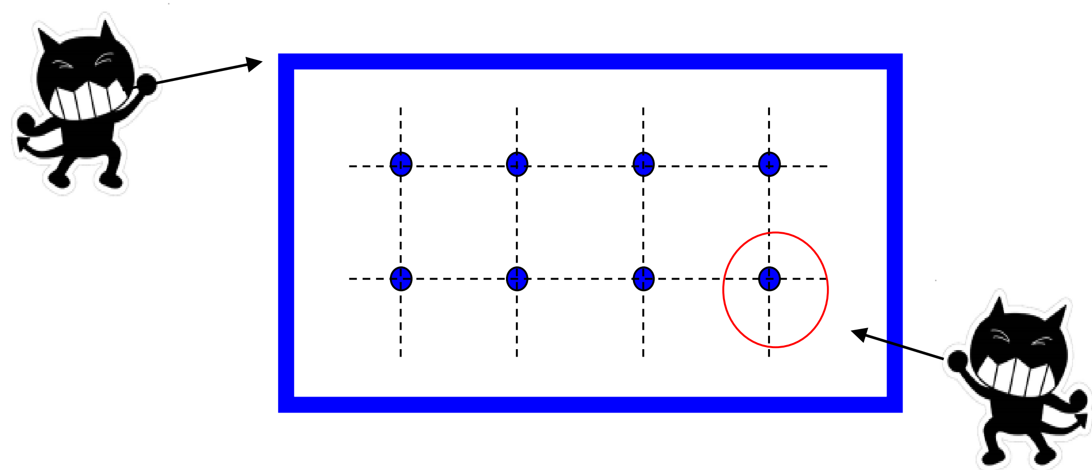


Quantum error correction

Decoherence is the primary reason why we cannot build large scale quantum computers.



Arute et al. *Nature* **574**, 505 (2019)

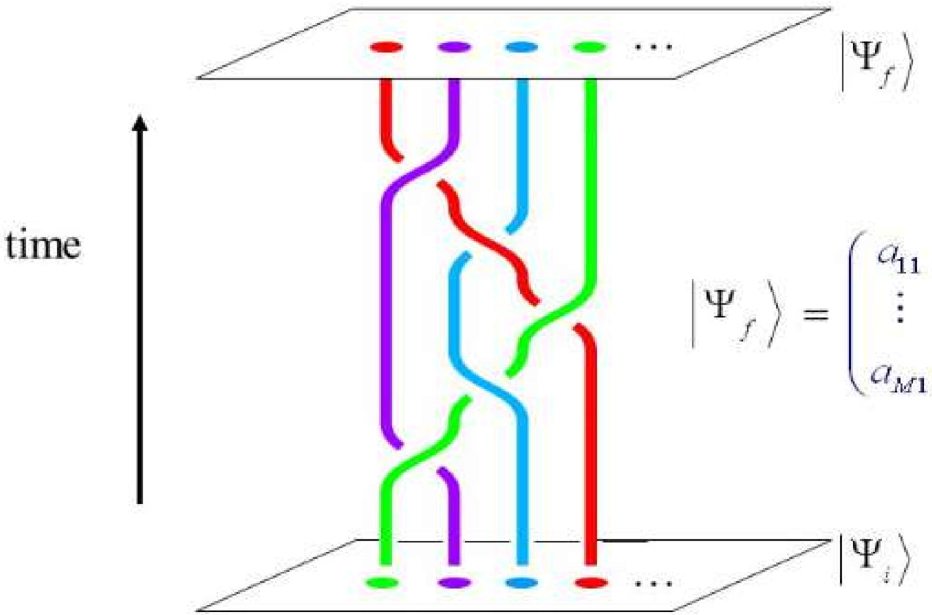


Quantum error correction is the standard path to get to large scale quantum computers.

Bit flip code: $|0\rangle \to |0_L\rangle \equiv |000\rangle$ and $|1\rangle \to |1_L\rangle \equiv |111\rangle$

Topological quantum computing

Is there a more robust way of performing quantum computing?



$$|\Psi_f\rangle = \begin{pmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{M1} & \cdots & a_{MM} \end{pmatrix} |\Psi_i\rangle$$

Alexei Kitaev, Caltech

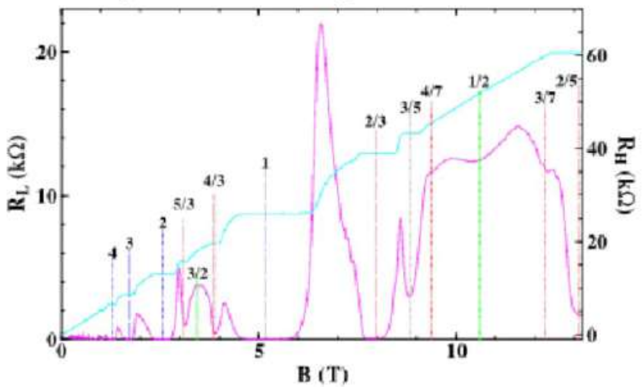
Quantum gates → Braids

Using anyons to store quantum information instead of qubits, the quantum information is more robust to perturbations, since it does not change the topological properties of the states.

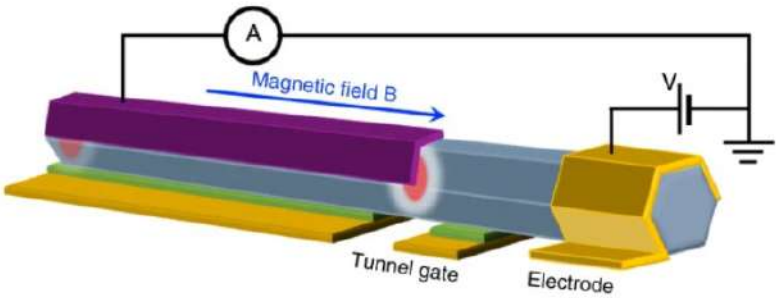
Topological quantum computing approaches

Physical anyonic systems

Integer and Fractional Quantum Hall Effects



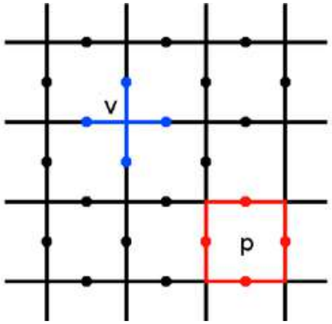
Semiconductor nanowire systems



Zhang, Nature Comm. 10, 5128 (2019)

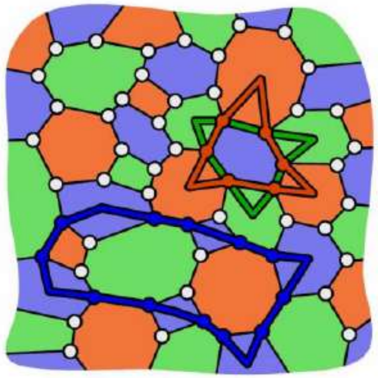
Artificial models

Toric codes



$$H = -\prod_{i=vertex} X_i - \prod_{i=plaquette} Z_i$$

Color codes



Kitaev chain

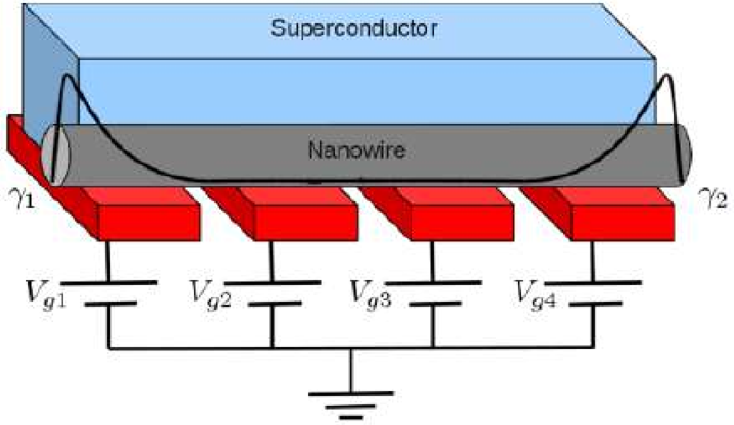
A model that possesses anyons is the Kitaev chain, defined as

$$H = -\mu \sum_n a_n^\dagger a_n - t \sum_n \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \Delta \sum_n \left(a_n a_{n+1} + a_{n+1}^\dagger a_n^\dagger \right)$$

Electron potential

Electron hopping

BCS superconducting terms



$$\{a_n, a_m\} = 0$$

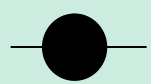
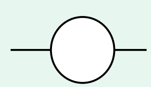
$$\{a_n, a_m^\dagger\} = \delta_{nm}$$

Example: one site

To see why the Kitaev chain has Majorana modes, first consider a single fermion with a Hamiltonian

$$H = \mu a^{\dagger} a$$

There are two states

Energy	Fermion	
μ	$a^{\dagger}\|0\rangle$	
0	$\|0\rangle$	

Example: one site

We can write this using Jordan Wigner transformation

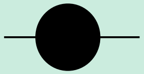
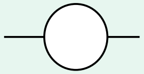
$$H = \mu\left(\sigma^z + I\right)/2$$

$$\sigma_j^+ = e^{\left(-i\pi \sum_{k=1}^{j-1} a_k^\dagger a_k\right)} \cdot a_j^\dagger$$

$$\sigma_j^- = e^{\left(+i\pi \sum_{k=1}^{j-1} a_k^\dagger a_k\right)} \cdot a_j$$

$$\sigma_j^z = 2a_j^\dagger a_j - I$$

There are two states

Energy	Fermion	Spin
μ	$a^\dagger \lvert 0\rangle$ 	$\lvert\uparrow\rangle$
0	$\lvert 0\rangle$ 	$\lvert\downarrow\rangle$

Example: one site

Yet another way to write this is using Majorana operators

$$\gamma_L = a + a^\dagger$$

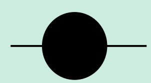
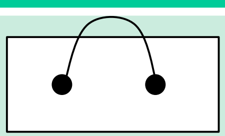
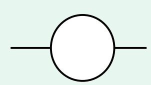
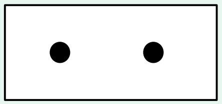
$$\gamma_R = -ia + ia^\dagger$$

$$a^\dagger = (\gamma_L - i\gamma_R)/2$$

$$\{\gamma_n, \gamma_{n'}\} = 2\delta_{nn'}$$

$$H = \mu\left(I + i\gamma_L\gamma_R\right)/2$$

There are two states

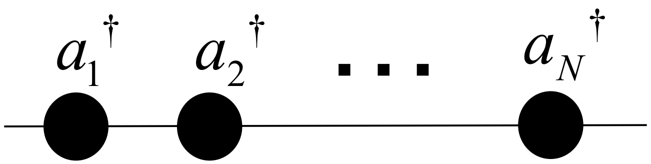
Energy	Fermion	Spin	Majorana
μ	$a^\dagger \lvert 0\rangle$ 	$\lvert\uparrow\rangle$	 $\frac{1}{2}(\gamma_L - i\gamma_R)\lvert 0\rangle$
0	$\lvert 0\rangle$ 	$\lvert\downarrow\rangle$	$\lvert 0\rangle$ 

Quasifermions

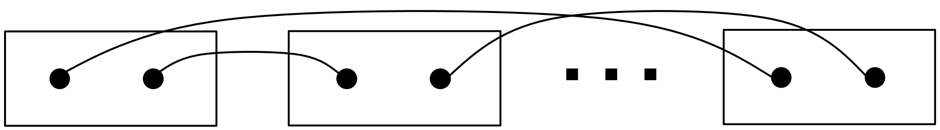
On a single site, writing fermions in terms of Majoranas is just a change of variables. But you can form new types of non-physical fermions using Majorana operators

$$f_{\alpha\beta}^{\dagger} = \frac{1}{2}(\gamma_\alpha - i\gamma_\beta)$$

Original Fermions



Majorana picture



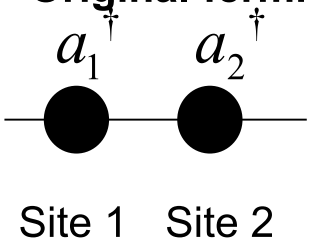
Such fermions obey fermion anti-commutation relations, as long as the pairs are unique

$$\left\{f_p^{\dagger}, f_{p'}\right\} = \delta_{pp'}$$

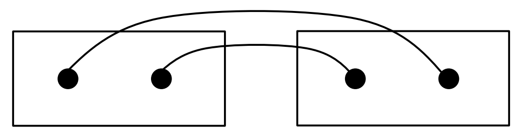
2 site system

For example for a 2 fermion system

Original fermions



Majorana picture

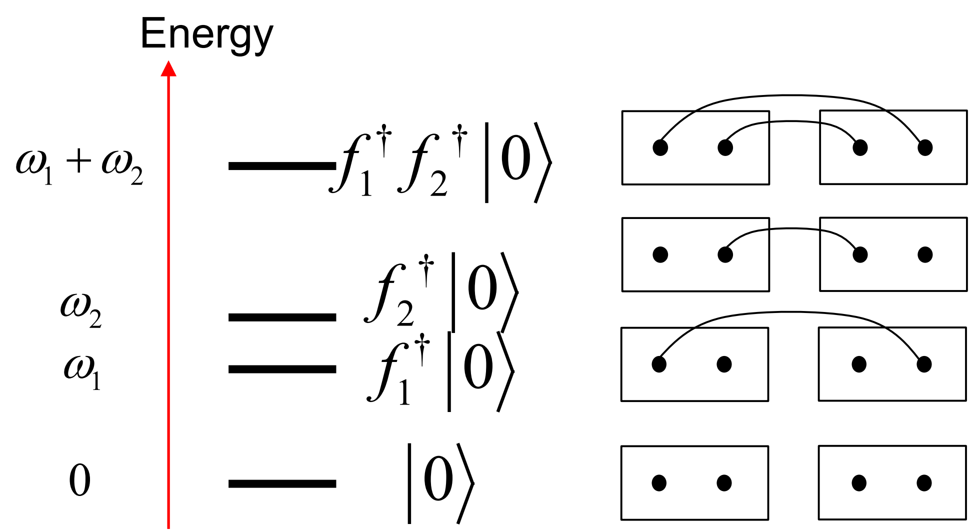


$$f_1^\dagger = \tfrac{1}{2}\left(\gamma_{1,L} - i\gamma_{2,R}\right)$$

$$f_2^\dagger = \tfrac{1}{2}\left(\gamma_{1,R} - i\gamma_{2,L}\right)$$

We can construct a Hamiltonian using the new quasifermions

$$H = \omega_1 f_1^\dagger f_1 + \omega_2 f_2^\dagger f_2$$

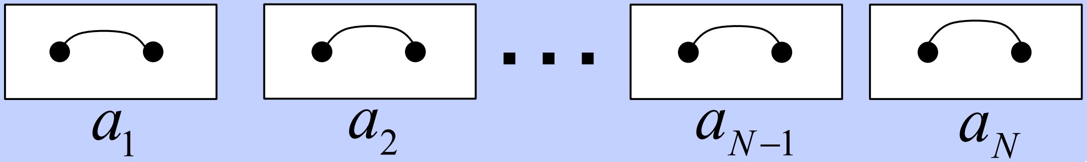


N site Kitaev chain

$$H = \mu \sum_{n=1}^{N} a_n^\dagger a_n - t \sum_{n=1}^{N-1} \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \Delta \sum_{n=1}^{N-1} \left(a_n a_{n+1} + a_{n+1}^\dagger a_n^\dagger \right)$$

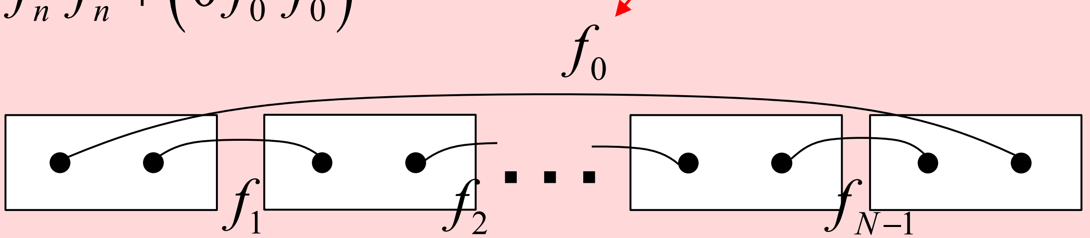
$$\mu > 0, \Delta = t = 0$$

$$H = \mu \sum_{n=1}^{N} \left(I + i\gamma_{n,L}\gamma_{n,R} \right) / 2 = \mu \sum_{n=1}^{N} a_n^\dagger a_n$$



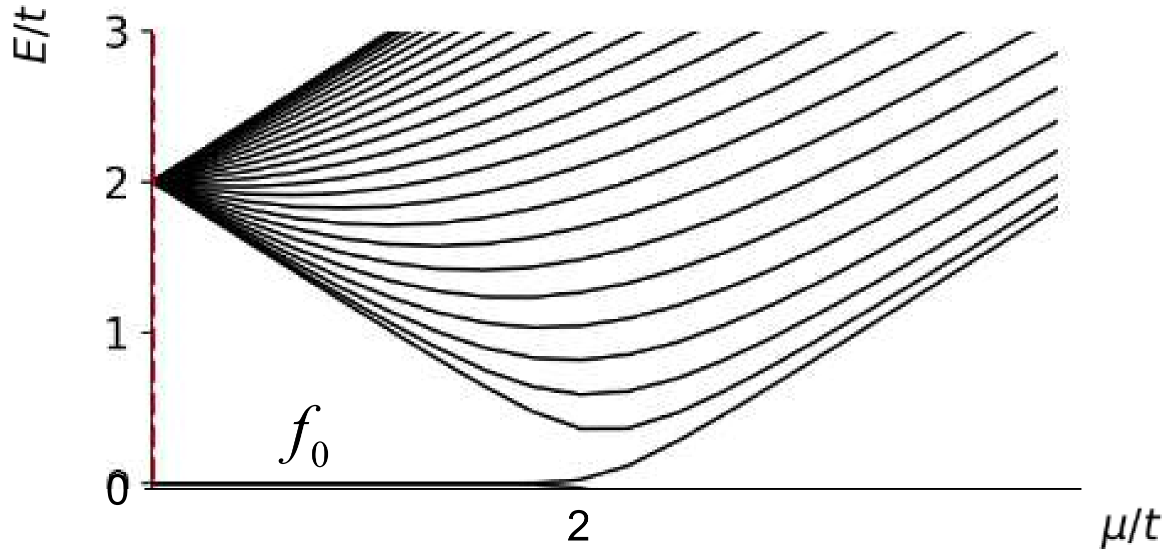
$$\mu = 0, \Delta = t$$

$$H = t \sum_{n=1}^{N-1} \left(I + i\gamma_{n,R}\gamma_{n+1,R} \right) = t \sum_{n=1}^{N-1} f_n^\dagger f_n + \left(0 f_0^\dagger f_0 \right)$$



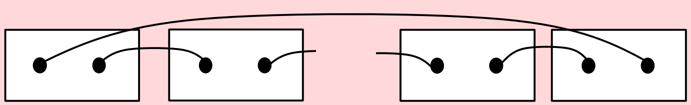
Spectrum of Kitaev chain

Energy coefficients of each mode



Topological phase

$$H = t\sum_{n=1}^{N-1}\left(I + i\gamma_{n,R}\gamma_{n+1,R}\right) = t\sum_{n=1}^{N-1} f_n^\dagger f_n + \left(0 f_0^\dagger f_0\right)$$



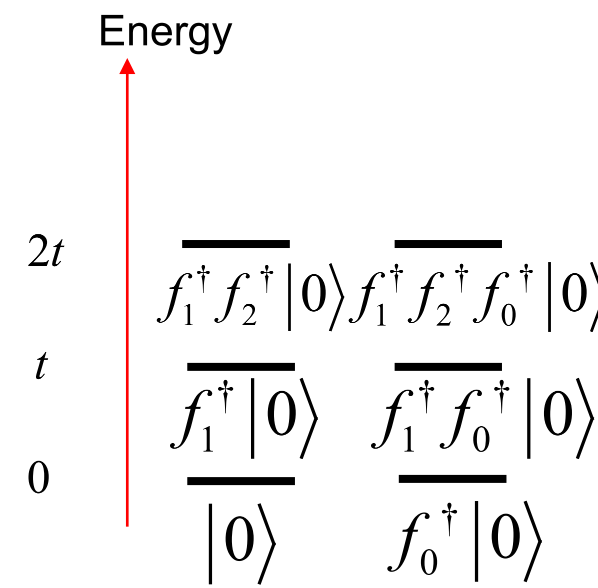
Normal phase

$$H = \mu\sum_{n=1}^{N}\left(I + i\gamma_{n,L}\gamma_{n,R}\right)/2 = \mu\sum_{n=1}^{N} a_n^\dagger a_n$$

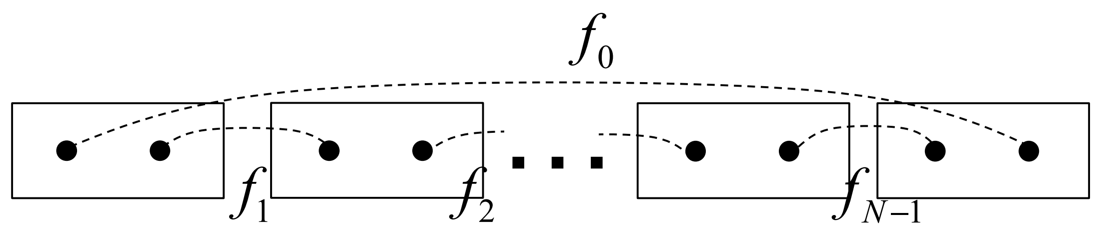
Logical states

The Majorana Zero modes are used as the logical states

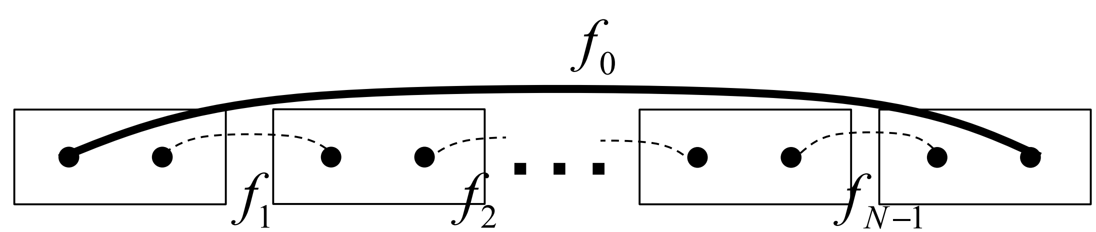
$$H = t\sum_{n=1}^{N-1}\left(I + i\gamma_{n,R}\gamma_{n+1,R}\right) = t\sum_{n=1}^{N-1} f_n^{\dagger} f_n + \left(0 f_0^{\dagger} f_0\right)$$



$$\left|0_L\right\rangle = \left|0\right\rangle$$

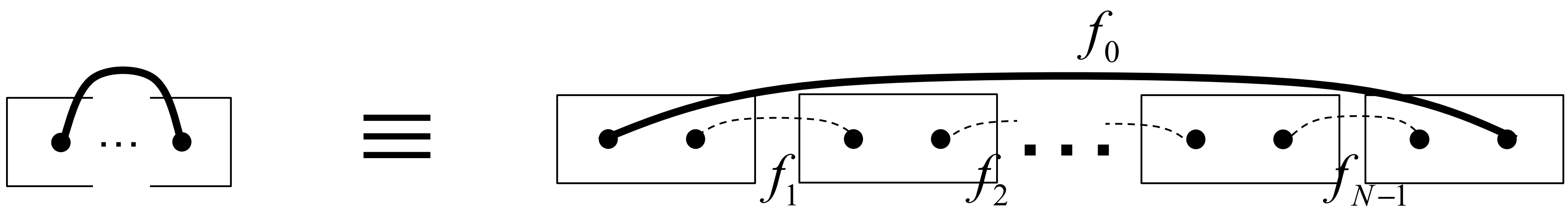


$$\left|1_L\right\rangle = f_0^{\dagger}\left|0\right\rangle$$



Multiple qubits

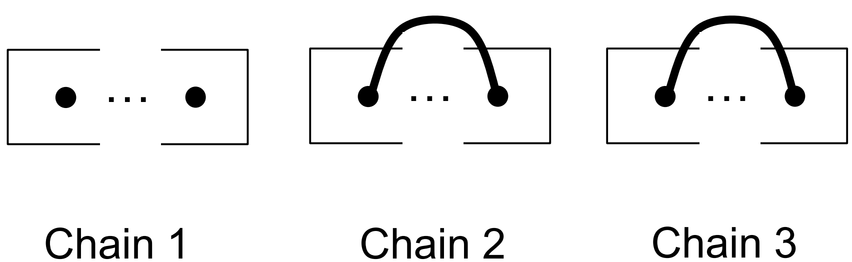
One Kitaev chain corresponds to a single qubit. Since only the edge modes matter, abbreviate



Then by using M chains we can make multiple qubits. A logical state is formed by either occupying the MZM modes

e.g.

$$|011_L\rangle$$



Braiding

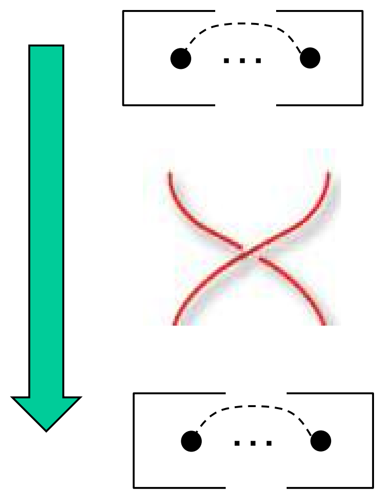
The Majorana modes can be braided using the operator

$$B_{nm} = \exp(\tfrac{\pi}{4}\gamma_n\gamma_m) = \frac{1}{\sqrt{2}}\left(I + \gamma_n\gamma_m\right)$$

The fact that these are braids can be found by evaluating

$$B_{nm}\gamma_n B_{nm}^{\dagger} = -\gamma_m$$

$$B_{nm}\gamma_m B_{nm}^{\dagger} = \gamma_n$$



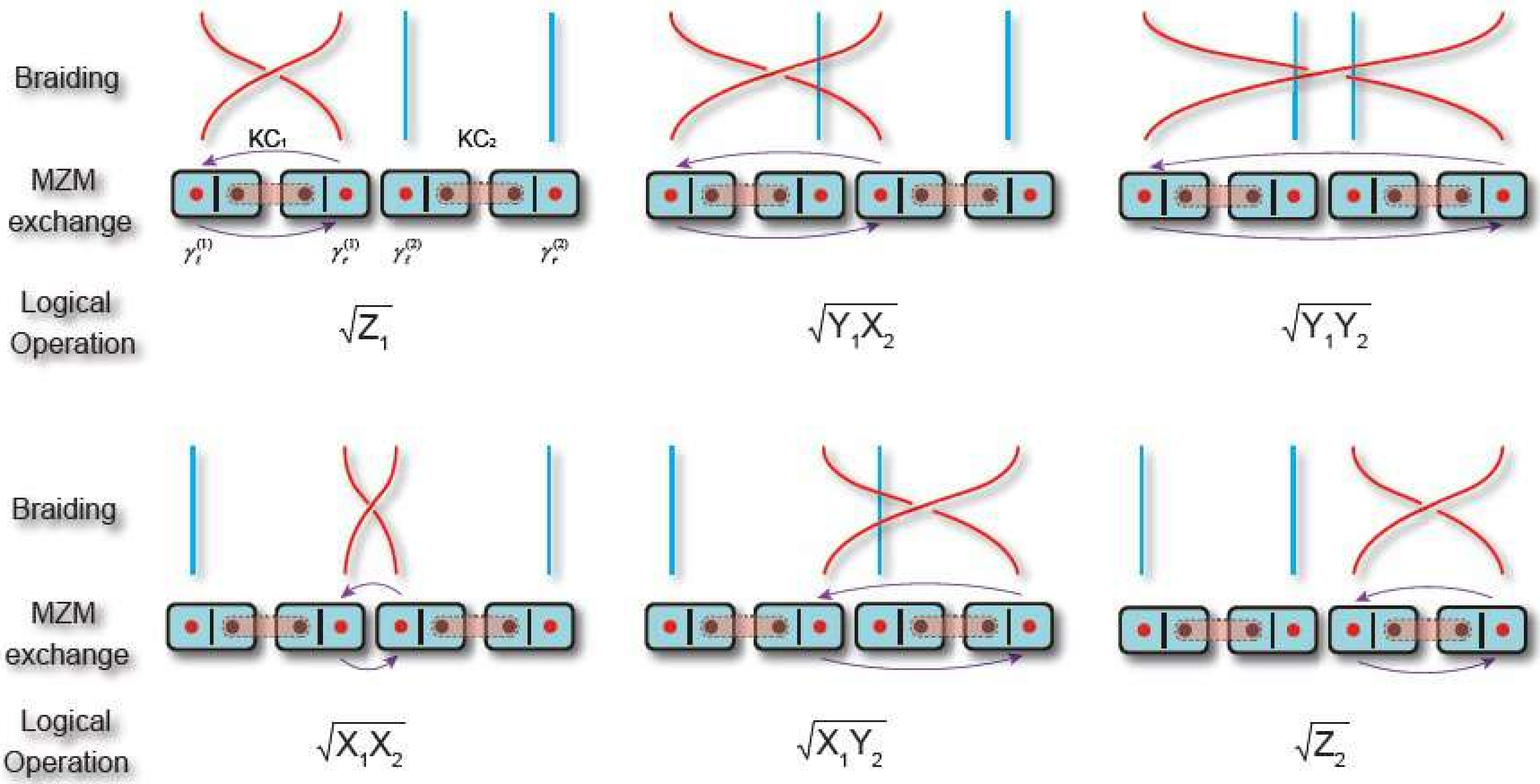
$$a\left|0_L\right\rangle + b\left|1_L\right\rangle$$

$$a'\left|0_L\right\rangle + b'\left|1_L\right\rangle$$

The braiding operation will in general change the quantum state on the chains

Logical gates for braids

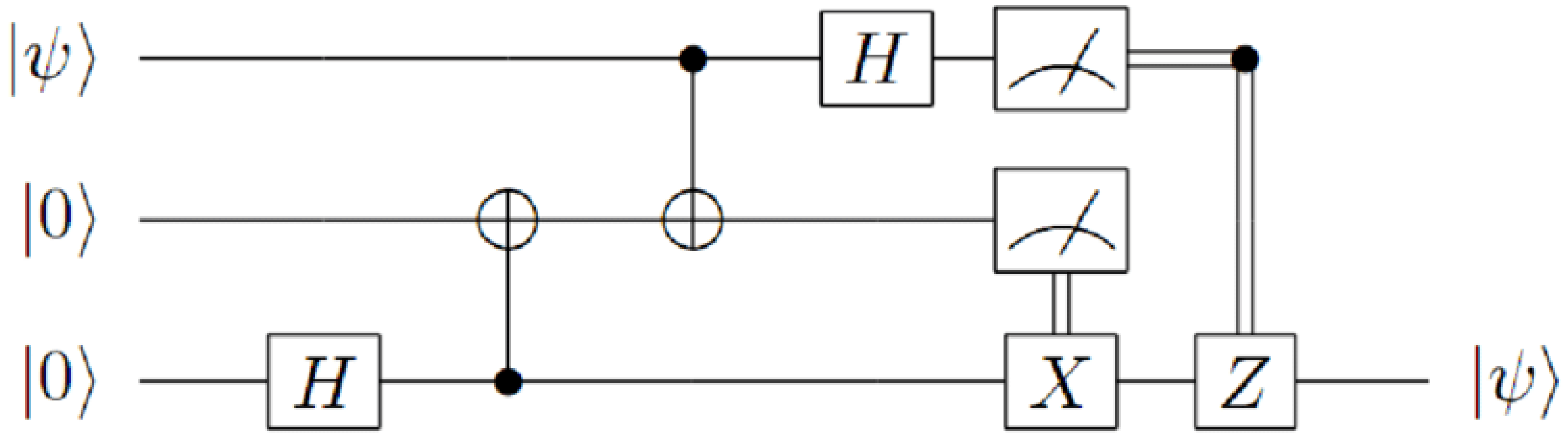
On 2 chains, there are a total of 6 possible braids



These are all Clifford operations

Quantum teleportation

Although Clifford operations are insufficient for universal quantum computing, and can be simulated efficiently on a classical computer (Gottesman-Knill theorem), they can still do something non-trivial.



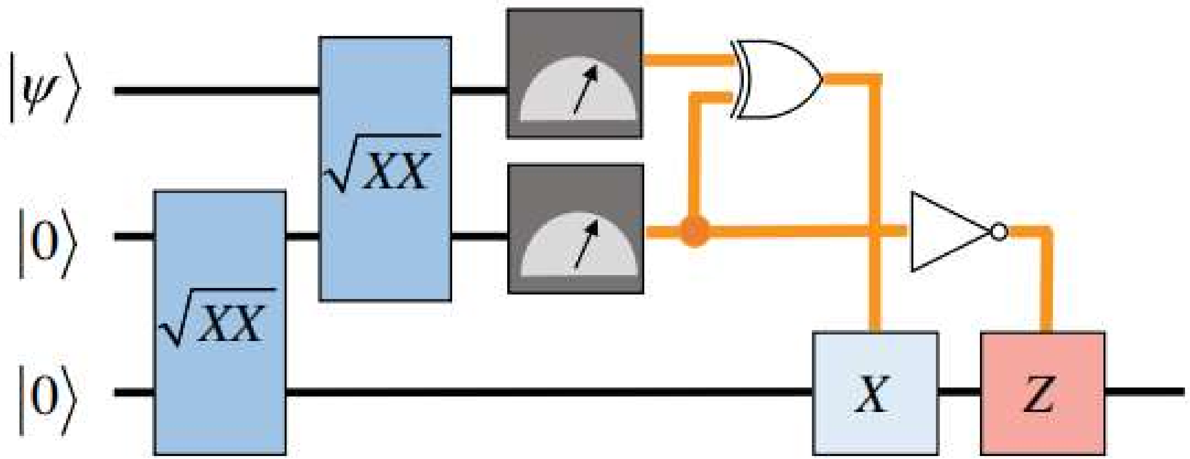
Bell state preparation

Bell measurement

Classical correction

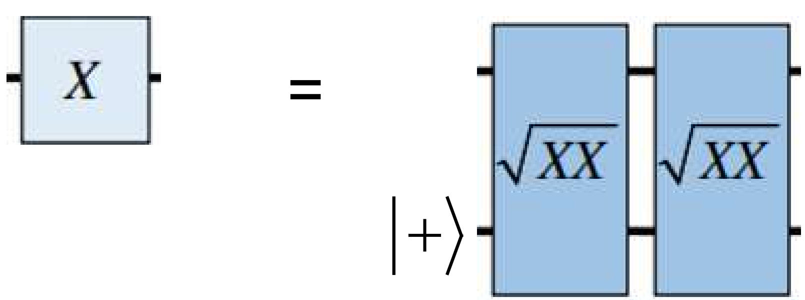
Teleportation with braid gates

Using the available gate operations we can make an alternative teleportation circuit

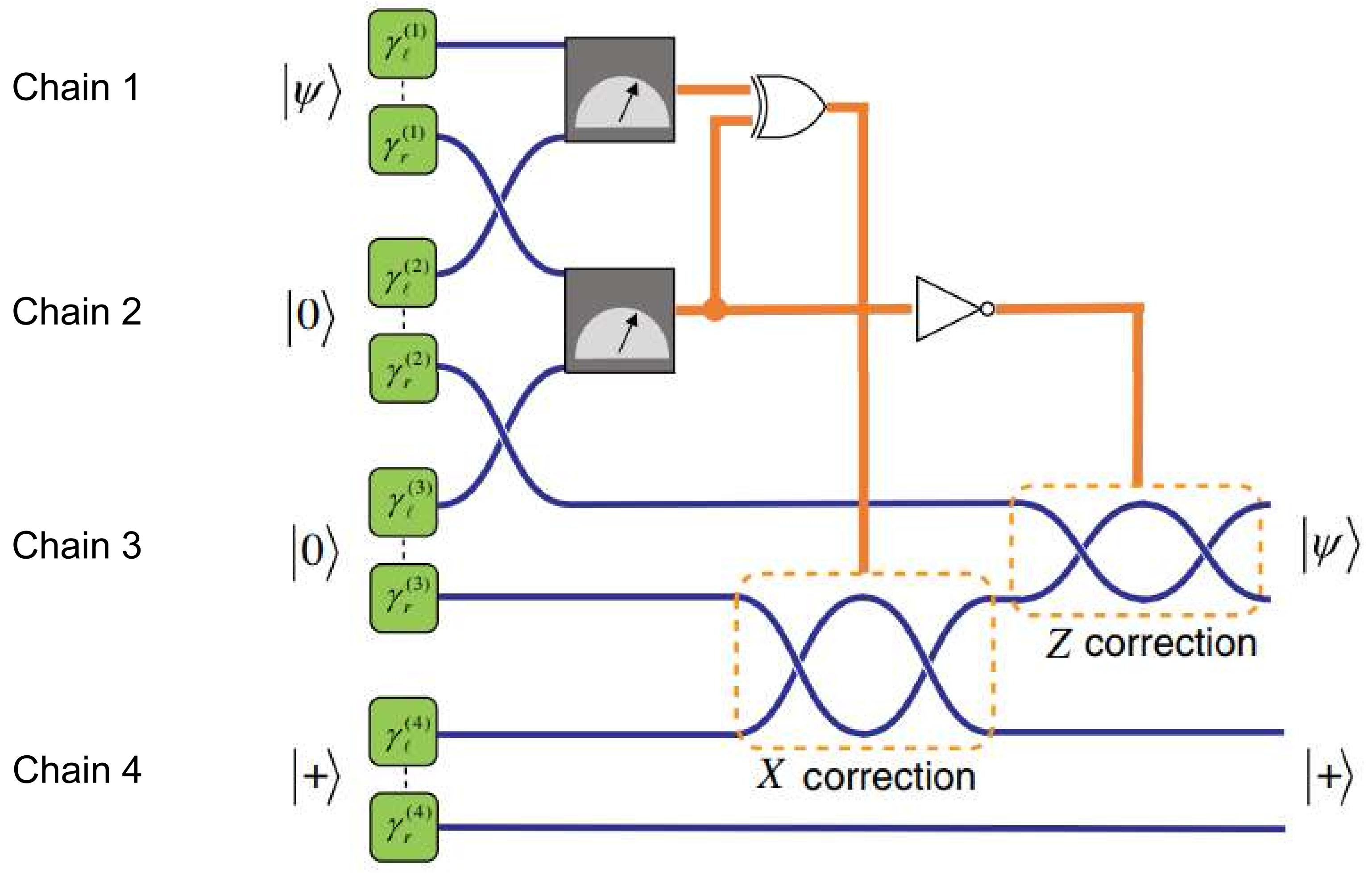


How to make X gate?

$$Z = \sqrt{Z}\sqrt{Z}$$



Teleportation by braiding



Performing this sequence allows a way of teleporting by braiding!

Quantum simulation of Kitaev chain

Since we do not have a physical Kitaev chain to perform braiding with, we can perform an equivalent sequence on a spin chain.

$$H = -\mu \sum_n a_n^\dagger a_n - t \sum_n \left(a_{n+1}^\dagger a_n + a_n^\dagger a_{n+1} \right) + \Delta \sum_n \left(a_n a_{n+1} + a_{n+1}^\dagger a_n^\dagger \right)$$

Jordan Wigner transformation

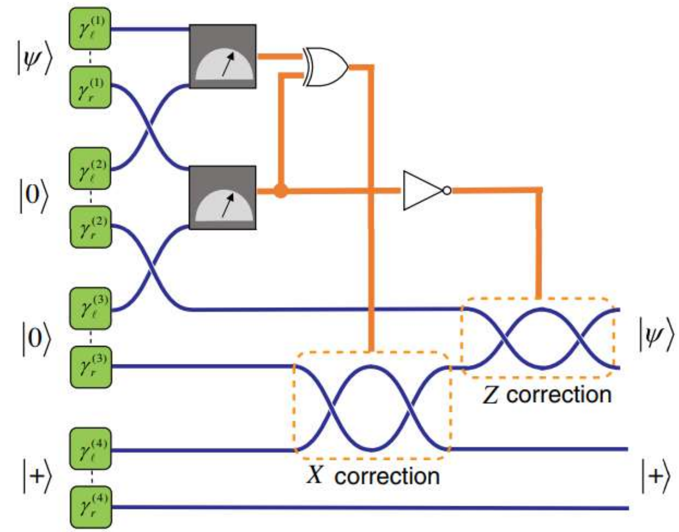
$$H = -\mu \sum_n \sigma_n^z - 2t \sum_n \sigma_n^x \sigma_{n+1}^x$$

The two logical states in the spin representation are

$$|0_L\rangle = \frac{1}{\sqrt{2}}(|+\cdots+\rangle + |-\cdots-\rangle)$$
$$|1_L\rangle = \frac{1}{\sqrt{2}}(|+\cdots+\rangle - |-\cdots-\rangle)$$

Encoder

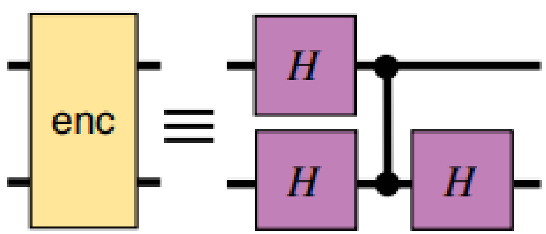
Our braiding teleportation circuit requires preparing particular types of state initially, then also making measurements in the logical basis.



The two qubit encoder

$|0\rangle$

$|\psi\rangle = a|0\rangle + b|0\rangle$

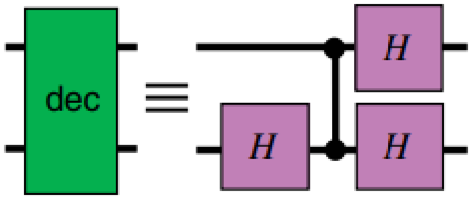


$$\frac{a}{\sqrt{2}}\left(|++\rangle + |--\rangle\right) + \frac{b}{\sqrt{2}}\left(|++\rangle - |--\rangle\right)$$

Decoder

The decoder is just the reverse of the encoder

$$\frac{a}{\sqrt{2}}\left(|++\rangle+|--\rangle\right)+\frac{b}{\sqrt{2}}\left(|++\rangle-|--\rangle\right)$$



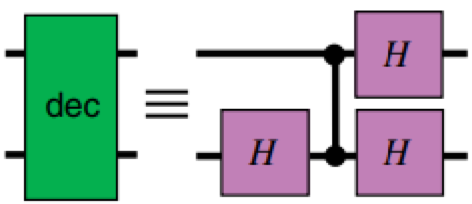
$|0\rangle$

$|\psi\rangle = a|0\rangle + b|0\rangle$

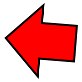
Apart from performing a quantum simulation, this has a practical benefit that it can detect errors!

e.g. Z error

$$\frac{a}{\sqrt{2}}\left(|-+\rangle+|+-\rangle\right)+\frac{b}{\sqrt{2}}\left(|-+\rangle-|+-\rangle\right)$$



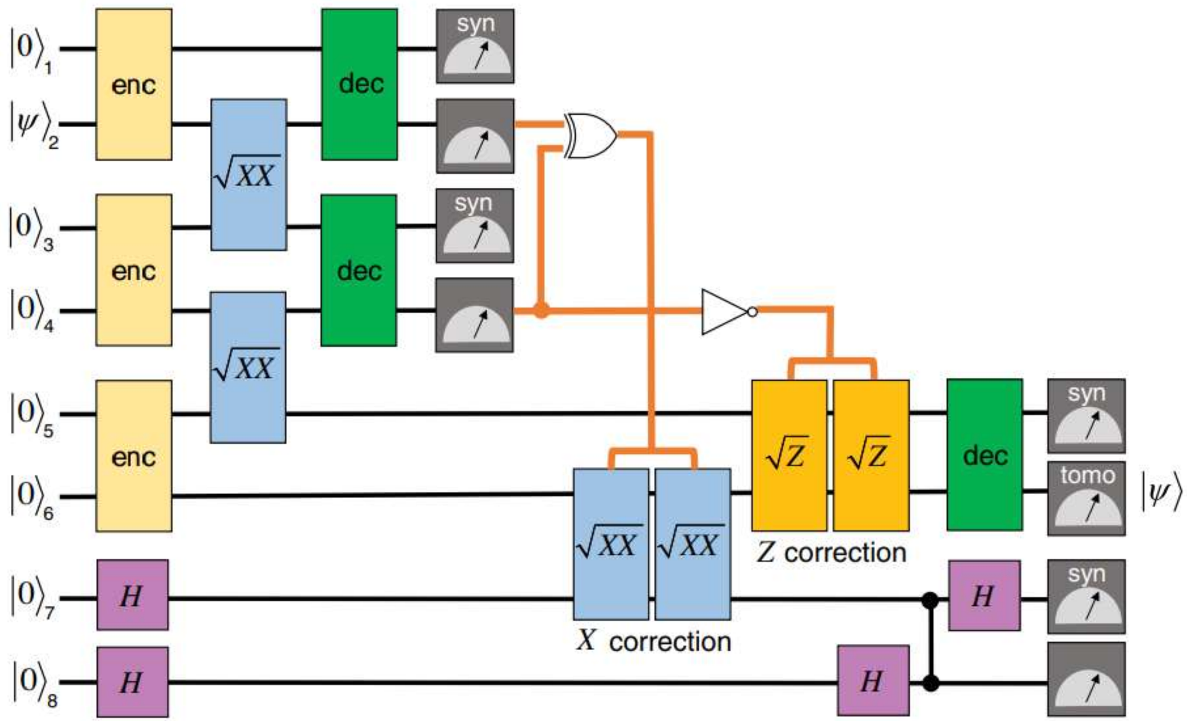
$|1\rangle$



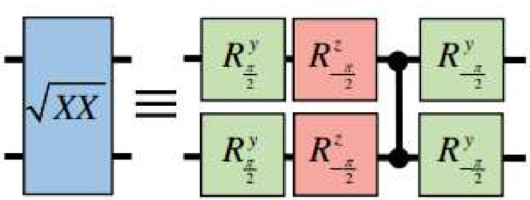
Error!

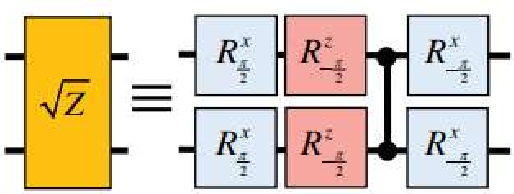
We can deal with such errors by discarding cases where errors are detected.

Qubit version of braiding circuit



where

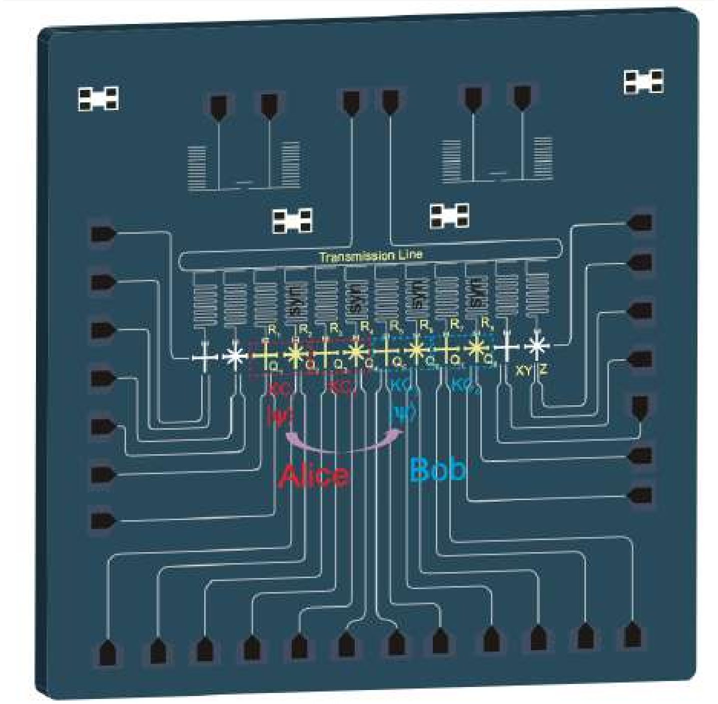




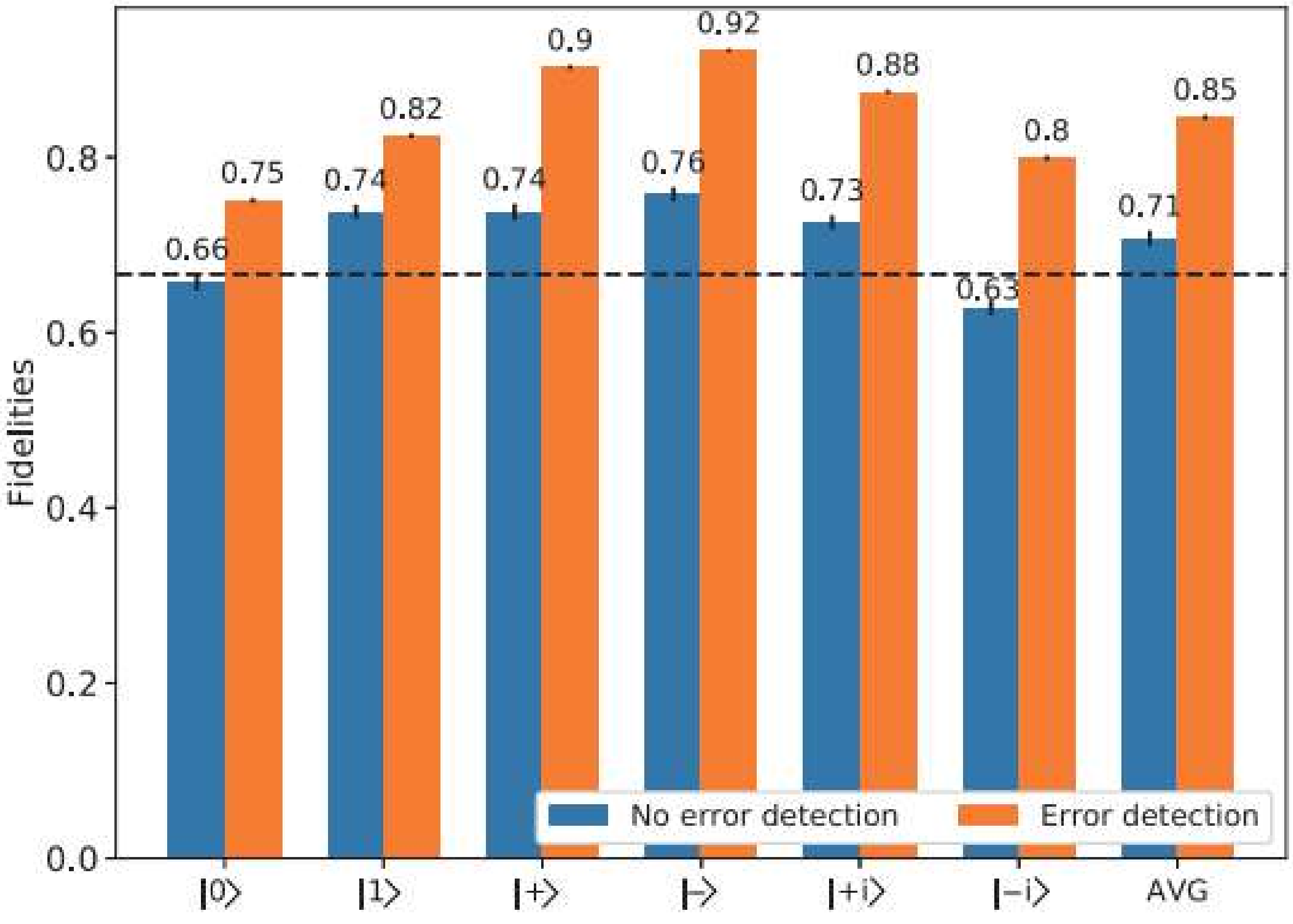
Superconducting quantum processor

Processor specification

- 12 superconducting Xmon qubits
- Arranged in line
- Fixed nearest-neighbor capacitive coupling



Experimental fidelities

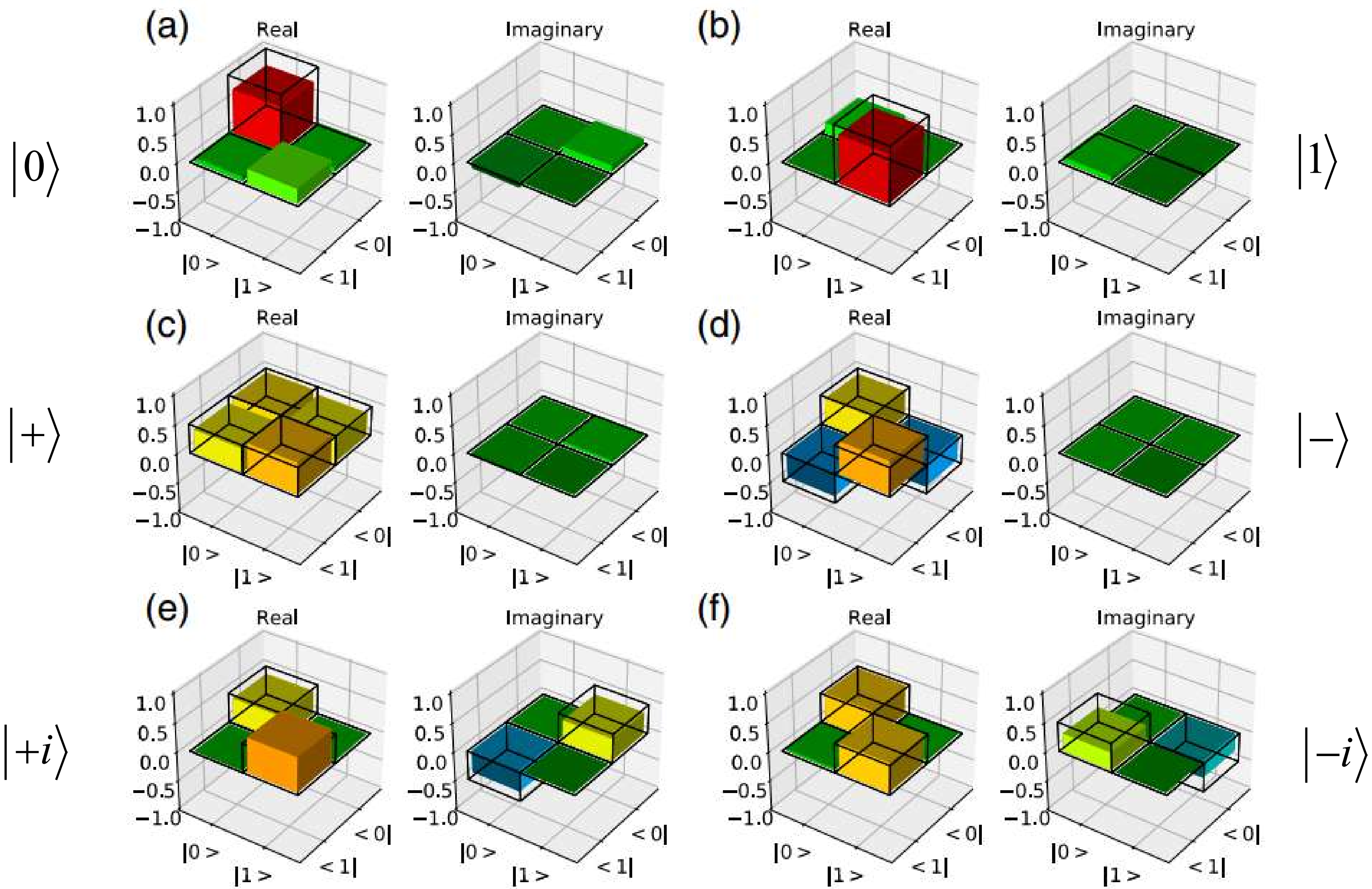


6 input states on average perform better than classical bound of F=2/3.

With error detection, superior performance is obtained.

Huang, Narozniak, TB et al. Phys. Rev. Lett. **126**, 090502 (2021)

Tomography



$|0\rangle$
$|1\rangle$
$|+\rangle$
$|-\rangle$
$|+i\rangle$
$|-i\rangle$
(a)
(b)
(c)
(d)
(e)
(f)
Real
Imaginary
1.0
0.5
0.0
−0.5
−1.0
|0 >
|1 >
< 0|
< 1|