Generalized global symmetries from string theory

Federico Bonetti

UNIVERSIDAD
DE MURCIA

f SéNeCa(+)
Agencia de Ciencia y Tecnología
Región de Murcia

based on 2112.02092 with Apruzzi, Garcia Etxebarria, Hosseini, Schafer-Nameki
2412.07842 with Del Zotto, Minasian

GTP Seminar — NYUAD — 19/02/2025

Introduction and motivation

- A key challenge in theoretical physics is to study QFTs in **strongly coupled regimes**
- We need tools beyond perturbative analysis
- We can aim to learn valuable lessons by studying **controlled examples**
- Analytic control can stem from a higher degree of **symmetry**
- In this talk: interplay between two ingredients to construct and study examples of strongly coupled models

Generalized global symmetries in QFT

Top-down construction of (S)QFTs in string theory/M-theory

Generalized global symmetries

- The textbook notion of global symmetry has been vastly generalized building on a key idea [Gaiotto, Kapustin, Seiberg, Willet 14]

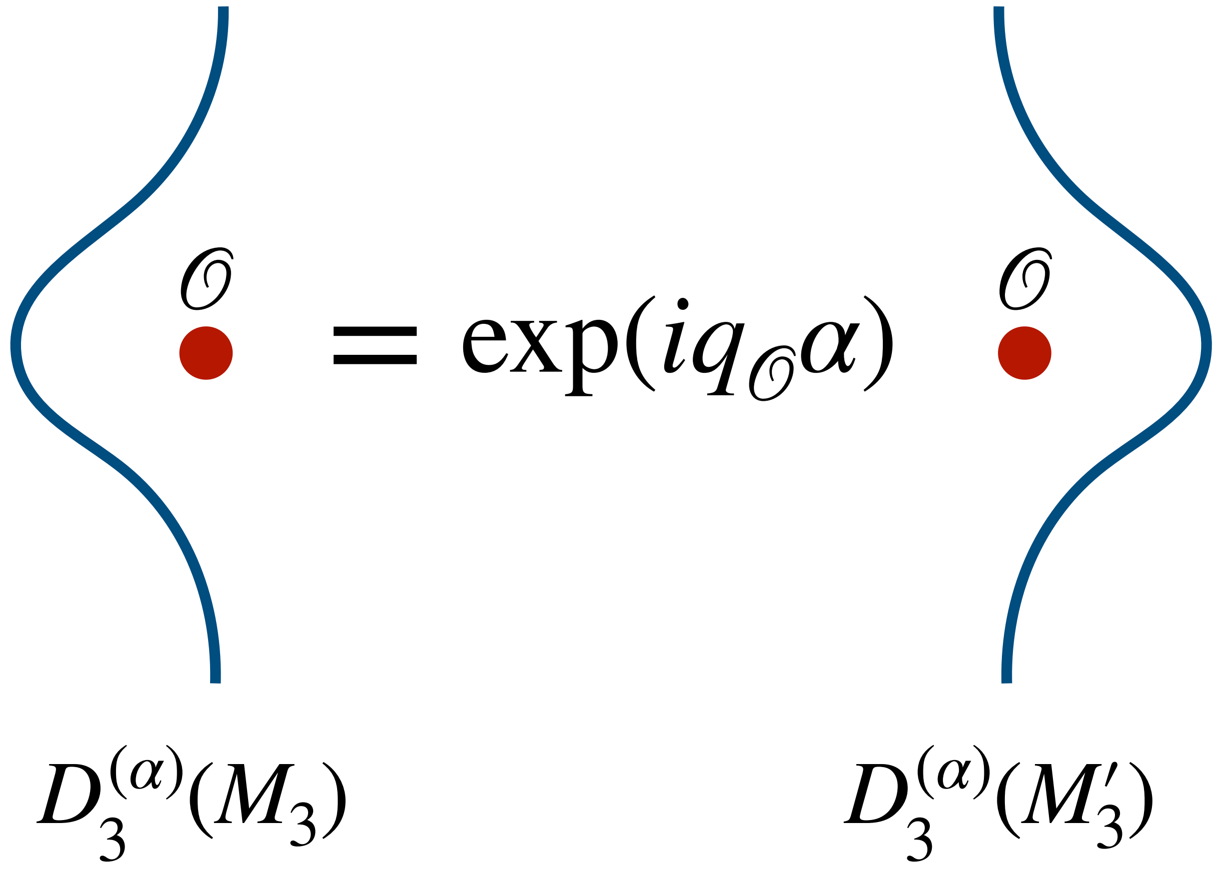
Global symmetries ↔ *Topological operators*

- Example: usual continuous $U(1)$ global symmetry (0-form symmetry)

conserved current $\partial_\mu j^\mu = 0 \;,\; d*j = 0 \quad \Rightarrow$ codimension-1 top. op. $D_3^{(\alpha)} = \exp i\alpha \int_{M_3} *j \;, \quad \alpha \in [0,2\pi)$

- In Minkowski spacetime we take M_3 to be a spatial slice at constant time.
  In a Wick-rotated setting, we can put our QFT on non-trivial manifolds and take any closed M_3

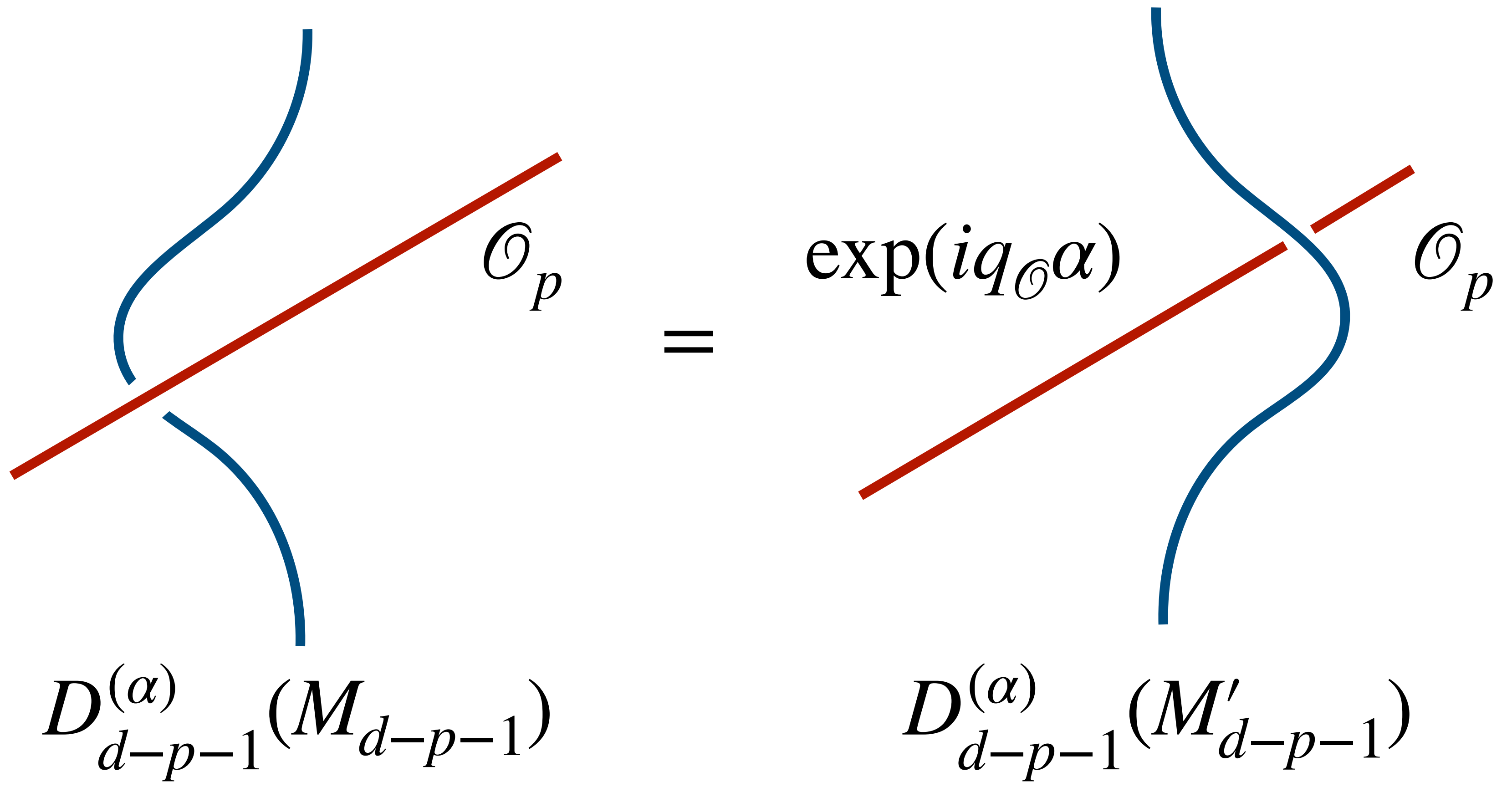
- If we sweep the topological operator past a local operator,
  we implement the symmetry action



Generalized global symmetries

[Gaiotto, Kapustin, Seiberg, Willet 14]

- **p-form symmetry**:
  - the topological operator has codimension $p+1$
  - the symmetry acts on operators of dimension p



- Example: $U(1)$ 1-form symmetries in 4d Maxwell theory (pure $U(1)$ gauge theory)

conserved electric 2-form current:	$* j_2^{\text{el}} \propto * F_2$	$d * j_2^{\text{el}} = 0$	by EOM	measures charge of Wilson line op.
conserved magnetic 2-form current:	$* j_2^{\text{mag}} \propto F_2$	$d * j_2^{\text{mag}} = 0$	by Bianchi id.	measures charge of 't Hooft line op.

- Example: center symmetry in pure non-Abelian $SU(N)$ gauge theory (1-form symmetry)

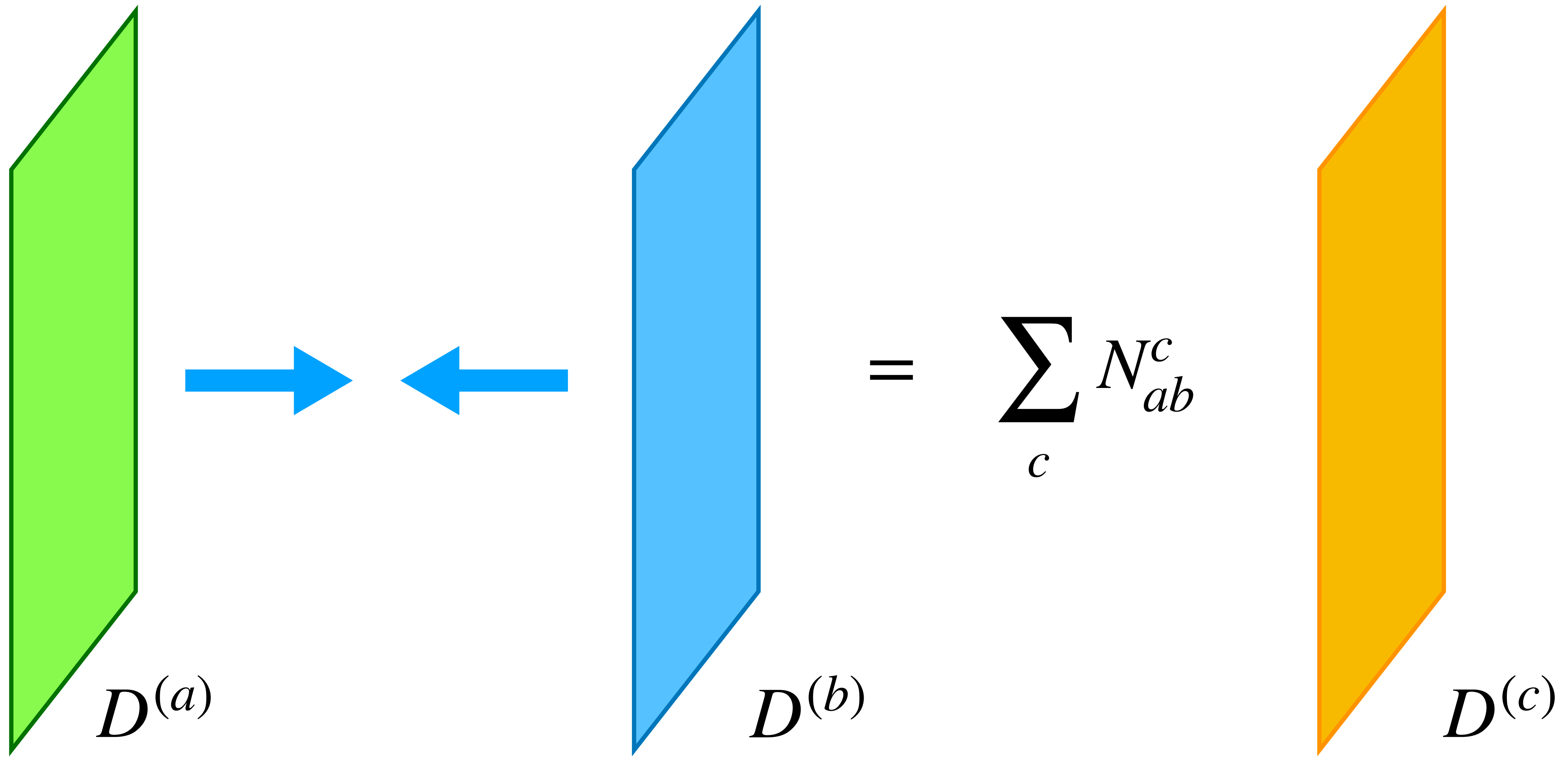
center$(SU(N)) = \mathbb{Z}_N$ Wilson line in rep. $\mathbf{R}$ has charge $= N$-ality of $\mathbf{R} \in \{0,1,\ldots,N-1\}$

Generalized global symmetries

The topological point of view allows for a rich variety of symmetry structures

- **Higher-groups**: non-trivial “mixtures” of two or more p-form symmetries with different p’s
- Instead of a group, we can have an **algebra**

fusion algebra of topological defects:



- In an algebra, elements can fail to have an inverse ⇒ **“non-invertible symmetry”**
- This structure is familiar from 2d QFTs (e.g. Verlinde lines in rational CFTs)
- More recently, non-invertible symmetries have been constructed in many non-trivial QFTs in various higher dimensions

e.g reviews [Cordova, Dumitrescu, Intriligator, Shao 22; McGreevy 22; Gomes 23; Schafer-Nameki 23; Brennan, Hong 23; Bhardwaj, Bottini, Fraser-Taliente, Gladden, Gould, Platschorre, Tillim 23; Shao 23; Carqueville, Del Zotto, Runkel 23]

Generalized global symmetries

Example: **2d Ising CFT**

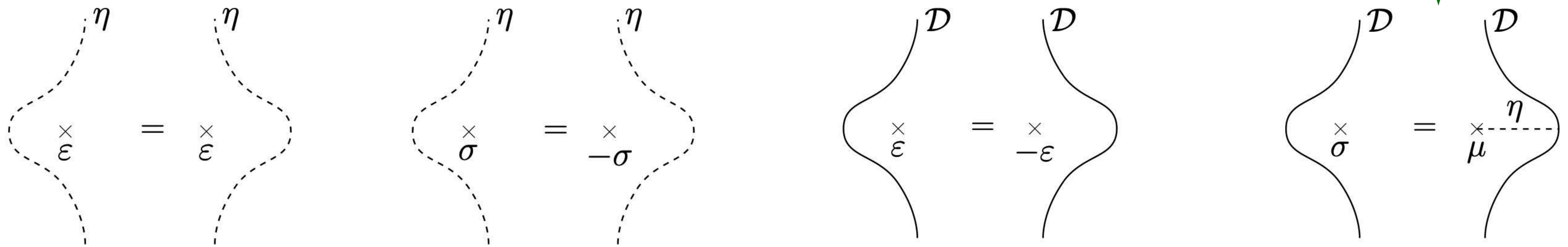
- Primary local operators identity, $\epsilon\,(\frac{1}{2},\frac{1}{2})$, $\sigma\,(\frac{1}{16},\frac{1}{16})$
- A usual $\mathbb{Z}_2$ 0-form symmetry: topological line operator η
- Non-invertible 0-form symmetry (**Kramers-Wannier** duality at critical point): topological line operator $\mathscr{D}$
- Fusion algebra

$$\eta \otimes \eta = 1 \qquad \eta \otimes \mathscr{D} = \mathscr{D} \otimes \eta = \mathscr{D} \qquad \mathscr{D} \otimes \mathscr{D} = 1 \oplus \eta$$

Beyond group-like fusion!

The non-inv. symmetry maps the local op. σ to the non-local op. μ!

- Action of topological lines on local operators



taken from Shao's Lectures

Generalized global symmetries

Some applications of generalized global symmetries

- Organize the **spectrum** of operators, including both local and extended operators (“symmetry multiplets”)
- Constrain **correlators** of (local/extended) operators (a la Ward identities)
- Constrain RG flows/IR phases using **’t Hooft anomaly matching**
- Extend Landau’s paradigm: gapped and gapless **phases** can be organized based on spontaneous symmetry breaking patterns of generalized symmetries

The majority of constructions/studies of generalized global symmetries is in a weakly-coupled/Lagrangian setting

- Can we find/study generalized symmetry structures in strongly coupled systems?
- A fruitful approach is to use stringy constructions

From string theory to quantum field theories

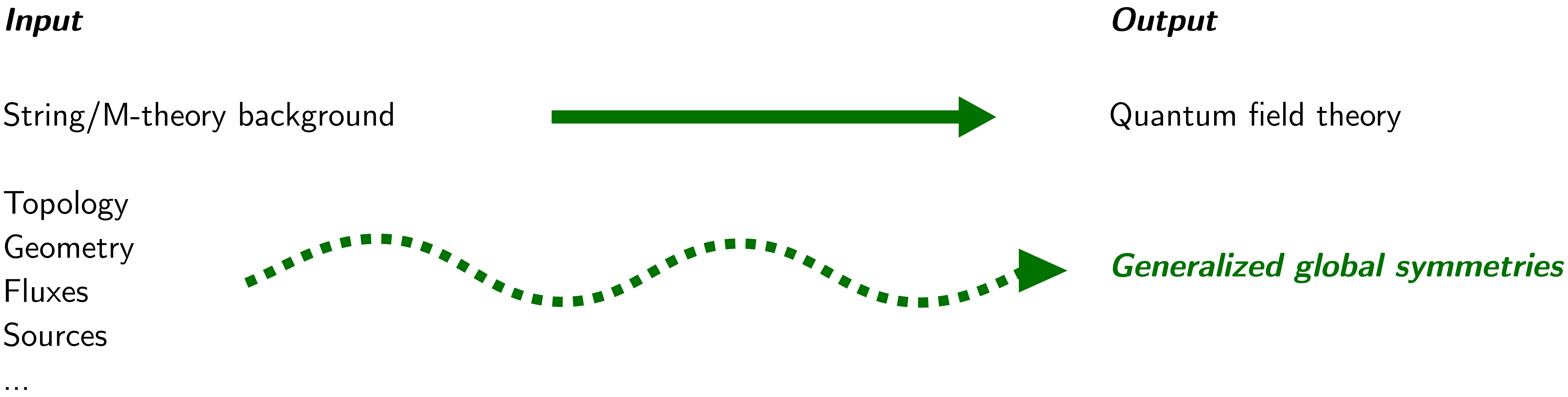
- String/M-theory is a theory of quantum gravity. In this talk, however, we regard it as a tool to study QFTs
- Three main avenues for constructing QFTs in string/M-theory
- **Brane constructions**
  Branes are extended solitonic objects in string/M-theory. They carry non-trivial localized degrees of freedom that can give rise to interacting worldvolume theories. Example: a stack of N D-branes supports $U(N)$ gauge theory
- **Geometric engineering**
  We put string/M-theory on a background $\mathbb{R}^{1,d-1} \times X$, where X is non-compact and has an isolated singularity at P_0. We can realize a non-trivial QFT living on $\mathbb{R}^{1,d-1} \times \{P_0\}$
- **AdS/CFT (holography)**

  non-gravitational field theory on M_d $\xleftrightarrow{\text{duality}}$ *string/M-theory on spacetime with asymptotic boundary* M_d

- These frameworks allow us to access strongly coupled QFTs

From string theory to quantum field theories

- These approaches are different but share some common qualitative features



Input

String/M-theory background

Topology
Geometry
Fluxes
Sources
...

Output

Quantum field theory

Generalized global symmetries

- Broad question:
  How do we extract the generalized global symmetry structures of the QFT from the topology, geometry, fluxes, sources of the string/M-theory background?

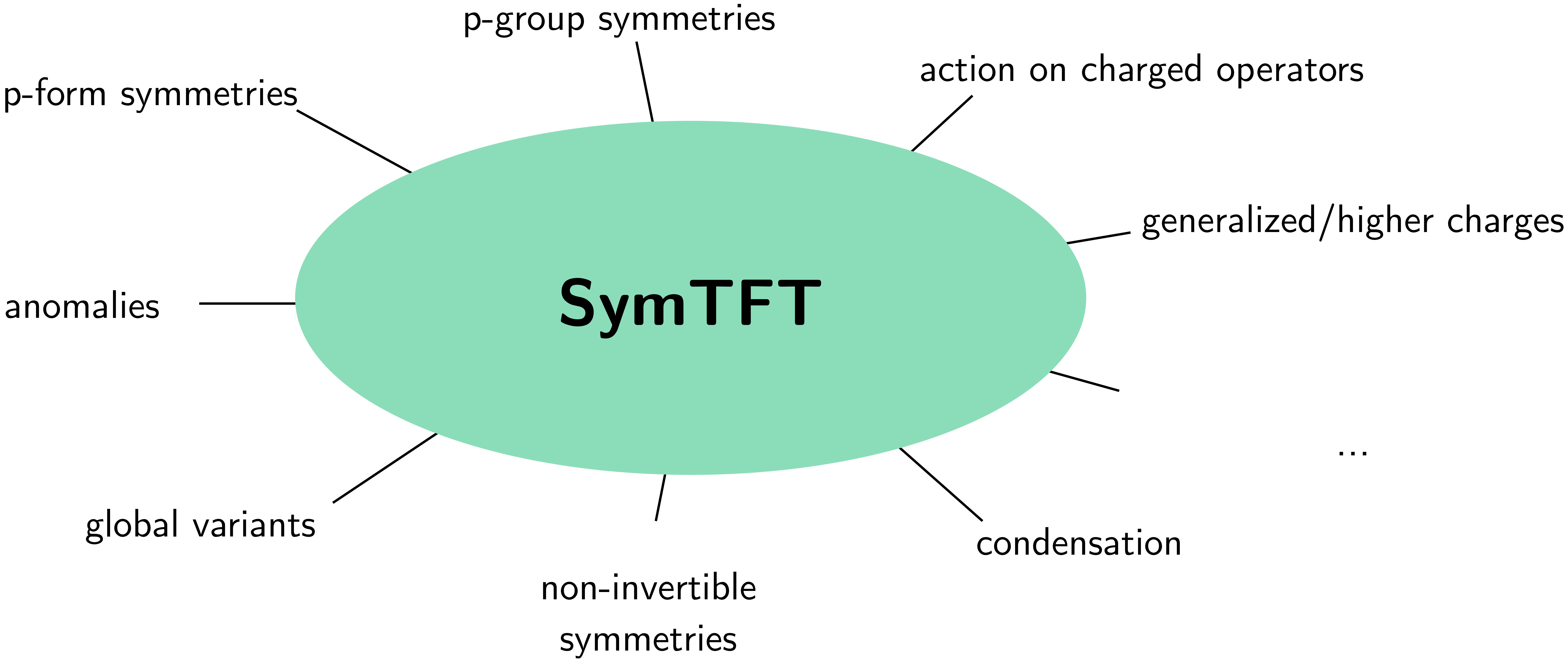
- Powerful tool/formalism: SymTFT or Sandwich Construction

Outline

1. Introduction
2. Brief review of the SymTFT construction
3. Strategies to extract SymTFTs from string constructions
4. Example: M-theory geometric engineering
5. Example: $AdS_7 \times \mathbb{RP}^4$
6. Conclusions and outlook

Brief review of the SymTFT construction

SymTFT as organizing principle



physical QFT $\mathscr{T}$ in d dimensions → symmetry topological field theory in $d+1$

[Freed, Teleman 12; Freed 14; Ji, Wen 19; Gaiotto, Kulp 20; Apruzzi, **FB**, Garcia-Etxebarria, Hosseini, Schafer-Nameki 21; Freed, Moore, Teleman 22; ...]

SymTFT as organizing principle

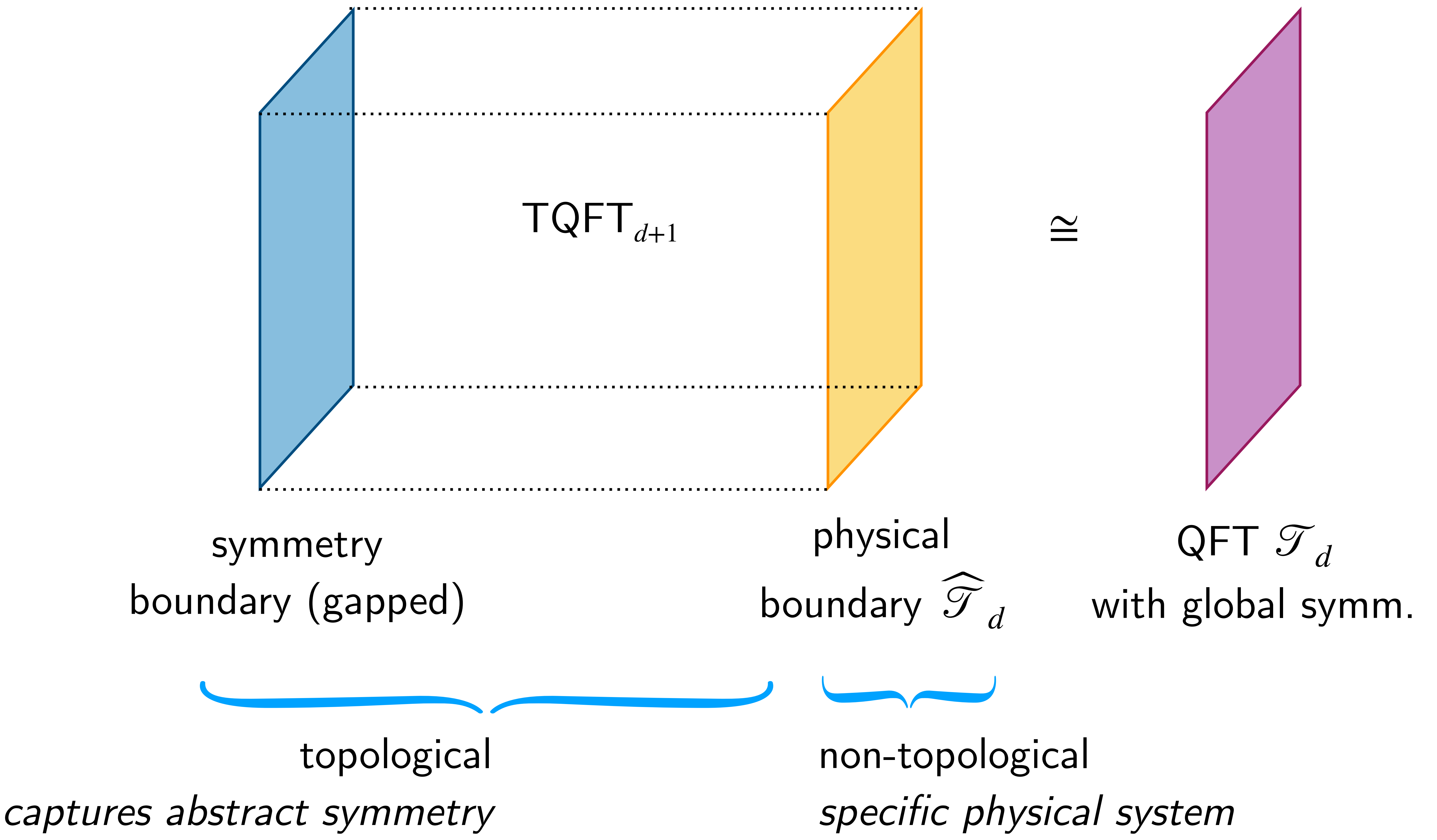
- The SymTFT sandwich construction is a proposal to achieve a separation between [Freed, Moore, Teleman 22]
  - "abstract" symmetry structure S
  - specific physical system $\mathscr{T}$ on which the symmetry structure acts

- The SymTFT approach has been particularly fruitful for finite symmetries (including non-invertible)
  - "multiplets" of non-invertible symmetries [Bhardwaj, Schafer-Nameki 23; Bartsch, Bullimore, Ferrari, Grigoletto, Pearson]
  - anomalies for non-invertible symmetries [Cordova, Hsin, Zhang 23; Antinucci, Benini, Copetti, Galati, Rizi 23; ...]
  - gapped and gapless phases with non-invertible symmetries [Bhardwaj, Schafer-Nameki, ...]
  - ...

- Proposals are available to extend it to continuous symmetries

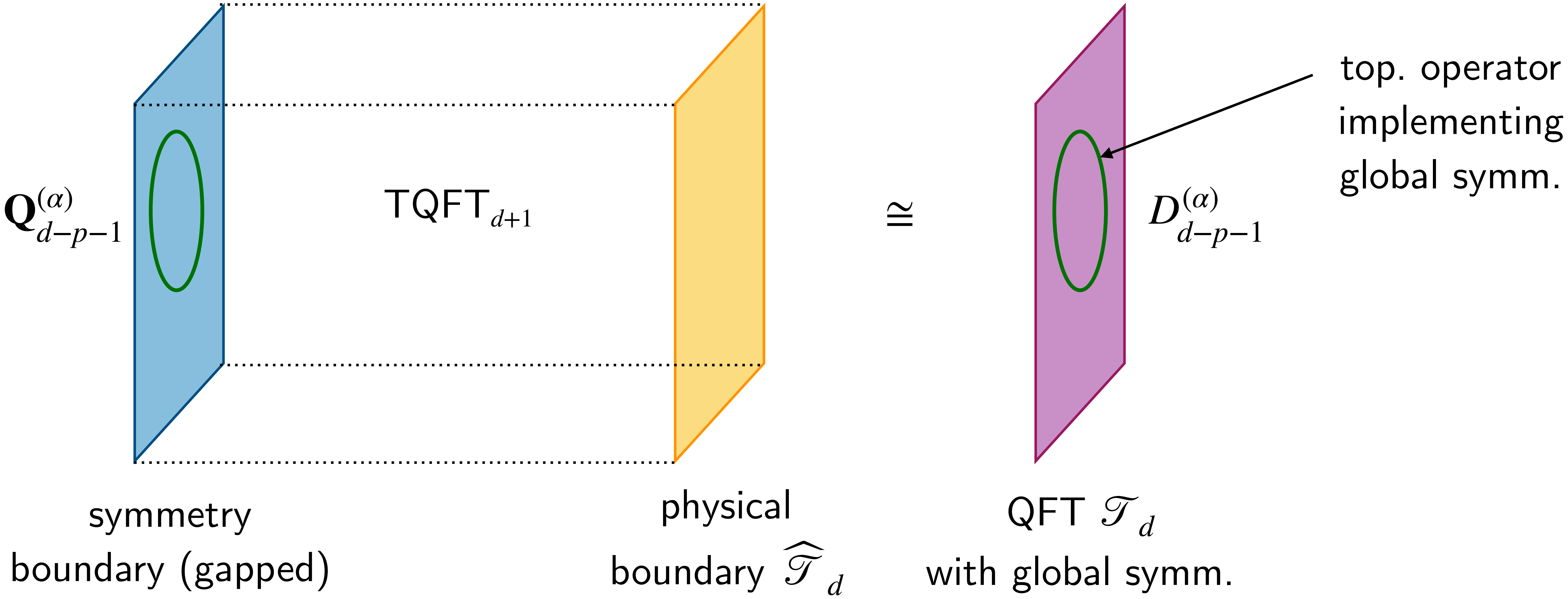
[Brennan, Sun 24; Antinucci, Benini 24; **FB**, Del Zotto, Minasian 24; Apruzzi, Bedogna, Dondi 24]

SymTFT construction



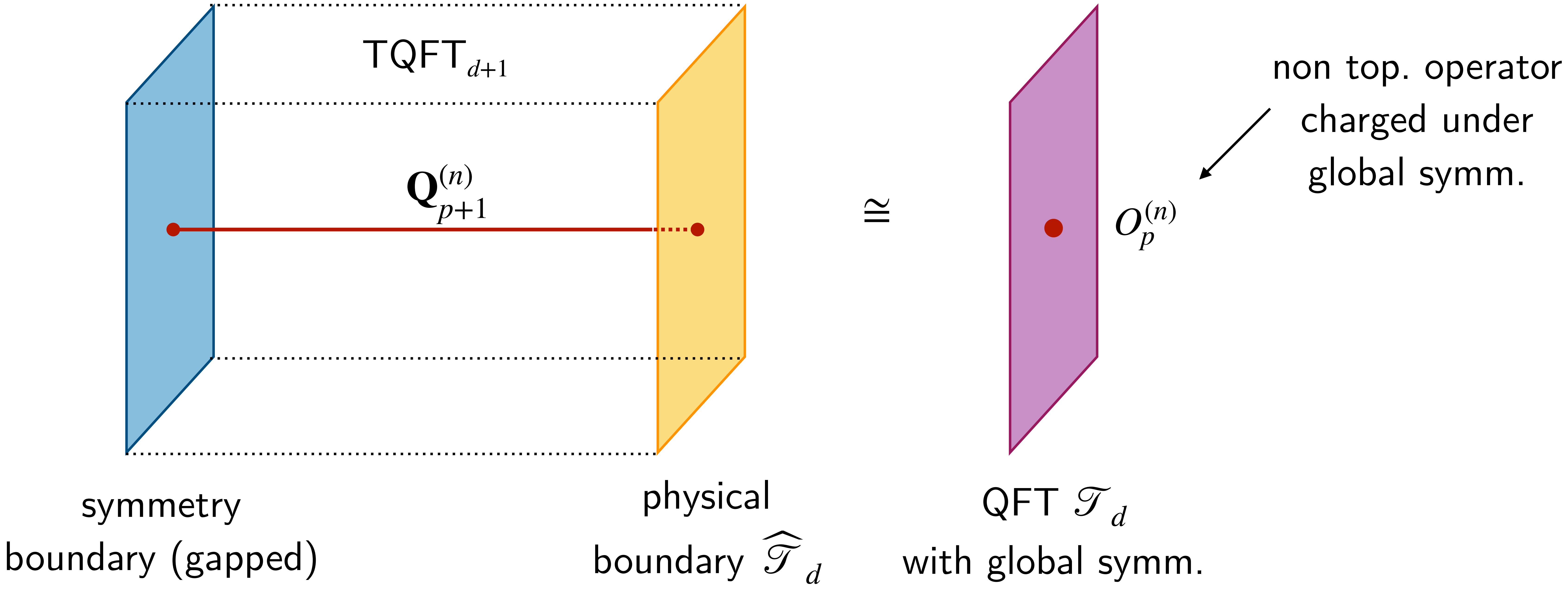
- The physical theory of interest is realized as interval compactification of a TQFT in one higher dimension with one gapped boundary and one physical boundary (non necessarily gapped)

SymTFT construction



- Topological operators parallel to the boundaries are mapped to the topological operators that implement the global symmetries of the QFT $\mathscr{T}_d$

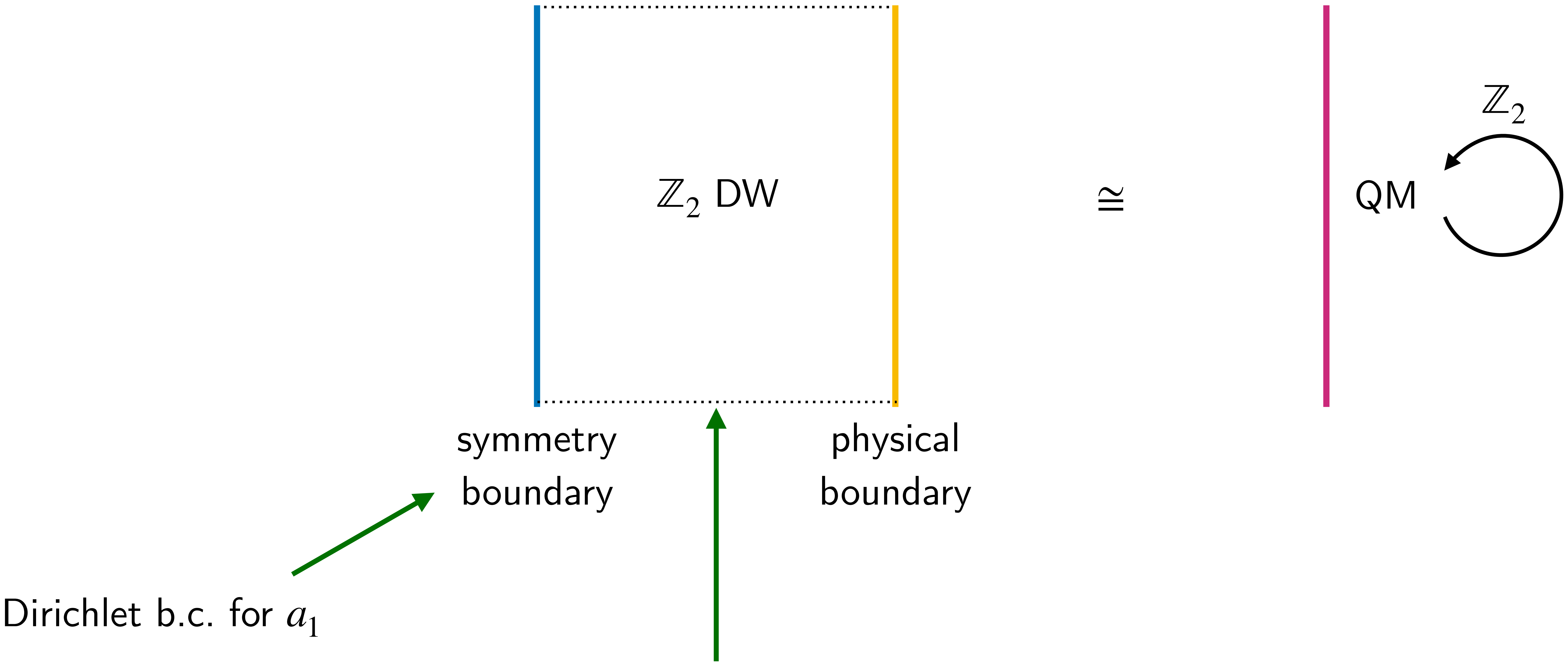
SymTFT construction



- Topological operators stretched between the two boundaries yield non-topological operators in $\mathscr{T}_d$ that are charged under the global symmetry

Example

- Quantum mechanical system (1d QFT) with an ordinary global $\mathbb{Z}_2$ symmetry
- Sandwich picture: coupling to finite $\mathbb{Z}_2$ gauge theory in 2d (Dijkgraaf-Witten theory)



$\mathbb{Z}_2$ DW

$\cong$

QM

$\mathbb{Z}_2$

symmetry boundary

physical boundary

Dirichlet b.c. for a_1

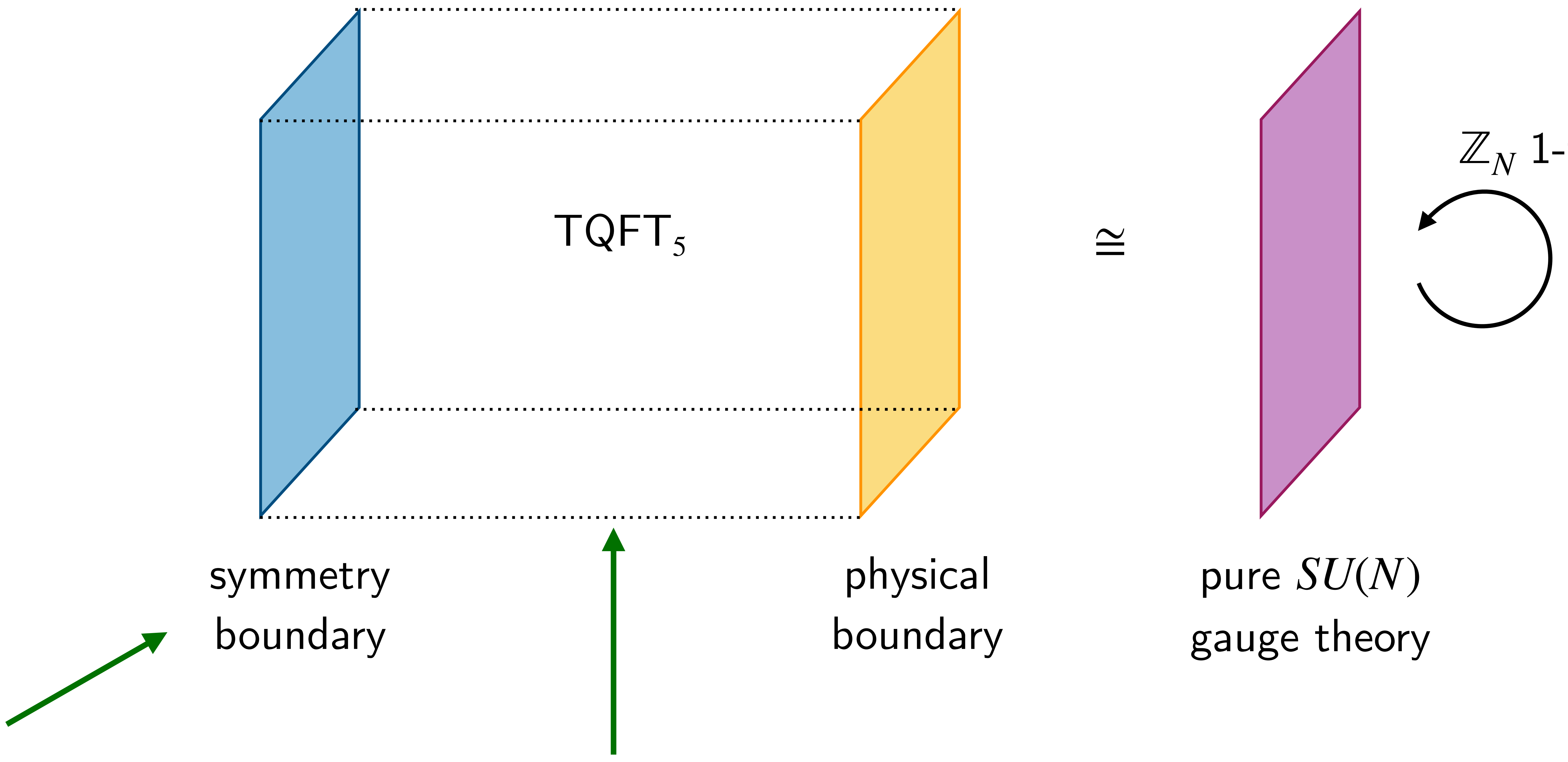
$$S = 2\pi \int_{M_2} \frac{1}{2} c_0 \cup \delta a_1$$

$$a_1 \in C^1(M_2; \mathbb{Z}_2),\ c_0 \in C^0(M_2; \mathbb{Z}_2)$$

[...; Freed, Moore, Teleman 22]

Example

- 4d pure gauge theory with gauge group $SU(N)$: global $\mathbb{Z}_N$ 1-form symmetry
- Sandwich picture: coupling to finite $\mathbb{Z}_2$ 2-form gauge theory in 5d



TQFT_5 $\cong$

$\mathbb{Z}_N$ 1-form symmetry

symmetry boundary

physical boundary

pure $SU(N)$ gauge theory

Dirichlet b.c. for b_2

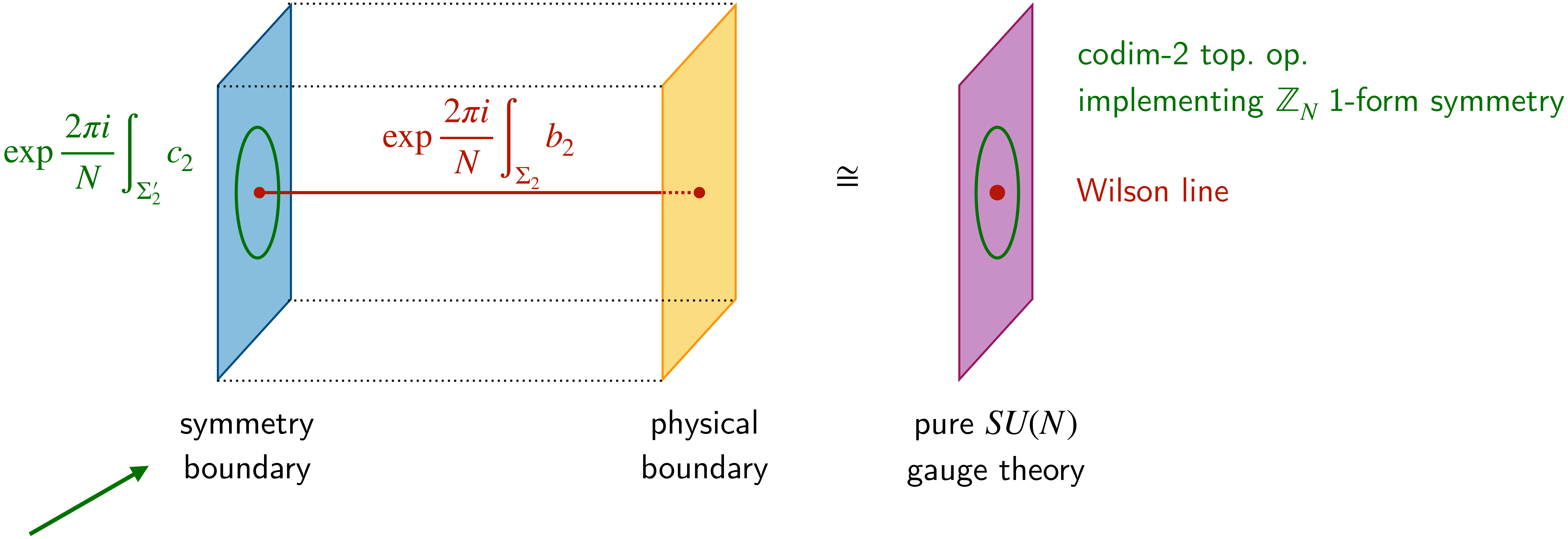
$$S = 2\pi \int_{M_5} \frac{1}{N} c_2 \cup \delta b_2$$

$$b_2, c_2 \in C^2(M_5; \mathbb{Z}_2)$$

[... Witten 95; ...; Gaiotto, Kapustin, Seiberg, Willet 14; ... Freed, Moore, Teleman 22]

Example

- 4d pure gauge theory with gauge group $SU(N)$: global $\mathbb{Z}_N$ 1-form symmetry
- Sandwich picture: coupling to finite $\mathbb{Z}_2$ 2-form gauge theory in 5d



Dirichlet b.c. for b_2

$$S = 2\pi \int_{M_5} \frac{1}{N} c_2 \cup \delta b_2$$

$$b_2, c_2 \in C^2(M_5; \mathbb{Z}_2)$$

[... Witten 95; ...; Gaiotto, Kapustin, Seiberg, Willet 14; ... Freed, Moore, Teleman 22]

Bulk TQFT for group like symmetries

- For finite Abelian symmetries

$$S = 2\pi \int_{M_{d+1}} \left(\underbrace{\frac{1}{N_1} c_{d-p_1-1} \cup \delta a_{p_1+1} + \frac{1}{N_2} c_{d-p_2-1} \cup \delta a_{p_2+1}} + \ldots + \underbrace{\mathscr{A}[a_{p_1+1}, a_{p_2+2}, \ldots]} \right)$$

Quadratic terms

Encode the fact that:

a_{p_1+1} is a discrete $\mathbb{Z}_{N_1}$ gauge field

a_{p_2+1} is a discrete $\mathbb{Z}_{N_2}$ gauge field

…

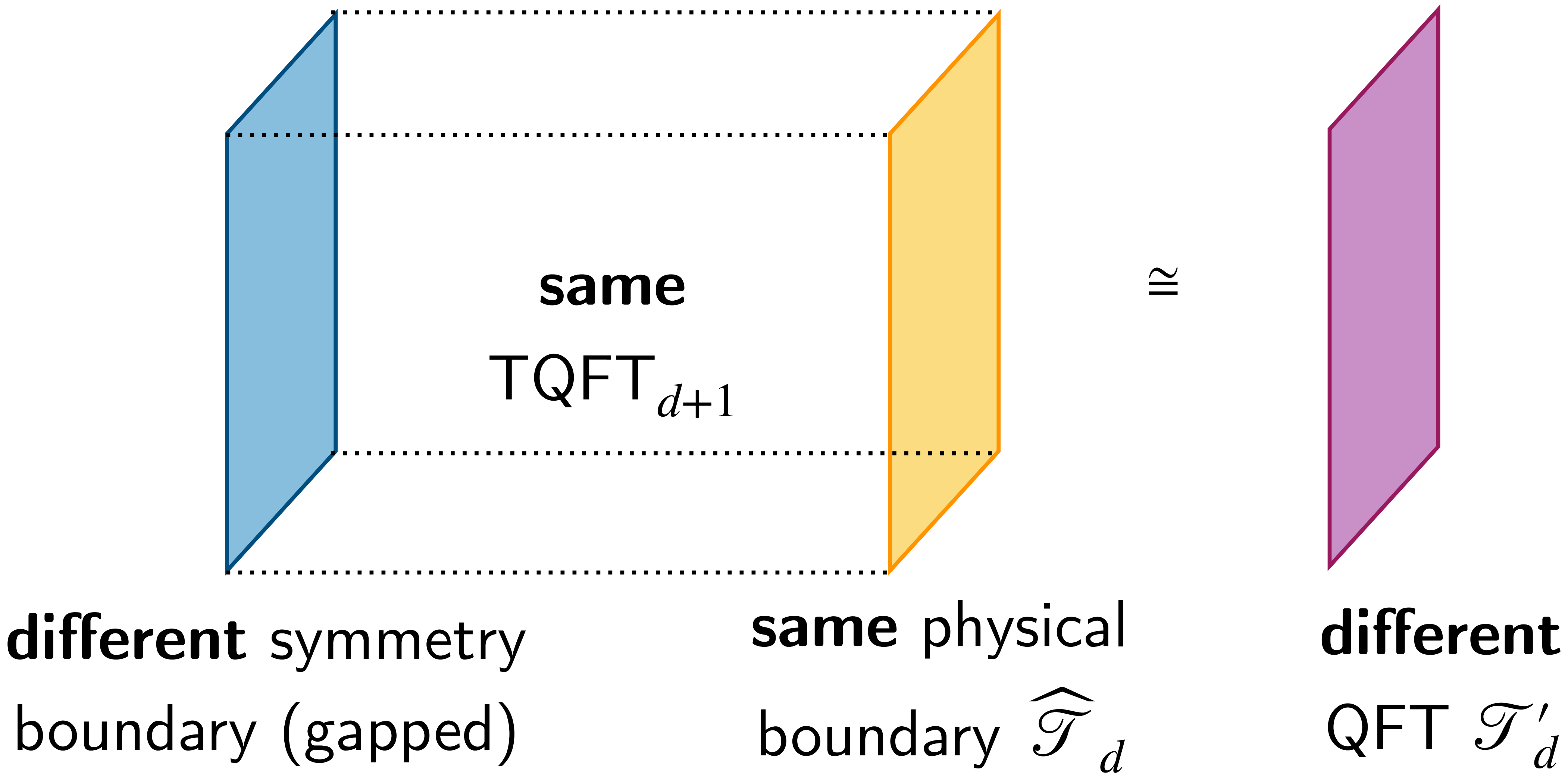
"twist" terms

Related to potential anomalies among the global symmetries associated to the discrete gauge fields a_{p_1+1}, a_{p_2+1}, etc.

Global variants from the SymTFT

- Global variants are different version of a QFT obtained via topological manipulations (e.g. gauging a finite symmetry)
- All global variants share the same bulk SymTFT and the same physical boundary. The symmetry boundary changes

$$S = 2\pi \int_{M_{d+1}} \left(\frac{1}{N_1} c_{d-p_1-1} \cup \delta a_{p_1+1} + \frac{1}{N_2} c_{d-p_2-1} \cup \delta a_{p_2+1} + \ldots \right.$$
$$\left. + \mathscr{A}[a_{p_1+1}, a_{p_2+2}, \ldots] \right)$$



- Example: Dirichlet boundary conditions for a fields. Gives QFT with group like symmetries and mixed anomalies (from $\mathscr{A}$)
- Depending on $\mathscr{A}$, it might be possible to choose different topological boundary conditions
- In this way a mixed anomaly can give rise to a **higher group** or a **non-invertible symmetry** [Tachikawa 17; Kaidi, Ohmori, Zheng 21]

Strategies to extract SymTFTs from string constructions

Strategies

- Setting: a QFT realized by a string/M-theory background
- Broadly speaking, we have two main strategies to extract information about the SymTFT of the QFT
  1. Extract topological couplings in the **Lagrangian of the SymTFT**
  2. Extract directly some of the topological **operators of the SymTFT**

Strategy 1: Lagrangian of the SymTFT

- At low energies, string/M-theory is captured by a supergravity theory
- We study the relevant supergravity theory in the string/M-theory background that defines the QFT of interest
- For example:

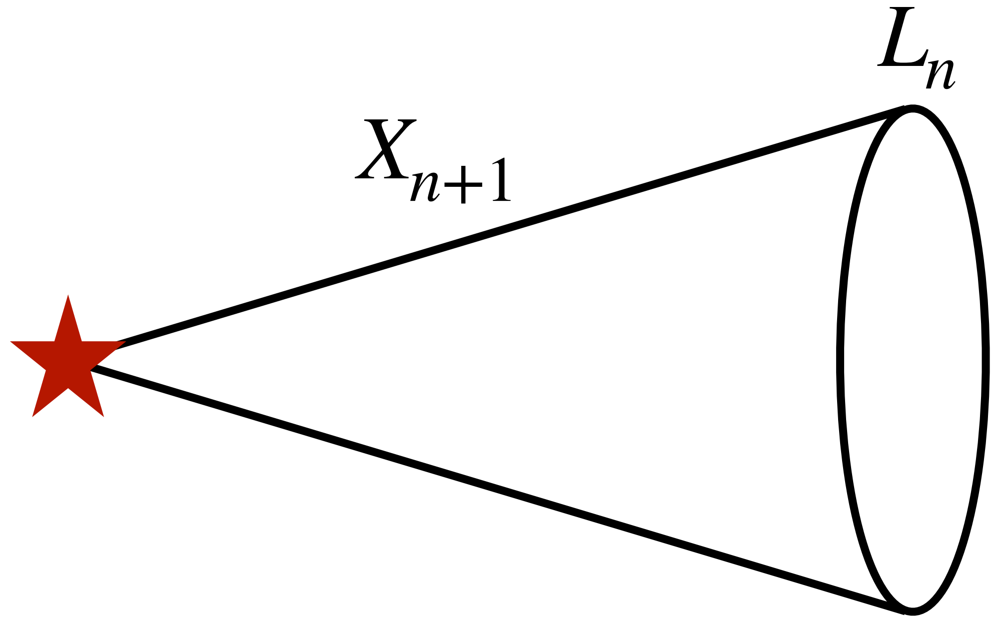
Geometric engineering

$M_D = \mathbb{R}^{1,d-1} \times X_{n+1}$

X_{n+1} = non-compact singular space

L_n = link of singularity (smooth, closed)

We study D-dimensional supergravity on L_n

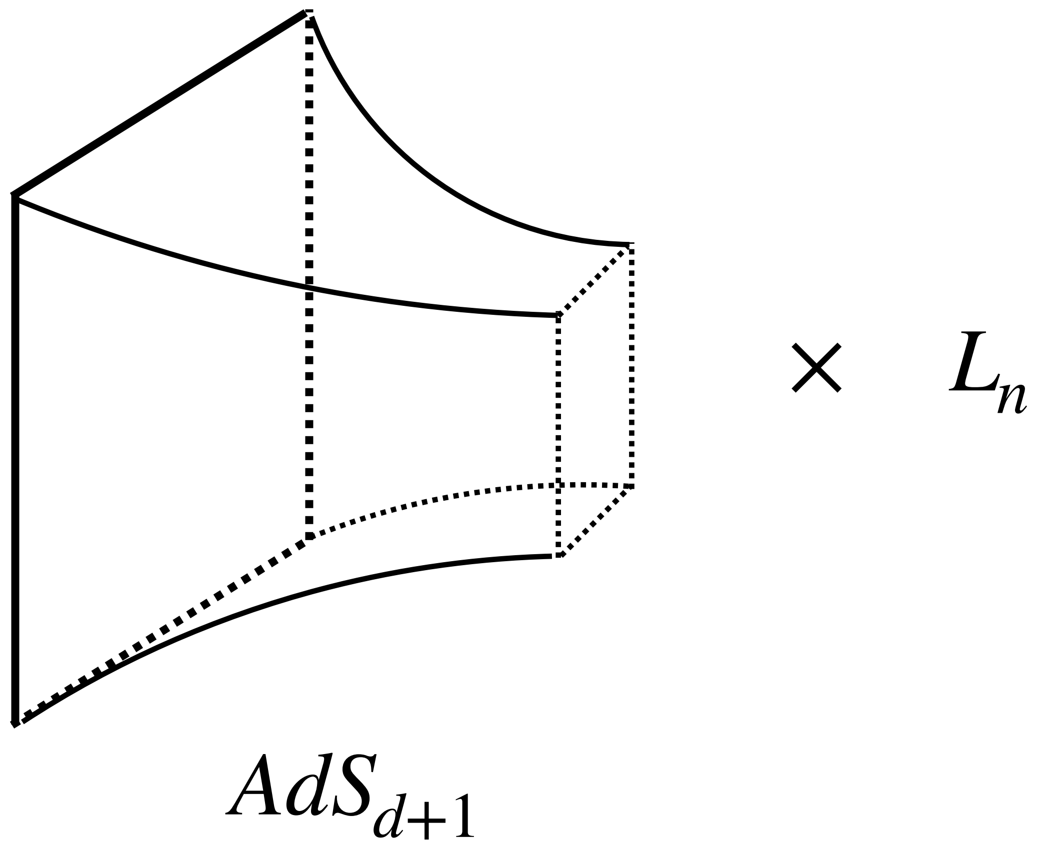


AdS/CFT

$M_D = AdS_{d+1} \times L_n$

L_n = internal space (smooth, closed)

We study D-dimensional supergravity on L_n



$(D = d + n + 1)$

Strategy 1: Lagrangian of the SymTFT

- In this talk, we focus on finite Abelian symmetries
- A finite Abelian symmetry in the QFT corresponds to a discrete gauge field in the SymTFT Lagrangian
- Tasks:
  1) Identify possible origins of discrete gauge fields from supergravity on L_n. For example:
     - ❖ Continuous gauge field with topological mass terms (BF terms)
     - ❖ Torsion in the (co)homology of L_n
  2) Compute topological terms in the SymTFT Lagrangian involving the fields identified in 1)

[Witten 98; Maldacena, Moore, Seiberg 01; Belov, Moore 04; ...; Bergman, Tachikawa, Zafrir 20; Bah, **FB**, Minasian 20; Apruzzi, **FB**, Garcia-Etxebarria, Hosseini, Schafer-Nameki 21; Bergman, Hirano 22; ...]

Strategy 2: operators of the SymTFT

- We can extract some of the operators of the SymTFT using **branes** in string/M-theory

[Apruzzi, Bah, **FB**, Schäfer-Nameki 22; García Etxebarria 22; Heckman, Hübner, Torres, Zhang 22; Heckman, Hübner, Torres, Yu, Zhang 22; Cvetič, Heckman, Hübner, Torres 23; Etheredge, García Etxebarria, Heidenreich, Rauch 23; Apruzzi, **FB**, Gould, Schäfer-Nameki 23; Bah, Leung, Waddleton 23; Baume, Heckman, Hübner, Torres, Turner, Yu 23; Yu 23; Heckman, Hübner, Murdia 24; Heckman, McNamara, Montero, Sharon, Vafa, Valenzuela 24; Cvetič, Donagi, Heckman, Hübner, Torres 24; Argurio, Benini, Bertolini, Galati, Niro 24; Franco, Yu 24; Gutperle, Li, Rathore, Roumpedakis 24; Bergman, Garcia-Valdecasas, Mignosa, Rodriguez-Gomez 24; Waddleton 24; García Etxebarria, Huertas, Uranga 24; Tian, Wang 24; Najjar, Santilli, Wang 24; ...]

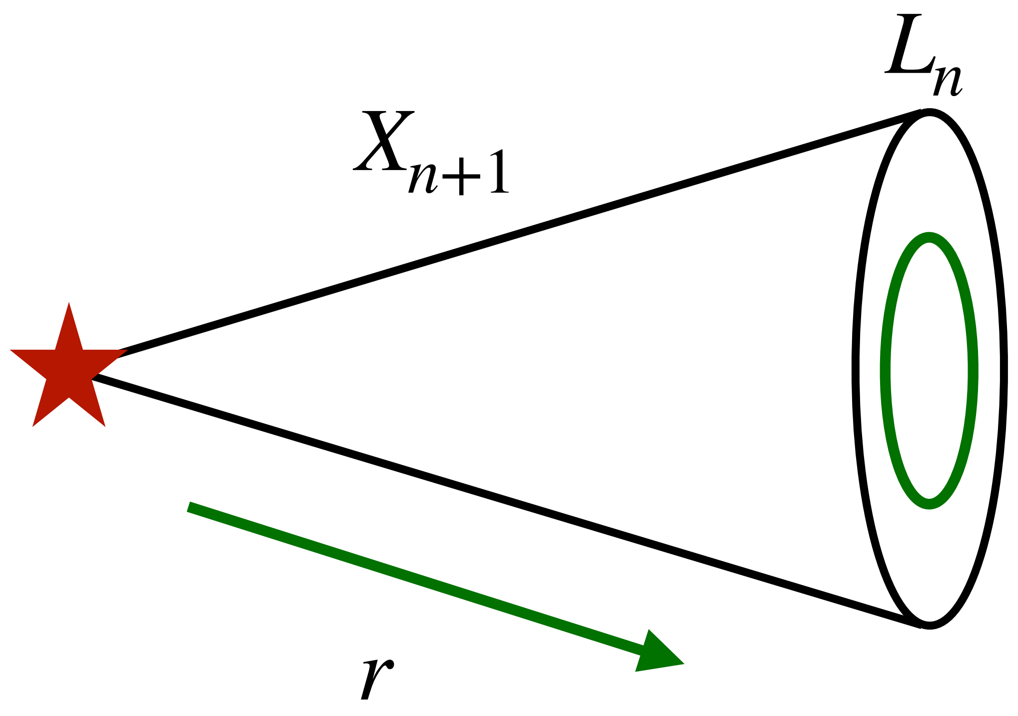
- Branes are solitonic objects with finite tension, non-trivial dynamics, non-topological worldvolume theories, etc...

- How can we extract topological information from them?

Strategy 2: operators of the SymTFT

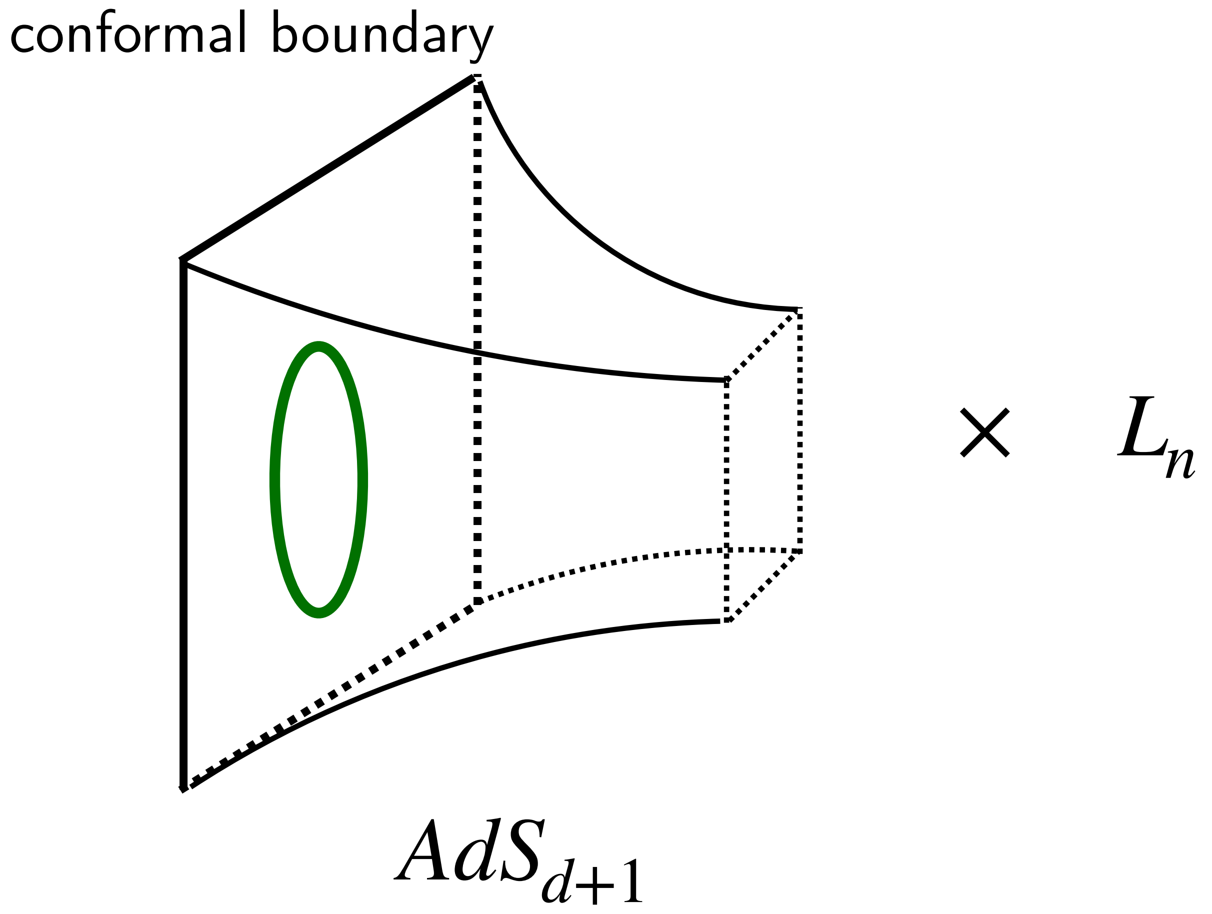
- We consider "branes at infinity". Their effective tension diverges. Non-topological modes on the branes freeze out.

Geometric engineering



Brane wrapping a cycle in L_n at fixed r_*
in the limit $r_* \to 0$

AdS/CFT



Brane wrapping a cycle in L_n at fixed r_*
in the limit $r_* \to$ conformal boundary

- This approach captures: **linking**, **fusion**, **action** of non-invertible symmetries, etc.

Strategy 2: operators of the SymTFT

- Concrete example: worldvolume action of a D-brane approaching conformal boundary of AdS

$$S_{\text{DBI}} = -T_p \int_{W_{p+1}} d^{p+1}\xi e^{-\Phi} \sqrt{-\det\left(g_{\alpha\beta} + \mathscr{F}_{\alpha\beta}\right)} \qquad S_{\text{WZ}} = \int_{W_{p+1}} \left[\sum_q C_q e^{\mathscr{F}_2} \sqrt{\frac{\widehat{A}(T)}{\widehat{A}(N)}}\right]_{p+1} \qquad \mathscr{F}_2 = \frac{1}{2\pi}(da_1 + B_2)$$

- Remarks:
  - If the brane approaches the asymptotic boundary at infinity (parallel to it), its effective tension scales to infinity

$$ds^2_{d+1} = \frac{r^2}{L^2} d\vec{x}^2 + \frac{L^2}{r^2} dr^2 \qquad r \to \infty \qquad T_{\text{eff}} \sim r^{p+1}$$

  - DBI term freezes out
  - In contrast, the WZ term remains non-trivial in this limit. We highlight:
    - The coupling to the RR p-form potentials
    - The presence of a dynamical $U(1)$ gauge field on the worldvolume of the D-brane

Example: M-theory geometric engineering

Geometric engineering: M-theory on threefold singularity

- Let us focus on a concrete class of examples in geometric engineering

- Setup: M-theory on a canonical Calabi-Yau threefold singularity X_6. External spacetime: $\mathbb{R}^{1,4}$

- Engineers a 5d superconformal field theory (SCFT) [Morrison, Seiberg 96; Intriligator, Morrison, Seiberg 97; ...]

- X_6 admits a crepant resolution X_6^{res}. M-theory on X_6^{res} describes the (extended) Coulomb branch of the SCFT
  - Description in terms of 5d $\mathcal{N} = 1$ vector multiplets and hypermultiplets
  - On special loci on the extended Coulom branch we sometimes find a non-Abelian gauge theory description
  - The origin of the extended Coulomb branch is the SCFT point

Example

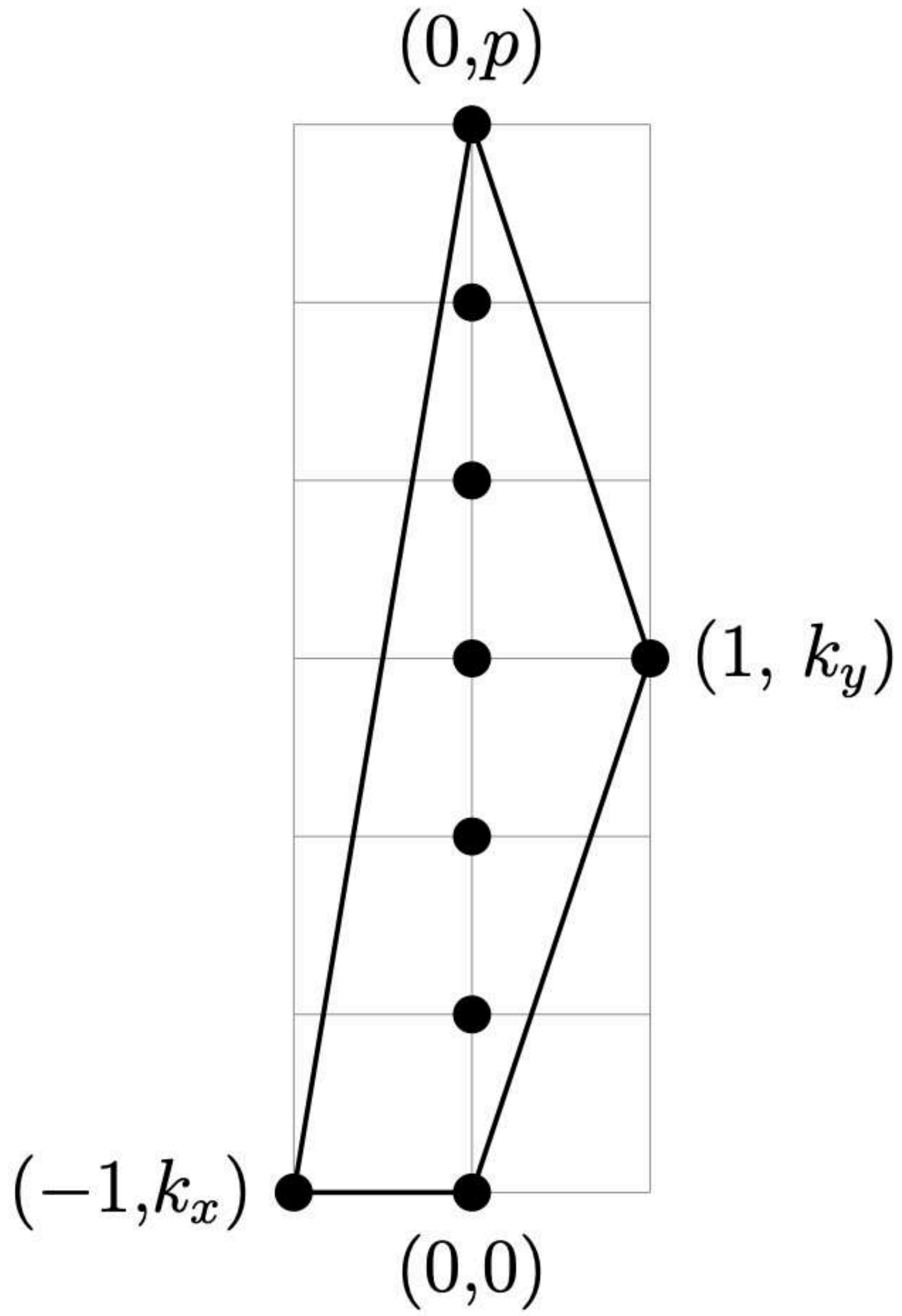
- A well-studied family of canonical CY threefold singularities are toric CY cones
- Described by a 2d toric diagram.
- This geometry yields a 5d SCFT that admits a non-Abelian gauge theory description

$$SU(p)_q : \text{ 5d } \mathcal{N} = 1 \ SU(p) \text{ gauge theory with a CS coupling at level } q$$

- Here $q = p - (k_x + k_y)$
- In this example: the non-Abelian gauge theory description enjoys a 1-form symmetry

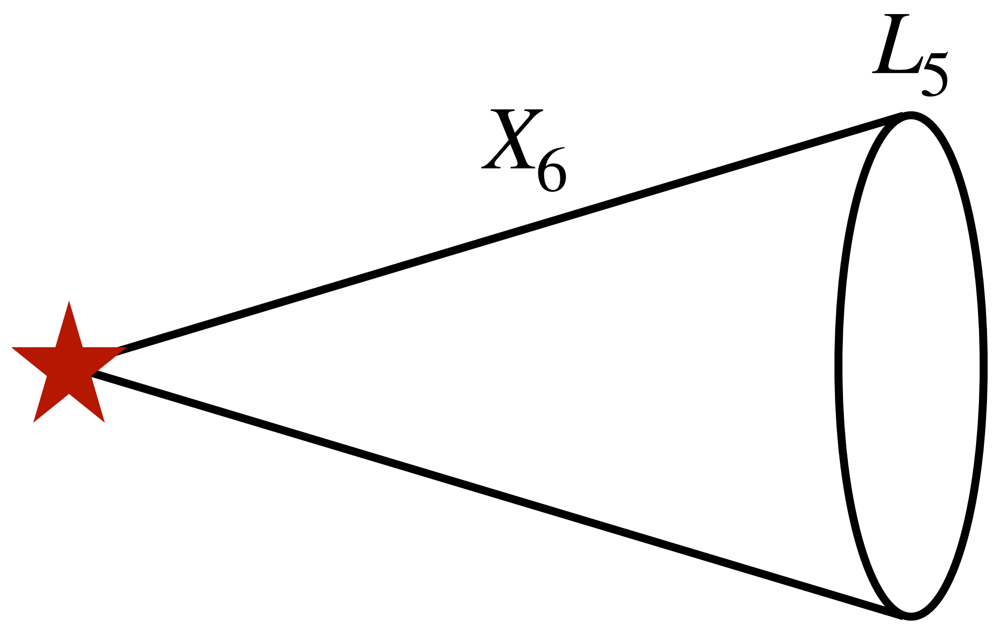
$$\text{1-form symmetry group} = \mathbb{Z}_{\gcd(p,q)}$$

- More broadly: 1-form symmetries are ubiquitous in 5d SCFTs engineered in M-theory



1-form symmetry and geometry

Q1: *How do we determine the 1-form symmetry group of the SCFT from the geometry?*



1-form symmetry group $= \mathrm{Tor}\, H^2(L_5)$

Q2: *Can we derive potential anomalies for this 1-form symmetry from the geometry?*

anomaly coefficient ↔ torsional linking pairing $\ell_{L_5}(t_2, t_2 \cup t_2)$

$\ell_{L_5} : \mathrm{Tor}\, H^2(L_5) \times H^4(L_5) \to \mathbb{Q}/\mathbb{Z}$

$t_2 =$ generator of $\mathrm{Tor}\, H^2(L_5)$

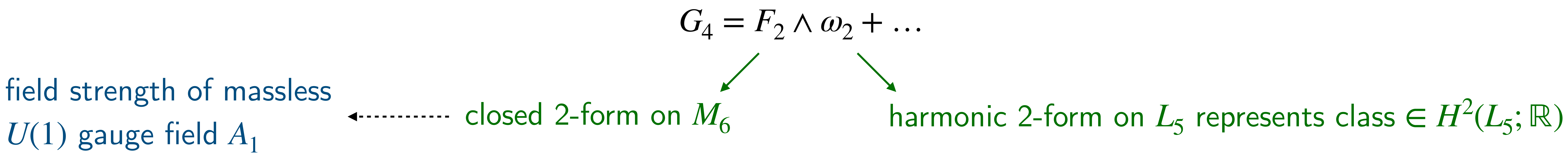
To derive these facts, we study 11d supergravity on the base L_5 of the Calabi-Yau cone (L_5 is the link of the singularity)

11d supergravity on L_5

- The bosonic fields of 11d supergravity are the metric and a 3-form potential C_3 with field strength

$$G_4 = dC_3$$

- We are interested in the low-energy modes of 11d supergravity on L_5. We borrow intuition from the *Kaluza-Klein* program
- A set of low-energy modes comes from expanding G_4 onto representatives of cohomology classes of L_5



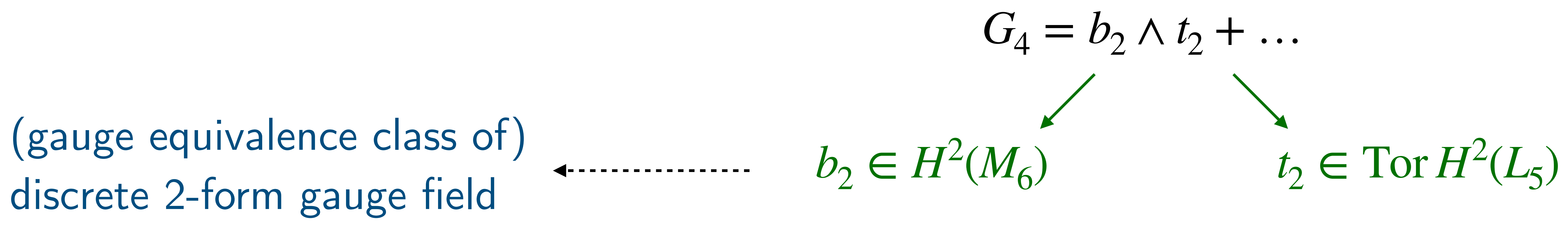
$$G_4 = F_2 \wedge \omega_2 + \dots$$

field strength of massless $U(1)$ gauge field A_1

closed 2-form on M_6

harmonic 2-form on L_5 represents class $\in H^2(L_5; \mathbb{R})$

- Our goal is to capture a finite 1-form symmetry of the SCFT. We extend the usual Kaluza-Klein program to include torsional elements in cohomology. Schematically



$$G_4 = b_2 \wedge t_2 + \dots$$

(gauge equivalence class of) discrete 2-form gauge field

$b_2 \in H^2(M_6)$

$t_2 \in \mathrm{Tor}\, H^2(L_5)$

How can we describe more precisely this expansion onto torsional classes?

An application of differential cohomology

- We can model (gauge equivalence classes of) configurations of the 3-form potential of M-theory using a class

$$\breve{G}_4 \in \breve{H}^4(M_{11})$$

- Differential refinement of integral cohomology group $H^4(M_{11})$

- $\breve{G}_4$ combines information about the topological class of G_4 and about the 3-form "connection" and its curvature
  - $I : \breve{H}^4(M_{11}) \to H^4(M_{11}) \qquad I(\breve{G}_4) = a_4$ "forgets" the differential refinement
  - $R : \breve{H}^4(M_{11}) \to \Omega^4(M_{11}) \qquad R(\breve{G}_4) = G_4$ extracts the curvature (closed 4-form with integral periods)

- Compatibility condition

$$\varrho(a_4) = [G_4]_{\text{de Rham}} \qquad\qquad \varrho : H^\bullet(M_{11}) \to H^\bullet(M_{11};\mathbb{R})$$

- Caveats:
  - We work on an orientable, Spin spacetime
  - We consider M-theory backgrounds in which there is no half-integral quantization of G_4 $(w_4(M_{11}) = 0)$ [Witten 96]

see also [Freed, Moore, Segal 06; Monnier 13; ...; Sati 18; Fiorenza, Sati, Schreiber 20,21,...]

An application of differential cohomology

Some useful general facts about $\check{H}^p(M)$ (M is smooth, closed, orientable)

- The map $I : \check{H}^p(M) \to H^p(M)$ is surjective: we can uplift any integral class to a differential cohomology class
- There is a well-defined notion of product $\check{H}^p(M) \times \check{H}^q(M) \to \check{H}^{p+q}(M)$ with the properties:
  - Graded commutativity $\breve{\alpha}_p \cdot \breve{\beta}_q = (-1)^{pq} \breve{\beta}_q \cdot \breve{\alpha}_p$
  - Compatibility with the maps $I : \check{H}^p(M) \to H^p(M)$ and $R : \check{H}^p(M) \to \Omega^p(M)$

$$I(\breve{\alpha}_p \cdot \breve{\beta}_q) = I(\breve{\alpha}_p) \cup I(\breve{\beta}_q) \qquad R(\breve{\alpha}_p \cdot \breve{\beta}_q) = R(\breve{\alpha}_p) \wedge R(\breve{\beta}_q)$$

- There is a notion of integration $\int_M : \check{H}^p(M) \to \check{H}^{p-\dim M}(\mathrm{pt})$. In particular
  - $p = \dim M$ integral is valued in $\check{H}^0(\mathrm{pt}) \cong \mathbb{Z}$
  - $p = \dim M + 1$ integral is valued in $\check{H}^1(\mathrm{pt}) \cong \mathbb{R}/\mathbb{Z}$

Expansion of G_4 onto torsional classes

We can now make more precise the notion of “Kaluza-Klein expansion onto torsional classes”

- Total spacetime is a product $M_{11} = M_6 \times L_5$
- Identify a set of generators $t_2 \in \mathrm{Tor}\, H^2(L_5)$. Suppose $nt_2 = 0$
- Use surjectivity of $I : \breve{H}^2(L_5) \to H^2(L_5)$ to uplift t_2 to some $\breve{t}_2 \in \breve{H}^2(L_5)$
- Consider terms in the 4-form flux of the form

$$\breve{G}_4 = \breve{b}_2 \cdot \breve{t}_2 + \dots \qquad \breve{b}_2 \in \breve{H}^2(M_6)$$

- One can show that $\breve{b}_2 \cdot \breve{t}_2$ (for a fixed $\breve{t}_2$) depends only on $I(\breve{b}_2) = b_2 \in H^2(M_6)$ and is invariant under

$$b_2 \to b_2 + nx_2 \text{ for any } x_2 \in H^2(M_6)$$

- Lesson: expanding $\breve{G}_4$ onto $\breve{t}_2$ we get a discrete $\mathbb{Z}_n$ 2-form gauge field in the external 6d spacetime

Technical assumption: we take external spacetime M_6 to be without torsion in (co)homology

Comparison with the screening argument

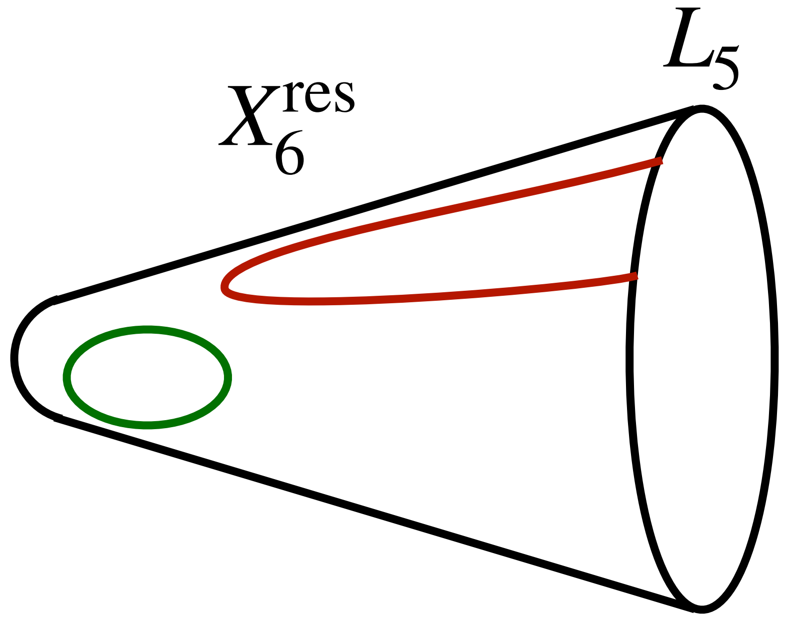
- We have confirmed the statement

$$\text{1-form symmetry group of 5d SCFT } \cong \operatorname{Tor} H^2(L_5)$$

- This point of view reproduces the same results as the "screening argument"

$$\text{1-form symmetry group of 5d SCFT} \cong \text{line operators modulo screening by dynamical particles}$$

- In the resolved Calabi-Yau geometry $X_6^{\rm res}$:
  - M2-branes extending in the radial direction (relative 2-cycles in $H_2(X_6^{\rm res}; L_5)$) give line operators
  - M2-branes wrapping compact 2-cycles in $H_2(X_6^{\rm res})$ give dynamical particles



- We then have

$$\text{1-form symmetry group of 5d SCFT } \cong \frac{H_2(X_6^{\rm res}, L_5)}{H_2(X_6^{\rm res})} \cong H_1(L_5)$$

- Indeed $H_1(L_5) \cong \operatorname{Tor} H^2(L_5)$ for the geometries under consideration

Technical assumptions: $H_1(X_6) = 0$, $b_1(L_5) = 0$

[Albertini, Del Zotto, Garcia Etxebarria, Hosseini 20; Morrison, Schafer-Nameki, Willet 20]

Cubic couplings

- The expansion $\breve{G}_4 = \breve{b}_2 \cdot \breve{t}_2 + \ldots$ also allows us to derive some topological couplings in the external 6d spacetime
- They originate from the topological couplings in the 11d low-energy effective action of M-theory

$$S \supset -\frac{1}{(2\pi)^2}\frac{1}{6}\int_{M_{11}} C_3 \wedge G_4 \wedge G_4 - \int_{M_{11}} C_3 \wedge X_8$$

$$X_8 = \frac{1}{192}\left(p_1^2 - 4p_2\right)$$

two-derivative CS term

higher-derivative correction
needed for consistency

[Vafa, Witten 95; Duff, Liu, Minasian 95; Witten 96; ...]

- We reformulate these couplings using the $\mathbb{R}/\mathbb{Z}$ valued integral of differential cohomology

$$S \supset -\frac{1}{6}\int_{M_{11}} \breve{G}_4 \cdot \breve{G}_4 \cdot \breve{G}_4 - \int_{M_{11}} \breve{G}_4 \cdot \breve{X}_8$$

Cubic couplings

- As in a standard dimensional reduction, we plug $\breve{G}_4 = \breve{b}_2 \cdot \breve{t}_2 + \ldots$ into the action and collect relevant terms

$$-\frac{1}{6}\int_{M_6} \breve{b}_2 \cdot \breve{b}_2 \cdot \breve{b}_2 \int_{L_5} \breve{t}_2 \cdot \breve{t}_2 \cdot \breve{t}_2 + \frac{1}{96}\int_{M_6} \breve{b}_2 \cdot \breve{p}_1(M_6) \int_{L_5} \breve{t}_2 \cdot \breve{p}_1(L_5)$$

- The two terms can be combined by a congruence relation of the schematic form $4b_2^3 = b_2 p_1 \text{ mod } 24$

- In total we obtain the following cubic coupling in external spacetime

$$\alpha \int_{M_6} \breve{b}_2 \cdot \breve{b}_2 \cdot \breve{b}_2 \qquad\qquad \alpha = -\frac{1}{6}\int_{L_5} \breve{t}_2 \cdot \breve{t}_2 \cdot \breve{t}_2 + \frac{1}{24}\int_{L_5} \breve{t}_2 \cdot \breve{p}_1(L_5) \ \text{ mod } 1$$

- Interpretation of α: refinement of the $\mathbb{R}/\mathbb{Z}$ valued linking pairing

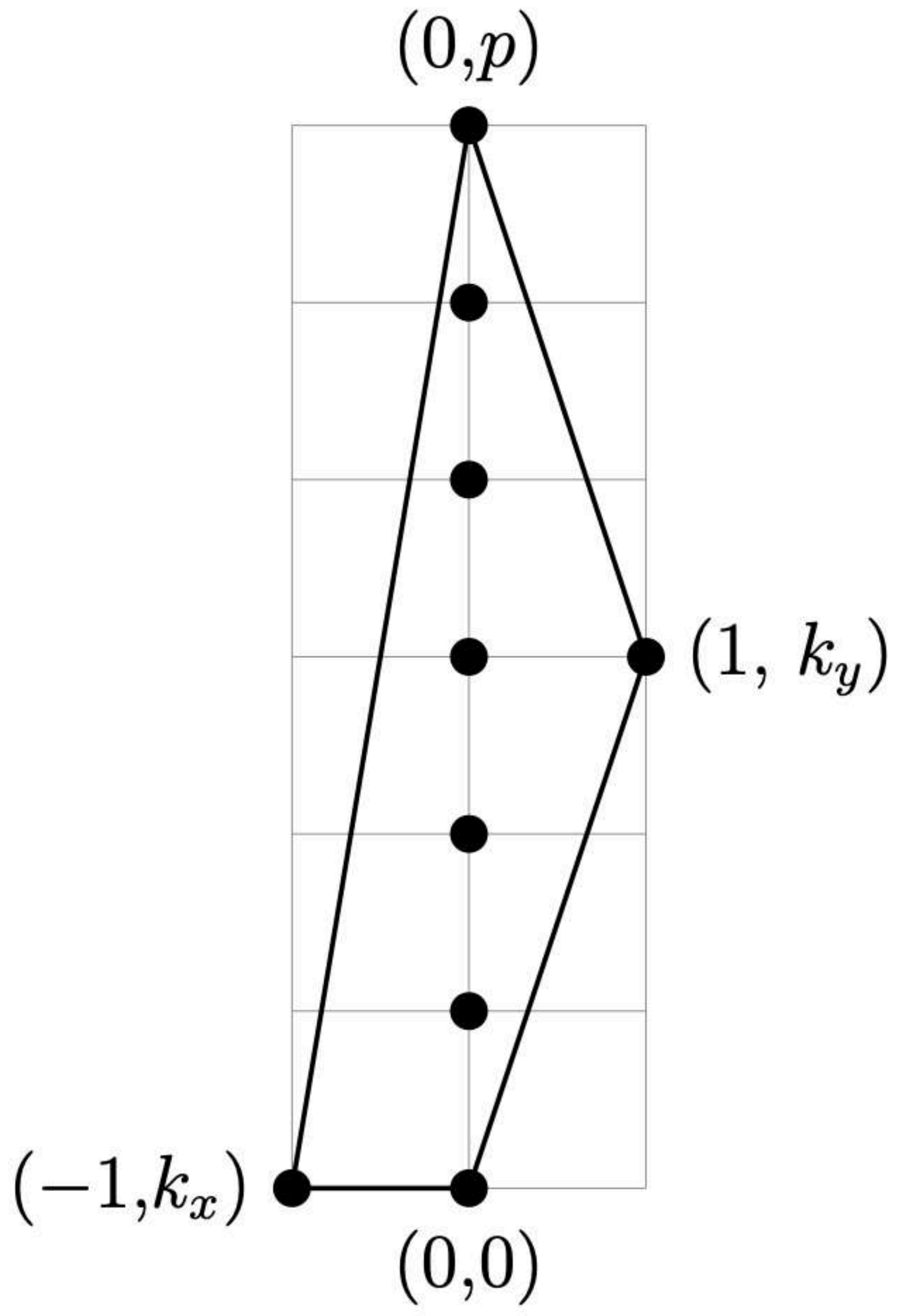
$$\ell_{L_5}(t_2, t_2 \cup t_2) = \int_{L_5} \breve{t}_2 \cdot \breve{t}_2 \cdot \breve{t}_2 \quad \text{mod } 1$$

$\ell_{L_5} : \mathrm{Tor}\, H^2(L_5) \times H^4(L_5) \to \mathbb{Q}/\mathbb{Z}$

$t_2 \in \mathrm{Tor}\, H^2(L_5)$

Example — revisited

$$q = p - (k_x + k_y)$$



- How do we compute α?

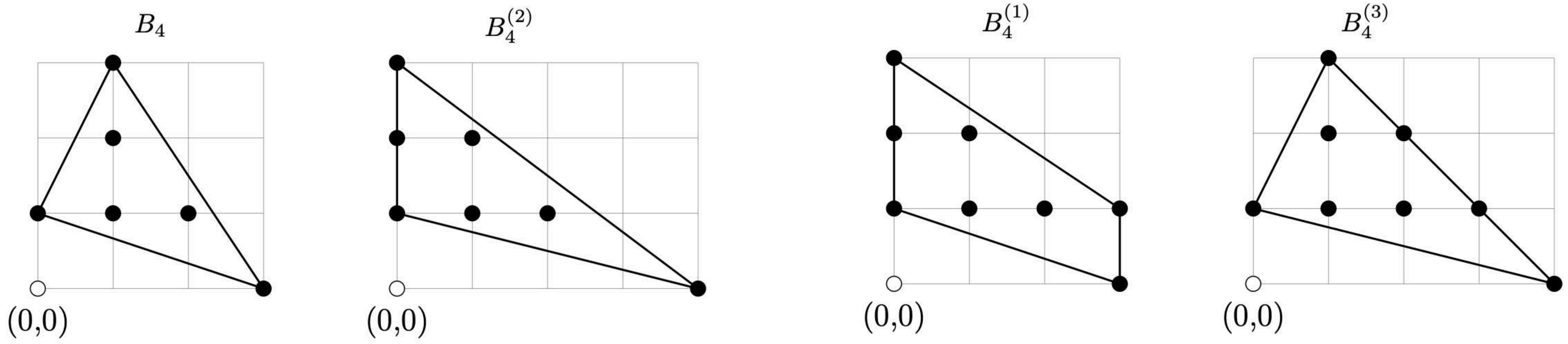
- It can be extracted from intersection numbers of compact 2- and 4-cycles in the resolved Calabi-Yau
  building on [Gordon, Litherland 78]

- For the example of $SU(p)_q$ we get

$$\alpha = \frac{qp(p-1)(p-2)}{6\,\mathrm{gcd}(p,q)^3} \bmod 1$$

- Matches field theory analysis [Benetti Genolini, Tizzano 20; Gukov, Hsin, Pei 20]

- The geometric computation extends immediately to examples that do not admit a Lagrangian description



[Morrison, Schafer-Nameki, Willet 20; Eckhard, Schafer-Nameki, Wang 20]

Example: $AdS_7 \times \mathbb{RP}^4$

Example: $AdS_7 \times \mathbb{RP}^4$

[Dasgupta, Mukhi 95; Witten 95; Hori 98;
Ahn, Kim, Yang 98; Gimon 98; Hanany, Kol 00]

- M-theory on $AdS_7 \times \mathbb{RP}^4$
- Near-horizon limit of stack of M5-branes on $\mathbb{Z}_2$ orbifold singularity (OM5-plane)
- Dual to 6d (2,0) SCFT of type D_N

- This setup offers opportunity to study finite symmetries of a 6d (2,0) SCFT from the gravity dual
    - $\mathbb{Z}_2$ 0-form symmetry
    - $\mathbb{Z}_2$ 2-form symmetry

A cubic term from $AdS_7 \times \mathbb{RP}^4$

- Same strategy as before: we study 11d supergravity on $\mathbb{RP}^4$
- We focus on torsional cohomology classes
- Technical complication: $\mathbb{RP}^4$ is non-orientable; G_4 is odd under parity; we expand it onto classes twisted by orientation bundle

$$H^\bullet(\mathbb{RP}^4;\widetilde{\mathbb{Z}}) = \quad 0 \quad , \quad \underset{\text{generator: } t_1}{\underset{\nearrow}{\mathbb{Z}_2}} \quad , \quad 0 \quad , \quad \underset{\text{generator: } t_3}{\underset{\nwarrow}{\mathbb{Z}_2}} \quad , \quad \mathbb{Z}$$

- Relevant terms

$$\breve{G}_4 = \breve{a}_1 \cdot \breve{t}_3 + \breve{a}_3 \cdot \breve{t}_1 \qquad\qquad a_1 \leftrightarrow \mathbb{Z}_2 \text{ 0-form symmetry} \qquad a_3 \leftrightarrow \mathbb{Z}_2 \text{ 2-form symmetry}$$

- From the cubic CGG coupling in 11d we obtain a cubic coupling in 7d

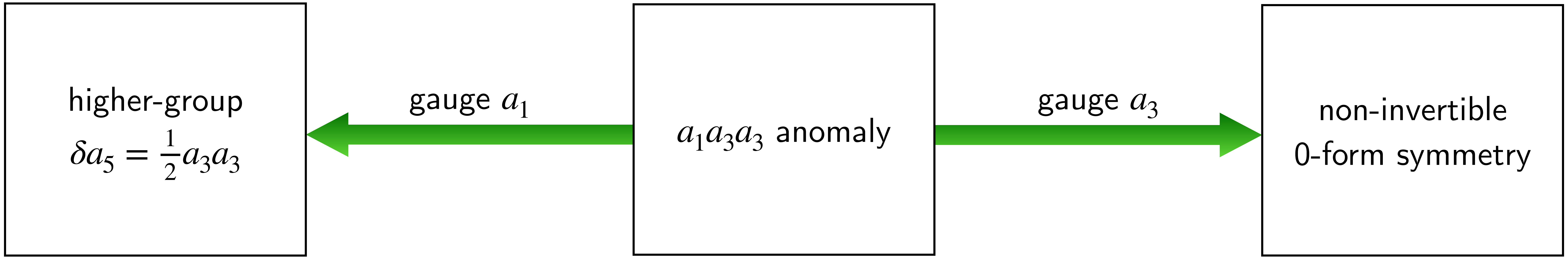
$$S \supset \frac{1}{4}\int_{M_7} a_1 \cup a_3 \cup a_3$$

More precisely the coupling involves a quadratic refinement of the pairing $a_3 \cup a_3$ [Browder 69; Brown 72]

Non-invertible symmetries from $AdS_7 \times \mathbb{RP}^4$

- The presence of a cubic coupling in 7d signals a mixed anomaly between the 0-form and 2-form symmetries
- A mixed anomaly of the form $a_1 a_3 a_3$ allows us to access different variants of the 6d theory with non-trivial generalized symmetries

[Tachikawa 17; Kaidi, Ohmori, Zheng 21]



higher-group
$\delta a_5 = \frac{1}{2} a_3 a_3$

gauge a_1

$a_1 a_3 a_3$ anomaly

gauge a_3

non-invertible
0-form symmetry

- Cfr. non-invertible symmetries in 6d SCFTs in [Lawrie, Yu, Zhang 23; Apruzzi, Schafer-Nameki, Warman 24]

Alternative picture: branes in $AdS_7 \times \mathbb{RP}^4$

[**FB**, Del Zotto, Minasian 24]

- We can also confirm the above analysis by studying "branes at infinity" in AdS_7
- They wrap cycles in $\mathbb{RP}^4$
- Relevant branes:

M2 on $\Sigma_3 \times \widetilde{\mathrm{pt}}$ M5 on $\Sigma_3' \times \mathbb{RP}^3$

M2 on $\Sigma_1 \times \mathbb{RP}^2$ M5 on $\Sigma_5 \times \mathbb{RP}^1$

Homology of $\mathbb{RP}^4$

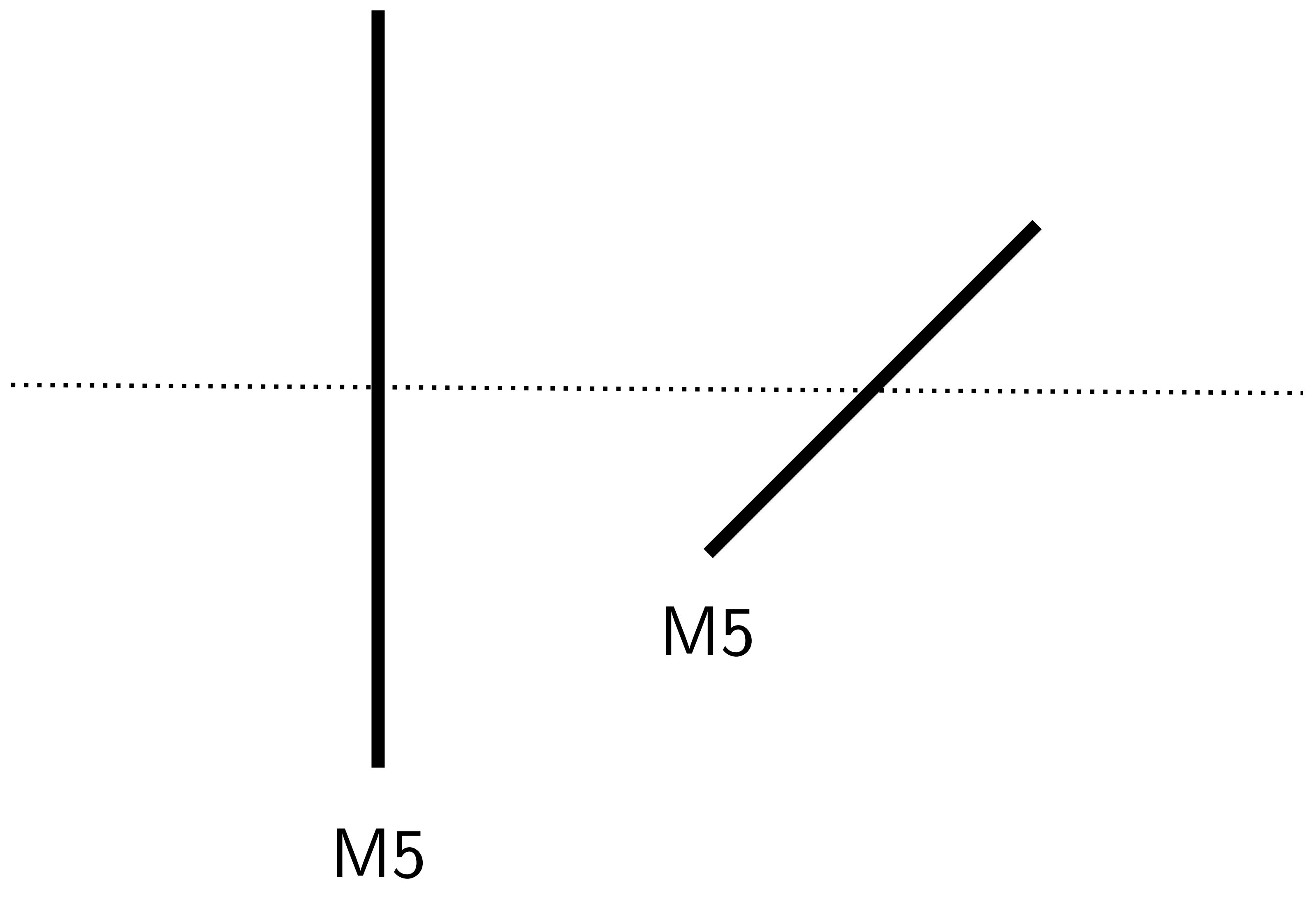
	untwisted	twisted
free	pt	$\mathbb{RP}^4$
torsional	$\mathbb{RP}^1$, $\mathbb{RP}^3$	$\widetilde{\mathrm{pt}}$, $\mathbb{RP}^2$

- In particular, we can identify a "smoking gun" of non-invertible symmetry making use of **Hanany-Witten transitions**

A brief reminder on Hanany-Witten transitions

- We study a configuration of two M5-branes that share 2 directions of spacetime
- In the remaining 9 dimensions, they appear as 4d objects and can link
- When they are moved past each other, an M2-brane is created, stretching between them

	0	1	2	3	4	5	6	
M5	X	X	X					$\mathbb{RP}^3$
M5	X	X		X	X	X	X	$\mathbb{RP}^1$
M2	X	X					X	$\widetilde{\mathrm{pt}}$



Can be seen from Bianchi identities with brane sources

$$dG_4 = \delta_5^{\mathrm{M5}}$$

$$dG_7 = \frac{1}{2}G_4^2 + X_8 + \delta_8^{\mathrm{M2}}$$

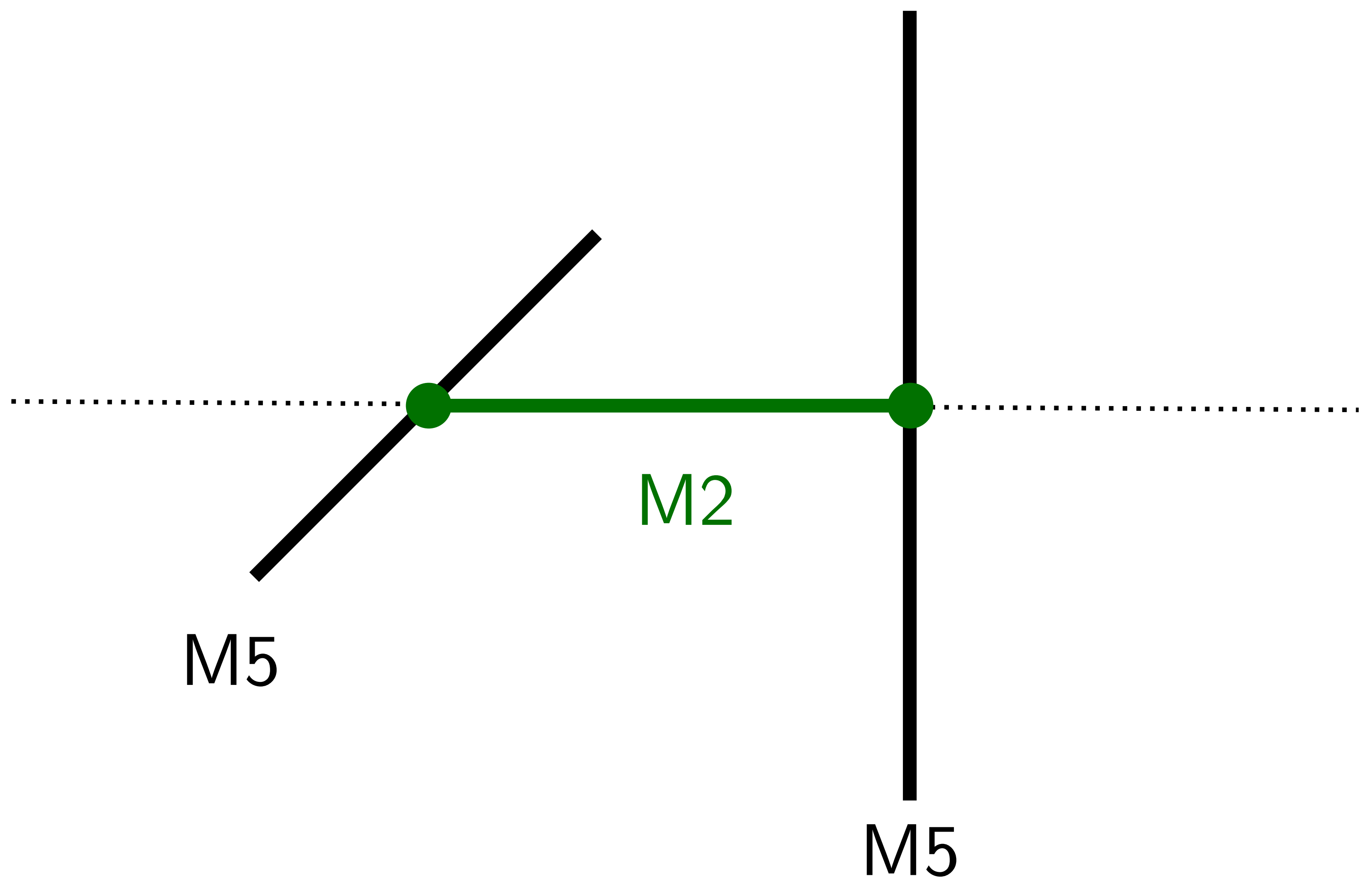
$$d\delta_8^{\mathrm{M2}} = -\,G_4\delta_5^{\mathrm{M5}}$$

[Hanany, Witten; ...; Marolf; ...]

A brief reminder on Hanany-Witten transitions

- We study a configuration of two M5-branes that share 2 directions of spacetime
- In the remaining 9 dimensions, they appear as 4d objects and can link
- When they are moved past each other, an M2-brane is created, stretching between them

	0	1	2	3	4	5	6	
M5	X	X	X					$\mathbb{RP}^3$
M5	X	X		X	X	X	X	$\mathbb{RP}^1$
M2	X	X					X	$\widetilde{\mathrm{pt}}$



Can be seen from Bianchi identities with brane sources

$$dG_4 = \delta_5^{\text{M5}}$$

$$dG_7 = \frac{1}{2}G_4^2 + X_8 + \delta_8^{\text{M2}}$$

$$d\delta_8^{\text{M2}} = -\,G_4\delta_5^{\text{M5}}$$

[Hanany, Witten; ...; Marolf; ...]

Application to $AdS_7 \times \mathbb{RP}^4$

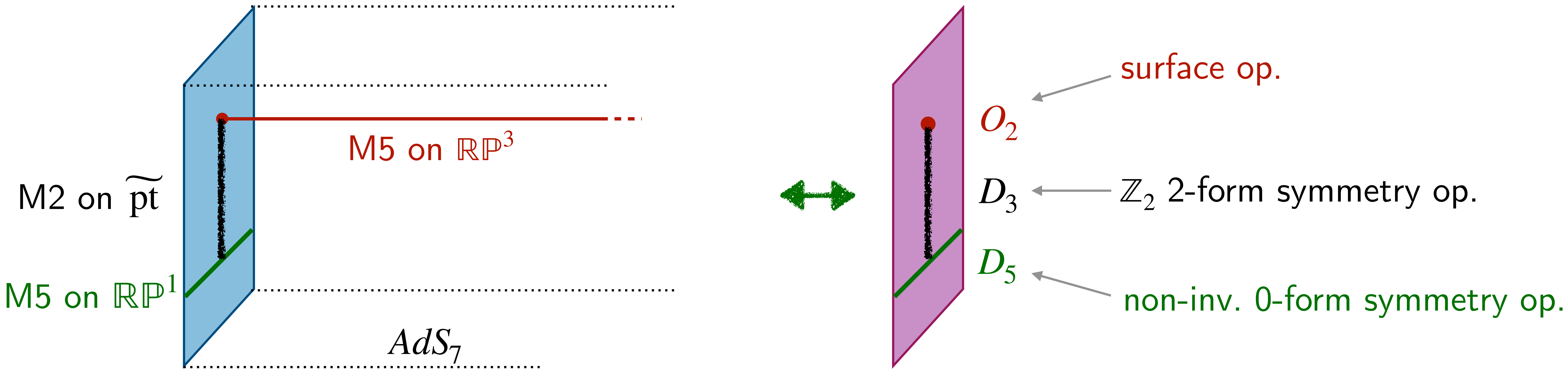
[**FB**, Del Zotto, Minasian 24]

- Relevant branes:

M2 on $\Sigma_3 \times \widetilde{\mathrm{pt}}$ M5 on $\Sigma_3' \times \mathbb{RP}^3$

M2 on $\Sigma_1 \times \mathbb{RP}^2$ M5 on $\Sigma_5 \times \mathbb{RP}^1$

	0	1	2	3	4	5	6	
M5	X	X	X					$\mathbb{RP}^3$
M5	X	X		X	X	X	X	$\mathbb{RP}^1$
M2	X	X					X	$\widetilde{\mathrm{pt}}$



M5 on $\mathbb{RP}^3$

M2 on $\widetilde{\mathrm{pt}}$

M5 on $\mathbb{RP}^1$

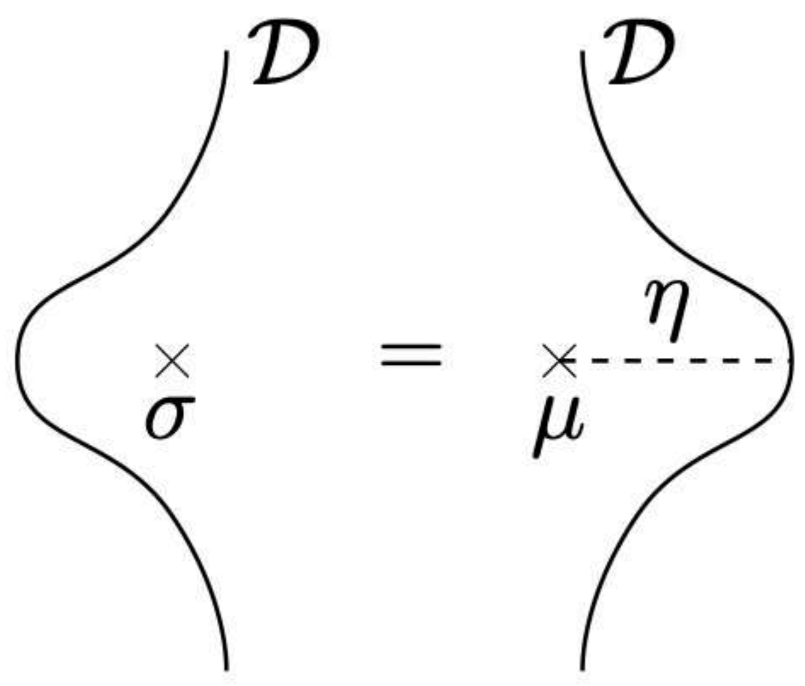
AdS_7

O_2 surface op.

D_3 $\mathbb{Z}_2$ 2-form symmetry op.

D_5 non-inv. 0-form symmetry op.

6d (2,0) SCFT



analogous to
2d critical Ising!

agrees with analysis in [Lawrie, Yu, Zhang 23; Apruzzi, Schäfer-Nameki, Warman 24]

Conclusions and outlook

Conclusions and outlook

Conclusions

- The SymTFT construction is a powerful tool to describe symmetry structures in field theory
- For QFTs realized by a top-down string/M-theory construction

supergravity on link ⟶ Lagrangian of the SymTFT

branes at infinity ⟶ topological operators of the SymTFT

- This framework can capture: anomalies, higher group structures, non-invertible symmetries and their actions

Outlook

- Further extend dictionary between topology/geometry on the string side, and symmetry structures on the QFT side (e.g. discrete isometries of the link)
- Explore more systematically how higher-structures of symmetries (morphisms, associators, ...) can be realized from branes
- Explore possible connections between categorical symmetries and classification of D-brane charges (K-theory)

Thank you!