A reduced semantics for deciding trace equivalence using constraint systems

POST'14

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joint work with	David Baelde LSV	*and*	Stéphanie Delaune LSV

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Our Contribution

Reduce search space of equivalence checking using POR ideas by eliminating a lot of redundancies (for simple processes).

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Terms

$\mathcal{T}$: set of terms + equational theory. *e.g.,* $\mathsf{dec}(\mathsf{enc}(m, k), k) = m$.

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- $P_c ::= 0 \mid \mathtt{in}(c,x).P_c \mid \mathtt{out}(c,m).P_c \mid \mathtt{if}\ T\ \mathtt{then}\ P_c\ \mathtt{else}\ P_c$
- $P_s ::= P_{c_1} | P_{c_2} | \ldots P_{c_n} \qquad c_i \neq c_j$

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Semantics

$$(\{\mathtt{out}(c,m).P\} \uplus \mathcal{P}; \Phi) \xrightarrow{\nu w.\mathtt{out}(c,w)} (\{P\} \uplus \mathcal{P}; \Phi \cup \{w \triangleright m\})$$
if $T \ \wedge\ w$ fresh in Φ

$$(\{\mathtt{in}(c,x).P\} \uplus \mathcal{P}; \Phi) \xrightarrow{\mathtt{in}(c,t)} (\{P[x \mapsto u]\} \cup \mathcal{P}; \Phi)$$
if $t\Phi = u \ \wedge\ fv(t) \subseteq \mathrm{dom}(\Phi)$

Trace Equivalence

Properties:

1. Reachability (*e.g.,* secret, authentification) and
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Trace equivalence

- $\Phi \sim \Phi' \iff (\forall M, N,\ M\Phi = N\Phi \iff M\Phi' = N\Phi')$
- $A \approx B \iff \forall A \xrightarrow{s} A',\ \exists B',\ B \xrightarrow{s} B' \wedge \Phi_{A'} \sim \Phi_{B'}$ and conversely.

Big Picture

Goal

- Motivation: Improve algorithms checking trace equivalence for simple processes
- How: Dramatically decrease the number of interleavings to consider via a reduced semantics

$$\boxed{\begin{array}{c}\longrightarrow\\ \approx\end{array}} \xrightarrow[\text{Thm 1: } \approx = \approx_c]{\textbf{Compression}} \boxed{\begin{array}{c}\longrightarrow_c\\ \approx_c\end{array}} \xrightarrow[\approx_c = \approx_s]{\text{Symbolic}} \boxed{\begin{array}{c}\longmapsto_c\\ \approx_s\end{array}} \xrightarrow[\text{Thm 2: } \approx_s = \approx_d^2]{\textbf{Reduction}} \boxed{\begin{array}{c}\longmapsto_d\\ \approx_d^2\end{array}}$$

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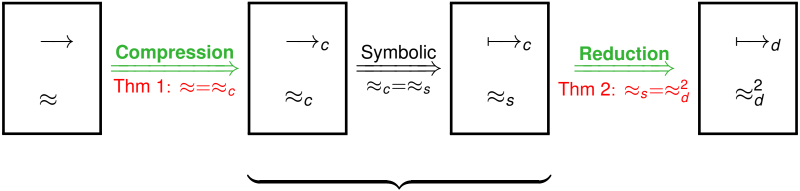
Grouping actions:

- generalization of the idea "force to perform output as soon as possible"
- $\rightarrow_c$ only explores specific traces
- Theorem 1: $\approx=\approx_c$

Big Picture



Symbolic semantics:

- classic step adapted for $\rightarrow_c$

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Analyze dependencies:

- force one order for independent (parallel) actions
- analyze dependencies "on the fly"
- $\longmapsto_d$ explores even less traces
- Theorem 2: $\approx_s=\approx_d^2$

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Grouping actions into *blocks*

$$\mathtt{in}(c,\_)\ldots\mathtt{in}(c,\_).\mathtt{out}(c,\_)\ldots\mathtt{out}(c,\_)$$

via a compressed semantics $\rightarrow_c$.

Compression - Example

Basic rules of $\rightarrow_c$:

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Example

Consider $P = P_1 \mid P_2$ with $P_i = \mathtt{in}(c_i, x).\mathtt{in}(c_i, y).\mathtt{out}(c_i, \langle x, y\rangle)$.

- Semantics $\rightarrow_c$ explores only two interleavings of 6 actions:

  $\mathtt{in}(c_1, x_1).\mathtt{in}(c_1, y_1).\mathtt{out}(c_1, w_1).\mathtt{in}(c_2, x_2).\mathtt{in}(c_2, y_2).\mathtt{out}(c_2, w_2)$

  and

  $\mathtt{in}(c_2, x_2).\mathtt{in}(c_2, y_2).\mathtt{out}(c_2, w_2).\mathtt{in}(c_1, x_1).\mathtt{in}(c_1, y_1).\mathtt{out}(c_1, w_1)$

- semantics $\rightarrow$ explores 20 such interleavings.

Compression - Result

The semantics $\rightarrow_c$ induces a compressed trace equivalence $\approx_c$

Theorem 1

$$A \approx B \iff A \approx_c B$$

Key ideas

- symmetric: remove same interleavings on both sides
- completeness: in any execution, we can swap two actions on different channels (simple processes)

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Benefits

- first optimization that decreases (possibly exponentially many) interleavings to consider
- easy to implement
- allow us to reason with *macro-actions i.e.,* blocks
  $\rightsquigarrow$ reduced semantics

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Inputs messages: infinitely branching $\rightsquigarrow$ symbolic calculus.

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System of Constraints

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$$P = \mathtt{out}(c,k).\mathtt{in}(c,x).\mathtt{out}(c,\langle k,x\rangle).\mathtt{in}(c,y)$$

leads to

$$\mathcal{S} = \{\{w\} \vdash^?_X x, \{w,w'\} \vdash^?_Y y\}$$
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Symbolic process

$$(\mathcal{P}; \Phi; \mathcal{S})$$

Symbolic Calculus - 2

Semantics

$$(\{\mathtt{out}(c,m).P\} \uplus \mathcal{P}; \Phi; \mathcal{S}) \xmapsto{\nu w.\mathtt{out}(c,w)} (\{P\} \uplus \mathcal{P}; \Phi \cup \{w \rhd m\}; \mathcal{S})$$

if w fresh in ϕ

$$(\{\mathtt{in}(c,x).P\} \uplus \mathcal{P}; \Phi; \mathcal{S}) \xmapsto{\mathtt{in}(c,X)} (\mathcal{P}; \Phi; \mathcal{S} \cup \{\mathrm{dom}(\phi) \vdash^?_X x\})$$

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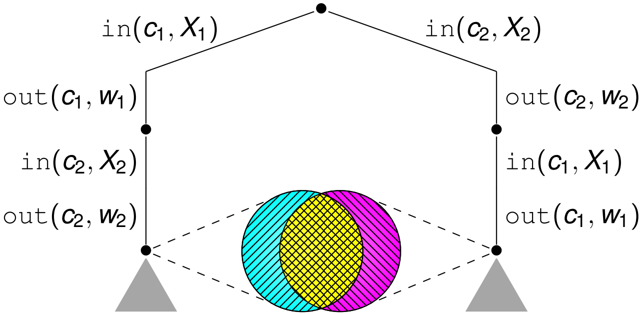
- There already exist several procedures checking equivalence between constraint systems
- Goal: starting with $\longmapsto_c$ (compressed symbolic semantics), reduces the number of interleavings to explore

$$P = \mathtt{in}(c_1, x_1).\mathtt{out}(c_1, k_1).P_1 \mid \mathtt{in}(c_2, x_2).\mathtt{out}(c_2, k_2).P_2$$



Sebastian Mödersheim, Luca Vigano, and David Basin.
Constraint differentiation: Search-space reduction for the constraint-based analysis of security protocols.
Journal of Computer Security, 18(4):575–618, 2010.

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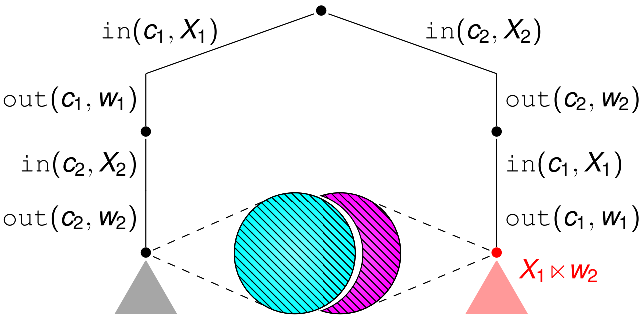

Dependency constraint: X_1 must depend on w_2.

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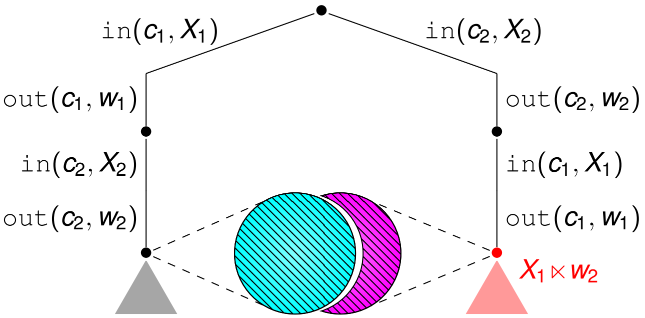

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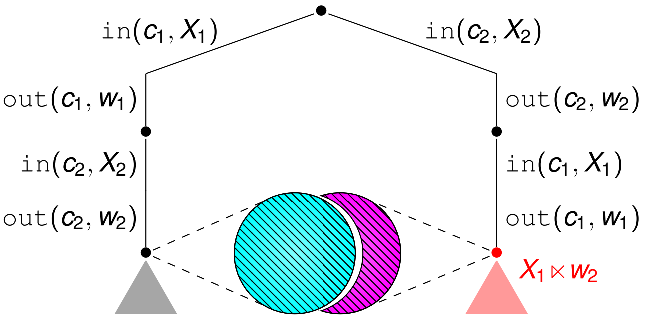

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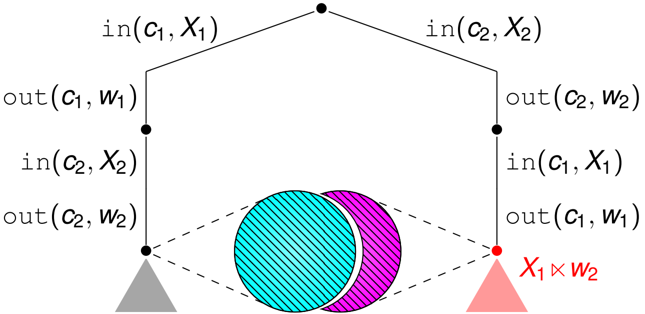

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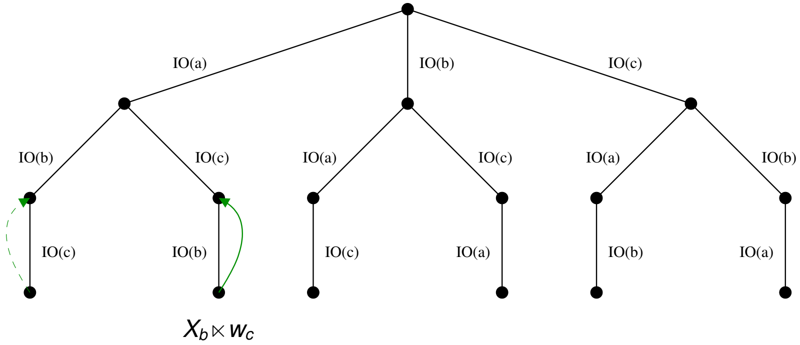
We can add constraints on the fly thanks to an order $<$.

- symmetry: Eliminate same traces on both sides
- Do not remove too much information (intruder can observe the order).

$$P = IO(a)|IO(b)|IO(c) \quad \text{where} \quad IO(l) = \texttt{in}(c_l, X_l).\texttt{out}(c_l, w_l)$$



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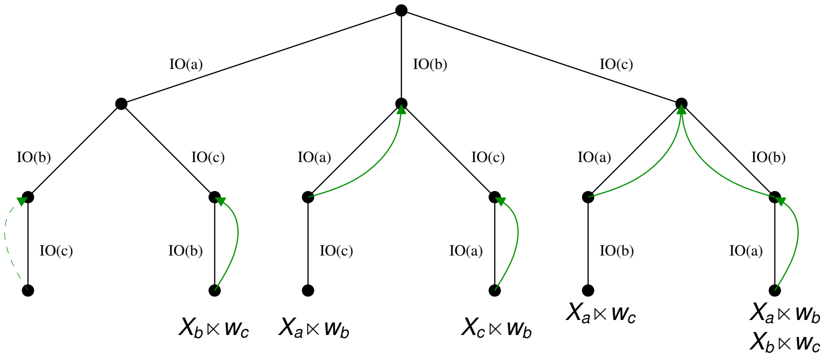

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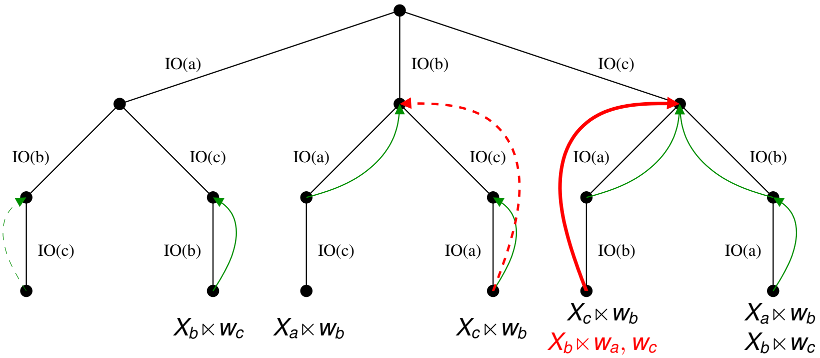

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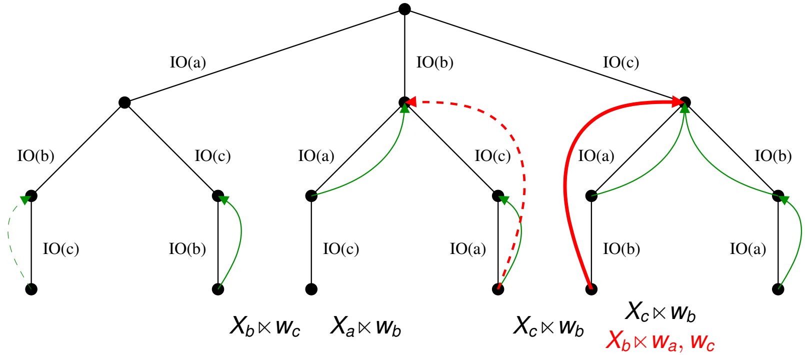

A block on c is executed following t, one input of the block must depend on one output of $\mathrm{dep}(t, c) = \{w_1, w_2 \ldots w_n\}$ if

- $t = t_1.IO(c_1).IO(c_2)\ldots IO(c_n)$
- $c < c_1$;
- $c_2, \ldots, c_n < c$

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Two possible semantics for $\mathrm{Sol}(X \ltimes w)$:

- second order: w occurs in the recipe $X\Theta$
- first order: for all recipe R, if $R\Phi = (X\Theta)\Phi$ then w occurs in R

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Theorem 2

$$\approx_d^2 = \approx_s \qquad \approx_d^1 = \approx_s$$

Idea of the proof

- $[t]$: set of traces modulo valid permutations;
- $\mathrm{Min}([t])$: lexico. minimum of the class.

Lemma 1

If $\rightarrow_c$ explores t from P then it also explores all traces of $[t]$ (+ same resulting frames).

Lemma 2

- If $\rightarrow_c$ explores t from P then $\mapsto_d$ explores $\mathrm{Min}([t])$ (+ same resulting frames);
- $\mapsto_d$ explores no other trace of $[t]$ from P.

Outline

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- New optimizations: compression and reduction;
- applied to trace equivalence checking;
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Tool	Protocol	Size	Ref (s)	Comp (s)	Red (s)
APTE	PA 1 Sess.	2/9/5	0.164	0.012	0.004
	PA 2 Sess.	4/15/5	> 237h	16.72	11.856
	PA 3 Sess.	6/21/5	> 237h	379696	91266
SPEC	2 par	2/22/10	$>$ 20 hours	13853	122.27
	7 par	7/14/2	$>$ 20 hours	13853	370.65

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Any question?

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Benchmarks

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	BAC 1 S./1	4/52/6	13.98	0.02	0.008
	Simple 3 par	3/6/2	0.060	0.004	0.0040
	Simple 5 par	5/10/2	178.8	0.124	0.024
	Simple 7 par	7/14/2	> 163h	8.512	0.196
	Simple 10 par	7/14/2	> 163h	664	1.05
	Complex 4 par	4/10/4	99.87	0.55	0.136
	Complex 7 par	7/16/4	> 163h	198077	363.08
SPEC	2 par	2/22/10	$>$ 20 hours	13853	122.27
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	WMF 1S	3/16/3	65.20	8.01	8.09
	WMF 2S $\perp$	6/24/3	7742.24	3.21	3.30

```
### Description of the role of Alice
let process_Alice k_a k_b =
  new N_a;
  in(a,a);
  out(a,aenc((N_a,pk(k_a)),pk(k_b)));
  in(a,x).

### Description of the role of Bob
let process_Bob k_a k_b =
  in(b,x);
  let (na,pka) = adec(x,k_b) in
  if pka = pk(k_a)
  then new N_b; out(b,aenc((na,N_b,pk(k_b)),pk(k_a)))
  else new N; out(b,aenc(N,pk(k_a))).

### Main
let instance1 =
  new k_a ; new k_b ; new k_c ; out(c,pk(k_a)) ; out(c,pk(k_b)) ;
  out(c,pk(k_c)); ( process_Alice k_a k_b | process_Bob k_a k_b ).

let instance2 =
  new k_a ; new k_b ; new k_c ; out(c,pk(k_a)) ; out(c,pk(k_b)) ;
  out(c,pk(k_c)); ( process_Alice k_c k_b | process_Bob k_c k_b ).

equivalence instance1 and instance2.
```

Outline

Informal Analogy: Focusing - Compression

- Compression: complete (wrt. equivalence) reduction of search space
- Focusing: complete (wrt. provability) reduction of search space

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Processes	LL formulae	polarity
$\mathtt{in}(c, x).P$	$\exists x.A$	synchronous
$\mathtt{out}(c, t).P$	$\forall x.A$	asynchronous
$P_1 \mid P_2$	$P_1 \parr P_2$	asynchronous
$!^{a, \overrightarrow{c}} P$	$?P$	asynchronous

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compressed execution	focused derivation
completeness of $\approx_c$	completeness of focused proof system

Focused semantics

Focused execution: alternation of two phases

1. *Asynchronous phase:*
2. *Synchronous phase:*

Focused semantics

Focused execution: alternation of two phases

1. *Asynchronous phase:* **When:** $\exists$ output process.
   What: Only output actions are available.
2. *Synchronous phase:*

Focused semantics

Focused execution: alternation of two phases

1. *Asynchronous phase:* **When:** $\exists$ output process.
   What: Only output actions are available.
2. *Synchronous phase:* **When:** all processes start with an input or !.
   What: Choose one input process (or replicate one !): its is now under focus. Force to perform all its inputs until it reveals an asynchronous action.

Results (work in progress)

Even more effective compression handling REPLICATION and nested parallel compositions for determinate processes.

$$\text{if } A \xrightarrow{t_r} A_1 \text{ and } A \xrightarrow{t_r} A_2 \text{ then } A_1 = A_2.$$

Proof of completeness following the informal analogy and:

Dale Miller and Alexis Saurin.
From proofs to focused proofs: a modular proof of focalization in linear logic.
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- Strictly better: does the same for simple processes.
- Very modular: can be applied to any π-calculus-like.