Group 5

Early Papers on Quantum Field Theory (1929–1930)

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Whenever we look back at the development of physical theory in the period between 1925 and 1930 we feel the joy and the shock of the miraculous. From our present modest expectations about the rate of progress in the understanding and solution of fundamental problems and about the amount of work a single person can possibly master per year, it seems already an extraordinary harvest for a few men to establish a consistent quantum theory in the regime of nonrelativistic mechanics and an ample task for many years to understand the correct interpretation of the formalism, to discuss the strange and novel features of its conceptual structure, to apply the theory to the analysis of the immense experimental material in atomic spectroscopy, collision process, molecular structure .... Yet, almost immediately after the birth of quantum mechanics and side by side with the problems mentioned above other fundamental questions were tackled: the incorporation of the principles of special relativity into quantum physics, the quantum theory of the Maxwell field up to the establishment of a coherent relativistic theory of electromagnetism interacting with matter. The papers of Heisenberg and Pauli in 1929 and 1930 (Nos. 1 and 2, pp. 8 – 68 and 69 – 91 below[1]) mark in a certain sense the successful conclusion of this effort. Although it took the following decade to work out the applications of electrodynamics in the lowest order of approximation, another decade to develop a usable systematic perturbation theory, a third decade to understand the interpretation of quantum field theories without recourse to perturbation expansions, and although some of the fundamental difficulties of the formalism encountered by Heisenberg and Pauli remain unresolved even now, one can say that by 1930 the language was created whose grammar and basic vocabulary is used to this day. The quantum theory of the Maxwell-Dirac fields, whose equations were written down in the papers mentioned, not only turned out to be extraordinarily successful in its own regime but became the prototype of present day quantum field theories in all "high energy" physics.

The essential tenets were:

1) A classical field theory is analogous to a mechanical system with continuously many degrees of freedom. Thus a quantum field theory should result if one regards the field quantities of the classical theory as non-commuting objects ("q-numbers"). The commutation relations can be guessed from the Lagrangian formalism in the same manner as in quantum mechanics; the equations of motion remain formally unchanged by the transition to quantum theory.

To implement this program, Pauli had been interested since early 1927 in the generalization of calculus to a continuum of variables, a subject frequently al-

[1] All page numbers refer to pages in this volume. (Editor)

luded to as "*Volterra-Mathematik*" in the correspondence between Pauli and Heisenberg in those years. Actually it turned out that very little of functional analysis was needed for the formulation of the basic equations. The replacement of differential quotients by variational derivatives and of the Kronecker symbol by Dirac's δ-function sufficed. The hard part of functional analysis, namely the integration process, was not needed in the formulation and could be avoided even in the solution as long as one relied on perturbation expansions.

2) Matter should be described – even on the classical level – by a relativistically covariant wave field. Specifically, in the case of electrons, the Dirac equation appeared to be the appropriate wave equation. In the quantum theory of a Dirac field, where the field quantities become non-commuting objects, the Pauli principle can be incorporated in the way shown by Jordan and Wigner [1]: The classical Poisson brackets should be replaced not by commutator but by anticommutator brackets.

Writing down the classical Lagrangian involving the electromagnetic vector potential Φ_μ and the Dirac field Ψ_α so that it yields the interacting Maxwell-Dirac equations on the classical level, an unexpected difficulty arose in the canonical quantization procedure. In fact, this difficulty appeared already in the Hamiltonian treatment of electrodynamics without interaction with charged matter. It turned out that the Lagrangian is degenerate in the sense that one of the canonical momenta vanishes and thus cannot satisfy the canonical commutation relations. This problem held up the writing of the 1929 paper for at least a year until Heisenberg found a trick to overcome it by adding the term $\varepsilon \partial \Phi_\mu / \partial x_\mu$ to the Lagrangian and arguing that at the end of calculations the limit $\varepsilon \to 0$ could be taken.

Before discussing in more detail the contents of the group of papers (Nos. 1, 2) let us look briefly at related work by other authors in this period. The idea that the Maxwell field could be regarded as a mechanical system with infinitely many degrees of freedom was, of course, general knowledge. The development of a quantum theory of radiation based on the representation of the radiation field by an infinite collection of quantum mechanical oscillators had already been suggested in the last section of the famous *Dreimännerarbeit* (No. 4 of Group 3, see AI, pp. 446–455), where Jordan discussed the theory of fluctutations of the radiation field in a cavity; it was carried further in Dirac's papers on the emission, absorption, and dispersion of light [2]. In this context Dirac developed time dependent perturbation theory, giving a general formula for transition probabilities per unit time, a formula so central in all applications during the next decades that Fermi called it later "the golden rule of quantum mechanics". Nevertheless in order to incorporate relativistic invariance it appeared necessary to focus on the field quantities as functions in space-time and formulate the basic equations without expansion of the field into normal modes. That this could be done was demonstrated for the free Maxwell field in a paper by Jordan and Pauli [3]. But the 4-dimensional formalism used there could not easily be generalized to the interacting case. Therefore the quest for a Hamiltonian formulation arose.

Much more mysterious than the first tenet was the second one, the quantization of the matter field. In a letter dated February 23, 1927 Heisenberg wrote to Pauli: "That one should quantize the Maxwell equations to obtain the photons etc. à la Dirac I gladly believe, but one should then later on perhaps also quantize the de Broglie waves to obtain charge, mass and statistics of electrons and nuclei". ([4], p. 376: *"Daß man die Maxwell'schen Gleichungen quanteln soll, um die Lichtquanten usw. à la Dirac zu bekommen, glaub ich schon, aber man soll dann vielleicht doch auch später die de Broglie Wellen quanteln, um Ladung, Masse und Statistik der Elektronen und Kerne zu bekommen"*.) Indeed, just at that time Jordan and Klein did carry through the (canonical) quantization of matter wave fields with the surprising result that the quantized wave field was equivalent to a description of an arbitrary number of noninteracting particles satisfying Bose statistics [5]. This was followed by the paper of Jordan and Wigner showing how the quantization rules could be adapted to describe a many body system with Fermi statistics [1].

In January 1928 Dirac presented his relativistic wave equation of the electron [6]. This provided new impetus and new puzzles. On February 15, 1928, Pauli wrote to Kronig: "Now Dirac's paper has been published. It is marvellous how everything fits. Mr. Gordon could verify without difficulties that ... the old Sommerfeld formula [fine structure of H-spectrum] ... follows rigorously". ([4], p. 435: *"Nun ist ja die Dirac'sche Arbeit erschienen. Es ist wunderbar, wie das alles stimmt. Herr Gordon konnte ohne Schwierigkeiten nachrechnen, daß ... die alte Sommerfeldsche Formel ... in Strenge folgt"*.) Two days later Pauli wrote a long letter to Dirac ([4], pp. 435 – 438) explaining in detail the present status of the Heisenberg-Pauli program, asking for Dirac's opinion about the principal difficulty encountered: the self energy of an electron. The end of the letter referred to the new puzzle raised by Dirac's paper concerning states of positive and negative charges and transitions between them. These two difficulties were to become dominant themes in the following years. (Compare the introduction by A. Pais to the next group of publications of Heisenberg below.) The impact of the second can be illustrated by Heisenberg's remark in a letter to Jordan in April 1928: "In complete apathy and despair about the present status (or should one rather say pigsty) of physics, which Dirac's beautiful but incorrect papers transformed into a hopeless maze of formulas ..." ([7]: *"In völliger Apathie und Verzweiflung über den gegenwärtigen Stand (oder sollte man sagen: Saustall) der Physik, der durch Dirac's ebenso schöne wie unrichtige Arbeiten in ein hoffnungsloses Chaos von Formeln ... verwandelt wurde ..."*).

In spite of the fact that the two major difficulties could not be resolved, Heisenberg and Pauli decided in January 1929 to complete and publish their work on quantum electrodynamics after Heisenberg had finally succeeded in overcoming the formal difficulty posed by the degeneracy of the Lagrangian by the method already mentioned. (No. 1, p. 8 – 68 below) A tremendous amount of work had to be done in a short time because of Heisenberg's pending departure for a longer stay in the USA in March 1929 [8]. Specifically, the proof of Lorentz invariance of the commutation relations was rather cumbersome in the first paper, the approximation methods used to make contact with the quantum mechanics of several electrons in configuration space were an agony. (In this respect the parallel work by Fermi [9] provided a more elegant method: the elimination of the longitudinal field.) Finally the authors wanted to present at least one example of an application to a hitherto untreated problem and chose the radiation associated with the tunelling of a charged particle through a barrier. This choice is particularly interesting because behind it stood the hope that this effect might be responsible for the continuous energy spectrum of β-decay electrons for which experimental evidence started to accumulate at the time.

Part II of the paper submitted in September 1929 has as its central theme the invariance properties and associated conservation laws of quantum electrodynamics (No. 2, pp. 69 – 91 below). The proof of Lorentz invariance becomes much more transparent, following some suggestions by von Neumann. Also included are a beautiful discussion of gauge invariance and charge conservation, a demonstration that the term added to the Lagrangian in the first paper in order to make the Hamiltonian formalism work does not affect any relations between gauge invariant quantities so that this artifact could actually be avoided, a very clear exposition of the relation to Fermi's treatment of the unobservable parts of the electromagnetic potentials [9], and a new discussion of the transformation to configuration space (partly due to Oppenheimer). Interestingly enough the authors remark in passing that gauge invariance does not forbid the annihilation of oppositely charged particles and write down a conceivable interaction term which would procedure transitions from electron + proton to pure radiation.

With the second paper of Heisenberg and Pauli quantum electrodynamics reached a degree of completeness. The formalism was developed as far as possible without the solution of the fundamental problems of "self energy" and "Dirac jumps". Both problems were recurrent themes in the following years (compare A. Pais, loc. cit.).

We shall mention here only the first paper by Heisenberg on the self energy problem (see paper No. 1 of the following Group 6, pp. 106 – 115 below). It is remarkable both because of its brave and ingeneous attempt to solve the coupled equations in the limit of zero (bare) electron mass, an attempt which unfortunately did not prove fruitful at the time, and because the idea of a fundamental length – which occupied much of Heisenberg's thoughts in later years – was briefly disucced and dismissed there: "It seems – for the time being – better not to introduce the length r_0 in the theory but to stick to the relativistic invariance." (see p. 107: "*Es erscheint also einstweilen richtiger, die Länge r_0 nicht in die Grundlagen der Theorie einzuführen, sondern an der relativistischen Invarianz festzuhalten.*")

References

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2 P.A.M. Dirac: The quantum theory of emission and absorption of radiation. Proc. Roy. Soc. (London) **A114**, 243 – 265 (1927);
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3 P. Jordan, W. Pauli: Zur Quantenelektrodynamik ladungsfreier Felder. Z. Phys. **47**, 151 – 173 (1928)

4 W. Pauli: *Wissenschaftlicher Briefwechsel/Scientific Correspondence, Volume I: 1919 – 1929* (Springer-Verlag, New York, Heidelberg, Berlin 1979)

5 P. Jordan, O. Klein: Zum Mehrkörperproblem der Quantentheorie. Z. Phys. **45**, 751 – 765 (1927)

6 P.A.M. Dirac: The quantum theory of the electron. Proc. Roy. Soc. (London) **A117**, 610 – 624 (1928)

7 Letter from Heisenberg to Jordan. I am indebted to Dr. H. Rechenberg for this quotation.

8 Compare the letter of Pauli to O. Klein, dated February 18 and March 16, 1929 ([4], pp. 488 – 492, 494 – 495). Concerning the clumsiness of some arguments in the paper (No. 1) he wrote: "This is the curse of overseas trips by European physicists." ([4], p. 494: "*Das ist der Fluch der Amerika-Reisen europäischer Physiker.*")

9 E. Fermi: Sopra l'elettrodinamica quantistica. Rend. Accad. Lincei **9**, 881 – 887 (1929)