A DETERMINATION OF THE NUCLEAR SIZE OF COMET HALE–BOPP (C/1995 O1) *

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Abstract. The signal of the nucleus was digitally extracted from six images of the innermost coma of this comet, obtained with the Hubble Space Telescope's Wide-Field Planetary Camera 2 in the planetary mode between October 23, 1995 and October 17, 1996. Two different anisotropic, power-type laws were used to filter out the contribution from the dust coma: one peaking at the center of the elliptical surface brightness distribution (law A), the other peaking at its focus (law B). The nuclear R magnitudes in the Cousins system, reduced to a zero phase angle and to 1 AU from Earth and the Sun with a phase coefficient of 0.035 mag/deg and an inverse square distance power law, are found to average 9.46 $\pm$ 0.07 and 9.48 $\pm$ 0.18 when law A and law B are applied, respectively. These results become 9.49 $\pm$ 0.07 and 9.51 $\pm$ 0.17, when the nucleus signal on the October 1995 image is assumed to consist of a sum of the contributions from two unresolved nuclear components. In either scenario, no systematic variations are apparent in the nuclear brightness with time, which suggests the absence of any significant contamination of the extracted nuclear signal by the coma. Assuming a geometric albedo of 4 percent, the corresponding effective nuclear diameter amounts to 71 $\pm$ 4 km (formal error). This result substantially exceeds the size estimates published by Weaver et al., which are based only on the October 1995 observation and which were obtained with the help of a different reduction method. Runs in which a power-type law fitting the contribution from the coma was assumed to hold all the way to a small fraction of a pixel from the nucleus led to distinctly inferior solutions and yielded spurious values $\ll$70 km for the nuclear diameter.

Keywords: Comet Hale–Bopp, deconvolution technique, nuclear magnitude, nuclear size

1. Introduction

Considerable activity of comet Hale–Bopp at large heliocentric distances gave rise to early speculations that this comet could become a spectacular object (Marsden, 1995). Yet, the only nuclear-size determination published before 1998 was by Weaver et al. (1997), based on their inspection of the image taken with the planetary mode charge coupled device (CCD) of the Wide-Field Planetary Camera 2 (WFPC-2) of the Hubble Space Telescope (HST) on October 23, 1995. After an extrapolated contribution from the coma had been removed, the residual intensity in the brightest pixel was assumed to be due to scattered light from the nucleus of a geometric albedo of 4 percent. Weaver et al. concluded that the nucleus was at least

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TABLE I

Basic information on the employed HST observations and their geometry

Date 1995/96 (UTC)	Time from perihelion, $t - T$ (days)	Distance from Earth (AU)	Distance from Sun (AU)	Phase angle	Position angle of radius vector	Averaged frame exposure time (s)
Oct. 23.27067	−525.8674	6.709	6.351	8°.16	90°.7	180
May 20.45470	−315.6833	3.679	4.354	10.81	256.6	20
June 22.48838	−282.6497	3.029	4.009	4.45	231.2	20
July 25.59949	−249.5386	2.744	3.652	8.24	112.6	16
Sept.23.17578	−189.9623	2.946	2.979	19.49	90.8	14
Oct. 17.63281	−165.5052	3.041	2.689	18.76	81.7	10

27 km in diameter but that it could be as large as 42 km in diameter, depending on the systematic effects involved. This is some 3–4 times as large as the nucleus of Halley's comet and several tens of times as massive.

2. The Observations

I report on the results of my analysis of the digital charts of six images taken between October 23, 1995 and October 17, 1996, in the planetary mode CCD of the HST's WFPC-2 (a pixel size of 0.0455 arcsec). Applied throughout was an F675W filter; the conversion of the filter magnitudes to the R magnitudes in the Cousins system is discussed by Weaver et al. (1997). The images, kindly provided by H. A. Weaver after they had been digitally cleaned, are averages of a few exposures. Table I lists the observation time, the geometry of the Sun–Earth-comet configuration, and the averaged frame exposure time for each set of images taken. Available in practice were signal distributions in fields of 15 pixels, or nearly 0.7 arcsec, across and centered on the pixel of peak brightness. The samples employed are made up of 157 pixels. The location of each pixel is described by its coordinates $\{X, Y\}$.

3. The Deconvolution Technique

The applied technique that allows one to extract the nucleus signal from a digital chart is described elsewhere (Sekanina, 1995), except for one major improvement mentioned below. The observed surface brightness distribution was available as an array of pixel signals measured in CCD analog-to-digital intensity units (ADU per pixel^2). A background signal of 3 ADU per pixel^2 was subtracted. The net pixel signals consist of a convolved sum of these contributions: (i) from an extended source (the coma), (ii) from the central nucleus, and (iii) from additional point sources, some of which could represent genuine companions or nuclear fragments, while others might turn out to be fictitious condensations of light, of instrumental or unknown origin.

The model point spread function (PSF) of a point source such as the nucleus is approximated by a quasi-Gaussian law. The PSF's surface brightness distribution $b_{\rm psf}(x, y)$ at a location, whose coordinates are $\{x, y\}$ (in arcsec from the PSF's peak) and whose angular distance from the peak is $(x^2 + y^2)^{1/2}$, is expressed (in ADU per arcsec^2) as

$$b_{\rm psf}(x, y) = b_* \exp\left[-\left(\frac{x^2 + y^2}{2\sigma_{\rm psf}^2}\right)^{\nu_{\rm psf}}\right], \tag{1}$$

where $\sigma_{\rm psf} > 0$ is the PSF's dispersion parameter (in arcsec), $\nu_{\rm psf} > 0$ is a dimensionless constant, and $b_* = b_{\rm psf}(0, 0)$ is the peak surface brightness. The integrated brightness I_* (in ADU), of the point source is

$$I_* = 2\pi b_* \sigma_{\rm psf}^2 \nu_{\rm psf}^{-1} \Gamma(\nu_{\rm psf}^{-1}), \tag{2}$$

where $\Gamma(z)$ denotes the Gamma function of argument z. The appropriate parametric values for the F675W filter are $\sigma_{\rm psf} = 0.008126$ arcsec and $\nu_{\rm psf} = 0.318$. Each point source is then characterized by three constants: the total signal I_* and the pixel coordinates X_* and Y_* of the PSF's peak.

For the surface brightness distribution of an extended source, $b_{\rm ext}(x, y)$, two laws (after convolution with the PSF) are applied. They both allow for anisotropy, with the emission extending the farthest along the x axis. Law A is described by an ellipse, with the peak surface brightness b_0 situated at its center,

$$b_{\rm ext}(x, y) = \frac{b_0}{1 + \left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{y}{\sigma_y}\right)^2\right]^{\nu/2}}, \tag{3}$$

while law B has its peak brightness at one of the foci,

$$b_{\rm ext}(x, y) = \frac{b_0}{1 + \left[\frac{\sigma_x (x^2 + y^2)^{1/2} - x(\sigma_x^2 - \sigma_y^2)^{1/2}}{\sigma_y^2}\right]^{\nu}}, \tag{4}$$

where σ_x and σ_y are the maximum and minimum dispersions of the surface brightness distribution along, respectively, the x and y axes ($\sigma_x \geq \sigma_y$) and ν is the exponent of the power law. The coordinates $\{x, y\}$ are reckoned from the ellipse's

TABLE II
The brightness and size of the nucleus of comet Hale–Bopp

Date 1995/96 (UTC)	Coma law employed	Apparent magnitude $R(r, \Delta, \phi)$	Normalized magnitude $R(1, 1, 0)$	Nuclear diameter (km)	Mean residual (ADU)	Number of point sources
Oct. 23.27[a]	A	17.80	9.37	75.8	± 20.5	2
	B	17.79	9.36	76.2	± 21.0	2
May 20.45	A	15.79	9.39	75.1	± 19.1	4
	B	15.64	9.24	80.4	± 20.1	2
June 22.49	A	15.06	9.48	72.0	± 26.9	6
	B	15.06	9.48	72.0	± 29.7	4
July 25.60	A	14.85	9.56	69.3	± 17.3	9
	B	14.87	9.57	68.9	± 19.5	7
Sept. 23.18	A	14.86	9.46	72.7	± 23.8	6
	B	14.84	9.44	73.2	± 50.6	3
Oct. 17.63	A	14.70	9.48	72.0	± 33.8	2
	B	14.98	9.76	63.4	± 28.6	6

[a] If the point source consists of contributions from two unresolved nuclear components, the primary and a satellite (Sekanina, 1997–1999), the brightness of the primary requires a correction of +0.21 mag; the nuclear diameters become 68.8 and 69.2 km, respectively.

center, when law A is employed, or from its focus in the case of law B. Each extended source is fully described by seven constants: the peak surface brightness b_0, the pixel coordinates of its location $\{X_0, Y_0\}$, the dispersions σ_x and σ_y, the exponent ν, and the angle θ that the direction of the least steeply declining brightness (the x axis) subtends with the X axis.

The deconvolving optimization procedure is an iterative least-squares differential-correction technique, which was described in detail by Sekanina (1995). Solving for $N_{\rm pt}$ point sources and $N_{\rm ext}$ extended sources involves the determination of $(3N_{\rm pt} + 7N_{\rm ext})$ parameters.

4. Results and Conclusions

The integrated signals of the nucleus yielded by the applied deconvolution procedure are presented in Table II. The columns list the observation time, the assumed distribution law for the extended source (coma), the apparent R magnitude of the nucleus in the Cousins system (at a heliocentric distance r, a geocentric distance Δ, and a phase angle ϕ), the nuclear R magnitude normalized to $r = \Delta = 1$ AU and $\phi = 0°$ (assuming the phase law has a slope of 0.035 mag/deg), the effective nuclear diameter (at an assumed geometric albedo of 4 percent), the mean pixel-

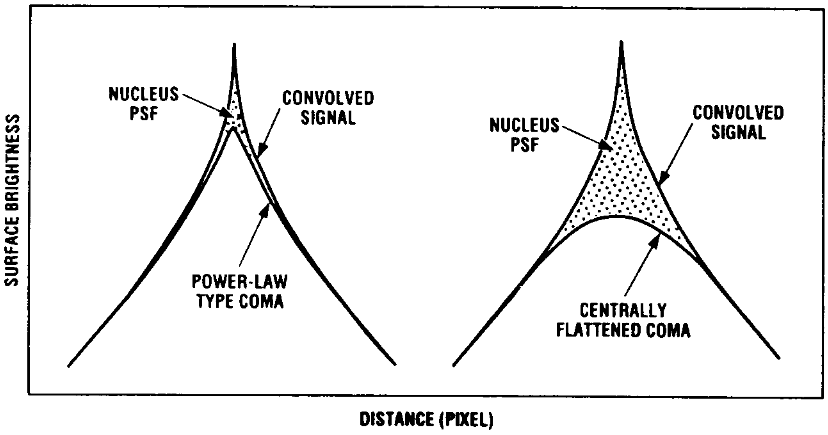

Figure 1. Schematic representation of two scenarios for a convolved signal distribution of the nucleus whose position coincides with the brightness maximum of a coma. The peak convolved signal of a given magnitude requires the nucleus (the shaded area) to be fainter when the coma closely follows a power law nearly to its very center (on the left) than when it progressively flattens out toward its maximum (on the right).

signal residual for the best converging solution, and the number of point sources introduced.

The table offers consistent results for the nuclear dimensions throughout the entire 12-month period, independent of the coma law employed. The average normalized R magnitude of the nucleus is 9.46 ± 0.07 from the runs based on law A and 9.48 ± 0.18 from those based on law B. If the signal of the central peak on the October 1995 image involves a sum of the contributions from two unresolved components – the primary and a satellite whose brightness is 0.21 the primary's brightness (Sekanina, 1997–1999) – then the average normalized R magnitude of the primary nucleus becomes 9.49 ± 0.07 and 9.51 ± 0.17 from the runs based, respectively, on law A and law B. The average effective nuclear diameter is found to amount to 71 ± 4 km, indicating that Hale–Bopp, a true giant especially among Earth-approaching cometary nuclei, is seven times as large and several hundred times as massive as Halley, if the nuclear geometric albedos of the two comets are identical.

The derived size significantly exceeds the value estimated by Weaver et al. (1997). This discrepancy is unquestionably due to differences in accounting for the contribution from the extended source in the nucleus proximity, as shown in Figure 1. The surface brightness of the coma on the left-hand side follows a power law nearly to its very center, while on the right-hand side it levels off at much larger distances. Obviously, the peak convolved signal of a given magnitude requires a fainter point source on the left than on the right. Tests of the applied deconvolution technique on synthetic images, kindly provided by H. A. Weaver, indicate that the effective dispersion $\sigma_{\rm eff} = \sqrt{\sigma_x \sigma_y}$ for the coma signal distributions

TABLE III

Comparison of solutions with two point sources and one extended source (assumed law A) fitting the brightness distribution of comet Hale–Bopp on the image taken October 23, 1995

Parameters forced	Effective nuclear diameter (km)		Coma center from nucleus		Effective dispersion	Law's exponent	Mean residual	Maximum systematic
	1 comp.	2 comp.	km	P.A.	σ_{eff} (arcsec)	ν	(ADU)	residuals (ADU)
None	75.8	68.8	485	7°	0.160	1.24	± 20.5	0
σ_{eff}	77.1	70.0	520	8	(0.040)	0.73	± 23.5	30
σ_{eff}, X_0, Y_0	67.8	61.6	(0)	–	(0.040)	0.86	± 37.7	70
σ_{eff}, X_0, Y_0	64.6	58.6	(0)	–	(0.020)	0.83	± 37.5	70
σ_{eff}, X_0, Y_0, ν	62.8	57.0	(0)	–	(0.040)	(1.00)	± 38.5	70
σ_{eff}, X_0, Y_0, ν	53.6	48.7	(0)	–	(0.020)	(1.00)	± 38.7	80
σ_{eff}, X_0, Y_0, ν	52.6	47.8	(0)	–	(0.040)	(1.20)	± 43.3	90
σ_{eff}, X_0, Y_0, ν	44.5	40.4	(0)	–	(0.030)	(1.20)	± 43.1	80
σ_{eff}, X_0, Y_0, ν	26.7	24.2	(0)	–	(0.020)	(1.20)	± 41.8	80

(after convolution with the PSF) that closely follow a power law to the center is typically 0.02–0.04 arcsec, less than the pixel size. As illustrated in Table III for the image of October 23, 1995, the observations require much higher values of σ_{eff}, due probably to severe fragmentation of dust particles shortly after their ejection. Since the applied technique allows one to force any parameter, the resulting effects on the quality of fit can be examined. In Table III, several solutions with some of the parameters forced (in parentheses) are compared with the fully optimized solution. The diameter of the primary nucleus is determined for both options considered in Table II (one nucleus vs. two unresolved components).

The results listed in Table III lead one to the following conclusions: (i) both the dispersion σ_{eff} and the law's power ν can have a major effect on the calculated value of the nuclear size, which diminishes with decreasing σ_{eff} and with increasing ν; (ii) the quality of fit by the various solutions with some of the parameters forced deteriorates significantly relative to the fully optimized solution; and (iii) the mean residual of the fit is generally correlated with the derived nuclear size: the worse the fit, the smaller the nucleus. It is concluded from the results in Tables II and III that the effective diameter of comet Hale–Bopp's primary nucleus is most probably $\sim$70 km.

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