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A new version of trial equation method for a complex nonlinear system arising in optical fibers

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Abstract

In this study, the dissipation problem of nonlinear pulse in mono mode optical fibers which is governed by the Fokas system (FS) is considered. The solutions of this system have an important role in comprehending the different wave structures in physical settings. Therefore, a new version of the trial equation method (NVTEM) is employed to present the new exact wave solutions of the FS. The advantage of the NVTEM is to use different root possibilities of a polynomial which shape the solutions of the related model. Primarily this system is converted to a nonlinear ordinary differential equation (NODE) via the traveling wave transform to apply the proposed method. Various exact wave solutions to the FS are obtained such as rational function, exponential function, hyperbolic function, and Jacobi elliptic function solutions. Thus, we have revealed solutions featly which are unlike the wave solutions previously found by other analytical methods. The present results depict the formation and development of such waves and their interactions. The exhibition of the solutions is given by 3D plots together with the corresponding 2D plots. The outcomes have shown that the proposed technique is abundant in achieving different wave solutions of many nonlinear differential equations in the field of optics.

Keywords Fokas system · New version of trial equation method · Exact traveling wave solution · Analytical methods

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1 Introduction

The nonlinear evolution equations (NLEEs) are not tools only in mathematics but also in mechanics, physics, biology, and material sciences. They are used to model various phenomena in nonlinear sciences. Thus, searching for the exact solutions to such equations has attracted the attention of scientists. Several useful methods have been developed and applied to NLEEs. The extended simple equation method, the Hirota bilinear method (Akram et al. 2023), the extended tanh-function method (Ahmad and Rani 2024), the enhanced direct algebraic method (Arnous et al. 2024), the new extended hyperbolic function method, the Sine-Gordon equation expansion method, the unified method and extended sinh-Gordon equation expansion method, the new extended direct algebraic method (Bilal et al. 2022a, b, 2023, 2024), $\left(G'/G^2\right)$-expansion function method, the generalized tanh method, the generalized Kudryashov method, the generalized exponential rational function method, the expansion function method (Bilal et al. 2021a, b, 2022c; Rehman et al. 2022), the extended Fan-sub equation method, the novel ϕ^6-model expansion method (Bilal et al. 2021a, b, 2022d), the extended rational sine-cosine/sinh-cosh and advance expansion function techniques (Seadawy et al. 2021), the extended trial equation method (Nadeem and Iambor 2023) and so on. On the other hand, complex nonlinear partial differential equations (CNPDEs) are employed to represent a wide family of nonlinear systems in applied sciences. In particular, electromagnetic wave propagation, light pulse propagation, and short pulse propagation in optical fibers are modeled via several CNPDEs, such as the Kundu-Eckhaus equation (Bekir and Zahran 2020), complex Ginzburg-Landau equation (Isah and Yokus 2023), Schrödinger equation (Liu and Feng 2023), complex Fokas-Lenells equation (Khater et al. 2021), Hirota Maccari system (Yokus and Baskonus 2022), complex Radhakrishnan-Kundu-Lakshmanan equation (Kudryashov 2022), Kundu–Mukherjee–Naskar equation (Wang et al. 2023), Lakshmanan-Porsezian-Daniel equation (Peng et al. 2022), Triki-Biswas equation (González-Gaxiola 2022), Gerdjikov Ivanov equation (Iqbal et al. 2023), Biswas-Arshed equation (Cinar et al. 2023), new Hamiltonian amplitude equation (Taghizadeh and Mirzazadeh 2011), Davey-Stewartson equation and complex coupled Maccari equation (Sulaiman et al. 2021), Hirota equation (Demiray and Pandir 2016). Therefore, it has become an important issue to investigate the dynamic attitude and exact traveling wave solutions of CNPDEs. In this context, the FS is considered as in Eq. (1) (Ali et al. 2023; Alotaibi et al. 2023; Kaplan et al. 2023):

$$\begin{aligned} iw_t + r_1 w_{xx} + r_2 ws = 0, \\ r_3 s_y - r_4(|w|^2)_x = 0, \end{aligned} \tag{1}$$

where r_1, r_2, r_3, r_4 are nonzero constants and $w = w(x, y, t)$ and $s = s(x, y, t)$ are complex functions symbolizing the propagation of nonlinear pulses in mono mode optical fibers. This model was suggested by Fokas (1994) and Shulman (1983) as an extension of the nonlinear Schrödinger equation and it simulates the dynamics of waves through single-mode fiber optics. Equation (1) is reduced to nonlinear Schrodinger equation when y approaches to x.

This important class of nonlinear differential equations has been investigated in many studies to obtain exact wave solutions, such as the generalized Kudryashov and modified Kudryashov procedures (Kaplan et al. 2023), Painlevé approach and semi-inverse variational principle (Alotaibi et al. 2023), Sardar sub-equation approach, Bernoulli sub-ODE method, and generic Kudryashov's method (Ali et al. 2023), the method of planar dynamical system (Tang and Li 2023), the complete discriminant system method of polynomials

(Zhang et al. 2023), the Jacobi elliptic function expansion method (Tarla et al. 2022), the Exp-function method (Wang 2022), the generalized projective Riccati equation method and the two variables $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion method (Sadaf et al. 2022), the extended rational sine-cosine and sinh-cosh methods (Wang et al. 2022), Hirota's bilinear method combined with the Kadomtsev-Petviashvili (KP) hierarchy reduction method (Rao et al. 2019), and the Cole-Hopf transformation (Wang 2023).

This research is motivated to reveal the different exact wave structures of Eq. (1) by the NVTEM which has not been applied before. The advantages of the proposed method are that it acquires the rational, exponential, hyperbolic, and Jacobi elliptic type function solutions of an NLEE, which is not the case for some methods such as the tanh-function method, G'/G-expansion method, Kudryashov method, and F-expansion method. Moreover, this scheme serves as a functioning and simple algorithm to evaluate the exact solutions of NLEEs governing many physical phenomena such as optics, fluid dynamics, mechanics, and mathematical biological models. Various novel and efficient solutions have been established through this approach. The variety and diversity of the reported solution aid in analyzing the propagation of pulses in optical fibers.

The structure of the paper is as follows within the scope of our aim: In Sect. 2 the description of the NVTEM is given. In Sect. 3 the exact wave solutions of the FS are investigated through NVTEM, and the results are presented. In Sect. 4 the behaviors of the obtained solutions are observed graphically. In Sect. 5 the results are discussed and summarized.

2 Representation of the method

In this section, the steps of the NVTEM are given to gain the wave solutions of a nonlinear partial differential equation (NPDE). This method is a developed version of the trial equation method which was first proposed by Liu (2005a; b). The extended trial equation method and the NVTEM are improved as a variation of the trial equation method by Pandır et al. (2012; 2013a; 2013b).

Step 1 We consider a NPDE as follows,

$$P\left(u, u_x, u_y, u_t, u_{xx}, u_{xy}, \dots\right) = 0. \tag{2}$$

Employing the traveling wave transform,

$$u(x, y, t) = U(\xi),\ \xi = c_1 x + c_2 y - c_3 t, \tag{3}$$

where c_1, c_2, c_3 are nonzero constants, Eq. (2) is transformed into a NODE as in Eq. (4),

$$G\left(U, \frac{dU}{d\xi}, \frac{d^2U}{d\xi^2}, \dots\right) = 0. \tag{4}$$

Step 2 This method assumes a solution for Eq. (4) as in the form,

$$U(\xi) = A_0 + \sum_{i=1}^{M} A_i P^i(\xi) + B_i P^{-i}(\xi), \tag{5}$$

where A_0, A_i, B_i, $(i = 1, 2, \ldots, M)$ are nonzero constants and M will be determined by balancing the highest order derivative and highest nonlinear term in Eq. (4). Besides, the function $P(\xi)$ satisfies the equation in Eq. (6).

$$(P')^2(\xi) = h_0 + h_1 P(\xi) + h_2 P^2(\xi) + \cdots h_N P^N(\xi). \tag{6}$$

Step 3 The solution in Eq. (5) is substituted into Eq. (4) and the required derivatives are evaluated using Eq. (6). This gives us an expression including the powers of the function $P(\xi)$ and equating the coefficients of these powers, an algebraic equation system for A_0, A_i, B_i, $(i = 1, 2, \ldots, M)$, h_0, $h_1, \ldots, h_N, c_1$, c_2, c_3 and for the other parameters in Eq. (2) is obtained to be solved by Mathematica software.

Step 4 Finally the function $P(\xi)$ in Eq. (5) is evaluated from Eq. (6) as,

$$\pm(\xi - EE) = \int \frac{dP}{\sqrt{h_0 + h_1 P(\xi) + h_2 P^2(\xi) + \cdots h_N P^N(\xi)}}, \tag{7}$$

where EE is the integration constant. The desired exact wave solutions of Eq. (2) is obtained by considering the function $P(\xi)$ from Eq. (7).

3 Application of NVTEM to FS

In this section the NVTEM is applied to the FS in Eq. (1). First, the traveling wave transformation is used as in the following,

$$w(x, y, t) = \psi(\eta) e^{i\theta},\ s(x, y, t) = \phi(\eta),\ \eta = x + y - vt, \tag{8}$$

where $\theta = k_1 x + k_2 y + k_3 t + k_4$, k_1, k_2, k_3, k_4 are constants and v is for the speed of wave frame. Then the following results are obtained,

$$(-v + 2r_1 k_1) i\psi' - k_3 \psi + r_1 \psi'' - r_1 k_1^2 \psi + r_2 \psi \phi = 0, \tag{9}$$

$$r_3 \phi' - 2r_4 \psi \psi' = 0. \tag{10}$$

Integrating Eq. (10) with zero integration constant we obtain,

$$\phi(\eta) = \frac{r_4}{r_3} \psi^2(\eta). \tag{11}$$

Eq. (11) is substituted into Eq. (9) and by writing $v = 2r_1 k_1$, Eq. (12) is obtained as follows,

$$r_1 \psi'' - (k_3 + r_1 k_1^2) \psi + \frac{r_2 r_4}{r_3} \psi^3 = 0. \tag{12}$$

Balancing the terms ψ'' and ψ^3 in Eq. (12), $M = 1$ is evaluated. Then the NVTEM proposes a solution for Eq. (12) as in the form,

$$\psi(\eta) = A_0 + A_1 P(\eta) + \frac{B_1}{P(\eta)}. \tag{13}$$

Eq. (13) is substituted in Eq. (12) by considering $N = 4$. Thus the derivative ψ'' is determined by considering the following equation,

$$(P')^2(\eta) = h_0 + h_1 P(\eta) + h_2 P^2(\eta) + h_3 P^3(\eta) + h_4 P^4(\eta). \tag{14}$$

The algebraic equation system which is obtained by equating the coefficients of the powers of function $P(\eta)$, is solved via Mathematica software. The $P(\eta)$ function in Eq. (13) is determined from Eq. (7) as,

$$\pm(\eta - EE) = \int \frac{dP}{\sqrt{h_0 + h_1 P(\eta) + h_2 P^2(\eta) + h_3 P^3(\eta) + h_4 P^4(\eta)}}. \tag{15}$$

If we suppose that $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the polynomial $h_0 + h_1 P(\eta) + h_2 P^2(\eta) + h_3 P^3(\eta) + h_4 P^4(\eta)$ then the following results are obtained for the integral in Eq. (15),

$$\pm(\eta - EE) = -\frac{1}{P - \alpha_1}, \tag{16}$$

$$\pm(\eta - EE) = \frac{2}{\alpha_1 - \alpha_2}\sqrt{\frac{P - \alpha_2}{P - \alpha_1}},\ \alpha_2 > \alpha_1 \tag{17}$$

$$\pm(\eta - EE) = \frac{1}{\alpha_1 - \alpha_2}\ln\left|\frac{P - \alpha_1}{P - \alpha_2}\right|,\ \alpha_1 > \alpha_2 \tag{18}$$

$$\begin{aligned}\pm(\eta - EE) = {} & \frac{2}{\sqrt{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}} \\ & \ln\left|\frac{\sqrt{(P - \alpha_2)(\alpha_1 - \alpha_3)} - \sqrt{(P - \alpha_3)(\alpha_1 - \alpha_2)}}{\sqrt{(P - \alpha_2)(\alpha_1 - \alpha_3)} + \sqrt{(P - \alpha_3)(\alpha_1 - \alpha_2)}}\right|,\ \alpha_1 > \alpha_2 > \alpha_3\end{aligned} \tag{19}$$

$$\pm(\eta - EE) = \frac{2F(\phi, l)}{\sqrt{(\alpha_1 - \alpha_2)(\alpha_2 - \alpha_4)}},\ \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4, \tag{20}$$

where $F(\phi, l) = \int_0^{\phi} \frac{d\zeta}{\sqrt{1 - l^2 \sin^2(\zeta)}}$, $\phi = \arcsin\sqrt{\frac{(P-\alpha_1)(\alpha_2-\alpha_4)}{(P-\alpha_2)(\alpha_1-\alpha_4)}}$ and $l^2 = \frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$. The following cases are evaluated and the corresponding exact wave solutions of Eq. (1) are presented. *Case-1:*

$$\begin{aligned} & A_0 = 0,\ A_1 = -\frac{i\sqrt{r_1 r_3 h_4}}{\sqrt{r_2 r_4}},\ B_1 = \frac{2i(k_3 + k_1^2 r_1)\sqrt{r_3}}{3\sqrt{r_1 r_2 r_4 h_4}}, \\ & h_0 = \frac{2(k_3 + k_1^2 r_1)^2}{9 r_1^2 h_4},\ h_1 = 0,\ h_2 = -\frac{2(k_3 + k_1^2 r_1)}{3 r_1},\ h_3 = 0. \end{aligned} \tag{21}$$

In this case, the integral in Eq. (22)

$$\pm(\eta - EE) = \int \frac{dP}{\sqrt{h_0 + h_2 P^2(\eta) + h_4 P^4(\eta)}}, \tag{22}$$

is considered and the function $P(\eta)$ is evaluated as in the following,

$$P(\eta) = -sn\left[\sigma_2(EE-\eta), \frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1, \tag{23}$$

where $\sigma_1 = \sqrt{\frac{-h_2-\sqrt{h_2^2-4h_0h_4}}{2h_4}}$ and $\sigma_2 = \sqrt{\frac{-h_2+\sqrt{h_2^2-4h_0h_4}}{2h_4}}$. Thus the exact wave solutions of Eq. (1) are obtained by substituting Eq. (23) into Eq. (13) together with the coefficients in Eq. (21) as,

$$w_1(x,y,t) = e^{i\theta}\left(-A_1 sn\left[\sigma_2(EE-\eta), \frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1 + \frac{B_1}{sn\left[\sigma_2(EE-\eta), \frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1}\right), \tag{24}$$

$$s_1(x,y,t) = \frac{r_4}{r_3}\left(-A_1 sn\left[\sigma_2(EE-\eta), \frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1 + \frac{B_1}{sn\left[\sigma_2(EE-\eta), \frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1}\right)^2, \tag{25}$$

where $\eta = x + y - vt$ and $\theta = k_1x + k_2y + k_3t + k_4$. Equations (24–25) are the Jacobi elliptic function solutions of Eq. (1) for Case-1.

Case-2:

$$\begin{aligned}
&A_0 = -\sqrt{\frac{13}{3}}\frac{\sqrt{-(k_3+k_1^2r_1)r_3}}{\sqrt{r_2r_4}}, A_1 = -\frac{i\sqrt{r_1r_3h_4}}{\sqrt{r_2r_4}}, B_1 = \frac{28i(k_3+k_1^2r_1)\sqrt{r_3}}{3\sqrt{r_1r_2r_4h_4}},\\
&h_0 = \frac{392(k_3+k_1^2r_1)^2}{9r_1^2h_4}, h_1 = -16i\sqrt{\frac{13}{3}}\frac{\sqrt{(-(k_3+k_1^2r_1)r_3)^3}}{\sqrt{(r_1r_3)^3}\sqrt{h_4}},\\
&h_2 = -\frac{28(k_3+k_1^2r_1)}{3r_1}, h_3 = -4i\sqrt{\frac{13}{3}}\frac{\sqrt{(-(k_3+k_1^2r_1)r_3h_4)}}{\sqrt{r_1r_3}}.
\end{aligned} \tag{26}$$

For Case-2 the exact wave solutions of Eq. (1) are obtained by evaluating the function $P(\eta)$ from Eqs. (16–20) as in the following.

$$w_{2,1}(x,y,t) = e^{i\theta}\left(A_0 + A_1\left(\alpha_1 - \frac{1}{\eta-EE}\right) + \frac{B_1}{\left(\alpha_1 - \frac{1}{\eta-EE}\right)}\right), \tag{27}$$

$$s_{2,1}(x,y,t) = \frac{r_4}{r_3}\left(A_0 + A_1\left(\alpha_1 - \frac{1}{\eta-EE}\right) + \frac{B_1}{\left(\alpha_1 - \frac{1}{\eta-EE}\right)}\right)^2, \tag{28}$$

$$w_{2,2}(x,y,t)=e^{i\theta}\left(\begin{array}{c}A_0+A_1\left(\alpha_1+\dfrac{4(\alpha_2-\alpha_1)}{4-(\alpha_2-\alpha_1)^2(\eta-EE)^2}\right)\\+\dfrac{B_1}{\left(\alpha_1+\frac{4(\alpha_2-\alpha_1)}{4-(\alpha_2-\alpha_1)^2(\eta-EE)^2}\right)}\end{array}\right),\tag{29}$$

$$s_{2,2}(x,y,t)=\frac{r_4}{r_3}\left(\begin{array}{c}A_0+A_1\left(\alpha_1+\dfrac{4(\alpha_2-\alpha_1)}{4-(\alpha_2-\alpha_1)^2(\eta-EE)^2}\right)\\+\dfrac{B_1}{\left(\alpha_1+\frac{4(\alpha_2-\alpha_1)}{4-(\alpha_2-\alpha_1)^2(\eta-EE)^2}\right)}\end{array}\right)^2,\tag{30}$$

$$w_{2,3}(x,y,t)=e^{i\theta}\left(A_0+A_1\left(\frac{\alpha_2e^{(\alpha_1-\alpha_2)(\eta-EE)}-\alpha_1}{e^{(\alpha_1-\alpha_2)(\eta-EE)}-1}\right)+\frac{B_1}{\left(\frac{\alpha_2e^{(\alpha_1-\alpha_2)(\eta-EE)}-\alpha_1}{e^{(\alpha_1-\alpha_2)(\eta-EE)}-1}\right)}\right),\tag{31}$$

$$s_{2,3}(x,y,t)=\frac{r_4}{r_3}\left(A_0+A_1\left(\frac{\alpha_2e^{(\alpha_1-\alpha_2)(\eta-EE)}-\alpha_1}{e^{(\alpha_1-\alpha_2)(\eta-EE)}-1}\right)+\frac{B_1}{\left(\frac{\alpha_2e^{(\alpha_1-\alpha_2)(\eta-EE)}-\alpha_1}{e^{(\alpha_1-\alpha_2)(\eta-EE)}-1}\right)}\right)^2,\tag{32}$$

$$w_{2,4}(x,y,t)=e^{i\theta}\left(\begin{array}{c}A_0\\+A_1\left(\alpha_1-\dfrac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh(\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}(\eta-EE))}\right)\\+\dfrac{B_1}{\left(\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh(\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}(\eta-EE))}\right)}\end{array}\right),\tag{33}$$

$$s_{2,4}(x,y,t)=\frac{r_4}{r_3}\left(\begin{array}{c}A_0\\+A_1\left(\alpha_1-\dfrac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh(\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}(\eta-EE))}\right)\\+\dfrac{B_1}{\left(\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh(\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}(\eta-EE))}\right)}\end{array}\right)^2,\tag{34}$$

$$w_{2,5}(x,y,t)=e^{i\theta}\left(\begin{array}{c}A_0\\+A_1\left(\alpha_2+\dfrac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\eta-EE),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right)}\right)\\+\dfrac{B_1}{\left(\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\eta-EE),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right)}\right)}\end{array}\right),\tag{35}$$

$$s_{2,5}(x,y,t)=\frac{r_4}{r_3}\left(\begin{array}{c}A_0\\ +A_1\left(\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\eta-EE),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right)}\right)\\ +\frac{B_1}{\left(\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left(\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}(\eta-EE),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right)}\right)}\end{array}\right)^2, \tag{36}$$

Eqs. (35–36) are the Jacobi elliptic function solutions of Eq. (1) for Case-2. *Case-3:*

$$\begin{aligned}&A_0=0,\ A_1=-\frac{i\sqrt{r_1r_3h_4}}{\sqrt{r_2r_4}},\ h_0=\frac{-r_2r_4B_1^2}{2r_1r_3},\ h_1=0,h_2=\frac{i\sqrt{r_2r_4h_4}B_1}{r_1r_3},\\&h_3=0,\ k_1=-\frac{\sqrt{-2k_3r_3-3i\sqrt{r_1r_2r_3r_4h_4}B_1}}{\sqrt{2r_1r_3}}.\end{aligned} \tag{37}$$

The function $P(\eta)$ is as in Eq. (23) since the result $h_1=h_3=0$ causes the integral in Eq. (22). Then the wave solutions of Eq. (1) are evaluated as below by considering the coefficients in (37).

$$w_3(x,y,t)=e^{i\theta}\left(-A_1sn\left[\sigma_2(EE-\eta),\frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1+\frac{B_1}{sn\left[\sigma_2(EE-\eta),\frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1}\right), \tag{38}$$

$$s_3(x,y,t)=\frac{r_4}{r_3}\left(-A_1sn\left[\sigma_2(EE-\eta),\frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1+\frac{B_1}{sn\left[\sigma_2(EE-\eta),\frac{\sigma_1^2}{\sigma_2^2}\right]\sigma_1}\right)^2, \tag{39}$$

where σ_1 and σ_2 are defined before and the values h_0, h_2, h_4 in σ_1, σ_2 are as in Eq. (37). Equations (38–39) are the Jacobi elliptic function solutions of Eq. (1) for Case-3.

Case-4:

$$\begin{aligned}&A_1=-\frac{i\sqrt{r_1r_3h_4}}{\sqrt{r_2r_4}},\ B_1=-\frac{28i\sqrt{r_2r_4}A_0^2}{13\sqrt{r_1r_3h_4}},\ h_0=\frac{392r_2^2r_4^2A_0^4}{169r_1^2r_3^2h_4},\ h_1=\frac{48i\sqrt{r_2^3r_4^3}A_0^3}{13\sqrt{r_1^3r_3^3h_4}},\\&h_2=\frac{28r_2r_4A_0^2}{13r_1r_3},\ h_3=\frac{4i\sqrt{r_2r_4h_4}A_0}{\sqrt{r_1r_3}},\ k_1=-\frac{\sqrt{-13k_3r_3-3r_2r_4A_0^2}}{\sqrt{13r_1r_3}}.\end{aligned} \tag{40}$$

In Case-4 the coefficients A_0, A_1, B_1 are different from zero. Therefore the form of the solutions of Eq. (1) will be as in Eqs. (27–36). It can be chosen as $A_0=-\alpha_1A_1$ and $A_0=-\alpha_2A_1$ to obtain the rational function solution, combined soliton solution, and the combined Jacobi elliptic function solution of Eq. (1) which is not possible in Case-2 due to the fixed value of A_0.

If $A_0=-\alpha_1A_1$ and $EE=0$, the rational function solutions are obtained as,

$$w_{4,1}(x,y,t) = e^{i\theta}\left(\frac{i\sqrt{r_1r_3h_4}(-13+\eta\alpha_1(13+28\eta\alpha_1))}{13\sqrt{r_2r_4}\eta(-1+\eta\alpha_1)}\right), \tag{41}$$

$$s_{4,1}(x,y,t) = -\frac{r_1h_4(-13+\eta\alpha_1(13+28\eta\alpha_1))^2}{169r_2\eta^2(-1+\eta\alpha_1)^2}, \tag{42}$$

$$w_{4,2}(x,y,t) = e^{i\theta}\left(i\frac{\sqrt{r_1r_3h_4}}{13\sqrt{r_2r_4}}\left(-\frac{52(\alpha_2-\alpha_1)}{4-\eta^2(\alpha_1-\alpha_2)^2}+\frac{28\alpha_1^2}{\alpha_1+\frac{4(\alpha_2-\alpha_1)}{4-\eta^2(\alpha_1-\alpha_2)^2}}\right)\right), \tag{43}$$

$$s_{4,2}(x,y,t) = -\frac{r_1h_4}{169r_2}\left(-\frac{52(\alpha_2-\alpha_1)}{4-\eta^2(\alpha_1-\alpha_2)^2}+\frac{28\alpha_1^2}{\alpha_1+\frac{4(\alpha_2-\alpha_1)}{4-\eta^2(\alpha_1-\alpha_2)^2}}\right)^2, \tag{44}$$

and the traveling wave solution in Eqs. (45–46) together with the soliton solution in Eqs. (47–48) are obtained:

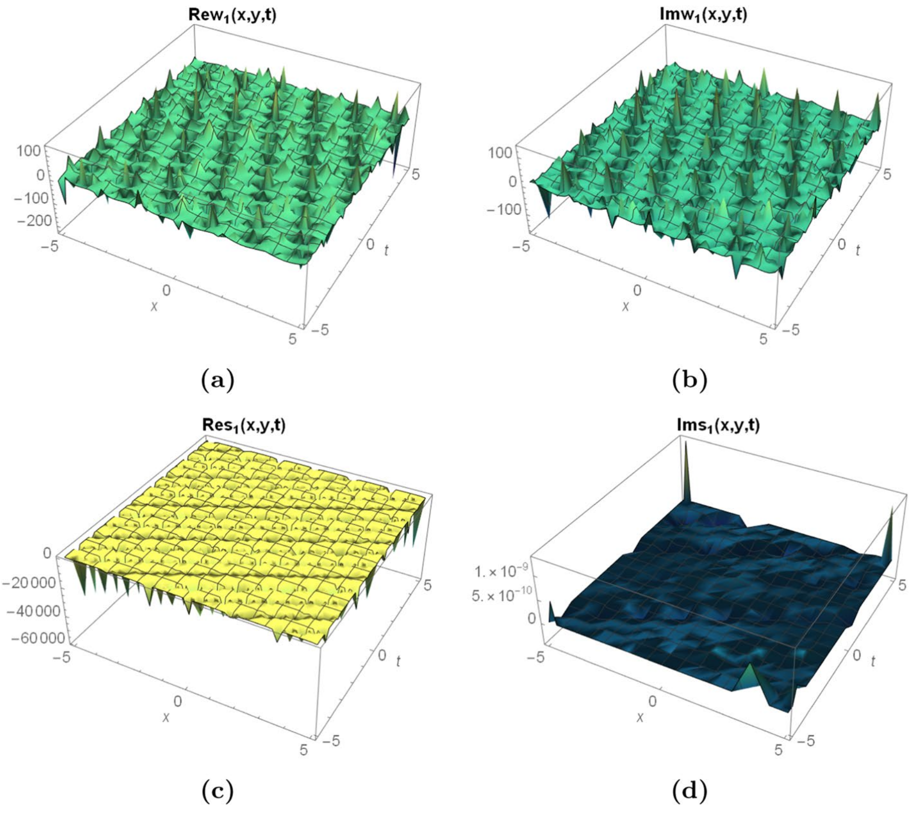

Fig. 1 The 3D-depiction of w_1 and s_1 for $r_2 = 2.3, r_3 = 3, r_4 = 5.1, r_1 = 6.2, h_4 = 3, k_1 = 3.6, k_2 = 2, k_3 = 1.4, k_4 = 2, EE = 5, y = 0.85, v = 2$

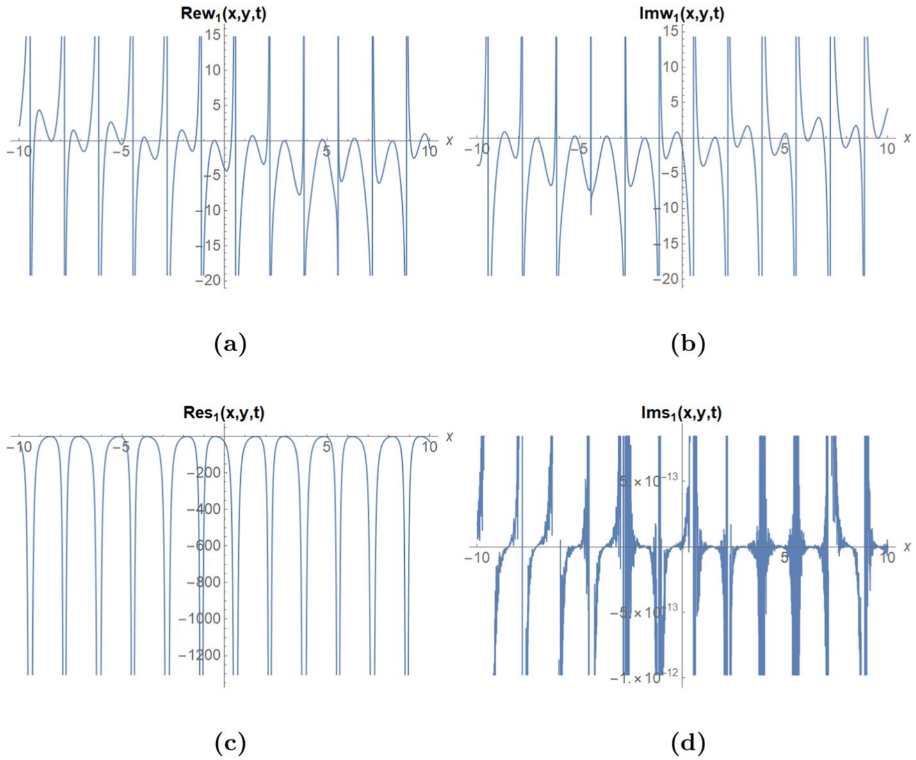

Fig. 2 The 2D-depiction of w_1 and s_1 for $r_2 = 2.3, r_3 = 3, r_4 = 5.1, r_1 = 6.2, h_4 = 3, k_1 = 3.6, k_2 = 2, k_3 = 1.4, k_4 = 2, EE = 5,\ y = 0.85, \nu = 2,\ t = 1$

$$w_{4,3}(x,y,t) = e^{i\theta}\left(\pm\alpha_2 A_1 \coth\left[\frac{\alpha_1-\alpha_2}{2}\eta\right] \pm \frac{B_1}{2}\tanh\left[\frac{\alpha_1-\alpha_2}{2}\eta\right]\right), \tag{45}$$

$$s_{4,3}(x,y,t) = \frac{r_4}{r_3}\left(\pm\alpha_2 A_1 \coth\left[\frac{\alpha_1-\alpha_2}{2}\eta\right] \pm \frac{B_1}{2}\tanh\left[\frac{\alpha_1-\alpha_2}{2}\eta\right]\right)^2, \tag{46}$$

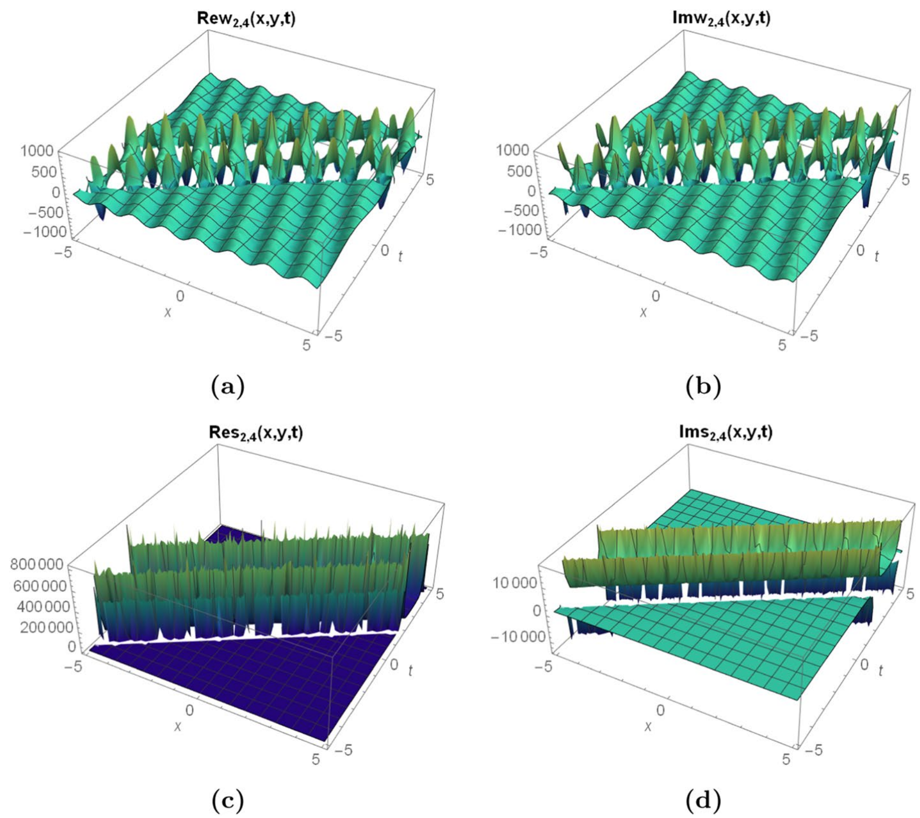

Fig. 3 The 3D-depiction of $w_{2,4}$ and $s_{2,4}$ for $r_2 = 3.3, r_4 = -1.86, r_1 = 3.2, r3 = -2.6, h_4 = -2.1, k_1 = 4.6, k_2 = 2.48, k_3 = 1.3, k_4 = 2.5, EE = 0.7, \alpha_1 = 2, \alpha_2 = 1.5, \alpha_3 = 1,\ y = 0.85, v = 1.6$

$$w_{4,4}(x,y,t) = e^{i\theta}\left(\frac{A_1\sigma_3}{\sigma_4 + \cosh\left[\sigma_5\eta\right]} + \frac{B_1}{\alpha_1 - \frac{\sigma_3}{\sigma_4+\cosh\left[\sigma_5\eta\right]}}\right), \tag{47}$$

$$s_{4,4}(x,y,t) = \frac{r_4}{r_3}\left(\frac{A_1\sigma_3}{\sigma_4 + \cosh\left[\sigma_5\eta\right]} + \frac{B_1}{\alpha_1 - \frac{\sigma_3}{\sigma_4+\cosh\left[\sigma_5\eta\right]}}\right)^2, \tag{48}$$

where $\sigma_3 = \frac{-2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{(\alpha_3-\alpha_2)}$, $\sigma_4 = \frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2}$, $\sigma_5 = \sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}$.

If $A_0 = -\alpha_2 A_1$, the Jacobi elliptic function solutions of Eq. (1) are evaluated as in Eqs. (49–50) for Case-4.

$$w_{4,5}(x,y,t) = e^{i\theta}\left(\frac{\sigma_6 A_1}{\sigma_7 + sn^2\left[\sigma_8\eta,\sigma_9\right]} + \frac{B_1}{\alpha_2 + \frac{\sigma_6 A_1}{\sigma_7+sn^2\left[\sigma_8\eta,\sigma_9\right]}}\right), \tag{49}$$

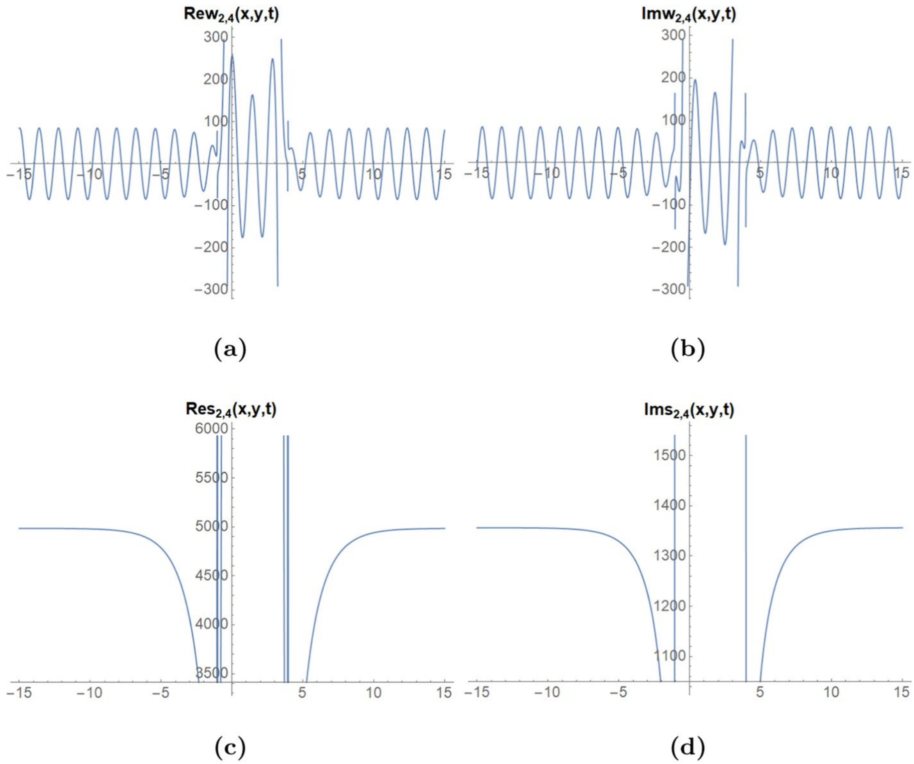

Fig. 4 The 2D-depiction of $w_{2,4}$ and $s_{2,4}$ for $r_2 = 3.3, r_4 = -1.86, r_1 = 3.2, r3 = -2.6, h_4 = -2.1, k_1 = 4.6$, $k_2 = 2.48, k_3 = 1.3, k_4 = 2.5, EE = 0.7, \alpha_1 = 2, \alpha_2 = 1.5, \alpha_3 = 1,\ y = 0.85, v = 1.6, t = 1$

$$s_{4,5}(x,y,t) = \frac{r_4}{r_3}\left(\frac{\sigma_6 A_1}{\sigma_7 + sn^2\left[\sigma_8\eta, \sigma_9\right]} + \frac{B_1}{\alpha_2 + \frac{\sigma_6 A_1}{\sigma_7 + sn^2\left[\sigma_8\eta, \sigma_9\right]}}\right)^2, \tag{50}$$

where $\sigma_6 = \frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_1-\alpha_4}$, $\sigma_7 = \frac{\alpha_4-\alpha_2}{\alpha_1-\alpha_4}$, $\sigma_8 = \frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2}$, $\sigma_9 = \frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}$.

4 Results and discussion

The long-time behavior of a solution to a NLEE is significant in applied sciences. Thus, the dynamical behaviors of the solution pair $w_1 - s_1$ in Eqs. (24–25), $w_{2,4} - s_{2,4}$ in Eqs. (33–34), $w_{2,5} - s_{2,5}$ in Eqs. (35–36), $w_3 - s_3$ in Eqs. (38–39) and $w_{4,3} - s_{4,3}$ in Eqs. (45–46) are presented in Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 for some fixed values of parameters. Our findings are new wave types when compared to the results in (Ali et al. 2023; Alotaibi

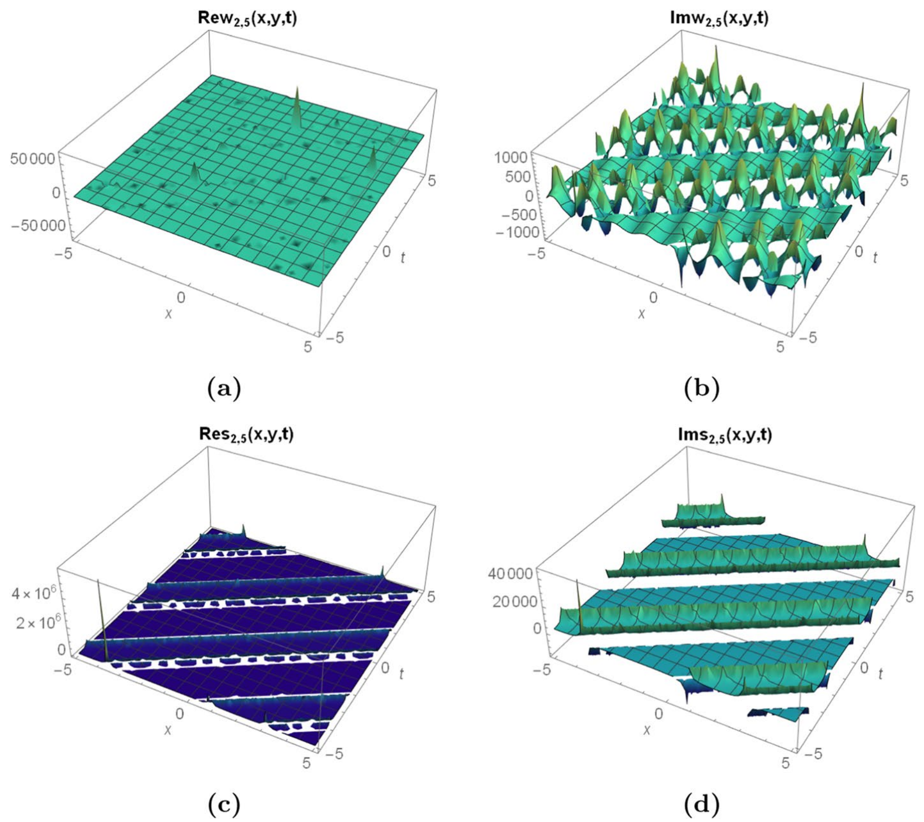

Fig. 5 The 3D-depiction of $w_{2,5}$ and $s_{2,5}$ for $r_2 = 3.3, r_4 = -1.86, r_1 = 3.2, r3 = -2.6, h_4 = -2.1, k_1 = 4.6, k_2 = 2.48, k_3 = 1.3, k_4 = 2.5, EE = 0.7, \alpha_1 = 3, \alpha_2 = 2.5, \alpha_3 = 1.5, \alpha_4 = 1, y = 0.85, \nu = 1.6$

et al. 2023; Kaplan et al. 2023; Rao et al. 2019; Sadaf et al. 2022; Tang and Li 2023; Tarla et al. 2022; Wang 2022; Wang et al. 2022; Wang 2023; Zhang et al. 2023). The NVTEM has given different results than the other analytical methods used in the literature for FS. We have presented Jacobi elliptic function solutions which are not reported in (Ali et al. 2023; Alotaibi et al. 2023; Kaplan et al. 2023; Rao et al. 2019; Sadaf et al. 2022; Wang 2022; Wang et al. 2022; Wang 2023). The graphical representation also demonstrates distinct characteristics of the current solutions.

In Figs. 1, 2, 5, 6, 7 and 8 the combined Jacobi elliptic function solutions are illustrated. In Figs. 3, 4, 9 and 10 the hyperbolic type solutions are presented.

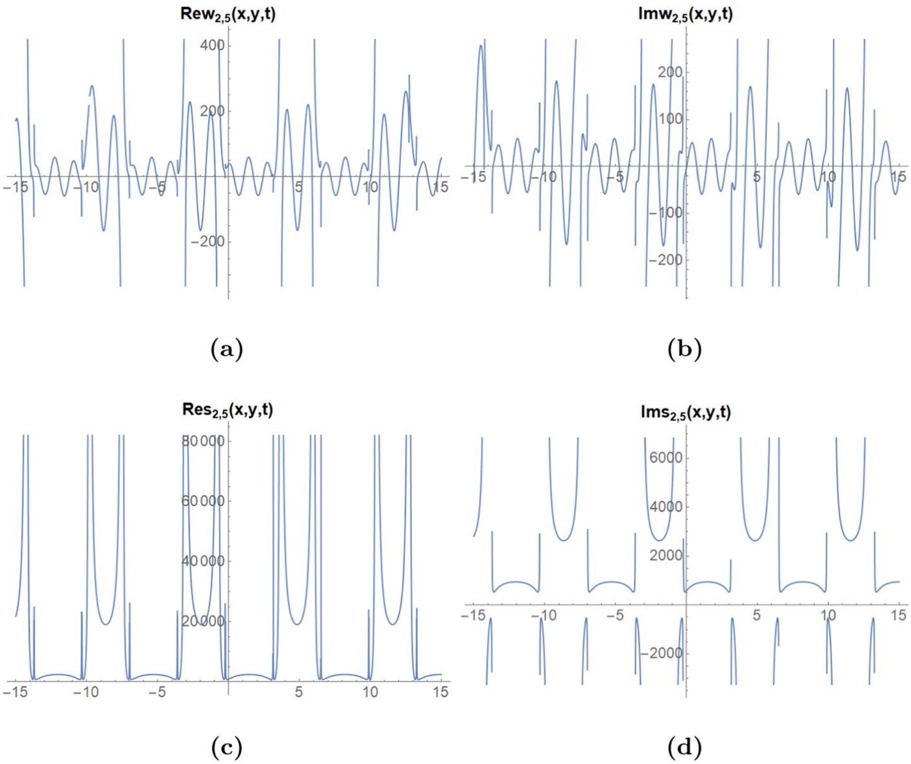

Fig. 6 The 2D-depiction of $w_{2,5}$ and $s_{2,5}$ for $r_2 = 3.3, r_4 = -1.86, r_1 = 3.2, r3 = -2.6, h_4 = -2.1, k_1 = 4.6,$ $k_2 = 2.48, k_3 = 1.3, k_4 = 2.5, EE = 0.7, \alpha_1 = 3, \alpha_2 = 2.5, \alpha_3 = 1.5, \alpha_4 = 1, y = 0.85, v = 1.6, t = 1$

5 Conclusions

In this paper, many novel computational wave solutions of the FS that describe the propagation of the nonlinear pulses in mono-mode optical fibers are investigated through NVTEM. The rational, exponential, hyperbolic, and Jacobi elliptic function types are evaluated for this complex system. All these solutions are fresh in the frame of NVTEM and they are verified with the aid of Mathematica. The benefit of the proposed method is that it presents different waveforms by considering the roots of a *N*th-order polynomial and as a result, the current results demonstrate various superposed waveforms. Therefore, this research illustrates how the NVTEM is an effective and straightforward method to be applied to many NLEEs. The evolvement of some obtained solutions is presented in 3D and 2D graphs for a better comprehension of their physical behaviors. The current method is capable of capturing the different wave solutions of various nonlinear partial differential equations in applied sciences. This research offers beneficial information about the FS and NVTEM for the related physical analysis.

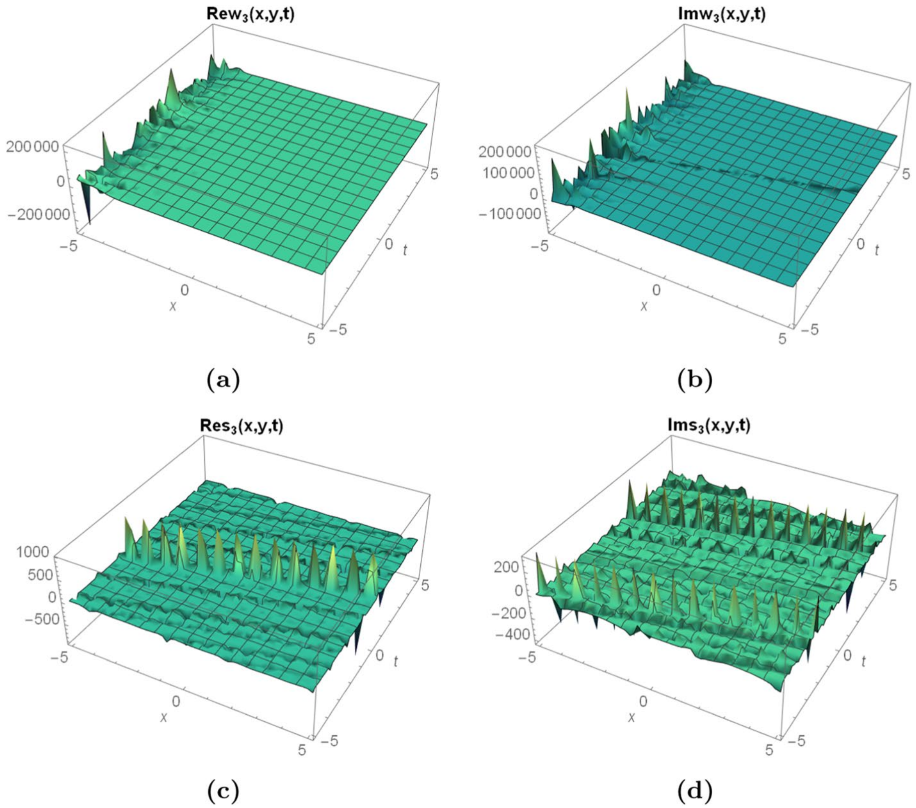

Fig. 7 The 3D-depiction of w_3 and s_3 for $r_2 = 2.3, r_4 = 2.1, r_1 = 4.2, r_3 = 3, h_4 = 2, k_2 = 4, k_3 = 5.4, k_4 = 3, EE = 0.7, y = 0.85, v = 4, B_1 = 4.5$

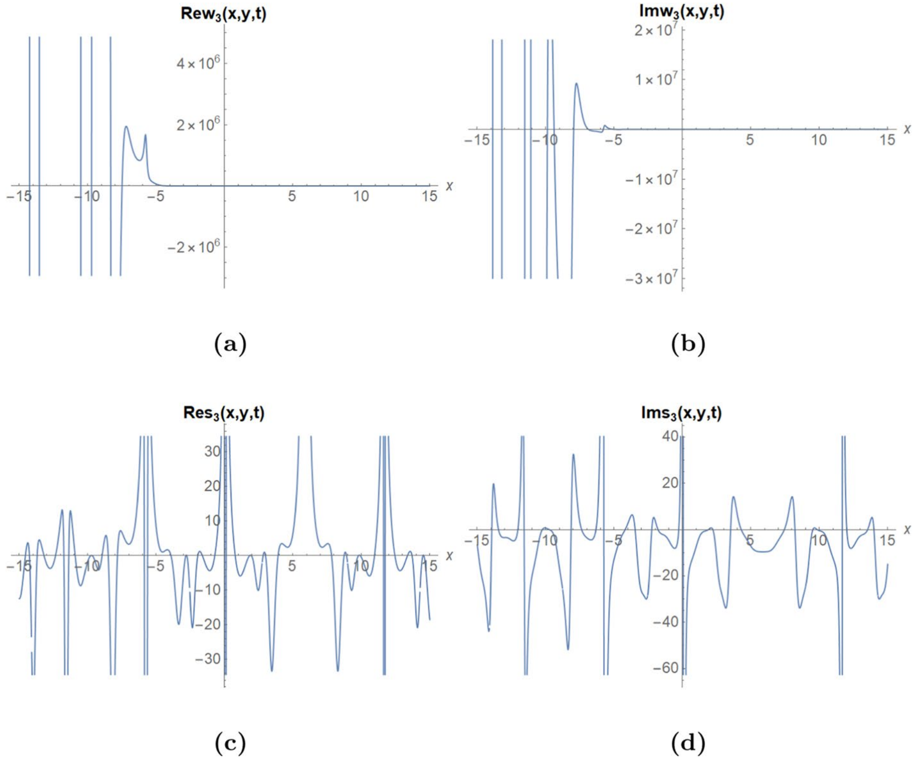

Fig. 8 The 2D-depiction of w_3 and s_3 for $r_2 = 2.3, r_4 = 2.1, r_1 = 4.2, r_3 = 3, h_4 = 2, k_2 = 4, k_3 = 5.4, k_4 = 3, EE = 0.7, y = 0.85, v = 4, B_1 = 4.5, t = 1$

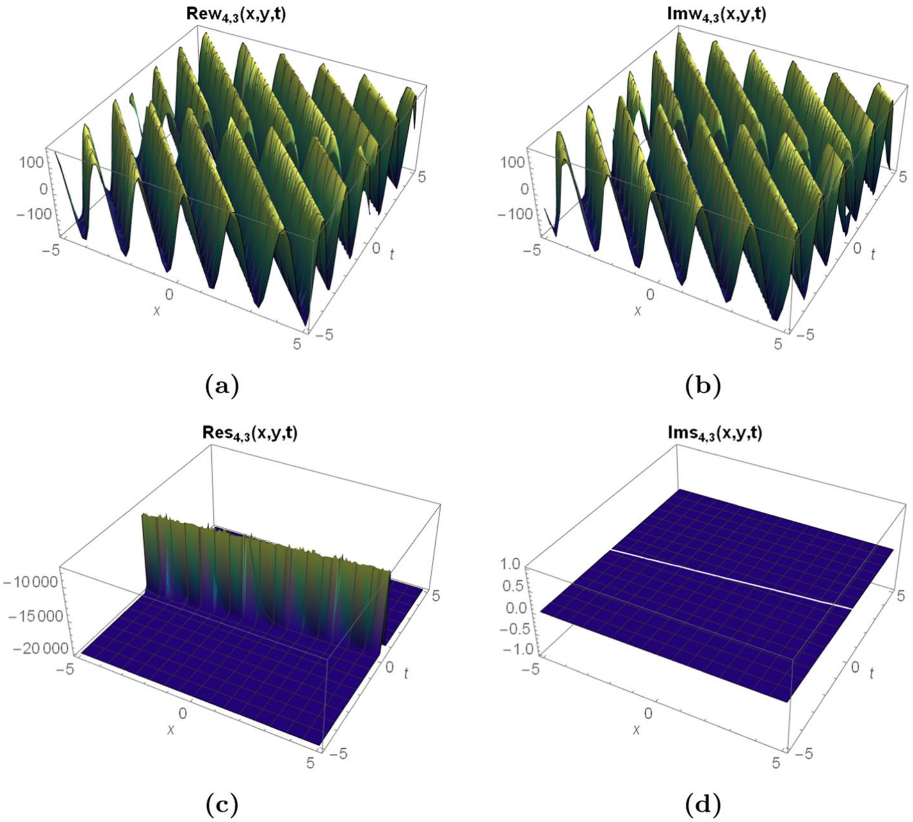

Fig. 9 The 3D-depiction of $w_{4,3}$ and $s_{4,3}$ for $r_2 = 7.2, r_4 = -2.5, r_1 = 2, r_3 = 3.4, h_4 = 3, k_2 = 1.48, k_3 = -3.3, k_4 = 2.5, \alpha_1 = 3.8, \alpha_2 = 0.2, y = 0.85, \nu = 10$

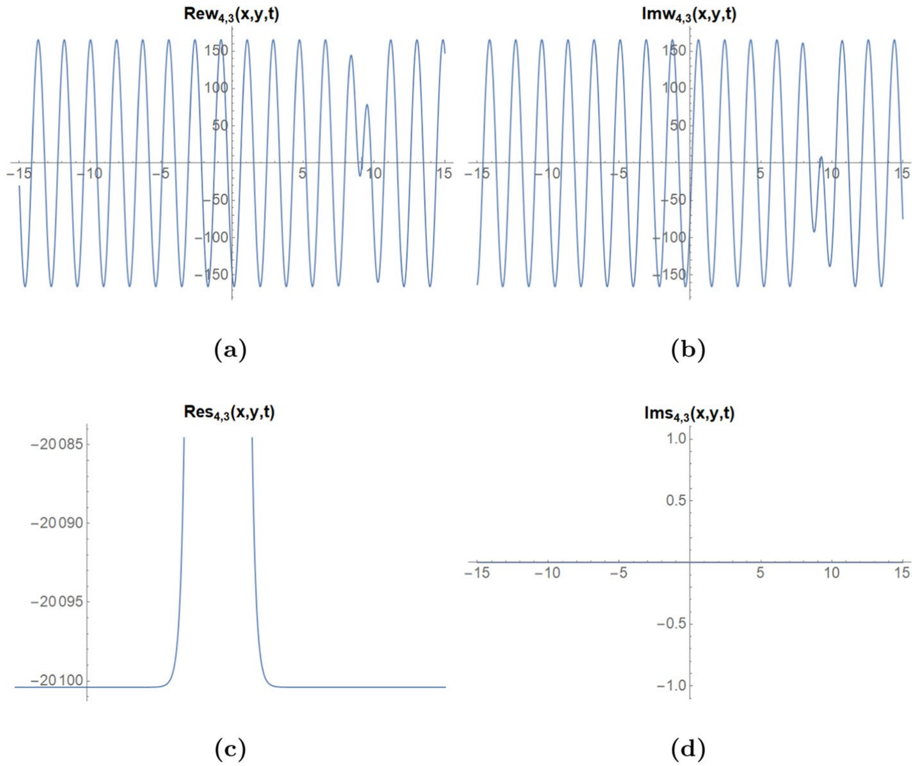

Fig. 10 The 2D-depiction of $w_{4,3}$ and $s_{4,3}$ for $r_2 = 7.2, r_4 = -2.5, r_1 = 2, r_3 = 3.4, h_4 = 3, k_2 = 1.48, k_3 = -3.3, k_4 = 2.5, \alpha_1 = 3.8, \alpha_2 = 0.2, y = 0.85, \nu = 10, t = 1$

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Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

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