



# Venture capital project selection based on interval number grey target decision model

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Published online: 4 January 2021  
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## Abstract

In this study, aiming at the multi-attribute decision-making problem with incomplete and uncertain attribute weight information and attribute value of interval numbers, a grey target decision-making model of interval numbers based on positive and negative clouts is proposed. Firstly, in this model, the linear transformation operator of interval number is used to normalize the original decision information, and the positive and negative clouts of interval number are designed. Secondly, after the space projection distance between each scheme and the positive and negative clouts is considered comprehensively, the off-target distance is taken as the basis of vector analysis in space to obtain a new comprehensive off-target distance. The existing interval number grey target decision-making model ignores the important influence of interval distribution and the correlation between the attributes in scheme evaluation, and there are some fuzzy errors when setting the weight of attributes. In order to solve the above problems, this paper combined with the uncertainty analysis of the attribute weights, a goal programming is constructed for the objective function based on the comprehensive off-target distance minimization to solve the attribute weight vector, and finally determine the order of the scheme. Finally, the feasibility and effectiveness of the proposed grey target decision model are verified by an example of venture capital projects. Compared with traditional models, the improved model fully considers the characteristics of interval data and the correlation between the attributes.

**Keywords** Positive and negative clouts · Interval number · Grey target decision · Comprehensive off-target distance · Goal programming

## 1 Introduction

Multi-attribute decision-making (MADM) is widely applied in the fields of society, economy, technology and engineering design. At present, there are many mature techniques and analysis methods for the MADM problem of attribute weight and attribute value with determined values. However, due to the uncertainty of objective things, the complexity of decision-making problems and the fuzziness of human thinking, it is difficult for people to give accurate values for the evaluation of things in the decision-making process, but expressing by interval numbers or fuzzy numbers. In addition, it is sometimes difficult for people to obtain complete information of attribute weight because of the limitations of understanding. As a result, many scholars have done a lot of research and put forward many methods, which need to be further explored. The actions that should be taken according to the actual situation and the predetermined goals are decision-making,

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Communicated by V. Loia.

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which is not only an important part of all kinds of management activities, but also runs through everyone's work, learning and life process. In a broad sense, decision-making refers to the whole process of a series of activities, such as raising questions, collecting information, determining objectives, formulating alternatives, evaluating and selecting schemes, as well as implementing, feeding back and revising. In a narrow sense, decision-making can be understood as choosing a plan under uncertain conditions, that is to say, making a decision, which to a large extent depends on the experience, attitude and determination of the individual decision maker who also needs to bear risks. The theory of grey target was first put forward by Professor Deng Ju-long, the founder of grey system theory. Its basic idea is to make decision when the decision model contains grey element or the general decision model is combined with grey model. Later, Liu Si-feng reflected the merits and demerits of situation effect vector in the grey target decision-making model by using the value of off-target distance (Si-feng et al. 2015). On the basis of Deng Ju-long and Liu Si-feng, the following scholars have conducted in-depth research and expansion on grey target decision-making. Grey target decision-making measures the strengths and weaknesses of each decision object based on the distance from the bull's-eye. Compared with traditional decision-making methods, it has decision-making ideas easy to understand and enjoys wide applications.

At present, the theoretical research on the grey target decision-making model mainly focuses on the following two aspects: the optimization of modeling methods and the expansion of modeling objects. First of all, in terms of optimization of modeling methods, Yan et al. (Shu-li et al. 2015) proposed a grey target decision-making method for three-parameter interval grey number group based on prospect theory in consideration of the influence of index satisfaction domain and risk attitude on decision results. Aiming at a class of group decision-making problems in which the attribute value, attribute weight and decision maker weight are all interval grey numbers, Dai and Li (2014) introduced the concepts of group positive, negative clouts and group deviation from clouts and put forward the group decision-making method of grey multi-attribute deviation from clouts. Bo et al. (2013) evaluated the advantages and disadvantages of the decision-making scheme by comparing the area size of the graph enclosed by the index values and the line of the clout in the index set, so as to weaken the influence of the extreme index values in the modeling object on the calculation results of the off-target distance to a certain extent, and established a cobweb grey target decision-making model. Shu-li and Si-feng (2014) proposed a group grey target decision-making method based on the prospect theory taking into account the influence of the expected grey target of the decision

maker on the group decision-making, in which the expected grey target was used as the reference point to define the prospect value function, and the prospect value was normalized by using the linear transformation operator for bonus and forfeit, which can fully reflect whether the evaluation value was in the target. Ma (Jin-shan 2019) presented a generalized grey target decision method for mixed attributes. Secondly, in the extension of modeling objects, Li-ping et al. (2019) proposed the interval number grey target decision-making model based on multi-dimensional association sampling, introduced the fuzzy set-valued statistical method to determine the index weight, and expanded the grey target decision-making model weight from the real number sequence to the interval number sequence, which reduced the fuzzy error in the expert decision-making process. Si-feng et al. (2010) extended the grey target decision-making model from real number sequences to interval number sequences and established the grey target decision-making model based on interval number. Based on the theory of variable weight and grey target, Zhuang et al. (2019) proposed a dynamic evaluation method, which solved the performance evaluation problem of hybrid multi-attribute command and control system with accurate real number, interval number and triangular fuzzy number. Rong-bo et al. (2018) proposed an improved grey target decision model based on moment estimation method. Gao-feng et al. (2015) presented an intuitionistic fuzzy linear programming model, and proposed a solution method of intuitionistic fuzzy linear programming based on total exact function. Gao et al. (2020) introduced a novel target threat assessment method based on three-way decisions under intuitionistic fuzzy MADM environment.

In the aspect of determining the weight of interval number MADM, Duan (Chuan-qing 2017) transformed the attribute value of intuitionistic fuzzy number (IFN) into dual-interval numbers, and determined the attribute weight according to the difference between attribute values of decision-making scheme. Xiao-di et al. (2016) established a model to describe attributes according to the mean value, variance of attribute values and the correlation between attributes. Sheng et al. (2018) put forward an improved interval intuitionistic fuzzy entropy calculation formula, which integrates membership, non-membership and hesitation, to calculate the weight value of decision attribute index and improve the objectivity of decision index weight. The above research provides some ideas for solving the problem of grey target decision-making. However, it is also be found that there are relatively few researches on the grey target decision-making problem with interval grey number form of decision information and incomplete attribute weight. Therefore, the corresponding grey target decision-making model is proposed in this paper to meet the needs. The constructed interval number grey target decision-

making model uses an interval number linear transformation operator to transform the effect sample matrix into a standardized decision matrix, which calculates the positive and negative bull's-eye distances of the effect vector and the comprehensive bull's-eye distances of each program. The model solves attribute weight vector according to the principle of maximum entropy. By considering interval distribution and attribute relevance, it weakens the influence of attribute extreme values on the decision results, so that the evaluation results are more scientific and accurate.

## 2 Preliminaries

### 2.1 Grey target decision

In 1982, Professor Deng Ju-long, a Chinese scholar, established the grey system theory. After that, the grey system theory has been widely used, and derived a series of grey decision-making methods, such as grey target decision-making, which is the application and embodiment of the non-uniqueness principle in the grey system theory in the decision-making theory. In other words, it is basically to find out the standard index sequence closest to the target value in a set of index sequences composed of decision objects, and call it the clout of grey target. The index sequence composed of other decision objects and the clout together constitute the grey target, and their grey correlation with the clout is called the off-target proximity (Yan-hua et al. 2012). The evaluation and ranking of decision objects are mainly based on the proximity of the clout. Generally speaking, the grey target decision-making is to find a clout as the optimal mode in the grey target and then compare the decision points in the grey target with the clout points to determine the relatively optimal countermeasures.

**Definition 1** The totality of events within a certain research range is referred to event set within the research range, which is denoted as

$$A = \{a_1, a_2, \dots, a_n\}$$

where  $a_i$  is the  $i$ -th event,  $i = 1, 2, \dots, n$ ; all the corresponding possible countermeasures are collectively referred to as countermeasure set, which is denoted as

$$B = \{b_1, b_2, \dots, b_m\}$$

where  $b_j$  is the  $j$ -th countermeasure,  $j = 1, 2, \dots, m$ .

**Definition 2** Let  $S = \{s_{ij} = (a_i, b_j) | a_i \in A, b_j \in B\}$  be a decision scheme set, where  $u_{ij}^{(k)}$  is the measure of effectiveness of decision scheme  $s_{ij}$  in  $k$  targets, and  $R$  is the set of real numbers, then

$$u_{ij}^{(k)} : S \rightarrow R, \quad s_{ij} \rightarrow u_{ij}^{(k)}$$

is the effectiveness mapping of  $S$  in  $k$  targets.  $k$  is a certain number of targets; suppose that  $k$  objective is better under greater effect value,  $u_{ij}^{(k)} > u_{ih}^{(k)}$ , then it is said that the countermeasure  $b_j$  with regard to event  $a_i$  under the objective  $k$  is superior to  $b_h$ , which is denoted as  $b_j \succ b_h$ . Similarly, it is possible to define the countermeasure advantage class which is better under smaller target effect value.

**Definition 3** (Huan-hong et al. 2019): Let  $d_1^{(k)}, d_2^{(k)}$  be the upper, lower critical values of the measure of effectiveness of decision scheme  $s_{ij}$  in  $k$  targets, then  $S^1 = \{r | d_1^{(k)} \leq r \leq d_2^{(k)}\}$  is the one-dimensional decision-making grey target in  $k$  targets, and  $u_{ij}^{(k)} \in [d_1^{(k)}, d_2^{(k)}]$  is the promising result in  $k$  targets, thus the corresponding decision scheme  $s_{ij}$  is desirable in  $k$  targets, and  $b_j$  is a desirable decision of accident  $a_i$  in  $k$  targets.

In fact, the grey target decision-making is the area where the satisfactory effect lies in the sense of relative optimization. In many cases, it is impossible to get absolute optimum, so people usually ask for a satisfactory result. Of course, the decision grey target can be gradually reduced and finally transformed into a point according to the needs, which is the optimal effect, and the corresponding decision plan is the optimal plan.

**Definition 4** Let  $r_0 = (r_0^{(1)}, r_0^{(2)}, \dots, r_0^{(s)})$  be the optimal effect vector, then

$$R^s = \{(r^{(1)}, r^{(2)}, \dots, r^{(s)}) | (r^{(1)} - r_0^{(1)})^2 + (r^{(2)} - r_0^{(2)})^2 + \dots + (r^{(s)} - r_0^{(s)})^2 \leq R^2\} \tag{1}$$

is a  $s$ -dimensional spherical grey target with  $r_0 = (r_0^{(1)}, r_0^{(2)}, \dots, r_0^{(s)})$  as the clout and  $R$  as the radius.

**Definition 5** (Lin-ming et al. 2017): Let  $r_0 = (r_0^{(1)}, r_0^{(2)}, \dots, r_0^{(s)})$  be the clout, when  $r_1 = (r_1^{(1)}, r_1^{(2)}, \dots, r_1^{(s)}) \in R^s$ , then

$$|r_1 - r_0| = [(r_1^{(1)} - r_0^{(1)})^2 + (r_1^{(2)} - r_0^{(2)})^2 + \dots + (r_1^{(s)} - r_0^{(s)})^2]^{\frac{1}{2}} \tag{2}$$

is the target distance of vector  $r_1$ . The value of the target distance reflects the merits and demerits of the effect vector of the decision-making scheme.

## 2.2 Interval grey number and operation rules

In the study of grey system, the number of known value range but unknown exact value is called grey number, which is the basic unit of grey system. In the application, grey number actually refers to the uncertain number in a certain interval or a general number set, usually expressed by  $\otimes$ . The grey numbers with both lower bound  $\underline{a}$  and upper bound  $\bar{a}$  are called interval grey numbers, which are recorded as  $a(\otimes) \in [\underline{a}, \bar{a}]$ .

**Definition 6** (Dutta and Tripathy 2016): Let  $a(\otimes) \in [\underline{a}, \bar{a}]$  and  $b(\otimes) \in [\underline{b}, \bar{b}]$  be any two interval grey numbers and  $k$  is a positive real number, then the operation rules are as follows:

1.  $a(\otimes) + b(\otimes) \in [\underline{a} + \underline{b}, \bar{a} + \bar{b}]$ ;
2.  $a(\otimes) - b(\otimes) \in [\underline{a} - \bar{b}, \bar{a} - \underline{b}]$ ;
3.  $a(\otimes) \cdot b(\otimes) \in [\min\{\underline{a}\underline{b}, \underline{a}\bar{b}, \bar{a}\underline{b}, \bar{a}\bar{b}\}, \max\{\underline{a}\underline{b}, \underline{a}\bar{b}, \bar{a}\underline{b}, \bar{a}\bar{b}\}]$ ;
4.  $a(\otimes)/b(\otimes) \in [\min\{\frac{\underline{a}}{\bar{b}}, \frac{\underline{a}}{\underline{b}}, \frac{\bar{a}}{\bar{b}}, \frac{\bar{a}}{\underline{b}}\}, \max\{\frac{\underline{a}}{\bar{b}}, \frac{\underline{a}}{\underline{b}}, \frac{\bar{a}}{\bar{b}}, \frac{\bar{a}}{\underline{b}}\}]$ ;
5.  $k + a(\otimes) \in [k + \underline{a}, k + \bar{a}]$ ;
6.  $k \cdot a(\otimes) \in [k\underline{a}, k\bar{a}]$ .

**Definition 7** (Dang 2013): Let  $a(\otimes) \in [\underline{a}, \bar{a}]$  and  $b(\otimes) \in [\underline{b}, \bar{b}]$  be any two interval grey numbers, then the distance between the interval grey numbers  $a(\otimes)$  and  $b(\otimes)$  is:

$$d(a(\otimes), b(\otimes)) = 2^{-\frac{1}{2}}[(\underline{a} - \underline{b})^2 + (\bar{a} - \bar{b})^2]^{\frac{1}{2}} \tag{3}$$

## 3 The establishment of interval number grey target decision model

### 3.1 Normalization of decision matrix

Because the attribute values in the sample effectiveness decision matrix are in different measurement units and subject to different standards, in order to facilitate the unified processing, the theory of vague set and set pair analysis can be used, and the idea of bonus and forfeit can be used for reference, so as to generate the linear transformation operator  $[-1, 1]$  of bonus and forfeit to deal with the attribute dimensionless, thus obtaining the normalized decision matrix.

If  $x_{ij}$  is the evaluation value for the  $j$ -th attribute  $C_j$  of scheme  $S_i$ , which is an interval grey number, i.e.,  $x_{ij}(\otimes) \in [\underline{x}_{ij}, \bar{x}_{ij}]$ , where  $0 \leq \underline{x}_{ij} \leq \bar{x}_{ij}; i = 1, 2, \dots, n$ ; and  $j = 1, 2, \dots, m$ .

Let

$$z_j = \frac{1}{2n} \sum_{i=1}^n (x_{ij}, \bar{x}_{ij}) \tag{4}$$

$$i = 1, 2, \dots, n, \quad j = 1, 2, \dots, m$$

The different dimensions of attributes in the decision matrix will have adverse effects on the decision results, so the attribute values in the decision matrix are treated dimensionless (Zhu and Hipel 2012).

1. If  $x_{ij}$  is a benefit-type attribute, then

$$[\underline{y}_{ij}, \bar{y}_{ij}] = \left[ \frac{\underline{x}_{ij} - z_j}{|z_j|}, \frac{\bar{x}_{ij} - z_j}{|z_j|} \right] \tag{5}$$

2. If  $x_{ij}$  is a cost-type attribute, then

$$[\underline{y}_{ij}, \bar{y}_{ij}] = \left[ \frac{z_j - \bar{x}_{ij}}{|z_j|}, \frac{z_j - \underline{x}_{ij}}{|z_j|} \right] \tag{6}$$

After dimensionalization, the transformed matrix  $D$  is:

$$D = ([\underline{y}_{ij}, \bar{y}_{ij}])_{n \times m} \tag{7}$$

In this way,  $\underline{y}_{ij}$  may be less than -1 and  $\bar{y}_{ij}$  may be greater than 1. Therefore, the following transformation may be used to normalize matrix  $D$  to get the normalized decision matrix  $R$ .

$$R = ([\underline{r}_{ij}, \bar{r}_{ij}])_{n \times m} \tag{8}$$

where

$$[\underline{r}_{ij}, \bar{r}_{ij}] = \left[ \frac{\underline{y}_{ij}}{\max_j(|\underline{y}_{ij}|, |\bar{y}_{ij}|)}, \frac{\bar{y}_{ij}}{\max_j(|\underline{y}_{ij}|, |\bar{y}_{ij}|)} \right] \tag{9}$$

The above transformation is called a  $[-1, 1]$  linear transformation operator (Jie et al. 2010).

Thus,  $\underline{r}_{ij}, \bar{r}_{ij} \in [-1, 1]$ . The above transformation can be carried out for each attribute to obtain the consistency effect measurement matrix of effect sample value of scheme  $S_i$  for attribute  $C_j$  (Wen-jie and Hong-ping 2019).

$$R = \begin{bmatrix} [\underline{r}_{11}, \bar{r}_{11}] & [\underline{r}_{12}, \bar{r}_{12}] & \cdots & [\underline{r}_{1m}, \bar{r}_{1m}] \\ [\underline{r}_{21}, \bar{r}_{21}] & [\underline{r}_{22}, \bar{r}_{22}] & \cdots & [\underline{r}_{2m}, \bar{r}_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{r}_{n1}, \bar{r}_{n1}] & [\underline{r}_{n2}, \bar{r}_{n2}] & \cdots & [\underline{r}_{nm}, \bar{r}_{nm}] \end{bmatrix} \tag{10}$$

### 3.2 Grey target decision of positive and negative clouts

**Definition 8** Let  $r_j^+ = \max\{\frac{(\underline{r}_{ij} + \bar{r}_{ij})}{2} | 1 \leq i \leq n\}$ ,  $j = 1, 2, \dots, m$ , the corresponding decision value is recorded as  $[\underline{r}_j^+, \bar{r}_j^+]$ , then

$$r^+ = \{r_1^+, r_2^+, \dots, r_m^+\} = \{[L_{i1}^+, \bar{r}_{i1}^+], [L_{i2}^+, \bar{r}_{i2}^+], \dots, [L_{im}^+, \bar{r}_{im}^+]\} \tag{11}$$

is the optimal effect vector of grey target decision, known as the interval number positive clout. Here, + means positive bull's-eye, *j* means serial number of the attribute, and  $r_j^+$  means positive bull's-eye about the interval number of the *j*-th attribute.

**Definition 9** (Garg and Arora 2018; Yue and Ze-shui 2019): Let  $r_j^- = \min\{\frac{(L_{ij} + \bar{r}_{ij})}{2} | 1 \leq i \leq n\}$ ,  $j = 1, 2, \dots, m$ , the corresponding decision value is recorded as  $[L_{ij}^-, \bar{r}_{ij}^-]$ , then

$$r^- = \{r_1^-, r_2^-, \dots, r_m^-\} = \{[L_{i1}^-, \bar{r}_{i1}^-], [L_{i2}^-, \bar{r}_{i2}^-], \dots, [L_{im}^-, \bar{r}_{im}^-]\} \tag{12}$$

is the worst effect vector of grey target decision, known as the interval number negative clout.

where attribute weight is  $W = (\omega_1, \omega_2, \dots, \omega_m)$ , and  $\sum_{j=1}^m \omega_j = 1$ .

**Definition 10**

$$\varepsilon_i^+ = 2^{-\frac{1}{2}}[\omega_1(L_{i1} - L_{i1}^+)^2] + \omega_1(\bar{r}_{i1} - \bar{r}_{i1}^+)^2 + \dots + \omega_m(\bar{r}_{im} - \bar{r}_{im}^+)^2 \tag{13}$$

is the positive clout distance of effect vector  $r_i$ .

$$\varepsilon_i^- = 2^{-\frac{1}{2}}[\omega_1(L_{i1} - L_{i1}^-)^2] + \omega_1(\bar{r}_{i1} - \bar{r}_{i1}^-)^2 + \dots + \omega_m(\bar{r}_{im} - \bar{r}_{im}^-)^2 \tag{14}$$

is the negative clout distance of effect vector  $r_i$ .

**Definition 11** (Mei-juan et al. 2017)

$$\varepsilon_i^0 = 2^{-\frac{1}{2}}[\omega_1(L_{i1}^+ - L_{i1}^-)^2] + \omega_1(\bar{r}_{i1}^+ - \bar{r}_{i1}^-)^2 + \dots + \omega_m(\bar{r}_{im}^+ - \bar{r}_{im}^-)^2 \tag{15}$$

is the distance between the positive and negative clouts.

According to the definition in Shu-li and Si-feng (2014), the distances  $\varepsilon_i^+$ ,  $\varepsilon_i^-$  and  $\varepsilon_i^0$  fall on the same straight line or form a triangle. Therefore, the projection size of the distance between the positive and negative clouts can be used to get the optimal countermeasure of the event, that is, the larger the projection, the better the corresponding countermeasure. Let the angle between the positive off-target distance and connection between positive and negative clouts as  $\theta$ , then according to the cosine law

$$(\varepsilon_i^+)^2 + (\varepsilon_i^0)^2 - 2\varepsilon_i^+ \varepsilon_i^0 \cos \theta = (\varepsilon_i^-)^2$$

Since the positive and negative off-target distances  $\varepsilon_i^+$  and  $\varepsilon_i^-$  are both vectors, considering the projection of the off-target distance on the line between the positive and negative clouts, the comprehensive off-target distance  $\varepsilon_i$  is:

$$\varepsilon_i = \varepsilon_i^+ \cos \theta = \frac{(\varepsilon_i^+)^2 + (\varepsilon_i^0)^2 - (\varepsilon_i^-)^2}{2\varepsilon_i^0} \tag{16}$$

In order to make the decision-making information more scientific and reasonable, both positive and negative clouts are considered in the comprehensive off-target distance, and the off-target distance is used as a vector (Zheng-hua et al. 2019; Khameneh and Kılıçman 2019).

### 3.3 Determining attribute weight based on information entropy theory

According to the theory of information entropy, information entropy can measure the ordering degree of the system (Zu-yun 2013). Therefore, the smaller the difference between different experts on the evaluation value of the same attribute, the greater the entropy of the evaluation data of the attribute, indicating that the more unified the expert opinions, the more effective the evaluation data of the attribute. On the contrary, when different experts have different evaluation values for the same attribute, the smaller the information entropy of the evaluation data of the attribute is, indicating that the more inconsistent the expert opinions are, and the less effective the evaluation data of the attribute are.

The difference degree of evaluation data of attribute  $C_j$  is measured by information entropy; then, the information entropy of evaluation data of attribute  $C_j$  is  $H_j$ :

$$H_j = - \sum_{i=1}^n u_{ij} \ln u_{ij} \tag{17}$$

According to the extremum principle of entropy, the more consistent the evaluation value  $u_{ij}$  of attribute  $C_j$  is, the greater the value of information entropy  $H_j$  is (Ji-bin et al. 2020).

If the attribute weight vector  $W = (\omega_1, \omega_2, \dots, \omega_m)$  is not completely determined, the sequence is a grey connotation sequence, and the grey entropy can be defined:

$$H_{\otimes}(\omega) = - \sum_{j=1}^m \omega_j \ln \omega_j \tag{18}$$

According to the principle of maximum entropy,  $\omega_j(j = 1, 2, \dots, m)$  should be adjusted to reduce the uncertainty of weight vector  $W = (\omega_1, \omega_2, \dots, \omega_m)$  so that  $H_{\otimes}(\omega)$  is maximized. At the same time, the weight  $\omega_j(j = 1, 2, \dots, m)$  is adjusted to minimize the overall off-target



distance, so the following multi-objective optimization model can be established (Wei et al. 2017):

$$\begin{cases} \min \sum_{i=1}^n \varepsilon_i = \sum_{i=1}^n \frac{(\varepsilon_i^+)^2 + (\varepsilon_i^0)^2 - (\varepsilon_i^-)^2}{2\varepsilon_i^0} \\ \max H_{\otimes}(\omega) = - \sum_{j=1}^m \omega_j \ln \omega_j \\ \text{s.t. } \sum_{j=1}^m \omega_j = 1, \quad \omega_j \geq 0, \quad j = 1, 2, \dots, m \end{cases} \quad (19)$$

In order to solve the multi-objective optimization model, it can be transformed into a single-objective optimization model according to the fair competition of each scheme (Zarbakshnia et al. 2020).

$$\begin{cases} \min \left\{ \lambda \sum_{i=1}^n \frac{(\varepsilon_i^+)^2 + (\varepsilon_i^0)^2 - (\varepsilon_i^-)^2}{2\varepsilon_i^0} + (1 - \lambda) \sum_{j=1}^m \omega_j \ln \omega_j \right\} \\ \text{s.t. } \sum_{j=1}^m \omega_j = 1, \quad \omega_j \geq 0, \quad j = 1, 2, \dots, m \end{cases} \quad (20)$$

where  $0 < \lambda < 1$ . In consideration of the fair competition of optimized objective function,  $\lambda = 0.5$  is generally adopted. The model is solved by Visual Studio programming, and the attribute weight vector  $W = (\omega_1, \omega_2, \dots, \omega_m)$  is obtained. Finally, by substituting it into Eq. 16, we can get the comprehensive off-target distance  $\varepsilon_i$ . The alternatives are sorted according to the value of  $\varepsilon_i$ . the smaller it is, the better the corresponding alternatives are.

### 4 Decision-making problems and methods of interval number multi-attribute grey target based on positive and negative clouts

#### 4.1 Problem description

If in the MADM problem, there are  $n$  decision-making schemes, which make up the decision-making scheme set  $S = \{S_1, S_2, \dots, S_n\}$ , where  $S_i$  is the  $i$ -th scheme,  $i = 1, 2, \dots, n$ ;  $m$  evaluation indicators (attributes) constitute the attribute set  $C = \{C_1, C_2, \dots, C_m\}$ , where  $C_j$  is the  $j$ -th attribute, and  $j = 1, 2, \dots, m$ . The attribute weight vector  $W = (\omega_1, \omega_2, \dots, \omega_m)$  is not completely certain, but  $\omega_j \geq 0$  and  $\sum_{j=1}^m \omega_j = 1$ . The decision information is not a specific exact number, but an interval grey number. The attribute value of scheme  $S_i$  to attribute  $C_j$  is  $x_{ij}(\otimes) \in [\underline{x}_{ij}, \bar{x}_{ij}]$ , where  $0 \leq \underline{x}_{ij} \leq \bar{x}_{ij}, i = 1, 2, \dots, n; j = 1, 2, \dots, m$ , then the sample effectiveness matrix  $X$  of scheme set  $S$  to attribute set  $C$  is (Ashraf et al. 2018; Das et al. 2020):

$$X = \begin{bmatrix} x_{11}(\otimes) & x_{12}(\otimes) & \cdots & x_{1m}(\otimes) \\ x_{21}(\otimes) & x_{22}(\otimes) & \cdots & x_{2m}(\otimes) \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1}(\otimes) & x_{n2}(\otimes) & \cdots & x_{nm}(\otimes) \end{bmatrix}$$

The above matrix can be converted into:

$$X = \begin{bmatrix} [\underline{x}_{11}, \bar{x}_{11}] & [\underline{x}_{12}, \bar{x}_{12}] & \cdots & [\underline{x}_{1m}, \bar{x}_{1m}] \\ [\underline{x}_{21}, \bar{x}_{21}] & [\underline{x}_{22}, \bar{x}_{22}] & \cdots & [\underline{x}_{2m}, \bar{x}_{2m}] \\ \vdots & \vdots & \ddots & \vdots \\ [\underline{x}_{n1}, \bar{x}_{n1}] & [\underline{x}_{n2}, \bar{x}_{n2}] & \cdots & [\underline{x}_{nm}, \bar{x}_{nm}] \end{bmatrix}$$

#### 4.2 Steps of interval number multi-attribute grey target decision

To sum up, the specific steps of interval number multi-attribute grey target decision-making based on positive and negative clouts are as follows:

**Step 1:** According to the MADM problem, the sample effect matrix  $X$  is transformed into the normalized decision matrix  $R$  by using the  $[-1, 1]$  interval number linear transformation operator.

**Step 2:** By using Eqs. 11–12, the positive and negative clout numbers  $r^+$  and  $r^-$  of grey target decision-making are determined, respectively.

**Step 3:** According to Eqs. 13–14, the positive and negative off-target distances  $\varepsilon_i^+$  and  $\varepsilon_i^-$  of effect vector  $z_i$  are calculated, respectively, and the distance between positive and negative clouts  $\varepsilon_i^0$  is calculated according to Eq. 15.

**Step 4:** According to the principle of maximum entropy and the idea of comprehensive off-target distance minimization, a multi-objective optimization model for solving grey entropy is established by using Eq. 18–Eq. 19. Then, the multi-objective optimization model is transformed into a single-objective optimization model, such as Eq. 20, which is solved by software programming to obtain the attribute weight vector  $W = (\omega_1, \omega_2, \dots, \omega_m)$ .

**Step 5:** Eq. 16 is used to determine the comprehensive off-target distance  $\varepsilon_i$ , and the alternatives are sorted and optimized according to the value of  $\varepsilon_i$ .

### 5 Instance analysis

Venture capital is an investment process to promote the commercialization and industrialization of high-tech achievements as soon as possible, so as to obtain high capital gains. However, there are many uncertainties in the process of venture capital, which bring great risks to the investment and its return. If there are 5 alternatives  $S_i (i = 1, 2, \dots, 5)$  for the project investment of a venture

capital company, the project (scheme) is evaluated from the perspective of risk factors. The risk factors mainly include six attributes: (1) market risk ( $C_1$ ), difficulty in determining competitiveness, prediction of diffusion speed of innovative products, determination of market acceptance ability, etc., and the uncertainty of whether to win the market competitive advantage due to environmental factors; (2) technology risk ( $C_2$ ), brought by many factors, such as the inherent deficiency of new ideas and new technology itself, the immaturity and imperfection of technology, and the rapid emergence of alternative new technology; (3) management risk ( $C_3$ ), brought by the quality of management, management ability and various personnel factors; (4) environmental risk ( $C_4$ ), caused by the change of social political and economic environment; (5) production risk ( $C_5$ ), produced in the production process brought by the unforeseen obstacles in the production process, instruments, equipment, raw materials and other aspects of the enterprise; (6) financial risk ( $C_6$ ), as a result of the change of financial market to the investment of enterprises. The attribute weight information given by the decision maker is not completely determined, and the known weight information is:  $0.15 \leq \omega_1 \leq 0.18, 0.16 \leq \omega_2 \leq 0.17, 0.17 \leq \omega_3 \leq 0.18, 0.14 \leq \omega_4 \leq 0.19, 0.13 \leq \omega_5 \leq 0.16, 0.16 \leq \omega_6 \leq 0.20$ ; and  $\sum_{j=1}^6 \omega_j = 1$ . According to the decision maker's risk attitude, the best investment plan was determined.

The above six attributes are all cost attributes, and the attribute values of each scheme under each attribute are shown in Table 1.

1. According to the  $[-1, 1]$  interval number linear transformation operator, the interval number effect matrix is transformed into a dimensionless decision matrix, that is, the dimensionless transformation matrix, as shown in Table 2.

By Eq. 8–Eq. 9, the transformation matrix  $D$  is normalized to obtain the normalized decision matrix  $R$ , as shown in Table 3.

2. According to the normalized decision matrix  $R$ , the positive and negative clouts of grey target decision-making are obtained by Eq. 11–12.

The interval number positive clout  $r^+$  is

$$r^+ = \{[0.026, 0.923], [0.364, 0.818], [0.392, 0.772], [0.316, 0.711], [0.429, 0.857], [0.475, 0.803]\}$$

The interval number negative clout  $r^-$  is

$$r^- = \{[-1.000, -0.231], [-1.000, -0.091], [-1.000, -0.367], [-1.000, -0.605], [-1.000, -0.714], [-1.000, -0.016]\}$$

3. y Eq. 13 - Eq. 14, the positive and negative off-target distances of the effect vector  $r_i$  are calculated, respectively.

The positive off-target distance  $\varepsilon_i^+$  is:

$$\begin{cases} \varepsilon_1^+ = 2^{-\frac{1}{2}}[0.0657\omega_1 + 1.1938\omega_2 + 3.2367\omega_3 + 0.0173\omega_4 + 1.0612\omega_5 + 1.8275\omega_6]^{\frac{1}{2}} \\ \varepsilon_2^+ = 2^{-\frac{1}{2}}[0.0657\omega_1 + 2.6860\omega_2 + 0.0801\omega_3 + 0.2770\omega_4 + 0.6939\omega_5]^{\frac{1}{2}} \\ \varepsilon_3^+ = 2^{-\frac{1}{2}}[0.2794\omega_1 + 0.9774\omega_3 + 0.0173\omega_4 + 4.5102\omega_5 + 2.8487\omega_6]^{\frac{1}{2}} \\ \varepsilon_4^+ = 2^{-\frac{1}{2}}[1.0684\omega_1 + 0.4591\omega_2 + 3.4626\omega_4 + 0.4300\omega_6]^{\frac{1}{2}} \\ \varepsilon_5^+ = 2^{-\frac{1}{2}}[2.3833\omega_1 + 0.7805\omega_2 + 1.1857\omega_3 + 1.4716\omega_4 + 0.5306\omega_5 + 1.9618\omega_6]^{\frac{1}{2}} \end{cases}$$

The negative off-target distance  $\varepsilon_i^-$  is:

$$\begin{cases} \varepsilon_1^- = 2^{-\frac{1}{2}}[1.9231\omega_1 + 0.2984\omega_2 + 3.1337\omega_4 + 1.3265\omega_5 + 0.2687\omega_6]^{\frac{1}{2}} \\ \varepsilon_2^- = 2^{-\frac{1}{2}}[1.8573\omega_1 + 2.3874\omega_3 + 2.3546\omega_4 + 1.8163\omega_5 + 2.8487\omega_6]^{\frac{1}{2}} \\ \varepsilon_3^- = 2^{-\frac{1}{2}}[1.2163\omega_1 + 2.6860\omega_2 + 0.6569\omega_3 + 3.1337\omega_4]^{\frac{1}{2}} \\ \varepsilon_4^- = 2^{-\frac{1}{2}}[0.4274\omega_1 + 0.9412\omega_2 + 3.2367\omega_3 + 4.5102\omega_5 + 1.3437\omega_6]^{\frac{1}{2}} \\ \varepsilon_5^- = 2^{-\frac{1}{2}}[0.5739\omega_2 + 0.5127\omega_3 + 0.4328\omega_4 + 2.5510\omega_5 + 0.1344\omega_6]^{\frac{1}{2}} \end{cases}$$

The distance  $\varepsilon_i^0$  between positive and negative clouts is calculated by Eq. 15:

$$\varepsilon_i^0 = 2^{-\frac{1}{2}}[2.3833\omega_1 + 2.6860\omega_2 + 3.2367\omega_3 + 3.4626\omega_4 + 4.5102\omega_5 + 2.8487\omega_6]^{\frac{1}{2}}$$

4. The single-objective optimization model is determined by Eq. 20 in combination with the information of positive off-target distance, negative off-target distance, distance between positive and negative clouts and incomplete attribute weight.

**Table 1** Interval number effect sample matrix  $X$

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
$S_1$	[0.21,0.30]	[0.26,0.31]	[0.30,0.35]	[0.22,0.26]	[0.25,0.30]	[0.24,0.32]
$S_2$	[0.23,0.28]	[0.28,0.34]	[0.23,0.25]	[0.22,0.29]	[0.24,0.29]	[0.22,0.24]
$S_3$	[0.25,0.29]	[0.22,0.25]	[0.26,0.30]	[0.23,0.25]	[0.32,0.34]	[0.27,0.33]
$S_4$	[0.25,0.35]	[0.24,0.29]	[0.21,0.24]	[0.32,0.35]	[0.21,0.24]	[0.22,0.28]
$S_5$	[0.30,0.36]	[0.25,0.30]	[0.26,0.31]	[0.28,0.32]	[0.22,0.29]	[0.25,0.32]

**Table 2** Dimensionless transformation matrix *D*

	<i>C</i> <sub>1</sub>	<i>C</i> <sub>2</sub>	<i>C</i> <sub>3</sub>	<i>C</i> <sub>4</sub>	<i>C</i> <sub>5</sub>	<i>C</i> <sub>6</sub>
<i>S</i> <sub>1</sub>	[− 0.064,0.255]	[− 0.131,0.051]	[− 0.292,− 0.107]	[0.051,0.197]	[− 0.111,0.074]	[− 0.190,0.108]
<i>S</i> <sub>2</sub>	[0.007,0.184]	[− 0.241,− 0.022]	[0.077,0.151]	[− 0.058,0.197]	[− 0.074,0.111]	[0.108,0.182]
<i>S</i> <sub>3</sub>	[− 0.028,0.113]	[0.088,0.197]	[−0.107,0.041]	[0.088,0.161]	[− 0.259,− 0.185]	[− 0.227,− 0.004]
<i>S</i> <sub>4</sub>	[− 0.241,0.113]	[− 0.058,0.124]	[0.114,0.225]	[− 0.277,− 0.168]	[0.111,0.222]	[− 0.041,0.182]
<i>S</i> <sub>5</sub>	[− 0.277,− 0.064]	[− 0.095,0.088]	[− 0.144,0.041]	[− 0.168,− 0.022]	[− 0.074,0.185]	[− 0.190,0.071]

**Table 3** Normalized decision matrix *R*

	<i>C</i> <sub>1</sub>	<i>C</i> <sub>2</sub>	<i>C</i> <sub>3</sub>	<i>C</i> <sub>4</sub>	<i>C</i> <sub>5</sub>	<i>C</i> <sub>6</sub>
<i>S</i> <sub>1</sub>	[− 0.231,0.923]	[− 0.545,0.212]	[− 1.000,− 0.367]	[0.184,0.711]	[− 0.429,0.286]	[− 0.836,0.475]
<i>S</i> <sub>2</sub>	[0.026,0.667]	[− 1.000,− 0.091]	[0.266,0.519]	[− 0.211,0.711]	[− 0.286,0.429]	[0.475,0.803]
<i>S</i> <sub>3</sub>	[− 0.103,0.410]	[0.364,0.818]	[− 0.367,0.139]	[0.316,0.579]	[− 1.000,− 0.714]	[− 1.000,− 0.016]
<i>S</i> <sub>4</sub>	[− 0.872,0.410]	[− 0.242,0.515]	[0.392,0.772]	[− 1.000,− 0.605]	[0.429,0.857]	[− 0.180,0.803]
<i>S</i> <sub>5</sub>	[− 1.000,− 0.231]	[− 0.394,0.364]	[− 0.494,0.139]	[− 0.605,− 0.079]	[− 0.286,0.714]	[− 0.836,0.311]

$$\left\{ \begin{array}{l} \min \left\{ \lambda \sum_{i=1}^n \frac{(\varepsilon_i^+)^2 + (\varepsilon_i^0)^2 - (\varepsilon_i^-)^2}{2\varepsilon_i^0} + (1 - \lambda) \sum_{j=1}^m \omega_j \ln \omega_j \right\} \\ 0.15 \leq \omega_1 \leq 0.18; \quad 0.16 \leq \omega_2 \leq 0.17 \\ 0.17 \leq \omega_3 \leq 0.18; \quad 0.14 \leq \omega_4 \leq 0.19 \\ 0.13 \leq \omega_5 \leq 0.16; \quad 0.16 \leq \omega_6 \leq 0.20 \\ \sum_{j=1}^m \omega_j = 1, \quad j = 1, 2, \dots, m \\ 0 < \lambda < 1 \end{array} \right.$$

Software programming is used to solve the above optimization model, and the attribute weight vector *W* is obtained as follows:

$$W = (0.179, 0.160, 0.176, 0.189, 0.135, 0.160)$$

5. Eq. 16 is used to determine the comprehensive off-target distance  $\varepsilon_i$ , and the alternatives are sorted according to the value of  $\varepsilon_i$ .

$$\varepsilon_1 = 0.6280, \quad \varepsilon_2 = 0.3684, \quad \varepsilon_3 = 0.6140, \\ \varepsilon_4 = 0.5009, \varepsilon_5 = 0.7853$$

Thus,  $\varepsilon_2 < \varepsilon_4 < \varepsilon_3 < \varepsilon_1 < \varepsilon_5$ ; hence, the schemes are sorted as  $S_2 \succ S_4 \succ S_3 \succ S_1 \succ S_5$ . After calculation and analysis, it is found that the best investment scheme is *S*<sub>2</sub>, which is consistent with the conclusion in Ref. (Yong et al. 2013), so the method is feasible and effective. Based on the theory of information entropy and the idea of minimal comprehensive bull’s-eye distance, a multi-objective optimization model is established and transformed into a single-objective optimization model. Therefore, by solving

the linear programming, the weights can be objectively obtained when the attribute weights are not completely determined, so that the scheme can be effectively evaluated.

At the same time, decision makers usually give decision-making information in the form of interval grey number because of the complexity of decision-making environment information and uncertainty, it is difficult to make decisions according to the existing methods. The grey target decision-making model based on positive and negative clouts can solve this problem well, because the distance between the scheme and the positive and negative clouts is considered, the off-target distance is defined as a vector on the basis of spatial analysis, and a multi-objective optimization model is established to solve the problem. In order to maintain linearity of the original data, this paper proposes a new data standardization method suitable for interval grey numbers based on the characteristics of interval grey numbers. The interval number grey target decision-making model is used to analyze the effectiveness of venture capital projects, and effective ranking and selection of the best are realized, which will help venture capital companies understand the difference in effectiveness of different venture capital projects.

Obviously, compared with the cited literature, the model proposed herein boasts the following advantages:

- (1) The former method uses interval number linear transformation operator to normalize the original decision information, designs positive and negative bull’s-eye of the interval number and constructs



positive and negative ideal program, respectively. In contrast, this paper not only designs the positive and negative bull's-eye of the grey target decision-making interval, but also comprehensively considers the spatial projection distance of each program from the positive and negative bull's-eye. By using the bull's-eye distance that can reflect the spatial projection distance relationship as a vector for spatial analysis, a new comprehensive bull's-eye distance is derived. Therefore, the decision-making information obtained by the latter is more in line with the actual situation and also more scientific and reasonable.

- (2) The former method provides the attribute weight space in advance in view of the situation where the attribute weight information is not completely determined, and then uses prospect theory and genetic algorithm programming to establish and solve the optimization model, thereby obtaining the optimal weight vector. It is not difficult to see that the obtained weight vector values are distributed in the upper and lower limits of the attribute interval, rather than evenly distributed within the interval. The model is susceptible to the influence of extreme values, resulting in inaccurate decision-making results. In contrast, this paper constructs a multi-objective programming model based on the principle of maximum entropy and minimal comprehensive bull's-eye distance to determine the attribute weight vector. Therefore, the decision results of the latter are more objective and convincing compared to the former.

## 6 Conclusions

Grey target decision-making is one of the important methods to solve the problem of MADM. In this study, aiming at the complexity and uncertainty of the actual decision-making environment, an interval number grey target decision-making model based on positive and negative clouts is proposed, in which the interval number linear transformation operator is used to normalize the multi-attribute grey target decision-making values, and the concepts of positive and negative clouts and the positive and negative off-target distances of grey target decision-making are introduced. On the basis of it, the calculation method of comprehensive off-target distance is put forward by combining with spatial analysis, and the attribute weight vector is solved by introducing the information entropy theory, and the schemes are sorted accordingly. With better application and practical decision-making value, the

achievements of this research can provide an effective scientific method for solving the interval number grey target decision-making problem. This model is mainly suitable for multi-attribute decision-making problems when there are many decision-making objects, the decision information is an interval grey number, and the attribute weight is not completely determined. In future research, the decision information can be extended to application in multi-attribute decision-making problem under background with uncertain linguistic variables and fuzzy numbers. In addition, as business changes and technological developments lead to the problem of attribute overlap, in the development of evaluation practice, evaluation data should be continuously accumulated to dynamically revise the evaluation attributes and weights.

**Acknowledgements** We would like to thank the referees for their valuable comments and suggestions.

**Author Contributions** All authors have contributed to this research equally.

**Funding** This work was supported in part by the Scientific Research Fund of Hunan Provincial Education Department under Grant 20A080 ("Research on intuitionistic fuzzy multi-attribute decision-making method considering decision maker's preference and its application"), in part by the Social Science Achievement Evaluation Committee Project of Hunan Provincial under Grant XSP20YBZ031, and in part by the Natural Science Foundation of China under Grant 61802120.

## Compliance with ethical standards

**Conflict of interest** The authors declare no conflict of interest.

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