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Dark matter in a constrained E_6 inspired SUSY model

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Abstract: We investigate dark matter in a constrained E_6 inspired supersymmetric model with an exact custodial symmetry and compare with the CMSSM. The breakdown of E_6 leads to an additional $U(1)_N$ symmetry and a discrete matter parity. The custodial and matter symmetries imply there are two stable dark matter candidates, though one may be extremely light and contribute negligibly to the relic density. We demonstrate that a predominantly Higgsino, or mixed bino-Higgsino, neutralino can account for all of the relic abundance of dark matter, while fitting a 125 GeV SM-like Higgs and evading LHC limits on new states. However we show that the recent LUX 2016 limit on direct detection places severe constraints on the mixed bino-Higgsino scenarios that explain all of the dark matter. Nonetheless we still reveal interesting scenarios where the gluino, neutralino and chargino are light and discoverable at the LHC, but the full relic abundance is not accounted for. At the same time we also show that there is a huge volume of parameter space, with a predominantly Higgsino dark matter candidate that explains all the relic abundance, that will be discoverable with XENON1T. Finally we demonstrate that for the E_6 inspired model the exotic leptoquarks could still be light and within range of future LHC searches.

Keywords: Cosmology of Theories beyond the SM, Supersymmetric Standard Model

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Contents

1 Introduction

A plethora of astrophysical and cosmological observations provide strong evidence for the presence of non-baryonic, non-luminous matter, so called dark matter (DM), that constitutes about 25% of the energy density of the Universe [1]. So far its microscopic composition remains unknown. However it is clear that dark matter can not consist of any standard model (SM) particles. Therefore its existence represents the strongest piece of evidence for physics beyond the SM.

Models with softly broken supersymmetry (SUSY) are currently the best motivated extensions of the SM. Within these models the quadratic divergences, which give rise to the destabilization of the electroweak (EW) scale, get cancelled [2–5]. Models with softly broken SUSY also provide an attractive framework for the incorporation of the gravitational interactions. Indeed, a partial unification of the SM gauge interactions with gravity can be attained within models based on the ($N = 1$) local SUSY (supergravity). Nevertheless ($N = 1$) supergravity (SUGRA) is a non-renormalizable theory. The ($N = 1$) SUGRA models can arise from ten dimensional $E_8 \times E_8'$ heterotic string theory [6]. The compactification of the extra dimensions in this theory results in breaking $E_8 \to E_6$ [7–9]. The remaining E_8' constitutes a hidden sector that gives rise to spontaneous breakdown of local SUSY. The hidden sector and visible sectors interact only gravitationally, which allows for the breaking of local SUSY in the hidden sector to be communicated to the visible sector and results in a set of soft SUSY breaking interactions.

When R-parity is conserved the lightest SUSY particle (LSP) in the models with softly broken SUSY is stable and therefore can play the role of dark matter [10]. Moreover in the simplest SUSY extension of the SM, i.e., the minimal supersymmetric standard model (MSSM), the SM gauge couplings extrapolated to high energies using the renormalization group (RG) equations (RGEs) converge to a common value at some high energy scale $M_X \sim 10^{16}$ GeV [11–14]. This permits to embed the SM gauge group into Grand Unified Theories (GUTs) [15] based on E_6 or its subgroups such as SU(5) and SO(10).

In this context it is especially important to explore the implications for dark matter and collider phenomenology within well motivated E_6 inspired SUSY extensions of the SM. The breakdown of E_6 may lead to a variety of SUSY models at low energies. In particular, a set of the simplest E_6 inspired SUSY extensions of the SM includes supersymmetric models based on the SM gauge group, like the MSSM, as well as extensions of the MSSM with an extra U(1) gauge symmetry. Within the class of the E_6 inspired U(1) extensions of the MSSM, there is a unique choice of Abelian $U(1)_N$ gauge symmetry that allows zero charges for right-handed neutrinos and this is the $U(1)'$ that appears in the exceptional supersymmetric standard model (E_6SSM) [16, 17]. This choice ensures that the right-handed neutrinos can be superheavy, so that a high scale see-saw mechanism can be used to generate the mass hierarchy in the lepton sector, providing a comprehensive understanding of the neutrino oscillations data. Successful leptogenesis is also a distinctive feature of the E_6SSM because the heavy Majorana right-handed neutrinos may decay into final states with lepton number $L = \pm 1$, creating a lepton asymmetry in the early Universe [18, 19]. Since sphalerons violate $B + L$ but conserve $B - L$, this lepton asymmetry gets converted into the observed baryon asymmetry of the Universe through the EW phase transition. In this case substantial values of the CP-asymmetries can be generated even for the lightest right-handed neutrino masses $M_1 \sim 10^6$ GeV so that successful thermal leptogenesis may be achieved without encountering a gravitino problem [19].

To ensure anomaly cancellation the matter content of the E_6SSM is extended to include three $\mathbf{27}$ representations of E_6. In addition the low energy spectrum can be supplemented by a $SU(2)_W$ doublet L_4 and anti-doublet $\overline{L}_4$ from extra $\mathbf{27}'$ and $\overline{\mathbf{27}'}$ to preserve the unification of the SM gauge couplings at high energies [20]. Thus the E_6SSM contains extra exotic matter beyond the MSSM. Over the last ten years, several variants of the E_6SSM have been proposed [16, 17, 21–31]. The E_6 inspired SUSY models with an extra $U(1)_N$ gauge symmetry have been thoroughly investigated as well. For example, the possibility

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of mixing between doublet and singlet neutrinos [32], the effects of $Z - Z'$ mixing [33], the neutralino sector [33–35], the implications of the exotic states for the dark matter [36], the renormalization group flow [20, 34] and EW symmetry breaking (EWSB) in the model [34, 37, 38] have all been studied. More recently, the RG flow of the Yukawa couplings and the theoretical upper bound on the lightest Higgs boson mass were explored in the vicinity of the quasi-fixed point [39, 40] that appears as a result of the intersection of the invariant and quasi-fixed lines [41]. Detailed studies of the E$_6$SSM have established that the additional exotic matter and Z' in the model would lead to distinctive LHC signatures [16, 17, 22, 25, 42–47], as well as result in non-standard Higgs decays for sufficiently light exotics [30, 40, 48–53]. In this SUSY model the particle spectrum has been examined in refs. [54–57], including the effects of threshold corrections from heavy states [58]. The renormalization of the vacuum expectation values (VEVs) that lead to EWSB in the model has also been calculated [59, 60], and the fine tuning in the model has been studied [61, 62].

Although the presence of exotic matter in the E$_6$SSM may lead to spectacular collider signatures it also gives rise to non-diagonal flavor transitions and rapid proton decay. In principle, an approximate Z_2^H symmetry can be imposed to suppress flavor changing processes in these U(1) extensions of the MSSM while the most dangerous baryon and lepton number violating operators can be forbidden by another exact Z_2 symmetry which plays a similar role to the R-parity in the MSSM [16, 17]. Using the method proposed in [63–65] it was shown that the LSP and next-to-lightest SUSY particle (NLSP) in the E$_6$SSM have masses below 60–65 GeV [49]. As a consequence these states can give rise to unacceptably large branching ratios of the exotic decays of the SM-like Higgs boson into the LSP and NLSP. In order to suppress such exotic Higgs decays and to prevent the decays of the lightest MSSM-like neutralino into the LSP and NLSP in models with approximate Z_2^H symmetry an additional Z_2^S symmetry needs to be postulated [26]. All discrete symmetries mentioned above do not commute with E_6 and the imposition of such symmetries to ameliorate phenomenological problems, which generically arise because of the presence of the exotic matter at low energies, is an undesirable feature of the models under consideration.

Here we focus on the investigation of the U(1)$_N$ extension of the MSSM (SE$_6$SSM) in which a single discrete $\tilde{Z}_2^H$ symmetry forbids tree-level flavor-changing transitions and the most dangerous operators that violate baryon and lepton numbers [28, 30, 39]. In a recent letter [66] we specified a set of benchmark points representing scenarios with a 125 GeV SM-like Higgs, which are consistent with the LHC limits on SUSY particles and measured dark matter abundance, within the constrained version of the above SE$_6$SSM (CSE$_6$SSM). As in any other constrained SUSY model, the soft SUSY-breaking scalar masses, gaugino masses, the trilinear and bilinear scalar couplings in the CSE$_6$SSM are each assumed to be universal at the scale M_X, where all gauge couplings coincide, i.e., $m_i^2(M_X) = m_0^2$, $M_i(M_X) = M_{1/2}$, $A_i(M_X) = A_0$ and $B_i(M_X) = B$. The benchmark scenarios presented in ref. [66] lead to large spin-independent (SI) dark matter-nucleon scattering cross section observable soon at XENON1T experiment and new physics signatures that may be observable at the 13 TeV LHC. These new signatures should allow to distinguish the SUSY model under consideration from the simplest SUSY extensions of the SM. At the same time in this

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letter we did not examine the CSE_6SSM parameter space thoroughly and did not provide full details of our calculations. We also did not include the full set of the two-loop RGEs which were used in our analysis.

In this article we present the results of the comprehensive analysis of the CSE_6SSM parameter space which is consistent with the 125 GeV SM-like Higgs, measured dark matter density and present LHC limits on sparticle masses. As in the MSSM the matter parity in the SE_6SSM is preserved. Therefore in both models the lightest R-parity odd state, i.e., LSP, is absolutely stable. In most scenarios that have been explored within the MSSM and its extensions the LSP is the lightest neutralino. In the CMSSM the lightest neutralino state is predominantly a linear superposition of the Higgsino and bino. Since the lightest neutralinos are heavy weakly interacting massive particles (WIMPs) they explain well the large scale structure of the Universe [67] and can provide the correct relic abundance of dark matter as long as the mass of the lightest neutralino is below the TeV scale [10]. The conservation of $\tilde{Z}_2^H$ symmetry and matter parity in the SE_6SSM results in the lightest neutralino as well as the lightest exotic state being stable. In the simplest phenomenologically viable scenarios the lightest exotic states have masses substantially lower than 1 eV forming hot dark matter in the Universe. The results of our analysis indicate that in this case the lightest neutralino in the CSE_6SSM, which is either mostly Higgsino or a mixed bino-Higgsino state, can account for all or some of the observed cold dark matter relic density.

We perform a scan of the parameter space of the CSE_6SSM enforcing successful EW symmetry breaking and imposing theoretical and low energy experimental constraints mentioned above. We also compute the dark matter density and SI neutralino-nucleon scattering cross section as well as examine their dependence on the parameters of the CSE_6SSM. The obtained results are compared with the corresponding ones in the CMSSM. We show that present LUX bounds set sufficiently stringent constraints on the mixing between bino and Higgsino states, for cases where they give a substantial contribution to the observed dark matter density. We therefore find that if the relic density is to be explained with Higgsino dark matter in either the CMSSM or CSE_6SSM, then the lightest neutralino must be a relatively pure Higgsino state with a highly restricted level of bino mixing, and this is what we find in most of the allowed parameter space. As a consequence the observed dark matter abundance can be reproduced only if the mass of lightest neutralino is relatively close to 1 TeV. In this scenario all sparticles are so heavy that it won't be possible to discover these states at the LHC. If the lightest neutralino is considerably lighter than 1 TeV then this state can account for only a small fraction of the measured dark matter density in the allowed part of parameter space within both the CSE_6SSM and CMSSM. At the same time we argue that the scenarios with relatively small masses of lightest neutralino and low relic dark matter abundance can still lead to the spectrum of SUSY particles that may be observed at the 13 TeV LHC. In the CSE_6SSM the set of states detectable at the LHC includes gluino, chargino and neutralino states as well as exotic fermions. In the most part of the allowed CSE_6SSM parameter space the lightest neutralino has sufficiently large direct detection cross section which should be observable soon at the XENON1T experiment.

The paper is organized as follows. In the next section we briefly review the E_6 inspired $U(1)_N$ extension of the MSSM with exact custodial $\tilde{Z}_2^H$ symmetry and define the CSE_6SSM.

In section 3 we consider the breakdown of gauge symmetry within this SUSY model. In section 4 the analytical expressions for the mass matrices and masses of all new states that appear in the SE$_6$SSM are specified. In section 5 we discuss the implications of the SUSY model under consideration for dark matter and summarize the results of our studies. section 6 is reserved for our conclusions. Appendix A contains the complete system of the two-loop RGEs that we use in our analysis.

2 The SE$_6$SSM

In orbifold GUT models the breakdown of E_6 gauge symmetry can lead to the SM gauge group along with two additional $\mathrm{U}(1)'$ factors [28], i.e.,

$$E_6 \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_\chi \times \mathrm{U}(1)_\psi, \tag{2.1}$$

where $\mathrm{U}(1)_\psi$ and $\mathrm{U}(1)_\chi$ are associated with the subgroups $E_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)_\psi \supset \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \times \mathrm{U}(1)_\psi$. Further symmetry breaking can then result in a low-energy model with a single additional $\mathrm{U}(1)'$ that is a linear combination of $\mathrm{U}(1)_\psi$ and $\mathrm{U}(1)_\chi$,

$$\mathrm{U}(1)' = \mathrm{U}(1)_\chi \cos\theta_{E_6} + \mathrm{U}(1)_\psi \sin\theta_{E_6}. \tag{2.2}$$

In this case, the value of the mixing angle θ_{E_6} characterizes the resulting $\mathrm{U}(1)'$ at low-energies, and several choices of symmetry breaking pattern have been considered (for reviews, see for example refs. [68–70]). In U(1) extensions with E_6 inspired charges, gauge anomalies automatically cancel provided that the low-energy matter content fills in complete representations of E_6. The SM particle content can be accommodated if each generation is embedded within a fundamental **27**-plet of E_6, which requires the introduction of extra matter to form complete multiplets. In addition to the SM fermions, each of these **27**-plets ($\mathbf{27}_i$, $i = 1, 2, 3$) contains a pair of $\mathrm{SU}(2)_L$ doublets, H_u, H_d, a pair of color triplets, D_i, $\overline{D}_i$, a right-handed neutrino, N_i^c, and a SM singlet, S_i. In general, both N_i^c and S_i carry non-zero $\mathrm{U}(1)'$ charges. The doublets H_i^u and H_i^d may be identified as Higgs or inert Higgs doublets, the distinction being that the latter do not develop VEVs. The states $\overline{D}_i$ and D_i have electric charge $\pm 1/3$ and carry $B - L$ charge twice that of ordinary quarks, and therefore may either be diquarks or leptoquarks.

The potential for interesting phenomenology associated with these exotic states, along with at least one Z' boson, has provided substantial motivation for studying E_6 inspired models [71–80]. Possible signatures of the exotic states at colliders have been studied [81], as well as limits on the Z' mass [82]. In addition to observing these exotic states, an underlying E_6 GUT might leave identifiable fingerprints on the ordinary MSSM mass spectrum, such as in the pattern of first and second generation sfermion masses [83]. Further motivation for studying this class of models has come from the fact that they are able to address several weaknesses of the MSSM. The extended gauge sector and the presence of additional singlets, some of which may get VEVs, allows for the solution of the MSSM μ-problem [84] in a way similar to in the next-to-MSSM (NMSSM) [85, 86]. These same features also lead to a theoretical upper bound on the lightest Higgs boson mass that is larger than can be achieved in the MSSM, and indeed in the NMSSM [38, 43, 87, 88]. The accompanying enlarged Higgs [16, 87, 88] and neutralino [33–35, 88–96] sectors have been extensively studied. It has been proposed that the extra D-terms could also solve the tachyon problems encountered in anomaly mediated SUSY breaking scenarios [97], while the inclusion of appropriate family symmetries could provide an explanation for the hierarchy of fermion masses and mixings [23, 24, 98, 99]. Many further implications of these models have also been considered, including for EWSB [34, 37, 38, 100–103], neutrino physics [32, 104], leptogenesis [18, 19] and EW baryogenesis [105, 106], the muon anomalous magnetic moment [107, 108], electric dipole moments [89, 90], lepton flavor violating processes [91] and the possibility of CP-violation in the extended Higgs sector [109].

As was noted above, in the rank-5 models described by eq. (2.2) both the singlets S_i and the right-handed neutrinos N_i^c are charged under the additional $\mathrm{U}(1)'$ in general. However, for the choice of $\theta_{E_6} = \arctan\sqrt{15}$, the right-handed neutrinos are uncharged under the resulting $\mathrm{U}(1)'$, denoted $\mathrm{U}(1)_N$. In this case, a large Majorana mass is allowed for the N_i^c and a see-saw mechanism can be used to explain the observed neutrino masses, while also allowing for an explanation of the baryon asymmetry via leptogenesis [18, 19].

In this article we study a $\mathrm{U}(1)_N$ extension of the MSSM in which tree-level flavor-changing transitions and the most dangerous baryon and lepton number violating operators are forbidden by a single discrete $\tilde{Z}_2^H$ symmetry [28, 30, 39]. In the SUSY models under consideration [28], E_6 is assumed to be broken directly to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_\chi \times \mathrm{U}(1)_\psi$ at or near the GUT scale, M_X, which can be achieved in 5 or 6 dimensional orbifold GUT models. The additional $\mathrm{U}(1)_\chi \times \mathrm{U}(1)_\psi$ is then broken near M_X to $\mathrm{U}(1)_N \times Z_2^M$, where the matter parity Z_2^M is defined by

$$Z_2^M = (-1)^{3(B-L)}. \tag{2.3}$$

Below M_X, the three complete **27**-plets of E_6 are taken to be accompanied by a set of pairs of multiplets M_l, $\overline{M}_l$, coming from incomplete $\mathbf{27}'$ and $\overline{\mathbf{27}'}$ representations, respectively. Note that anomalies still cancel, since the fields from M_l and $\overline{M}_l$ carry opposite U(1) charges. A single exact $\tilde{Z}_2^H$, commuting with E_6, may then be imposed under which all components of the **27**-plets are odd, thereby forbidding both interactions that generate large flavor changing neutral currents (FCNCs) and those that would lead to rapid proton decay. Doing so precludes any of the components of the **27**-plets from getting VEVs to break EW symmetry, so that, for example, all of the **27**-plet Higgs states H_i^u, H_i^d are inert and cannot be identified with the usual MSSM Higgs doublets. But, at the same time the multiplets M_l and $\overline{M}_l$ may be either even or odd under $\tilde{Z}_2^H$, allowing some of them to get VEVs for spontaneous symmetry breaking. In the model considered here we include two pairs of $\mathrm{SU}(2)_L$ doublets, H_u and $\overline{H}_u$, H_d and $\overline{H}_d$, as well as a pair of singlets S and $\overline{S}$. The fields H_u, H_d, S and $\overline{S}$ are postulated to be even under $\tilde{Z}_2^H$ symmetry and are responsible for the breaking of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_N \to \mathrm{U}(1)_{\mathrm{em}}$ at the TeV scale.[1] The doublets

[1]The initial breaking of $\mathrm{U}(1)_\psi \times \mathrm{U}(1)_\chi \to \mathrm{U}(1)_N \times Z_2^M$ can be achieved with the VEVs of a multiplet pair N_H^c and $\overline{N}_H^c$ with the quantum numbers of right-handed neutrinos. These VEVs may also be responsible for the generation of Majorana masses for the **27**-plet right-handed neutrinos; the full details of the construction can be found in ref. [28].

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$\overline{H}_u$ and $\overline{H}_d$ are odd under $\tilde{Z}_2^H$, so that they can mix with a combination of the **27**-plet states, defined to be the third generation H_3^u, H_3^d. In this case they may form vectorlike states with masses of order M_X, and so may be integrated out of the low-energy spectrum.

With only this set of multiplets, the imposed $\tilde{Z}_2^H$ would forbid any renormalizable operators allowing the exotic quarks to decay. Such long-lived exotics would be produced in the early Universe and would lead to estimated concentrations [110, 111] in excess of the observed limits on heavy isotopes [112–114]. To avoid this, a pair of $\tilde{Z}_2^H$ even $SU(2)_L$ doublets L_4 and $\overline{L}_4$ with the quantum numbers of leptons are also included at the TeV scale that couple to the exotic D_i, $\overline{D}_i$ and allow the exotic quarks to decay. This choice also implies that D_i and $\overline{D}_i$ are leptoquarks in this scenario.

In addition to the above sets of multiplets, in the model considered here we also include a pure singlet superfield $\hat{\phi}$ in the spectrum below the GUT scale, which is uncharged under all of the gauge symmetries [30]. This superfield is likewise taken to be even under $\tilde{Z}_2^H$ so that the superpotential may contain a term proportional to $\hat{\phi}\hat{S}\hat{\overline{S}}$, to stabilize the scalar potential, and the scalar component of $\hat{\phi}$ is allowed to develop a non-zero VEV. The fields H_u, H_d, S, $\overline{S}$ and $\hat{\phi}$ are all expected to get masses at or below the TeV scale. Thus after integrating out superheavy states the low-energy matter content in this model, which we refer to as the SE$_6$SSM, consists of the superfields shown in table 1. At low-energies and neglecting suppressed non-renormalizable interactions, the superpotential can then be written

$$\begin{aligned} W = {} & \lambda \hat{S}\hat{H}_d \cdot \hat{H}_u - \sigma\hat{\phi}\hat{S}\hat{\overline{S}} + \frac{\kappa_\phi}{3}\hat{\phi}^3 + \frac{\mu_\phi}{2}\hat{\phi}^2 + \Lambda_F\hat{\phi} + \tilde{\lambda}_{\alpha\beta}\hat{S}\hat{H}_\alpha^d \cdot \hat{H}_\beta^u + \kappa_{ij}\hat{S}\hat{D}_i\hat{\overline{D}}_j \\ & + \tilde{f}_{i\alpha}\hat{S}_i\hat{H}_u \cdot \hat{H}_\alpha^d + f_{i\alpha}\hat{S}_i\hat{H}_\alpha^u \cdot \hat{H}_d - g_{ij}^D\hat{Q}_i \cdot \hat{L}_4\hat{\overline{D}}_j - h_{i\alpha}^E\hat{e}_i^c\hat{H}_\alpha^d \cdot \hat{L}_4 + \mu_L\hat{L}_4 \cdot \hat{\overline{L}}_4 \\ & + \tilde{\sigma}\hat{\phi}\hat{\overline{L}}_4 \cdot \hat{L}_4 + y_{ij}^U\hat{u}_i^c\hat{H}_u \cdot \hat{Q}_j + y_{ij}^D\hat{d}_i^c\hat{Q}_j \cdot \hat{H}_d + y_{ij}^E\hat{e}_i^c\hat{L}_j \cdot \hat{H}_d. \end{aligned} \tag{2.4}$$

We denote superfields with hats, and adopt the convention $\hat{A} \cdot \hat{B} \equiv \epsilon_{\alpha\beta}\hat{A}^\alpha\hat{B}^\beta = \hat{A}^2\hat{B}^1 - \hat{A}^1\hat{B}^2$ for the SU(2) dot product. The exact $\tilde{Z}_2^H$ symmetry forbids all terms of the form $\mathbf{27} \times \mathbf{27} \times \mathbf{27}$, so that the allowed trilinear interactions involving non-singlet fields are of the form $\mathbf{27}' \times \mathbf{27}' \times \mathbf{27}'$ or $\mathbf{27}' \times \mathbf{27} \times \mathbf{27}$. By making appropriate rotations of the superfields $(\hat{H}_\alpha^d, \hat{H}_\alpha^u)$ and $(\hat{D}_i, \hat{\overline{D}}_i)$, the trilinear couplings $\tilde{\lambda}_{\alpha\beta}$ and κ_{ij} are chosen to be flavor diagonal, while the other new couplings are $\tilde{f}_{i\alpha}$, $f_{i\alpha}$, g_{ij}^D and $h_{i\alpha}^E$ are not, in general. The superpotential also contains several bilinear terms, such as those of the form $\mathbf{27}' \times \overline{\mathbf{27}'}$. The corresponding couplings, for example μ_L, may be generated[2] through the Giudice-Masiero mechanism [116].

As well as being invariant under the single imposed $\tilde{Z}_2^H$ symmetry, the superpotential is also invariant under the residual Z_2^M symmetry resulting from the breakdown of $U(1)_\psi \times U(1)_\chi \to U(1)_N \times Z_2^M$. The presence of multiple Z_2 symmetries suggests that it is not unreasonable to expect multiple stable states that may play the role of DM. For our analysis, it is convenient to define a combination of these two Z_2 symmetries by $\tilde{Z}_2^H = Z_2^M \times Z_2^E$. The transformation properties of each field under this Z_2^E symmetry are also

[2]For example, in a SUGRA model this term can be induced after the breakdown of local SUSY if the Kähler potential contains an extra term of the form $\left[Z_L(\hat{L}_4\hat{\overline{L}}_4) + h.c.\right]$ [115].

	$\hat{Q}_i$	$\hat{u}^c_i$	$\hat{d}^c_i$	$\hat{L}_i$	$\hat{e}^c_i$	$\hat{D}_i$	$\hat{\overline{D}}_i$	$\hat{S}_i$	$\hat{H}^u_\alpha$	$\hat{H}^d_\alpha$	$\hat{H}_u$	$\hat{H}_d$	$\hat{S}$	$\hat{\overline{S}}$	$\hat{L}_4$	$\hat{\overline{L}}_4$
$\mathrm{SU}(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathrm{SU}(2)_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\overline{\mathbf{2}}$
$\sqrt{\frac{5}{3}}Q^Y_i$	$\frac{1}{6}$	$-\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	1	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
$\sqrt{40}Q^N_i$	1	1	2	2	1	-2	-3	5	-2	-3	-2	-3	5	-5	2	-2
$\tilde{Z}^H_2$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$
Z^M_2	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$-$
Z^E_2	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$-$	$-$

Table 1. Summary of the chiral superfields present at low-energies, showing their representations and charges under the gauge symmetries and their transformation properties under the discrete symmetries defined in the text. Here and throughout this paper, the generation index $i = 1, 2, 3$, while $\alpha = 1, 2$. Note that the pure singlet field $\hat{\phi}$ is omitted from the table, as it transforms trivially under all of the symmetries.

shown in table 1, and henceforth we shall refer to states that are odd under Z^E_2 as exotics. Since the Lagrangian is separately invariant under $\tilde{Z}^H_2$ and Z^M_2, it is also the case that transformations under Z^E_2 leave the Lagrangian invariant. In particular, this means that the lightest Z^E_2-odd, exotic state is absolutely stable and so can potentially be a DM candidate. The automatically conserved matter parity Z^M_2, meanwhile, is equivalent to R-parity and also implies the existence of a stable state, as in the MSSM. Examination of the possible cases shows that these two states are in fact distinct, so that the model contains two DM candidates. In the case that the stable, lightest Z^E_2 odd state is R-parity even,[3] then the lightest R-parity odd state must be stable, as usual. Conversely, if the lightest Z^E_2 odd state is also the lightest R-parity odd state, then either the lightest R-parity even, Z^E_2 odd state or the lightest R-parity odd, Z^E_2 even state (depending on which is lighter) is absolutely stable.

By applying the method described in ref. [63–65], it has previously been found that the lightest inert neutralinos can have masses no larger than $60 - 65\,\mathrm{GeV}$ [49–51]. These states then tend to be the lightest exotic states in the spectrum, and are predominantly combinations of the fermionic components of the inert singlet superfields $\hat{S}_i$. Substantial masses for these inert singlinos, of more than $\sim 1\,\mathrm{eV}$, are ruled out by measurements of the SM-like Higgs branching ratios and the DM relic density. The simplest viable solution is instead for the inert singlino masses to be much lighter than $1\,\mathrm{eV}$, which can be achieved provided that the couplings $\tilde{f}_{i\alpha}, f_{i\alpha} \lesssim 10^{-6}$. This results in the inert singlinos forming hot dark matter, giving a negligible contribution to the observed relic density.[4]

[3]Or, more precisely, if it is not the lightest R-parity odd state.

[4]The presence of very light neutral fermions in the particle spectrum may also lead to some interesting implications for the neutrino physics (see, for example, ref. [117]).

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In this case, the second DM candidate should account fully or partially for the DM density, with the latter possibility requiring either additional DM candidates or a non-standard thermal history of the Universe to be consistent with measurements. The sub-eV inert singlinos are both the lightest exotic and lightest R-parity odd states in the spectrum. This implies that the lightest R-parity even exotic state or the lightest R-parity odd, Z_2^E even state is a possible second DM candidate. As can be read from table 1, the possible exotic candidates are the exotic squarks arising from the superfields $(\hat{D}_i, \hat{\overline{D}}_i)$, the inert Higgs scalars coming from the mixing of $(\hat{S}_i, \hat{H}^u_\alpha, \hat{H}^d_\alpha)$, or the fermionic components of $(\hat{L}_4, \hat{\overline{L}}_4)$. The masses of these states are required to be sufficiently heavy to have evaded detection to date. In particular, for large values of the SUSY breaking scale M_S the scalars receive large soft SUSY breaking masses and can be of similar mass to the ordinary squarks. The fermionic components of $(\hat{L}_4, \hat{\overline{L}}_4)$, meanwhile, receive a supersymmetric mass contribution from the superpotential bilinear term $\mu_L \hat{L}_4 \cdot \hat{\overline{L}}_4$, which is not constrained by the requirement of successful EWSB and need not be small.[5] In the model studied here this means that the lightest R-parity odd, Z_2^E even state tends to be the stable state, corresponding to the lightest neutralino with $Z_2^E = +1$. Depending on the composition of this state, it may then account for some or all of the DM relic density, as in the MSSM. In the following we shall focus on cases where the lightest neutralino is a mixed bino-Higgsino state, or pure Higgsino; we will find that this leads to a DM candidate that is also MSSM-like in its interactions and predictions for the DM relic density.

As usual in low-energy SUSY models, the relevant masses and mixings of interest in the neutralino sector are governed both by the superpotential interactions in eq. (2.4) as well as a subset of the soft SUSY breaking interactions. Including the standard set of soft scalar masses, soft trilinears, and soft gaugino masses, the full set of soft SUSY breaking terms that we consider is

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = {}& m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_{\overline{S}}^2 |\overline{S}|^2 + m_{\Sigma_{ij}}^2 S_i^\dagger S_j + m_\phi^2 |\phi|^2 \\
&+ m_{H_{2,\alpha\beta}}^2 (H^u_\alpha)^\dagger H^u_\beta + m_{H_{1,\alpha\beta}}^2 (H^d_\alpha)^\dagger H^d_\beta + m_{D_{ij}}^2 D_i^\dagger D_j + m_{\overline{D}_{ij}}^2 \overline{D}_i^\dagger \overline{D}_j + m_{L_4}^2 |L_4|^2 \\
&+ m_{\overline{L}_4}^2 |\overline{L}_4|^2 + m_{Q_{ij}}^2 \tilde{Q}_i^\dagger \tilde{Q}_j + m_{u^c_{ij}}^2 (\tilde{u}^c_i)^\dagger \tilde{u}^c_j + m_{d^c_{ij}}^2 (\tilde{d}^c_i)^\dagger \tilde{d}^c_j + m_{L_{ij}}^2 \tilde{L}_i^\dagger \tilde{L}_j \\
&+ m_{e^c_{ij}}^2 (\tilde{e}^c_i)^\dagger \tilde{e}^c_j + \left(\mu_L B_L L_4 \cdot \overline{L}_4 + \frac{\mu_\phi B_\phi}{2} \phi^2 + \Lambda_S \phi + h.c. \right) \\
&+ \Big(T_\lambda S H_d \cdot H_u - T_\sigma \phi S \overline{S} + T^\kappa_{ij} S D_i \overline{D}_j + T^U_{ij} \tilde{u}^c_i H_u \cdot \tilde{Q}_j + T^D_{ij} \tilde{d}^c_i \tilde{Q}_j \cdot H_d \\
&+ T^E_{ij} \tilde{e}^c_i \tilde{L} \cdot H_d + T^{\tilde{\lambda}}_{\alpha\beta} S H^d_\alpha \cdot H^u_\beta + T^{\tilde{f}}_{i\alpha} S_i H_u \cdot H^d_\alpha + T^f_{i\alpha} S_i H^u_\alpha \cdot H_d \\
&+ T_{\tilde{\sigma}} \phi \overline{L}_4 \cdot L_4 + \frac{T_{\kappa_\phi}}{3} \phi^3 - T^{g^D}_{ij} \tilde{Q}_i \cdot L_4 \overline{D}_j - T^{h^E}_{i\alpha} \tilde{e}^c_i H^d_\alpha \cdot L_4 + h.c. \Big) \\
&+ \frac{1}{2} \Big(M_1 \tilde{B} \tilde{B} + M_2 \tilde{W} \tilde{W} + M_3 \tilde{G} \tilde{G} + M_1' \tilde{B}' \tilde{B}' + 2 M_{11} \tilde{B} \tilde{B}' + h.c. \Big).
\end{aligned} \tag{2.5}$$

[5]The principal constraints on the value of μ_L come from requiring that gauge unification still occurs, which restricts $\mu_L \lesssim 100\,\text{TeV}$ [20], and that the states associated with $\hat{L}_4$ and $\hat{\overline{L}}_4$ are light enough so that the exotic leptoquarks D_i, $\overline{D}_i$ decay sufficiently quickly.

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The general soft SUSY breaking Lagrangian, in which all of the soft parameters are treated as independent, introduces a large number of additional free parameters on top of the extra couplings already present in the superpotential. The number of free parameters can be much reduced by considering a constrained model in which certain relations are assumed to hold between the soft parameters at some high scale.

The CSE$_6$SSM is defined by imposing boundary conditions at the GUT scale M_X where all gauge couplings coincide. In the SE$_6$SSM, since all of the low-energy matter content can be placed in complete SU(5) multiplets with the exception of the doublets $\hat{L}_4$ and $\hat{\overline{L}}_4$, gauge coupling unification still occurs at the two-loop level for any value of $\alpha_3(M_Z)$, the strong coupling evaluated at the scale M_Z, consistent with the measured value [20, 28]. Therefore, at the GUT scale M_X we take

$$g_1(M_X) \approx g_1'(M_X) \approx g_2(M_X) \approx g_3(M_X), \tag{2.6}$$

where g_1, g_1', g_2 and g_3 are the GUT-normalized $\mathrm{U}(1)_Y$, $\mathrm{U}(1)_N$, $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_C$ gauge couplings, respectively. This allows for the $\mathrm{U}(1)_N$ gauge coupling g_1' to be fixed. The presence of multiple U(1) symmetries implies the possibility of kinetic mixing between the U(1) field strengths [118, 119]. In practice, this mixing can be handled by working in a rotated basis for the U(1) gauge fields where the mixing leads instead to non-zero off-diagonal gauge couplings, i.e., in covariant derivatives one finds terms of the form $Q_{\hat{\Phi}}^T G A_\mu$ with

$$Q_{\hat{\Phi}} = \begin{pmatrix} Q_{\hat{\Phi}}^Y \\ Q_{\hat{\Phi}}^N \end{pmatrix}, \quad G = \begin{pmatrix} g_1 & g_{11} \\ 0 & g_1' \end{pmatrix}, \quad A_\mu = \begin{pmatrix} B_\mu \\ B_\mu' \end{pmatrix}. \tag{2.7}$$

This field redefinition is also responsible for the appearance of the mixed gaugino soft mass, M_{11}, in the last bracketed term of eq. (2.5). It is natural to expect that at M_X, the kinetic mixing should vanish so that $g_{11}(M_X) = 0$, $M_{11}(M_X) = 0$. However, even if this holds at M_X, in general non-zero mixing terms will be generated at low-energies by RG running [120, 121]. Previous analyses [20, 122] suggest that in this particular model, provided that the off-diagonal gauge coupling vanishes at M_X, it remains very small at all scales below M_X as well, $g_{11} \sim 0.02 \ll g_1, g_1'$. Therefore in our analysis we neglect the effects of gauge kinetic mixing, setting $g_{11}(M_X) = 0$, $M_{11}(M_X) = 0$ and taking them to vanish at scales below this. Nevertheless, it is important to note that in general the effects of this kinetic mixing can be non-negligible [122–124]; it is small here as the only non-vanishing contribution to the mixing comes from the $(\hat{L}_4, \hat{\overline{L}}_4)$ multiplet pair.

The remaining soft masses satisfy high-scale relations analogous to those applied in the CMSSM. The soft scalar masses squared are taken to be flavor diagonal with diagonal elements set to the common value m_0^2 at M_X, and similarly the gaugino masses (with the exception of M_{11}, as noted above) are assumed to unify to the value $M_{1/2}$ at this scale. The values of the soft breaking trilinears are related to a single common trilinear parameter A_0 by

$$\begin{aligned} T_\lambda(M_X) &= \lambda(M_X)A_0\,, & T_\sigma(M_X) &= \sigma(M_X)A_0\,, \\ T^\kappa_{ij}(M_X) &= \kappa_{ij}(M_X)A_0\,, & T^U_{ij}(M_X) &= y^U_{ij}(M_X)A_0\,, \\ T^D_{ij}(M_X) &= y^D_{ij}(M_X)A_0\,, & T^E_{ij}(M_X) &= y^E_{ij}(M_X)A_0\,, \\ T^{\tilde\lambda}_{\alpha\beta}(M_X) &= \tilde\lambda_{\alpha\beta}(M_X)A_0\,, & T^{\tilde f}_{i\alpha}(M_X) &= \tilde f_{i\alpha}(M_X)A_0\,, \\ T^f_{i\alpha}(M_X) &= f_{i\alpha}(M_X)A_0\,, & T_{\tilde\sigma}(M_X) &= \tilde\sigma(M_X)A_0\,, \\ T_{\kappa_\phi}(M_X) &= \kappa(M_X)A_0\,, & T^{g^D}_{ij}(M_X) &= g^D_{ij}(M_X)A_0\,, \\ T^{h^E}_{i\alpha}(M_X) &= h^E_{i\alpha}(M_X)A_0\,. \end{aligned} \tag{2.8}$$

Similarly, the soft breaking bilinears are assumed to unify, $B_L(M_X) = B_\phi(M_X) = B_0$. The parameter B_0 is taken to be independent of A_0; to do so we assume that these soft terms are also generated via a Giudice-Masiero term, as used to produce the superpotential bilinears. The soft breaking tadpole Λ_S is not required to be related to other soft parameters by the high-scale boundary condition.

With this choice of boundary conditions, the remaining unfixed parameters in the CSE$_6$SSM consist of the new superpotential couplings, namely $\lambda(M_X)$, $\sigma(M_X)$, $\kappa_\phi(M_X)$, $\mu_\phi(M_X)$, $\Lambda_F(M_X)$, $\tilde\lambda_{\alpha\beta}(M_X)$, $\kappa_{ij}(M_X)$, $\tilde f_{i\alpha}(M_X)$, $f_{i\alpha}(M_X)$, $g^D_{ij}(M_X)$, $h^E_{i\alpha}(M_X)$, $\mu_L(M_X)$ and $\tilde\sigma(M_X)$, and the soft breaking parameters m_0, $M_{1/2}$, A_0, B_0 and Λ_S. To simplify our analysis, in the following we assume that all of these parameters are real. Once these high-scale parameters, together with the MSSM gauge and Yukawa couplings are specified, the model at low-energies is studied by integrating the RGEs given in appendix A from M_X to the EWSB scale.

3 Gauge symmetry breaking

The Higgs fields H_u, H_d, S, $\overline{S}$ and ϕ develop non-zero VEVs breaking $SU(2)_L \times U(1)_Y \times U(1)_N \to U(1)_{\rm em}$. The relevant part of the scalar potential reads

$$\begin{aligned} V = {}& \lambda^2|S|^2\left(|H_d|^2+|H_u|^2\right) + \sigma^2|\phi|^2|S|^2 + |\lambda H_d\cdot H_u - \sigma\phi\overline{S}|^2 \\ &+ |\kappa_\phi\phi^2 + \mu_\phi\phi + \Lambda_F - \sigma S\overline{S}|^2 + \frac{\bar g^2}{8}\left(|H_u|^2-|H_d|^2\right)^2 + \frac{g_2^2}{2}|H_d^\dagger H_u|^2 \\ &+ \frac{g_1'^2}{2}\left(Q_{H_d}|H_d|^2 + Q_{H_u}|H_u|^2 + Q_S|S|^2 - Q_S|\overline{S}|^2\right)^2 \\ &+ m^2_{H_d}|H_d|^2 + m^2_{H_u}|H_u|^2 + m^2_S|S|^2 + m^2_{\overline{S}}|\overline{S}|^2 + m^2_\phi|\phi|^2 \\ &+ \left(\frac{T_{\kappa_\phi}}{3}\phi^3 + \frac{\mu_\phi}{2}B_\phi\phi^2 + \Lambda_S\phi + T_\lambda S H_d\cdot H_u - T_\sigma\phi S\overline{S} + h.c.\right) + \Delta V, \end{aligned} \tag{3.1}$$

where $\bar g^2 = g_2^2 + 3g_1^2/5$ and ΔV contains the loop corrections to the effective potential. We denote by Q_Φ the $U(1)_N$ charge of the field Φ. In the presence of kinetic mixing these charges should be replaced by effective $U(1)_N$ charges [16].

At the physical minimum of this potential, the VEVs of the Higgs fields are taken to be of the form

$$\begin{aligned} &\langle H_d\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad\qquad \langle H_u\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix} \\ &\langle S\rangle = \frac{s_1}{\sqrt{2}}, \qquad \langle \overline{S}\rangle = \frac{s_2}{\sqrt{2}}, \qquad \langle \phi\rangle = \frac{\varphi}{\sqrt{2}}. \end{aligned} \tag{3.2}$$

The corresponding conditions for these non-zero VEVs to be a stationary point of the potential are,[6]

$$\begin{aligned} \frac{\partial V}{\partial v_1} &= m_{H_d}^2 v_1 - \frac{T_\lambda}{\sqrt{2}} s_1 v_2 + \frac{\lambda^2}{2}\left(v_2^2+s_1^2\right) v_1 + \frac{\lambda\sigma}{2} v_2 s_2 \varphi + \frac{\bar{g}^2}{8}\left(v_1^2 - v_2^2\right) v_1 \\ &+ \frac{g_1'^2}{2}\left(Q_{H_d} v_1^2 + Q_{H_u} v_2^2 + Q_S s_1^2 - Q_S s_2^2\right) Q_{H_d} v_1 + \frac{\partial \Delta V}{\partial v_1} = 0, \end{aligned} \tag{3.3a}$$

$$\begin{aligned} \frac{\partial V}{\partial v_2} &= m_{H_u}^2 v_2 - \frac{T_\lambda}{\sqrt{2}} s_1 v_1 + \frac{\lambda^2}{2}\left(v_1^2+s_1^2\right) v_2 + \frac{\lambda\sigma}{2} v_1 s_2 \varphi - \frac{\bar{g}^2}{8}\left(v_1^2 - v_2^2\right) v_2 \\ &+ \frac{g_1'^2}{2}\left(Q_{H_d} v_1^2 + Q_{H_u} v_2^2 + Q_S s_1^2 - Q_S s_2^2\right) Q_{H_u} v_2 + \frac{\partial \Delta V}{\partial v_2} = 0, \end{aligned} \tag{3.3b}$$

$$\begin{aligned} \frac{\partial V}{\partial s_1} &= m_S^2 s_1 - \frac{T_\lambda}{\sqrt{2}} v_1 v_2 - \frac{T_\sigma}{\sqrt{2}} \varphi s_2 + \frac{\sigma^2}{2}\varphi^2 s_1 + \sigma s_2 \left(\frac{\sigma}{2} s_1 s_2 - \frac{\kappa_\phi}{2}\varphi^2 - \frac{\mu_\phi}{\sqrt{2}}\varphi - \Lambda_F\right) \\ &+ \frac{\lambda^2}{2}\left(v_1^2+v_2^2\right) s_1 + \frac{g_1'^2}{2}\left(Q_{H_d} v_1^2 + Q_{H_u} v_2^2 + Q_S s_1^2 - Q_S s_2^2\right) Q_S s_1 + \frac{\partial \Delta V}{\partial s_1} = 0, \end{aligned} \tag{3.3c}$$

$$\begin{aligned} \frac{\partial V}{\partial s_2} &= m_{\overline{S}}^2 s_2 - \frac{T_\sigma}{\sqrt{2}} \varphi s_1 + \frac{\sigma^2}{2}\varphi^2 s_2 + \frac{\lambda\sigma}{2}\varphi v_1 v_2 + \sigma s_1 \left(\frac{\sigma}{2} s_1 s_2 - \frac{\kappa_\phi}{2}\varphi^2 - \frac{\mu_\phi}{\sqrt{2}}\varphi - \Lambda_F\right) \\ &- \frac{g_1'^2}{2}\left(Q_{H_d} v_1^2 + Q_{H_u} v_2^2 + Q_S s_1^2 - Q_S s_2^2\right) Q_S s_2 + \frac{\partial \Delta V}{\partial s_2} = 0, \end{aligned} \tag{3.3d}$$

$$\begin{aligned} \frac{\partial V}{\partial \varphi} &= m_\phi^2 \varphi - \frac{T_\sigma}{\sqrt{2}} s_1 s_2 + \mu_\phi B_\phi \varphi + \sqrt{2}\Lambda_S + \frac{T_{\kappa_\phi}}{\sqrt{2}}\varphi^2 + \frac{\sigma^2}{2}\left(s_1^2+s_2^2\right)\varphi + \frac{\lambda\sigma}{2} s_2 v_1 v_2 \\ &- 2\left(\frac{\sigma}{2} s_1 s_2 - \frac{\kappa_\phi}{2}\varphi^2 - \frac{\mu_\phi}{\sqrt{2}}\varphi - \Lambda_F\right)\left(\kappa_\phi \varphi + \frac{\mu_\phi}{\sqrt{2}}\right) + \frac{\partial \Delta V}{\partial \varphi} = 0. \end{aligned} \tag{3.3e}$$

Of the 14 degrees of freedom associated with this set of Higgs fields, after EWSB four massless Goldstone modes are swallowed to generate masses for the physical $W^\pm$, Z and Z' bosons. The masses of the charged gauge bosons remain the same as in the MSSM. The neutral gauge boson masses are rather different, since the fields H_u^0 and H_d^0 are charged under both U(1) groups and therefore there is $Z - Z'$ mixing even when gauge kinetic mixing is neglected. It is convenient to define the combinations of the VEVs,

$$v^2 = v_1^2 + v_2^2, \quad \tan\beta = \frac{v_2}{v_1}, \quad s^2 = s_1^2 + s_2^2, \quad \tan\theta = \frac{s_2}{s_1}. \tag{3.4}$$

The tree-level masses M_{Z_1}, M_{Z_2} of the physical Z and Z' bosons are then found by diagonalizing the squared mass matrix

$$M_{ZZ'}^2 = \begin{pmatrix} M_Z^2 & \Delta^2 \\ \Delta^2 & M_{Z'}^2 \end{pmatrix}, \tag{3.5}$$

[6]Here we are using the shorthand $\partial V/\partial\langle\Phi\rangle \equiv \partial V/\partial\Phi|_{\Phi=\langle\Phi\rangle}$.

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where $M_Z^2 = \bar{g}^2 v^2/4$ and

$$M_{Z'}^2 = g_1'^2 v^2 \left(Q_{H_d}^2 \cos^2\beta + Q_{H_u}^2 \sin^2\beta\right) + g_1'^2 Q_S^2 s^2,$$
$$\Delta^2 = \frac{\bar{g} g_1'}{2} v^2 \left(Q_{H_d} \cos^2\beta - Q_{H_u} \sin^2\beta\right).$$

The mixing between the two gauge bosons is strongly constrained by EW precision measurements [125], while LHC searches currently place lower bounds on the mass of the extra Z' in $\mathrm{U}(1)_N$ models of $M_{Z_2} \gtrsim 3.4\,\mathrm{TeV}$ [126]. The physical Z' mass can be made acceptably large provided that the combination of the SM singlet VEVs is large, $s \gtrsim 9\,\mathrm{TeV}$. This leads to negligible mixing between the physical states Z_1 and Z_2, with a mixing angle $\lesssim 10^{-4}$, so that the light state Z_1 is approximately the SM Z boson with $M_{Z_1} \approx M_Z = \bar{g}v/2$ and $v \approx 246\,\mathrm{GeV}$, while the heavier gauge boson has its mass set by the singlet VEVs with $M_{Z_2} \approx M_{Z'} \approx g_1' Q_S s$.

The presence of the singlet fields involved in EWSB means that the set of EWSB conditions, eq. (3.3), is somewhat larger than in the MSSM. In the MSSM, there are two such conditions, which read

$$\frac{\partial V}{\partial v_1} = (|\mu|^2 + m_{H_d}^2) v_1 + \frac{\bar{g}^2}{8}(v_1^2 - v_2^2) v_1 - \mu B v_2 + \frac{\partial \Delta V}{\partial v_1} = 0, \tag{3.6a}$$
$$\frac{\partial V}{\partial v_2} = (|\mu|^2 + m_{H_u}^2) v_2 - \frac{\bar{g}^2}{8}(v_1^2 - v_2^2) v_2 - \mu B v_1 + \frac{\partial \Delta V}{\partial v_2} = 0. \tag{3.6b}$$

Imposing the EWSB conditions allows for a subset of the model parameters to be fixed. Conventionally in the CMSSM, the two parameters fixed by eq. (3.6) are chosen to be μ and B. However, this choice is not unique, nor is it always the most convenient. In particular, when studying scenarios for dark matter, it is ideal to be able to vary μ directly, as this controls the Higgsino masses and therefore permits the composition of the lightest neutralino to be directly chosen. In all of our results below, in both models we allow $\mu_{(\mathrm{eff})}$ to remain free and instead fix m_0 using the EWSB conditions. This can be done by expressing the soft masses in terms of the GUT scale parameters using semi-analytic solutions to the RGEs, as detailed in section 5 below. In the MSSM, the remaining EWSB condition can be used to fix B_0, while in the SE$_6$SSM there are still four conditions available.

In this paper we primarily examine the part of the parameter space where all SUSY particles are considerably lighter than $M_{Z'}$. This corresponds to s_1, s_2 and φ being much larger than the SUSY breaking scale M_S. These VEVs are fixed using two of the EWSB conditions to determine $\tan\theta$ and φ, with the value of s being a free input parameter. The remaining two conditions can be used to fix the GUT scale parameters $\Lambda_F(M_X)$ and $\Lambda_S(M_X)$. The appropriate stationary points of the scalar potential in eq. (3.1) arise if $\Lambda_F \gg M_S^2$ and $\Lambda_S \gg M_S^3$. In this case the structure of the potential is further simplified if the dimensionless couplings κ_ϕ and σ are small. Then in the leading approximation the quartic part of the scalar potential in eq. (3.1) is just given by

$$\frac{g_1'^2}{2} \tilde{Q}_S^2 \left(|S|^2 - |\overline{S}|^2\right)^2 \tag{3.7}$$

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so that in the limit $|\langle S\rangle|, |\langle\overline{S}\rangle| \to \infty$ the SM singlet VEVs tend to lie approximately along the D-flat direction $s_1 \approx s_2$. The inclusion of non-zero couplings σ and κ_ϕ stabilize the potential along this direction resulting in large SM singlet VEVs, i.e.,

$$|\varphi| \sim |s_1| \approx |s_2| \sim \sqrt{\frac{2\Lambda_F}{\sigma}}\,. \tag{3.8}$$

For the ratio of the SM singlet VEVs s_2/s_1 one can obtain a more accurate estimate using the minimization conditions eq. (3.3c) and eq. (3.3d),

$$\tan^2\theta \simeq \frac{m_S^2 + \frac{\sigma^2}{2}\varphi^2 + \frac{g_1'^2}{2}\tilde{Q}_S^2 s^2}{m_{\overline{S}}^2 + \frac{\sigma^2}{2}\varphi^2 + \frac{g_1'^2}{2}\tilde{Q}_S^2 s^2}. \tag{3.9}$$

If the VEVs of the SM singlets φ, s_1 and s_2 are rather large due to the large values of parameters Λ_F and Λ_S then $M_{Z'} \gg M_S$ and from eq. (3.9) it follows that $\tan\theta \simeq 1$. This is in marked difference to the situation in the simplest variants of the E_6SSM where the EWSB conditions imply that $M_{Z'} \sim M_S$, forcing the SUSY spectrum to be substantially heavier than is required, for example, in the MSSM by collider searches, due to the large lower bound on $M_{Z'}$. In our numerical studies we take advantage of this behavior to search for solutions with a heavy Z' with a mass well above current limits and a somewhat lighter SUSY scale than could be achieved in the simplest E_6 inspired extensions of the MSSM.

After fixing the parameters m_0, $\tan\theta$, φ, $\Lambda_F(M_X)$ and $\Lambda_S(M_X)$, the remaining parameters listed after eq. (2.8) are still free, up to the constraint of requiring a viable mass spectrum. In our analysis we mostly focus on the scenarios with small Yukawa couplings λ, σ, $\tilde{\lambda}_{\alpha\beta}$, κ_{ij}, $\tilde{f}_{i\alpha}$ and $f_{i\alpha}$ that can lead to a set of relatively light exotic fermions which might be discovered at the LHC. Consequently for the high-scale boundary condition $m_S^2(M_X) = m_{\overline{S}}^2(M_X) = m_0^2$, the running of m_S^2 and $m_{\overline{S}}^2$ is such that at the EWSB scale $m_S^2 \simeq m_{\overline{S}}^2$. Thus the value of $\tan\theta$ is always extremely close to unity.

4 Particle spectrum

The extension of the Higgs sector responsible for the breaking of $U(1)_N$ and EW symmetry also modifies the masses of the physical states in the spectrum compared to those found in the simplest variants of the E_6SSM. The masses of the MSSM sfermions are almost unchanged. The smallness of the first and second generation Yukawa couplings leads to negligible mixing between the left- and right-handed states, so that their masses may be summarized as [56]

$$\left(m_{\tilde{d}_{L\alpha}}^{\overline{\mathrm{DR}}}\right)^2 \approx m_{Q_{\alpha\alpha}}^2 + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta + \Delta_Q, \tag{4.1}$$

$$\left(m_{\tilde{d}_{R\alpha}}^{\overline{\mathrm{DR}}}\right)^2 \approx m_{d^c_{\alpha\alpha}}^2 - \frac{1}{3} M_Z^2 \sin^2\theta_W \cos 2\beta + \Delta_{d^c}, \tag{4.2}$$

$$\left(m_{\tilde{u}_{L\alpha}}^{\overline{\mathrm{DR}}}\right)^2 \approx m_{Q_{\alpha\alpha}}^2 + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta + \Delta_Q, \tag{4.3}$$

$$\left(m_{\tilde{u}_{R\alpha}}^{\overline{\mathrm{DR}}}\right)^2 \approx m_{u^c_{\alpha\alpha}}^2 + \frac{2}{3} M_Z^2 \sin^2\theta_W \cos 2\beta + \Delta_{u^c}, \tag{4.4}$$

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$$\left(m^{\overline{\rm DR}}_{\tilde{e}_{L\alpha}}\right)^2 \approx m^2_{L\alpha\alpha} + \left(-\frac{1}{2} + \sin^2\theta_W\right) M_Z^2 \cos 2\beta + \Delta_L, \tag{4.5}$$

$$\left(m^{\overline{\rm DR}}_{\tilde{e}_{R\alpha}}\right)^2 \approx m^2_{e^c_{\alpha\alpha}} - M_Z^2 \sin^2\theta_W \cos 2\beta + \Delta_{e^c}, \tag{4.6}$$

$$\left(m^{\overline{\rm DR}}_{\tilde{\nu}_i}\right)^2 \approx m^2_{Lii} + \frac{1}{2} M_Z^2 \cos 2\beta + \Delta_L, \tag{4.7}$$

The only differences appear in the form of the $U(1)_N$ D-term contributions Δ_Φ, which now have a contribution from the extra singlet $\overline{S}$ and read

$$\Delta_\Phi = \frac{g_1'^2}{2} Q_\Phi v^2 \left(Q_{H_d}\cos^2\beta + Q_{H_u}\sin^2\beta\right) + \frac{g_1'^2}{2} Q_\Phi Q_S s^2 \cos 2\theta. \tag{4.8}$$

Compared to the E_6SSM, this D-term contribution is significantly smaller at large s, due to the suppression by $\cos 2\theta$, while the sign of the contribution to the masses remains the same.

The same is true of the third generation squarks and sleptons. Due to the large third generation Yukawa couplings, the left-right mixing is in general non-negligible so that the third generation sfermion masses follow from diagonalizing 2×2 mass matrices (in the absence of flavor off-diagonal soft terms as considered here). The stop, sbottom and stau masses are found to be

$$\begin{aligned}\left(m^{\overline{\rm DR}}_{\tilde{t}_{1,2}}\right)^2 = \frac{1}{2}\Bigg\{ & m^2_{Q_{33}} + m^2_{u^c_{33}} + \frac{1}{2}M_Z^2\cos 2\beta + \Delta_Q + \Delta_{u^c} + 2\left(m_t^{\overline{\rm DR}}\right)^2 \\ & \mp \sqrt{\left[m^2_{Q_{33}} - m^2_{u^c_{33}} + \left(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta + \Delta_Q - \Delta_{u^c}\right]^2 + 4X_t^2}\Bigg\},\end{aligned} \tag{4.9}$$

$$\begin{aligned}\left(m^{\overline{\rm DR}}_{\tilde{b}_{1,2}}\right)^2 = \frac{1}{2}\Bigg\{ & m^2_{Q_{33}} + m^2_{d^c_{33}} - \frac{1}{2}M_Z^2\cos 2\beta + \Delta_Q + \Delta_{d^c} + 2\left(m_b^{\overline{\rm DR}}\right)^2 \\ & \mp \sqrt{\left[m^2_{Q_{33}} - m^2_{d^c_{33}} + \left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right)M_Z^2\cos 2\beta + \Delta_Q - \Delta_{d^c}\right]^2 + 4X_b^2}\Bigg\},\end{aligned} \tag{4.10}$$

$$\begin{aligned}\left(m^{\overline{\rm DR}}_{\tilde{\tau}_{1,2}}\right)^2 = \frac{1}{2}\Bigg\{ & m^2_{L_{33}} + m^2_{e^c_{33}} - \frac{1}{2}M_Z^2\cos 2\beta + \Delta_L + \Delta_{e^c} + 2\left(m_\tau^{\overline{\rm DR}}\right)^2 \\ & \mp \sqrt{\left[m^2_{L_{33}} - m^2_{e^c_{33}} + \left(-\frac{1}{2} + 2\sin^2\theta_W\right)M_Z^2\cos 2\beta + \Delta_L - \Delta_{e^c}\right]^2 + 4X_\tau^2}\Bigg\},\end{aligned} \tag{4.11}$$

where the top, bottom and tau running masses are $m_t^{\overline{\rm DR}} = y^U_{33} v \sin\beta/\sqrt{2}$, $m_b^{\overline{\rm DR}} = y^D_{33} v \cos\beta/\sqrt{2}$, $m_\tau^{\overline{\rm DR}} = y^E_{33} v\cos\beta/\sqrt{2}$ and the mixing parameters are given by $X_t = \frac{T^U_{33} v}{\sqrt{2}}\sin\beta - \frac{\lambda y^U_{33} v s}{2}\cos\beta\cos\theta$, $X_b = \frac{T^D_{33} v}{\sqrt{2}}\cos\beta - \frac{\lambda y^D_{33} v s}{2}\sin\beta\cos\theta$ and $X_\tau = \frac{T^E_{33} v}{\sqrt{2}}\cos\beta -$

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$\frac{\lambda y^E_{33} vs}{2} \sin\beta\cos\theta$. Mixing between the left- and right-handed states allows the third generation sfermions to be lighter than their first and second generation counterparts, as usual. To be phenomenologically viable the squarks and sleptons cannot be too light, so that we require the SUSY breaking scale $M_S \gtrsim 1\,\text{TeV}$ with $M_S \gg M_Z$.

These formulas, as well as those in the following sections, give the running $\overline{\text{DR}}$ masses with all parameters appearing in them evaluated at a renormalization scale Q; the above formulas also assume no significant flavor mixing. They are useful for gaining an analytical understanding of the spectrum, but it should be emphasized that in our numerical calculations we make use of the general tree-level mass matrices for all states. To calculate the physical spectrum, we also include the full one-loop self-energy corrections to all of the mass matrices; further details about our numerical procedure are given in section 5. Such corrections are particularly important for accurately estimating the physical gluino mass,

$$m_{\tilde{g}} = M_3(M_S) + \Delta^{\tilde{g}}(M_S), \tag{4.12}$$

for which the one-loop corrections $\Delta^{\tilde{g}}$ can be quite large, of up to 20%–30%. Pair production of gluinos would lead to a significant enhancement in $pp \to q\bar{q}q\bar{q} + E_T^{\text{miss}} + X$, with X denoting any number of light quark or gluon jets [56]. This signature can be used to discover the model when $m_{\tilde{g}}$ is within the LHC reach, or exclude regions of SE_6SSM parameter space where this is the case. As the SE_6SSM contains the same colored states as in the E_6SSM, the form of these radiative corrections $\Delta^{\tilde{g}}$ is unchanged between the two models.

4.1 The chargino and neutralino sector

On the other hand, the predictions for the masses of some other remaining states, that is, the neutralino sector, the exotic states and the Higgs sector, are rather different in the SE_6SSM compared to the E_6SSM. At the same time because the supermultiplet of the Z' boson and the additional singlet superfields in the Higgs sector are electrically neutral, the fermion components of these superfields do not mix with chargino states, $\tilde{\chi}^{\pm}_{1,2}$. Therefore the tree-level chargino mass matrix and its eigenvalues are almost identical to the ones in the MSSM, the only difference being that $\mu \to \mu_{\text{eff}}$, where

$$\mu_{\text{eff}} = \frac{\lambda s_1}{\sqrt{2}} = \frac{\lambda s}{\sqrt{2}} \cos\theta. \tag{4.13}$$

By contrast, the neutral fermion components of $\hat{H}_u$, $\hat{H}_d$, $\hat{S}$, $\hat{\overline{S}}$ and $\hat{\phi}$ as well as the neutral gauginos may all mix, leading to a $Z_2^E = +1$ neutralino sector that is twice as large as the MSSM neutralino sector. The neutralino mass eigenstates, $\tilde{\chi}^0_i$, $i = 1, \ldots, 8$, are linear combinations of the neutral Higgsino and singlino fields $\tilde{H}^0_u$, $\tilde{H}^0_d$, $\tilde{S}$, $\tilde{\overline{S}}$, $\tilde{\phi}$, the bino $\tilde{B}$, the neutral $\text{SU}(2)_L$ gaugino $\tilde{W}_3$, and the $\text{U}(1)_N$ gaugino $\tilde{B}'$, and are obtained by diagonalizing the mass matrix

$$\text{diag}\left(m^{\overline{\text{DR}}}_{\tilde{\chi}^0_1}, \ldots, m^{\overline{\text{DR}}}_{\tilde{\chi}^0_8}\right) = N^* M_{\tilde{\chi}^0} N^\dagger. \tag{4.14}$$

The 8×8 tree-level mass matrix in the basis $(\tilde{H}_d^0, \tilde{H}_u^0, \tilde{W}_3, \tilde{B}, \tilde{B}', \tilde{S}\cos\theta - \tilde{\overline{S}}\sin\theta, \tilde{S}\sin\theta + \tilde{\overline{S}}\cos\theta, \tilde{\phi})$ can be written in terms of 4×4 sub-matrices as

$$M_{\tilde{\chi}^0} = \begin{pmatrix} A & C^T \\ C & B \end{pmatrix}. \tag{4.15}$$

The upper left sub-matrix has the same structure as the neutralino mass matrix in the MSSM with $\mu \to \mu_{\text{eff}}$,

$$A = \begin{pmatrix} 0 & -\mu_{\text{eff}} & \frac{g_2 v}{2}\cos\beta & -\frac{g_1 v}{2}\sqrt{\frac{3}{5}}\cos\beta \\ -\mu_{\text{eff}} & 0 & -\frac{g_2 v}{2}\sin\beta & \frac{g_1 v}{2}\sqrt{\frac{3}{5}}\sin\beta \\ \frac{g_2 v}{2}\cos\beta & -\frac{g_2 v}{2}\sin\beta & M_2 & 0 \\ -\frac{g_1 v}{2}\sqrt{\frac{3}{5}}\cos\beta & \frac{g_1 v}{2}\sqrt{\frac{3}{5}}\sin\beta & 0 & M_1 \end{pmatrix}. \tag{4.16}$$

The remaining two sub-matrices then contain the mass terms for the additional SM singlet neutralinos and their mixings with the MSSM-like neutralino sector,

$$B = \begin{pmatrix} M_1' & g_1' Q_S s & 0 & 0 \\ g_1' Q_S s & \frac{\sigma\varphi}{\sqrt{2}}\sin 2\theta & -\frac{\sigma\varphi}{\sqrt{2}}\cos 2\theta & 0 \\ 0 & -\frac{\sigma\varphi}{\sqrt{2}}\cos 2\theta & -\frac{\sigma\varphi}{\sqrt{2}}\sin 2\theta & -\frac{\sigma s}{\sqrt{2}} \\ 0 & 0 & -\frac{\sigma s}{\sqrt{2}} & \mu_\phi + \sqrt{2}\kappa_\phi\varphi \end{pmatrix}, \tag{4.17}$$

$$C = \begin{pmatrix} Q_{H_d} g_1' v\cos\beta & Q_{H_u} g_1' v\sin\beta & 0 & 0 \\ -\frac{\lambda v}{\sqrt{2}}\sin\beta\cos\theta & -\frac{\lambda v}{\sqrt{2}}\cos\beta\cos\theta & 0 & 0 \\ -\frac{\lambda v}{\sqrt{2}}\sin\beta\sin\theta & -\frac{\lambda v}{\sqrt{2}}\cos\beta\sin\theta & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.18}$$

As noted above, we neglect the mixed gaugino soft mass M_{11} arising from U(1) mixing.

For general values of the parameters and VEVs, the neutralino mass matrix of the SE$_6$SSM is clearly more complicated than its counterpart in the MSSM. In the parameter space that we consider here, however, the mass matrix has a rather simple structure so that the MSSM-like neutralinos and the states beyond the MSSM tend not to mix. Inspection of eq. (4.17) shows that two of the neutralinos, those that are a mixture of $\tilde{B}'$ and $\tilde{S}\cos\theta - \tilde{\overline{S}}\sin\theta$, have their masses set by the large value of $M_{Z'}$. For large values of the singlet VEV, the value of μ_{eff} would be similarly large unless λ is taken to be sufficiently small. For large values of μ_{eff} the states that are superpositions of $\tilde{H}_u^0$ and $\tilde{H}_d^0$ become very heavy, leading to two very heavy pure Higgsino neutralinos that cannot account for the relic dark matter density. Therefore we restrict our attention to small values of λ so that $\mu_{\text{eff}} \lesssim 1$ TeV. When $\lambda \ll \sigma$ while σ is rather small and $M_{Z'} \gg M_S$ as implied by eq. (3.8), the aforementioned states with masses set by $M_{Z'}$ become very heavy and decouple from the rest of the spectrum. For very large s, eq. (3.9) implies that $\tan\theta \approx 1$ to high precision, and we find this is indeed the case in our numerical results below, allowing us to express the masses of these states approximately as

$$m^{\overline{\rm DR}}_{\tilde{\chi}^0_{7,8}} \approx M_{Z'} \left[\frac{\sqrt{2}M_1' + \sigma\varphi}{2\sqrt{2}M_{Z'}} \pm \sqrt{1 + \frac{\left(\sqrt{2}M_1' - \sigma\varphi\right)^2}{8M_{Z'}^2}} \right] \sim M_{Z'}. \tag{4.19}$$

When $M_S \gg M_Z$ and λ is small, the mixing of the remaining extra states, which are a mixture of $\tilde{S}\sin\theta + \tilde{\overline{S}}\cos\theta$ and $\tilde{\phi}$, and the MSSM-like neutralinos is also highly suppressed. The masses of these states are then approximately given by

$$m^{\overline{\rm DR}}_{\tilde{\chi}^0_{5,6}} \approx \frac{1}{2} \left[\mu_\phi + \frac{\varphi}{\sqrt{2}}(2\kappa_\phi - \sigma) \pm \sqrt{2\sigma^2 s^2 + \left(\mu_\phi + \frac{\varphi}{\sqrt{2}}(2\kappa_\phi + \sigma)\right)^2} \right] \sim M_S. \tag{4.20}$$

For large values of $M_S \gtrsim 1$ TeV, these states will be heavy and, due to the lack of significant mixing, can also be ignored in the first approximation as far as determining the mass of the DM candidate goes. Provided this is the case, the neutralino DM candidate is expected to be predominantly MSSM-like, that is, a mixture of $\tilde{H}_d$, $\tilde{H}_u$, $\tilde{W}_3$ and $\tilde{B}$, with mass given by the lightest eigenvalue of the 4×4 sub-matrix in eq. (4.16). In particular, since this matrix is identical to the MSSM neutralino mass matrix (with $\mu \to \mu_{\rm eff}$), when $M_S \gg M_Z$ the masses of the four lightest neutralinos are determined by $\mu_{\rm eff}$, M_1 and M_2 as they are in the MSSM. In the CSE$_6$SSM, the condition of universal gaugino masses at M_X further implies that

$$M_1 \approx 1.1M_1' \approx 0.5M_2 \approx 0.3M_3 \approx 0.2M_{1/2}, \tag{4.21}$$

so that the MSSM-like neutralino sector in our case depends only on the two parameters $\mu_{\rm eff}$ and $M_{1/2}$. These values can also be compared to the relations found in the CMSSM,

$$M_1 \approx 0.5M_2 \approx 0.15M_3 \approx 0.4M_{1/2}, \tag{4.22}$$

which are quite different due to the modified RG flow in the SE$_6$SSM.

4.2 The exotic sector

The states that are odd under Z_2^E do not mix with the ordinary MSSM states or the Higgs fields, forming a separate sector containing the second DM candidate as well as additional exotic states, some of which may generate spectacular collider signals. As discussed above, the DM candidate in this sector is expected to be an almost massless inert singlino, which is the lightest of the inert neutralinos. The inert neutralino sector is formed by the fermion components ($\tilde{S}_i$, $\tilde{H}^u_\alpha$ and $\tilde{H}^d_\alpha$) of the superfields $\hat{S}_i$, $\hat{H}^u_\alpha$ and $\hat{H}^d_\alpha$. The scalar components of the corresponding superfields also mix to form a set of inert charged and neutral Higgs scalars. The general inert neutralino and neutral inert Higgs mass matrices are 7×7 matrices. In the basis $((\tilde{H}^{d,0}_1 + \tilde{H}^{u,0}_1)/\sqrt{2}, (\tilde{H}^{u,0}_1 - \tilde{H}^{d,0}_1)/\sqrt{2}, (\tilde{H}^{d,0}_2 + \tilde{H}^{u,0}_2)/\sqrt{2}, (\tilde{H}^{u,0}_2 - \tilde{H}^{d,0}_2)/\sqrt{2}, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3)$, the inert neutralino mass matrix is of the form

$$M_{\tilde{\chi}^0_I} = \begin{pmatrix} A_I & C_I^T \\ C_I & 0 \end{pmatrix}, \tag{4.23}$$

where

$$A_I = \mathrm{diag}\left(-\mu_{\tilde{H}^0_{I1}}, \mu_{\tilde{H}^0_{I1}}, -\mu_{\tilde{H}^0_{I2}}, \mu_{\tilde{H}^0_{I2}}\right) \tag{4.24}$$

contains the tree-level masses of the inert Higgsinos, $\mu_{\tilde{H}^0_{I\alpha}} = \tilde{\lambda}_{\alpha\alpha} s \cos\theta/\sqrt{2}$, in the absence of mixing with the inert singlinos, while the mixing is given by the 3×4 sub-matrix C_I with elements

$$\begin{aligned} (C_I)_{i1} &= \frac{v}{2}\left(f_{i1}\cos\beta + \tilde{f}_{i1}\sin\beta\right), \quad (C_I)_{i2} = \frac{v}{2}\left(f_{i1}\cos\beta - \tilde{f}_{i1}\sin\beta\right) \\ (C_I)_{i3} &= \frac{v}{2}\left(f_{i2}\cos\beta + \tilde{f}_{i2}\sin\beta\right), \quad (C_I)_{i4} = \frac{v}{2}\left(f_{i2}\cos\beta - \tilde{f}_{i2}\sin\beta\right). \end{aligned} \tag{4.25}$$

The couplings of the inert singlinos are required to satisfy $f_{i\alpha}, \tilde{f}_{i\alpha} \lesssim 10^{-6}$ to yield almost massless hot DM candidates. Then, provided that $\tilde{\lambda}_{\alpha\beta} \gtrsim 10^{-6}$, the mixing between the inert Higgsinos and the inert singlinos is entirely negligible, and the inert neutralinos correspond to two degenerate pairs of inert Higgsinos with tree-level masses given by eq. (4.24) and three almost massless inert singlinos. The inert charginos similarly have tree-level masses given by $\mu_{\tilde{H}^\pm_{I\alpha}} = |\mu_{\tilde{H}^0_{I\alpha}}|$.

When the couplings $f_{i\alpha}$, $\tilde{f}_{i\alpha}$ are negligibly small, the mass matrix associated with the scalar components of the superfields $\hat{S}_i$, $\hat{H}^u_\alpha$ and $\hat{H}^d_\alpha$ also simplifies in a similar fashion. In this case, the mixing between the neutral inert Higgs scalars (H^u_α and H^d_α) and the inert singlets S_i can be ignored and the corresponding mass matrix decomposes into a 3×3 singlet mass matrix and a 4×4 mass matrix for the inert Higgs scalars[7]. The family-diagonal structure of the couplings $\tilde{\lambda}_{\alpha\beta}$, as well as the fact that the off-diagonal soft scalar masses vanish at the GUT scale, ensures that the mixing between generations is very small. Thus the mass matrix for the inert singlets is approximately diagonal, with the tree-level masses for the inert singlet scalars given by

$$\left(m^{\overline{\mathrm{DR}}}_{S_{Ii}}\right)^2 = m^2_{\Sigma_{ii}} + \Delta_{S_i}. \tag{4.26}$$

For $\tan\theta \approx 1$, the inert singlet masses are therefore $\sim M_S$, and so are somewhat lighter than $M_{Z'}$. In the absence of generation mixing, the inert Higgs mass matrix can be written as two 2×2 matrices, yielding the tree-level masses

$$\begin{aligned} \left(m^{\overline{\mathrm{DR}}}_{H^0_{\alpha 1,2}}\right)^2 = \frac{1}{2}\Bigg[& m^2_{H_{1,\alpha\alpha}} + m^2_{H_{2,\alpha\alpha}} + \Delta_{H^d_\alpha} + \Delta_{H^u_\alpha} + 2\mu^2_{\tilde{H}^0_{I\alpha}} \\ & \mp \sqrt{\left(m^2_{H_{1,\alpha\alpha}} - m^2_{H_{2,\alpha\alpha}} + M_Z^2\cos 2\beta + \Delta_{H^d_\alpha} - \Delta_{H^u_\alpha}\right)^2 + 4X^2_{H_\alpha}}\,\Bigg], \end{aligned} \tag{4.27}$$

[7]Strictly speaking, for non-zero f and $\tilde{f}$ couplings, the inert neutral Higgs sector should actually be decomposed into CP-eigenstates. This leads to 7 CP-even scalars and 7 CP-odd scalars. When the couplings $f_{i\alpha}$ and $\tilde{f}_{i\alpha}$ are neglected, these states instead form 7 complex scalar mass eigenstates described by the mentioned 3×3 and 4×4 mass matrices.

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where $X_{H_\alpha} = \frac{T^{\tilde{\lambda}}_{\alpha\alpha} s}{\sqrt{2}} \cos\theta - \frac{\tilde{\lambda}_{\alpha\alpha}}{4}\left(\lambda v^2 \sin 2\beta + 2\sigma\varphi s \sin\theta\right)$. The masses of the inert charged Higgs states are likewise

$$\left(m^{\overline{\rm DR}}_{H^\pm_{\alpha 1,2}}\right)^2 = \frac{1}{2}\Bigg[m^2_{H_{1,\alpha\alpha}} + m^2_{H_{2,\alpha\alpha}} + \Delta_{H^d_\alpha} + \Delta_{H^u_\alpha} + 2\mu^2_{\tilde{H}^\pm_{I\alpha}} \mp \sqrt{\left(m^2_{H_{1,\alpha\alpha}} - m^2_{H_{2,\alpha\alpha}} - M_Z^2 \cos 2\theta_W \cos 2\beta + \Delta_{H^d_\alpha} - \Delta_{H^u_\alpha}\right)^2 + 4X^2_{H_\alpha}}\Bigg]. \tag{4.28}$$

The contribution to the mixing proportional to $\sigma\varphi s \sim M_S M_{Z'}$ can be of the order of the soft mass contributions to the masses. To prevent this potentially dangerous term from causing tachyonic states, the inert Higgs couplings $\tilde{\lambda}_{\alpha\beta}$ cannot be too large. In practice, in our numerical study we take these couplings to be not much larger than λ, e.g., $\tilde{\lambda}_{\alpha\beta} \sim 10^{-3}$, to satisfy this requirement. Doing so implies that the mixing is rather small so that the inert scalars tend to have masses of order M_S. At the same time, small values of the Yukawas $\tilde{\lambda}_{\alpha\beta}$ imply that the inert Higgsinos and charginos can be light, with masses not much heavier than the lightest $Z_2^E = +1$ neutralino, in which case they may be observable in LHC searches. The exact $\tilde{Z}_2^H$ symmetry forbids the Yukawa couplings of the inert Higgs and singlet superfields to ordinary quark and lepton superfields. In the E_6 models with only an approximate Z_2 symmetry responsible for suppressing FCNCs, such couplings in general are permitted along with those for the ordinary Higgs fields, leading to the inert Higgsinos and charginos decaying predominantly into third generation fermion-sfermion pairs [16]. The absence of these couplings in the SE$_6$SSM due to $\tilde{Z}_2^H$ symmetry means that the decay channels of the inert Higgsinos are rather different in this model. Pair production of the Z_2^E and R-parity odd inert Higgsinos and charginos can occur through off-shell W and Z bosons. They then decay into an inert singlino and an on-shell W or Z boson, or a Z_2^E even Higgs boson, through the mixing induced by the $f_{i\alpha}$ and $\tilde{f}_{i\alpha}$ superpotential couplings. When both of the produced states decay into gauge bosons it is expected that they should lead to enhancements in the rates of $pp \to ZZ + E_T^{\rm miss} + X$, $pp \to WZ + E_T^{\rm miss} + X$ and $pp \to WW + E_T^{\rm miss} + X$.

The choice of flavor diagonal couplings κ_{ij} also means that there is no substantial mixing between generations of the exotic leptoquarks, D_i and $\overline{D}_i$. The 6×6 mass matrix for the scalar leptoquarks reduces to three 2×2 matrices, giving the tree-level masses

$$\left(m^{\overline{\rm DR}}_{\tilde{D}_{i1,2}}\right)^2 = \frac{1}{2}\Bigg[m^2_{D_{ii}} + m^2_{\overline{D}_{ii}} + \Delta_D + \Delta_{\overline{D}} + 2\mu^2_{D_i} \mp \sqrt{\left(m^2_{D_{ii}} - m^2_{\overline{D}_{ii}} + \frac{2}{3}M_Z^2 \sin^2\theta_W \cos 2\beta + \Delta_D - \Delta_{\overline{D}}\right)^2 + 4X^2_{D_i}}\Bigg], \tag{4.29}$$

where $X_{D_i} = \frac{T^\kappa_{ii} s}{\sqrt{2}} \cos\theta - \frac{\kappa_{ii}}{4}\left(\lambda v^2 \sin 2\beta + 2\sigma\varphi s \sin\theta\right)$ and the corresponding spin-1/2 leptoquark masses are $\mu_{D_i} = \kappa_{ii} s \cos\theta/\sqrt{2}$. The same potentially dangerous contribution to the mixing that occurs in the inert Higgs mass matrices is also present here. To ensure that this does not lead to an instability of the physical vacuum, we require the couplings κ_{ij} to be small as well, $\kappa_{ij} \sim 10^{-3}$. As is the case for the inert Higgs states, this leads to the scalar leptoquark $\tilde{D}_i$ being heavier, with masses of the order of M_S, while the exotic fermions D_i can be light. These exotic fermion states are colored and, once past threshold, can be pair produced at the 13 TeV LHC. They subsequently decay with missing energy via a decay chain involving an initial decay into an ordinary squark (quark) and an exotic L_4 fermion (scalar) component, through the couplings g^D_{ij}. This is followed by a decay involving the couplings $h^E_{i\alpha}$ of the exotic L_4 state into a lepton and inert Higgs or singlet (inert neutralino). If a hierarchy exists in the sizes of the couplings g^D_{ij} and $h^E_{i\alpha}$ as is present in the SM Yukawas, then such a process leads to an enhancement in signals with third generation final states, namely in $pp \to t\bar{t}\tau^+\tau^- + E^{\rm miss}_t + X$ and $pp \to b\bar{b}\tau^+\tau^- + E^{\rm miss}_T + X$.

For the branching ratio of these leptoquark decays to be significant, and also for the lifetimes of the exotic leptoquarks to be sufficiently short, the states associated with $\hat{L}_4$ and $\hat{\overline{L}}_4$ should not be too heavy. The fermion and scalar components of $\hat{L}_4$ and $\hat{\overline{L}}_4$ form a set of exotic lepton and slepton states that do not mix with the other exotic fields. The fermion components lead to a pair of charged and neutral states $\tilde{L}^\pm_4$ and $\tilde{L}^0_{4,1}$, $\tilde{L}^0_{4,2}$ with degenerate tree-level masses given by

$$\mu_{\tilde{L}^\pm_4} = \mu_{\tilde{L}^0_4} = \mu_L - \frac{\tilde{\sigma}\varphi}{\sqrt{2}}. \tag{4.30}$$

The tree-level masses of the neutral exotic sleptons are given by

$$\left(m^{\overline{\rm DR}}_{L^0_{4_{1,2}}}\right)^2 = \frac{1}{2}\Bigg[m^2_{L_4} + m^2_{\overline{L}_4} + \Delta_{L_4} + \Delta_{\overline{L}_4} + 2\mu^2_{\tilde{L}^0_4} \mp \sqrt{\left(m^2_{L_4} - m^2_{\overline{L}_4} + M^2_Z\cos 2\beta + \Delta_{L_4} - \Delta_{\overline{L}_4}\right)^2 + 4X^2_{L_4}}\Bigg], \tag{4.31}$$

where the mixing parameter is

$$X_{L_4} = \mu_L B_L - \frac{T_{\tilde{\sigma}}\varphi}{\sqrt{2}} + \tilde{\sigma}\left(\frac{\sigma}{4}s^2\sin 2\theta - \frac{\kappa_\phi}{2}\varphi^2 - \frac{\mu_\phi}{\sqrt{2}}\varphi - \Lambda_F\right), \tag{4.32}$$

while those for the charged exotic sleptons read

$$\left(m^{\overline{\rm DR}}_{L^\pm_{4_{1,2}}}\right)^2 = \frac{1}{2}\Bigg[m^2_{L_4} + m^2_{\overline{L}_4} + \Delta_{L_4} + \Delta_{\overline{L}_4} + 2\mu^2_{\tilde{L}^\pm_4} \mp \sqrt{\left(m^2_{L_4} - m^2_{\overline{L}_4} - M^2_Z\cos 2\theta_W\cos 2\beta + \Delta_{L_4} - \Delta_{\overline{L}_4}\right)^2 + 4X^2_{L_4}}\Bigg]. \tag{4.33}$$

By tuning the above mixing parameter, the exotic sleptons could be made light enough so that the exotic D fermions decay rapidly enough. Alternatively, these states are allowed to be heavier than the spin-1/2 leptoquarks provided that the couplings g^D and h^E are taken to be sufficiently large. In the numerical results below, we find that taking values for these couplings of $\sim 10^{-2}$ lead to lifetimes of the exotic fermions short enough to be consistent with constraints from Big Bang nucleosynthesis. At the same time, the impact of the couplings g^D and h^E on the mass spectrum and DM predictions is negligible for these small values of the couplings. Consequently they may be safely varied in this range without having any substantial impact on the other sectors.

4.3 The Higgs sector

The Higgs sector of the SE$_6$SSM is substantially different from the simplest version of the E$_6$SSM, for which the spectrum of the Higgs bosons was explored in ref. [16]. In the simplest case the sector responsible for the breakdown of the $SU(2)_L \times U(1)_Y \times U(1)_N$ gauge symmetry includes just H_u, H_d and S resulting in three CP-even, one CP-odd and two charged states. One CP-even Higgs state, which is predominantly SM singlet field, is always almost degenerate with the Z' gauge boson. The qualitative pattern of the Higgs spectrum in the simplest variant of the E$_6$SSM depends on the coupling λ which is a coupling of the SM singlet superfield $\hat{S}$ to the Higgs doublets $\hat{H}_u$ and $\hat{H}_d$, i.e., $\lambda \hat{S} \hat{H}_u \hat{H}_d$, as in the SE$_6$SSM. If $\lambda < g_1'$ the singlet dominated CP-even state is very heavy and decouples which makes the rest of the Higgs spectrum indistinguishable from the one in the MSSM. When $\lambda \gtrsim g_1'$ the spectrum of the Higgs bosons has a very hierarchical structure, which is similar to the one that appears in the NMSSM with the approximate Peccei-Quinn (PQ) symmetry [127–131]. As a result the mass matrix of the CP-even Higgs sector can be diagonalized using the perturbation theory [131–134]. In this case the heaviest CP-even, CP-odd and charged states are almost degenerate and lie beyond the multi-TeV range whereas the mass of the second lightest CP-even Higgs state is set by the Z' boson mass.

As was mentioned before in the SE$_6$SSM the sector responsible for the breakdown of gauge symmetry involves five multiplets of scalar fields H_u, H_d, S, $\overline{S}$ and ϕ that give rise to ten physical degrees of freedom in the Higgs sector which form a set of charged and neutral Higgs bosons. The unbroken $U(1)_{\rm em}$ symmetry ensures that the charged components of H_u and H_d do not mix with the other Higgs and singlet fields. Two massive charged Higgs states are formed by the linear combination

$$H^+ = H_d^{-*} \sin\beta + H_u^+ \cos\beta, \tag{4.34}$$

with a mass given by

$$\left(m_{H^\pm}^{\overline{\rm DR}}\right)^2 = \frac{\sqrt{2}s}{\sin 2\beta}\left(T_\lambda \cos\theta - \frac{\lambda\sigma\varphi}{\sqrt{2}}\sin\theta\right) - \frac{\lambda^2}{2}v^2 + \frac{g_2^2}{4}v^2. \tag{4.35}$$

The linear combination orthogonal to eq. (4.34) constitutes the longitudinal degrees of freedom of the $W^\pm$ bosons.

In the absence of CP-violation in the Higgs sector, the real and imaginary parts of the neutral components of the Higgs and singlets fields do not mix, leading to three physical CP-odd Higgs bosons and five CP-even states. The Goldstone states that are absorbed by the Z and Z' bosons are mixtures of the imaginary parts of H_d^0, H_u^0, S and $\overline{S}$,

$$\begin{aligned} G &= \sqrt{2}\left(\operatorname{Im} H_d^0 \cos\beta - \operatorname{Im} H_u^0 \sin\beta\right), \\ G' &= \sqrt{2}\left(\operatorname{Im} S \cos\theta - \operatorname{Im} \overline{S} \sin\theta\right)\cos\gamma - \sqrt{2}\left(\operatorname{Im} H_u^0 \cos\beta + \operatorname{Im} H_d^0 \sin\beta\right)\sin\gamma, \end{aligned} \tag{4.36}$$

where

$$\tan\gamma = \frac{v}{2s}\sin 2\beta. \tag{4.37}$$

For phenomenologically viable scenarios with $s \gg v$, $\tan\gamma$ goes to zero. Expressed in terms of the field basis (P_1, P_2, P_3), where

$$\begin{aligned} P_1 &= \sqrt{2}\left(\mathrm{Im}\, H_u^0 \cos\beta + \mathrm{Im}\, H_d^0 \sin\beta\right)\cos\gamma + \sqrt{2}\left(\mathrm{Im}\, S\cos\theta - \mathrm{Im}\,\overline{S}\sin\theta\right)\sin\gamma, \\ P_2 &= \sqrt{2}\left(\mathrm{Im}\, S\sin\theta + \mathrm{Im}\,\overline{S}\cos\theta\right), \\ P_3 &= \sqrt{2}\,\mathrm{Im}\,\phi, \end{aligned} \tag{4.38}$$

the pseudoscalar mass matrix $\tilde{M}^2$ has elements

$$\begin{aligned} \tilde{M}_{11}^2 &= \frac{\sqrt{2}s}{\sin 2\beta\cos^2\gamma}\left(T_\lambda\cos\theta - \frac{\lambda\sigma\varphi}{\sqrt{2}}\sin\theta\right), \\ \tilde{M}_{12}^2 = \tilde{M}_{21}^2 &= \frac{v}{\sqrt{2}\cos\gamma}\left(T_\lambda\sin\theta + \frac{\lambda\sigma\varphi}{\sqrt{2}}\cos\theta\right), \\ \tilde{M}_{13}^2 = \tilde{M}_{31}^2 &= \frac{\lambda\sigma v s}{2\cos\gamma}\sin\theta, \\ \tilde{M}_{22}^2 &= \frac{2\sigma\varphi}{\sin 2\theta}\left(\frac{\kappa_\phi}{2}\varphi + \frac{\mu_\phi}{\sqrt{2}} + \frac{\Lambda_F}{\varphi}\right) + \frac{v^2\sin 2\beta}{\sqrt{2}s\sin 2\theta}\left(T_\lambda\sin^3\theta - \frac{\lambda\sigma\varphi}{\sqrt{2}}\cos^3\theta\right) \\ &\quad + \frac{\sqrt{2}T_\sigma\varphi}{\sin 2\theta}, \\ \tilde{M}_{23}^2 = \tilde{M}_{32}^2 &= \frac{T_\sigma s}{\sqrt{2}} - \sigma s\left(\kappa_\phi\varphi + \frac{\mu_\phi}{\sqrt{2}}\right) - \frac{\lambda\sigma}{4}v^2\sin 2\beta\cos\theta, \\ \tilde{M}_{33}^2 &= \frac{T_\sigma s^2}{2\sqrt{2}\varphi}\sin 2\theta - 2\mu_\phi B_\phi - \frac{3T_{\kappa_\phi}}{\sqrt{2}}\varphi - \frac{\sqrt{2}}{\varphi}\left(\mu_\phi\Lambda_F + \Lambda_S\right) + \sigma\kappa_\phi s^2\sin 2\theta - \frac{\kappa_\phi\mu_\phi}{\sqrt{2}}\varphi \\ &\quad - 4\kappa_\phi\Lambda_F + \frac{\sigma\mu_\phi s^2}{2\sqrt{2}\varphi}\sin 2\theta - \frac{\lambda\sigma s}{4\varphi}v^2\sin\theta\sin 2\beta. \end{aligned} \tag{4.39}$$

In the parameter space of interest here, the structure of the full 3×3 matrix is such that it can be approximately diagonalized analytically. Because $M_{Z'}, M_S \gg M_Z$ and we restrict our attention to small values of λ, the mixings between P_1 and P_2, P_3 are rather small and may be safely neglected. In this approximation, the mass of one CP-odd state is set by $\tilde{M}_{11}$. Thus it has almost the same mass as the charged Higgs states. The masses of two other CP-odd states are set by $\tilde{m}_+$ and $\tilde{m}_-$ which are given by

$$\left(\tilde{m}_\pm^{\overline{\mathrm{DR}}}\right)^2 \approx \frac{1}{2}\left\{\tilde{M}_{22}^2 + \tilde{M}_{33}^2 \pm \sqrt{(\tilde{M}_{22}^2 - \tilde{M}_{33}^2)^2 + 4\tilde{M}_{23}^4}\right\}. \tag{4.40}$$

As follows from eq. (4.40) in some cases $\tilde{m}_-$ can be rather small so that the lightest CP-odd state A_1 becomes the lightest particle in the spectrum. This happens, for example, in the limit $\kappa_\phi, \mu_\phi, \Lambda_F, \Lambda_S \to 0$, when $m_{A_1}^{\overline{\mathrm{DR}}}$ vanishes and the superpotential possesses a global $\mathrm{U}(1)_{PQ}$ PQ symmetry which is spontaneously broken by the VEVs s_1, s_2 and φ. For small but non-vanishing $\mathrm{U}(1)_{PQ}$ violating couplings, the state A_1 is a light pseudo-Goldstone boson of the approximate PQ symmetry and can be lighter than the SM-like Higgs. In this case, the decay $h_1 \to A_1 A_1$ is kinematically allowed and can in principle lead to non-negligible branching fractions for non-standard decays of the SM Higgs [30]. Even for larger values of the couplings κ_ϕ, μ_ϕ, Λ_F and Λ_S, m_{A_1} may be small provided that the remaining parameters in eq. (4.40) are tuned so that $\tilde{m}_- \to 0$. It is important to note that in either case, the vanishing of $m_{A_1}^{\overline{\rm DR}} \approx \tilde{m}_-$ does not also require that the lightest neutralino mass becomes small, as occurs for example in the PQ-symmetric NMSSM. Indeed, from eq. (4.19) and eq. (4.20) it is clear that the singlino dominated states should remain heavy, while $m_{\tilde{\chi}_1^0}$ is governed by the values of the gaugino masses and $\mu_{\rm eff}$. This means that by varying the other Lagrangian parameters for fixed $M_{1/2}$ and $\mu_{\rm eff}$, the value of m_{A_1} can be chosen independently of $m_{\tilde{\chi}_1^0}$. In particular, for a given $m_{\tilde{\chi}_1^0}$ this allows for the possibility of resonant annihilations $\tilde{\chi}_1^0\,\tilde{\chi}_1^0 \to A_1 \to$ *SM particles* with $m_{A_1} \approx 2m_{\tilde{\chi}_1^0}$, leading to regions of parameter space in which the well-known A-funnel mechanism is responsible for setting the DM relic density [135–137].

The real parts of H_d^0, H_u^0, S, $\overline{S}$ and ϕ form five physical CP-even Higgs states, h_i, related by the unitary transformation

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \end{pmatrix} = U_h \begin{pmatrix} \mathrm{Re}\, H_d^0 \\ \mathrm{Re}\, H_u^0 \\ \mathrm{Re}\, S \\ \mathrm{Re}\, \overline{S} \\ \mathrm{Re}\, \phi \end{pmatrix}, \tag{4.41}$$

where U_h diagonalizes the CP-even Higgs mass matrix, M^2. In the basis $(S_1, S_2, S_3, S_4, S_5)$, where

$$\begin{aligned} \sqrt{2}\,\mathrm{Re}\, S &= S_1 \cos\theta + S_2 \sin\theta + s_1, \\ \sqrt{2}\,\mathrm{Re}\, \overline{S} &= -S_1 \sin\theta + S_2 \cos\theta + s_2, \\ \sqrt{2}\,\mathrm{Re}\, \phi &= S_3 + \varphi, \\ \sqrt{2}\,\mathrm{Re}\, H_d^0 &= S_5 \cos\beta - S_4 \sin\beta + v_1, \\ \sqrt{2}\,\mathrm{Re}\, H_u^0 &= S_5 \sin\beta + S_4 \cos\beta + v_2, \end{aligned} \tag{4.42}$$

and using the EWSB conditions eq. (3.3) to eliminate the soft Higgs masses, this has elements

$$\begin{aligned} M_{11}^2 &= g_1'^2 Q_S^2 s^2 - \frac{\sigma^2 s^2}{2} \sin^2 2\theta + \sqrt{2} T_\sigma \varphi \sin 2\theta + \left(\kappa_\phi \sigma \varphi^2 + \sqrt{2}\sigma\mu_\phi\varphi + 2\sigma\Lambda_F\right) \sin 2\theta \\ &\quad + \frac{T_\lambda}{2\sqrt{2}s} v^2 \cos\theta \sin 2\beta - \frac{\lambda\sigma\varphi}{4s} v^2 \sin\theta \sin 2\beta, \\ M_{12}^2 &= M_{21}^2 = \frac{\sigma^2 s^2}{4} \sin 4\theta - \sqrt{2} T_\sigma \varphi \cos 2\theta - \left(\kappa_\phi \sigma \varphi^2 + \sqrt{2}\sigma\mu_\phi\varphi + 2\sigma\Lambda_F\right) \cos 2\theta \\ &\quad + \frac{T_\lambda}{2\sqrt{2}s} v^2 \sin\theta \sin 2\beta + \frac{\lambda\sigma\varphi}{4s} v^2 \cos\theta \sin 2\beta, \\ M_{13}^2 &= M_{31}^2 = \sigma^2 \varphi s \cos 2\theta - \frac{\lambda\sigma}{4} v^2 \sin\theta \sin 2\beta, \end{aligned}$$

$$
\begin{aligned}
M_{14}^2 = M_{41}^2 &= \frac{g_1'^2}{2} Q_S (Q_{H_u} - Q_{H_d}) sv \sin 2\beta - \frac{T_\lambda}{\sqrt{2}} v \cos\theta \cos 2\beta \\
&\quad - \frac{\lambda\sigma}{2} \varphi v \sin\theta \cos 2\beta, \\
M_{15}^2 = M_{51}^2 &= g_1'^2 Q_S (Q_{H_d} \cos^2\beta + Q_{H_u} \sin^2\beta) sv - \frac{T_\lambda}{\sqrt{2}} v \cos\theta \sin 2\beta + \lambda^2 vs \cos^2\theta \\
&\quad - \frac{\lambda\sigma}{2} \varphi v \sin\theta \sin 2\beta, \\
M_{22}^2 &= \frac{\sigma^2 s^2}{2} \sin^2 2\theta + \frac{\sqrt{2} T_\sigma \varphi}{\sin 2\theta} \cos^2 2\theta + \left(\kappa_\phi \sigma \varphi^2 + \sqrt{2}\sigma\mu_\phi\varphi + 2\sigma\Lambda_F\right) \frac{\cos^2 2\theta}{\sin 2\theta} \\
&\quad + \frac{T_\lambda v^2}{2\sqrt{2} s \cos\theta} \sin^2\theta \sin 2\beta - \frac{\lambda\sigma\varphi v^2}{4 s \sin\theta} \cos^2\theta \sin 2\beta, \\
M_{23}^2 = M_{32}^2 &= -\frac{T_\sigma}{\sqrt{2}} s + \sigma^2 \varphi s \sin 2\theta - \sigma s \left(\kappa_\phi \varphi + \frac{\mu_\phi}{\sqrt{2}}\right) \\
&\quad + \frac{\lambda\sigma}{4} v^2 \cos\theta \sin 2\beta, \\
M_{24}^2 = M_{42}^2 &= \left(-\frac{T_\lambda}{\sqrt{2}} v \sin\theta + \frac{\lambda\sigma}{2} \varphi v \cos\theta\right) \cos 2\beta, \\
M_{25}^2 = M_{52}^2 &= \frac{\lambda^2}{2} sv \sin 2\theta + \left(-\frac{T_\lambda}{\sqrt{2}} v \sin\theta + \frac{\lambda\sigma}{2} \varphi v \cos\theta\right) \sin 2\beta, \\
M_{33}^2 &= \frac{T_\sigma s^2}{2\sqrt{2}\varphi} \sin 2\theta - \sqrt{2} \frac{\Lambda_S}{\varphi} + \frac{T_{\kappa_\phi}}{\sqrt{2}} \varphi + \mu_\phi \left(\frac{\sigma s^2}{2\sqrt{2}\varphi} \sin 2\theta + 3 \frac{\kappa_\phi \varphi}{\sqrt{2}} - \frac{\sqrt{2}\Lambda_F}{\varphi}\right) \\
&\quad + 2\kappa_\phi^2 \varphi^2 - \frac{\lambda\sigma s}{4\varphi} v^2 \sin\theta \sin 2\beta, \\
M_{34}^2 = M_{43}^2 &= \frac{\lambda\sigma}{2} sv \sin\theta \cos 2\beta, \\
M_{35}^2 = M_{53}^2 &= \frac{\lambda\sigma}{2} sv \sin\theta \sin 2\beta, \\
M_{44}^2 &= \frac{\sqrt{2} s}{\sin 2\beta} \left(T_\lambda \cos\theta - \frac{\lambda\sigma\varphi}{\sqrt{2}} \sin\theta\right) + \left(\frac{\bar{g}^2}{4} - \frac{\lambda^2}{2}\right) v^2 \sin^2 2\beta \\
&\quad + \frac{g_1'^2}{4} (Q_{H_u} - Q_{H_d})^2 v^2 \sin^2 2\beta, \\
M_{45}^2 = M_{54}^2 &= \left(\frac{\lambda^2}{4} - \frac{\bar{g}^2}{8}\right) v^2 \sin 4\beta + \frac{g_1'^2}{2} v^2 (Q_{H_u} - Q_{H_d}) \\
&\quad \times (Q_{H_d} \cos^2\beta + Q_{H_u} \sin^2\beta) \sin 2\beta, \\
M_{55}^2 &= \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \frac{\bar{g}^2}{4} v^2 \cos^2 2\beta + g_1'^2 v^2 (Q_{H_d} \cos^2\beta + Q_{H_u} \sin^2\beta)^2.
\end{aligned}
\tag{4.43}
$$

With the exceptions of M_{45}^2, M_{54}^2 and M_{55}^2, the size of the mass matrix elements is determined by the singlet VEVs s and φ. For small values of λ such that $\lambda s \sim \sigma s \sim \sigma\varphi \sim M_S$, it is therefore expected that all but the lightest state have masses of the order of the SUSY scale or heavier. In particular, for $\lambda \sim \sigma \to 0$ the element $M_{11}^2 \sim M_{Z'} \gg M_S$, while all other matrix elements are substantially smaller. Thus the mass of the heaviest CP-even state is approximately degenerate with the Z' mass. After neglecting all terms which are proportional to λv in eqs. (4.43) it is easy to see that in the limit $M_S \gg M_Z$ the mass of another CP-even state is set by M_{44}, i.e., this state is almost degenerate with the charged Higgs states, while the masses of two other CP-even states are determined by m_+ and m_-,

$$\left(m_{\pm}^{\overline{\rm DR}}\right)^2 \approx \frac{1}{2}\left\{M_{22}^2 + M_{33}^2 \pm \sqrt{(M_{22}^2 - M_{33}^2)^2 + 4M_{23}^4}\right\} . \tag{4.44}$$

The mass of the lightest state, on the other hand, is bounded from above by the smallest element M_{55}^2, i.e., the tree-level lightest CP-even Higgs mass satisfies

$$\left(m_{h_1}^{\overline{\rm DR}}\right)^2 \leq \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \frac{\bar{g}^2}{4} v^2 \cos^2 2\beta + g_1'^2 v^2 (Q_{H_d} \cos^2\beta + Q_{H_u} \sin^2\beta)^2. \tag{4.45}$$

Consequently $h_1 \approx S_5$ is always light, and for $M_S \gg M_Z$ is SM-like in its interactions. While the upper bound eq. (4.45) is larger than in the MSSM, it is still the case that radiative corrections are important for reaching $m_{h_1} \approx 125\,\text{GeV}$. Moreover, the by-now very precise measurement of the Higgs mass, $m_{h_1}^{\text{exp.}} = 125.09 \pm 0.21 \pm 0.11\,\text{GeV}$ [138], strongly constrains the parameter space of SUSY models and necessitates a reliable calculation of the physical Higgs mass. In principle, the physical Higgs masses can be determined from the poles in the propagator after including the one-loop self-energies by solving

$$\det\left[p_i^2 \mathbf{1} - M^2(M_S) + \Sigma_h(p_i^2)\right] = 0 \tag{4.46}$$

with $m_{h_i}^2 = \text{Re}(p_i^2)$ and where $M^2(M_S)$ is the tree-level Higgs mass matrix, evaluated here at M_S, and $\Sigma_h(p^2)$ denotes the self-energies. The required one-loop self-energies are automatically included in the tools used in our numerical studies, described below. However, for large values of $M_S \gg M_Z$ this strategy leads to large logarithmic contributions to the Higgs masses due to heavy states, which should be resummed to get an accurate estimate for the Higgs mass. In our case, the discussion above indicates that the SUSY spectrum is split, containing many heavy scalars, notably the MSSM sfermions and the exotic scalars, as well as light neutralinos and exotic fermions. Such scenarios are well handled by an effective field theory (EFT) approach to calculate the lightest Higgs mass, in which the large logarithms are resummed. In the MSSM, the largest of these contributions is usually associated with the third generation sfermions, and in particular the stops. In the SE$_6$SSM, there are also contributions from the heavy exotic scalars that should be accounted for. Because the exotic Yukawa couplings $\tilde{\lambda}_{\alpha\beta}$ and κ_{ij} are very small in the models we consider, these logarithmic corrections to the Higgs mass are very small and can be neglected compared to the contributions from the stops and other MSSM sfermions.[8] In our results below, to obtain the light CP-even Higgs mass we therefore make use of the known EFT calculation in the MSSM, which includes the dominant contributions to the Higgs mass. While a complete EFT calculation including the exotic states would be more accurate,[9] we expect that in this case the accuracy of our calculation should not be significantly reduced due to the small size of the exotic contributions.

[8]We have confirmed numerically that the contributions in eq. (4.46) from the exotic states are negligible compared to those from the stops and sbottoms.

[9]Such a calculation has been presented very recently [139], but this was not available when the scans presented here were performed.

5 Results

5.1 Scan procedure

To study scenarios in the CSE$_6$SSM that are able to account for the observed relic DM density with a MSSM-like DM candidate, a dedicated CSE$_6$SSM spectrum generator was created using FlexibleSUSY-1.1.0 [140, 141] and SARAH-4.5.6 [142–145]. The generated code,[10] which internally also relies on some routines from SOFTSUSY [146, 147], provides a precise determination of the mass spectrum by making use of the full two-loop RGEs and one-loop self-energies for all of the masses. Leading two-loop contributions to the CP-odd and CP-even Higgs masses taken from the known NMSSM [148] and MSSM [149–153] expressions were initially also included,[11] since the additional contributions from new states are expected to be small by virtue of their small couplings.

However, as noted above, for the solutions presented below this fixed order Higgs mass calculation suffers from the effects of large logarithmic contributions that are not resummed and so a MSSM EFT calculation is employed to predict m_{h_1} instead. To do so, at the SUSY scale defined by $M_S = \sqrt{m_{\tilde{t}_1}^{\overline{\text{DR}}} m_{\tilde{t}_2}^{\overline{\text{DR}}}}$, with $m_{\tilde{t}_\alpha}^{\overline{\text{DR}}}$ given by eq. (4.9), we performed a simple tree-level matching to the MSSM. In this simple matching procedure, the $\overline{\text{DR}}$ MSSM soft scalar masses $m_{Q_{ii}}^2$, $m_{u_{ii}^c}$, $m_{d_{ii}^c}$, $m_{L_{ii}}$, $m_{e_{ii}^c}$, gaugino masses M_1, M_2, M_3 and soft trilinear $A_t \equiv T_{33}^U / y_{33}^U$ are set at M_S to their values obtained in the CSE$_6$SSM after running from M_X. The MSSM μ parameter is set to its effective value at M_S, eq. (4.13), while an effective MSSM pseudoscalar mass, $(m_A)_{\text{eff}}$, is obtained from the effective soft bilinear

$$(B\mu)_{\text{eff}} = \frac{T_\lambda s}{\sqrt{2}} \cos\theta - \frac{\lambda\sigma}{2} s\varphi \sin\theta. \tag{5.1}$$

The lightest CP-even Higgs mass was then calculated using SUSYHD-1.0.2 [156] to obtain a more accurate estimate for the SM-like Higgs mass. The remaining heavy CP-even Higgs masses were computed using the ordinary fixed order approach.

As mentioned above, for the purposes of studying the MSSM-like DM candidate it is most convenient to directly vary the parameters $M_{1/2}$ and μ_{eff}. For this reason, we implemented a solver algorithm in FlexibleSUSY that makes use of the semi-analytic solutions to the RGEs. A similar algorithm has previously been used in studies of the constrained E$_6$SSM, where it was described in ref. [56]. The main advantage of this algorithm over the standard two-scale fixed point iteration is that by expanding all of the soft parameters at low-energies using the semi-analytic solutions, the EWSB conditions can be used to fix a subset of the input high-scale parameters in terms of the remaining input parameters. In particular, the low-energy soft Higgs and singlet masses can be written in the form

$$m_\Phi^2(M_S) = a_\Phi(M_S) m_0^2 + b_\Phi(M_S) M_{1/2}^2 + c_\Phi(M_S) M_{1/2} A_0 + d_\Phi(M_S) A_0^2, \tag{5.2}$$

[10] All of the code used in the following analysis is made available at https://doi.org/10.5281/zenodo.215628.

[11] While full two-loop corrections to the Higgs masses, in the gaugeless limit, can be calculated for a general model in SARAH [154, 155], this capability was not available in FlexibleSUSY at the time our numerical study was done.

for $\Phi = H_d, H_u, S, \overline{S}, \phi$. Imposing the EWSB conditions eq. (3.3) then allows m_0 to be fixed, as desired, along with $\tan\theta$, φ, $\Lambda_F(M_X)$ and $\Lambda_S(M_X)$. The parameters λ and $M_{1/2}$ remain free parameters that can be varied to set the mass and composition of $\tilde{\chi}_1^0$.

To satisfy the limits on the Z' mass, we take advantage of the mechanism described below eq. (3.8) to set $M_{Z'}$ well above the current limits, and so we set $M_{Z'} \approx 240\,\text{TeV}$. This requires a very large value of $s = 650\,\text{TeV}$ at the SUSY scale. Acceptably small values of $\mu_{\text{eff}} \lesssim 1\,\text{TeV}$ for reproducing the DM relic density are then achieved for very small $|\lambda|$, though μ_{eff} is still large enough to evade limits from LEP. In this study we focus on scenarios in which the LSP is either a mixed bino-Higgsino or pure Higgsino dark matter candidate. To do so, we considered $|\lambda(M_X)| = 9.15181 \times 10^{-4}$ and $|\lambda(M_X)| = 2.4 \times 10^{-3}$, for both $\lambda < 0$ and $\lambda > 0$. Because $\tan\theta \approx 1$ for such large values of s, this corresponds to $|\mu_{\text{eff}}(M_X)| \approx 347\,\text{GeV}$ and $|\mu_{\text{eff}}(M_X)| \approx 898\,\text{GeV}$, giving values at the SUSY scale of $|\mu_{\text{eff}}(M_S)| \approx 417\,\text{GeV}$ and $|\mu_{\text{eff}}(M_S)| \approx 1046\,\text{GeV}$, respectively.[12]

To prevent tachyonic states in the exotic sector, the exotic couplings cannot be too large, and for our scans we chose fixed values satisfying $\tilde{\lambda}_{\alpha\beta}(M_X), \kappa_{ij}(M_X) \leq 3 \times 10^{-3}$. Additionally, to simplify our analysis we took these couplings to be family universal with $\tilde{\lambda}_{\alpha\beta}(M_X) = \tilde{\lambda}_0 \delta_{\alpha\beta}$ and $\kappa_{ij}(M_X) = \kappa_0 \delta_{ij}$. A SUSY scale somewhat below $M_{Z'}$ was obtained by choosing small $\sigma(M_X) = 2 \times 10^{-2}$. Light inert singlinos in the spectrum were ensured by choosing extremely small values for the couplings $\tilde{f}_{i\alpha}$ and $f_{i\alpha}$, while for simplicity we set the couplings $\tilde{\sigma}(M_X)$, $\mu_\phi(M_X)$, $g^D_{ij}(M_X)$ and $h^E_{i\alpha}(M_X)$ to zero. We stress that the impact of the latter two sets of couplings on the quantities we investigate is numerically negligible. We have checked that their values could also be increased to satisfy constraints on the exotic lifetimes without altering our results. We also chose $\kappa_\phi(M_X) = 10^{-2}$, and $\mu_L(M_X) = 10\,\text{TeV}$. While the above fixed couplings impact the mass spectrum, they do not play a significant role in the predictions for dark matter, for the scenarios considered here in which the dark matter candidate is the lightest MSSM-like neutralino, and hence we do not scan over them.

For each parameter point in the scan, the GUT scale M_X at which these values are set is defined to be the scale at which $g_1(M_X) = g_2(M_X)$. This condition is solved iteratively, as described in ref. [140]. We do not require that $g_3(M_X)$ is also unified, but this will be approximately fulfilled due to the inclusion of the $\hat{L}_4$ and $\hat{\overline{L}}_4$ states. This is similar to what occurs in the E_6SSM [20].

For $\lambda \ll \bar{g}$, the tree-level upper bound on the SM-like Higgs mass is maximized for large $\tan\beta$. We took $\tan\beta(M_Z) = 10$ to saturate this limit. As in the CMSSM, the transformation $M_{1/2} \to -M_{1/2}$, $A_0 \to -A_0$, $B_0 \to -B_0$ and $\mu_{\text{eff}} \to -\mu_{\text{eff}}$ leaves our results invariant. We use this symmetry to fix $M_{1/2} \geq 0$. Setting $B_0 = 0$, we scanned over $M_{1/2}$ and A_0 by uniformly sampling in the intervals [0 TeV, 20 TeV] and [−20 TeV, 20 TeV],

[12] The values of $|\mu_{\text{eff}}|$ given are the mean values over all of the obtained valid solutions. The exact values of $|\mu_{\text{eff}}(M_S)|$ and $|\mu_{\text{eff}}(M_X)|$ vary over the parameter space scanned, since $\tan\theta$ varies slightly over the scanned region, as it is an EWSB output parameter, and the RG evolution also changes slightly due to sparticle threshold corrections. For the smaller value of $|\lambda(M_X)|$, the solutions we present have 409 GeV $\leq |\mu_{\text{eff}}(M_S)| \leq 425\,\text{GeV}$, and 344 GeV $\leq |\mu_{\text{eff}}(M_X)| \leq 349\,\text{GeV}$. For the larger $|\lambda(M_X)|$ value we obtain solutions with 1032 GeV $\leq |\mu_{\text{eff}}(M_S)| \leq 1063\,\text{GeV}$, and 892 GeV $\leq |\mu_{\text{eff}}(M_X)| \leq 903\,\text{GeV}$.

	$\lambda(M_X) = \pm 9.15181 \times 10^{-4}$	$\lambda(M_X) = \pm 2.4 \times 10^{-3}$
$\sigma(M_X)$	2×10^{-2}	2×10^{-2}
$\kappa_\phi(M_X)$	10^{-2}	10^{-2}
$\tilde{\lambda}_{\alpha\beta}(M_X) = \tilde{\lambda}_0 \delta_{\alpha\beta}$	10^{-3}	3×10^{-3}
$\kappa_{ij}(M_X) = \kappa_0 \delta_{ij}$	10^{-3}	1.4×10^{-3}, 3×10^{-3}
$\tilde{f}_{11}(M_X)$, $\tilde{f}_{22}(M_X)$, $\tilde{f}_{31}(M_X)$	10^{-7}	10^{-7}
$f_{11}(M_X)$, $f_{22}(M_X)$, $f_{32}(M_X)$	10^{-7}	10^{-7}
$\mu_L(M_X)$ [TeV]	10	10
$s(M_S)$ [TeV]	650	650
$M_{1/2}$ [TeV]	$[0, 20]$	$[0, 20]$
A_0 [TeV]	$[-20, 20]$	$[-20, 20]$
$\tan\beta(M_Z)$	10	10

Table 2. Summary of the fixed parameter values and allowed ranges used in the CSE$_6$SSM for the two values of $|\lambda(M_X)|$ considered. The free parameters $\tilde{\sigma}(M_X)$, $\mu_\phi(M_X)$, B_0, $g^D_{ij}(M_X)$, $h^E_{i\alpha}(M_X)$ and the $\tilde{f}_{i\alpha}(M_X)$, $f_{i\alpha}(M_X)$ not shown are set to zero in both cases. The parameters m_0, $\tan\theta$, φ, Λ_F and Λ_S are fixed by the requirement of correct EWSB. In the CMSSM, the same ranges are taken for $M_{1/2}$ and A_0 for the comparison scans with $\mu(M_S) = \pm 417$ GeV and $\mu(M_S) = \pm 1046$ GeV, and we set $\tan\beta(M_Z) = 10$ as well. The EWSB conditions are used to fix m_0 and B_0 in the CMSSM.

respectively, to find solutions with the correct Higgs mass and an allowed DM relic density. The relic density and direct detection cross section were calculated numerically with micrOMEGAs-4.1.8 [157–163], using CalcHEP [164] model files automatically generated with SARAH. The values of the CSE$_6$SSM parameters used are summarized in table 2.

For this choice of parameters the lightest neutralino is expected to be MSSM-like in its composition and couplings. At the same time, the spectrum and the RG flow of couplings in the CSE$_6$SSM is very different to that in the CMSSM. While the two models may in this limit make very similar predictions concerning DM, the ranges of parameter space in which this occurs and their collider signatures can therefore be quite distinct. This makes it interesting to compare the CSE$_6$SSM and CMSSM directly. To do this comparison, we also generated a CMSSM spectrum generator using FlexibleSUSY and SARAH as described above, and modified it to make use of the semi-analytic solver algorithm. The MSSM EWSB conditions were used to fix the common soft scalar mass m_0 and soft breaking bilinear B_0 at the GUT scale, and $M_{1/2}$ and A_0 were scanned over the same ranges as in the CSE$_6$SSM. This was done for values of $\mu(M_S)$ fixed to the mean values obtained in the CSE$_6$SSM, that is, $|\mu(M_S)| = 417$ GeV and $|\mu(M_S)| = 1046$ GeV, respectively. The same fixed value of $\tan\beta(M_Z) = 10$ was used. In this way we are able to present a more direct comparison of the two models, in which analogous parameters are approximately matched between the two.[13] The CMSSM solutions that we obtained have a heavy SUSY scale as

[13]We emphasise that our approach in the CMSSM differs from the conventional approach in the literature, in which μ would be determined by the EWSB conditions and m_0 is an input parameter.

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well, so that we again used SUSYHD to compute the lightest Higgs mass. The predicted DM relic density and direct detection cross section were calculated in micrOMEGAs using model files generated by SARAH.[14]

In both models, valid points were selected by imposing the theoretical constraints that the point should have a valid spectrum with correct EWSB and no tachyonic states. We required that all couplings remain perturbative up to the GUT scale. Since we perform only a naïve matching to the MSSM in the EFT calculation, we allowed for an uncertainty of $\pm 3\,\text{GeV}$ in the result for m_{h_1}, which is somewhat larger than is reported by SUSYHD. For the CSE$_6$SSM we accepted points with calculated light Higgs masses satisfying $122\ \text{GeV} \leq m_{h_1} \leq 128\,\text{GeV}$, and for comparison we allowed the same range of Higgs masses in the CMSSM. A point predicting a relic density Ωh^2 greater than that determined by Planck observations [165],

$$(\Omega h^2)_{\text{exp.}} = 0.1188 \pm 0.0010, \tag{5.3}$$

is effectively ruled out if one assumes a standard cosmological history. Points with a predicted relic density that does not exceed this value are not ruled out in the same way, though in this case additional contributions to DM are required. In our scans we excluded all points that have a predicted relic density $(\Omega h^2)_{\text{th.}} > (\Omega h^2)_{\text{exp.}}$.

To make a clear comparison of the impact of collider bounds on the CSE$_6$SSM and CMSSM, model specific limits should be applied to each. However in the CSE$_6$SSM the RGEs drive the sfermions to masses which are substantially larger than the gaugino masses, creating a hierarchical spectrum that persists even with the decoupling of the Z' mass from the rest of the spectrum. This means that typically LHC collider limits come from the gaugino sector, especially the gluino which is produced through strong interactions. The gluino decays in an MSSM-like manner and as a result the gluino mass limit set in the CMSSM in the heavy sfermion limit should, to a reasonable approximation, apply to the gluino in the CSE$_6$SSM also.[15] So that the reader can see where current and future collider limits should constrain the models we will show explicit gluino mass contours in each model, along with contours for the physical first generation squark mass, $m_{\tilde{u}_6}$. Note that this is approximately degenerate with the remaining first and second generation squark masses, i.e., $m_{\tilde{q}_{1,2}} \approx m_{\tilde{u}_6}$.

5.2 Mixed bino-Higgsino dark matter

We first consider cases with a light Higgsino mass term of $|\mu_{(\text{eff})}(M_S)| \approx 417\,\text{GeV}$. The results obtained in the CSE$_6$SSM and the CMSSM for this value of $|\mu_{(\text{eff})}|$ are compared in figure 1 and figure 2.

[14] We checked that the results obtained this way were in very good agreement with those found from using the MSSM implementation already available in micrOMEGAs, provided some care was taken to define the quark mass parameters consistently in the calculation of the direct detection cross sections.

[15] A more thorough treatment involves reinterpreting existing searches, for which a variety of tools, such as CheckMATE [166], MadAnalysis [167], SModelS [168] or Fastlim [169] are available. However since the situation is fairly simple in this case, with very heavy sfermions, we consider this unnecessary here and beyond the scope of our analysis.

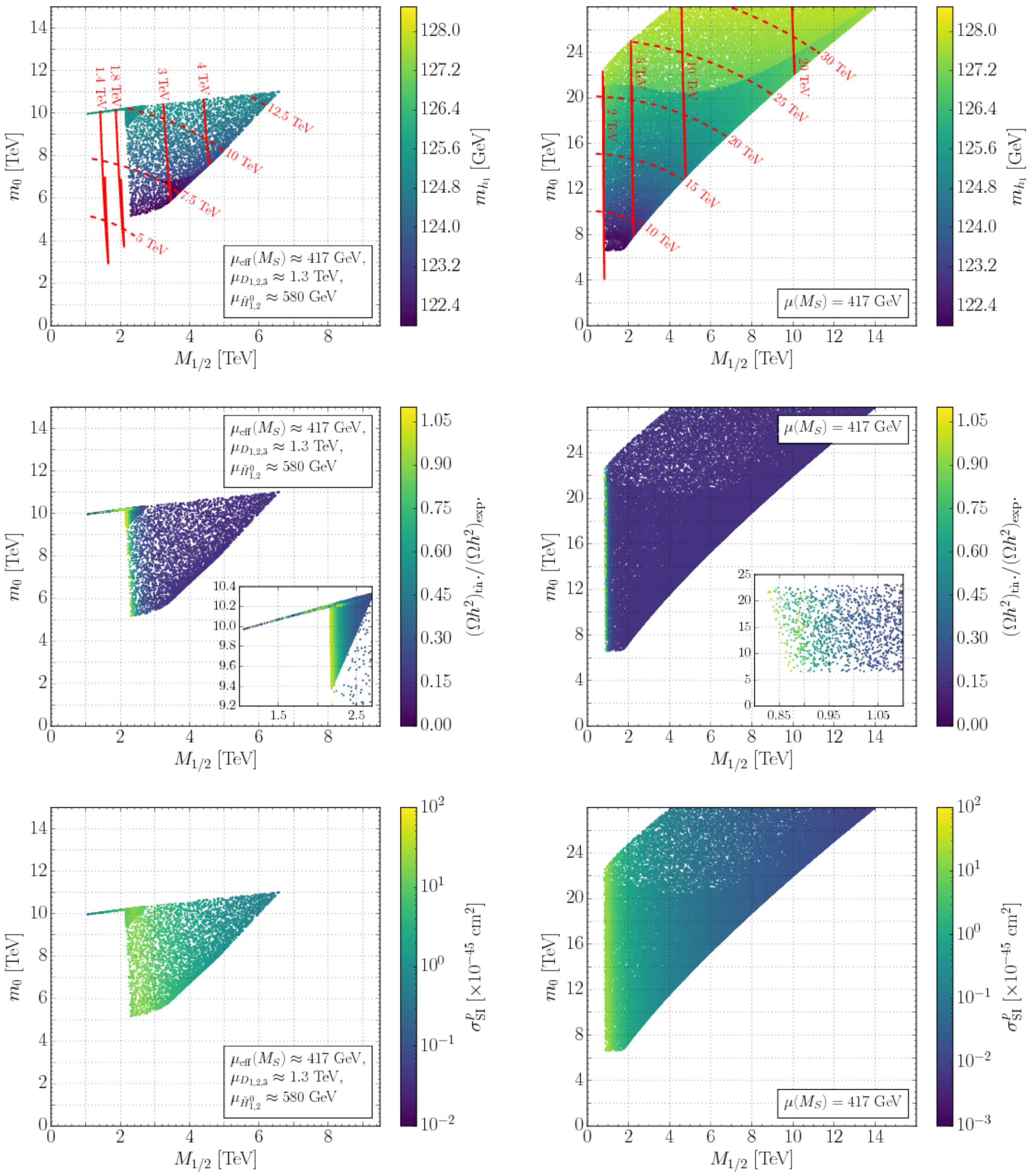

Figure 1. Contour plots in the $M_{1/2} - m_0$ plane of the lightest CP-even Higgs mass (top row), DM relic density (middle row) and proton SI cross section (bottom row) in the CSE$_6$SSM with $\mu_{\rm eff}(M_X) \approx 347\,\text{GeV}$ (left column) and CMSSM with $\mu(M_S) = 417\,\text{GeV}$ (right column). In the top row, we also show contours of the gluino (solid lines) and squark (dashed lines) masses. At large values of $M_{1/2}$, where the $\tilde{\chi}^0_1$ is a light Higgsino, the relic density saturates with $(\Omega h^2)_{\rm th.}/(\Omega h^2)_{\rm exp.} \approx 0.15$.

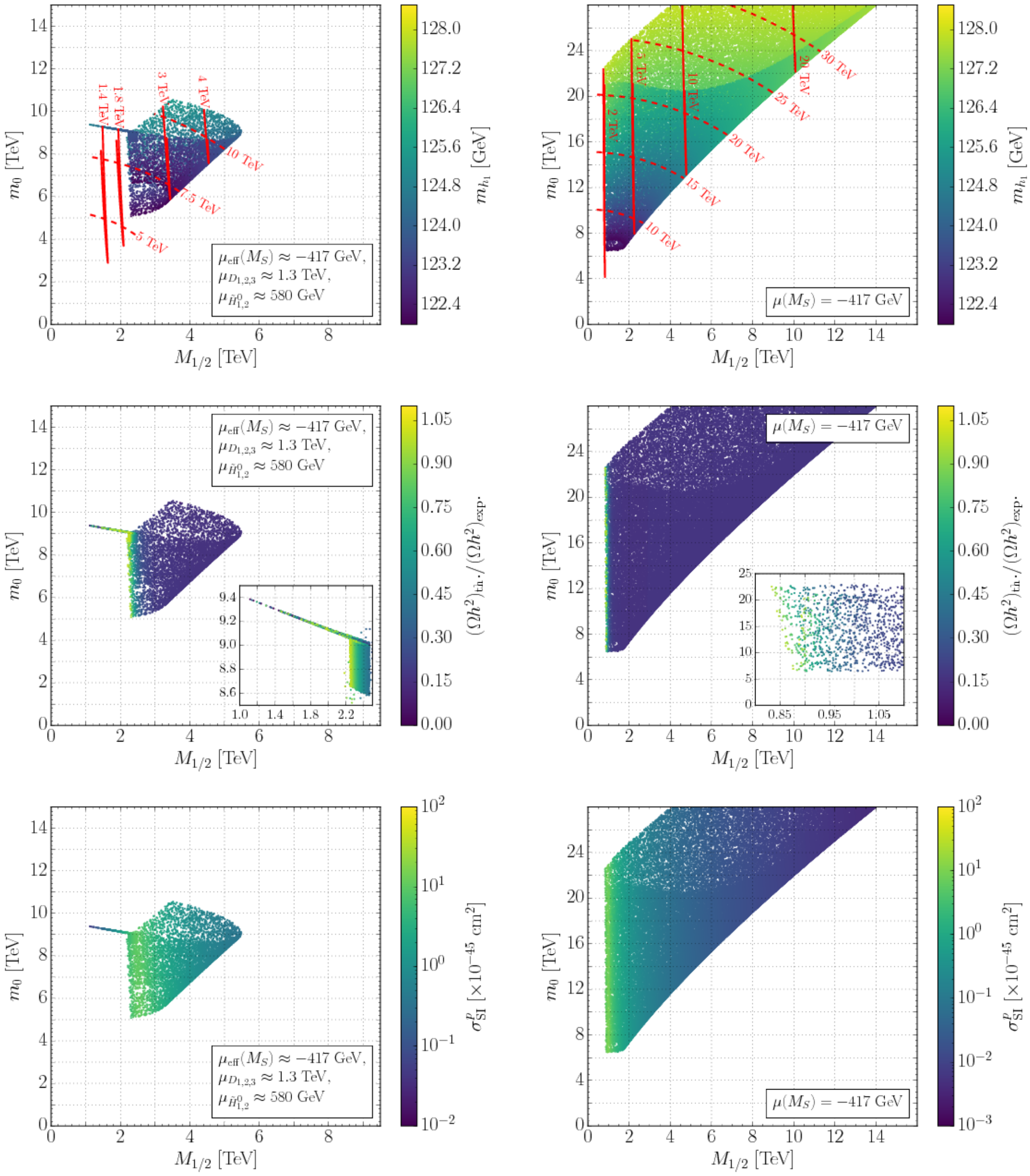

Figure 2. Contour plots in the $M_{1/2} - m_0$ plane of the lightest CP-even Higgs mass (top row), DM relic density (middle row) and proton SI cross section (bottom row) in the CSE$_6$SSM with $\mu_{\rm eff}(M_X) \approx -347\,\text{GeV}$ (left column) and CMSSM with $\mu(M_S) = -417\,\text{GeV}$ (right column). In the top row, we show contours of the gluino (solid lines) and squark (dashed lines) masses. As for the positive $\mu_{\rm eff}$ case, at large $M_{1/2}$ the relic density reaches a limiting value of $(\Omega h^2)_{\rm th.}/(\Omega h^2)_{\rm exp.} \approx 0.15$.

In the top row of figure 1 we compare the mass of the SM-like Higgs in the two models. In both we find solutions consistent with $m_{h_1} \approx 125\,\text{GeV}$, but the allowed regions in the $M_{1/2}-m_0$ plane clearly differ quite substantially. For such large values of s and small values of λ the tree-level mass of the lightest CP-even Higgs in the SE$_6$SSM is approximately the same as it is in the MSSM, $(m_{h_1}^{\overline{\text{DR}}})^2 \approx M_Z^2 \cos^2 2\beta$, as follows from approximately diagonalizing the mass matrix in eq. (4.43). Without substantial tree-level contributions from the additional F- and D-terms, a 125 GeV Higgs is achieved with large radiative corrections in the CSE$_6$SSM as well as in the CMSSM. In principle, large enough loop corrections result from either large sparticle masses, particularly stop masses, or large stop mixing. However, increasing A_0 or $M_{1/2}$ to generate large mixings for fixed $\mu_{(\text{eff})}$ leads to the value of m_0 increasing as needed to satisfy the EWSB conditions. As a result in the solutions we obtain $m_0 > A_0, M_{1/2}$ and large enough radiative corrections must arise from sufficiently heavy sparticle masses instead. The effect of the Higgs mass constraint can be clearly seen in the top row of figure 1 and figure 2, where the requirement $m_{h_1} \geq 122\,\text{GeV}$ imposes the lower bound on m_0 for small values of $M_{1/2}$.

The right-most boundary of the solution region is a consequence of determining m_0^2 from the EWSB conditions. When the soft masses and SUSY scale are large and $|\mu_{(\text{eff})}| \ll M_{1/2}$, as is the case here, the resulting function for $m_0^2(M_{1/2}, A_0)$ has a minimum at each $M_{1/2}$ with $m_{0,\text{min}}^2(M_{1/2}) > 0$.[16] Hence when $\mu_{(\text{eff})}$ is fixed, we do not find points with values of m_0 below this boundary for each given value of $M_{1/2}$. This can be contrasted with the usual procedure in the CMSSM, where lower values of m_0^2 can be found by varying $|\mu|$ and $B\mu$.

In the CMSSM, the Higgs mass constraint $m_{h_1} \leq 128\,\text{GeV}$ also puts an upper bound on the possible values of $M_{1/2}$. This is shown in figure 3, where we plot m_{h_1} in the $M_{1/2} - A_0$ plane in both models for $\mu_{(\text{eff})} > 0$. The upper bound on m_{h_1} cuts off the solution region at large values of $M_{1/2}$ in figure 3 in the CMSSM. In comparison, in the CSE$_6$SSM the region at large $M_{1/2}$ is ruled out by the presence of tachyonic states. The lower right region of the CSE$_6$SSM $M_{1/2} - A_0$ plane in figure 3 is excluded by tachyonic pseudoscalars A_i, while the uppermost boundary is due to tachyonic CP-even Higgs states. This corresponds to the much more restrictive upper bound on m_0 in the CSE$_6$SSM in figure 1 compared to the CMSSM. The same is true for $\mu_{(\text{eff})} < 0$ in figure 2, though the position of the boundary is modified, leading to the much smaller range of acceptable m_0 values in the CSE$_6$SSM for this value of $|\mu_{\text{eff}}|$. It should be noted, however, that these results are obtained for a single value of s. It is expected that if s and λ are allowed to vary while maintaining fixed μ_{eff}, additional solutions would be obtained, as is found in the constrained E$_6$SSM [56, 57]. It is important to emphasize that in the CSE$_6$SSM there is still additional parameter space available, and that the constraints shown here apply only for a single value of $M_{Z'}$ in the model.

The large values of m_0 required result in a large SUSY scale and all scalars except the SM-like Higgs h_1, and the lightest pseudoscalar A_1 in the CSE$_6$SSM, are very heavy. In

[16] For example, at tree-level and neglecting small D-term contributions the EWSB conditions lead to an expression of the form $m_0^2 = \xi_1 M_{1/2}^2 + \xi_2 M_{1/2} A_0 + \xi_3 A_0^2 + \xi_0 |\mu_{(\text{eff})}|^2$ where the coefficients $\xi_1, \xi_3 > 0$ and $\xi_0, \xi_2 < 0$ for $\tan\beta = 10$ are set by the RG flow. Because $\xi_1 - \xi_2^2/(4\xi_3) > 0$, for fixed $|\mu_{(\text{eff})}| \ll M_{1/2}$ it is easy to show that there is a non-trivial lower bound on the value of m_0^2.

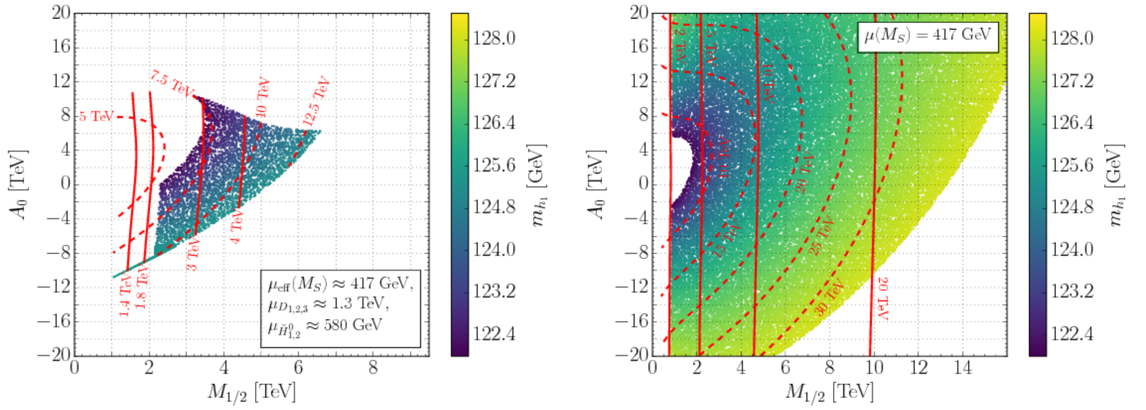

Figure 3. Contour plots of the lightest CP-even Higgs mass in the $M_{1/2}-A_0$ plane in the CSE$_6$SSM with $\mu_{\rm eff}(M_X) \approx 347$ GeV (left) and the CMSSM with $\mu(M_S) = 417$ GeV (right). Also shown are contours of the gluino (solid lines) and squark (dashed lines) masses for both models.

the top row of figure 1 and figure 2 we show contours of the gluino and first and second generation squark masses. The viable solutions that we find in the CSE$_6$SSM all have squark masses $m_{\tilde{q}_{1,2}} \geq 5.4$ TeV, while in our CMSSM solutions $m_{\tilde{q}_{1,2}} \geq 6.5$ TeV, so that these states are not observable at the LHC. On the other hand, the small exotic couplings lead to light exotic fermions. For $|\mu_{\rm eff}(M_X)| \approx 347$ GeV, the choice of $\kappa_0 = 10^{-3}$ leads to exotic D fermion masses of ≈ 1.3 TeV. Similarly, setting $\tilde{\lambda}_0 = 10^{-3}$ leads to inert Higgsinos with masses ≈ 580 GeV. Both sets of states are therefore light enough to be produced at the LHC and would be detectable via the signatures discussed in section 4. Given the increasingly large SUSY scale required by LHC searches in constrained models, this makes searches targeting the exotic spin-1/2 leptoquark and inert Higgsino states attractive for still being able to probe the CSE$_6$SSM parameter space. Because the exotic couplings cannot be too large in the scenarios considered here, improved limits on these states would strongly constrain the solutions we have found with very small values of $|\mu_{\rm eff}|$.

In addition to the restriction on the allowed values of m_0, there is also a lower bound on $M_{1/2}$ in both models, which is determined by the relic density constraint. The behaviour in the CMSSM in this case is well understood. When M_1 is sufficiently large, $\tilde{\chi}_1^0$ is a nearly pure, light Higgsino that is underabundant [170]. The opposite limit, with small $M_{1/2}$ and $M_1 \lesssim \mu$, leads to an almost pure bino LSP that is overabundant, due to its small annihilation cross section. Therefore requiring $\Omega h^2 \leq 0.1188$ amounts to placing a lower bound on $M_{1/2}$ for fixed μ.

Since $\mu_{(\rm eff)}$ is small in this case, an acceptable relic density is achieved with relatively low values of $M_{1/2}$. The minimal allowed value of $M_{1/2}$ in the CMSSM, $M_{1/2} \approx 0.85$ TeV, leads to $M_1 \approx \mu$ and the LSP is a so-called "well-tempered" highly mixed bino-Higgsino state [171] that saturates the relic density. This region is evident in the middle rows of figure 1 and figure 2 as an extremely narrow strip at the minimum value of $M_{1/2}$ (shown in greater detail in the insets) where $(\Omega h^2)_{\rm th.} \approx 0.1188$, while for larger $M_{1/2}$ the Higgsino DM candidate leads to $(\Omega h^2)_{\rm th.} \ll 0.1188$. From comparing the left and right panels in

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the middle rows of figure 1 and figure 2 it is clear that similar behaviour occurs for the $Z_2^E = +1$ DM candidate in the CSE$_6$SSM. From eq. (4.21) and eq. (4.22) it follows that the necessary value of M_1 occurs for smaller values of $M_{1/2}$ in the CMSSM.

The low allowed values of $M_{1/2}$ imply that in the light $\mu_{(\mathrm{eff})}$ scenario the gluino as well as the ordinary neutralino and chargino states can be light. Though the location of the well-tempered strip differs in the two models, the masses of the gluino, neutralino and charginos are rather similar. For example, in both models in this strip $m_{\tilde{\chi}_1^0} \approx 370\,\mathrm{GeV}$. In the CMSSM, we find that $m_{\tilde{g}} \gtrsim 2.1\,\mathrm{TeV}$, the minimum value occurring in the well-tempered region. A very similar result can be seen in the CSE$_6$SSM, with $m_{\tilde{g}} \gtrsim 2\,\mathrm{TeV}$ except for a narrow line of solutions where the gluino can be as light as $m_{\tilde{g}} \approx 1\,\mathrm{TeV}$.

For these solutions, the bino DM candidate is viable due to the A-funnel mechanism. In the CMSSM, m_A is only light enough so that $m_A \approx 2m_{\tilde{\chi}_1^0}$ at large $\tan\beta \gtrsim 50$ [136]. Because we only considered $\tan\beta(M_Z) = 10$ in our scans, $m_A > 6\,\mathrm{TeV}$ is always very heavy in our CMSSM results and the A-funnel region is not accessible. In the CSE$_6$SSM, for a given value of $\tan\beta$ and $M_{1/2}$ one can make $m_{A_1} \approx 2m_{\tilde{\chi}_1^0}$ light by fine tuning A_0 appropriately. This corresponds to the lower boundary of the solution region in figure 3. Therefore even for $\tan\beta(M_Z) = 10$ light bino DM can satisfy the relic density constraint in the CSE$_6$SSM. This does, however, imply a substantial fine tuning; in our scans, additional points were sampled from this region to overcome this.

In either the bulk or A-funnel regions, the gluino is thus observable at run II or at the high luminosity LHC (HL-LHC); indeed, gluino masses under 2 TeV are already rather close to the limits based on the most recent $\sqrt{s} = 13\,\mathrm{TeV}$ data and so LHC searches will soon be probing this part of the parameter space. Similarly, both models also predict light neutralinos and charginos with masses of a few hundred GeV. To be precise, our CMSSM solutions satisfy 366 GeV $\leq m_{\tilde{\chi}_1^0} \leq 452\,\mathrm{GeV}$, 428 GeV $\leq m_{\tilde{\chi}_2^0} \leq 453\,\mathrm{GeV}$ and 419 GeV $\leq m_{\tilde{\chi}_1^\pm} \leq 453\,\mathrm{GeV}$, while in the CSE$_6$SSM the ranges are 182 GeV $\leq m_{\tilde{\chi}_1^0} \leq 426\,\mathrm{GeV}$, 335 GeV $\leq m_{\tilde{\chi}_2^0} \leq 438\,\mathrm{GeV}$, and 335 GeV $\leq m_{\tilde{\chi}_1^\pm} \leq 431\,\mathrm{GeV}$. This suggests the neutralinos and charginos could also be discoverable at the HL-LHC [172] in the small $\mu_{(\mathrm{eff})}$ case. The overall picture for the solutions presented with $|\mu(M_S)| \approx 417\,\mathrm{GeV}$ is of a split spectrum, with unobservably heavy scalars but light exotic fermions and EW-inos, as well as a sufficiently light gluino. This scenario would therefore predict interesting collider phenomenology in tandem with accounting for the observed DM relic density.

However, while small values of $\mu_{(\mathrm{eff})}$ permit the neutralinos and gluino to be observable at the LHC, models with a highly mixed bino-Higgsino DM candidate are strongly constrained by null results from direct detection experiments. In the bottom rows of figure 1 and figure 2 we show the $\tilde{\chi}_1^0$-proton SI cross section for each sign of $\mu_{(\mathrm{eff})}$. In the region where $(\Omega h^2)_{\mathrm{th.}}$ matches the observed value, the direct detection cross section peaks at $\sim 10^{-45} - 10^{-44}$ cm^2 and is above the 90% exclusion limits set by LUX [173, 174]. In both the CSE$_6$SSM and CMSSM, the SI cross section in this part of the parameter space is dominated by t-channel exchange of the lightest CP-even Higgs h_1. Thus in the leading approximation the SI part of χ_1^0-nucleon cross section takes the form

$$\sigma_{SI} = \frac{4m_r^2 m_N^2}{\pi v^2 m_{h_1}^4} \left| g_{h_1\chi_1\chi_1} F^N \right|^2 ,$$
$$m_r = \frac{m_{\chi_1^0} m_N}{m_{\chi_1^0} + m_N} , \qquad F^N = \sum_{q=u,d,s} f_{Tq}^N + \frac{2}{27} \sum_{Q=c,b,t} f_{TQ}^N , \tag{5.4}$$

where

$$m_N f_{Tq}^N = \langle N | m_q \bar{q} q | N \rangle , \qquad f_{TQ}^N = 1 - \sum_{q=u,d,s} f_{Tq}^N , \tag{5.5}$$

while[17] $f_{Tu}^N \simeq 0.0153$, $f_{Td}^N \simeq 0.0191$ and $f_{Ts}^N \simeq 0.0447$. The size of the cross section in eq. (5.4) is set by the $h_1\,\tilde{\chi}_1^0\,\tilde{\chi}_1^0$ coupling $g_{h_1\chi_1\chi_1}$, which is given by

$$g_{h_1\chi_1\chi_1} = \frac{1}{2} \left(\sqrt{\frac{3}{5}} g_1 N_{14} - g_2 N_{13} \right) \left[N_{11} (U_h)_{11} - N_{12} (U_h)_{12} \right] , \tag{5.6}$$

where the neutralino mixing matrix elements N_{ij} are defined[18] in eq. (4.14) and the Higgs mixing matrix U_h is defined by eq. (4.41). In the CSE$_6$SSM, the contributions to this coupling involving the singlet mixing components N_{1j}, $j = 5, 6, 7, 8$, are negligible in our case and can be ignored. In the highly mixed case with $|\mu| \approx M_1$ and $N_{13} \lesssim N_{14}$, the products $N_{11}N_{14}$ and $N_{12}N_{14}$ that appear above are large and the SI cross section is enhanced [178]. Therefore points with a mixed bino-Higgsino DM candidate that saturates the relic abundance are excluded, for both[19] signs of $\mu_{(\text{eff})}$. As $M_{1/2}$ is increased (decreased) so that $\tilde{\chi}_1^0$ has a smaller (larger) bino component, the SI cross section decreases as $N_{14} \to 0$ ($N_{11}, N_{12} \to 0$). Additionally, the reduction in Ωh^2 for larger values of $M_{1/2}$ implies a reduction in the local number density of WIMPs and thereby weakens the limits from direct detection. We estimate the extent to which this occurs by rescaling the given limits by the predicted relic abundance, so that a given set of values $(m_{\tilde{\chi}_1^0}, \sigma_{\text{SI}}^p)$ is not excluded if

$$\sigma_{\text{SI}}^p \leq \frac{(\Omega h^2)_{\text{exp.}}}{(\Omega h^2)_{\text{th.}}} \sigma_{\text{SI}}^{p,\text{LUX}}(m_{\tilde{\chi}_1^0}), \tag{5.7}$$

where $\sigma_{\text{SI}}^{p,\text{LUX}}(m_{\tilde{\chi}_1^0})$ is the LUX limit at the WIMP mass $m_{\tilde{\chi}_1^0}$. Thus points away from the well-tempered strip may still avoid the direct detection limits. In the CSE$_6$SSM, the presence of the A-funnel region also allows for solutions with $(\Omega h^2)_{\text{th.}} \approx (\Omega h^2)_{\text{exp.}}$ and a predicted SI cross section below current limits for $\lambda < 0$. Nevertheless, as discussed below future limits are expected to probe a substantial portion of the remaining parameter space. Therefore scenarios with small $\mu_{(\text{eff})}$ and a mixed bino-Higgsino $\tilde{\chi}_1^0$ are very tightly constrained.

[17] The values of these hadronic matrix elements are the default values used in micrOMEGAs, as determined in ref. [162] from lattice results. A review of some recent determinations of the required sigma terms $\sigma_{\pi N}$ and σ_s has been given in ref. [175], while an extraction of these quantities from phenomenological inputs using chiral effective field theory has been presented in refs. [176, 177].

[18] Note that in this convention N_{11} and N_{12} specify the Higgsino mixing, while N_{13} and N_{14} give the wino and bino mixing respectively.

[19] For $\mu_{(\text{eff})} < 0$ the SI cross section is slightly smaller, due to a cancellation between the contributions from the up- and down-type Higgsinos, but this is not significant enough to evade the current limits.

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5.3 Pure Higgsino dark matter

Scenarios with a heavy, pure Higgsino DM candidate are less constrained by direct detection limits due to both the weaker limits at high WIMP masses and the suppression of the SI scattering cross section for a pure Higgsino LSP [179]. Analyses of the CMSSM parameter space that also account for limits from collider searches suggest that this part of the parameter space is favored by experimental constraints [180], though scenarios with a relatively light LSP can still fit the data [181]. To see that this is also true in the CSE$_6$SSM, in figure 4 and figure 5 we compare the CSE$_6$SSM with $|\mu_{\text{eff}}(M_S)| \approx 1046\,\text{GeV}$ to the CMSSM with $|\mu(M_S)| = 1046\,\text{GeV}$.

As in the previous case with small $\mu_{(\text{eff})}$, the region in which we find solutions in the CSE$_6$SSM is much smaller than in the CMSSM. The upper bound on m_0 again arises from tachyonic CP-even and CP-odd Higgs states that occur as $|A_0|$ is increased. At the same time, the minimum value of $M_{1/2}$ that satisfies the relic density constraint is much larger. This is because a relic density consistent with eq. (5.3) requires $\tilde{\chi}_1^0$ to be nearly purely Higgsino with $m_{\tilde{\chi}_1^0} \approx 1\,\text{TeV}$, which is achieved for $|M_1| \gtrsim |\mu_{(\text{eff})}| \approx 1\,\text{TeV}$. The condition of universal gaugino masses at M_X then means that the gluino is now very heavy along with the sfermions. In the CSE$_6$SSM we find solutions with $m_{\tilde{g}} \geq 3.8\,\text{TeV}$, compared to the minimum value of $m_{\tilde{g}} \geq 5.7\,\text{TeV}$ in the CMSSM scan. The prospects for an LHC discovery in this scenario are fairly poor in the CMSSM, as the gluino and all sfermions would be out of reach at run II.

For the CSE$_6$SSM points shown in figure 4 and figure 5 we considered slightly larger exotic couplings with $\kappa_0 = \tilde{\lambda}_0 = 3 \times 10^{-3}$. The couplings are required to be large enough to ensure that $\tilde{\chi}_1^0$ is still the stable second DM candidate, rather than one of the exotic sector possibilities. The exotic fermions are correspondingly heavier, with masses satisfying $3\text{ TeV} \leq \mu_{D_i} \leq 3.3\,\text{TeV}$ and $1.63\text{ TeV} \leq \mu_{\tilde{H}^0_{I\alpha}} \leq 1.67\,\text{TeV}$, which also makes them unlikely to be observable at run II or at the HL-LHC. Note however that, in addition to being able to vary $M_{Z'}$, there is also some freedom to vary the exotic couplings to obtain lighter exotic states. We illustrate this in figure 6, where we plot the valid solutions with $\kappa_0 = 1.4 \times 10^{-3}$, giving D fermion masses of $\mu_{D_i} \in [1.5\text{ TeV}, 1.6\text{ TeV}]$, comparable with the potential exclusion reach for third generation squarks at the HL-LHC [182]. For fixed $|\lambda(M_X)| = 2.4 \times 10^{-3}$ the effect of this is to slightly increase the minimum allowed value of $M_{1/2}$ outside of the A-funnel region. This is due to an increase in the calculated $(\Omega h^2)_{\text{th.}}$, which was already rather close to the value from Planck observations. The larger value of the relic density in turn arises because of the increase in $\mu_{\text{eff}}(M_S)$ that results for smaller values of κ_0 in the RG running; this can be seen, for example, from eq. (A.18). A compensating small reduction in $\lambda(M_X)$ can be used to maintain the low-energy value of μ_{eff} and therefore $(\Omega h^2)_{\text{th.}}$, in which case the smaller values of $M_{1/2}$ shown in figure 4 and figure 5 continue to be allowed. The presence of light exotics is an important possible signature that allows the model to be discovered when the SUSY breaking scale is very heavy, as well as distinguishing the E_6 inspired model from the CMSSM.

As can be seen in the middle rows of figure 4 and figure 5, and in figure 6, the prediction for the relic density in the CSE$_6$SSM remains similar to that in the CMSSM. In both models

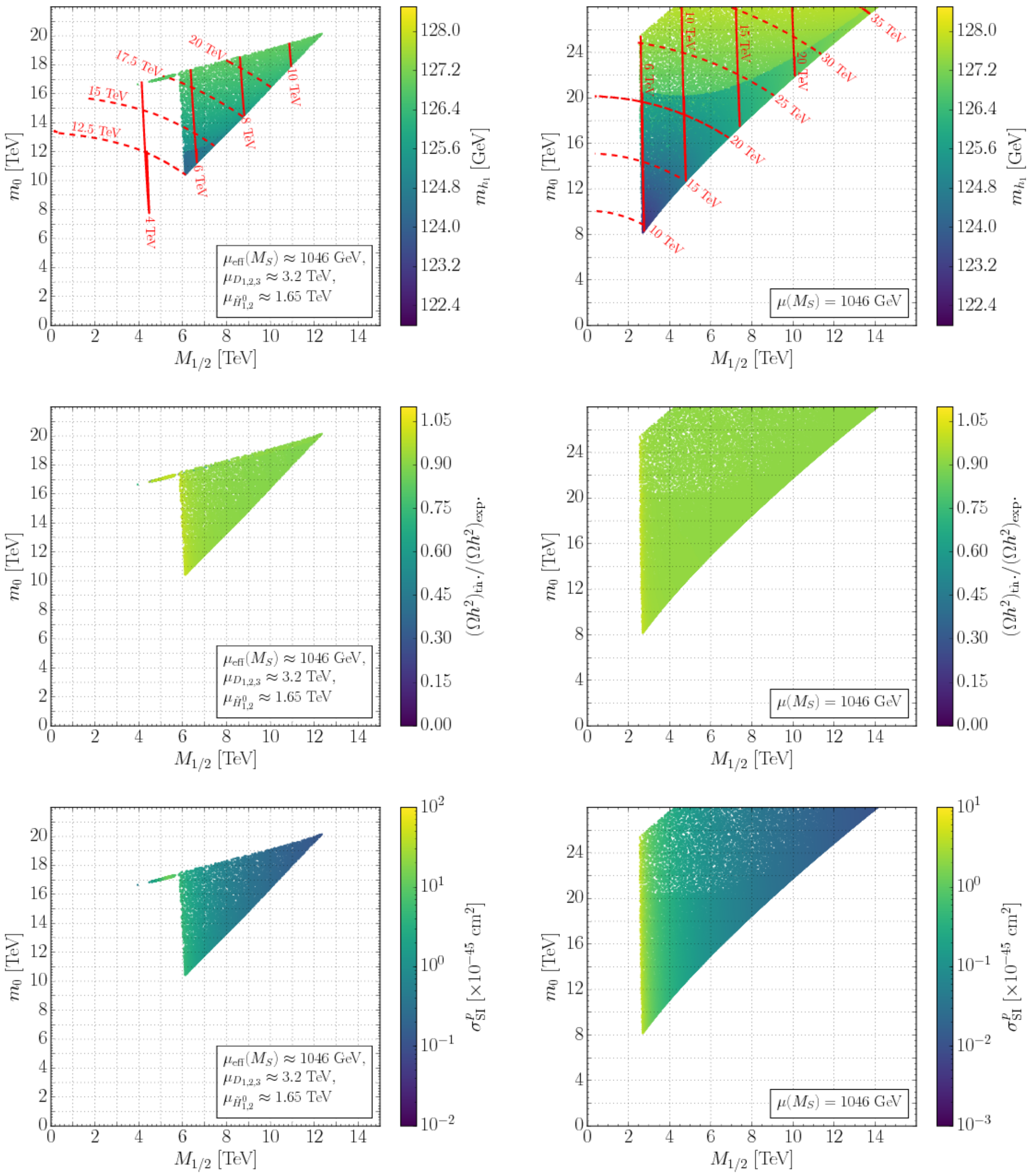

Figure 4. Contour plots in the $M_{1/2} - m_0$ plane of the lightest CP-even Higgs mass (top row), DM relic density (middle row) and proton SI cross section (bottom row) in the CSE$_6$SSM with $\mu_{\rm eff}(M_X) \approx 898\,$GeV (left column) and CMSSM with $\mu(M_S) = 1046\,$GeV (right column). In the top row, we show contours of the gluino (solid lines) and squark (dashed lines) masses.

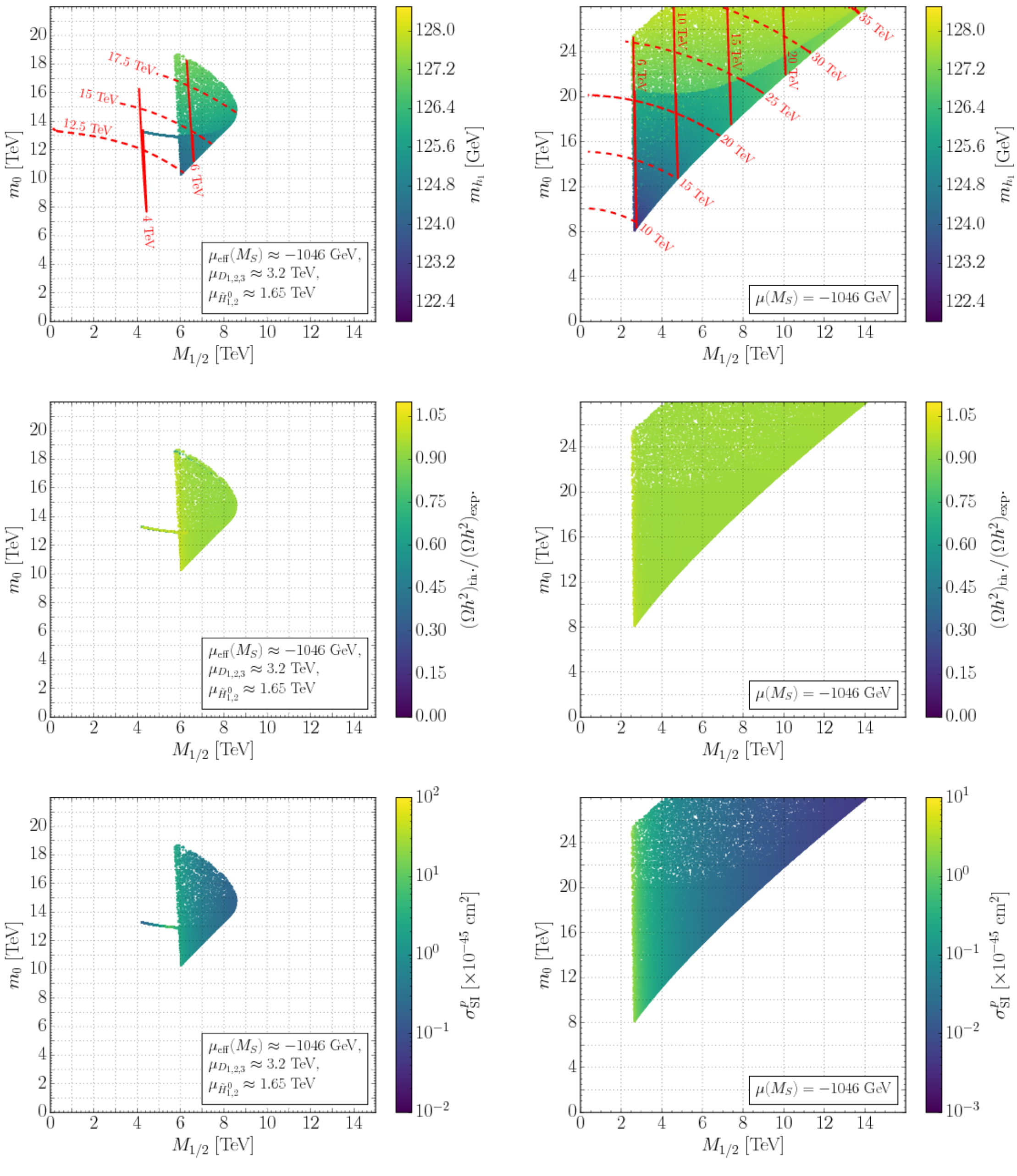

Figure 5. Contour plots in the $M_{1/2} - m_0$ plane of the lightest CP-even Higgs mass (top row), DM relic density (middle row) and proton SI cross section (bottom row) in the CSE$_6$SSM with $\mu_{\rm eff}(M_X) \approx -898$ GeV (left column) and CMSSM with $\mu(M_S) = -1046$ GeV (right column). In the top row, we show contours of the gluino (solid lines) and squark (dashed lines) masses.

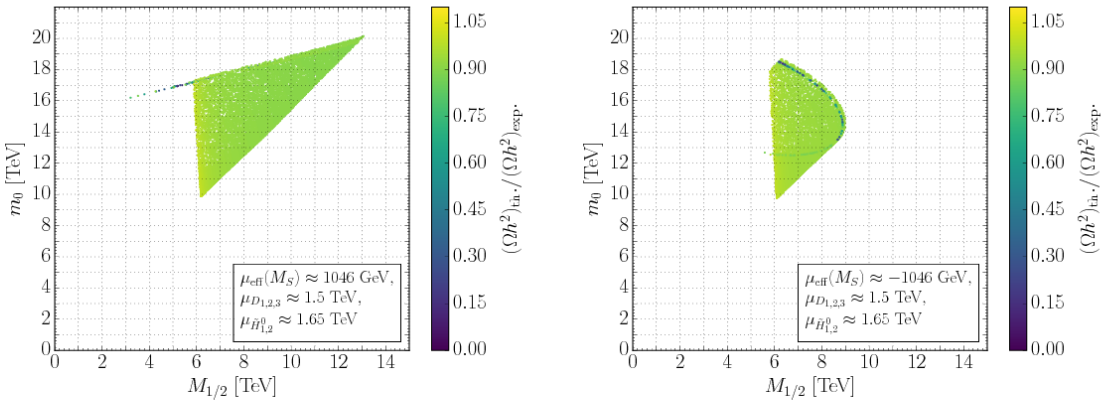

Figure 6. Contour plots in the $M_{1/2} - m_0$ plane of the DM relic density in the CSE$_6$SSM with $\mu_{\rm eff}(M_S) \approx 1046$ GeV (left) and $\mu_{\rm eff}(M_S) \approx -1046$ GeV (right), with reduced values of the exotic Yukawa couplings $\kappa_{ij}(M_X)$ such that $\mu_{D_i} \approx 1.5$ TeV.

a Higgsino with a mass of approximately 1 TeV saturates the observed value in eq. (5.3). The narrow A-funnel region at lower $M_{1/2}$ is again accessible in the CSE$_6$SSM by tuning A_0 to reduce m_{A_1}. As large mixings are no longer required to reproduce the relic density for $|\mu_{(\rm eff)}| \approx 1$ TeV, a large fraction of the solutions found have a predicted SI cross section below the current LUX limits. Points in both models with $M_{1/2}$ where the LSP transitions from being pure bino to pure Higgsino, i.e., where $M_1 \approx \mu_{(\rm eff)}$ near the lower bound on $M_{1/2}$, present a larger cross section that is in excess of the LUX limits. Therefore even for heavy $\mu_{(\rm eff)}$ in the CMSSM and CSE$_6$SSM constraints can be put on the parameter space by direct detection searches. At larger $M_{1/2}$ (that is, where M_1 is significantly larger than $\mu_{(\rm eff)}$) the models currently evade the SI direct detection limits, and are very unlikely to be probed by direct collider searches in the near future if the exotic fermions in the CSE$_6$SSM are not light. However, this part of the CSE$_6$SSM, and CMSSM, parameter space will be constrained by results from XENON1T, as we now discuss in more detail.

5.4 Impact of current and future searches

In figure 7 we show the current and future regions probed by LUX and XENON1T for $|\mu_{(\rm eff)}(M_S)| \approx 417$ GeV in both models. As described above, the existing 2015 LUX limits already essentially exclude the well-tempered bino-Higgsino solution region at low $m_{\tilde{g}}$, i.e., low $M_{1/2}$, where the SI cross section is enhanced by large mixings. The effect of the new 2016 limit is to extend this exclusion to larger gluino masses, despite the reduction in the predicted relic density and SI cross section. This is as expected from the results of dedicated MSSM studies [183, 184]. XENON1T [185] is projected to exclude (or discover) even larger values of $m_{\tilde{g}}$. In this CMSSM scenario, XENON1T can potentially exclude $\tilde{g}$ masses up to 4–5 TeV.

The exclusions set by direct detection searches in the CSE$_6$SSM are to some extent similar to those in the CMSSM. In particular, outside of the A-funnel region in the CSE$_6$SSM, the LUX limits exclude gluino masses $m_{\tilde{g}} \lesssim 3$ TeV for $\mu_{(\rm eff)} > 0$ and $m_{\tilde{g}} \lesssim 2.5$ TeV for

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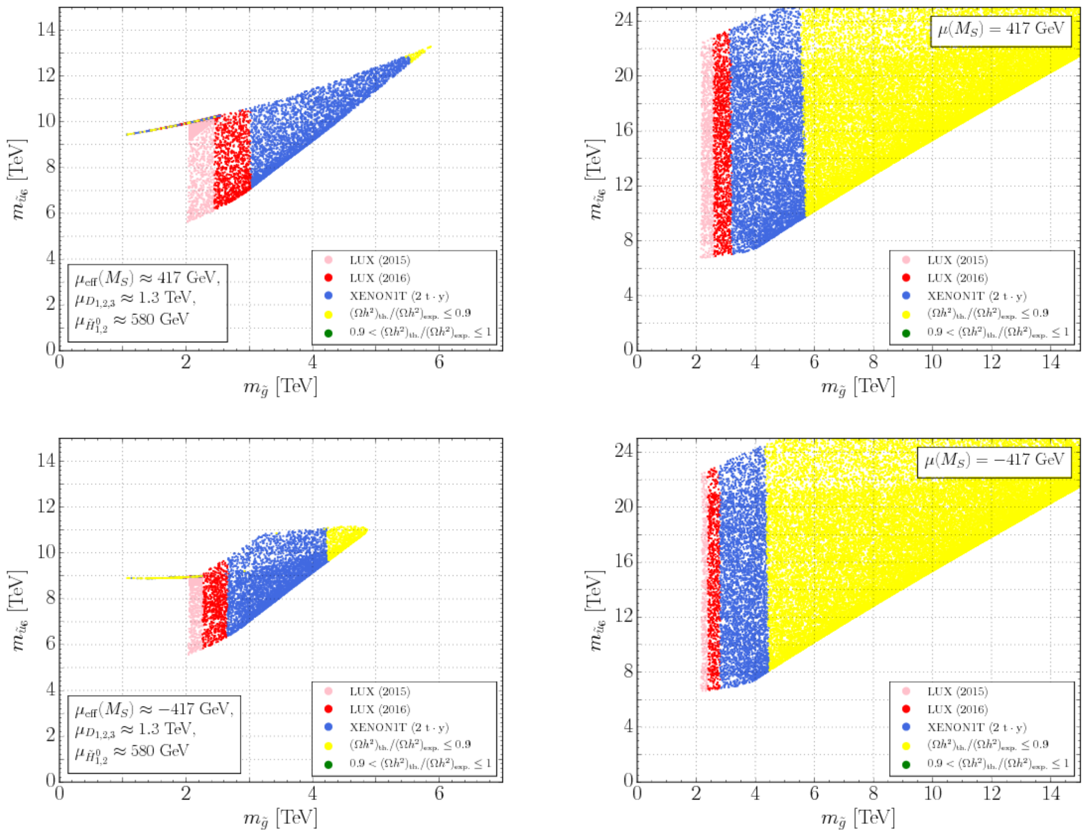

Figure 7. Plots of direct detection and collider constraints in the $m_{\tilde{g}} - m_{\tilde{u}_6}$ plane in the CSE$_6$SSM with $|\mu_{\rm eff}(M_X)| \approx 347\,$GeV (left column) and the CMSSM with $|\mu(M_S)| = 417\,$GeV (right column). In the top row, $\mu_{\rm (eff)}(M_X) > 0$, and in the bottom row $\mu_{\rm (eff)}(M_X) < 0$. In each plot, we show points that have a SI cross section in excess of the 2015 [173] and 2016 [174] LUX limits (pink and red, respectively) and points that are not currently excluded but are within the projected reach [185] of XENON1T (blue). In each case, the exclusion limit is determined according to eq. (5.7). Finally, points that are not excluded by any limits but that predict a relic density that is less than 90% of the measured value are shown in yellow, while those points with $0.9 < (\Omega h^2)_{\rm th.}/(\Omega h^2)_{\rm exp.} \leq 1$ are shown in green.

$\mu_{\rm (eff)} < 0$ in both models. Similarly, XENON1T will be able to probe gluino masses up to 4–5 TeV in the CSE$_6$SSM as well. This accounts for a large fraction of as yet unexcluded solutions in the CSE$_6$SSM.

However, as can be seen from the left column of figure 7, some points in the A-funnel region will still not be excluded by LUX or XENON1T. These points have a suppressed SI cross section or do not saturate the relic density bound, or both. This is also true in both models for those points not excluded at large $m_{\tilde{g}}$. Points close to the well-tempered region, where the amount of mixing is still relatively large, only escape being excluded if they lead to an extremely small relic density. If it is required that the LSP explains a substantial fraction of the observed relic abundance, for example $(\Omega h^2)_{\rm th.}/(\Omega h^2)_{\rm exp.} > 0.1$, then these points are removed. This is illustrated in figure 8, where we show the variation in the bino

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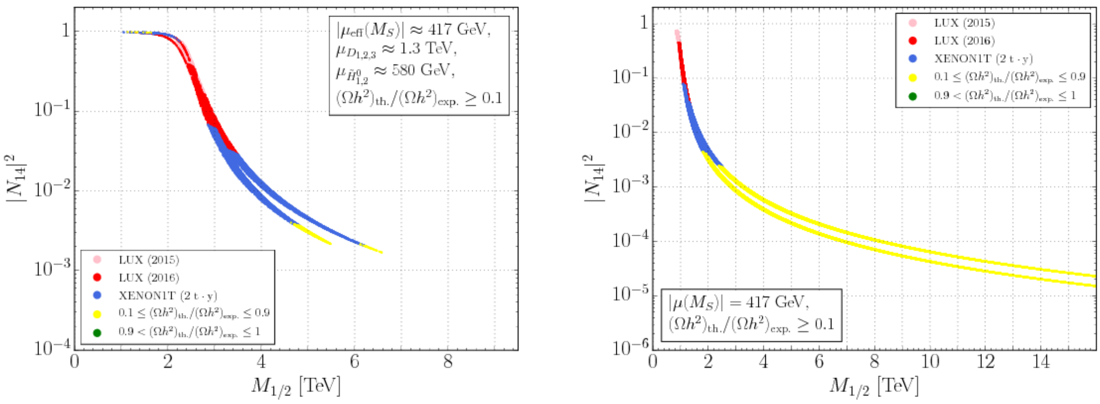

Figure 8. Plots showing points excluded by direct detection constraints in the $M_{1/2} - |N_{14}|^2$ plane in the CSE$_6$SSM (left) and CMSSM (right) for $|\mu_{(\rm eff)}(M_S)| \approx 417\,$GeV, after also requiring that the LSP accounts for at least 10% of the observed relic density. The scaling of the limits and the colour coding is otherwise the same as in figure 7.

fraction for points satisfying this criterion. The effect of the direct detection limits is to heavily restrict the amount of mixing allowed. The surviving points are forced to either be almost pure bino, at small $M_{1/2}$, or almost pure Higgsino at large $M_{1/2}$ and hence having a heavy SUSY spectrum.

While the A-funnel points will not be observable at XENON1T, the fact that $m_{\tilde{g}} \lesssim 2\,$TeV for these solutions means that most are in reach of LHC searches targeting gluinos. This highlights the complementary nature of collider and direct detection searches; similar observations have been made for the CMSSM (see, for example, ref. [186]). Given the similarity of the lightest $Z_2^E = +1$ neutralinos in the CSE$_6$SSM to the ordinary MSSM neutralino sector, it is not so surprising that this continues to hold. In particular, results from XENON1T will be able to constrain the CSE$_6$SSM (and CMSSM) at much higher SUSY scales than are expected to be reached at the LHC. We conclude from this that direct detection searches, if no WIMPs are observed, will be able to place indirect limits on the sparticle masses much higher than can be achieved at run II, when the neutralino does not annihilate via special mechanisms such as the A-funnel. Thus direct detection limits are a particularly strong constraint on the CSE$_6$SSM parameter space.

The solutions that we find with a heavy Higgsino DM candidate lead to gluino and MSSM sfermion masses beyond the exclusion reach at run II. This is shown in figure 9. Consequently there are effectively no constraints on this part of parameter space coming from collider limits, at least in the CMSSM. In the CSE$_6$SSM, the possibility of light exotic fermions, as in figure 6, would allow for the model to be discovered even if all MSSM-like states and exotic scalars are heavy. However, if these states are also heavy then limits from direct detection searches are much more effective at constraining the parameter space.

Prior to the most recent LUX limits, all of our solutions with heavy $|\mu_{(\rm eff)}|$ were consistent with direct detection limits. This is no longer true for the new 2016 LUX limits, which now exclude points with $M_1 \approx \mu_{(\rm eff)}$. Therefore the current direct detection lim-

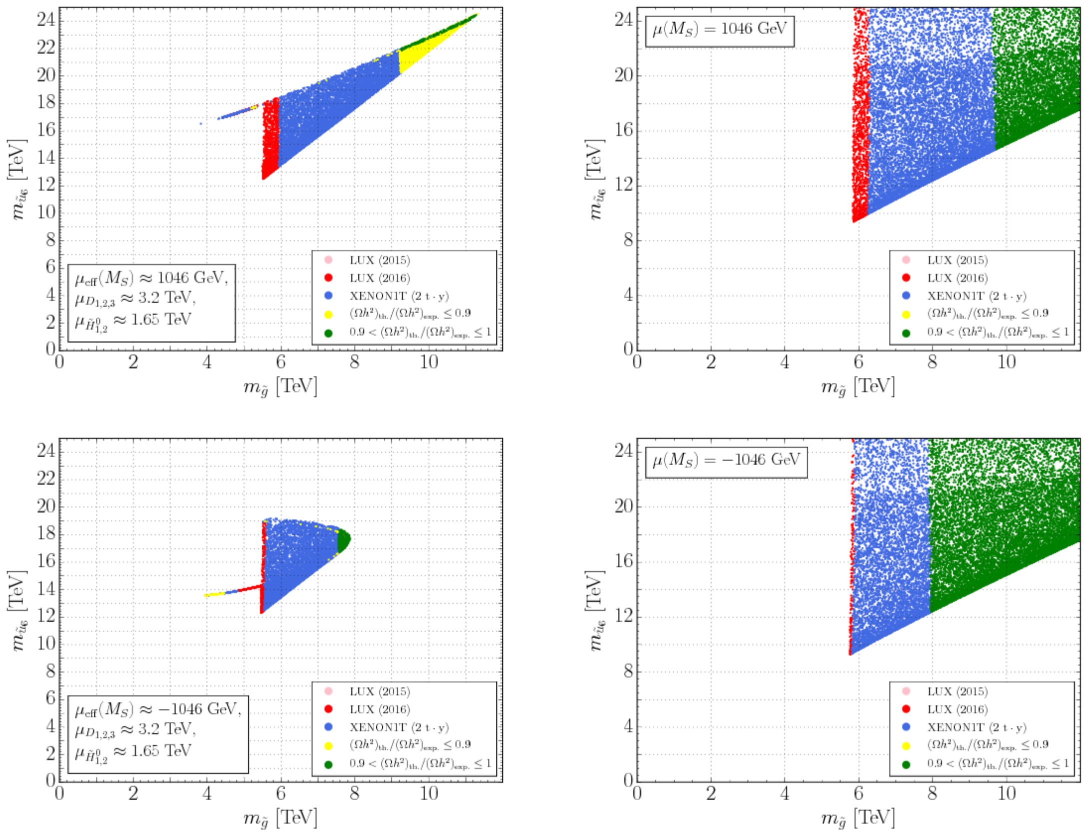

Figure 9. Plots of constraints in the $m_{\tilde{g}} - m_{\tilde{u}_6}$ plane in the CSE$_6$SSM with $|\mu_{\rm eff}(M_X)| \approx 898$ GeV (left column) and the CMSSM with $|\mu(M_S)| = 1046$ GeV (right column). In the top row, $\mu_{(\rm eff)}(M_X) > 0$, and in the bottom row $\mu_{(\rm eff)}(M_X) < 0$. The color coding is the same as in figure 7.

its are already probing the heavy $|\mu_{(\rm eff)}|$ parameter space. Scenarios with a highly mixed bino-Higgsino $\tilde{\chi}^0_1$ accounting for at least 10% of the relic abundance are again all excluded by the current limits. This is shown in figure 10. Thus in the case that the LSP is relevant for addressing the DM problem, direct detection limits place stringent constraints on the allowable bino-Higgsino admixture. More extensive coverage of the valid, low mixing regions will require results from XENON1T, however.

It is clear that in the CSE$_6$SSM, results from XENON1T will place very strong constraints on the parameter space, as it should be possible to cover almost all of the allowed region. As for the previous small $|\mu_{(\rm eff)}|$ case, the surviving regions are the A-funnel region and at very large $m_{\tilde{g}}$. In this scenario the A-funnel region cannot be searched for directly at the LHC; from the left column of figure 9 it can be seen that the gluino mass is always greater than ≈ 4 TeV. An interesting question is to what extent indirect DM detection experiments or results from flavor physics can constrain the CSE$_6$SSM here; we leave this for a future study. On the other hand, for very heavy spectra without light exotic fermions neither collider searches nor results from XENON1T will constrain the CSE$_6$SSM or the

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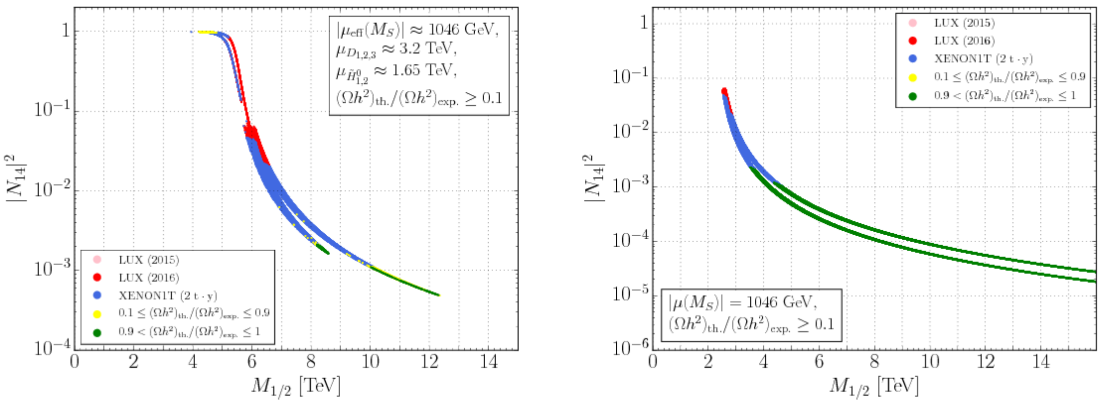

Figure 10. Plots showing points excluded by direct detection constraints in the $M_{1/2} - |N_{14}|^2$ plane in the CSE$_6$SSM (left) and CMSSM (right) for $|\mu_{(\rm eff)}(M_S)| \approx 1046\,\text{GeV}$, after also requiring that the LSP accounts for at least 10% of the observed relic density. The scaling of the limits and the colour coding is the same as in figure 7.

CMSSM. Even more sensitive direct detection experiments, such as results from LZ, will be required to directly search for these scenarios.

It should be noted that the large number of solutions for which $(\Omega h^2)_{\rm th.}$ is indicated as being less than 90% of the Planck value in figure 9 still account for a very large fraction of the observed relic abundance. Small changes in $\lambda(M_X)$, or $\mu(M_X)$ in the CMSSM, are enough to closely reproduce the value in eq. (5.3) without significantly changing any other results, unlike in the light Higgsino case where the DM candidate is severely underabundant assuming a standard freeze-out scenario. At large $M_{1/2}$ the relic density is still fully accounted for by the Higgsino DM candidate. Unfortunately, while these scenarios can explain the observed DM density entirely, the expected collider phenomenology is rather uninteresting as all states are too heavy to be observable.

6 Conclusions

We have studied dark matter and LHC phenomenology implications in both the CMSSM and a constrained version of an E_6 inspired model (CSE$_6$SSM). The SE$_6$SSM is a string inspired alternative to the MSSM, where the break down of the E_6 gauge group leads to a discrete R-parity and a $\mathrm{U}(1)_N$ gauge extension surviving to the TeV scale that forbids the μ-term of the MSSM. The charges allow the standard see-saw mechanism for neutrino masses and a leptogenesis explanation of the matter-anti-matter asymmetry. The model contains exotic states at low energies needed to fill three generations of complete **27**-plet representations of E_6 and ensure anomaly cancellation, and can give rise to spectacular collider signatures. A single additional discrete symmetry which commutes with E_6 is imposed to forbid FCNCs and this along with R-parity lead to multiple dark matter candidates. In this paper we focused on scenarios where the lightest exotic particle is an extremely light singlino which forms hot dark matter, but contributes negligibly to the relic density. We

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showed that the relic density can instead be explained entirely by the lightest MSSM-like neutralino.

We have performed a detailed exploration of the parameter space of both the CMSSM and CSE_6SSM and compared the results. We find that in both models one may fit the observed relic density with a pure Higgsino neutralino that has a mass around 1 TeV. Alternatively this can be achieved with a mixed bino-Higgsino dark matter candidate, requiring a fine tuning of M_1 and $\mu_{(\mathrm{eff})}$ to obtain the well-tempered strip and this can work for lighter neutralino masses ($\approx 400\,\mathrm{GeV}$ in our example). However recent direct detection results have placed strong limits on this mixing, placing a significant tension between fitting the observed relic density and evading direct detection limits. Indeed we find that the recent LUX 2016 direct detection limits constrain Higgsino-bino mixing such that it rules out this well-tempered strip for both models for light and heavy neutralinos.

However we also found that the CSE_6SSM can have special A-funnel solutions where the correct relic density can be achieved for lighter $M_{1/2}$, a scenario that is only possible in the CMSSM for a much larger $\tan\beta$ than is considered here. Such scenarios exist for both the heavier and lighter Higgsino masses considered. For lighter Higgsino masses this A-funnel region, which can escape direct detection limits even from the future results of XENON1T, will be probed by the LHC run II. This demonstrates an important complementarity between collider searches and experiments for the direct detection of dark matter.

Such special regions aside however it is now rather difficult to explain dark matter in the lighter scenarios. Nonetheless if one requires only that the relic density is not too large then many scenarios are still viable and have phenomenology that will be probed with run II of the LHC. Since the sfermions will still be very heavy the main signatures arise from the production of gluinos, charginos and neutralinos, with MSSM-like signatures. On the other hand the leptoquarks in the CSE_6SSM can be light enough to detect even when the SUSY scale is very heavy. These exotic states would lead to considerable enhancement of $p\,p \to t\,\bar{t}\,\tau^+\,\tau^- + E_t^{\mathrm{miss}} + X$ and $p\,p \to b\,\bar{b}\,\tau^+\,\tau^- + E_T^{\mathrm{miss}} + X$, where X stands for any light quark or gluon jets.

Heavier scenarios with a Higgsino dark matter candidate of around 1 TeV are also not currently constrained so much by direct detection and it is possible to fit the relic density in both the CMSSM and CSE_6SSM for a wide range of the parameter space. These scenarios have a rather heavy spectrum which is not accessible to the LHC, however they will be probed by future direct detection experiments, such as XENON1T which will be able to probe most of the viable solutions we have found in the CSE_6SSM. Therefore the future impact of XENON1T on these models will be very significant.

Acknowledgments

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A RGEs

In our analysis, the SUSY preserving and soft SUSY breaking parameters at M_S are obtained from the GUT scale boundary conditions by running them using two-loop RGEs. These RGEs were automatically derived using SARAH-4.5.6, which makes use of the general results given in refs. [187–189]. For completeness, in this appendix we summarize the complete set of RGEs used to obtain our results. For a general parameter p, the RG equation for p is expressed in terms of the one- and two-loop β functions, $\beta_p^{(1)}$ and $\beta_p^{(2)}$ respectively, according to

$$\frac{\mathrm{d}p(t)}{\mathrm{d}t} = \beta_p = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4}, \tag{A.1}$$

where $t = \ln Q/M_X$ gives the scale at which p is evaluated.

A.1 Gauge couplings

In general, kinetic mixing of the $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_N$ leads to a set of RGEs for the Abelian gauge couplings involving a set of off-diagonal gauge couplings. In the triangle basis of eq. (2.7), these RGEs can be written

$$\frac{\mathrm{d}G}{\mathrm{d}t} = G \times B, \tag{A.2}$$

where the matrix of β functions is

$$B = \begin{pmatrix} \beta_{g_1} g_1^2 & 2 g_1 g_1' \beta_{g_{11}} + 2 g_1 g_{11} \beta_{g_1} \\ 0 & g_1'^2 \beta_{g_1'} + 2 g_1' g_{11} \beta_{g_{11}} + g_{11}^2 \beta_{g_1} \end{pmatrix}. \tag{A.3}$$

The off-diagonal β function $\beta_{g_{11}}$ is rather small, with $\beta_{g_{11}}^{(1)} = -\sqrt{6}/5$ at one-loop. As discussed in section 2, the effects of kinetic mixing are therefore small if g_{11} vanishes at the GUT scale, and so we neglect it. When this is done, the two-loop RGEs for the diagonal Abelian gauge couplings are

$$\beta_{g_1}^{(1)} = \frac{48}{5} g_1^3, \tag{A.4}$$

$$\begin{aligned}\beta_{g_1}^{(2)} = g_1^3 \Big[& \frac{234}{25} g_1^2 + \frac{81}{25} g_1'^2 + \frac{54}{5} g_2^2 + 24 g_3^2 - \frac{6}{5}|\lambda|^2 - \frac{6}{5}|\tilde{\sigma}|^2 - \frac{26}{5}\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\ & - \frac{14}{5}\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - \frac{18}{5}\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - \frac{4}{5}\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - \frac{6}{5}\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\ & - \frac{6}{5}\mathrm{Tr}\Big(f f^\dagger\Big) - \frac{6}{5}\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - \frac{14}{5}\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \frac{18}{5}\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Big],\end{aligned} \tag{A.5}$$

$$\beta_{g_1'}^{(1)} = \frac{213}{20} g_1'^3, \tag{A.6}$$

$$\begin{aligned}\beta_{g_1'}^{(2)} = g_1'^3 \Big[& \frac{81}{25} g_1^2 + \frac{2457}{200} g_1'^2 + \frac{51}{5} g_2^2 + 24 g_3^2 - \frac{19}{5}|\lambda|^2 - \frac{5}{2}|\sigma|^2 \\ & - \frac{4}{5}|\tilde{\sigma}|^2 - \frac{9}{5}\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - \frac{21}{5}\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - \frac{7}{5}\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)\end{aligned}$$

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$$- \frac{57}{10}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - \frac{19}{5}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - \frac{19}{5}\,\mathrm{Tr}\Big(ff^\dagger\Big)$$

$$- \frac{19}{5}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - \frac{21}{5}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \frac{7}{5}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg]. \tag{A.7}$$

The β functions for the $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_C$ gauge couplings are the same irrespective of whether or not the kinetic mixing is taken into account. They are

$$\beta_{g_2}^{(1)} = 4g_2^3, \tag{A.8}$$

$$\beta_{g_2}^{(2)} = g_2^3\Bigg[\frac{18}{5}g_1^2 + \frac{17}{5}g_1'^2 + 46g_2^2 + 24g_3^2 - 2|\lambda|^2 - 2|\tilde{\sigma}|^2 - 6\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)$$

$$- 6\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\,\mathrm{Tr}\Big(ff^\dagger\Big)$$

$$- 2\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 6\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg], \tag{A.9}$$

$$\beta_{g_3}^{(1)} = 0, \tag{A.10}$$

$$\beta_{g_3}^{(2)} = g_3^3\Bigg[3g_1^2 + 3g_1'^2 + 9g_2^2 + 48g_3^2 - 4\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 4\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)$$

$$- 2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 4\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big)\Bigg]. \tag{A.11}$$

A.2 Superpotential trilinear couplings

When gauge kinetic mixing is neglected, the running of the dimensionless superpotential couplings is described by the following two-loop β functions:

$$\beta_{y^D}^{(1)} = y^D\Bigg(3y^{D\dagger}y^D + y^{U\dagger}y^U + g^{D*}g^{DT}\Bigg) + y^D\Bigg[-\frac{7}{15}g_1^2 - \frac{7}{10}g_1'^2 - 3g_2^2 - \frac{16}{3}g_3^2$$

$$+ |\lambda|^2 + 3\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) + \mathrm{Tr}\Big(y^E y^{E\dagger}\Big) + \mathrm{Tr}\Big(ff^\dagger\Big)\Bigg], \tag{A.12}$$

$$\beta_{y^D}^{(2)} = y^D\Bigg\{y^{U\dagger}y^U\Bigg[\frac{4}{5}g_1^2 + \frac{1}{5}g_1'^2 - |\lambda|^2 - 3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - \mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\Bigg]$$

$$+ y^{D\dagger}y^D\Bigg[\frac{4}{5}g_1^2 + \frac{6}{5}g_1'^2 + 6g_2^2 - 3|\lambda|^2 - 9\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)$$

$$- 3\,\mathrm{Tr}\Big(ff^\dagger\Big)\Bigg] + g^{D*}g^{DT}\Bigg[\frac{2}{5}g_1^2 + \frac{3}{5}g_1'^2 - |\tilde{\sigma}|^2 - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg]$$

$$- 4y^{D\dagger}y^D y^{D\dagger}y^D - 2y^{U\dagger}y^U y^{D\dagger}y^D - 2y^{U\dagger}y^U y^{U\dagger}y^U - 2g^{D*}g^{DT}y^{D\dagger}y^D$$

$$- 2g^{D*}g^{DT}g^{D*}g^{DT} - g^{D*}\kappa^T\kappa^* g^{DT}\Bigg\} + y^D\Bigg\{\frac{413}{90}g_1^4 + \frac{77}{10}g_1'^4 + \frac{33}{2}g_2^4$$

$$+ \frac{128}{9}g_3^4 - \frac{7}{30}g_1^2 g_1'^2 + g_1^2 g_2^2 + \frac{8}{9}g_1^2 g_3^2 + \frac{3}{2}g_1'^2 g_2^2 + \frac{4}{3}g_1'^2 g_3^2 + 8g_2^2 g_3^2 - 3|\lambda|^4$$

$$
\begin{aligned}
&-\frac{2}{5}g_1^2\left[\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)-3\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)\right]+g_1'^2\left[-\frac{3}{5}\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)-\frac{1}{5}\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)\right.\\
&\left.+\,\mathrm{Tr}\Big(ff^\dagger\Big)\right]+16g_3^2\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)+|\lambda|^2\Big[g_1'^2-|\sigma|^2-2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)\\
&-3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)-\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)\Big]-3\,\mathrm{Tr}\Big(ff^\dagger ff^\dagger\Big)-2\,\mathrm{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big)\\
&-3\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{DT}y^{D*}\Big)-2\,\mathrm{Tr}\Big(h^E h^{E\dagger}y^E y^{E\dagger}\Big)-9\,\mathrm{Tr}\Big(y^D y^{D\dagger}y^D y^{D\dagger}\Big)\\
&\left.-3\,\mathrm{Tr}\Big(y^D y^{U\dagger}y^U y^{D\dagger}\Big)-3\,\mathrm{Tr}\Big(y^E y^{E\dagger}y^E y^{E\dagger}\Big)-\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger f^T f^*\Big)\right\}, \qquad \text{(A.13)}
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(1)}_{y^U}=\;&y^U\Big(y^{D\dagger}y^D+3y^{U\dagger}y^U+g^{D*}g^{DT}\Big)+y^U\left[-\frac{13}{15}g_1^2-\frac{3}{10}g_1'^2-3g_2^2-\frac{16}{3}g_3^2\right.\\
&\left.+|\lambda|^2+3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)+\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)\right], \qquad \text{(A.14)}
\end{aligned}
$$

$$
\begin{aligned}
\beta^{(2)}_{y^U}=\;&y^U\left\{y^{U\dagger}y^U\left[\frac{2}{5}g_1^2+\frac{3}{5}g_1'^2+6g_2^2-3|\lambda|^2-9\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)-3\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)\right]\right.\\
&+y^{D\dagger}y^D\left[\frac{2}{5}g_1^2+\frac{3}{5}g_1'^2-|\lambda|^2-3\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)-\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)-\mathrm{Tr}\Big(ff^\dagger\Big)\right]\\
&+g^{D*}g^{DT}\left[\frac{2}{5}g_1^2+\frac{3}{5}g_1'^2-|\tilde\sigma|^2-3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big)-\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\right]\\
&-2y^{D\dagger}y^D y^{D\dagger}y^D-2y^{D\dagger}y^D y^{U\dagger}y^U-4y^{U\dagger}y^U y^{U\dagger}y^U-2g^{D*}g^{DT}y^{U\dagger}y^U\\
&\left.-2g^{D*}g^{DT}g^{D*}g^{DT}-g^{D*}\kappa^T\kappa^* g^{DT}\right\}+y^U\left\{\frac{3913}{450}g_1^4+\frac{81}{25}g_1'^4+\frac{33}{2}g_2^4\right.\\
&+\frac{128}{9}g_3^4+\frac{161}{300}g_1^2g_1'^2+g_1^2g_2^2+\frac{136}{45}g_1^2g_3^2+\frac{3}{4}g_1'^2g_2^2+\frac{8}{15}g_1'^2g_3^2+8g_2^2g_3^2\\
&-3|\lambda|^4+\frac{4}{5}g_1^2\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)+\frac{3}{2}g_1'^2\left[-\frac{1}{5}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)+\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)\right]\\
&+16g_3^2\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)+|\lambda|^2\left[\frac{3}{2}g_1'^2-|\sigma|^2-2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)-3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\right.\\
&\left.-3\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)-\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)-\mathrm{Tr}\Big(ff^\dagger\Big)\right]-2\,\mathrm{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big)\\
&-3\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\tilde f\tilde f^\dagger\Big)-\mathrm{Tr}\Big(\tilde f h^{E\dagger}h^E\tilde f^\dagger\Big)-\mathrm{Tr}\Big(\tilde f\tilde\lambda^\dagger\tilde\lambda\tilde f^\dagger\Big)-3\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big)\\
&\left.-3\,\mathrm{Tr}\Big(y^D y^{U\dagger}y^U y^{D\dagger}\Big)-9\,\mathrm{Tr}\Big(y^U y^{U\dagger}y^U y^{U\dagger}\Big)\right\}, \qquad \text{(A.15)}
\end{aligned}
$$

$$\beta^{(1)}_{y^E} = \left(2h^E h^{E\dagger} + 3y^E y^{E\dagger}\right) y^E + y^E\left[-\frac{9}{5}g_1^2 - \frac{7}{10}g_1'^2 - 3g_2^2 + |\lambda|^2 + 3\,\mathrm{Tr}\left(y^D y^{D\dagger}\right)\right.$$

$$\left. + \mathrm{Tr}\left(y^E y^{E\dagger}\right) + \mathrm{Tr}\left(f f^\dagger\right)\right], \tag{A.16}$$

$$\beta^{(2)}_{y^E} = \left\{ y^E y^{E\dagger}\left[\frac{3}{2}g_1'^2 + 6g_2^2 - 3|\lambda|^2 - 9\,\mathrm{Tr}\left(y^D y^{D\dagger}\right) - 3\,\mathrm{Tr}\left(y^E y^{E\dagger}\right) - 3\,\mathrm{Tr}\left(f f^\dagger\right)\right]\right.$$

$$+ h^E h^{E\dagger}\left[-\frac{6}{5}g_1^2 + \frac{6}{5}g_1'^2 + 6g_2^2 - 2|\tilde{\sigma}|^2 - 6\,\mathrm{Tr}\left(g^D g^{D\dagger}\right) - 2\,\mathrm{Tr}\left(h^E h^{E\dagger}\right)\right]$$

$$- 2h^E \tilde{f}^\dagger \tilde{f} h^{E\dagger} - 2h^E h^{E\dagger} h^E h^{E\dagger} - 2h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} - 2y^E y^{E\dagger} h^E h^{E\dagger}$$

$$\left. - 4y^E y^{E\dagger} y^E y^{E\dagger}\right\} y^E + y^E\left\{\frac{189}{10}g_1^4 + \frac{77}{10}g_1'^4 + \frac{33}{2}g_2^4 + \frac{3}{20}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2\right.$$

$$+ \frac{39}{20}g_1'^2 g_2^2 - 3|\lambda|^4 + \frac{2}{5}g_1^2\left[- \mathrm{Tr}\left(y^D y^{D\dagger}\right) + 3\,\mathrm{Tr}\left(y^E y^{E\dagger}\right)\right]$$

$$+ g_1'^2\left[-\frac{3}{5}\,\mathrm{Tr}\left(y^D y^{D\dagger}\right) - \frac{1}{5}\,\mathrm{Tr}\left(y^E y^{E\dagger}\right) + \mathrm{Tr}\left(f f^\dagger\right)\right] + 16g_3^2\,\mathrm{Tr}\left(y^D y^{D\dagger}\right)$$

$$+ |\lambda|^2\left[g_1'^2 - |\sigma|^2 - 2\,\mathrm{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) - 3\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right) - 3\,\mathrm{Tr}\left(y^U y^{U\dagger}\right) - \mathrm{Tr}\left(\tilde{f}\tilde{f}^\dagger\right)\right]$$

$$- 3\,\mathrm{Tr}\left(f f^\dagger f f^\dagger\right) - 2\,\mathrm{Tr}\left(f f^\dagger \tilde{f}\tilde{f}^\dagger\right) - 3\,\mathrm{Tr}\left(g^D g^{D\dagger} y^{DT} y^{D*}\right)$$

$$- 2\,\mathrm{Tr}\left(h^E h^{E\dagger} y^E y^{E\dagger}\right) - 9\,\mathrm{Tr}\left(y^D y^{D\dagger} y^D y^{D\dagger}\right) - 3\,\mathrm{Tr}\left(y^D y^{U\dagger} y^U y^{D\dagger}\right)$$

$$\left. - 3\,\mathrm{Tr}\left(y^E y^{E\dagger} y^E y^{E\dagger}\right) - \mathrm{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\right)\right\}, \tag{A.17}$$

$$\beta^{(1)}_{\lambda} = \lambda\left[4|\lambda|^2 + |\sigma|^2 - \frac{3}{5}g_1^2 - \frac{19}{10}g_1'^2 - 3g_2^2 + 3\,\mathrm{Tr}\left(y^U y^{U\dagger}\right) + 3\,\mathrm{Tr}\left(y^D y^{D\dagger}\right)\right.$$

$$\left. + \mathrm{Tr}\left(y^E y^{E\dagger}\right) + 2\,\mathrm{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) + 3\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right) + \mathrm{Tr}\left(f f^\dagger\right) + \mathrm{Tr}\left(\tilde{f}\tilde{f}^\dagger\right)\right], \tag{A.18}$$

$$\beta^{(2)}_{\lambda} = \lambda\left\{\frac{297}{50}g_1^4 + \frac{551}{25}g_1'^4 + \frac{33}{2}g_2^4 + \frac{27}{100}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{39}{20}g_1'^2 g_2^2 - 10|\lambda|^4\right.$$

$$- 2|\sigma|^2\left(|\kappa_\phi|^2 + |\sigma|^2 + |\tilde{\sigma}|^2\right) + \frac{2}{5}g_1^2\left[2\,\mathrm{Tr}\left(y^U y^{U\dagger}\right) - \mathrm{Tr}\left(y^D y^{D\dagger}\right)\right.$$

$$\left. + 3\,\mathrm{Tr}\left(y^E y^{E\dagger}\right) + 2\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right) + 3\,\mathrm{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right)\right] - g_1'^2\left[\frac{3}{10}\,\mathrm{Tr}\left(y^U y^{U\dagger}\right)\right.$$

$$+ \frac{3}{5}\,\mathrm{Tr}\left(y^D y^{D\dagger}\right) + \frac{1}{5}\,\mathrm{Tr}\left(y^E y^{E\dagger}\right) + \frac{6}{5}\,\mathrm{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) + \frac{9}{5}\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right)$$

$$\left. - \mathrm{Tr}\left(f f^\dagger\right) - \frac{3}{2}\,\mathrm{Tr}\left(\tilde{f}\tilde{f}^\dagger\right)\right] + 6g_2^2\,\mathrm{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) + 16g_3^2\left[\mathrm{Tr}\left(y^U y^{U\dagger}\right)\right.$$

$$
\begin{aligned}
&+\operatorname{Tr}\Big(y^D y^{D\dagger}\Big)+\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\Bigg]+|\lambda|^2\Bigg[\frac{6}{5}g_1^2+\frac{13}{10}g_1'^2+6g_2^2-2|\sigma|^2\\
&-9\operatorname{Tr}\Big(y^U y^{U\dagger}\Big)-9\operatorname{Tr}\Big(y^D y^{D\dagger}\Big)-3\operatorname{Tr}\Big(y^E y^{E\dagger}\Big)-6\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\\
&-4\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)-3\operatorname{Tr}\Big(ff^\dagger\Big)-3\operatorname{Tr}\Big(\tilde f\tilde f^\dagger\Big)\Bigg]-3\operatorname{Tr}\Big(ff^\dagger ff^\dagger\Big)\\
&-4\operatorname{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big)-3\operatorname{Tr}\Big(\tilde f\tilde f^\dagger\tilde f\tilde f^\dagger\Big)-\operatorname{Tr}\Big(\tilde f h^{E\dagger}h^E\tilde f^\dagger\Big)-3\operatorname{Tr}\Big(\tilde f\tilde\lambda^\dagger\tilde\lambda\tilde f^\dagger\Big)\\
&-3\operatorname{Tr}\Big(g^D g^{D\dagger}y^{DT}y^{D*}\Big)-3\operatorname{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big)-6\operatorname{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big)\\
&-2\operatorname{Tr}\Big(h^E h^{E\dagger}y^E y^{E\dagger}\Big)-2\operatorname{Tr}\Big(h^E\tilde\lambda^\dagger\tilde\lambda h^{E\dagger}\Big)-9\operatorname{Tr}\Big(y^D y^{D\dagger}y^D y^{D\dagger}\Big)\\
&-6\operatorname{Tr}\Big(y^D y^{U\dagger}y^U y^{D\dagger}\Big)-3\operatorname{Tr}\Big(y^E y^{E\dagger}y^E y^{E\dagger}\Big)-9\operatorname{Tr}\Big(y^U y^{U\dagger}y^U y^{U\dagger}\Big)\\
&-6\operatorname{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big)-4\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\tilde\lambda\tilde\lambda^\dagger\Big)-3\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger f^T f^*\Big)\Bigg\},
\end{aligned}
\tag{A.19}
$$

$$
\begin{aligned}
\beta_{\tilde\lambda}^{(1)}=\;&\tilde\lambda\Bigg[2|\lambda|^2+|\sigma|^2-\frac{3}{5}g_1^2-\frac{19}{10}g_1'^2-3g_2^2+2\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+3\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\Bigg]\\
&+\tilde\lambda\Big(\tilde f^\dagger\tilde f+h^{E\dagger}h^E+2\tilde\lambda^\dagger\tilde\lambda\Big)+f^T f^*\tilde\lambda,
\end{aligned}
\tag{A.20}
$$

$$
\begin{aligned}
\beta_{\tilde\lambda}^{(2)}=\;&\tilde\lambda\Bigg\{\tilde\lambda^\dagger\tilde\lambda\Bigg[\frac{5}{2}g_1'^2-4|\lambda|^2-2|\sigma|^2-4\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)-6\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\Bigg]\\
&+h^{E\dagger}h^E\Bigg[\frac{6}{5}g_1^2-\frac{1}{5}g_1'^2-|\tilde\sigma|^2-3\operatorname{Tr}\Big(g^D g^{D\dagger}\Big)-\operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg]\\
&+\tilde f^\dagger\tilde f\Bigg[g_1'^2-|\lambda|^2-3\operatorname{Tr}\Big(y^U y^{U\dagger}\Big)-\operatorname{Tr}\Big(\tilde f\tilde f^\dagger\Big)\Bigg]-2\tilde f^\dagger ff^\dagger\tilde f-2\tilde f^\dagger\tilde f\tilde f^\dagger\tilde f\\
&-\tilde f^\dagger\tilde f\tilde\lambda^\dagger\tilde\lambda-2h^{E\dagger}h^E h^{E\dagger}h^E-h^{E\dagger}h^E\tilde\lambda^\dagger\tilde\lambda-2h^{E\dagger}y^E y^{E\dagger}h^E-2\tilde\lambda^\dagger\tilde\lambda\tilde\lambda^\dagger\tilde\lambda\Bigg\}\\
&+\Bigg\{f^T f^*\Bigg[\frac{3}{2}g_1'^2-|\lambda|^2-3\operatorname{Tr}\Big(y^D y^{D\dagger}\Big)-\operatorname{Tr}\Big(y^E y^{E\dagger}\Big)-\operatorname{Tr}\Big(ff^\dagger\Big)\Bigg]\\
&-\tilde\lambda\tilde\lambda^\dagger f^T f^*-2f^T f^* f^T f^*-2f^T\tilde f^*\tilde f^T f^*\Bigg\}\tilde\lambda+\tilde\lambda\Bigg\{\frac{297}{50}g_1^4+\frac{551}{25}g_1'^4+\frac{33}{2}g_2^4\\
&+\frac{27}{100}g_1^2g_1'^2+\frac{9}{5}g_1^2g_2^2+\frac{39}{20}g_1'^2g_2^2-4|\lambda|^4-2|\sigma|^2\Big(|\kappa_\phi|^2+|\sigma|^2+|\tilde\sigma|^2\Big)\\
&+\frac{2}{5}g_1^2\Bigg[3\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\Bigg]-\frac{3}{5}g_1'^2\Bigg[2\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+3\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\Bigg]\\
&+6g_2^2\operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+16g_3^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)+|\lambda|^2\Bigg[\frac{6}{5}g_1^2-\frac{6}{5}g_1'^2+6g_2^2-6\operatorname{Tr}\Big(y^U y^{U\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 6\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(f f^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\bigg] \\
&- 2\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger\Big) - 6\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big) \\
&- 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big) - 4\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big)\bigg\},
\end{aligned} \tag{A.21}
$$

$$
\begin{aligned}
\beta_\kappa^{(1)} &= \kappa\bigg[2|\lambda|^2 + |\sigma|^2 - \frac{4}{15}g_1^2 - \frac{19}{10}g_1'^2 - \frac{16}{3}g_3^2 + 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] \\
&\quad 2\kappa\bigg(\kappa^\dagger\kappa + g^{D\dagger}g^D\bigg),
\end{aligned} \tag{A.22}
$$

$$
\begin{aligned}
\beta_\kappa^{(2)} &= \kappa\bigg\{\kappa^\dagger\kappa\bigg[\frac{5}{2}g_1'^2 - 4|\lambda|^2 - 2|\sigma|^2 - 4\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] \\
&\quad + g^{D\dagger}g^D\bigg[\frac{2}{5}g_1^2 - \frac{2}{5}g_1'^2 + 6g_2^2 - 2|\tilde{\sigma}|^2 - 6\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] \\
&\quad - 2g^{D\dagger}g^D g^{D\dagger}g^D - 2g^{D\dagger}g^D\kappa^\dagger\kappa - 2g^{D\dagger}y^{DT}y^{D*}g^D - 2g^{D\dagger}y^{UT}y^{U*}g^D \\
&\quad - 2\kappa^\dagger\kappa\kappa^\dagger\kappa\bigg\} + \kappa\bigg\{\frac{584}{225}g_1^4 + \frac{551}{25}g_1'^4 + \frac{128}{9}g_3^4 + \frac{19}{75}g_1^2 g_1'^2 + \frac{64}{45}g_1^2 g_3^2 \\
&\quad + \frac{52}{15}g_1'^2 g_3^2 - 4|\lambda|^4 - 2|\sigma|^2\bigg(|\kappa_\phi|^2 + |\sigma|^2 + |\tilde{\sigma}|^2\bigg) \\
&\quad + \frac{2}{5}g_1^2\bigg[3\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] - \frac{3}{5}g_1'^2\bigg[2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] \\
&\quad + 6g_2^2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 16g_3^2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + |\lambda|^2\bigg[\frac{6}{5}g_1^2 - \frac{6}{5}g_1'^2 + 6g_2^2 - 6\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&\quad - 6\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(f f^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\bigg] \\
&\quad - 2\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger\Big) - 6\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big) \\
&\quad - 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big) - 4\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big)\bigg\},
\end{aligned} \tag{A.23}
$$

$$
\beta_\sigma^{(1)} = \sigma\bigg[2|\kappa_\phi|^2 + 2|\lambda|^2 + 3|\sigma|^2 + 2|\tilde{\sigma}|^2 + 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - \frac{5}{2}g_1'^2\bigg], \tag{A.24}
$$

$$
\begin{aligned}
\beta_\sigma^{(2)} &= \sigma\bigg\{\frac{119}{4}g_1'^4 - 4|\lambda|^4 - 6|\sigma|^4 - 4|\tilde{\sigma}|^4 - 8|\kappa_\phi|^4 \\
&\quad + \frac{2}{5}g_1^2\bigg[3\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] - \frac{3}{5}g_1'^2\bigg[2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg]
\end{aligned}
$$

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$$
\begin{aligned}
&+ 6g_2^2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 16g_3^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + |\sigma|^2 \bigg[\frac{5}{2}g_1'^2 - 2|\tilde{\sigma}|^2 - 8|\kappa_\phi|^2 - 4\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 6\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] + |\tilde{\sigma}|^2\bigg[\frac{6}{5}g_1^2 + \frac{4}{5}g_1'^2 + 6g_2^2 - 2|\sigma|^2 - 8|\kappa_\phi|^2 - 6\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 2\operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + |\lambda|^2\bigg[\frac{6}{5}g_1^2 - \frac{6}{5}g_1'^2 + 6g_2^2 - 4|\sigma|^2 - 6\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 6\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2\operatorname{Tr}\Big(f f^\dagger\Big) - 2\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\bigg] \\
&- 2\operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger\Big) - 6\operatorname{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) - 2\operatorname{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big) - 6\operatorname{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big) \\
&- 4\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big)\bigg\},
\end{aligned}
\tag{A.25}
$$

$$
\beta_{\kappa_\phi}^{(1)} = 3\kappa_\phi\Big(2|\kappa_\phi|^2 + 2|\tilde{\sigma}|^2 + |\sigma|^2\Big),
\tag{A.26}
$$

$$
\begin{aligned}
\beta_{\kappa_\phi}^{(2)} = \kappa_\phi\bigg\{ &- 24|\kappa_\phi|^4 - 6|\sigma|^4 - 12|\tilde{\sigma}|^4 + |\sigma|^2\bigg[\frac{15}{2}g_1'^2 - 6|\lambda|^2 - 12|\kappa_\phi|^2 \\
&- 6\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 9\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] + |\tilde{\sigma}|^2\bigg[\frac{18}{5}g_1^2 + \frac{12}{5}g_1'^2 + 18g_2^2 - 24|\kappa_\phi|^2 \\
&- 18\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 6\operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\bigg]\bigg\},
\end{aligned}
\tag{A.27}
$$

$$
\begin{aligned}
\beta_{\tilde{\sigma}}^{(1)} = \tilde{\sigma}\bigg[&2|\kappa_\phi|^2 + |\sigma|^2 + 4|\tilde{\sigma}|^2 - \frac{3}{5}g_1^2 - \frac{2}{5}g_1'^2 - 3g_2^2 + 3\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\bigg],
\end{aligned}
\tag{A.28}
$$

$$
\begin{aligned}
\beta_{\tilde{\sigma}}^{(2)} = \tilde{\sigma}\bigg\{&\frac{297}{50}g_1^4 + \frac{217}{50}g_1'^4 + \frac{33}{2}g_2^4 + \frac{18}{25}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{6}{5}g_1'^2 g_2^2 - 8|\kappa_\phi|^4 - 2|\sigma|^4 \\
&- 10|\tilde{\sigma}|^4 + \frac{2}{5}g_1^2\bigg[-\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 3\operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + \frac{3}{10}g_1'^2\bigg[3\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + 16g_3^2\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + |\tilde{\sigma}|^2\bigg[\frac{6}{5}g_1^2 + \frac{4}{5}g_1'^2 + 6g_2^2 - 12|\kappa_\phi|^2 \\
&- 9\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 3\operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + |\sigma|^2\bigg[\frac{5}{2}g_1'^2 - 2|\lambda|^2 - 4|\kappa_\phi|^2 - 2|\tilde{\sigma}|^2 \\
&- 2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 3\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\bigg] - \operatorname{Tr}\Big(\tilde{f}h^{E\dagger}h^E\tilde{f}^\dagger\Big) - 9\operatorname{Tr}\Big(g^D g^{D\dagger} g^D g^{D\dagger}\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&- 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 3\,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) \\
&- 3\,\mathrm{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) - \mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big)\Bigg\},
\end{aligned} \tag{A.29}
$$

$$
\begin{aligned}
\beta_{g^D}^{(1)} = {}& g^D \left[|\tilde{\sigma}|^2 + 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - \frac{7}{15} g_1^2 - \frac{7}{10} g_1'^2 - 3 g_2^2 - \frac{16}{3} g_3^2\right] \\
&+ g^D \left(3 g^{D\dagger} g^D + \kappa^\dagger \kappa\right) + \left(y^{DT} y^{D*} + y^{UT} y^{U*}\right) g^D,
\end{aligned} \tag{A.30}
$$

$$
\begin{aligned}
\beta_{g^D}^{(2)} = {}& g^D \Bigg\{\kappa^\dagger \kappa \left[g_1'^2 - 2|\lambda|^2 - |\sigma|^2 - 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\right] \\
&+ g^{D\dagger} g^D \left[\frac{4}{5} g_1^2 + \frac{1}{5} g_1'^2 + 6 g_2^2 - 3|\tilde{\sigma}|^2 - 9\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\right] \\
&- 4 g^{D\dagger} g^D g^{D\dagger} g^D - \kappa^\dagger \kappa g^{D\dagger} g^D - \kappa^\dagger \kappa \kappa^\dagger \kappa\Bigg\} + \Bigg\{y^{DT} y^{D*} \bigg[\frac{2}{5} g_1^2 + \frac{3}{5} g_1'^2 - |\lambda|^2 \\
&- 3\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - \mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - \mathrm{Tr}\Big(f f^\dagger\Big)\bigg] + y^{UT} y^{U*} \bigg[\frac{4}{5} g_1^2 + \frac{1}{5} g_1'^2 - |\lambda|^2 \\
&- 3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - \mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\bigg] - 2 g^D g^{D\dagger} y^{DT} y^{D*} - 2 g^D g^{D\dagger} y^{UT} y^{U*} \\
&- 2 y^{DT} y^{D*} y^{DT} y^{D*} - 2 y^{UT} y^{U*} y^{UT} y^{U*}\Bigg\} g^D + g^D \Bigg\{\frac{413}{90} g_1^4 + \frac{77}{10} g_1'^4 + \frac{33}{2} g_2^4 \\
&+ \frac{128}{9} g_3^4 + \frac{41}{60} g_1^2 g_1'^2 + g_1^2 g_2^2 + \frac{8}{9} g_1^2 g_3^2 + \frac{3}{4} g_1'^2 g_2^2 + \frac{8}{3} g_1'^2 g_3^2 + 8 g_2^2 g_3^2 \\
&- |\tilde{\sigma}|^2 \left(2|\kappa_\phi|^2 + |\sigma|^2 + 3|\tilde{\sigma}|^2\right) + \frac{2}{5} g_1^2 \left[-\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + 3\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\right] \\
&+ \frac{3}{10} g_1'^2 \left[3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\right] + 16 g_3^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- \mathrm{Tr}\Big(\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger\Big) - 9\,\mathrm{Tr}\Big(g^D g^{D\dagger} g^D g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) \\
&- 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 3\,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger}\Big) \\
&- 2\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) - \mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big)\Bigg\},
\end{aligned} \tag{A.31}
$$

$$
\begin{aligned}
\beta_{h^E}^{(1)} = {}& h^E \left[|\tilde{\sigma}|^2 + 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - \frac{9}{5} g_1^2 - \frac{7}{10} g_1'^2 - 3 g_2^2\right] \\
&+ h^E \left(\tilde{f}^\dagger \tilde{f} + 3 h^{E\dagger} h^E + \tilde{\lambda}^\dagger \tilde{\lambda}\right) + 2 y^E y^{E\dagger} h^E,
\end{aligned} \tag{A.32}
$$

$$\beta^{(2)}_{h^E} = h^E\Big\{h^{E\dagger}h^E\Big[g_1'^2 + 6g_2^2 - 3|\tilde{\sigma}|^2 - 9\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Big]$$
$$+ \tilde{\lambda}^\dagger\tilde{\lambda}\Big[g_1'^2 - 2|\lambda|^2 - |\sigma|^2 - 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\Big]$$
$$+ \tilde{f}^\dagger\tilde{f}\Big[g_1'^2 - |\lambda|^2 - 3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - \mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\Big] - 2\tilde{f}^\dagger f f^\dagger \tilde{f} - 2\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger\tilde{f}$$
$$- 2\tilde{f}^\dagger\tilde{f}h^{E\dagger}h^E - 4h^{E\dagger}h^E h^{E\dagger}h^E - 2\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}h^E - \tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda} - \tilde{\lambda}^\dagger f^T f^*\tilde{\lambda}\Big\}$$
$$+ \Big\{y^E y^{E\dagger}\Big[-\frac{6}{5}g_1^2 + \frac{6}{5}g_1'^2 + 6g_2^2 - 2|\lambda|^2 - 6\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)$$
$$- 2\,\mathrm{Tr}\Big(ff^\dagger\Big)\Big] - 2h^E h^{E\dagger}y^E y^{E\dagger} - 2y^E y^{E\dagger}y^E y^{E\dagger}\Big\}h^E + h^E\Big\{\frac{189}{10}g_1^4 + \frac{77}{10}g_1'^4$$
$$+ \frac{33}{2}g_2^4 + \frac{3}{20}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{39}{20}g_1'^2 g_2^2 - |\tilde{\sigma}|^2\Big(2|\kappa_\phi|^2 + 2|\sigma|^2 + 3|\tilde{\sigma}|^2\Big)$$
$$+ \frac{2}{5}g_1^2\Big[3\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - \mathrm{Tr}\Big(g^D g^{D\dagger}\Big)\Big] + \frac{3}{10}g_1'^2\Big[3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Big]$$
$$+ 16g_3^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \mathrm{Tr}\Big(\tilde{f}h^{E\dagger}h^E\tilde{f}^\dagger\Big) - 9\,\mathrm{Tr}\Big(g^D g^{D\dagger}g^D g^{D\dagger}\Big)$$
$$- 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{DT}y^{D*}\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big) - 3\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big)$$
$$- 3\,\mathrm{Tr}\Big(h^E h^{E\dagger}h^E h^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger}y^E y^{E\dagger}\Big) - \mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big)\Big\}, \quad \text{(A.33)}$$

$$\beta^{(1)}_{\tilde{f}} = \tilde{f}\Big[|\lambda|^2 + 3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) + \mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - \frac{3}{5}g_1^2 - \frac{19}{10}g_1'^2 - 3g_2^2\Big]$$
$$+ \Big(2ff^\dagger + 3\tilde{f}\tilde{f}^\dagger\Big)\tilde{f} + \tilde{f}\Big(h^{E\dagger}h^E + \tilde{\lambda}^\dagger\tilde{\lambda}\Big), \quad \text{(A.34)}$$

$$\beta^{(2)}_{\tilde{f}} = \Big\{\tilde{f}\tilde{f}^\dagger\Big[\frac{6}{5}g_1^2 - \frac{1}{5}g_1'^2 + 6g_2^2 - 3|\lambda|^2 - 9\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 3\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\Big]$$
$$+ ff^\dagger\Big[\frac{6}{5}g_1^2 - \frac{6}{5}g_1'^2 + 6g_2^2 - 2|\lambda|^2 - 6\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)$$
$$- 2\,\mathrm{Tr}\Big(ff^\dagger\Big)\Big] - 2ff^\dagger ff^\dagger - 2f\tilde{\lambda}^*\tilde{\lambda}^T f^\dagger - 2\tilde{f}\tilde{f}^\dagger ff^\dagger - 4\tilde{f}\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger\Big\}\tilde{f}$$
$$+ \tilde{f}\Big\{h^{E\dagger}h^E\Big[\frac{6}{5}g_1^2 - \frac{1}{5}g_1'^2 - |\tilde{\sigma}|^2 - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Big]$$
$$+ \tilde{\lambda}^\dagger\tilde{\lambda}\Big[g_1'^2 - 2|\lambda|^2 - |\sigma|^2 - 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\Big] - 2h^{E\dagger}h^E\tilde{f}^\dagger\tilde{f}$$

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$$- 2h^{E\dagger}h^{E}h^{E\dagger}h^{E} - 2h^{E\dagger}y^{E}y^{E\dagger}h^{E} - 2\tilde{\lambda}^{\dagger}\tilde{\lambda}\tilde{f}^{\dagger}\tilde{f} - \tilde{\lambda}^{\dagger}\tilde{\lambda}\tilde{\lambda}^{\dagger}\tilde{\lambda} - \tilde{\lambda}^{\dagger}f^{T}f^{*}\tilde{\lambda}\bigg\}$$

$$+ \tilde{f}\bigg\{\frac{297}{50}g_1^4 + \frac{551}{25}g_1'^4 + \frac{33}{2}g_2^4 + \frac{27}{100}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{39}{20}g_1'^2 g_2^2 - 3|\lambda|^4$$

$$+ \frac{4}{5}g_1^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) + \frac{3}{2}g_1'^2\bigg[-\frac{1}{5}\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) + \operatorname{Tr}\Big(\tilde{f}\tilde{f}^{\dagger}\Big)\bigg] + 16g_3^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big)$$

$$+ |\lambda|^2\bigg[\frac{3}{2}g_1'^2 - |\sigma|^2 - 3\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)$$

$$- 3\operatorname{Tr}\Big(\kappa\kappa^{\dagger}\Big) - \operatorname{Tr}\Big(ff^{\dagger}\Big)\bigg] - 2\operatorname{Tr}\Big(ff^{\dagger}\tilde{f}\tilde{f}^{\dagger}\Big) - 3\operatorname{Tr}\Big(\tilde{f}\tilde{f}^{\dagger}\tilde{f}\tilde{f}^{\dagger}\Big)$$

$$- \operatorname{Tr}\Big(\tilde{f}h^{E\dagger}h^{E}\tilde{f}^{\dagger}\Big) - \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^{\dagger}\tilde{\lambda}\tilde{f}^{\dagger}\Big) - 3\operatorname{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big)$$

$$- 3\operatorname{Tr}\Big(y^D y^{U\dagger}y^U y^{D\dagger}\Big) - 9\operatorname{Tr}\Big(y^U y^{U\dagger}y^U y^{U\dagger}\Big)\bigg\}, \tag{A.35}$$

$$\beta_f^{(1)} = f\bigg[|\lambda|^2 + 3\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) + \operatorname{Tr}\Big(ff^{\dagger}\Big) - \frac{3}{5}g_1^2 - \frac{19}{10}g_1'^2 - 3g_2^2\bigg]$$

$$+ \Big(3ff^{\dagger} + 2\tilde{f}\tilde{f}^{\dagger}\Big)f + f\tilde{\lambda}^{*}\tilde{\lambda}^{T}, \tag{A.36}$$

$$\beta_f^{(2)} = f\bigg\{\tilde{\lambda}^{*}\tilde{\lambda}^{T}\bigg[\frac{3}{2}g_1'^2 - 2|\lambda|^2 - |\sigma|^2 - 2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big) - 3\operatorname{Tr}\Big(\kappa\kappa^{\dagger}\Big)\bigg]$$

$$- \tilde{\lambda}^{*}\tilde{f}^{T}\tilde{f}^{*}\tilde{\lambda}^{T} - \tilde{\lambda}^{*}h^{ET}h^{E*}\tilde{\lambda}^{T} - 2\tilde{\lambda}^{*}\tilde{\lambda}^{T}f^{\dagger}f - \tilde{\lambda}^{*}\tilde{\lambda}^{T}\tilde{\lambda}^{*}\tilde{\lambda}^{T}\bigg\}$$

$$+ \bigg\{\tilde{f}\tilde{f}^{\dagger}\bigg[\frac{6}{5}g_1^2 - \frac{6}{5}g_1'^2 + 6g_2^2 - 2|\lambda|^2 - 6\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 2\operatorname{Tr}\Big(\tilde{f}\tilde{f}^{\dagger}\Big)\bigg]$$

$$+ ff^{\dagger}\bigg[\frac{6}{5}g_1^2 + \frac{3}{10}g_1'^2 + 6g_2^2 - 3|\lambda|^2 - 9\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 3\operatorname{Tr}\Big(y^E y^{E\dagger}\Big)$$

$$- 3\operatorname{Tr}\Big(ff^{\dagger}\Big)\bigg] - 4ff^{\dagger}ff^{\dagger} - 2ff^{\dagger}\tilde{f}\tilde{f}^{\dagger} - 2\tilde{f}\tilde{f}^{\dagger}\tilde{f}\tilde{f}^{\dagger} - 2\tilde{f}h^{E\dagger}h^{E}\tilde{f}^{\dagger}$$

$$- 2\tilde{f}\tilde{\lambda}^{\dagger}\tilde{\lambda}\tilde{f}^{\dagger}\bigg\}f + f\bigg\{\frac{297}{50}g_1^4 + \frac{551}{25}g_1'^4 + \frac{33}{2}g_2^4 + \frac{27}{100}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{39}{20}g_1'^2 g_2^2$$

$$- 3|\lambda|^4 + \frac{2}{5}g_1^2\bigg[-\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + 3\operatorname{Tr}\Big(y^E y^{E\dagger}\Big)\bigg] + g_1'^2\bigg[-\frac{3}{5}\operatorname{Tr}\Big(y^D y^{D\dagger}\Big)$$

$$- \frac{1}{5}\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) + \operatorname{Tr}\Big(ff^{\dagger}\Big)\bigg] + 16g_3^2\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + |\lambda|^2\bigg[g_1'^2 - |\sigma|^2$$

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$$- 3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - \mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)\Bigg] - 3\,\mathrm{Tr}\Big(ff^\dagger ff^\dagger\Big)$$

$$- 2\,\mathrm{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big)$$

$$- 9\,\mathrm{Tr}\Big(y^D y^{D\dagger} y^D y^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(y^E y^{E\dagger} y^E y^{E\dagger}\Big)$$

$$- \mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger f^T f^*\Big)\Big\}. \tag{A.37}$$

A.3 Superpotential bilinear and linear couplings

The β functions of the bilinear superpotential parameters μ_ϕ and μ_L read

$$\beta^{(1)}_{\mu_\phi} = 2\mu_\phi\Bigg(2|\kappa_\phi|^2 + 2|\tilde\sigma|^2 + |\sigma|^2\Bigg), \tag{A.38}$$

$$\beta^{(2)}_{\mu_\phi} = \mu_\phi\Bigg\{ - 16|\kappa_\phi|^4 - 8|\tilde\sigma|^4 - 4|\sigma|^4 + |\tilde\sigma|^2\Bigg[\frac{12}{5}g_1^2 + \frac{8}{5}g_1'^2 + 12g_2^2 - 16|\kappa_\phi|^2$$

$$- 12\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 4\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg] + |\sigma|^2\Bigg[5g_1'^2 - 4|\lambda|^2 - 8|\kappa_\phi|^2$$

$$- 4\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\Bigg]\Bigg\}, \tag{A.39}$$

$$\beta^{(1)}_{\mu_L} = \mu_L\Bigg[2|\tilde\sigma|^2 + 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - \frac{3}{5}g_1^2 - \frac{2}{5}g_1'^2 - 3g_2^2\Bigg], \tag{A.40}$$

$$\beta^{(2)}_{\mu_L} = \mu_L\Bigg\{\frac{297}{50}g_1^4 + \frac{217}{50}g_1'^4 + \frac{33}{2}g_2^4 + \frac{18}{25}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{6}{5}g_1'^2 g_2^2$$

$$- 2|\tilde\sigma|^2\Bigg(2|\kappa_\phi|^2 + 3|\tilde\sigma|^2 + |\sigma|^2\Bigg) + \frac{2}{5}g_1^2\Bigg[- \mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + 3\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg]$$

$$+ \Bigg(\frac{3}{10}g_1'^2 - |\tilde\sigma|^2\Bigg)\Bigg[3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg] + 16g_3^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big)$$

$$- \mathrm{Tr}\Big(\tilde f h^{E\dagger} h^E \tilde f^\dagger\Big) - 9\,\mathrm{Tr}\Big(g^D g^{D\dagger} g^D g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big)$$

$$- 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 3\,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger}\Big)$$

$$- 2\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) - \mathrm{Tr}\Big(h^E \tilde\lambda^\dagger \tilde\lambda h^{E\dagger}\Big)\Bigg\}, \tag{A.41}$$

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while that for the linear superpotential parameter Λ_F is

$$\beta^{(1)}_{\Lambda_F} = \Lambda_F \Big(2|\kappa_\phi|^2 + 2|\tilde{\sigma}|^2 + |\sigma|^2 \Big), \tag{A.42}$$

$$\begin{aligned}\beta^{(2)}_{\Lambda_F} = \Lambda_F \Bigg\{ &- 8|\kappa_\phi|^4 - 4|\tilde{\sigma}|^2 - 2|\sigma|^4 + |\tilde{\sigma}|^2 \Bigg[\frac{6}{5} g_1^2 + \frac{4}{5} g_1'^2 + 6 g_2^2 - 8|\kappa_\phi|^2 \\ &- 6\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \Bigg] + |\sigma|^2 \Bigg[\frac{5}{2} g_1'^2 - 2|\lambda|^2 - 4|\kappa_\phi|^2 \\ &- 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \Bigg] \Bigg\}. \end{aligned} \tag{A.43}$$

A.4 Gaugino masses

The two-loop β functions for the soft gaugino masses are

$$\beta^{(1)}_{M_1} = \frac{96}{5} g_1^2 M_1, \tag{A.44}$$

$$\begin{aligned}\beta^{(2)}_{M_1} = g_1^2 \Bigg[&\frac{936}{25} g_1^2 M_1 + \frac{162}{25} g_1'^2 (M_1 + M_1') + \frac{108}{5} g_2^2 (M_1 + M_2) + 48 g_3^2 (M_1 + M_3) \\ &- \frac{52}{5}\,\mathrm{Tr}\Big(M_1 y^U y^{U\dagger} - y^{U\dagger} T^U\Big) - \frac{28}{5}\,\mathrm{Tr}\Big(M_1 y^D y^{D\dagger} - y^{D\dagger} T^D\Big) \\ &- \frac{36}{5}\,\mathrm{Tr}\Big(M_1 y^E y^{E\dagger} - y^{E\dagger} T^E\Big) - \frac{12}{5} \lambda^* \Big(M_1 \lambda - T_\lambda\Big) \\ &- \frac{12}{5}\,\mathrm{Tr}\Big(M_1 \tilde{\lambda}\tilde{\lambda}^\dagger - \tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - \frac{8}{5}\,\mathrm{Tr}\Big(M_1 \kappa\kappa^\dagger - \kappa^\dagger T^\kappa\Big) - \frac{12}{5} \tilde{\sigma}^* \Big(M_1 \tilde{\sigma} - T_{\tilde{\sigma}}\Big) \\ &- \frac{12}{5}\,\mathrm{Tr}\Big(M_1 f f^\dagger - f^\dagger T^f\Big) - \frac{12}{5}\,\mathrm{Tr}\Big(M_1 \tilde{f}\tilde{f}^\dagger - \tilde{f}^\dagger T^{\tilde{f}}\Big) \\ &- \frac{28}{5}\,\mathrm{Tr}\Big(M_1 g^D g^{D\dagger} - g^{D\dagger} T^{g^D}\Big) - \frac{36}{5}\,\mathrm{Tr}\Big(M_1 h^E h^{E\dagger} - h^{E\dagger} T^{h^E}\Big) \Bigg], \end{aligned} \tag{A.45}$$

$$\beta^{(1)}_{M_2} = 8 g_2^2 M_2, \tag{A.46}$$

$$\begin{aligned}\beta^{(2)}_{M_2} = g_2^2 \Bigg[&\frac{36}{5} g_1^2 (M_1 + M_2) + \frac{34}{5} g_1'^2 (M_1' + M_2) + \frac{184}{5} g_2^2 M_2 + 48 g_3^2 (M_2 + M_3) \\ &- 12\,\mathrm{Tr}\Big(M_2 y^U y^{U\dagger} - y^{U\dagger} T^U\Big) - 12\,\mathrm{Tr}\Big(M_2 y^D y^{D\dagger} - y^{D\dagger} T^D\Big) \\ &- 4\,\mathrm{Tr}\Big(M_2 y^E y^{E\dagger} - y^{E\dagger} T^E\Big) - 4\lambda^* \Big(M_2 \lambda - T_\lambda\Big) - 4\,\mathrm{Tr}\Big(M_2 \tilde{\lambda}\tilde{\lambda}^\dagger - \tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) \\ &- 4\tilde{\sigma}^* \Big(M_2 \tilde{\sigma} - T_{\tilde{\sigma}}\Big) - 4\,\mathrm{Tr}\Big(M_2 f f^\dagger - f^\dagger T^f\Big) - 4\,\mathrm{Tr}\Big(M_2 \tilde{f}\tilde{f}^\dagger - \tilde{f}^\dagger T^{\tilde{f}}\Big) \\ &- 12\,\mathrm{Tr}\Big(M_2 g^D g^{D\dagger} - g^{D\dagger} T^{g^D}\Big) - 4\,\mathrm{Tr}\Big(M_2 h^E h^{E\dagger} - h^{E\dagger} T^{h^E}\Big) \Bigg], \end{aligned} \tag{A.47}$$

$$\beta^{(1)}_{M_3} = 0, \tag{A.48}$$

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$$\beta^{(2)}_{M_3} = g_3^2\Big[6g_1^2(M_1+M_3)+6g_1'^2(M_1'+M_3)+18g_2^2(M_2+M_3)+192g_3^2M_3$$
$$-8\,\mathrm{Tr}\Big(M_3y^Uy^{U\dagger}-y^{U\dagger}T^U\Big)-8\,\mathrm{Tr}\Big(M_3y^Dy^{D\dagger}-y^{D\dagger}T^D\Big)$$
$$-4\,\mathrm{Tr}\Big(M_3\kappa\kappa^\dagger-\kappa^\dagger T^\kappa\Big)-8\,\mathrm{Tr}\Big(M_3g^Dg^{D\dagger}-g^{D\dagger}T^{g^D}\Big)\Big], \quad \text{(A.49)}$$

$$\beta^{(1)}_{M_1'} = \frac{213}{10}g_1'^2M_1', \quad \text{(A.50)}$$

$$\beta^{(2)}_{M_1'} = g_1'^2\Big[\frac{162}{25}g_1^2(M_1+M_1')+\frac{2457}{50}g_1'^2M_1'+\frac{102}{5}g_2^2(M_1'+M_2)+48g_3^2(M_1'+M_3)$$
$$-\frac{18}{5}\,\mathrm{Tr}\Big(M_1'y^Uy^{U\dagger}-y^{U\dagger}T^U\Big)-\frac{42}{5}\,\mathrm{Tr}\Big(M_1'y^Dy^{D\dagger}-y^{D\dagger}T^D\Big)$$
$$-\frac{14}{5}\,\mathrm{Tr}\Big(M_1'y^Ey^{E\dagger}-y^{E\dagger}T^E\Big)-\frac{38}{5}\lambda^*\Big(M_1'\lambda-T_\lambda\Big)$$
$$-\frac{38}{5}\,\mathrm{Tr}\Big(M_1'\tilde\lambda\tilde\lambda^\dagger-\tilde\lambda^\dagger T^{\tilde\lambda}\Big)-\frac{57}{5}\,\mathrm{Tr}\Big(M_1'\kappa\kappa^\dagger-\kappa^\dagger T^\kappa\Big)-\frac{8}{5}\tilde\sigma^*\Big(M_1'\tilde\sigma-T_{\tilde\sigma}\Big)$$
$$-5\sigma^*\Big(M_1'\sigma-T_\sigma\Big)-\frac{38}{5}\,\mathrm{Tr}\Big(M_1'ff^\dagger-f^\dagger T^f\Big)-\frac{38}{5}\,\mathrm{Tr}\Big(M_1'\tilde f\tilde f^\dagger-\tilde f^\dagger T^{\tilde f}\Big)$$
$$-\frac{42}{5}\,\mathrm{Tr}\Big(M_1'g^Dg^{D\dagger}-g^{D\dagger}T^{g^D}\Big)-\frac{14}{5}\,\mathrm{Tr}\Big(M_1'h^Eh^{E\dagger}-h^{E\dagger}T^{h^E}\Big)\Big]. \quad \text{(A.51)}$$

As mentioned above, kinetic mixing in this class of E_6 inspired models is small and so we neglect the mixed gaugino mass M_{11}.

A.5 Soft-breaking trilinear scalar couplings

The two-loop RGEs for the soft scalar trilinear couplings read

$$\beta^{(1)}_{T^D} = 4y^Dy^{D\dagger}T^D+2y^Dy^{U\dagger}T^U+2y^Dg^{D*}T^{g^DT}+5T^Dy^{D\dagger}y^D+T^Dy^{U\dagger}y^U$$
$$+T^Dg^{D*}g^{DT}-\frac{7}{15}g_1^2T^D-\frac{7}{10}g_1'^2T^D-3g_2^2T^D-\frac{16}{3}g_3^2T^D+|\lambda|^2T^D$$
$$+T^D\,\mathrm{Tr}\Big(ff^\dagger\Big)+3T^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)+T^D\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)+y^D\Big[2\lambda^*T_\lambda$$
$$+2\,\mathrm{Tr}\Big(f^\dagger T^f\Big)+2\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)+6g_2^2M_2+6\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)+\frac{14}{15}g_1^2M_1$$
$$+\frac{32}{3}g_3^2M_3+\frac{7}{5}g_1'^2M_1'\Big], \quad \text{(A.52)}$$

$$\beta^{(2)}_{T^D} = \frac{6}{5}g_1^2y^Dy^{D\dagger}T^D+\frac{9}{5}g_1'^2y^Dy^{D\dagger}T^D+6g_2^2y^Dy^{D\dagger}T^D-4|\lambda|^2y^Dy^{D\dagger}T^D$$
$$-\frac{8}{5}g_1^2M_1y^Dy^{U\dagger}y^U-\frac{2}{5}g_1'^2M_1'y^Dy^{U\dagger}y^U+\frac{8}{5}g_1^2y^Dy^{U\dagger}T^U+\frac{2}{5}g_1'^2y^Dy^{U\dagger}T^U$$
$$-2|\lambda|^2y^Dy^{U\dagger}T^U-\frac{4}{5}g_1^2M_1y^Dg^{D*}g^{DT}-\frac{6}{5}g_1'^2M_1'y^Dg^{D*}g^{DT}$$
$$+\frac{4}{5}g_1^2y^Dg^{D*}T^{g^DT}+\frac{6}{5}g_1'^2y^Dg^{D*}T^{g^DT}-2|\tilde\sigma|^2y^Dg^{D*}T^{g^DT}+\frac{6}{5}g_1^2T^Dy^{D\dagger}y^D$$

$$
\begin{aligned}
&+\frac{9}{5}g_1'^2T^Dy^{D\dagger}y^D+12g_2^2T^Dy^{D\dagger}y^D-5|\lambda|^2T^Dy^{D\dagger}y^D+\frac{4}{5}g_1^2T^Dy^{U\dagger}y^U\\
&+\frac{1}{5}g_1'^2T^Dy^{U\dagger}y^U-|\lambda|^2T^Dy^{U\dagger}y^U+\frac{2}{5}g_1^2T^Dg^{D*}g^{DT}+\frac{3}{5}g_1'^2T^Dg^{D*}g^{DT}\\
&-|\tilde{\sigma}|^2T^Dg^{D*}g^{DT}-6y^Dy^{D\dagger}y^Dy^{D\dagger}T^D-8y^Dy^{D\dagger}T^Dy^{D\dagger}y^D\\
&-2y^Dy^{U\dagger}y^Uy^{D\dagger}T^D-4y^Dy^{U\dagger}y^Uy^{U\dagger}T^U-4y^Dy^{U\dagger}T^Uy^{D\dagger}y^D\\
&-4y^Dy^{U\dagger}T^Uy^{U\dagger}y^U-2y^Dg^{D*}g^{DT}y^{D\dagger}T^D-4y^Dg^{D*}g^{DT}g^{D*}T^{g^DT}\\
&-2y^Dg^{D*}\kappa^T\kappa^*T^{g^DT}-4y^Dg^{D*}T^{g^DT}y^{D\dagger}y^D-4y^Dg^{D*}T^{g^DT}g^{D*}g^{DT}\\
&-2y^Dg^{D*}T^{\kappa T}\kappa^*g^{DT}-6T^Dy^{D\dagger}y^Dy^{D\dagger}y^D-4T^Dy^{U\dagger}y^Uy^{D\dagger}y^D\\
&-2T^Dy^{U\dagger}y^Uy^{U\dagger}y^U-4T^Dg^{D*}g^{DT}y^{D\dagger}y^D-2T^Dg^{D*}g^{DT}g^{D*}g^{DT}\\
&-T^Dg^{D*}\kappa^T\kappa^*g^{DT}+\frac{413}{90}g_1^4T^D-\frac{7}{30}g_1^2g_1'^2T^D+\frac{77}{10}g_1'^4T^D+g_1^2g_2^2T^D\\
&+\frac{3}{2}g_1'^2g_2^2T^D+\frac{33}{2}g_2^4T^D+\frac{8}{9}g_1^2g_3^2T^D+\frac{4}{3}g_1'^2g_3^2T^D+8g_2^2g_3^2T^D+\frac{128}{9}g_3^4T^D\\
&+g_1'^2|\lambda|^2T^D-3|\lambda|^4T^D-|\sigma|^2|\lambda|^2T^D-2\lambda^*y^Dy^{U\dagger}y^UT_\lambda\\
&-2\tilde{\sigma}^*y^Dg^{D*}g^{DT}T_{\tilde{\sigma}}-4y^Dy^{D\dagger}T^D\,\mathrm{Tr}\Big(ff^\dagger\Big)-5T^Dy^{D\dagger}y^D\,\mathrm{Tr}\Big(ff^\dagger\Big)\\
&+g_1'^2T^D\,\mathrm{Tr}\Big(ff^\dagger\Big)-2y^Dy^{U\dagger}T^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-T^Dy^{U\dagger}y^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\\
&-|\lambda|^2T^D\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-6y^Dg^{D*}T^{g^DT}\,\mathrm{Tr}\Big(g^Dg^{D\dagger}\Big)-3T^Dg^{D*}g^{DT}\,\mathrm{Tr}\Big(g^Dg^{D\dagger}\Big)\\
&-2y^Dg^{D*}T^{g^DT}\,\mathrm{Tr}\Big(h^Eh^{E\dagger}\Big)-T^Dg^{D*}g^{DT}\,\mathrm{Tr}\Big(h^Eh^{E\dagger}\Big)\\
&-12y^Dy^{D\dagger}T^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-15T^Dy^{D\dagger}y^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-\frac{2}{5}g_1^2T^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&-\frac{3}{5}g_1'^2T^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)+16g_3^2T^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-4y^Dy^{D\dagger}T^D\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)\\
&-5T^Dy^{D\dagger}y^D\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)+\frac{6}{5}g_1^2T^D\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-\frac{1}{5}g_1'^2T^D\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)\\
&-6y^Dy^{U\dagger}T^U\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-3T^Dy^{U\dagger}y^U\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-3|\lambda|^2T^D\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)\\
&-3|\lambda|^2T^D\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-2|\lambda|^2T^D\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big)-2y^Dy^{U\dagger}y^U\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big)\\
&-6y^Dg^{D*}g^{DT}\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big)-2y^Dg^{D*}g^{DT}\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big)\\
&-\frac{2}{5}y^Dy^{D\dagger}y^D\Big[15\lambda^*T_\lambda+15\,\mathrm{Tr}\Big(f^\dagger T^f\Big)+15\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)+30g_2^2M_2\\
&+45\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)+4g_1^2M_1+6g_1'^2M_1'\Big]-6y^Dy^{U\dagger}y^U\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)\\
&-3T^D\,\mathrm{Tr}\Big(ff^\dagger ff^\dagger\Big)-2T^D\,\mathrm{Tr}\Big(ff^\dagger\tilde{f}\tilde{f}^\dagger\Big)-3T^D\,\mathrm{Tr}\Big(g^Dg^{D\dagger}y^{DT}y^{D*}\Big)\\
&-2T^D\,\mathrm{Tr}\Big(h^Eh^{E\dagger}y^Ey^{E\dagger}\Big)-9T^D\,\mathrm{Tr}\Big(y^Dy^{D\dagger}y^Dy^{D\dagger}\Big)-3T^D\,\mathrm{Tr}\Big(y^Dy^{U\dagger}y^Uy^{D\dagger}\Big)\\
&-3T^D\,\mathrm{Tr}\Big(y^Ey^{E\dagger}y^Ey^{E\dagger}\Big)-T^D\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^Tf^*\Big)-\frac{1}{45}y^D\Big(826g_1^4M_1-21g_1^2g_1'^2M_1\\
&+90g_1^2g_2^2M_1+80g_1^2g_3^2M_1-21g_1^2g_1'^2M_1'+1386g_1'^4M_1'+135g_1'^2g_2^2M_1'\\
&+120g_1'^2g_3^2M_1'+80g_1^2g_3^2M_3+120g_1'^2g_3^2M_3+720g_2^2g_3^2M_3+2560g_3^4M_3
\end{aligned}
$$

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$$
\begin{aligned}
&+ 90g_1^2 g_2^2 M_2 + 135g_1'^2 g_2^2 M_2 + 2970g_2^4 M_2 + 720g_2^2 g_3^2 M_2 + 540\lambda^* |\lambda|^2 T_\lambda \\
&+ 90g_1'^2 M_1' \operatorname{Tr}\Big(f f^\dagger\Big) - 36g_1^2 M_1 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 54g_1'^2 M_1' \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&+ 1440g_3^2 M_3 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + 108g_1^2 M_1 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 18g_1'^2 M_1' \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 90g_1'^2 \operatorname{Tr}\Big(f^\dagger T^f\Big) + 36g_1^2 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) + 54g_1'^2 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) \\
&- 1440g_3^2 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) - 108g_1^2 \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) + 18g_1'^2 \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) \\
&+ 90\lambda^* \Big\{ T_\lambda \Big[2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 3\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 3\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - g_1'^2 + |\sigma|^2 \\
&+ \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\Big] + \lambda\Big[2\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 3\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 3\operatorname{Tr}\Big(y^{U\dagger} T^U\Big) + g_1'^2 M_1' \\
&+ \sigma^* T_\sigma + \operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big)\Big]\Big\} + 540\operatorname{Tr}\Big(f f^\dagger T^f f^\dagger\Big) + 180\operatorname{Tr}\Big(f f^\dagger T^{\tilde{f}} \tilde{f}^\dagger\Big) \\
&+ 180\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^f f^\dagger\Big) + 180\operatorname{Tr}\Big(h^E h^{E\dagger} T^E y^{E\dagger}\Big) + 1620\operatorname{Tr}\Big(y^D y^{D\dagger} T^D y^{D\dagger}\Big) \\
&+ 270\operatorname{Tr}\Big(y^D y^{U\dagger} T^U y^{D\dagger}\Big) + 180\operatorname{Tr}\Big(y^E y^{E\dagger} T^{h^E} h^{E\dagger}\Big) + 540\operatorname{Tr}\Big(y^E y^{E\dagger} T^E y^{E\dagger}\Big) \\
&+ 270\operatorname{Tr}\Big(y^U y^{D\dagger} T^D y^{U\dagger}\Big) + 90\operatorname{Tr}\Big(f^\dagger T^f \tilde{\lambda}^* \tilde{\lambda}^T\Big) + 270\operatorname{Tr}\Big(g^{D\dagger} y^{DT} y^{D*} T^{g^D}\Big) \\
&+ 270\operatorname{Tr}\Big(y^{D\dagger} T^D g^{D*} g^{DT}\Big) + 90\operatorname{Tr}\Big(\tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}}\Big)\Big),
\end{aligned} \tag{A.53}
$$

$$
\begin{aligned}
\beta_{T^{h^E}}^{(1)} &= 2h^E \tilde{f}^\dagger T^{\tilde{f}} + 4h^E h^{E\dagger} T^{h^E} + 2h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} + 2y^E y^{E\dagger} T^{h^E} + T^{h^E} \tilde{f}^\dagger \tilde{f} \\
&+ 5T^{h^E} h^{E\dagger} h^E + T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} + 4T^E y^{E\dagger} h^E - \frac{9}{5} g_1^2 T^{h^E} - \frac{7}{10} g_1'^2 T^{h^E} - 3g_2^2 T^{h^E} \\
&+ |\tilde{\sigma}|^2 T^{h^E} + 3T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + h^E \Bigg[2\operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&+ 2\tilde{\sigma}^* T_{\tilde{\sigma}} + 6g_2^2 M_2 + 6\operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + \frac{18}{5} g_1^2 M_1 + \frac{7}{5} g_1'^2 M_1' \Bigg],
\end{aligned} \tag{A.54}
$$

$$
\begin{aligned}
\beta_{T^{h^E}}^{(2)} &= 2g_1'^2 h^E \tilde{f}^\dagger T^{\tilde{f}} - 2|\lambda|^2 h^E \tilde{f}^\dagger T^{\tilde{f}} - 2g_1'^2 M_1' h^E h^{E\dagger} h^E - 12g_2^2 M_2 h^E h^{E\dagger} h^E \\
&+ \frac{6}{5} g_1^2 h^E h^{E\dagger} T^{h^E} + \frac{4}{5} g_1'^2 h^E h^{E\dagger} T^{h^E} + 6g_2^2 h^E h^{E\dagger} T^{h^E} - 4|\tilde{\sigma}|^2 h^E h^{E\dagger} T^{h^E} \\
&- 2g_1'^2 M_1' h^E \tilde{\lambda}^\dagger \tilde{\lambda} + 2g_1'^2 h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 4|\lambda|^2 h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 2|\sigma|^2 h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} \\
&+ \frac{12}{5} g_1^2 M_1 y^E y^{E\dagger} h^E - \frac{12}{5} g_1'^2 M_1' y^E y^{E\dagger} h^E - 12g_2^2 M_2 y^E y^{E\dagger} h^E \\
&- \frac{6}{5} g_1^2 y^E y^{E\dagger} T^{h^E} + \frac{6}{5} g_1'^2 y^E y^{E\dagger} T^{h^E} + 6g_2^2 y^E y^{E\dagger} T^{h^E} - 2|\lambda|^2 y^E y^{E\dagger} T^{h^E} \\
&+ g_1'^2 T^{h^E} \tilde{f}^\dagger \tilde{f} - |\lambda|^2 T^{h^E} \tilde{f}^\dagger \tilde{f} - \frac{6}{5} g_1^2 T^{h^E} h^{E\dagger} h^E + \frac{11}{5} g_1'^2 T^{h^E} h^{E\dagger} h^E \\
&+ 12g_2^2 T^{h^E} h^{E\dagger} h^E - 5|\tilde{\sigma}|^2 T^{h^E} h^{E\dagger} h^E + g_1'^2 T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} - 2|\lambda|^2 T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} \\
&- |\sigma|^2 T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} - \frac{12}{5} g_1^2 T^E y^{E\dagger} h^E + \frac{12}{5} g_1'^2 T^E y^{E\dagger} h^E + 12g_2^2 T^E y^{E\dagger} h^E \\
&- 4|\lambda|^2 T^E y^{E\dagger} h^E - 4h^E \tilde{f}^\dagger f f^\dagger T^{\tilde{f}} - 4h^E \tilde{f}^\dagger \tilde{f} \tilde{f}^\dagger T^{\tilde{f}} - 2h^E \tilde{f}^\dagger \tilde{f} h^{E\dagger} T^{h^E} \\
&- 4h^E \tilde{f}^\dagger T^f f^\dagger \tilde{f} - 4h^E \tilde{f}^\dagger T^{\tilde{f}} \tilde{f}^\dagger \tilde{f} - 4h^E \tilde{f}^\dagger T^{\tilde{f}} h^{E\dagger} h^E - 6h^E h^{E\dagger} h^E h^{E\dagger} T^{h^E}
\end{aligned}
$$

$$
\begin{aligned}
& -4h^E h^{E\dagger} y^E y^{E\dagger} T^{h^E} - 8h^E h^{E\dagger} T^{h^E} h^{E\dagger} h^E - 4h^E h^{E\dagger} T^E y^{E\dagger} h^E \\
& -2h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} T^{h^E} - 2h^E \tilde{\lambda}^\dagger \tilde{\lambda} \tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 4h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} h^{E\dagger} h^E - 2h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} \tilde{\lambda}^\dagger \tilde{\lambda} \\
& -2h^E \tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}} - 2h^E \tilde{\lambda}^\dagger T^{fT} f^* \tilde{\lambda} - 2y^E y^{E\dagger} y^E y^{E\dagger} T^{h^E} - 4y^E y^{E\dagger} T^E y^{E\dagger} h^E \\
& -2T^{h^E} \tilde{f}^\dagger f f^\dagger \tilde{f} - 2T^{h^E} \tilde{f}^\dagger \tilde{f} \tilde{f}^\dagger \tilde{f} - 4T^{h^E} \tilde{f}^\dagger \tilde{f} h^{E\dagger} h^E - 6T^{h^E} h^{E\dagger} h^E h^{E\dagger} h^E \\
& -2T^{h^E} h^{E\dagger} y^E y^{E\dagger} h^E - 4T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} h^E - T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} \tilde{\lambda}^\dagger \tilde{\lambda} - T^{h^E} \tilde{\lambda}^\dagger f^T f^* \tilde{\lambda} \\
& -4T^E y^{E\dagger} y^E y^{E\dagger} h^E + \frac{189}{10} g_1^4 T^{h^E} + \frac{3}{20} g_1^2 g_1'^2 T^{h^E} + \frac{77}{10} g_1'^4 T^{h^E} + \frac{9}{5} g_1^2 g_2^2 T^{h^E} \\
& + \frac{39}{20} g_1'^2 g_2^2 T^{h^E} + \frac{33}{2} g_2^4 T^{h^E} - 2|\tilde{\sigma}|^2 |\kappa_\phi|^2 T^{h^E} - |\tilde{\sigma}|^2 |\sigma|^2 T^{h^E} - 3|\tilde{\sigma}|^4 T^{h^E} \\
& -4\lambda^* h^E \tilde{\lambda}^\dagger \tilde{\lambda} T_\lambda - 4\lambda^* y^E y^{E\dagger} h^E T_\lambda - 2\sigma^* h^E \tilde{\lambda}^\dagger \tilde{\lambda} T_\sigma - 6\tilde{\sigma}^* h^E h^{E\dagger} h^E T_{\tilde{\sigma}} \\
& -2y^E y^{E\dagger} T^{h^E} \operatorname{Tr}\Big(f f^\dagger\Big) - 4T^E y^{E\dagger} h^E \operatorname{Tr}\Big(f f^\dagger\Big) - 2h^E \tilde{f}^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) \\
& -T^{h^E} \tilde{f}^\dagger \tilde{f} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) - 12h^E h^{E\dagger} T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 15T^{h^E} h^{E\dagger} h^E \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
& -\frac{2}{5} g_1^2 T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + \frac{9}{10} g_1'^2 T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 16g_3^2 T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
& -4h^E h^{E\dagger} T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 5T^{h^E} h^{E\dagger} h^E \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + \frac{6}{5} g_1^2 T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
& + \frac{3}{10} g_1'^2 T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 6y^E y^{E\dagger} T^{h^E} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12T^E y^{E\dagger} h^E \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
& -2y^E y^{E\dagger} T^{h^E} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 4T^E y^{E\dagger} h^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 6h^E \tilde{f}^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
& -3T^{h^E} \tilde{f}^\dagger \tilde{f} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 6h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} \operatorname{Tr}\Big(\kappa \kappa^\dagger\Big) - 3T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\kappa \kappa^\dagger\Big) \\
& -4h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} \operatorname{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^\dagger\Big) - 2T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^\dagger\Big) - 4y^E y^{E\dagger} h^E \operatorname{Tr}\Big(f^\dagger T^f\Big) \\
& -18h^E h^{E\dagger} h^E \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 6h^E h^{E\dagger} h^E \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
& -12y^E y^{E\dagger} h^E \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) - 4y^E y^{E\dagger} h^E \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) - 2h^E \tilde{f}^\dagger \tilde{f} \Big[3 \operatorname{Tr}\Big(y^{U\dagger} T^U\Big) \\
& + g_1'^2 M_1' + \lambda^* T_\lambda + \operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big)\Big] - 6h^E \tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) - 4h^E \tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) \\
& -T^{h^E} \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger\Big) - 9T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger} g^D g^{D\dagger}\Big) - 3T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) \\
& -3T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 3T^{h^E} \operatorname{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 3T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger}\Big) \\
& -2T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) - T^{h^E} \operatorname{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big) - \frac{1}{10} h^E \Bigg[756 g_1^4 M_1 \\
& + 3g_1^2 g_1'^2 M_1 + 36 g_1^2 g_2^2 M_1 + 3g_1^2 g_1'^2 M_1' + 308 g_1'^4 M_1' + 39 g_1'^2 g_2^2 M_1' + 36 g_1^2 g_2^2 M_2 \\
& + 39 g_1'^2 g_2^2 M_2 + 660 g_2^4 M_2 + 120 \tilde{\sigma}^* |\tilde{\sigma}|^2 T_{\tilde{\sigma}} + 40 \kappa_\phi^* \tilde{\sigma}^* \Big(\kappa_\phi T_{\tilde{\sigma}} + \tilde{\sigma} T_{\kappa_\phi}\Big) \\
& + 20\sigma^* \tilde{\sigma}^* \Big(\sigma T_{\tilde{\sigma}} + \tilde{\sigma} T_\sigma\Big) - 8g_1^2 M_1 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 18 g_1'^2 M_1' \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
& + 320 g_3^2 M_3 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 24 g_1^2 M_1 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + 6 g_1'^2 M_1' \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+8g_1^2\operatorname{Tr}\Big(g^{D\dagger}T^{g^D}\Big)-18g_1'^2\operatorname{Tr}\Big(g^{D\dagger}T^{g^D}\Big)-320g_3^2\operatorname{Tr}\Big(g^{D\dagger}T^{g^D}\Big)\\
&-24g_1^2\operatorname{Tr}\Big(h^{E\dagger}T^{h^E}\Big)-6g_1'^2\operatorname{Tr}\Big(h^{E\dagger}T^{h^E}\Big)+20\operatorname{Tr}\Big(\tilde{f}h^{E\dagger}T^{h^E}\tilde{f}^\dagger\Big)\\
&+360\operatorname{Tr}\Big(g^Dg^{D\dagger}T^{g^D}g^{D\dagger}\Big)+60\operatorname{Tr}\Big(g^D\kappa^\dagger T^\kappa g^{D\dagger}\Big)+20\operatorname{Tr}\Big(h^E\tilde{f}^\dagger T^{\tilde{f}}h^{E\dagger}\Big)\\
&+120\operatorname{Tr}\Big(h^Eh^{E\dagger}T^{h^E}h^{E\dagger}\Big)+40\operatorname{Tr}\Big(h^Eh^{E\dagger}T^Ey^{E\dagger}\Big)+20\operatorname{Tr}\Big(h^E\tilde{\lambda}^\dagger T^{\tilde{\lambda}}h^{E\dagger}\Big)\\
&+40\operatorname{Tr}\Big(y^Ey^{E\dagger}T^{h^E}h^{E\dagger}\Big)+60\operatorname{Tr}\Big(\kappa g^{D\dagger}T^{g^D}\kappa^\dagger\Big)+20\operatorname{Tr}\Big(\tilde{\lambda}h^{E\dagger}T^{h^E}\tilde{\lambda}^\dagger\Big)\\
&+60\operatorname{Tr}\Big(g^{D\dagger}y^{DT}y^{D*}T^{g^D}\Big)+60\operatorname{Tr}\Big(g^{D\dagger}y^{UT}y^{U*}T^{g^D}\Big)\\
&+60\operatorname{Tr}\Big(y^{D\dagger}T^Dg^{D*}g^{DT}\Big)+60\operatorname{Tr}\Big(y^{U\dagger}T^Ug^{D*}g^{DT}\Big)\bigg],
\end{aligned}
\tag{A.55}
$$

$$
\begin{aligned}
\beta_{T^E}^{(1)}=\;&2h^Eh^{E\dagger}T^E+4y^Ey^{E\dagger}T^E+4T^{h^E}h^{E\dagger}y^E+5T^Ey^{E\dagger}y^E-\frac{9}{5}g_1^2T^E\\
&-\frac{7}{10}g_1'^2T^E-3g_2^2T^E+|\lambda|^2T^E+T^E\operatorname{Tr}\Big(ff^\dagger\Big)+3T^E\operatorname{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&+T^E\operatorname{Tr}\Big(y^Ey^{E\dagger}\Big)+y^E\bigg[2\lambda^*T_\lambda+2\operatorname{Tr}\Big(f^\dagger T^f\Big)+2\operatorname{Tr}\Big(y^{E\dagger}T^E\Big)+6g_2^2M_2\\
&+6\operatorname{Tr}\Big(y^{D\dagger}T^D\Big)+\frac{18}{5}g_1^2M_1+\frac{7}{5}g_1'^2M_1'\bigg],
\end{aligned}
\tag{A.56}
$$

$$
\begin{aligned}
\beta_{T^E}^{(2)}=\;&-\frac{6}{5}g_1^2h^Eh^{E\dagger}T^E+\frac{6}{5}g_1'^2h^Eh^{E\dagger}T^E+6g_2^2h^Eh^{E\dagger}T^E-2|\tilde{\sigma}|^2h^Eh^{E\dagger}T^E\\
&-3g_1'^2M_1'y^Ey^{E\dagger}y^E-12g_2^2M_2y^Ey^{E\dagger}y^E+\frac{6}{5}g_1^2y^Ey^{E\dagger}T^E+\frac{9}{5}g_1'^2y^Ey^{E\dagger}T^E\\
&+6g_2^2y^Ey^{E\dagger}T^E-4|\lambda|^2y^Ey^{E\dagger}T^E-\frac{12}{5}g_1^2T^{h^E}h^{E\dagger}y^E+\frac{12}{5}g_1'^2T^{h^E}h^{E\dagger}y^E\\
&+12g_2^2T^{h^E}h^{E\dagger}y^E-4|\tilde{\sigma}|^2T^{h^E}h^{E\dagger}y^E-\frac{6}{5}g_1^2T^Ey^{E\dagger}y^E+\frac{27}{10}g_1'^2T^Ey^{E\dagger}y^E\\
&+12g_2^2T^Ey^{E\dagger}y^E-5|\lambda|^2T^Ey^{E\dagger}y^E-2h^E\tilde{f}^\dagger\tilde{f}h^{E\dagger}T^E-4h^E\tilde{f}^\dagger T^{\tilde{f}}h^{E\dagger}y^E\\
&-2h^Eh^{E\dagger}h^Eh^{E\dagger}T^E-4h^Eh^{E\dagger}T^{h^E}h^{E\dagger}y^E-2h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}T^E\\
&-4h^E\tilde{\lambda}^\dagger T^{\tilde{\lambda}}h^{E\dagger}y^E-4y^Ey^{E\dagger}h^Eh^{E\dagger}T^E-6y^Ey^{E\dagger}y^Ey^{E\dagger}T^E\\
&-4y^Ey^{E\dagger}T^{h^E}h^{E\dagger}y^E-8y^Ey^{E\dagger}T^Ey^{E\dagger}y^E-4T^{h^E}\tilde{f}^\dagger\tilde{f}h^{E\dagger}y^E\\
&-4T^{h^E}h^{E\dagger}h^Eh^{E\dagger}y^E-4T^{h^E}\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}y^E-2T^Ey^{E\dagger}h^Eh^{E\dagger}y^E\\
&-6T^Ey^{E\dagger}y^Ey^{E\dagger}y^E+\frac{189}{10}g_1^4T^E+\frac{3}{20}g_1^2g_1'^2T^E+\frac{77}{10}g_1'^4T^E+\frac{9}{5}g_1^2g_2^2T^E\\
&+\frac{39}{20}g_1'^2g_2^2T^E+\frac{33}{2}g_2^4T^E+g_1'^2|\lambda|^2T^E-3|\lambda|^4T^E-|\sigma|^2|\lambda|^2T^E\\
&-6\lambda^*y^Ey^{E\dagger}y^ET_\lambda-4y^Ey^{E\dagger}T^E\operatorname{Tr}\Big(ff^\dagger\Big)-5T^Ey^{E\dagger}y^E\operatorname{Tr}\Big(ff^\dagger\Big)\\
&+g_1'^2T^E\operatorname{Tr}\Big(ff^\dagger\Big)-|\lambda|^2T^E\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-6h^Eh^{E\dagger}T^E\operatorname{Tr}\Big(g^Dg^{D\dagger}\Big)\\
&-12T^{h^E}h^{E\dagger}y^E\operatorname{Tr}\Big(g^Dg^{D\dagger}\Big)-2h^Eh^{E\dagger}T^E\operatorname{Tr}\Big(h^Eh^{E\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 4T^{h^E} h^{E\dagger} y^E \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 12 y^E y^{E\dagger} T^E \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 15 T^E y^{E\dagger} y^E \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - \frac{2}{5} g_1^2 T^E \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - \frac{3}{5} g_1'^2 T^E \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&+ 16 g_3^2 T^E \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 4 y^E y^{E\dagger} T^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 5 T^E y^{E\dagger} y^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&+ \frac{6}{5} g_1^2 T^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - \frac{1}{5} g_1'^2 T^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 3|\lambda|^2 T^E \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 3|\lambda|^2 T^E \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 2|\lambda|^2 T^E \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 6 y^E y^{E\dagger} y^E \operatorname{Tr}\Big(f^\dagger T^f\Big) \\
&+ \frac{4}{5} h^E h^{E\dagger} y^E \Big[-15 g_2^2 M_2 - 15 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + 3 g_1^2 M_1 - 3 g_1'^2 M_1' \\
&- 5 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) - 5\tilde\sigma^* T_{\tilde\sigma}\Big] - 18 y^E y^{E\dagger} y^E \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) \\
&- 6 y^E y^{E\dagger} y^E \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) - 3 T^E \operatorname{Tr}\Big(f f^\dagger f f^\dagger\Big) - 2 T^E \operatorname{Tr}\Big(f f^\dagger \tilde f \tilde f^\dagger\Big) \\
&- 3 T^E \operatorname{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 2 T^E \operatorname{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) \\
&- 9 T^E \operatorname{Tr}\Big(y^D y^{D\dagger} y^D y^{D\dagger}\Big) - 3 T^E \operatorname{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) \\
&- 3 T^E \operatorname{Tr}\Big(y^E y^{E\dagger} y^E y^{E\dagger}\Big) - T^E \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger f^T f^*\Big) - \frac{1}{10} y^E \Big(756 g_1^4 M_1 \\
&+ 3 g_1^2 g_1'^2 M_1 + 36 g_1^2 g_2^2 M_1 + 3 g_1^2 g_1'^2 M_1' + 308 g_1'^4 M_1' + 39 g_1'^2 g_2^2 M_1' \\
&+ 36 g_1^2 g_2^2 M_2 + 39 g_1'^2 g_2^2 M_2 + 660 g_2^4 M_2 + 120 \lambda^* |\lambda|^2 T_\lambda + 20 g_1'^2 M_1' \operatorname{Tr}\Big(f f^\dagger\Big) \\
&- 8 g_1^2 M_1 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12 g_1'^2 M_1' \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + 320 g_3^2 M_3 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&+ 24 g_1^2 M_1 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 4 g_1'^2 M_1' \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 20 g_1'^2 \operatorname{Tr}\Big(f^\dagger T^f\Big) \\
&+ 8 g_1^2 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) + 12 g_1'^2 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) - 320 g_3^2 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) \\
&- 24 g_1^2 \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) + 4 g_1'^2 \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) + 20 \lambda^* \Big\{ T_\lambda \Big[2 \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) \\
&+ 3 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 3 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - g_1'^2 + |\sigma|^2 + \operatorname{Tr}\Big(\tilde f \tilde f^\dagger\Big)\Big] + \lambda \Big[2 \operatorname{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big) \\
&+ 3 \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 3 \operatorname{Tr}\Big(y^{U\dagger} T^U\Big) + g_1'^2 M_1' + \sigma^* T_\sigma + \operatorname{Tr}\Big(\tilde f^\dagger T^{\tilde f}\Big)\Big]\Big\} \\
&+ 120 \operatorname{Tr}\Big(f f^\dagger T^f f^\dagger\Big) + 40 \operatorname{Tr}\Big(f f^\dagger T^{\tilde f} \tilde f^\dagger\Big) + 40 \operatorname{Tr}\Big(\tilde f \tilde f^\dagger T^f f^\dagger\Big) \\
&+ 40 \operatorname{Tr}\Big(h^E h^{E\dagger} T^E y^{E\dagger}\Big) + 360 \operatorname{Tr}\Big(y^D y^{D\dagger} T^D y^{D\dagger}\Big) + 60 \operatorname{Tr}\Big(y^D y^{U\dagger} T^U y^{D\dagger}\Big) \\
&+ 40 \operatorname{Tr}\Big(y^E y^{E\dagger} T^{h^E} h^{E\dagger}\Big) + 120 \operatorname{Tr}\Big(y^E y^{E\dagger} T^E y^{E\dagger}\Big) + 60 \operatorname{Tr}\Big(y^U y^{D\dagger} T^D y^{U\dagger}\Big) \\
&+ 20 \operatorname{Tr}\Big(f^\dagger T^f \tilde\lambda^* \tilde\lambda^T\Big) + 60 \operatorname{Tr}\Big(g^{D\dagger} y^{DT} y^{D*} T^{g^D}\Big) + 60 \operatorname{Tr}\Big(y^{D\dagger} T^D g^{D*} g^{DT}\Big) \\
&+ 20 \operatorname{Tr}\Big(\tilde\lambda^\dagger f^T f^* T^{\tilde\lambda}\Big)\Big),
\end{aligned} \tag{A.57}
$$

$$
\begin{aligned}
\beta_{T_{\tilde\sigma}}^{(1)} &= \frac{6}{5} g_1^2 M_1 \tilde\sigma + \frac{4}{5} g_1'^2 M_1' \tilde\sigma + 6 g_2^2 M_2 \tilde\sigma - \frac{3}{5} g_1^2 T_{\tilde\sigma} - \frac{2}{5} g_1'^2 T_{\tilde\sigma} - 3 g_2^2 T_{\tilde\sigma} + 12 |\tilde\sigma|^2 T_{\tilde\sigma} \\
&+ 2\kappa_\phi^* \Big(2\tilde\sigma T_{\kappa_\phi} + \kappa_\phi T_{\tilde\sigma}\Big) + \sigma^* \Big(2\tilde\sigma T_\sigma + \sigma T_{\tilde\sigma}\Big) + 3 T_{\tilde\sigma} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ T_{\tilde\sigma} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + 6\tilde\sigma \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + 2\tilde\sigma \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big),
\end{aligned} \tag{A.58}
$$

$$
\begin{aligned}
\beta^{(2)}_{T_{\tilde\sigma}} = & -\frac{594}{25}g_1^4 M_1\tilde\sigma - \frac{36}{25}g_1^2 g_1'^2 M_1\tilde\sigma - \frac{18}{5}g_1^2 g_2^2 M_1\tilde\sigma - \frac{36}{25}g_1^2 g_1'^2 M_1'\tilde\sigma - \frac{434}{25}g_1'^4 M_1'\tilde\sigma \\
& - \frac{12}{5}g_1'^2 g_2^2 M_1'\tilde\sigma - \frac{18}{5}g_1^2 g_2^2 M_2\tilde\sigma - \frac{12}{5}g_1'^2 g_2^2 M_2\tilde\sigma - 66 g_2^4 M_2\tilde\sigma - 32\kappa_\phi^*\tilde\sigma|\kappa_\phi|^2 T_{\kappa_\phi} \\
& + \frac{297}{50}g_1^4 T_{\tilde\sigma} + \frac{18}{25}g_1^2 g_1'^2 T_{\tilde\sigma} + \frac{217}{50}g_1'^4 T_{\tilde\sigma} + \frac{9}{5}g_1^2 g_2^2 T_{\tilde\sigma} + \frac{6}{5}g_1'^2 g_2^2 T_{\tilde\sigma} + \frac{33}{2}g_2^4 T_{\tilde\sigma} \\
& - 8|\kappa_\phi|^4 T_{\tilde\sigma} - 50|\tilde\sigma|^4 T_{\tilde\sigma} - 2\sigma^*|\sigma|^2\Big(4\tilde\sigma T_\sigma + \sigma T_{\tilde\sigma}\Big) + \frac{4}{5}g_1^2 M_1\tilde\sigma\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
& - \frac{9}{5}g_1'^2 M_1'\tilde\sigma\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 32 g_3^2 M_3\tilde\sigma\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \frac{2}{5}g_1^2 T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
& + \frac{9}{10}g_1'^2 T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + 16 g_3^2 T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - \frac{12}{5}g_1^2 M_1\tilde\sigma\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
& - \frac{3}{5}g_1'^2 M_1'\tilde\sigma\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) + \frac{6}{5}g_1^2 T_{\tilde\sigma}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) + \frac{3}{10}g_1'^2 T_{\tilde\sigma}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
& - \frac{4}{5}g_1^2\tilde\sigma\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) + \frac{9}{5}g_1'^2\tilde\sigma\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) + 32 g_3^2\tilde\sigma\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) \\
& + \frac{12}{5}g_1^2\tilde\sigma\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) + \frac{3}{5}g_1'^2\tilde\sigma\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) - \frac{1}{5}|\tilde\sigma|^2\Big\{60\kappa_\phi^*\Big(2\tilde\sigma T_{\kappa_\phi} \\
& + 3\kappa_\phi T_{\tilde\sigma}\Big) - 3T_{\tilde\sigma}\Big[-15\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) + 30 g_2^2 - 45\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + 4g_1'^2 + 6g_1^2\Big] \\
& + 2\tilde\sigma\Big[15\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) + 30 g_2^2 M_2 + 45\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) + 4g_1'^2 M_1' + 6 g_1^2 M_1\Big]\Big\} \\
& - \frac{1}{2}\sigma^*\Big[10 g_1'^2 M_1'\sigma\tilde\sigma - 10 g_1'^2\tilde\sigma T_\sigma + 8\tilde\sigma|\tilde\sigma|^2 T_\sigma - 5g_1'^2\sigma T_{\tilde\sigma} + 12\sigma|\tilde\sigma|^2 T_{\tilde\sigma} \\
& + 8\kappa_\phi^*\Big(2\kappa_\phi\tilde\sigma T_\sigma + 2\sigma\tilde\sigma T_{\kappa_\phi} + \kappa_\phi\sigma T_{\tilde\sigma}\Big) + 4\lambda^*\Big(2\lambda\tilde\sigma T_\sigma + 2\sigma\tilde\sigma T_\lambda + \lambda\sigma T_{\tilde\sigma}\Big) \\
& + 12\tilde\sigma T_\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 6\sigma T_{\tilde\sigma}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 8\tilde\sigma T_\sigma\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) + 4\sigma T_{\tilde\sigma}\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) \\
& + 12\sigma\tilde\sigma\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 8\sigma\tilde\sigma\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)\Big] - T_{\tilde\sigma}\,\mathrm{Tr}\Big(\tilde f h^{E\dagger}h^E\tilde f^\dagger\Big) \\
& - 2\tilde\sigma\,\mathrm{Tr}\Big(\tilde f h^{E\dagger}T^{h^E}\tilde f^\dagger\Big) - 9T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}g^D g^{D\dagger}\Big) - 36\tilde\sigma\,\mathrm{Tr}\Big(g^D g^{D\dagger}T^{g^D}g^{D\dagger}\Big) \\
& - 3T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{DT}y^{D*}\Big) - 3T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big) - 3T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) \\
& - 6\tilde\sigma\,\mathrm{Tr}\Big(g^D\kappa^\dagger T^\kappa g^{D\dagger}\Big) - 2\tilde\sigma\,\mathrm{Tr}\Big(h^E\tilde f^\dagger T^{\tilde f}h^{E\dagger}\Big) - 3T_{\tilde\sigma}\,\mathrm{Tr}\Big(h^E h^{E\dagger}h^E h^{E\dagger}\Big) \\
& - 2T_{\tilde\sigma}\,\mathrm{Tr}\Big(h^E h^{E\dagger}y^E y^{E\dagger}\Big) - 12\tilde\sigma\,\mathrm{Tr}\Big(h^E h^{E\dagger}T^{h^E}h^{E\dagger}\Big) - 4\tilde\sigma\,\mathrm{Tr}\Big(h^E h^{E\dagger}T^E y^{E\dagger}\Big) \\
& - T_{\tilde\sigma}\,\mathrm{Tr}\Big(h^E\tilde\lambda^\dagger\tilde\lambda h^{E\dagger}\Big) - 2\tilde\sigma\,\mathrm{Tr}\Big(h^E\tilde\lambda^\dagger T^{\tilde\lambda}h^{E\dagger}\Big) - 4\tilde\sigma\,\mathrm{Tr}\Big(y^E y^{E\dagger}T^{h^E}h^{E\dagger}\Big) \\
& - 6\tilde\sigma\,\mathrm{Tr}\Big(\kappa g^{D\dagger}T^{g^D}\kappa^\dagger\Big) - 2\tilde\sigma\,\mathrm{Tr}\Big(\tilde\lambda h^{E\dagger}T^{h^E}\tilde\lambda^\dagger\Big) - 6\tilde\sigma\,\mathrm{Tr}\Big(g^{D\dagger}y^{DT}y^{D*}T^{g^D}\Big) \\
& - 6\tilde\sigma\,\mathrm{Tr}\Big(g^{D\dagger}y^{UT}y^{U*}T^{g^D}\Big) - 6\tilde\sigma\,\mathrm{Tr}\Big(y^{D\dagger}T^D g^{D*}g^{DT}\Big) \\
& - 6\tilde\sigma\,\mathrm{Tr}\Big(y^{U\dagger}T^U g^{D*}g^{DT}\Big), \qquad (A.59)
\end{aligned}
$$

$$
\beta^{(1)}_{T_{\kappa_\phi}} = 3\Big[2\tilde\sigma^*\Big(2\kappa_\phi T_{\tilde\sigma} + \tilde\sigma T_{\kappa_\phi}\Big) + 6|\kappa_\phi|^2 T_{\kappa_\phi} + \sigma^*\Big(2\kappa_\phi T_\sigma + \sigma T_{\kappa_\phi}\Big)\Big], \qquad (A.60)
$$

$$
\begin{aligned}
\beta_{T_{\kappa_\phi}}^{(2)} &= -\frac{3}{10}\Big\{20\sigma^*|\sigma|^2\Big(4\kappa_\phi T_\sigma + \sigma T_{\kappa_\phi}\Big) + 4\Big[100|\kappa_\phi|^4 T_{\kappa_\phi} + 10\tilde{\sigma}^*|\tilde{\sigma}|^2\Big(4\kappa_\phi T_{\tilde{\sigma}} \\
&\quad + \tilde{\sigma}T_{\kappa_\phi}\Big) + \tilde{\sigma}^*\Big(\tilde{\sigma}T_{\kappa_\phi}\Big[-15g_2^2 + 15\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 2g_1'^2 - 3g_1^2 \\
&\quad + 5\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) + 60|\kappa_\phi|^2\Big] + 2\kappa_\phi\Big\{T_{\tilde{\sigma}}\Big[-15g_2^2 + 15\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + 20|\kappa_\phi|^2 \\
&\quad - 2g_1'^2 - 3g_1^2 + 5\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\Big] + \tilde{\sigma}\Big[15g_2^2 M_2 + 15\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) + 2g_1'^2 M_1' \\
&\quad + 3g_1^2 M_1 + 5\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big)\Big]\Big\}\Big)\Big] + 5\sigma^*\Big(\sigma T_{\kappa_\phi}\Big[24|\kappa_\phi|^2 + 4|\lambda|^2 + 4\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&\quad - 5g_1'^2 + 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\Big] + 2\kappa_\phi\Big\{4\lambda^*\Big(\lambda T_\sigma + \sigma T_\lambda\Big) + T_\sigma\Big[4\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 5g_1'^2 \\
&\quad + 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 8|\kappa_\phi|^2\Big] + \sigma\Big[4\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 5g_1'^2 M_1' + 6\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)\Big]\Big\}\Big)\Big\}, \qquad \text{(A.61)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T_\sigma}^{(1)} &= 5g_1'^2 M_1'\sigma - \frac{5}{2}g_1'^2 T_\sigma + 9|\sigma|^2 T_\sigma + 2|\tilde{\sigma}|^2 T_\sigma + 2\kappa_\phi^*\Big(2\sigma T_{\kappa_\phi} + \kappa_\phi T_\sigma\Big) \\
&\quad + 2\lambda^*\Big(2\sigma T_\lambda + \lambda T_\sigma\Big) + 4\sigma\tilde{\sigma}^* T_{\tilde{\sigma}} + 3T_\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 2T_\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&\quad + 6\sigma\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 4\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big), \qquad \text{(A.62)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T_\sigma}^{(2)} &= -119g_1'^4 M_1'\sigma - \frac{12}{5}g_1^2 M_1\sigma|\tilde{\sigma}|^2 - \frac{8}{5}g_1'^2 M_1'\sigma|\tilde{\sigma}|^2 - 12g_2^2 M_2\sigma|\tilde{\sigma}|^2 \\
&\quad - 16\sigma|\tilde{\sigma}|^2\kappa_\phi^* T_{\kappa_\phi} - 32\kappa_\phi^*\sigma|\kappa_\phi|^2 T_{\kappa_\phi} + \frac{119}{4}g_1'^4 T_\sigma + \frac{6}{5}g_1^2|\tilde{\sigma}|^2 T_\sigma + \frac{4}{5}g_1'^2|\tilde{\sigma}|^2 T_\sigma \\
&\quad + 6g_2^2|\tilde{\sigma}|^2 T_\sigma - 8|\kappa_\phi|^4 T_\sigma - 30|\sigma|^4 T_\sigma - 8|\tilde{\sigma}|^2|\kappa_\phi|^2 T_\sigma - 4|\tilde{\sigma}|^4 T_\sigma \\
&\quad - 4\lambda^*|\lambda|^2\Big(4\sigma T_\lambda + \lambda T_\sigma\Big) + \frac{12}{5}g_1^2\sigma\tilde{\sigma}^* T_{\tilde{\sigma}} + \frac{8}{5}g_1'^2\sigma\tilde{\sigma}^* T_{\tilde{\sigma}} + 12g_2^2\sigma\tilde{\sigma}^* T_{\tilde{\sigma}} \\
&\quad - 16\sigma|\kappa_\phi|^2\tilde{\sigma}^* T_{\tilde{\sigma}} - 16\sigma\tilde{\sigma}^*|\tilde{\sigma}|^2 T_{\tilde{\sigma}} - 6|\tilde{\sigma}|^2 T_\sigma\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&\quad - 12\sigma\tilde{\sigma}^* T_{\tilde{\sigma}}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 2|\tilde{\sigma}|^2 T_\sigma\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 4\sigma\tilde{\sigma}^* T_{\tilde{\sigma}}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
&\quad - \frac{8}{5}g_1^2 M_1\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + \frac{18}{5}g_1'^2 M_1'\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 32g_3^2 M_3\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&\quad + \frac{4}{5}g_1^2 T_\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - \frac{9}{5}g_1'^2 T_\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 16g_3^2 T_\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&\quad - \frac{12}{5}g_1^2 M_1\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + \frac{12}{5}g_1'^2 M_1'\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 12g_2^2 M_2\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&\quad + \frac{6}{5}g_1^2 T_\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - \frac{6}{5}g_1'^2 T_\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 6g_2^2 T_\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&\quad - 12\sigma|\tilde{\sigma}|^2\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) - 4\sigma|\tilde{\sigma}|^2\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) - \frac{2}{5}\lambda^*\Big(2\sigma T_\lambda\Big[-3g_1^2 \\
&\quad + 3g_1'^2 - 15g_2^2 + 10|\sigma|^2 + 5\,\mathrm{Tr}\Big(ff^\dagger\Big) + 5\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + 15\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) \\
&\quad + 5\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) + 15\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)\Big] + \lambda\Big\{T_\sigma\Big[-3g_1^2 + 3g_1'^2 - 15g_2^2 + 30|\sigma|^2 \\
&\quad + 5\,\mathrm{Tr}\Big(ff^\dagger\Big) + 5\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + 15\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) + 5\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) + 15\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big)\Big] \\
&\quad + 2\sigma\Big[3g_1^2 M_1 - 3g_1'^2 M_1' + 15g_2^2 M_2 + 5\,\mathrm{Tr}\Big(f^\dagger T^f\Big) + 5\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) \\
&\quad + 15\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big) + 5\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big) + 15\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)\Big]\Big\}\Big) + \frac{8}{5}g_1^2\sigma\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&- \frac{18}{5} g_1'^2 \sigma \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 32 g_3^2 \sigma \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) + \frac{12}{5} g_1^2 \sigma \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) \\
&- \frac{12}{5} g_1'^2 \sigma \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 12 g_2^2 \sigma \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - \frac{1}{2}|\sigma|^2 \Big\{ 16 \kappa_\phi^* \Big(2\sigma T_{\kappa_\phi} + 3\kappa_\phi T_\sigma\Big) \\
&+ 3 T_\sigma \Big[12 \operatorname{Tr}\Big(\kappa \kappa^\dagger\Big) - 5 g_1'^2 + 8 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 8|\tilde{\sigma}|^2 \Big] + 2\sigma \Big[12 \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) \\
&+ 5 g_1'^2 M_1' + 8 \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 8 \tilde{\sigma}^* T_{\tilde{\sigma}} \Big] \Big\} - 2 T_\sigma \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda} \tilde{f}^\dagger\Big) - 4\sigma \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \tilde{f}^\dagger\Big) \\
&- 6 T_\sigma \operatorname{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 12\sigma \operatorname{Tr}\Big(g^D \kappa^\dagger T^\kappa g^{D\dagger}\Big) - 2 T_\sigma \operatorname{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big) \\
&- 4\sigma \operatorname{Tr}\Big(h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} h^{E\dagger}\Big) - 12\sigma \operatorname{Tr}\Big(\kappa g^{D\dagger} T^{g^D} \kappa^\dagger\Big) - 6 T_\sigma \operatorname{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big) \\
&- 24\sigma \operatorname{Tr}\Big(\kappa\kappa^\dagger T^\kappa \kappa^\dagger\Big) - 4\sigma \operatorname{Tr}\Big(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}} \tilde{\lambda}^\dagger\Big) - 4\sigma \operatorname{Tr}\Big(\tilde{\lambda} h^{E\dagger} T^{h^E} \tilde{\lambda}^\dagger\Big) \\
&- 4 T_\sigma \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 16\sigma \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \tilde{\lambda}^\dagger\Big) - 2 T_\sigma \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big) \\
&- 4\sigma \operatorname{Tr}\Big(f^\dagger T^f \tilde{\lambda}^* \tilde{\lambda}^T\Big) - 4\sigma \operatorname{Tr}\Big(\tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}}\Big), \qquad \text{(A.63)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T^{g^D}}^{(1)} =\ & 5 g^D g^{D\dagger} T^{g^D} + 2 g^D \kappa^\dagger T^\kappa + 4 T^{g^D} g^{D\dagger} g^D + T^{g^D} \kappa^\dagger \kappa + y^{DT} y^{D*} T^{g^D} \\
&+ y^{UT} y^{U*} T^{g^D} + 2 T^{DT} y^{D*} g^D + 2 T^{UT} y^{U*} g^D - \frac{7}{15} g_1^2 T^{g^D} - \frac{7}{10} g_1'^2 T^{g^D} \\
&- 3 g_2^2 T^{g^D} - \frac{16}{3} g_3^2 T^{g^D} + |\tilde{\sigma}|^2 T^{g^D} + 3 T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&+ g^D \Bigg[2 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) + 2 \tilde{\sigma}^* T_{\tilde{\sigma}} + 6 g_2^2 M_2 + 6 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + \frac{14}{15} g_1^2 M_1 \\
&+ \frac{32}{3} g_3^2 M_3 + \frac{7}{5} g_1'^2 M_1' \Bigg], \qquad \text{(A.64)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T^{g^D}}^{(2)} =\ & \frac{6}{5} g_1^2 g^D g^{D\dagger} T^{g^D} - \frac{1}{5} g_1'^2 g^D g^{D\dagger} T^{g^D} + 12 g_2^2 g^D g^{D\dagger} T^{g^D} - 5|\tilde{\sigma}|^2 g^D g^{D\dagger} T^{g^D} \\
&- 2 g_1'^2 M_1' g^D \kappa^\dagger \kappa + 2 g_1'^2 g^D \kappa^\dagger T^\kappa - 4|\lambda|^2 g^D \kappa^\dagger T^\kappa - 2|\sigma|^2 g^D \kappa^\dagger T^\kappa \\
&+ \frac{6}{5} g_1^2 T^{g^D} g^{D\dagger} g^D + \frac{4}{5} g_1'^2 T^{g^D} g^{D\dagger} g^D + 6 g_2^2 T^{g^D} g^{D\dagger} g^D - 4|\tilde{\sigma}|^2 T^{g^D} g^{D\dagger} g^D \\
&+ g_1'^2 T^{g^D} \kappa^\dagger \kappa - 2|\lambda|^2 T^{g^D} \kappa^\dagger \kappa - |\sigma|^2 T^{g^D} \kappa^\dagger \kappa - \frac{4}{5} g_1^2 M_1 y^{DT} y^{D*} g^D \\
&- \frac{6}{5} g_1'^2 M_1' y^{DT} y^{D*} g^D + \frac{2}{5} g_1^2 y^{DT} y^{D*} T^{g^D} + \frac{3}{5} g_1'^2 y^{DT} y^{D*} T^{g^D} \\
&- |\lambda|^2 y^{DT} y^{D*} T^{g^D} - \frac{8}{5} g_1^2 M_1 y^{UT} y^{U*} g^D - \frac{2}{5} g_1'^2 M_1' y^{UT} y^{U*} g^D \\
&+ \frac{4}{5} g_1^2 y^{UT} y^{U*} T^{g^D} + \frac{1}{5} g_1'^2 y^{UT} y^{U*} T^{g^D} - |\lambda|^2 y^{UT} y^{U*} T^{g^D} + \frac{4}{5} g_1^2 T^{DT} y^{D*} g^D \\
&+ \frac{6}{5} g_1'^2 T^{DT} y^{D*} g^D - 2|\lambda|^2 T^{DT} y^{D*} g^D + \frac{8}{5} g_1^2 T^{UT} y^{U*} g^D + \frac{2}{5} g_1'^2 T^{UT} y^{U*} g^D \\
&- 2|\lambda|^2 T^{UT} y^{U*} g^D - 6 g^D g^{D\dagger} g^D g^{D\dagger} T^{g^D} - 8 g^D g^{D\dagger} T^{g^D} g^{D\dagger} g^D \\
&- 4 g^D g^{D\dagger} y^{DT} y^{D*} T^{g^D} - 4 g^D g^{D\dagger} y^{UT} y^{U*} T^{g^D} - 4 g^D g^{D\dagger} T^{DT} y^{D*} g^D \\
&- 4 g^D g^{D\dagger} T^{UT} y^{U*} g^D - g^D \kappa^\dagger \kappa g^{D\dagger} T^{g^D} - 2 g^D \kappa^\dagger \kappa \kappa^\dagger T^\kappa - 2 g^D \kappa^\dagger T^\kappa g^{D\dagger} g^D \\
&- 2 g^D \kappa^\dagger T^\kappa \kappa^\dagger \kappa - 6 T^{g^D} g^{D\dagger} g^D g^{D\dagger} g^D - 2 T^{g^D} g^{D\dagger} y^{DT} y^{D*} g^D
\end{aligned}
$$

$$
\begin{aligned}
&- 2T^{g^D} g^{D\dagger} y^{UT} y^{U*} g^D - 2T^{g^D} \kappa^\dagger \kappa g^{D\dagger} g^D - T^{g^D} \kappa^\dagger \kappa \kappa^\dagger \kappa \\
&- 2y^{DT} y^{D*} y^{DT} y^{D*} T^{g^D} - 4y^{DT} y^{D*} T^{DT} y^{D*} g^D - 2y^{UT} y^{U*} y^{UT} y^{U*} T^{g^D} \\
&- 4y^{UT} y^{U*} T^{UT} y^{U*} g^D - 4T^{DT} y^{D*} y^{DT} y^{D*} g^D - 4T^{UT} y^{U*} y^{UT} y^{U*} g^D \\
&+ \frac{413}{90} g_1^4 T^{g^D} + \frac{41}{60} g_1^2 g_1'^2 T^{g^D} + \frac{77}{10} g_1'^4 T^{g^D} + g_1^2 g_2^2 T^{g^D} + \frac{3}{4} g_1'^2 g_2^2 T^{g^D} \\
&+ \frac{33}{2} g_2^4 T^{g^D} + \frac{8}{9} g_1^2 g_3^2 T^{g^D} + \frac{8}{3} g_1'^2 g_3^2 T^{g^D} + 8 g_2^2 g_3^2 T^{g^D} + \frac{128}{9} g_3^4 T^{g^D} \\
&- 2|\tilde\sigma|^2 |\kappa_\phi|^2 T^{g^D} - |\tilde\sigma|^2 |\sigma|^2 T^{g^D} - 3|\tilde\sigma|^4 T^{g^D} - 4\lambda^* g^D \kappa^\dagger \kappa T_\lambda \\
&- 2\lambda^* y^{DT} y^{D*} g^D T_\lambda - 2\lambda^* y^{UT} y^{U*} g^D T_\lambda - 2\sigma^* g^D \kappa^\dagger \kappa T_\sigma \\
&- y^{DT} y^{D*} T^{g^D} \operatorname{Tr}\Big(f f^\dagger\Big) - 2T^{DT} y^{D*} g^D \operatorname{Tr}\Big(f f^\dagger\Big) - y^{UT} y^{U*} T^{g^D} \operatorname{Tr}\Big(\tilde f \tilde f^\dagger\Big) \\
&- 2T^{UT} y^{U*} g^D \operatorname{Tr}\Big(\tilde f \tilde f^\dagger\Big) - 15 g^D g^{D\dagger} T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 12 T^{g^D} g^{D\dagger} g^D \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - \frac{2}{5} g_1^2 T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + \frac{9}{10} g_1'^2 T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ 16 g_3^2 T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 5 g^D g^{D\dagger} T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 4T^{g^D} g^{D\dagger} g^D \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&+ \frac{6}{5} g_1^2 T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + \frac{3}{10} g_1'^2 T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 3y^{DT} y^{D*} T^{g^D} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6T^{DT} y^{D*} g^D \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - y^{DT} y^{D*} T^{g^D} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 2T^{DT} y^{D*} g^D \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 3y^{UT} y^{U*} T^{g^D} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 6T^{UT} y^{U*} g^D \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 6 g^D \kappa^\dagger T^\kappa \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 3T^{g^D} \kappa^\dagger \kappa \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 4 g^D \kappa^\dagger T^\kappa \operatorname{Tr}\Big(\tilde\lambda \tilde\lambda^\dagger\Big) - 2T^{g^D} \kappa^\dagger \kappa \operatorname{Tr}\Big(\tilde\lambda \tilde\lambda^\dagger\Big) - 2y^{DT} y^{D*} g^D \operatorname{Tr}\Big(f^\dagger T^f\Big) \\
&- 2y^{UT} y^{U*} g^D \operatorname{Tr}\Big(\tilde f^\dagger T^{\tilde f}\Big) - \frac{2}{5} g^D g^{D\dagger} g^D \Big[15 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) + 15 \tilde\sigma^* T_{\tilde\sigma} \\
&+ 30 g_2^2 M_2 + 45 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + 4 g_1^2 M_1 + g_1'^2 M_1'\Big] - 6y^{DT} y^{D*} g^D \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) \\
&- 2y^{DT} y^{D*} g^D \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) - 6y^{UT} y^{U*} g^D \operatorname{Tr}\Big(y^{U\dagger} T^U\Big) - 6 g^D \kappa^\dagger \kappa \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) \\
&- 4 g^D \kappa^\dagger \kappa \operatorname{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big) - T^{g^D} \operatorname{Tr}\Big(\tilde f h^{E\dagger} h^E \tilde f^\dagger\Big) - 9T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger} g^D g^{D\dagger}\Big) \\
&- 3T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 3T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) \\
&- 3T^{g^D} \operatorname{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 3T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger}\Big) - 2T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) \\
&- T^{g^D} \operatorname{Tr}\Big(h^E \tilde\lambda^\dagger \tilde\lambda h^{E\dagger}\Big) - \frac{1}{90} g^D \Big[1652 g_1^4 M_1 + 123 g_1^2 g_1'^2 M_1 + 180 g_1^2 g_2^2 M_1 \\
&+ 160 g_1^2 g_3^2 M_1 + 123 g_1^2 g_1'^2 M_1' + 2772 g_1'^4 M_1' + 135 g_1'^2 g_2^2 M_1' + 480 g_1'^2 g_3^2 M_1' \\
&+ 160 g_1^2 g_3^2 M_3 + 480 g_1'^2 g_3^2 M_3 + 1440 g_2^2 g_3^2 M_3 + 5120 g_3^4 M_3 + 180 g_1^2 g_2^2 M_2 \\
&+ 135 g_1'^2 g_2^2 M_2 + 5940 g_2^4 M_2 + 1440 g_2^2 g_3^2 M_2 + 1080 \tilde\sigma^* |\tilde\sigma|^2 T_{\tilde\sigma} \\
&+ 360 \kappa_\phi^* \tilde\sigma^* \Big(\kappa_\phi T_{\tilde\sigma} + \tilde\sigma T_{\kappa_\phi}\Big) + 180 \sigma^* \tilde\sigma^* \Big(\sigma T_{\tilde\sigma} + \tilde\sigma T_\sigma\Big) - 72 g_1^2 M_1 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ 162 g_1'^2 M_1' \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 2880 g_3^2 M_3 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 216 g_1^2 M_1 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+54g_1'^2 M_1' \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + 72g_1^2 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 162g_1'^2 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) \\
&- 2880g_3^2 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 216g_1^2 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) - 54g_1'^2 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&+ 180 \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} T^{h^E} \tilde{f}^\dagger\Big) + 3240 \operatorname{Tr}\Big(g^D g^{D\dagger} T^{g^D} g^{D\dagger}\Big) + 540 \operatorname{Tr}\Big(g^D \kappa^\dagger T^\kappa g^{D\dagger}\Big) \\
&+ 180 \operatorname{Tr}\Big(h^E \tilde{f}^\dagger T^{\tilde{f}} h^{E\dagger}\Big) + 1080 \operatorname{Tr}\Big(h^E h^{E\dagger} T^{h^E} h^{E\dagger}\Big) + 360 \operatorname{Tr}\Big(h^E h^{E\dagger} T^E y^{E\dagger}\Big) \\
&+ 180 \operatorname{Tr}\Big(h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} h^{E\dagger}\Big) + 360 \operatorname{Tr}\Big(y^E y^{E\dagger} T^{h^E} h^{E\dagger}\Big) + 540 \operatorname{Tr}\Big(\kappa g^{D\dagger} T^{g^D} \kappa^\dagger\Big) \\
&+ 180 \operatorname{Tr}\Big(\tilde{\lambda} h^{E\dagger} T^{h^E} \tilde{\lambda}^\dagger\Big) + 540 \operatorname{Tr}\Big(g^{D\dagger} y^{DT} y^{D*} T^{g^D}\Big) \\
&+ 540 \operatorname{Tr}\Big(g^{D\dagger} y^{UT} y^{U*} T^{g^D}\Big) + 540 \operatorname{Tr}\Big(y^{D\dagger} T^D g^{D*} g^{DT}\Big) \\
&+ 540 \operatorname{Tr}\Big(y^{U\dagger} T^U g^{D*} g^{DT}\Big)\Big],
\end{aligned}
\tag{A.65}
$$

$$
\begin{aligned}
\beta_{T^\kappa}^{(1)} = {} & 4\kappa g^{D\dagger} T^{g^D} + 3\kappa\kappa^\dagger T^\kappa + 2T^\kappa g^{D\dagger} g^D + 3T^\kappa \kappa^\dagger \kappa - \frac{4}{15} g_1^2 T^\kappa - \frac{19}{10} g_1'^2 T^\kappa \\
& - \frac{16}{3} g_3^2 T^\kappa + 2|\lambda|^2 T^\kappa + |\sigma|^2 T^\kappa + 3T^\kappa \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 2T^\kappa \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
& + \kappa \bigg[2\sigma^* T_\sigma + 4\lambda^* T_\lambda + 4 \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 6 \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) + \frac{19}{5} g_1'^2 M_1' \\
& + \frac{32}{3} g_3^2 M_3 + \frac{8}{15} g_1^2 M_1 \bigg],
\end{aligned}
\tag{A.66}
$$

$$
\begin{aligned}
\beta_{T^\kappa}^{(2)} = {} & \frac{4}{5} g_1^2 \kappa g^{D\dagger} T^{g^D} - \frac{4}{5} g_1'^2 \kappa g^{D\dagger} T^{g^D} + 12 g_2^2 \kappa g^{D\dagger} T^{g^D} - 4|\tilde{\sigma}|^2 \kappa g^{D\dagger} T^{g^D} \\
& - 5 g_1'^2 M_1' \kappa\kappa^\dagger \kappa + \frac{7}{2} g_1'^2 \kappa\kappa^\dagger T^\kappa - 6|\lambda|^2 \kappa\kappa^\dagger T^\kappa - 3|\sigma|^2 \kappa\kappa^\dagger T^\kappa + \frac{2}{5} g_1^2 T^\kappa g^{D\dagger} g^D \\
& - \frac{2}{5} g_1'^2 T^\kappa g^{D\dagger} g^D + 6 g_2^2 T^\kappa g^{D\dagger} g^D - 2|\tilde{\sigma}|^2 T^\kappa g^{D\dagger} g^D + 4 g_1'^2 T^\kappa \kappa^\dagger \kappa \\
& - 6|\lambda|^2 T^\kappa \kappa^\dagger \kappa - 3|\sigma|^2 T^\kappa \kappa^\dagger \kappa - 4\kappa g^{D\dagger} g^D g^{D\dagger} T^{g^D} - 2\kappa g^{D\dagger} g^D \kappa^\dagger T^\kappa \\
& - 4\kappa g^{D\dagger} T^{g^D} g^{D\dagger} g^D - 4\kappa g^{D\dagger} T^{g^D} \kappa^\dagger \kappa - 4\kappa g^{D\dagger} y^{DT} y^{D*} T^{g^D} \\
& - 4\kappa g^{D\dagger} y^{UT} y^{U*} T^{g^D} - 4\kappa g^{D\dagger} T^{DT} y^{D*} g^D - 4\kappa g^{D\dagger} T^{UT} y^{U*} g^D - 3\kappa\kappa^\dagger \kappa\kappa^\dagger T^\kappa \\
& - 4\kappa\kappa^\dagger T^\kappa \kappa^\dagger \kappa - 2T^\kappa g^{D\dagger} g^D g^{D\dagger} g^D - 4T^\kappa g^{D\dagger} g^D \kappa^\dagger \kappa - 2T^\kappa g^{D\dagger} y^{DT} y^{D*} g^D \\
& - 2T^\kappa g^{D\dagger} y^{UT} y^{U*} g^D - 3T^\kappa \kappa^\dagger \kappa\kappa^\dagger \kappa + \frac{584}{225} g_1^4 T^\kappa + \frac{19}{75} g_1^2 g_1'^2 T^\kappa + \frac{551}{25} g_1'^4 T^\kappa \\
& + \frac{64}{45} g_1^2 g_3^2 T^\kappa + \frac{52}{15} g_1'^2 g_3^2 T^\kappa + \frac{128}{9} g_3^4 T^\kappa + \frac{6}{5} g_1^2 |\lambda|^2 T^\kappa - \frac{6}{5} g_1'^2 |\lambda|^2 T^\kappa \\
& + 6 g_2^2 |\lambda|^2 T^\kappa - 4|\lambda|^4 T^\kappa - 2|\sigma|^2 |\kappa_\phi|^2 T^\kappa - 2|\sigma|^4 T^\kappa - 2|\tilde{\sigma}|^2 |\sigma|^2 T^\kappa \\
& - 8\lambda^* \kappa\kappa^\dagger \kappa T_\lambda - 4\sigma^* \kappa\kappa^\dagger \kappa T_\sigma - 2|\lambda|^2 T^\kappa \operatorname{Tr}\Big(f f^\dagger\Big) - 2|\lambda|^2 T^\kappa \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
& - 12\kappa g^{D\dagger} T^{g^D} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 6T^\kappa g^{D\dagger} g^D \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
& - 4\kappa g^{D\dagger} T^{g^D} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 2T^\kappa g^{D\dagger} g^D \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 6|\lambda|^2 T^\kappa \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
& - 2|\lambda|^2 T^\kappa \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 6|\lambda|^2 T^\kappa \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 9\kappa\kappa^\dagger T^\kappa \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)
\end{aligned}
$$

$$
\begin{aligned}
&-9T^{\kappa}\kappa^{\dagger}\kappa\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)+\frac{4}{5}g_1^2T^{\kappa}\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)-\frac{9}{5}g_1'^2T^{\kappa}\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)+16g_3^2T^{\kappa}\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)\\
&-6\kappa\kappa^{\dagger}T^{\kappa}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)-6T^{\kappa}\kappa^{\dagger}\kappa\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)+\frac{6}{5}g_1^2T^{\kappa}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)-\frac{6}{5}g_1'^2T^{\kappa}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)\\
&+6g_2^2T^{\kappa}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)-\frac{4}{5}\kappa g^{D\dagger}g^{D}\Big[15g_2^2M_2+15\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big)+5\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big)\\
&+5\tilde{\sigma}^*T_{\tilde{\sigma}}+g_1^2M_1-g_1'^2M_1'\Big]-12\kappa\kappa^{\dagger}\kappa\,\mathrm{Tr}\Big(\kappa^{\dagger}T^{\kappa}\Big)-8\kappa\kappa^{\dagger}\kappa\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\Big)\\
&-2T^{\kappa}\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^{\dagger}\tilde{\lambda}\tilde{f}^{\dagger}\Big)-6T^{\kappa}\,\mathrm{Tr}\Big(g^{D}\kappa^{\dagger}\kappa g^{D\dagger}\Big)-2T^{\kappa}\,\mathrm{Tr}\Big(h^{E}\tilde{\lambda}^{\dagger}\tilde{\lambda}h^{E\dagger}\Big)\\
&-6T^{\kappa}\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\kappa\kappa^{\dagger}\Big)-4T^{\kappa}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)-2T^{\kappa}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}f^{T}f^{*}\Big)\\
&-\frac{2}{225}\kappa\Big(1168g_1^4M_1+57g_1^2g_1'^2M_1+320g_1^2g_3^2M_1+57g_1^2g_1'^2M_1'+9918g_1'^4M_1'\\
&+780g_1'^2g_3^2M_1'+320g_1^2g_3^2M_3+780g_1'^2g_3^2M_3+6400g_3^4M_3+1800\lambda^*|\lambda|^2T_{\lambda}\\
&+450|\tilde{\sigma}|^2\sigma^*T_{\sigma}+900\sigma^*|\sigma|^2T_{\sigma}+450\kappa_{\phi}^*\sigma^*\Big(\kappa_{\phi}T_{\sigma}+\sigma T_{\kappa_{\phi}}\Big)+450|\sigma|^2\tilde{\sigma}^*T_{\tilde{\sigma}}\\
&+180g_1^2M_1\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)-405g_1'^2M_1'\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)+3600g_3^2M_3\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)\\
&+270g_1^2M_1\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)-270g_1'^2M_1'\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)+1350g_2^2M_2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)\\
&+90\lambda^*\Big\{T_{\lambda}\Big[-15g_2^2+15\,\mathrm{Tr}\Big(y^{D}y^{D\dagger}\Big)+15\,\mathrm{Tr}\Big(y^{U}y^{U\dagger}\Big)-3g_1^2+3g_1'^2\\
&+5\,\mathrm{Tr}\Big(ff^{\dagger}\Big)+5\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^{\dagger}\Big)+5\,\mathrm{Tr}\Big(y^{E}y^{E\dagger}\Big)\Big]+\lambda\Big[3g_1^2M_1-3g_1'^2M_1'\\
&+15g_2^2M_2+5\,\mathrm{Tr}\Big(f^{\dagger}T^{f}\Big)+5\,\mathrm{Tr}\Big(\tilde{f}^{\dagger}T^{\tilde{f}}\Big)+15\,\mathrm{Tr}\Big(y^{D\dagger}T^{D}\Big)+5\,\mathrm{Tr}\Big(y^{E\dagger}T^{E}\Big)\\
&+15\,\mathrm{Tr}\Big(y^{U\dagger}T^{U}\Big)\Big]\Big\}-180g_1^2\,\mathrm{Tr}\Big(\kappa^{\dagger}T^{\kappa}\Big)+405g_1'^2\,\mathrm{Tr}\Big(\kappa^{\dagger}T^{\kappa}\Big)\\
&-3600g_3^2\,\mathrm{Tr}\Big(\kappa^{\dagger}T^{\kappa}\Big)-270g_1^2\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\Big)+270g_1'^2\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\Big)\\
&-1350g_2^2\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\Big)+450\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\tilde{f}^{\dagger}\Big)+1350\,\mathrm{Tr}\Big(g^{D}\kappa^{\dagger}T^{\kappa}g^{D\dagger}\Big)\\
&+450\,\mathrm{Tr}\Big(h^{E}\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}h^{E\dagger}\Big)+1350\,\mathrm{Tr}\Big(\kappa g^{D\dagger}T^{g^D}\kappa^{\dagger}\Big)+2700\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}T^{\kappa}\kappa^{\dagger}\Big)\\
&+450\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{f}^{\dagger}T^{\tilde{f}}\tilde{\lambda}^{\dagger}\Big)+450\,\mathrm{Tr}\Big(\tilde{\lambda}h^{E\dagger}T^{h^E}\tilde{\lambda}^{\dagger}\Big)+1800\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\tilde{\lambda}^{\dagger}\Big)\\
&+450\,\mathrm{Tr}\Big(f^{\dagger}T^{f}\tilde{\lambda}^*\tilde{\lambda}^{T}\Big)+450\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}f^{T}f^{*}T^{\tilde{\lambda}}\Big)\Big), \qquad \text{(A.67)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T^{\tilde{\lambda}}}^{(1)}=&\,2\tilde{\lambda}\tilde{f}^{\dagger}T^{\tilde{f}}+2\tilde{\lambda}h^{E\dagger}T^{h^E}+3\tilde{\lambda}\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}+T^{\tilde{\lambda}}\tilde{f}^{\dagger}\tilde{f}+T^{\tilde{\lambda}}h^{E\dagger}h^{E}+3T^{\tilde{\lambda}}\tilde{\lambda}^{\dagger}\tilde{\lambda}\\
&+f^{T}f^{*}T^{\tilde{\lambda}}+2T^{fT}f^{*}\tilde{\lambda}-\frac{3}{5}g_1^2T^{\tilde{\lambda}}-\frac{19}{10}g_1'^2T^{\tilde{\lambda}}-3g_2^2T^{\tilde{\lambda}}+2|\lambda|^2T^{\tilde{\lambda}}+|\sigma|^2T^{\tilde{\lambda}}\\
&+3T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\kappa\kappa^{\dagger}\Big)+2T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^{\dagger}\Big)+\tilde{\lambda}\Bigg[2\sigma^*T_{\sigma}+4\lambda^*T_{\lambda}+4\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\Big)\\
&+6g_2^2M_2+6\,\mathrm{Tr}\Big(\kappa^{\dagger}T^{\kappa}\Big)+\frac{19}{5}g_1'^2M_1'+\frac{6}{5}g_1^2M_1\Bigg], \qquad \text{(A.68)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{T^{\tilde\lambda}}^{(2)} = {} & 2g_1'^2\tilde\lambda\tilde f^\dagger T^{\tilde f} - 2|\lambda|^2\tilde\lambda\tilde f^\dagger T^{\tilde f} - \frac{12}{5}g_1^2 M_1\tilde\lambda h^{E\dagger}h^E + \frac{2}{5}g_1'^2 M_1'\tilde\lambda h^{E\dagger}h^E \\
& + \frac{12}{5}g_1^2\tilde\lambda h^{E\dagger}T^{h^E} - \frac{2}{5}g_1'^2\tilde\lambda h^{E\dagger}T^{h^E} - 2|\tilde\sigma|^2\tilde\lambda h^{E\dagger}T^{h^E} - 5g_1'^2 M_1'\tilde\lambda\tilde\lambda^\dagger\tilde\lambda \\
& + \frac{7}{2}g_1'^2\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda} - 6|\lambda|^2\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda} - 3|\sigma|^2\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda} + g_1'^2 T^{\tilde\lambda}\tilde f^\dagger\tilde f \\
& - |\lambda|^2 T^{\tilde\lambda}\tilde f^\dagger\tilde f + \frac{6}{5}g_1^2 T^{\tilde\lambda}h^{E\dagger}h^E - \frac{1}{5}g_1'^2 T^{\tilde\lambda}h^{E\dagger}h^E - |\tilde\sigma|^2 T^{\tilde\lambda}h^{E\dagger}h^E \\
& + 4g_1'^2 T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda - 6|\lambda|^2 T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda - 3|\sigma|^2 T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda - 3g_1'^2 M_1' f^T f^*\tilde\lambda \\
& + \frac{3}{2}g_1'^2 f^T f^* T^{\tilde\lambda} - |\lambda|^2 f^T f^* T^{\tilde\lambda} + 3g_1'^2 T^{fT} f^*\tilde\lambda - 2|\lambda|^2 T^{fT} f^*\tilde\lambda \\
& - 4\tilde\lambda\tilde f^\dagger f f^\dagger T^{\tilde f} - 4\tilde\lambda\tilde f^\dagger\tilde f\tilde f^\dagger T^{\tilde f} - \tilde\lambda\tilde f^\dagger\tilde f\tilde\lambda^\dagger T^{\tilde\lambda} - 4\tilde\lambda\tilde f^\dagger T^f f^\dagger\tilde f \\
& - 4\tilde\lambda\tilde f^\dagger T^{\tilde f}\tilde f^\dagger\tilde f - 2\tilde\lambda\tilde f^\dagger T^{\tilde f}\tilde\lambda^\dagger\tilde\lambda - 4\tilde\lambda h^{E\dagger}h^E h^{E\dagger}T^{h^E} - \tilde\lambda h^{E\dagger}h^E\tilde\lambda^\dagger T^{\tilde\lambda} \\
& - 4\tilde\lambda h^{E\dagger}y^E y^{E\dagger}T^{h^E} - 4\tilde\lambda h^{E\dagger}T^{h^E}h^{E\dagger}h^E - 2\tilde\lambda h^{E\dagger}T^{h^E}\tilde\lambda^\dagger\tilde\lambda \\
& - 4\tilde\lambda h^{E\dagger}T^E y^{E\dagger}h^E - 3\tilde\lambda\tilde\lambda^\dagger\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda} - 4\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda - 2\tilde\lambda\tilde\lambda^\dagger f^T f^* T^{\tilde\lambda} \\
& - 2\tilde\lambda\tilde\lambda^\dagger T^{fT} f^*\tilde\lambda - 2T^{\tilde\lambda}\tilde f^\dagger f f^\dagger\tilde f - 2T^{\tilde\lambda}\tilde f^\dagger\tilde f\tilde f^\dagger\tilde f - 2T^{\tilde\lambda}\tilde f^\dagger\tilde f\tilde\lambda^\dagger\tilde\lambda \\
& - 2T^{\tilde\lambda}h^{E\dagger}h^E h^{E\dagger}h^E - 2T^{\tilde\lambda}h^{E\dagger}h^E\tilde\lambda^\dagger\tilde\lambda - 2T^{\tilde\lambda}h^{E\dagger}y^E y^{E\dagger}h^E \\
& - 3T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda\tilde\lambda^\dagger\tilde\lambda - T^{\tilde\lambda}\tilde\lambda^\dagger f^T f^*\tilde\lambda - 2f^T f^* f^T f^* T^{\tilde\lambda} - 4f^T f^* T^{fT} f^*\tilde\lambda \\
& - 2f^T\tilde f^*\tilde f^T f^* T^{\tilde\lambda} - 4f^T\tilde f^* T^{\tilde f T} f^*\tilde\lambda - 4T^{fT} f^* f^T f^*\tilde\lambda - 4T^{fT}\tilde f^*\tilde f^T f^*\tilde\lambda \\
& - 8\lambda^*\tilde\lambda\tilde\lambda^\dagger\tilde\lambda T_\lambda - 2\lambda^* f^T f^*\tilde\lambda T_\lambda + \frac{297}{50}g_1^4 T^{\tilde\lambda} + \frac{27}{100}g_1^2 g_1'^2 T^{\tilde\lambda} + \frac{551}{25}g_1'^4 T^{\tilde\lambda} \\
& + \frac{9}{5}g_1^2 g_2^2 T^{\tilde\lambda} + \frac{39}{20}g_1'^2 g_2^2 T^{\tilde\lambda} + \frac{33}{2}g_2^4 T^{\tilde\lambda} + \frac{6}{5}g_1^2|\lambda|^2 T^{\tilde\lambda} - \frac{6}{5}g_1'^2|\lambda|^2 T^{\tilde\lambda} \\
& + 6g_2^2|\lambda|^2 T^{\tilde\lambda} - 4|\lambda|^4 T^{\tilde\lambda} - 2|\sigma|^2|\kappa_\phi|^2 T^{\tilde\lambda} - 2|\sigma|^4 T^{\tilde\lambda} - 2|\tilde\sigma|^2|\sigma|^2 T^{\tilde\lambda} \\
& - 4\sigma^*\tilde\lambda\tilde\lambda^\dagger\tilde\lambda T_\sigma - 2\tilde\sigma^*\tilde\lambda h^{E\dagger}h^E T_{\tilde\sigma} - f^T f^* T^{\tilde\lambda}\,\mathrm{Tr}\Big(f f^\dagger\Big) - 2T^{fT} f^*\tilde\lambda\,\mathrm{Tr}\Big(f f^\dagger\Big) \\
& - 2|\lambda|^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(f f^\dagger\Big) - 2\tilde\lambda\tilde f^\dagger T^{\tilde f}\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big) - T^{\tilde\lambda}\tilde f^\dagger\tilde f\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big) \\
& - 2|\lambda|^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big) - 6\tilde\lambda h^{E\dagger}T^{h^E}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 3T^{\tilde\lambda}h^{E\dagger}h^E\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
& - 2\tilde\lambda h^{E\dagger}T^{h^E}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - T^{\tilde\lambda}h^{E\dagger}h^E\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 3f^T f^* T^{\tilde\lambda}\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) \\
& - 6T^{fT} f^*\tilde\lambda\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 6|\lambda|^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - f^T f^* T^{\tilde\lambda}\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) \\
& - 2T^{fT} f^*\tilde\lambda\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 2|\lambda|^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 6\tilde\lambda\tilde f^\dagger T^{\tilde f}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
& - 3T^{\tilde\lambda}\tilde f^\dagger\tilde f\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 6|\lambda|^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 9\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
& - 9T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + \frac{4}{5}g_1^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - \frac{9}{5}g_1'^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
& + 16g_3^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 6\tilde\lambda\tilde\lambda^\dagger T^{\tilde\lambda}\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 6T^{\tilde\lambda}\tilde\lambda^\dagger\tilde\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) \\
& + \frac{6}{5}g_1^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - \frac{6}{5}g_1'^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) + 6g_2^2 T^{\tilde\lambda}\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&- 2f^T f^* \tilde{\lambda}\,\mathrm{Tr}\Big(f^\dagger T^{f}\Big) - 6\tilde{\lambda} h^{E\dagger} h^E\,\mathrm{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 2\tilde{\lambda} h^{E\dagger} h^E\,\mathrm{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&- 6f^T f^* \tilde{\lambda}\,\mathrm{Tr}\Big(y^{D\dagger} T^{D}\Big) - 2f^T f^* \tilde{\lambda}\,\mathrm{Tr}\Big(y^{E\dagger} T^{E}\Big) - 2\tilde{\lambda} \tilde{f}^\dagger \tilde{f}\Big[3\,\mathrm{Tr}\Big(y^{U\dagger} T^{U}\Big) \\
&+ g_1'^2 M_1' + \lambda^* T_\lambda + \mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big)\Big] - 12\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) - 8\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) \\
&- 2T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger\Big) - 6T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) - 2T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big) \\
&- 6T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big) - 4T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big) \\
&- \frac{1}{50}\tilde{\lambda}\Big(1188 g_1^4 M_1 + 27 g_1^2 g_1'^2 M_1 + 180 g_1^2 g_2^2 M_1 + 27 g_1^2 g_1'^2 M_1' + 4408 g_1'^4 M_1' \\
&+ 195 g_1'^2 g_2^2 M_1' + 180 g_1^2 g_2^2 M_2 + 195 g_1'^2 g_2^2 M_2 + 3300 g_2^4 M_2 + 800\lambda^*|\lambda|^2 T_\lambda \\
&+ 200|\tilde{\sigma}|^2\sigma^* T_\sigma + 400\sigma^*|\sigma|^2 T_\sigma + 200\kappa_\phi^*\sigma^*\Big(\kappa_\phi T_\sigma + \sigma T_{\kappa_\phi}\Big) + 200|\sigma|^2\tilde{\sigma}^* T_{\tilde{\sigma}} \\
&+ 80 g_1^2 M_1\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 180 g_1'^2 M_1'\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 1600 g_3^2 M_3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&+ 120 g_1^2 M_1\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 120 g_1'^2 M_1'\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 600 g_2^2 M_2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&+ 40\lambda^*\Big\{T_\lambda\Big[-15 g_2^2 + 15\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) + 15\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 3g_1^2 + 3g_1'^2 \\
&+ 5\,\mathrm{Tr}\Big(f f^\dagger\Big) + 5\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + 5\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big)\Big] + \lambda\Big[3 g_1^2 M_1 - 3 g_1'^2 M_1' \\
&+ 15 g_2^2 M_2 + 5\,\mathrm{Tr}\Big(f^\dagger T^{f}\Big) + 5\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) + 15\,\mathrm{Tr}\Big(y^{D\dagger} T^{D}\Big) \\
&+ 5\,\mathrm{Tr}\Big(y^{E\dagger} T^{E}\Big) + 15\,\mathrm{Tr}\Big(y^{U\dagger} T^{U}\Big)\Big]\Big\} - 80 g_1^2\,\mathrm{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) + 180 g_1'^2\,\mathrm{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) \\
&- 1600 g_3^2\,\mathrm{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) - 120 g_1^2\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 120 g_1'^2\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) \\
&- 600 g_2^2\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 200\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\tilde{f}^\dagger\Big) + 600\,\mathrm{Tr}\Big(g^D\kappa^\dagger T^{\kappa} g^{D\dagger}\Big) \\
&+ 200\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger T^{\tilde{\lambda}} h^{E\dagger}\Big) + 600\,\mathrm{Tr}\Big(\kappa g^{D\dagger} T^{g^D}\kappa^\dagger\Big) + 1200\,\mathrm{Tr}\Big(\kappa\kappa^\dagger T^{\kappa}\kappa^\dagger\Big) \\
&+ 200\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}}\tilde{\lambda}^\dagger\Big) + 200\,\mathrm{Tr}\Big(\tilde{\lambda} h^{E\dagger} T^{h^E}\tilde{\lambda}^\dagger\Big) + 800\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\tilde{\lambda}^\dagger\Big) \\
&+ 200\,\mathrm{Tr}\Big(f^\dagger T^{f}\tilde{\lambda}^*\tilde{\lambda}^T\Big) + 200\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}}\Big)\Big),
\end{aligned}
\tag{A.69}
$$

$$
\begin{aligned}
\beta_{T_\lambda}^{(1)} = {} & T_\lambda\Big[-\frac{3}{5}g_1^2 - \frac{19}{10}g_1'^2 - 3g_2^2 + 12|\lambda|^2 + |\sigma|^2 + \mathrm{Tr}\Big(f f^\dagger\Big) + \mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&+ 3\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) + \mathrm{Tr}\Big(y^E y^{E\dagger}\Big) + 3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) + 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big)\Big] \\
&+ \frac{1}{5}\lambda\Big[6 g_1^2 M_1 + 19 g_1'^2 M_1' + 30 g_2^2 M_2 + 10\sigma^* T_\sigma + 10\,\mathrm{Tr}\Big(f^\dagger T^{f}\Big) \\
&+ 10\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) + 30\,\mathrm{Tr}\Big(y^{D\dagger} T^{D}\Big) + 10\,\mathrm{Tr}\Big(y^{E\dagger} T^{E}\Big) + 30\,\mathrm{Tr}\Big(y^{U\dagger} T^{U}\Big) \\
&+ 30\,\mathrm{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) + 20\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big)\Big],
\end{aligned}
\tag{A.70}
$$

$$
\begin{aligned}
\beta_{T_\lambda}^{(2)} = {} & -\frac{594}{25} g_1^4 M_1 \lambda - \frac{27}{50} g_1^2 g_1'^2 M_1 \lambda - \frac{18}{5} g_1^2 g_2^2 M_1 \lambda - \frac{27}{50} g_1^2 g_1'^2 M_1' \lambda - \frac{2204}{25} g_1'^4 M_1' \lambda \\
&- \frac{39}{10} g_1'^2 g_2^2 M_1' \lambda - \frac{18}{5} g_1^2 g_2^2 M_2 \lambda - \frac{39}{10} g_1'^2 g_2^2 M_2 \lambda - 66 g_2^4 M_2 \lambda + \frac{297}{50} g_1^4 T_\lambda
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{27}{100}g_1^2g_1'^2T_\lambda+\frac{551}{25}g_1'^4T_\lambda+\frac{9}{5}g_1^2g_2^2T_\lambda+\frac{39}{20}g_1'^2g_2^2T_\lambda+\frac{33}{2}g_2^4T_\lambda-50|\lambda|^4T_\lambda\\
&-2|\sigma|^4T_\lambda-2|\tilde\sigma|^2|\sigma|^2T_\lambda-4\lambda|\tilde\sigma|^2\sigma^*T_\sigma-8\lambda\sigma^*|\sigma|^2T_\sigma-2\kappa_\phi^*\sigma^*\Big(2\kappa_\phi\lambda T_\sigma\\
&+2\lambda\sigma T_{\kappa_\phi}+\kappa_\phi\sigma T_\lambda\Big)-4\lambda|\sigma|^2\tilde\sigma^*T_{\tilde\sigma}-2g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(ff^\dagger\Big)+g_1'^2T_\lambda\,\mathrm{Tr}\Big(ff^\dagger\Big)\\
&-3g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)+\frac{3}{2}g_1'^2T_\lambda\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)+\frac{4}{5}g_1^2M_1\lambda\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&+\frac{6}{5}g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-32g_3^2M_3\lambda\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-\frac{2}{5}g_1^2T_\lambda\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&-\frac{3}{5}g_1'^2T_\lambda\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)+16g_3^2T_\lambda\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-\frac{12}{5}g_1^2M_1\lambda\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)\\
&+\frac{2}{5}g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)+\frac{6}{5}g_1^2T_\lambda\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-\frac{1}{5}g_1'^2T_\lambda\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)\\
&-\frac{8}{5}g_1^2M_1\lambda\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)+\frac{3}{5}g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-32g_3^2M_3\lambda\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)\\
&+\frac{4}{5}g_1^2T_\lambda\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-\frac{3}{10}g_1'^2T_\lambda\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)+16g_3^2T_\lambda\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)\\
&-\frac{8}{5}g_1^2M_1\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)+\frac{18}{5}g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-32g_3^2M_3\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\\
&+\frac{4}{5}g_1^2T_\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-\frac{9}{5}g_1'^2T_\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)+16g_3^2T_\lambda\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-\frac{12}{5}g_1^2M_1\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)\\
&+\frac{12}{5}g_1'^2M_1'\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)-12g_2^2M_2\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+\frac{6}{5}g_1^2T_\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)\\
&-\frac{6}{5}g_1'^2T_\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+6g_2^2T_\lambda\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)+2g_1'^2\lambda\,\mathrm{Tr}\Big(f^\dagger T^f\Big)+3g_1'^2\lambda\,\mathrm{Tr}\Big(\tilde f^\dagger T^{\tilde f}\Big)\\
&-\frac{4}{5}g_1^2\lambda\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)-\frac{6}{5}g_1'^2\lambda\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)+32g_3^2\lambda\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)\\
&+\frac{12}{5}g_1^2\lambda\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)-\frac{2}{5}g_1'^2\lambda\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)+\frac{8}{5}g_1^2\lambda\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)\\
&-\frac{3}{5}g_1'^2\lambda\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)+32g_3^2\lambda\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)+\frac{8}{5}g_1^2\lambda\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)\\
&-\frac{18}{5}g_1'^2\lambda\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)+32g_3^2\lambda\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)+\frac{12}{5}g_1^2\lambda\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)\\
&-\frac{12}{5}g_1'^2\lambda\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)+12g_2^2\lambda\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)-\frac{1}{10}|\lambda|^2\Big\{3T_\lambda\Big[-12g_1^2-13g_1'^2\\
&-60g_2^2+20|\sigma|^2+30\,\mathrm{Tr}\Big(ff^\dagger\Big)+30\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\Big)+90\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&+30\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)+90\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)+60\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)+40\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)\Big]\\
&+2\lambda\Big[12g_1^2M_1+13g_1'^2M_1'+60g_2^2M_2+20\sigma^*T_\sigma+30\,\mathrm{Tr}\Big(f^\dagger T^f\Big)\\
&+30\,\mathrm{Tr}\Big(\tilde f^\dagger T^{\tilde f}\Big)+90\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)+30\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)+90\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)\\
&+60\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)+40\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)\Big]\Big\}-3T_\lambda\,\mathrm{Tr}\Big(ff^\dagger ff^\dagger\Big)-4T_\lambda\,\mathrm{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big)\\
&-12\lambda\,\mathrm{Tr}\Big(ff^\dagger T^ff^\dagger\Big)-8\lambda\,\mathrm{Tr}\Big(ff^\dagger T^{\tilde f}\tilde f^\dagger\Big)-3T_\lambda\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger\tilde f\tilde f^\dagger\Big)\\
&-8\lambda\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger T^ff^\dagger\Big)-12\lambda\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger T^{\tilde f}\tilde f^\dagger\Big)-T_\lambda\,\mathrm{Tr}\Big(\tilde fh^{E\dagger}h^E\tilde f^\dagger\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 2\lambda \,\mathrm{Tr}\Big(\tilde{f} h^{E\dagger} T^{h^E} \tilde{f}^\dagger\Big) - 3T_\lambda \,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger\Big) - 6\lambda \,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\tilde{f}^\dagger\Big) \\
&- 3T_\lambda \,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 3T_\lambda \,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 6T_\lambda \,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) \\
&- 12\lambda \,\mathrm{Tr}\Big(g^D \kappa^\dagger T^\kappa g^{D\dagger}\Big) - 2\lambda \,\mathrm{Tr}\Big(h^E \tilde{f}^\dagger T^{\tilde{f}} h^{E\dagger}\Big) - 2T_\lambda \,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) \\
&- 4\lambda \,\mathrm{Tr}\Big(h^E h^{E\dagger} T^E y^{E\dagger}\Big) - 2T_\lambda \,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big) - 4\lambda \,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} h^{E\dagger}\Big) \\
&- 9T_\lambda \,\mathrm{Tr}\Big(y^D y^{D\dagger} y^D y^{D\dagger}\Big) - 36\lambda \,\mathrm{Tr}\Big(y^D y^{D\dagger} T^D y^{D\dagger}\Big) - 6T_\lambda \,\mathrm{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) \\
&- 12\lambda \,\mathrm{Tr}\Big(y^D y^{U\dagger} T^U y^{D\dagger}\Big) - 3T_\lambda \,\mathrm{Tr}\Big(y^E y^{E\dagger} y^E y^{E\dagger}\Big) - 4\lambda \,\mathrm{Tr}\Big(y^E y^{E\dagger} T^{h^E} h^{E\dagger}\Big) \\
&- 12\lambda \,\mathrm{Tr}\Big(y^E y^{E\dagger} T^E y^{E\dagger}\Big) - 12\lambda \,\mathrm{Tr}\Big(y^U y^{D\dagger} T^D y^{U\dagger}\Big) - 9T_\lambda \,\mathrm{Tr}\Big(y^U y^{U\dagger} y^U y^{U\dagger}\Big) \\
&- 36\lambda \,\mathrm{Tr}\Big(y^U y^{U\dagger} T^U y^{U\dagger}\Big) - 12\lambda \,\mathrm{Tr}\Big(\kappa g^{D\dagger} T^{g^D} \kappa^\dagger\Big) - 6T_\lambda \,\mathrm{Tr}\Big(\kappa\kappa^\dagger\kappa\kappa^\dagger\Big) \\
&- 24\lambda \,\mathrm{Tr}\Big(\kappa\kappa^\dagger T^\kappa \kappa^\dagger\Big) - 6\lambda \,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}}\tilde{\lambda}^\dagger\Big) - 4\lambda \,\mathrm{Tr}\Big(\tilde{\lambda} h^{E\dagger} T^{h^E}\tilde{\lambda}^\dagger\Big) \\
&- 4T_\lambda \,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 16\lambda \,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\tilde{\lambda}^\dagger\Big) - 3T_\lambda \,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big) \\
&- 6\lambda \,\mathrm{Tr}\Big(f^\dagger T^f \tilde{\lambda}^* \tilde{\lambda}^T\Big) - 6\lambda \,\mathrm{Tr}\Big(g^{D\dagger} y^{DT} y^{D*} T^{g^D}\Big) - 6\lambda \,\mathrm{Tr}\Big(g^{D\dagger} y^{UT} y^{U*} T^{g^D}\Big) \\
&- 6\lambda \,\mathrm{Tr}\Big(y^{D\dagger} T^D g^{D*} g^{DT}\Big) - 6\lambda \,\mathrm{Tr}\Big(y^{U\dagger} T^U g^{D*} g^{DT}\Big) - 6\lambda \,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}}\Big),
\end{aligned}
\tag{A.71}
$$

$$
\begin{aligned}
\beta^{(1)}_{T^{\tilde{f}}} &= 2 f f^\dagger T^{\tilde{f}} + 4\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} + 2\tilde{f} h^{E\dagger} T^{h^E} + 2\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} + 4T^f f^\dagger \tilde{f} + 5T^{\tilde{f}}\tilde{f}^\dagger\tilde{f} \\
&+ T^{\tilde{f}} h^{E\dagger} h^E + T^{\tilde{f}}\tilde{\lambda}^\dagger\tilde{\lambda} - \frac{3}{5} g_1^2 T^{\tilde{f}} - \frac{19}{10} g_1'^2 T^{\tilde{f}} - 3g_2^2 T^{\tilde{f}} + |\lambda|^2 T^{\tilde{f}} \\
&+ T^{\tilde{f}} \,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + 3T^{\tilde{f}} \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) + \tilde{f}\Bigg[2\lambda^* T_\lambda + 2\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) + 6g_2^2 M_2 \\
&+ 6\,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) + \frac{19}{5} g_1'^2 M_1' + \frac{6}{5} g_1^2 M_1\Bigg],
\end{aligned}
\tag{A.72}
$$

$$
\begin{aligned}
\beta^{(2)}_{T^{\tilde{f}}} &= \frac{6}{5} g_1^2 f f^\dagger T^{\tilde{f}} - \frac{6}{5} g_1'^2 f f^\dagger T^{\tilde{f}} + 6g_2^2 f f^\dagger T^{\tilde{f}} - 2|\lambda|^2 f f^\dagger T^{\tilde{f}} \\
&- \frac{12}{5} g_1^2 M_1 \tilde{f}\tilde{f}^\dagger\tilde{f} + \frac{2}{5} g_1'^2 M_1' \tilde{f}\tilde{f}^\dagger\tilde{f} - 12 g_2^2 M_2 \tilde{f}\tilde{f}^\dagger\tilde{f} + \frac{6}{5} g_1^2 \tilde{f}\tilde{f}^\dagger T^{\tilde{f}} \\
&+ \frac{4}{5} g_1'^2 \tilde{f}\tilde{f}^\dagger T^{\tilde{f}} + 6g_2^2 \tilde{f}\tilde{f}^\dagger T^{\tilde{f}} - 4|\lambda|^2 \tilde{f}\tilde{f}^\dagger T^{\tilde{f}} - \frac{12}{5} g_1^2 M_1 \tilde{f} h^{E\dagger} h^E \\
&+ \frac{2}{5} g_1'^2 M_1' \tilde{f} h^{E\dagger} h^E + \frac{12}{5} g_1^2 \tilde{f} h^{E\dagger} T^{h^E} - \frac{2}{5} g_1'^2 \tilde{f} h^{E\dagger} T^{h^E} - 2|\tilde{\sigma}|^2 \tilde{f} h^{E\dagger} T^{h^E} \\
&- 2g_1'^2 M_1' \tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda} + 2g_1'^2 \tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 4|\lambda|^2 \tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 2|\sigma|^2 \tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \\
&+ \frac{12}{5} g_1^2 T^f f^\dagger \tilde{f} - \frac{12}{5} g_1'^2 T^f f^\dagger \tilde{f} + 12 g_2^2 T^f f^\dagger \tilde{f} - 4|\lambda|^2 T^f f^\dagger \tilde{f} \\
&+ \frac{12}{5} g_1^2 T^{\tilde{f}}\tilde{f}^\dagger\tilde{f} - \frac{7}{5} g_1'^2 T^{\tilde{f}}\tilde{f}^\dagger\tilde{f} + 12 g_2^2 T^{\tilde{f}}\tilde{f}^\dagger\tilde{f} - 5|\lambda|^2 T^{\tilde{f}}\tilde{f}^\dagger\tilde{f} \\
&+ \frac{6}{5} g_1^2 T^{\tilde{f}} h^{E\dagger} h^E - \frac{1}{5} g_1'^2 T^{\tilde{f}} h^{E\dagger} h^E - |\tilde{\sigma}|^2 T^{\tilde{f}} h^{E\dagger} h^E + g_1'^2 T^{\tilde{f}}\tilde{\lambda}^\dagger\tilde{\lambda} \\
&- 2|\lambda|^2 T^{\tilde{f}}\tilde{\lambda}^\dagger\tilde{\lambda} - |\sigma|^2 T^{\tilde{f}}\tilde{\lambda}^\dagger\tilde{\lambda} - 2 f f^\dagger f f^\dagger T^{\tilde{f}} - 4 f f^\dagger T^f f^\dagger \tilde{f}
\end{aligned}
$$

$$
\begin{aligned}
&- 2f\tilde{\lambda}^*\tilde{\lambda}^T f^\dagger T^{\tilde{f}} - 4f\tilde{\lambda}^* T^{\tilde{\lambda}T} f^\dagger \tilde{f} - 4\tilde{f}\tilde{f}^\dagger f f^\dagger T^{\tilde{f}} - 6\tilde{f}\tilde{f}^\dagger \tilde{f}\tilde{f}^\dagger T^{\tilde{f}} \\
&- 4\tilde{f}\tilde{f}^\dagger T^f f^\dagger \tilde{f} - 8\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} \tilde{f}^\dagger \tilde{f} - 2\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger T^{\tilde{f}} - 4\tilde{f} h^{E\dagger} h^E h^{E\dagger} T^{h^E} \\
&- 4\tilde{f} h^{E\dagger} y^E y^{E\dagger} T^{h^E} - 4\tilde{f} h^{E\dagger} T^{h^E} \tilde{f}^\dagger \tilde{f} - 4\tilde{f} h^{E\dagger} T^{h^E} h^{E\dagger} h^E - 4\tilde{f} h^{E\dagger} T^E y^{E\dagger} h^E \\
&- 2\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}} - 2\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 4\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \tilde{f}^\dagger \tilde{f} - 2\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \tilde{\lambda}^\dagger \tilde{\lambda} \\
&- 2\tilde{f}\tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}} - 2\tilde{f}\tilde{\lambda}^\dagger T^{fT} f^* \tilde{\lambda} - 4T^f f^\dagger f f^\dagger \tilde{f} - 4T^f \tilde{\lambda}^* \tilde{\lambda}^T f^\dagger \tilde{f} \\
&- 2T^{\tilde{f}} \tilde{f}^\dagger f f^\dagger \tilde{f} - 6T^{\tilde{f}} \tilde{f}^\dagger \tilde{f}\tilde{f}^\dagger \tilde{f} - 4T^{\tilde{f}} h^{E\dagger} h^E \tilde{f}^\dagger \tilde{f} - 2T^{\tilde{f}} h^{E\dagger} h^E h^{E\dagger} h^E \\
&- 2T^{\tilde{f}} h^{E\dagger} y^E y^{E\dagger} h^E - 4T^{\tilde{f}} \tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger \tilde{f} - T^{\tilde{f}} \tilde{\lambda}^\dagger \tilde{\lambda}\tilde{\lambda}^\dagger \tilde{\lambda} - T^{\tilde{f}} \tilde{\lambda}^\dagger f^T f^* \tilde{\lambda} \\
&+ \frac{297}{50} g_1^4 T^{\tilde{f}} + \frac{27}{100} g_1^2 g_1'^2 T^{\tilde{f}} + \frac{551}{25} g_1'^4 T^{\tilde{f}} + \frac{9}{5} g_1^2 g_2^2 T^{\tilde{f}} + \frac{39}{20} g_1'^2 g_2^2 T^{\tilde{f}} \\
&+ \frac{33}{2} g_2^4 T^{\tilde{f}} + \frac{3}{2} g_1'^2 |\lambda|^2 T^{\tilde{f}} - 3|\lambda|^4 T^{\tilde{f}} - |\sigma|^2 |\lambda|^2 T^{\tilde{f}} - 6\lambda^* \tilde{f}\tilde{f}^\dagger \tilde{f} T_\lambda \\
&- 4\lambda^* \tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda} T_\lambda - 2\sigma^* \tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda} T_\sigma - 2\tilde{\sigma}^* \tilde{f} h^{E\dagger} h^E T_{\tilde{\sigma}} - 2f f^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(f f^\dagger\Big) \\
&- 4T^f f^\dagger \tilde{f} \operatorname{Tr}\Big(f f^\dagger\Big) - |\lambda|^2 T^{\tilde{f}} \operatorname{Tr}\Big(f f^\dagger\Big) - 4\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&- 5T^{\tilde{f}} \tilde{f}^\dagger \tilde{f} \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + \frac{3}{2} g_1'^2 T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 6\tilde{f} h^{E\dagger} T^{h^E} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 3T^{\tilde{f}} h^{E\dagger} h^E \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 2\tilde{f} h^{E\dagger} T^{h^E} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- T^{\tilde{f}} h^{E\dagger} h^E \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 6f f^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12T^f f^\dagger \tilde{f} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 3|\lambda|^2 T^{\tilde{f}} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 2f f^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 4T^f f^\dagger \tilde{f} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- |\lambda|^2 T^{\tilde{f}} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 12\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 15T^{\tilde{f}} \tilde{f}^\dagger \tilde{f} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&+ \frac{4}{5} g_1^2 T^{\tilde{f}} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - \frac{3}{10} g_1'^2 T^{\tilde{f}} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) + 16 g_3^2 T^{\tilde{f}} \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 6\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 3T^{\tilde{f}} \tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 3|\lambda|^2 T^{\tilde{f}} \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 4\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2T^{\tilde{f}} \tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2|\lambda|^2 T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 6\tilde{f}\tilde{f}^\dagger \tilde{f} \operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) - 6\tilde{f} h^{E\dagger} h^E \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 2\tilde{f} h^{E\dagger} h^E \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&- \frac{4}{5} f f^\dagger \tilde{f} \Big[15 g_2^2 M_2 + 15 \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) + 3 g_1^2 M_1 - 3 g_1'^2 M_1' + 5\lambda^* T_\lambda \\
&+ 5 \operatorname{Tr}\Big(f^\dagger T^f\Big) + 5 \operatorname{Tr}\Big(y^{E\dagger} T^E\Big)\Big] - 18\tilde{f}\tilde{f}^\dagger \tilde{f} \operatorname{Tr}\Big(y^{U\dagger} T^U\Big) - 6\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) \\
&- 4\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda} \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - 2T^{\tilde{f}} \operatorname{Tr}\Big(f f^\dagger \tilde{f}\tilde{f}^\dagger\Big) - 3T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger \tilde{f}\tilde{f}^\dagger\Big) \\
&- T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger\Big) - T^{\tilde{f}} \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger\Big) - 3T^{\tilde{f}} \operatorname{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) \\
&- 3T^{\tilde{f}} \operatorname{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) - 9T^{\tilde{f}} \operatorname{Tr}\Big(y^U y^{U\dagger} y^U y^{U\dagger}\Big) \\
&- \frac{1}{50} \tilde{f}\Big(1188 g_1^4 M_1 + 27 g_1^2 g_1'^2 M_1 + 180 g_1^2 g_2^2 M_1 + 27 g_1^2 g_1'^2 M_1' + 4408 g_1'^4 M_1' \\
&+ 195 g_1'^2 g_2^2 M_1' + 180 g_1^2 g_2^2 M_2 + 195 g_1'^2 g_2^2 M_2 + 3300 g_2^4 M_2 + 600\lambda^* |\lambda|^2 T_\lambda
\end{aligned}
$$

$$+ 150 g_1'^2 M_1' \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + 80 g_1^2 M_1 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 30 g_1'^2 M_1' \operatorname{Tr}\Big(y^U y^{U\dagger}\Big)$$
$$+ 1600 g_3^2 M_3 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 150 g_1'^2 \operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) - 80 g_1^2 \operatorname{Tr}\Big(y^{U\dagger} T^U\Big)$$
$$+ 30 g_1'^2 \operatorname{Tr}\Big(y^{U\dagger} T^U\Big) - 1600 g_3^2 \operatorname{Tr}\Big(y^{U\dagger} T^U\Big) + 50\lambda^*\Big\{T_\lambda\Big[2\operatorname{Tr}\Big(f f^\dagger\Big)$$
$$+ 2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) + 2|\sigma|^2 - 3 g_1'^2 + 4\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 6\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 6\operatorname{Tr}\Big(y^D y^{D\dagger}\Big)\Big]$$
$$+ \lambda\Big[2\operatorname{Tr}\Big(f^\dagger T^f\Big) + 2\operatorname{Tr}\Big(y^{E\dagger} T^E\Big) + 2\sigma^* T_\sigma + 3 g_1'^2 M_1' + 4\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big)$$
$$+ 6\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 6\operatorname{Tr}\Big(y^{D\dagger} T^D\Big)\Big]\Big\} + 200\operatorname{Tr}\Big(f f^\dagger T^{\tilde{f}} \tilde{f}^\dagger\Big) + 200\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^f f^\dagger\Big)$$
$$+ 600\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^{\tilde{f}}\tilde{f}^\dagger\Big) + 100\operatorname{Tr}\Big(\tilde{f} h^{E\dagger} T^{h^E}\tilde{f}^\dagger\Big) + 100\operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\tilde{f}^\dagger\Big)$$
$$+ 100\operatorname{Tr}\Big(h^E\tilde{f}^\dagger T^{\tilde{f}} h^{E\dagger}\Big) + 300\operatorname{Tr}\Big(y^D y^{U\dagger} T^U y^{D\dagger}\Big) + 300\operatorname{Tr}\Big(y^U y^{D\dagger} T^D y^{U\dagger}\Big)$$
$$+ 1800\operatorname{Tr}\Big(y^U y^{U\dagger} T^U y^{U\dagger}\Big) + 100\operatorname{Tr}\Big(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}}\tilde{\lambda}^\dagger\Big) + 300\operatorname{Tr}\Big(g^{D\dagger} y^{UT} y^{U*} T^{g^D}\Big)$$
$$+ 300\operatorname{Tr}\Big(y^{U\dagger} T^U g^{D*} g^{DT}\Big)\Big), \tag{A.73}$$
$$\beta_{T^f}^{(1)} = 4 f f^\dagger T^f + 2 f\tilde{\lambda}^* T^{\tilde{\lambda}T} + 2\tilde{f}\tilde{f}^\dagger T^f + 5 T^f f^\dagger f + T^f\tilde{\lambda}^*\tilde{\lambda}^T + 4 T^{\tilde{f}}\tilde{f}^\dagger f$$
$$- \frac{3}{5} g_1^2 T^f - \frac{19}{10} g_1'^2 T^f - 3 g_2^2 T^f + |\lambda|^2 T^f + T^f\operatorname{Tr}\Big(f f^\dagger\Big) + 3 T^f \operatorname{Tr}\Big(y^D y^{D\dagger}\Big)$$
$$+ T^f\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) + f\Bigg[2\lambda^* T_\lambda + 2\operatorname{Tr}\Big(f^\dagger T^f\Big) + 2\operatorname{Tr}\Big(y^{E\dagger} T^E\Big) + 6 g_2^2 M_2$$
$$+ 6\operatorname{Tr}\Big(y^{D\dagger} T^D\Big) + \frac{19}{5} g_1'^2 M_1' + \frac{6}{5} g_1^2 M_1\Bigg], \tag{A.74}$$
$$\beta_{T^f}^{(2)} = \frac{6}{5} g_1^2 f f^\dagger T^f + \frac{9}{5} g_1'^2 f f^\dagger T^f + 6 g_2^2 f f^\dagger T^f - 4|\lambda|^2 f f^\dagger T^f$$
$$- 3 g_1'^2 M_1' f\tilde{\lambda}^*\tilde{\lambda}^T + 3 g_1'^2 f\tilde{\lambda}^* T^{\tilde{\lambda}T} - 4|\lambda|^2 f\tilde{\lambda}^* T^{\tilde{\lambda}T} - 2|\sigma|^2 f\tilde{\lambda}^* T^{\tilde{\lambda}T}$$
$$- \frac{12}{5} g_1^2 M_1\tilde{f}\tilde{f}^\dagger f + \frac{12}{5} g_1'^2 M_1'\tilde{f}\tilde{f}^\dagger f - 12 g_2^2 M_2\tilde{f}\tilde{f}^\dagger f + \frac{6}{5} g_1^2\tilde{f}\tilde{f}^\dagger T^f$$
$$- \frac{6}{5} g_1'^2\tilde{f}\tilde{f}^\dagger T^f + 6 g_2^2\tilde{f}\tilde{f}^\dagger T^f - 2|\lambda|^2\tilde{f}\tilde{f}^\dagger T^f + \frac{12}{5} g_1^2 T^f f^\dagger f$$
$$- \frac{9}{10} g_1'^2 T^f f^\dagger f + 12 g_2^2 T^f f^\dagger f - 5|\lambda|^2 T^f f^\dagger f + \frac{3}{2} g_1'^2 T^f\tilde{\lambda}^*\tilde{\lambda}^T$$
$$- 2|\lambda|^2 T^f\tilde{\lambda}^*\tilde{\lambda}^T - |\sigma|^2 T^f\tilde{\lambda}^*\tilde{\lambda}^T + \frac{12}{5} g_1^2 T^{\tilde{f}}\tilde{f}^\dagger f - \frac{12}{5} g_1'^2 T^{\tilde{f}}\tilde{f}^\dagger f$$
$$+ 12 g_2^2 T^{\tilde{f}}\tilde{f}^\dagger f - 4|\lambda|^2 T^{\tilde{f}}\tilde{f}^\dagger f - 6 f f^\dagger f f^\dagger T^f - 4 f f^\dagger\tilde{f}\tilde{f}^\dagger T^f$$
$$- 8 f f^\dagger T^f f^\dagger f - 4 f f^\dagger T^{\tilde{f}}\tilde{f}^\dagger f - 2 f\tilde{\lambda}^*\tilde{f}^T\tilde{f}^* T^{\tilde{\lambda}T} - 2 f\tilde{\lambda}^* h^{ET} h^{E*} T^{\tilde{\lambda}T}$$
$$- 2 f\tilde{\lambda}^*\tilde{\lambda}^T f^\dagger T^f - 2 f\tilde{\lambda}^*\tilde{\lambda}^T\tilde{\lambda}^* T^{\tilde{\lambda}T} - 2 f\tilde{\lambda}^* T^{\tilde{f}T}\tilde{f}^*\tilde{\lambda}^T - 2 f\tilde{\lambda}^* T^{h^E T} h^{E*}\tilde{\lambda}^T$$
$$- 4 f\tilde{\lambda}^* T^{\tilde{\lambda}T} f^\dagger f - 2 f\tilde{\lambda}^* T^{\tilde{\lambda}T}\tilde{\lambda}^*\tilde{\lambda}^T - 2\tilde{f}\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger T^f - 4\tilde{f}\tilde{f}^\dagger T^{\tilde{f}}\tilde{f}^\dagger f$$
$$- 2\tilde{f} h^{E\dagger} h^E\tilde{f}^\dagger T^f - 4\tilde{f} h^{E\dagger} T^{h^E}\tilde{f}^\dagger f - 2\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger T^f - 4\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\tilde{f}^\dagger f$$
$$- 6 T^f f^\dagger f f^\dagger f - 2 T^f f^\dagger\tilde{f}\tilde{f}^\dagger f - T^f\tilde{\lambda}^*\tilde{f}^T\tilde{f}^*\tilde{\lambda}^T - T^f\tilde{\lambda}^* h^{ET} h^{E*}\tilde{\lambda}^T$$

$$
\begin{aligned}
&- 4T^f\tilde{\lambda}^*\tilde{\lambda}^T f^\dagger f - T^f\tilde{\lambda}^*\tilde{\lambda}^T\tilde{\lambda}^*\tilde{\lambda}^T - 4T^{\tilde{f}}\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger f - 4T^{\tilde{f}}h^{E\dagger}h^E\tilde{f}^\dagger f \\
&- 4T^{\tilde{f}}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger f + \frac{297}{50}g_1^4T^f + \frac{27}{100}g_1^2g_1'^2T^f + \frac{551}{25}g_1'^4T^f + \frac{9}{5}g_1^2g_2^2T^f \\
&+ \frac{39}{20}g_1'^2g_2^2T^f + \frac{33}{2}g_2^4T^f + g_1'^2|\lambda|^2T^f - 3|\lambda|^4T^f - |\sigma|^2|\lambda|^2T^f \\
&- 4\lambda^* f\tilde{\lambda}^*\tilde{\lambda}^T T_\lambda - 4\lambda^*\tilde{f}\tilde{f}^\dagger f T_\lambda - 2\sigma^* f\tilde{\lambda}^*\tilde{\lambda}^T T_\sigma - 4ff^\dagger T^f\,\mathrm{Tr}\Big(ff^\dagger\Big) \\
&- 5T^f f^\dagger f\,\mathrm{Tr}\Big(ff^\dagger\Big) + g_1'^2T^f\,\mathrm{Tr}\Big(ff^\dagger\Big) - 2\tilde{f}\tilde{f}^\dagger T^f\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&- 4T^{\tilde{f}}\tilde{f}^\dagger f\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - |\lambda|^2T^f\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 12ff^\dagger T^f\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) \\
&- 15T^f f^\dagger f\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) - \frac{2}{5}g_1^2T^f\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) - \frac{3}{5}g_1'^2T^f\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) \\
&+ 16g_3^2T^f\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) - 4ff^\dagger T^f\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big) - 5T^f f^\dagger f\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big) \\
&+ \frac{6}{5}g_1^2T^f\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big) - \frac{1}{5}g_1'^2T^f\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big) - 6\tilde{f}\tilde{f}^\dagger T^f\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big) \\
&- 12T^{\tilde{f}}\tilde{f}^\dagger f\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big) - 3|\lambda|^2T^f\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big) - 6f\tilde{\lambda}^*T^{\tilde{\lambda}T}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 3T^f\tilde{\lambda}^*\tilde{\lambda}^T\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 3|\lambda|^2T^f\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 4f\tilde{\lambda}^*T^{\tilde{\lambda}T}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 2T^f\tilde{\lambda}^*\tilde{\lambda}^T\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2|\lambda|^2T^f\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 4\tilde{f}\tilde{f}^\dagger f\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) \\
&- \frac{3}{5}ff^\dagger f\Big[10\lambda^*T_\lambda + 10\,\mathrm{Tr}\Big(f^\dagger T^f\Big) + 10\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big) + 20g_2^2M_2 \\
&+ 30\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big) + 4g_1^2M_1 + g_1'^2M_1'\Big] - 12\tilde{f}\tilde{f}^\dagger f\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big) \\
&- 6f\tilde{\lambda}^*\tilde{\lambda}^T\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big) - 4f\tilde{\lambda}^*\tilde{\lambda}^T\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - 3T^f\,\mathrm{Tr}\Big(ff^\dagger ff^\dagger\Big) \\
&- 2T^f\,\mathrm{Tr}\Big(ff^\dagger\tilde{f}\tilde{f}^\dagger\Big) - 3T^f\,\mathrm{Tr}\Big(g^Dg^{D\dagger}y^{DT}y^{D*}\Big) - 2T^f\,\mathrm{Tr}\Big(h^Eh^{E\dagger}y^Ey^{E\dagger}\Big) \\
&- 9T^f\,\mathrm{Tr}\Big(y^Dy^{D\dagger}y^Dy^{D\dagger}\Big) - 3T^f\,\mathrm{Tr}\Big(y^Dy^{U\dagger}y^Uy^{D\dagger}\Big) - 3T^f\,\mathrm{Tr}\Big(y^Ey^{E\dagger}y^Ey^{E\dagger}\Big) \\
&- T^f\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^Tf^*\Big) - \frac{1}{50}f\Big(1188g_1^4M_1 + 27g_1^2g_1'^2M_1 + 180g_1^2g_2^2M_1 \\
&+ 27g_1^2g_1'^2M_1' + 4408g_1'^4M_1' + 195g_1'^2g_2^2M_1' + 180g_1^2g_2^2M_2 + 195g_1'^2g_2^2M_2 \\
&+ 3300g_2^4M_2 + 600\lambda^*|\lambda|^2T_\lambda + 100g_1'^2M_1'\,\mathrm{Tr}\Big(ff^\dagger\Big) - 40g_1^2M_1\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) \\
&- 60g_1'^2M_1'\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) + 1600g_3^2M_3\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big) + 120g_1^2M_1\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big) \\
&- 20g_1'^2M_1'\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big) - 100g_1'^2\,\mathrm{Tr}\Big(f^\dagger T^f\Big) + 40g_1^2\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big) \\
&+ 60g_1'^2\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big) - 1600g_3^2\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big) - 120g_1^2\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big) \\
&+ 20g_1'^2\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big) + 100\lambda^*\Big\{T_\lambda\Big[2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 3\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 3\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big) \\
&- g_1'^2 + |\sigma|^2 + \mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\Big] + \lambda\Big[2\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 3\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big) + 3\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big) \\
&+ g_1'^2M_1' + \sigma^*T_\sigma + \mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big)\Big]\Big\} + 600\,\mathrm{Tr}\Big(ff^\dagger T^ff^\dagger\Big) \\
&+ 200\,\mathrm{Tr}\Big(ff^\dagger T^{\tilde{f}}\tilde{f}^\dagger\Big) + 200\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^ff^\dagger\Big) + 200\,\mathrm{Tr}\Big(h^Eh^{E\dagger}T^Ey^{E\dagger}\Big)
\end{aligned}
$$

$$+ 1800\,\mathrm{Tr}\Big(y^D y^{D\dagger} T^D y^{D\dagger}\Big) + 300\,\mathrm{Tr}\Big(y^D y^{U\dagger} T^U y^{D\dagger}\Big) + 200\,\mathrm{Tr}\Big(y^E y^{E\dagger} T^{h^E} h^{E\dagger}\Big)$$
$$+ 600\,\mathrm{Tr}\Big(y^E y^{E\dagger} T^E y^{E\dagger}\Big) + 300\,\mathrm{Tr}\Big(y^U y^{D\dagger} T^D y^{U\dagger}\Big) + 100\,\mathrm{Tr}\Big(f^\dagger T^f \tilde{\lambda}^* \tilde{\lambda}^T\Big)$$
$$+ 300\,\mathrm{Tr}\Big(g^{D\dagger} y^{DT} y^{D*} T^{g^D}\Big) + 300\,\mathrm{Tr}\Big(y^{D\dagger} T^D g^{D*} g^{DT}\Big)$$
$$+ 100\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger f^T f^* T^{\tilde{\lambda}}\Big)\Big), \tag{A.75}$$

$$\beta^{(1)}_{T^U} = 2y^U y^{D\dagger} T^D + 4y^U y^{U\dagger} T^U + 2y^U g^{D*} T^{g^D T} + T^U y^{D\dagger} y^D + 5T^U y^{U\dagger} y^U$$
$$+ T^U g^{D*} g^{DT} - \frac{13}{15} g_1^2 T^U - \frac{3}{10} g_1'^2 T^U - 3g_2^2 T^U - \frac{16}{3} g_3^2 T^U + |\lambda|^2 T^U$$
$$+ T^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + 3T^U\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) + y^U\Bigg[2\lambda^* T_\lambda + 2\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) + 6g_2^2 M_2$$
$$+ 6\,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) + \frac{26}{15} g_1^2 M_1 + \frac{32}{3} g_3^2 M_3 + \frac{3}{5} g_1'^2 M_1'\Bigg], \tag{A.76}$$

$$\beta^{(2)}_{T^U} = \frac{4}{5} g_1^2 y^U y^{D\dagger} T^D + \frac{6}{5} g_1'^2 y^U y^{D\dagger} T^D - 2|\lambda|^2 y^U y^{D\dagger} T^D - \frac{4}{5} g_1^2 M_1 y^U y^{U\dagger} y^U$$
$$- \frac{6}{5} g_1'^2 M_1' y^U y^{U\dagger} y^U - 12 g_2^2 M_2 y^U y^{U\dagger} y^U + \frac{6}{5} g_1^2 y^U y^{U\dagger} T^U + \frac{4}{5} g_1'^2 y^U y^{U\dagger} T^U$$
$$+ 6g_2^2 y^U y^{U\dagger} T^U - 4|\lambda|^2 y^U y^{U\dagger} T^U - \frac{4}{5} g_1^2 M_1 y^U g^{D*} g^{DT} - \frac{6}{5} g_1'^2 M_1' y^U g^{D*} g^{DT}$$
$$+ \frac{4}{5} g_1^2 y^U g^{D*} T^{g^D T} + \frac{6}{5} g_1'^2 y^U g^{D*} T^{g^D T} - 2|\tilde{\sigma}|^2 y^U g^{D*} T^{g^D T} + \frac{2}{5} g_1^2 T^U y^{D\dagger} y^D$$
$$+ \frac{3}{5} g_1'^2 T^U y^{D\dagger} y^D - |\lambda|^2 T^U y^{D\dagger} y^D + g_1'^2 T^U y^{U\dagger} y^U + 12 g_2^2 T^U y^{U\dagger} y^U$$
$$- 5|\lambda|^2 T^U y^{U\dagger} y^U + \frac{2}{5} g_1^2 T^U g^{D*} g^{DT} + \frac{3}{5} g_1'^2 T^U g^{D*} g^{DT} - |\tilde{\sigma}|^2 T^U g^{D*} g^{DT}$$
$$- 4y^U y^{D\dagger} y^D y^{D\dagger} T^D - 2y^U y^{D\dagger} y^D y^{U\dagger} T^U - 4y^U y^{D\dagger} T^D y^{D\dagger} y^D$$
$$- 4y^U y^{D\dagger} T^D y^{U\dagger} y^U - 6y^U y^{U\dagger} y^U y^{U\dagger} T^U - 8y^U y^{U\dagger} T^U y^{U\dagger} y^U$$
$$- 2y^U g^{D*} g^{DT} y^{U\dagger} T^U - 4y^U g^{D*} g^{DT} g^{D*} T^{g^D T} - 2y^U g^{D*} \kappa^T \kappa^* T^{g^D T}$$
$$- 4y^U g^{D*} T^{g^D T} y^{U\dagger} y^U - 4y^U g^{D*} T^{g^D T} g^{D*} g^{DT} - 2y^U g^{D*} T^{\kappa T} \kappa^* g^{DT}$$
$$- 2T^U y^{D\dagger} y^D y^{D\dagger} y^D - 4T^U y^{D\dagger} y^D y^{U\dagger} y^U - 6T^U y^{U\dagger} y^U y^{U\dagger} y^U$$
$$- 4T^U g^{D*} g^{DT} y^{U\dagger} y^U - 2T^U g^{D*} g^{DT} g^{D*} g^{DT} - T^U g^{D*} \kappa^T \kappa^* g^{DT} + \frac{3913}{450} g_1^4 T^U$$
$$+ \frac{161}{300} g_1^2 g_1'^2 T^U + \frac{81}{25} g_1'^4 T^U + g_1^2 g_2^2 T^U + \frac{3}{4} g_1'^2 g_2^2 T^U + \frac{33}{2} g_2^4 T^U + \frac{136}{45} g_1^2 g_3^2 T^U$$
$$+ \frac{8}{15} g_1'^2 g_3^2 T^U + 8 g_2^2 g_3^2 T^U + \frac{128}{9} g_3^4 T^U + \frac{3}{2} g_1'^2 |\lambda|^2 T^U - 3|\lambda|^4 T^U$$
$$- |\sigma|^2 |\lambda|^2 T^U - 6\lambda^* y^U y^{U\dagger} y^U T_\lambda - 2\tilde{\sigma}^* y^U g^{D*} g^{DT} T_{\tilde{\sigma}} - 2y^U y^{D\dagger} T^D\,\mathrm{Tr}\Big(f f^\dagger\Big)$$
$$- T^U y^{D\dagger} y^D\,\mathrm{Tr}\Big(f f^\dagger\Big) - |\lambda|^2 T^U\,\mathrm{Tr}\Big(f f^\dagger\Big) - 4y^U y^{U\dagger} T^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)$$
$$- 5T^U y^{U\dagger} y^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) + \frac{3}{2} g_1'^2 T^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 6y^U g^{D*} T^{g^D T}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big)$$
$$- 3T^U g^{D*} g^{DT}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 2y^U g^{D*} T^{g^D T}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)$$

$$
\begin{aligned}
&- T^U g^{D*} g^{DT}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 6 y^U y^{D\dagger} T^D\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 3 T^U y^{D\dagger} y^D\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 3|\lambda|^2 T^U\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 2 y^U y^{D\dagger} T^D\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- T^U y^{D\dagger} y^D\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - |\lambda|^2 T^U\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 12 y^U y^{U\dagger} T^U\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 15 T^U y^{U\dagger} y^U\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) + \frac{4}{5} g_1^2 T^U\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - \frac{3}{10} g_1'^2 T^U\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&+ 16 g_3^2 T^U\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 3|\lambda|^2 T^U\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 2|\lambda|^2 T^U\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 6 y^U y^{U\dagger} y^U\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) - 6 y^U g^{D*} g^{DT}\,\mathrm{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 2 y^U g^{D*} g^{DT}\,\mathrm{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&- \frac{2}{5} y^U y^{D\dagger} y^D \Big[15\,\mathrm{Tr}\Big(y^{D\dagger} T^D\Big) + 2 g_1^2 M_1 + 3 g_1'^2 M_1' + 5\lambda^* T_\lambda + 5\,\mathrm{Tr}\Big(f^\dagger T^f\Big) \\
&+ 5\,\mathrm{Tr}\Big(y^{E\dagger} T^E\Big)\Big] - 18 y^U y^{U\dagger} y^U\,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) - 2 T^U\,\mathrm{Tr}\Big(f f^\dagger \tilde{f} \tilde{f}^\dagger\Big) \\
&- 3 T^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger \tilde{f}\tilde{f}^\dagger\Big) - T^U\,\mathrm{Tr}\Big(\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger\Big) - T^U\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger\Big) \\
&- 3 T^U\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 3 T^U\,\mathrm{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) - 9 T^U\,\mathrm{Tr}\Big(y^U y^{U\dagger} y^U y^{U\dagger}\Big) \\
&- \frac{1}{450} y^U \Big(15652 g_1^4 M_1 + 483 g_1^2 g_1'^2 M_1 + 900 g_1^2 g_2^2 M_1 + 2720 g_1^2 g_3^2 M_1 \\
&+ 483 g_1^2 g_1'^2 M_1' + 5832 g_1'^4 M_1' + 675 g_1'^2 g_2^2 M_1' + 480 g_1'^2 g_3^2 M_1' + 2720 g_1^2 g_3^2 M_3 \\
&+ 480 g_1'^2 g_3^2 M_3 + 7200 g_2^2 g_3^2 M_3 + 25600 g_3^4 M_3 + 900 g_1^2 g_2^2 M_2 + 675 g_1'^2 g_2^2 M_2 \\
&+ 29700 g_2^4 M_2 + 7200 g_2^2 g_3^2 M_2 + 5400 \lambda^* |\lambda|^2 T_\lambda + 1350 g_1'^2 M_1'\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&+ 720 g_1^2 M_1\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 270 g_1'^2 M_1'\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) + 14400 g_3^2 M_3\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 1350 g_1'^2\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) - 720 g_1^2\,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) + 270 g_1'^2\,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) \\
&- 14400 g_3^2\,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) + 450 \lambda^* \Big\{T_\lambda \Big[2\,\mathrm{Tr}\Big(f f^\dagger\Big) + 2\,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) + 2|\sigma|^2 \\
&- 3 g_1'^2 + 4\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + 6\,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big)\Big] + \lambda \Big[2\,\mathrm{Tr}\Big(f^\dagger T^f\Big) \\
&+ 2\,\mathrm{Tr}\Big(y^{E\dagger} T^E\Big) + 2\sigma^* T_\sigma + 3 g_1'^2 M_1' + 4\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 6\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big) \\
&+ 6\,\mathrm{Tr}\Big(y^{D\dagger} T^D\Big)\Big]\Big\} + 1800\,\mathrm{Tr}\Big(f f^\dagger T^{\tilde{f}} \tilde{f}^\dagger\Big) + 1800\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^f f^\dagger\Big) \\
&+ 5400\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} \tilde{f}^\dagger\Big) + 900\,\mathrm{Tr}\Big(\tilde{f} h^{E\dagger} T^{h^E} \tilde{f}^\dagger\Big) + 900\,\mathrm{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} \tilde{f}^\dagger\Big) \\
&+ 900\,\mathrm{Tr}\Big(h^E \tilde{f}^\dagger T^{\tilde{f}} h^{E\dagger}\Big) + 2700\,\mathrm{Tr}\Big(y^D y^{U\dagger} T^U y^{D\dagger}\Big) + 2700\,\mathrm{Tr}\Big(y^U y^{D\dagger} T^D y^{U\dagger}\Big) \\
&+ 16200\,\mathrm{Tr}\Big(y^U y^{U\dagger} T^U y^{U\dagger}\Big) + 900\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}} \tilde{\lambda}^\dagger\Big) \\
&+ 2700\,\mathrm{Tr}\Big(g^{D\dagger} y^{UT} y^{U*} T^{g^D}\Big) + 2700\,\mathrm{Tr}\Big(y^{U\dagger} T^U g^{D*} g^{DT}\Big)\Big). \qquad \text{(A.77)}
\end{aligned}
$$

A.6 Soft-breaking bilinear and linear couplings

The β functions for the soft-breaking bilinears are given by

$$
\begin{aligned}
\beta^{(1)}_{B_\phi \mu_\phi} &= 2 B_\phi \mu_\phi \Big(2|\tilde{\sigma}|^2 + 4|\kappa_\phi|^2 + |\sigma|^2\Big) + 4\mu_\phi \Big(2\kappa_\phi^* T_{\kappa_\phi} + 2\tilde{\sigma}^* T_{\tilde{\sigma}} + \sigma^* T_\sigma\Big) \\
&\quad - 8\tilde{\sigma}^* \kappa_\phi B_L \mu_L, \qquad \text{(A.78)}
\end{aligned}
$$

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$$
\begin{aligned}
\beta^{(2)}_{B_\phi\mu_\phi} = B_\phi\mu_\phi\bigg\{ &-32|\kappa_\phi|^4 - 8|\tilde\sigma|^4 - 4|\sigma|^4 + |\tilde\sigma|^2\bigg[\frac{12}{5}g_1^2 + \frac{8}{5}g_1'^2 + 12g_2^2 - 32|\kappa_\phi|^2 \\
&- 12\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 4\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + |\sigma|^2\Big[5g_1'^2 - 4|\lambda|^2 - 16|\kappa_\phi|^2 \\
&- 4\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 6\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)\Big]\bigg\} - \mu_\phi\bigg\{16|\sigma|^2\sigma^* T_\sigma + 80|\kappa_\phi|^2\kappa_\phi^* T_{\kappa_\phi} \\
&+ 32|\tilde\sigma|^2\tilde\sigma^* T_{\tilde\sigma} + \frac{24}{5}g_1^2\tilde\sigma^*(\tilde\sigma M_1 - T_{\tilde\sigma}) + \frac{16}{5}g_1'^2\tilde\sigma^*(\tilde\sigma M_1' - T_{\tilde\sigma}) \\
&+ 24g_2^2\tilde\sigma^*(\tilde\sigma M_2 - T_{\tilde\sigma}) + 10g_1'^2\sigma^*(M_1'\sigma - T_\sigma) + 16\tilde\sigma^*\kappa_\phi^*\Big(2\tilde\sigma T_{\kappa_\phi} + 3\kappa_\phi T_{\tilde\sigma}\Big) \\
&+ 8\sigma^*\kappa_\phi^*\Big(2\sigma T_{\kappa_\phi} + 3\kappa_\phi T_\sigma\Big) + 8\lambda^*\sigma^*\Big(\lambda T_\sigma + \sigma T_\lambda\Big) \\
&+ 12\sigma^*\Big[T_\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) + \sigma\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)\Big] + 8\sigma^*\Big[T_\sigma\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) + \sigma\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)\Big] \\
&+ 24\tilde\sigma^*\Big[T_{\tilde\sigma}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \tilde\sigma\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big)\Big] \\
&+ 8\tilde\sigma^*\Big[T_{\tilde\sigma}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) + \tilde\sigma\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big)\Big]\bigg\} + 8\kappa_\phi\tilde\sigma^* B_L\mu_L\bigg[2|\tilde\sigma|^2 \\
&+ 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - \frac{12}{5}g_1^2 - \frac{8}{5}g_1'^2 - 12g_2^2\bigg] + 8\kappa_\phi\tilde\sigma^*\mu_L\bigg[2\tilde\sigma^* T_{\tilde\sigma} \\
&+ 3\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) + \mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) + \frac{12}{5}g_1^2 M_1 + \frac{8}{5}g_1'^2 M_1' + 12g_2^2 M_2\bigg],
\end{aligned}
\tag{A.79}
$$

$$
\begin{aligned}
\beta^{(1)}_{B_L\mu_L} = B_L\mu_L\bigg[&6|\tilde\sigma|^2 + 3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + \mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - \frac{3}{5}g_1^2 - \frac{2}{5}g_1'^2 - 3g_2^2\bigg] \\
&+ \mu_L\bigg[4\tilde\sigma^* T_{\tilde\sigma} + 6\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) + 2\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) + \frac{6}{5}g_1^2 M_1 + \frac{4}{5}g_1'^2 M_1' \\
&+ 6g_2^2 M_2\bigg] - 2\tilde\sigma\kappa_\phi^* B_\phi\mu_\phi,
\end{aligned}
\tag{A.80}
$$

$$
\begin{aligned}
\beta^{(2)}_{B_L\mu_L} = B_L\mu_L\bigg\{&\frac{297}{50}g_1^4 + \frac{217}{50}g_1'^4 + \frac{33}{2}g_2^4 + \frac{18}{25}g_1^2 g_1'^2 + \frac{9}{5}g_1^2 g_2^2 + \frac{6}{5}g_1'^2 g_2^2 \\
&- 14|\tilde\sigma|^4 + \frac{2}{5}g_1^2\bigg[-\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + 3\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + \frac{3}{10}g_1'^2\bigg[3\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ \mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] + 16g_3^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) + |\tilde\sigma|^2\bigg[\frac{48}{5}g_1^2 + \frac{32}{5}g_1'^2 + 48g_2^2 \\
&- 2|\sigma|^2 - 4|\kappa_\phi|^2 - 15\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 5\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big)\bigg] - \mathrm{Tr}\Big(\tilde f h^{E\dagger} h^E \tilde f^\dagger\Big) \\
&- 9\,\mathrm{Tr}\Big(g^D g^{D\dagger} g^D g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 3\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) \\
&- 3\,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 3\,\mathrm{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big)
\end{aligned}
$$

$$- \operatorname{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big)\Bigg\} - \mu_L \Bigg\{ \frac{594}{25} g_1^4 M_1 + \frac{434}{25} g_1'^4 M_1' + 66 g_2^4 M_2$$
$$+ \frac{36}{25} g_1^2 g_1'^2 (M_1 + M_1') + \frac{18}{5} g_1^2 g_2^2 (M_1 + M_2) + \frac{12}{5} g_1'^2 g_2^2 (M_1' + M_2)$$
$$- \frac{4}{5} g_1^2 \operatorname{Tr}\Big(M_1 g^D g^{D\dagger} - g^{D\dagger} T^{g^D}\Big) + \frac{9}{5} g_1'^2 \operatorname{Tr}\Big(M_1' g^D g^{D\dagger} - g^{D\dagger} T^{g^D}\Big)$$
$$+ 32 g_3^2 \operatorname{Tr}\Big(M_3 g^D g^{D\dagger} - g^{D\dagger} T^{g^D}\Big) + \frac{12}{5} g_1^2 \operatorname{Tr}\Big(M_1 h^E h^{E\dagger} - h^{E\dagger} T^{h^E}\Big)$$
$$+ \frac{3}{5} g_1'^2 \operatorname{Tr}\Big(M_1' h^E h^{E\dagger} - h^{E\dagger} T^{h^E}\Big) + |\tilde{\sigma}|^2 \Bigg[\frac{48}{5} g_1^2 M_1 + \frac{32}{5} g_1'^2 M_1'$$
$$+ 48 g_2^2 M_2 + 8 \kappa_\phi^* T_{\kappa_\phi} + 4 \sigma^* T_\sigma + 18 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + 6 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big)\Bigg]$$
$$+ \tilde{\sigma}^* T_{\tilde{\sigma}} \Bigg[8|\kappa_\phi|^2 + 4|\sigma|^2 + 32|\tilde{\sigma}|^2 + 6 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg]$$
$$+ 2 \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} T^{h^E} \tilde{f}^\dagger\Big) + 36 \operatorname{Tr}\Big(g^D g^{D\dagger} T^{g^D} g^{D\dagger}\Big) + 6 \operatorname{Tr}\Big(g^D \kappa^\dagger T^\kappa g^{D\dagger}\Big)$$
$$+ 2 \operatorname{Tr}\Big(h^E \tilde{f}^\dagger T^{\tilde{f}} h^{E\dagger}\Big) + 12 \operatorname{Tr}\Big(h^E h^{E\dagger} T^{h^E} h^{E\dagger}\Big) + 4 \operatorname{Tr}\Big(h^E h^{E\dagger} T^E y^{E\dagger}\Big)$$
$$+ 2 \operatorname{Tr}\Big(h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} h^{E\dagger}\Big) + 4 \operatorname{Tr}\Big(y^E y^{E\dagger} T^{h^E} h^{E\dagger}\Big) + 6 \operatorname{Tr}\Big(\kappa g^{D\dagger} T^{g^D} \kappa^\dagger\Big)$$
$$+ 2 \operatorname{Tr}\Big(\tilde{\lambda} h^{E\dagger} T^{h^E} \tilde{\lambda}^\dagger\Big) + 6 \operatorname{Tr}\Big(g^{D\dagger} y^{DT} y^{D*} T^{g^D}\Big) + 6 \operatorname{Tr}\Big(g^{D\dagger} y^{UT} y^{U*} T^{g^D}\Big)$$
$$+ 6 \operatorname{Tr}\Big(y^{D\dagger} T^D g^{D*} g^{DT}\Big) + 6 \operatorname{Tr}\Big(y^{U\dagger} T^U g^{D*} g^{DT}\Big)\Bigg\}$$
$$+ 4 \tilde{\sigma} \kappa_\phi^* \Big[B_\phi \mu_\phi \Big(2|\kappa_\phi|^2 + 2|\tilde{\sigma}|^2 + |\sigma|^2\Big)$$
$$+ \mu_\phi \Big(2\kappa_\phi^* T_{\kappa_\phi} + 2\tilde{\sigma}^* T_{\tilde{\sigma}} + \sigma^* T_\sigma\Big)\Big]. \tag{A.81}$$

The two-loop β function for the soft-breaking linear coupling Λ_S is

$$\beta_{\Lambda_S}^{(1)} = \Lambda_S \Big(2|\kappa_\phi|^2 + 2|\tilde{\sigma}|^2 + |\sigma|^2\Big) + 2\Lambda_F \Big(2\kappa_\phi^* T_{\kappa_\phi} + 2\tilde{\sigma}^* T_{\tilde{\sigma}} + \sigma^* T_\sigma\Big)$$
$$+ 2(B_\phi \mu_\phi) \mu_\phi \kappa_\phi^* + 2(B_\phi \mu_\phi)^* T_{\kappa_\phi} - 4(B_L \mu_L) \mu_\phi \tilde{\sigma}^* - 4(B_L \mu_L)^* T_{\tilde{\sigma}}$$
$$- 4\Big(m_{L_4}^2 + m_{\bar{L}_4}^2\Big) \tilde{\sigma} \mu_L^* + 4 m_\phi^2 \kappa_\phi \mu_\phi^*, \tag{A.82}$$
$$\beta_{\Lambda_S}^{(2)} = \Lambda_S \Bigg\{ |\sigma|^2 \Bigg[\frac{5}{2} g_1'^2 - 4|\kappa_\phi|^2 - 2|\lambda|^2 - 2|\sigma|^2 - 3 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big)\Bigg]$$
$$+ |\tilde{\sigma}|^2 \Bigg[\frac{6}{5} g_1^2 + \frac{4}{5} g_1'^2 + 6 g_2^2 - 8|\kappa_\phi|^2 - 4|\tilde{\sigma}|^2 - 6 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\Bigg]$$
$$- 8|\kappa_\phi|^4 \Bigg\} + \Lambda_F \Bigg\{ \sigma^* T_\sigma \Big[5 g_1'^2 - 8|\kappa_\phi|^2 - 4|\lambda|^2 - 4|\sigma|^2 - 6 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)$$
$$- 4 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big)\Big] - |\sigma|^2 \Big[5 g_1'^2 M_1' + 8 \kappa_\phi^* T_{\kappa_\phi} + 4 \lambda^* T_\lambda + 4 \sigma^* T_\sigma + 6 \operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big)$$

$$
\begin{aligned}
&+4\,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger}T^{\tilde{\lambda}}\Big)\Big]+\tilde{\sigma}^{*}T_{\tilde{\sigma}}\Big[\frac{12}{5}g_1^2+\frac{8}{5}g_1'^2+12g_2^2-16|\kappa_\phi|^2-8|\tilde{\sigma}|^2\\
&-12\,\mathrm{Tr}\Big(g^{D}g^{D\dagger}\Big)-4\,\mathrm{Tr}\Big(h^{E}h^{E\dagger}\Big)\Big]-|\tilde{\sigma}|^2\Big[\frac{12}{5}g_1^2M_1+\frac{8}{5}g_1'^2M_1'+12g_2^2M_2\\
&+16\kappa_\phi^{*}T_{\kappa_\phi}+8\tilde{\sigma}^{*}T_{\tilde{\sigma}}+12\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^{D}}\Big)+4\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^{E}}\Big)\Big]-32|\kappa_\phi|^2\kappa_\phi^{*}T_{\kappa_\phi}\Big\}\\
&-4(B_\phi\mu_\phi)^{*}\Big[T_{\kappa_\phi}\Big(4|\kappa_\phi|^2+2|\tilde{\sigma}|^2+|\sigma|^2\Big)+\kappa_\phi\sigma^{*}T_{\sigma}+2\kappa_\phi\tilde{\sigma}^{*}T_{\tilde{\sigma}}\Big]\\
&+(B_L\mu_L)^{*}\Big\{\frac{12}{5}g_1^2\Big(\tilde{\sigma}M_1-T_{\tilde{\sigma}}\Big)+\frac{8}{5}g_1'^2\Big(\tilde{\sigma}M_1'-T_{\tilde{\sigma}}\Big)+12g_2^2\Big(\tilde{\sigma}M_2-T_{\tilde{\sigma}}\Big)\\
&+16|\tilde{\sigma}|^2T_{\tilde{\sigma}}+12\Big[T_{\tilde{\sigma}}\,\mathrm{Tr}\Big(g^{D}g^{D\dagger}\Big)+\tilde{\sigma}\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^{D}}\Big)\Big]+4\Big[T_{\tilde{\sigma}}\,\mathrm{Tr}\Big(h^{E}h^{E\dagger}\Big)\\
&+\tilde{\sigma}\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^{E}}\Big)\Big]\Big\}-4\kappa_\phi^{*}\mu_\phi(B_\phi\mu_\phi)\Big(2|\kappa_\phi|^2+2|\tilde{\sigma}|^2+|\sigma|^2\Big)\\
&+\tilde{\sigma}^{*}\mu_\phi B_L\mu_L\Big[8|\tilde{\sigma}|^2+12\,\mathrm{Tr}\Big(g^{D}g^{D\dagger}\Big)+4\,\mathrm{Tr}\Big(h^{E}h^{E\dagger}\Big)-\frac{12}{5}g_1^2-\frac{8}{5}g_1'^2\\
&-12g_2^2\Big]+\tilde{\sigma}^{*}\mu_L\mu_\phi\Big[\frac{12}{5}g_1^2M_1+\frac{8}{5}g_1'^2M_1'+12g_2^2M_2+8\tilde{\sigma}^{*}T_{\tilde{\sigma}}\\
&+12\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^{D}}\Big)+4\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^{E}}\Big)\Big]-4\kappa_\phi^{*}\mu_\phi^2\Big[2\kappa_\phi^{*}T_{\kappa_\phi}+2\tilde{\sigma}^{*}T_{\tilde{\sigma}}+\sigma^{*}T_{\sigma}\Big]\\
&-4\mu_\phi^{*}\Big[\kappa_\phi|\sigma|^2\Big(3m_\phi^2+m_S^2+m_{\bar{S}}^2\Big)+2\kappa_\phi|\tilde{\sigma}|^2\Big(3m_\phi^2+m_{L_4}^2+m_{\bar{L}_4}^2\Big)\\
&+\kappa_\phi\Big(10m_\phi^2|\kappa_\phi|^2+4|T_{\kappa_\phi}|^2+|T_\sigma|^2+2|T_{\tilde{\sigma}}|^2\Big)+\sigma T_\sigma^{*}T_{\kappa_\phi}+2\tilde{\sigma}T_{\tilde{\sigma}}^{*}T_{\kappa_\phi}\Big]\\
&+\mu_L^{*}\Big\{\tilde{\sigma}\Big[16|T_{\tilde{\sigma}}|^2+16m_{L_4}^2|\tilde{\sigma}|^2+16m_{\bar{L}_4}^2|\tilde{\sigma}|^2+8m_\phi^2|\tilde{\sigma}|^2+24m_{L_4}^2\,\mathrm{Tr}\Big(g^{D}g^{D\dagger}\Big)\\
&+12m_{\bar{L}_4}^2\,\mathrm{Tr}\Big(g^{D}g^{D\dagger}\Big)+8m_{L_4}^2\,\mathrm{Tr}\Big(h^{E}h^{E\dagger}\Big)+4m_{\bar{L}_4}^2\,\mathrm{Tr}\Big(h^{E}h^{E\dagger}\Big)\\
&+12\,\mathrm{Tr}\Big(T^{g^{D}*}T^{g^{D}T}\Big)+\mathrm{Tr}\Big(T^{h^{E}*}T^{h^{E}T}\Big)+12\,\mathrm{Tr}\Big(g^{D}m_{\bar{D}}^2g^{D\dagger}\Big)\\
&+12\,\mathrm{Tr}\Big(g^{D}g^{D\dagger}m_Q^2\Big)+\mathrm{Tr}\Big(h^{E}h^{E\dagger}m_{e^c}^{2*}\Big)+\mathrm{Tr}\Big(h^{E}m_{H_1}^{2*}h^{E\dagger}\Big)\Big]\\
&+T_{\tilde{\sigma}}\Big[12\,\mathrm{Tr}\Big(T^{g^{D}*}g^{DT}\Big)+\mathrm{Tr}\Big(T^{h^{E}*}h^{ET}\Big)\Big]\\
&-\frac{12}{5}g_1^2\Big(\tilde{\sigma}m_{L_4}^2+\tilde{\sigma}m_{\bar{L}_4}^2+2\tilde{\sigma}|M_1|^2-M_1T_{\tilde{\sigma}}\Big)\\
&-\frac{8}{5}g_1'^2\Big(\tilde{\sigma}m_{L_4}^2+\tilde{\sigma}m_{\bar{L}_4}^2+2\tilde{\sigma}|M_1'|^2-M_1'T_{\tilde{\sigma}}\Big)\\
&-12g_2^2\Big(\tilde{\sigma}m_{L_4}^2+\tilde{\sigma}m_{\bar{L}_4}^2+2\tilde{\sigma}|M_2|^2-M_2T_{\tilde{\sigma}}\Big)\Big\}. \qquad \text{(A.83)}
\end{aligned}
$$

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A.7 Soft scalar masses

In writing down the two-loop β functions for the soft scalar masses, the following quantities are defined,

$$
\begin{aligned}
\Sigma_{1,1} = {} & \sqrt{\frac{3}{5}} g_1 \Big[-m^2_{H_d} - m^2_{L_4} + m^2_{\bar{L}_4} + m^2_{H_u} + \mathrm{Tr}\left(m^2_{d^c}\right) - \mathrm{Tr}\left(m^2_{D}\right) \\
& + \mathrm{Tr}\left(m^2_{\bar{D}}\right) + \mathrm{Tr}\left(m^2_{e^c}\right) - \mathrm{Tr}\left(m^2_{H_1}\right) + \mathrm{Tr}\left(m^2_{H_2}\right) - \mathrm{Tr}\left(m^2_{L}\right) \\
& + \mathrm{Tr}\left(m^2_{Q}\right) - 2\,\mathrm{Tr}\left(m^2_{u^c}\right) \Big],
\end{aligned}
\tag{A.84}
$$

$$
\begin{aligned}
\Sigma_{1,4} = {} & \frac{1}{\sqrt{40}} g'_1 \Big[-6m^2_{H_d} + 4m^2_{L_4} - 4m^2_{\bar{L}_4} - 4m^2_{H_u} + 5m^2_{S} - 5m^2_{\bar{S}} \\
& + 6\,\mathrm{Tr}\left(m^2_{d^c}\right) - 6\,\mathrm{Tr}\left(m^2_{D}\right) - 9\,\mathrm{Tr}\left(m^2_{\bar{D}}\right) + \mathrm{Tr}\left(m^2_{e^c}\right) - 6\,\mathrm{Tr}\left(m^2_{H_1}\right) \\
& - 4\,\mathrm{Tr}\left(m^2_{H_2}\right) + 4\,\mathrm{Tr}\left(m^2_{L}\right) + 6\,\mathrm{Tr}\left(m^2_{Q}\right) + 5\,\mathrm{Tr}\left(m^2_{\Sigma}\right) + 3\,\mathrm{Tr}\left(m^2_{u^c}\right) \Big],
\end{aligned}
\tag{A.85}
$$

$$
\begin{aligned}
\Sigma_{2,11} = {} & \frac{1}{10} g_1^2 \Big[3m^2_{H_d} + 3m^2_{L_4} + 3m^2_{\bar{L}_4} + 3m^2_{H_u} + 2\,\mathrm{Tr}\left(m^2_{d^c}\right) + 2\,\mathrm{Tr}\left(m^2_{D}\right) \\
& + 2\,\mathrm{Tr}\left(m^2_{\bar{D}}\right) + 6\,\mathrm{Tr}\left(m^2_{e^c}\right) + 3\,\mathrm{Tr}\left(m^2_{H_1}\right) + 3\,\mathrm{Tr}\left(m^2_{H_2}\right) + 3\,\mathrm{Tr}\left(m^2_{L}\right) \\
& + \mathrm{Tr}\left(m^2_{Q}\right) + 8\,\mathrm{Tr}\left(m^2_{u^c}\right) \Big],
\end{aligned}
\tag{A.86}
$$

$$
\begin{aligned}
\Sigma_{2,14} = {} & \frac{1}{10} \sqrt{\frac{3}{2}} g_1 g'_1 \Big[3m^2_{H_d} - 2m^2_{L_4} - 2m^2_{\bar{L}_4} - 2m^2_{H_u} + 2\,\mathrm{Tr}\left(m^2_{d^c}\right) + 2\,\mathrm{Tr}\left(m^2_{D}\right) \\
& - 3\,\mathrm{Tr}\left(m^2_{\bar{D}}\right) + \mathrm{Tr}\left(m^2_{e^c}\right) + 3\,\mathrm{Tr}\left(m^2_{H_1}\right) - 2\,\mathrm{Tr}\left(m^2_{H_2}\right) - 2\,\mathrm{Tr}\left(m^2_{L}\right) \\
& + \mathrm{Tr}\left(m^2_{Q}\right) - 2\,\mathrm{Tr}\left(m^2_{u^c}\right) \Big],
\end{aligned}
\tag{A.87}
$$

$$
\begin{aligned}
\Sigma_{3,1} = {} & \frac{1}{40\sqrt{15}} g_1 \Big[-18g_1^2 m^2_{H_d} - 27g_1'^2 m^2_{H_d} - 90g_2^2 m^2_{H_d} - 18g_1^2 m^2_{L_4} - 12g_1'^2 m^2_{L_4} \\
& - 90g_2^2 m^2_{L_4} + 18g_1^2 m^2_{\bar{L}_4} + 12g_1'^2 m^2_{\bar{L}_4} + 90g_2^2 m^2_{\bar{L}_4} + 18g_1^2 m^2_{H_u} + 12g_1'^2 m^2_{H_u} \\
& + 90g_2^2 m^2_{H_u} + 60\left(-m^2_{H_u} + m^2_{H_d}\right)|\lambda|^2 + 60\left(-m^2_{\bar{L}_4} + m^2_{L_4}\right)|\tilde{\sigma}|^2 \\
& + 8g_1^2\,\mathrm{Tr}\left(m^2_{d^c}\right) + 12g_1'^2\,\mathrm{Tr}\left(m^2_{d^c}\right) + 160g_3^2\,\mathrm{Tr}\left(m^2_{d^c}\right) - 8g_1^2\,\mathrm{Tr}\left(m^2_{D}\right) \\
& - 12g_1'^2\,\mathrm{Tr}\left(m^2_{D}\right) - 160g_3^2\,\mathrm{Tr}\left(m^2_{D}\right) + 8g_1^2\,\mathrm{Tr}\left(m^2_{\bar{D}}\right) + 27g_1'^2\,\mathrm{Tr}\left(m^2_{\bar{D}}\right) \\
& + 160g_3^2\,\mathrm{Tr}\left(m^2_{\bar{D}}\right) + 72g_1^2\,\mathrm{Tr}\left(m^2_{e^c}\right) + 3g_1'^2\,\mathrm{Tr}\left(m^2_{e^c}\right) - 18g_1^2\,\mathrm{Tr}\left(m^2_{H_1}\right) \\
& - 27g_1'^2\,\mathrm{Tr}\left(m^2_{H_1}\right) - 90g_2^2\,\mathrm{Tr}\left(m^2_{H_1}\right) + 18g_1^2\,\mathrm{Tr}\left(m^2_{H_2}\right) + 12g_1'^2\,\mathrm{Tr}\left(m^2_{H_2}\right) \\
& + 90g_2^2\,\mathrm{Tr}\left(m^2_{H_2}\right) - 18g_1^2\,\mathrm{Tr}\left(m^2_{L}\right) - 12g_1'^2\,\mathrm{Tr}\left(m^2_{L}\right) - 90g_2^2\,\mathrm{Tr}\left(m^2_{L}\right) \\
& + 2g_1^2\,\mathrm{Tr}\left(m^2_{Q}\right) + 3g_1'^2\,\mathrm{Tr}\left(m^2_{Q}\right) + 90g_2^2\,\mathrm{Tr}\left(m^2_{Q}\right) + 160g_3^2\,\mathrm{Tr}\left(m^2_{Q}\right) \\
& - 64g_1^2\,\mathrm{Tr}\left(m^2_{u^c}\right) - 6g_1'^2\,\mathrm{Tr}\left(m^2_{u^c}\right) - 320g_3^2\,\mathrm{Tr}\left(m^2_{u^c}\right) + 60m^2_{H_d}\,\mathrm{Tr}\left(f f^\dagger\right) \\
& - 60m^2_{H_u}\,\mathrm{Tr}\left(\tilde{f}\tilde{f}^\dagger\right) + 180m^2_{L_4}\,\mathrm{Tr}\left(g^D g^{D\dagger}\right) + 60m^2_{L_4}\,\mathrm{Tr}\left(h^E h^{E\dagger}\right) \\
& + 180m^2_{H_d}\,\mathrm{Tr}\left(y^D y^{D\dagger}\right) + 60m^2_{H_d}\,\mathrm{Tr}\left(y^E y^{E\dagger}\right) - 180m^2_{H_u}\,\mathrm{Tr}\left(y^U y^{U\dagger}\right)
\end{aligned}
$$

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$$
\begin{aligned}
&- 60\,\mathrm{Tr}\Big(f m_{H_2}^{2*} f^\dagger\Big) + 60\,\mathrm{Tr}\Big(\tilde{f} m_{H_1}^{2*} \tilde{f}^\dagger\Big) - 120\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^2 g^{D\dagger}\Big) \\
&- 60\,\mathrm{Tr}\Big(g^D g^{D\dagger} m_Q^2\Big) - 120\,\mathrm{Tr}\Big(h^E h^{E\dagger} m_{e^c}^{2*}\Big) + 60\,\mathrm{Tr}\Big(h^E m_{H_1}^{2*} h^{E\dagger}\Big) \\
&+ 60\,\mathrm{Tr}\Big(m_D^2 \kappa\kappa^\dagger\Big) - 60\,\mathrm{Tr}\Big(m_{\bar{D}}^2 \kappa^\dagger \kappa\Big) - 60\,\mathrm{Tr}\Big(m_{H_2}^2 \tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 120\,\mathrm{Tr}\Big(y^D y^{D\dagger} m_{d^c}^{2*}\Big) - 60\,\mathrm{Tr}\Big(y^D m_Q^{2*} y^{D\dagger}\Big) - 120\,\mathrm{Tr}\Big(y^E y^{E\dagger} m_{e^c}^{2*}\Big) \\
&+ 60\,\mathrm{Tr}\Big(y^E m_L^{2*} y^{E\dagger}\Big) + 240\,\mathrm{Tr}\Big(y^U y^{U\dagger} m_{u^c}^{2*}\Big) - 60\,\mathrm{Tr}\Big(y^U m_Q^{2*} y^{U\dagger}\Big) \\
&+ 60\,\mathrm{Tr}\Big(\tilde{\lambda} m_{H_1}^{2*} \tilde{\lambda}^\dagger\Big)\Big], \qquad \text{(A.88)}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{2,2} = \frac{1}{2}\Big[&3\,\mathrm{Tr}\Big(m_Q^2\Big) + m_{H_d}^2 + m_{L_4}^2 + m_{\bar{L}_4}^2 + m_{H_u}^2 + \mathrm{Tr}\Big(m_{H_1}^2\Big) + \mathrm{Tr}\Big(m_{H_2}^2\Big) \\
&+ \mathrm{Tr}\Big(m_L^2\Big)\Big], \qquad \text{(A.89)}
\end{aligned}
$$

$$
\Sigma_{2,3} = \frac{1}{2}\Big[2\,\mathrm{Tr}\Big(m_Q^2\Big) + \mathrm{Tr}\Big(m_{d^c}^2\Big) + \mathrm{Tr}\Big(m_D^2\Big) + \mathrm{Tr}\Big(m_{\bar{D}}^2\Big) + \mathrm{Tr}\Big(m_{u^c}^2\Big)\Big], \qquad \text{(A.90)}
$$

$$
\begin{aligned}
\Sigma_{2,41} = \frac{1}{10}\sqrt{\frac{3}{2}} g_1 g_1' \Big[&3m_{H_d}^2 - 2m_{L_4}^2 - 2m_{\bar{L}_4}^2 - 2m_{H_u}^2 + 2\,\mathrm{Tr}\Big(m_{d^c}^2\Big) + 2\,\mathrm{Tr}\Big(m_D^2\Big) \\
&- 3\,\mathrm{Tr}\Big(m_{\bar{D}}^2\Big) + \mathrm{Tr}\Big(m_{e^c}^2\Big) + 3\,\mathrm{Tr}\Big(m_{H_1}^2\Big) - 2\,\mathrm{Tr}\Big(m_{H_2}^2\Big) - 2\,\mathrm{Tr}\Big(m_L^2\Big) \\
&+ \mathrm{Tr}\Big(m_Q^2\Big) - 2\,\mathrm{Tr}\Big(m_{u^c}^2\Big)\Big], \qquad \text{(A.91)}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{2,44} = \frac{1}{40} g_1'^2 \Big[&18m_{H_d}^2 + 8m_{L_4}^2 + 8m_{\bar{L}_4}^2 + 8m_{H_u}^2 + 25m_S^2 + 25m_{\bar{S}}^2 + 12\,\mathrm{Tr}\Big(m_{d^c}^2\Big) \\
&+ 12\,\mathrm{Tr}\Big(m_D^2\Big) + 27\,\mathrm{Tr}\Big(m_{\bar{D}}^2\Big) + \mathrm{Tr}\Big(m_{e^c}^2\Big) + 18\,\mathrm{Tr}\Big(m_{H_1}^2\Big) + 8\,\mathrm{Tr}\Big(m_{H_2}^2\Big) \\
&+ 8\,\mathrm{Tr}\Big(m_L^2\Big) + 6\,\mathrm{Tr}\Big(m_Q^2\Big) + 25\,\mathrm{Tr}\Big(m_\Sigma^2\Big) + 3\,\mathrm{Tr}\Big(m_{u^c}^2\Big)\Big], \qquad \text{(A.92)}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{3,4} = -\frac{1}{80\sqrt{10}} g_1' \Big[&36g_1^2 m_{H_d}^2 + 54g_1'^2 m_{H_d}^2 + 180g_2^2 m_{H_d}^2 - 24g_1^2 m_{L_4}^2 - 16g_1'^2 m_{L_4}^2 \\
&- 120g_2^2 m_{L_4}^2 + 24g_1^2 m_{\bar{L}_4}^2 + 16g_1'^2 m_{\bar{L}_4}^2 + 120g_2^2 m_{\bar{L}_4}^2 + 24g_1^2 m_{H_u}^2 + 16g_1'^2 m_{H_u}^2 \\
&+ 120g_2^2 m_{H_u}^2 - 125g_1'^2 m_S^2 + 125g_1'^2 m_{\bar{S}}^2 - 40\Big(2m_{H_u}^2 + 3m_{H_d}^2 - 5m_S^2\Big)|\lambda|^2 \\
&+ 100\Big(-m_{\bar{S}}^2 + m_S^2\Big)|\sigma|^2 + 80m_{L_4}^2|\tilde{\sigma}|^2 - 80m_{\bar{L}_4}^2|\tilde{\sigma}|^2 - 16g_1^2\,\mathrm{Tr}\Big(m_{d^c}^2\Big) \\
&- 24g_1'^2\,\mathrm{Tr}\Big(m_{d^c}^2\Big) - 320g_3^2\,\mathrm{Tr}\Big(m_{d^c}^2\Big) + 16g_1^2\,\mathrm{Tr}\Big(m_D^2\Big) + 24g_1'^2\,\mathrm{Tr}\Big(m_D^2\Big) \\
&+ 320g_3^2\,\mathrm{Tr}\Big(m_D^2\Big) + 24g_1^2\,\mathrm{Tr}\Big(m_{\bar{D}}^2\Big) + 81g_1'^2\,\mathrm{Tr}\Big(m_{\bar{D}}^2\Big) + 480g_3^2\,\mathrm{Tr}\Big(m_{\bar{D}}^2\Big) \\
&- 24g_1^2\,\mathrm{Tr}\Big(m_{e^c}^2\Big) - g_1'^2\,\mathrm{Tr}\Big(m_{e^c}^2\Big) + 36g_1^2\,\mathrm{Tr}\Big(m_{H_1}^2\Big) + 54g_1'^2\,\mathrm{Tr}\Big(m_{H_1}^2\Big) \\
&+ 180g_2^2\,\mathrm{Tr}\Big(m_{H_1}^2\Big) + 24g_1^2\,\mathrm{Tr}\Big(m_{H_2}^2\Big) + 16g_1'^2\,\mathrm{Tr}\Big(m_{H_2}^2\Big) + 120g_2^2\,\mathrm{Tr}\Big(m_{H_2}^2\Big) \\
&- 24g_1^2\,\mathrm{Tr}\Big(m_L^2\Big) - 16g_1'^2\,\mathrm{Tr}\Big(m_L^2\Big) - 120g_2^2\,\mathrm{Tr}\Big(m_L^2\Big) - 4g_1^2\,\mathrm{Tr}\Big(m_Q^2\Big) \\
&- 6g_1'^2\,\mathrm{Tr}\Big(m_Q^2\Big) - 180g_2^2\,\mathrm{Tr}\Big(m_Q^2\Big) - 320g_3^2\,\mathrm{Tr}\Big(m_Q^2\Big) - 125g_1'^2\,\mathrm{Tr}\Big(m_\Sigma^2\Big) \\
&- 32g_1^2\,\mathrm{Tr}\Big(m_{u^c}^2\Big) - 3g_1'^2\,\mathrm{Tr}\Big(m_{u^c}^2\Big) - 160g_3^2\,\mathrm{Tr}\Big(m_{u^c}^2\Big) - 120m_{H_d}^2\,\mathrm{Tr}\Big(ff^\dagger\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&-80m_{H_u}^2\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)+240m_{L_4}^2\,\mathrm{Tr}\Big(g^Dg^{D\dagger}\Big)+80m_{L_4}^2\,\mathrm{Tr}\Big(h^Eh^{E\dagger}\Big)\\
&-360m_{H_d}^2\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-120m_{H_d}^2\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-240m_{H_u}^2\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)\\
&+300m_S^2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)+200m_S^2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big)+200\,\mathrm{Tr}\Big(ff^\dagger m_\Sigma^2\Big)\\
&-80\,\mathrm{Tr}\Big(fm_{H_2}^{2*}f^\dagger\Big)+200\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger m_\Sigma^2\Big)-120\,\mathrm{Tr}\Big(\tilde{f}m_{H_1}^{2*}\tilde{f}^\dagger\Big)\\
&-360\,\mathrm{Tr}\Big(g^Dm_{\bar{D}}^2g^{D\dagger}\Big)+120\,\mathrm{Tr}\Big(g^Dg^{D\dagger}m_Q^2\Big)+40\,\mathrm{Tr}\Big(h^Eh^{E\dagger}m_{e^c}^{2*}\Big)\\
&-120\,\mathrm{Tr}\Big(h^Em_{H_1}^{2*}h^{E\dagger}\Big)-120\,\mathrm{Tr}\Big(m_D^2\kappa\kappa^\dagger\Big)-180\,\mathrm{Tr}\Big(m_{\bar{D}}^2\kappa^\dagger\kappa\Big)\\
&-80\,\mathrm{Tr}\Big(m_{H_2}^2\tilde{\lambda}\tilde{\lambda}^\dagger\Big)+240\,\mathrm{Tr}\Big(y^Dy^{D\dagger}m_{d^c}^{2*}\Big)+120\,\mathrm{Tr}\Big(y^Dm_Q^{2*}y^{D\dagger}\Big)\\
&+40\,\mathrm{Tr}\Big(y^Ey^{E\dagger}m_{e^c}^{2*}\Big)+80\,\mathrm{Tr}\Big(y^Em_L^{2*}y^{E\dagger}\Big)+120\,\mathrm{Tr}\Big(y^Uy^{U\dagger}m_{u^c}^{2*}\Big)\\
&+120\,\mathrm{Tr}\Big(y^Um_Q^{2*}y^{U\dagger}\Big)-120\,\mathrm{Tr}\Big(\tilde{\lambda}m_{H_1}^{2*}\tilde{\lambda}^\dagger\Big)\Big], \qquad \text{(A.93)}
\end{aligned}
$$

The RGEs are then given by

$$
\begin{aligned}
\beta_{m_Q^2}^{(1)} ={}& -\frac{2}{15}g_1^2\mathbf{1}|M_1|^2-\frac{1}{5}g_1'^2\mathbf{1}|M_1'|^2-\frac{32}{3}g_3^2\mathbf{1}|M_3|^2-6g_2^2\mathbf{1}|M_2|^2+2m_{H_d}^2y^{D\dagger}y^D\\
&+2m_{H_u}^2y^{U\dagger}y^U+2T^{D\dagger}T^D+2T^{U\dagger}T^U+2m_{L_4}^2g^{D*}g^{DT}+2T^{g^D*}T^{g^DT}\\
&+m_Q^2y^{D\dagger}y^D+m_Q^2y^{U\dagger}y^U+m_Q^2g^{D*}g^{DT}+2y^{D\dagger}m_{d^c}^2y^D+y^{D\dagger}y^Dm_Q^2\\
&+2y^{U\dagger}m_{u^c}^2y^U+y^{U\dagger}y^Um_Q^2+2g^{D*}m_{\bar{D}}^2g^{DT}+g^{D*}g^{DT}m_Q^2+\frac{1}{\sqrt{15}}g_1\mathbf{1}\Sigma_{1,1}\\
&+\frac{1}{\sqrt{10}}g_1'\mathbf{1}\Sigma_{1,4}, \qquad \text{(A.94)}\\
\beta_{m_Q^2}^{(2)} ={}& \frac{32}{45}g_1^2g_3^2\mathbf{1}|M_3|^2+\frac{16}{15}g_1'^2g_3^2\mathbf{1}|M_3|^2+32g_2^2g_3^2\mathbf{1}|M_3|^2+\frac{160}{3}g_3^4\mathbf{1}|M_3|^2\\
&+\frac{2}{5}g_1^2g_2^2\mathbf{1}|M_2|^2+\frac{3}{5}g_1'^2g_2^2\mathbf{1}|M_2|^2+87g_2^4\mathbf{1}|M_2|^2+32g_2^2g_3^2\mathbf{1}|M_2|^2\\
&+\frac{16}{45}g_1^2g_3^2M_1\mathbf{1}M_3^*+\frac{8}{15}g_1'^2g_3^2M_1'\mathbf{1}M_3^*+16g_2^2g_3^2M_2\mathbf{1}M_3^*+\frac{1}{5}g_1^2g_2^2M_1\mathbf{1}M_2^*\\
&+\frac{3}{10}g_1'^2g_2^2M_1'\mathbf{1}M_2^*+16g_2^2g_3^2M_3\mathbf{1}M_2^*+\frac{4}{5}g_1^2m_{H_d}^2y^{D\dagger}y^D+\frac{6}{5}g_1'^2m_{H_d}^2y^{D\dagger}y^D\\
&-4m_{H_d}^2|\lambda|^2y^{D\dagger}y^D-2m_{H_u}^2|\lambda|^2y^{D\dagger}y^D-2m_S^2|\lambda|^2y^{D\dagger}y^D-2|T_\lambda|^2y^{D\dagger}y^D\\
&-2\lambda T_\lambda^*y^{D\dagger}T^D+\frac{8}{5}g_1^2m_{H_u}^2y^{U\dagger}y^U+\frac{2}{5}g_1'^2m_{H_u}^2y^{U\dagger}y^U-2m_{H_d}^2|\lambda|^2y^{U\dagger}y^U\\
&-4m_{H_u}^2|\lambda|^2y^{U\dagger}y^U-2m_S^2|\lambda|^2y^{U\dagger}y^U-2|T_\lambda|^2y^{U\dagger}y^U-2\lambda T_\lambda^*y^{U\dagger}T^U\\
&-\frac{4}{5}g_1^2M_1T^{D\dagger}y^D-\frac{6}{5}g_1'^2M_1'T^{D\dagger}y^D+\frac{4}{5}g_1^2T^{D\dagger}T^D+\frac{6}{5}g_1'^2T^{D\dagger}T^D\\
&-2|\lambda|^2T^{D\dagger}T^D-\frac{8}{5}g_1^2M_1T^{U\dagger}y^U-\frac{2}{5}g_1'^2M_1'T^{U\dagger}y^U+\frac{8}{5}g_1^2T^{U\dagger}T^U\\
&+\frac{2}{5}g_1'^2T^{U\dagger}T^U-2|\lambda|^2T^{U\dagger}T^U+\frac{4}{5}g_1^2m_{L_4}^2g^{D*}g^{DT}+\frac{6}{5}g_1'^2m_{L_4}^2g^{D*}g^{DT}\\
&-4m_{L_4}^2|\tilde{\sigma}|^2g^{D*}g^{DT}-2m_{\bar{L}_4}^2|\tilde{\sigma}|^2g^{D*}g^{DT}-2m_\phi^2|\tilde{\sigma}|^2g^{D*}g^{DT}-2|T_{\tilde{\sigma}}|^2g^{D*}g^{DT}\\
&+\frac{1}{150}g_1'^2M_1'^*\Big\{\Big[-11g_1^2\Big(2M_1'+M_1\Big)+160g_3^2M_1'+45g_2^2M_2+80g_3^2M_3
\end{aligned}
$$

$$
\begin{aligned}
&+90g_2^2M_1'+963g_1'^2M_1'\Big]\mathbf{1}+60\Big(2M_1'y^{U\dagger}y^U-3g^{D*}T^{g^DT}-3y^{D\dagger}T^D\\
&+6M_1'g^{D*}g^{DT}+6M_1'y^{D\dagger}y^D-y^{U\dagger}T^U\Big)\Big\}+\frac{1}{450}g_1^2M_1^*\Big(\Big\{10\Big[16g_3^2\Big(2M_1\\
&+M_3\Big)+9g_2^2\Big(2M_1+M_2\Big)\Big]+1734g_1^2M_1-33g_1'^2\Big(2M_1+M_1'\Big)\Big\}\mathbf{1}\\
&+360\Big(2M_1g^{D*}g^{DT}+2M_1y^{D\dagger}y^D-2y^{U\dagger}T^U+4M_1y^{U\dagger}y^U-g^{D*}T^{g^DT}\\
&-y^{D\dagger}T^D\Big)\Big)-2\tilde{\sigma}T_{\tilde{\sigma}}^*g^{D*}T^{g^DT}-\frac{4}{5}g_1^2M_1T^{g^D*}g^{DT}-\frac{6}{5}g_1'^2M_1'T^{g^D*}g^{DT}\\
&+\frac{4}{5}g_1^2T^{g^D*}T^{g^DT}+\frac{6}{5}g_1'^2T^{g^D*}T^{g^DT}-2|\tilde{\sigma}|^2T^{g^D*}T^{g^DT}+\frac{2}{5}g_1^2m_Q^2y^{D\dagger}y^D\\
&+\frac{3}{5}g_1'^2m_Q^2y^{D\dagger}y^D-|\lambda|^2m_Q^2y^{D\dagger}y^D+\frac{4}{5}g_1^2m_Q^2y^{U\dagger}y^U+\frac{1}{5}g_1'^2m_Q^2y^{U\dagger}y^U\\
&-|\lambda|^2m_Q^2y^{U\dagger}y^U+\frac{2}{5}g_1^2m_Q^2g^{D*}g^{DT}+\frac{3}{5}g_1'^2m_Q^2g^{D*}g^{DT}-|\tilde{\sigma}|^2m_Q^2g^{D*}g^{DT}\\
&+\frac{4}{5}g_1^2y^{D\dagger}m_{d^c}^2y^D+\frac{6}{5}g_1'^2y^{D\dagger}m_{d^c}^2y^D-2|\lambda|^2y^{D\dagger}m_{d^c}^2y^D+\frac{2}{5}g_1^2y^{D\dagger}y^Dm_Q^2\\
&+\frac{3}{5}g_1'^2y^{D\dagger}y^Dm_Q^2-|\lambda|^2y^{D\dagger}y^Dm_Q^2+\frac{8}{5}g_1^2y^{U\dagger}m_{u^c}^2y^U+\frac{2}{5}g_1'^2y^{U\dagger}m_{u^c}^2y^U\\
&-2|\lambda|^2y^{U\dagger}m_{u^c}^2y^U+\frac{4}{5}g_1^2y^{U\dagger}y^Um_Q^2+\frac{1}{5}g_1'^2y^{U\dagger}y^Um_Q^2-|\lambda|^2y^{U\dagger}y^Um_Q^2\\
&+\frac{4}{5}g_1^2g^{D*}m_{\bar{D}}^2g^{DT}+\frac{6}{5}g_1'^2g^{D*}m_{\bar{D}}^2g^{DT}-2|\tilde{\sigma}|^2g^{D*}m_{\bar{D}}^2g^{DT}+\frac{2}{5}g_1^2g^{D*}g^{DT}m_Q^2\\
&+\frac{3}{5}g_1'^2g^{D*}g^{DT}m_Q^2-|\tilde{\sigma}|^2g^{D*}g^{DT}m_Q^2-8m_{H_d}^2y^{D\dagger}y^Dy^{D\dagger}y^D-4y^{D\dagger}y^DT^{D\dagger}T^D\\
&-4y^{D\dagger}T^DT^{D\dagger}y^D-8m_{H_u}^2y^{U\dagger}y^Uy^{U\dagger}y^U-4y^{U\dagger}y^UT^{U\dagger}T^U-4y^{U\dagger}T^UT^{U\dagger}y^U\\
&-4T^{D\dagger}y^Dy^{D\dagger}T^D-4T^{D\dagger}T^Dy^{D\dagger}y^D-4T^{U\dagger}y^Uy^{U\dagger}T^U-4T^{U\dagger}T^Uy^{U\dagger}y^U\\
&-8m_{L_4}^2g^{D*}g^{DT}g^{D*}g^{DT}-4g^{D*}g^{DT}T^{g^D*}T^{g^DT}-2m_{L_4}^2g^{D*}\kappa^T\kappa^*g^{DT}\\
&-2m_S^2g^{D*}\kappa^T\kappa^*g^{DT}-2g^{D*}\kappa^TT^{\kappa*}T^{g^DT}-4g^{D*}T^{g^DT}T^{g^D*}g^{DT}\\
&-2g^{D*}T^{\kappa T}T^{\kappa*}g^{DT}-4T^{g^D*}g^{DT}g^{D*}T^{g^DT}-2T^{g^D*}\kappa^T\kappa^*T^{g^DT}\\
&-4T^{g^D*}T^{g^DT}g^{D*}g^{DT}-2T^{g^D*}T^{\kappa T}\kappa^*g^{DT}-2m_Q^2y^{D\dagger}y^Dy^{D\dagger}y^D\\
&-2m_Q^2y^{U\dagger}y^Uy^{U\dagger}y^U-2m_Q^2g^{D*}g^{DT}g^{D*}g^{DT}-m_Q^2g^{D*}\kappa^T\kappa^*g^{DT}\\
&-4y^{D\dagger}m_{d^c}^2y^Dy^{D\dagger}y^D-4y^{D\dagger}y^Dm_Q^2y^{D\dagger}y^D-4y^{D\dagger}y^Dy^{D\dagger}m_{d^c}^2y^D\\
&-2y^{D\dagger}y^Dy^{D\dagger}y^Dm_Q^2-4y^{U\dagger}m_{u^c}^2y^Uy^{U\dagger}y^U-4y^{U\dagger}y^Um_Q^2y^{U\dagger}y^U\\
&-4y^{U\dagger}y^Uy^{U\dagger}m_{u^c}^2y^U-2y^{U\dagger}y^Uy^{U\dagger}y^Um_Q^2-4g^{D*}m_{\bar{D}}^2g^{DT}g^{D*}g^{DT}\\
&-2g^{D*}m_{\bar{D}}^2\kappa^T\kappa^*g^{DT}-4g^{D*}g^{DT}m_Q^2g^{D*}g^{DT}-4g^{D*}g^{DT}g^{D*}m_{\bar{D}}^2g^{DT}\\
&-2g^{D*}g^{DT}g^{D*}g^{DT}m_Q^2-2g^{D*}\kappa^Tm_D^2\kappa^*g^{DT}-2g^{D*}\kappa^T\kappa^*m_{\bar{D}}^2g^{DT}\\
&-g^{D*}\kappa^T\kappa^*g^{DT}m_Q^2-2\lambda^*T^{D\dagger}y^DT_\lambda-2\lambda^*T^{U\dagger}y^UT_\lambda-2\tilde{\sigma}^*T^{g^D*}g^{DT}T_{\tilde{\sigma}}\\
&+6g_2^4\mathbf{1}\Sigma_{2,2}+\frac{32}{3}g_3^4\mathbf{1}\Sigma_{2,3}+\frac{2}{15}g_1^2\mathbf{1}\Sigma_{2,11}+\frac{1}{5}\sqrt{\frac{2}{3}}g_1g_1'\mathbf{1}\Sigma_{2,14}\\
&+\frac{1}{5}\sqrt{\frac{2}{3}}g_1g_1'\mathbf{1}\Sigma_{2,41}+\frac{1}{5}g_1'^2\mathbf{1}\Sigma_{2,44}+\frac{4}{\sqrt{15}}g_1\mathbf{1}\Sigma_{3,1}+2\sqrt{\frac{2}{5}}g_1'\mathbf{1}\Sigma_{3,4}
\end{aligned}
$$

$$
\begin{aligned}
&- 4m_{H_d}^2 y^{D\dagger} y^D \,\mathrm{Tr}\Big(f f^\dagger\Big) - 2T^{D\dagger} T^D \,\mathrm{Tr}\Big(f f^\dagger\Big) - m_Q^2 y^{D\dagger} y^D \,\mathrm{Tr}\Big(f f^\dagger\Big) \\
&- 2y^{D\dagger} m_{d^c}^2 y^D \,\mathrm{Tr}\Big(f f^\dagger\Big) - y^{D\dagger} y^D m_Q^2 \,\mathrm{Tr}\Big(f f^\dagger\Big) - 4m_{H_u}^2 y^{U\dagger} y^U \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) \\
&- 2T^{U\dagger} T^U \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) - m_Q^2 y^{U\dagger} y^U \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) - 2y^{U\dagger} m_{u^c}^2 y^U \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) \\
&- y^{U\dagger} y^U m_Q^2 \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) - 12m_{L_4}^2 g^{D*} g^{DT} \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 6T^{g^D*} T^{g^D T} \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 3m_Q^2 g^{D*} g^{DT} \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 6g^{D*} m_{\bar{D}}^2 g^{DT} \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 3g^{D*} g^{DT} m_Q^2 \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 4m_{L_4}^2 g^{D*} g^{DT} \,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 2T^{g^D*} T^{g^D T} \,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- m_Q^2 g^{D*} g^{DT} \,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 2g^{D*} m_{\bar{D}}^2 g^{DT} \,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- g^{D*} g^{DT} m_Q^2 \,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 12m_{H_d}^2 y^{D\dagger} y^D \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6T^{D\dagger} T^D \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 3m_Q^2 y^{D\dagger} y^D \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 6y^{D\dagger} m_{d^c}^2 y^D \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 3y^{D\dagger} y^D m_Q^2 \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 4m_{H_d}^2 y^{D\dagger} y^D \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 2T^{D\dagger} T^D \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- m_Q^2 y^{D\dagger} y^D \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 2y^{D\dagger} m_{d^c}^2 y^D \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - y^{D\dagger} y^D m_Q^2 \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 12m_{H_u}^2 y^{U\dagger} y^U \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 6T^{U\dagger} T^U \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 3m_Q^2 y^{U\dagger} y^U \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 6y^{U\dagger} m_{u^c}^2 y^U \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 3y^{U\dagger} y^U m_Q^2 \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 2T^{D\dagger} y^D \,\mathrm{Tr}\Big(f^\dagger T^f\Big) \\
&- 2T^{U\dagger} y^U \,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) - 6T^{g^D*} g^{DT} \,\mathrm{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 2T^{g^D*} g^{DT} \,\mathrm{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&- 6T^{D\dagger} y^D \,\mathrm{Tr}\Big(y^{D\dagger} T^D\Big) - 2T^{D\dagger} y^D \,\mathrm{Tr}\Big(y^{E\dagger} T^E\Big) - 6T^{U\dagger} y^U \,\mathrm{Tr}\Big(y^{U\dagger} T^U\Big) \\
&- 2y^{D\dagger} T^D \,\mathrm{Tr}\Big(T^{f*} f^T\Big) - 2y^{D\dagger} y^D \,\mathrm{Tr}\Big(T^{f*} T^{fT}\Big) - 2y^{U\dagger} T^U \,\mathrm{Tr}\Big(T^{\tilde{f}*} \tilde{f}^T\Big) \\
&- 2y^{U\dagger} y^U \,\mathrm{Tr}\Big(T^{\tilde{f}*} T^{\tilde{f}T}\Big) - 6g^{D*} T^{g^D T} \,\mathrm{Tr}\Big(T^{g^D*} g^{DT}\Big) \\
&- 6g^{D*} g^{DT} \,\mathrm{Tr}\Big(T^{g^D*} T^{g^D T}\Big) - 2g^{D*} T^{g^D T} \,\mathrm{Tr}\Big(T^{h^E*} h^{ET}\Big) \\
&- 2g^{D*} g^{DT} \,\mathrm{Tr}\Big(T^{h^E*} T^{h^E T}\Big) - 6y^{D\dagger} T^D \,\mathrm{Tr}\Big(T^{D*} y^{DT}\Big) \\
&- 6y^{D\dagger} y^D \,\mathrm{Tr}\Big(T^{D*} T^{DT}\Big) - 2y^{D\dagger} T^D \,\mathrm{Tr}\Big(T^{E*} y^{ET}\Big) - 2y^{D\dagger} y^D \,\mathrm{Tr}\Big(T^{E*} T^{ET}\Big) \\
&- 6y^{U\dagger} T^U \,\mathrm{Tr}\Big(T^{U*} y^{UT}\Big) - 6y^{U\dagger} y^U \,\mathrm{Tr}\Big(T^{U*} T^{UT}\Big) - 2y^{D\dagger} y^D \,\mathrm{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) \\
&- 2y^{D\dagger} y^D \,\mathrm{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big) - 2y^{U\dagger} y^U \,\mathrm{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) - 2y^{U\dagger} y^U \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger m_\Sigma^{2*}\Big) \\
&- 6g^{D*} g^{DT} \,\mathrm{Tr}\Big(g^D g^{D\dagger} m_Q^{2*}\Big) - 6g^{D*} g^{DT} \,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger}\Big) \\
&- 2g^{D*} g^{DT} \,\mathrm{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) - 2g^{D*} g^{DT} \,\mathrm{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2\Big) \\
&- 6y^{D\dagger} y^D \,\mathrm{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) - 2y^{D\dagger} y^D \,\mathrm{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&- 2y^{D\dagger}y^D \operatorname{Tr}\Big(m_L^2 y^{E\dagger}y^E\Big) - 6y^{D\dagger}y^D \operatorname{Tr}\Big(m_Q^2 y^{D\dagger}y^D\Big)\\
&- 6y^{U\dagger}y^U \operatorname{Tr}\Big(m_Q^2 y^{U\dagger}y^U\Big) - 6y^{U\dagger}y^U \operatorname{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big), \qquad \text{(A.95)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_L^2}^{(1)} =& -\frac{6}{5}g_1^2\mathbf{1}|M_1|^2 - \frac{4}{5}g_1'^2\mathbf{1}|M_1'|^2 - 6g_2^2\mathbf{1}|M_2|^2 + 2m_{H_d}^2 y^{E\dagger}y^E + 2T^{E\dagger}T^E\\
&+ m_L^2 y^{E\dagger}y^E + 2y^{E\dagger}m_{e^c}^2 y^E + y^{E\dagger}y^E m_L^2 - \sqrt{\frac{3}{5}}g_1\mathbf{1}\Sigma_{1,1} + \sqrt{\frac{2}{5}}g_1'\mathbf{1}\Sigma_{1,4}, \qquad \text{(A.96)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_L^2}^{(2)} =& \frac{18}{5}g_1^2g_2^2\mathbf{1}|M_2|^2 + \frac{12}{5}g_1'^2g_2^2\mathbf{1}|M_2|^2 + 87g_2^4\mathbf{1}|M_2|^2 + \frac{9}{5}g_1^2g_2^2M_1\mathbf{1}M_2^*\\
&+ \frac{6}{5}g_1'^2g_2^2M_1'\mathbf{1}M_2^* + \frac{12}{5}g_1^2m_{H_d}^2y^{E\dagger}y^E + \frac{3}{5}g_1'^2m_{H_d}^2y^{E\dagger}y^E - 4m_{H_d}^2|\lambda|^2y^{E\dagger}y^E\\
&- 2m_{H_u}^2|\lambda|^2y^{E\dagger}y^E - 2m_S^2|\lambda|^2y^{E\dagger}y^E - 2|T_\lambda|^2y^{E\dagger}y^E\\
&+ \frac{3}{25}g_1^2M_1^*\Big\{-20y^{E\dagger}T^E + 3\Big[2g_1'^2\Big(2M_1 + M_1'\Big) + 5g_2^2\Big(2M_1 + M_2\Big)\\
&+ 99g_1^2M_1\Big]\mathbf{1} + 40M_1y^{E\dagger}y^E\Big\} + \frac{3}{25}g_1'^2M_1'^*\Big\{\Big[10g_2^2\Big(2M_1' + M_2\Big)\\
&+ 217g_1'^2M_1' + 6g_1^2\Big(2M_1' + M_1\Big)\Big]\mathbf{1} + 10M_1'y^{E\dagger}y^E - 5y^{E\dagger}T^E\Big\}\\
&- 2\lambda T_\lambda^*y^{E\dagger}T^E - \frac{12}{5}g_1^2M_1T^{E\dagger}y^E - \frac{3}{5}g_1'^2M_1'T^{E\dagger}y^E + \frac{12}{5}g_1^2T^{E\dagger}T^E\\
&+ \frac{3}{5}g_1'^2T^{E\dagger}T^E - 2|\lambda|^2T^{E\dagger}T^E + \frac{6}{5}g_1^2m_L^2y^{E\dagger}y^E + \frac{3}{10}g_1'^2m_L^2y^{E\dagger}y^E\\
&- |\lambda|^2m_L^2y^{E\dagger}y^E + \frac{12}{5}g_1^2y^{E\dagger}m_{e^c}^2y^E + \frac{3}{5}g_1'^2y^{E\dagger}m_{e^c}^2y^E - 2|\lambda|^2y^{E\dagger}m_{e^c}^2y^E\\
&+ \frac{6}{5}g_1^2y^{E\dagger}y^Em_L^2 + \frac{3}{10}g_1'^2y^{E\dagger}y^Em_L^2 - |\lambda|^2y^{E\dagger}y^Em_L^2\\
&- 4m_{H_d}^2y^{E\dagger}h^Eh^{E\dagger}y^E - 4m_{L_4}^2y^{E\dagger}h^Eh^{E\dagger}y^E - 4y^{E\dagger}h^ET^{h^E\dagger}T^E\\
&- 8m_{H_d}^2y^{E\dagger}y^Ey^{E\dagger}y^E - 4y^{E\dagger}y^ET^{E\dagger}T^E - 4y^{E\dagger}T^{h^E}T^{h^E\dagger}y^E - 4y^{E\dagger}T^ET^{E\dagger}y^E\\
&- 4T^{E\dagger}h^Eh^{E\dagger}T^E - 4T^{E\dagger}y^Ey^{E\dagger}T^E - 4T^{E\dagger}T^{h^E}h^{E\dagger}y^E - 4T^{E\dagger}T^Ey^{E\dagger}y^E\\
&- 2m_L^2y^{E\dagger}h^Eh^{E\dagger}y^E - 2m_L^2y^{E\dagger}y^Ey^{E\dagger}y^E - 4y^{E\dagger}h^Em_{H_1}^2h^{E\dagger}y^E\\
&- 4y^{E\dagger}h^Eh^{E\dagger}m_{e^c}^2y^E - 2y^{E\dagger}h^Eh^{E\dagger}y^Em_L^2 - 4y^{E\dagger}m_{e^c}^2h^Eh^{E\dagger}y^E\\
&- 4y^{E\dagger}m_{e^c}^2y^Ey^{E\dagger}y^E - 4y^{E\dagger}y^Em_L^2y^{E\dagger}y^E - 4y^{E\dagger}y^Ey^{E\dagger}m_{e^c}^2y^E\\
&- 2y^{E\dagger}y^Ey^{E\dagger}y^Em_L^2 - 2\lambda^*T^{E\dagger}y^ET_\lambda + 6g_2^4\mathbf{1}\Sigma_{2,2} + \frac{6}{5}g_1^2\mathbf{1}\Sigma_{2,11}\\
&- \frac{2}{5}\sqrt{6}g_1g_1'\mathbf{1}\Sigma_{2,14} - \frac{2}{5}\sqrt{6}g_1g_1'\mathbf{1}\Sigma_{2,41} + \frac{4}{5}g_1'^2\mathbf{1}\Sigma_{2,44} - 4\sqrt{\frac{3}{5}}g_1\mathbf{1}\Sigma_{3,1}\\
&+ 4\sqrt{\frac{2}{5}}g_1'\mathbf{1}\Sigma_{3,4} - 4m_{H_d}^2y^{E\dagger}y^E\operatorname{Tr}\Big(ff^\dagger\Big) - 2T^{E\dagger}T^E\operatorname{Tr}\Big(ff^\dagger\Big)\\
&- m_L^2y^{E\dagger}y^E\operatorname{Tr}\Big(ff^\dagger\Big) - 2y^{E\dagger}m_{e^c}^2y^E\operatorname{Tr}\Big(ff^\dagger\Big) - y^{E\dagger}y^Em_L^2\operatorname{Tr}\Big(ff^\dagger\Big)\\
&- 12m_{H_d}^2y^{E\dagger}y^E\operatorname{Tr}\Big(y^Dy^{D\dagger}\Big) - 6T^{E\dagger}T^E\operatorname{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&- 3m_L^2y^{E\dagger}y^E\operatorname{Tr}\Big(y^Dy^{D\dagger}\Big) - 6y^{E\dagger}m_{e^c}^2y^E\operatorname{Tr}\Big(y^Dy^{D\dagger}\Big)
\end{aligned}
$$

$$- 3y^{E\dagger}y^E m_L^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 4m_{H_d}^2 y^{E\dagger}y^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big)$$
$$- 2T^{E\dagger}T^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - m_L^2 y^{E\dagger}y^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2y^{E\dagger}m_{e^c}^2 y^E \operatorname{Tr}\Big(y^E y^{E\dagger}\Big)$$
$$- y^{E\dagger}y^E m_L^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2T^{E\dagger}y^E \operatorname{Tr}\Big(f^\dagger T^f\Big) - 6T^{E\dagger}y^E \operatorname{Tr}\Big(y^{D\dagger}T^D\Big)$$
$$- 2T^{E\dagger}y^E \operatorname{Tr}\Big(y^{E\dagger}T^E\Big) - 2y^{E\dagger}T^E \operatorname{Tr}\Big(T^{f*}f^T\Big) - 2y^{E\dagger}y^E \operatorname{Tr}\Big(T^{f*}T^{fT}\Big)$$
$$- 6y^{E\dagger}T^E \operatorname{Tr}\Big(T^{D*}y^{DT}\Big) - 6y^{E\dagger}y^E \operatorname{Tr}\Big(T^{D*}T^{DT}\Big) - 2y^{E\dagger}T^E \operatorname{Tr}\Big(T^{E*}y^{ET}\Big)$$
$$- 2y^{E\dagger}y^E \operatorname{Tr}\Big(T^{E*}T^{ET}\Big) - 2y^{E\dagger}y^E \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) - 2y^{E\dagger}y^E \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big)$$
$$- 6y^{E\dagger}y^E \operatorname{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) - 2y^{E\dagger}y^E \operatorname{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big)$$
$$- 2y^{E\dagger}y^E \operatorname{Tr}\Big(m_L^2 y^{E\dagger}y^E\Big) - 6y^{E\dagger}y^E \operatorname{Tr}\Big(m_Q^2 y^{D\dagger}y^D\Big), \tag{A.97}$$

$$\beta_{m_{H_d}^2}^{(1)} = -\frac{6}{5}g_1^2|M_1|^2 - \frac{9}{5}g_1'^2|M_1'|^2 - 6g_2^2|M_2|^2 + 2m_{H_d}^2|\lambda|^2 + 2m_{H_u}^2|\lambda|^2$$
$$+ 2m_S^2|\lambda|^2 + 2|T_\lambda|^2 - \sqrt{\frac{3}{5}}g_1\Sigma_{1,1} - \frac{3}{\sqrt{10}}g_1'\Sigma_{1,4} + 2m_{H_d}^2 \operatorname{Tr}\Big(f f^\dagger\Big)$$
$$+ 6m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + 2m_{H_d}^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) + 2\operatorname{Tr}\Big(T^{f*}T^{fT}\Big)$$
$$+ 6\operatorname{Tr}\Big(T^{D*}T^{DT}\Big) + 2\operatorname{Tr}\Big(T^{E*}T^{ET}\Big) + 2\operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) + 2\operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big)$$
$$+ 6\operatorname{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) + 2\operatorname{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big) + 2\operatorname{Tr}\Big(m_L^2 y^{E\dagger}y^E\Big)$$
$$+ 6\operatorname{Tr}\Big(m_Q^2 y^{D\dagger}y^D\Big), \tag{A.98}$$

$$\beta_{m_{H_d}^2}^{(2)} = \frac{18}{5}g_1^2 g_2^2|M_2|^2 + \frac{27}{5}g_1'^2 g_2^2|M_2|^2 + 87g_2^4|M_2|^2 + 2g_1'^2 m_{H_d}^2|\lambda|^2 + 2g_1'^2 m_{H_u}^2|\lambda|^2$$
$$+ 2g_1'^2 m_S^2|\lambda|^2 + 2g_1'^2|T_\lambda|^2 + \frac{9}{5}g_1^2 g_2^2 M_1 M_2^* + \frac{27}{10}g_1'^2 g_2^2 M_1' M_2^* - 12m_{H_d}^2|\lambda|^4$$
$$- 12m_{H_u}^2|\lambda|^4 - 12m_S^2|\lambda|^4 - 2m_{H_d}^2|\sigma|^2|\lambda|^2 - 2m_{H_u}^2|\sigma|^2|\lambda|^2 - 2m_\phi^2|\sigma|^2|\lambda|^2$$
$$- 4m_S^2|\sigma|^2|\lambda|^2 - 2m_{\bar{S}}^2|\sigma|^2|\lambda|^2 - 2g_1'^2 M_1'\lambda T_\lambda^* - 24|\lambda|^2|T_\lambda|^2 - 2|\sigma|^2|T_\lambda|^2$$
$$- 2\sigma\lambda^* T_\sigma^* T_\lambda - 2\lambda\sigma^* T_\lambda^* T_\sigma - 2|\lambda|^2|T_\sigma|^2 + 6g_2^4\Sigma_{2,2} + \frac{6}{5}g_1^2\Sigma_{2,11}$$
$$+ \frac{3}{5}\sqrt{6}g_1 g_1'\Sigma_{2,14} + \frac{3}{5}\sqrt{6}g_1 g_1'\Sigma_{2,41} + \frac{9}{5}g_1'^2\Sigma_{2,44} - 4\sqrt{\frac{3}{5}}g_1\Sigma_{3,1}$$
$$- 6\sqrt{\frac{2}{5}}g_1'\Sigma_{3,4} + 2g_1'^2 m_{H_d}^2 \operatorname{Tr}\Big(f f^\dagger\Big) - 2m_{H_d}^2|\lambda|^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)$$
$$- 4m_{H_u}^2|\lambda|^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 2m_S^2|\lambda|^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 2|T_\lambda|^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)$$
$$- \frac{4}{5}g_1^2 m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - \frac{6}{5}g_1'^2 m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + 32g_3^2 m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big)$$
$$+ 64g_3^2|M_3|^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) + \frac{12}{5}g_1^2 m_{H_d}^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - \frac{2}{5}g_1'^2 m_{H_d}^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big)$$
$$- 6m_{H_d}^2|\lambda|^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 12m_{H_u}^2|\lambda|^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 6m_S^2|\lambda|^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big)$$
$$- 6|T_\lambda|^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 6m_{H_d}^2|\lambda|^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 6m_{H_u}^2|\lambda|^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)$$

$$
\begin{aligned}
&-12m_S^2|\lambda|^2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-6|T_\lambda|^2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big)-4m_{H_d}^2|\lambda|^2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)\\
&-4m_{H_u}^2|\lambda|^2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)-8m_S^2|\lambda|^2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)-4|T_\lambda|^2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big)\\
&-2\lambda T_\lambda^*\,\mathrm{Tr}\Big(\tilde f^\dagger T^{\tilde f}\Big)-32g_3^2M_3^*\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)+\frac{1}{50}g_1^2M_1^*\Big[1782g_1^2M_1\\
&-18g_1'^2M_1+180g_2^2M_1-9g_1'^2M_1'+90g_2^2M_2-80M_1\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&+240M_1\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)+40\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)-120\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)\Big]\\
&+\frac{1}{50}g_1'^2M_1'^*\Big[-9g_1^2M_1-18g_1^2M_1'+2997g_1'^2M_1'+270g_2^2M_1'+135g_2^2M_2\\
&+100\lambda^*\Big(2M_1'\lambda-T_\lambda\Big)+200M_1'\,\mathrm{Tr}\Big(ff^\dagger\Big)-120M_1'\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&-40M_1'\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-100\,\mathrm{Tr}\Big(f^\dagger T^f\Big)+60\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)+20\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)\Big]\\
&-6\lambda T_\lambda^*\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)-6\lambda T_\lambda^*\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)-4\lambda T_\lambda^*\,\mathrm{Tr}\Big(\tilde\lambda^\dagger T^{\tilde\lambda}\Big)\\
&-2g_1'^2M_1'\,\mathrm{Tr}\Big(T^{f*}f^T\Big)+2g_1'^2\,\mathrm{Tr}\Big(T^{f*}T^{fT}\Big)-2\lambda^*T_\lambda\,\mathrm{Tr}\Big(T^{\tilde f*}\tilde f^T\Big)\\
&-2|\lambda|^2\,\mathrm{Tr}\Big(T^{\tilde f*}T^{\tilde fT}\Big)+\frac{4}{5}g_1^2M_1\,\mathrm{Tr}\Big(T^{D*}y^{DT}\Big)+\frac{6}{5}g_1'^2M_1'\,\mathrm{Tr}\Big(T^{D*}y^{DT}\Big)\\
&-32g_3^2M_3\,\mathrm{Tr}\Big(T^{D*}y^{DT}\Big)-\frac{4}{5}g_1^2\,\mathrm{Tr}\Big(T^{D*}T^{DT}\Big)-\frac{6}{5}g_1'^2\,\mathrm{Tr}\Big(T^{D*}T^{DT}\Big)\\
&+32g_3^2\,\mathrm{Tr}\Big(T^{D*}T^{DT}\Big)-\frac{12}{5}g_1^2M_1\,\mathrm{Tr}\Big(T^{E*}y^{ET}\Big)+\frac{2}{5}g_1'^2M_1'\,\mathrm{Tr}\Big(T^{E*}y^{ET}\Big)\\
&+\frac{12}{5}g_1^2\,\mathrm{Tr}\Big(T^{E*}T^{ET}\Big)-\frac{2}{5}g_1'^2\,\mathrm{Tr}\Big(T^{E*}T^{ET}\Big)-6\lambda^*T_\lambda\,\mathrm{Tr}\Big(T^{U*}y^{UT}\Big)\\
&-6|\lambda|^2\,\mathrm{Tr}\Big(T^{U*}T^{UT}\Big)-6\lambda^*T_\lambda\,\mathrm{Tr}\Big(T^{\kappa*}\kappa^T\Big)-6|\lambda|^2\,\mathrm{Tr}\Big(T^{\kappa*}T^{\kappa T}\Big)\\
&-4\lambda^*T_\lambda\,\mathrm{Tr}\Big(T^{\tilde\lambda*}\tilde\lambda^T\Big)-4|\lambda|^2\,\mathrm{Tr}\Big(T^{\tilde\lambda*}T^{\tilde\lambda T}\Big)+2g_1'^2\,\mathrm{Tr}\Big(fm_{H_2}^2f^\dagger\Big)\\
&+2g_1'^2\,\mathrm{Tr}\Big(ff^\dagger m_\Sigma^{2*}\Big)-2|\lambda|^2\,\mathrm{Tr}\Big(\tilde fm_{H_1}^2\tilde f^\dagger\Big)-2|\lambda|^2\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger m_\Sigma^{2*}\Big)\\
&-\frac{4}{5}g_1^2\,\mathrm{Tr}\Big(m_{d^c}^2y^Dy^{D\dagger}\Big)-\frac{6}{5}g_1'^2\,\mathrm{Tr}\Big(m_{d^c}^2y^Dy^{D\dagger}\Big)+32g_3^2\,\mathrm{Tr}\Big(m_{d^c}^2y^Dy^{D\dagger}\Big)\\
&+\frac{12}{5}g_1^2\,\mathrm{Tr}\Big(m_{e^c}^2y^Ey^{E\dagger}\Big)-\frac{2}{5}g_1'^2\,\mathrm{Tr}\Big(m_{e^c}^2y^Ey^{E\dagger}\Big)-4|\lambda|^2\,\mathrm{Tr}\Big(m_{H_1}^2\tilde\lambda^\dagger\tilde\lambda\Big)\\
&+\frac{12}{5}g_1^2\,\mathrm{Tr}\Big(m_L^2y^{E\dagger}y^E\Big)-\frac{2}{5}g_1'^2\,\mathrm{Tr}\Big(m_L^2y^{E\dagger}y^E\Big)-\frac{4}{5}g_1^2\,\mathrm{Tr}\Big(m_Q^2y^{D\dagger}y^D\Big)\\
&-\frac{6}{5}g_1'^2\,\mathrm{Tr}\Big(m_Q^2y^{D\dagger}y^D\Big)+32g_3^2\,\mathrm{Tr}\Big(m_Q^2y^{D\dagger}y^D\Big)-6|\lambda|^2\,\mathrm{Tr}\Big(m_Q^2y^{U\dagger}y^U\Big)\\
&-6|\lambda|^2\,\mathrm{Tr}\Big(m_{u^c}^2y^Uy^{U\dagger}\Big)-6|\lambda|^2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger m_D^{2*}\Big)-6|\lambda|^2\,\mathrm{Tr}\Big(\kappa m_{\bar D}^{2*}\kappa^\dagger\Big)\\
&-4|\lambda|^2\,\mathrm{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger m_{H_2}^{2*}\Big)-12m_{H_d}^2\,\mathrm{Tr}\Big(ff^\dagger ff^\dagger\Big)-4m_{H_d}^2\,\mathrm{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big)\\
&-4m_{H_u}^2\,\mathrm{Tr}\Big(ff^\dagger\tilde f\tilde f^\dagger\Big)-12\,\mathrm{Tr}\Big(ff^\dagger T^fT^{f\dagger}\Big)-4\,\mathrm{Tr}\Big(ff^\dagger T^{\tilde f}T^{\tilde f\dagger}\Big)\\
&-12\,\mathrm{Tr}\Big(fT^{f\dagger}T^ff^\dagger\Big)-4\,\mathrm{Tr}\Big(fT^{f\dagger}T^{\tilde f}\tilde f^\dagger\Big)-2\,\mathrm{Tr}\Big(fT^{\tilde\lambda*}T^{\tilde\lambda T}f^\dagger\Big)\\
&-4\,\mathrm{Tr}\Big(\tilde f\tilde f^\dagger T^fT^{f\dagger}\Big)-4\,\mathrm{Tr}\Big(\tilde fT^{\tilde f\dagger}T^ff^\dagger\Big)-6m_{H_d}^2\,\mathrm{Tr}\Big(g^Dg^{D\dagger}y^{DT}y^{D*}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 6m_{L_4}^2 \operatorname{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*}\Big) - 6 \operatorname{Tr}\Big(g^D g^{D\dagger} T^{DT} T^{D*}\Big) \\
&- 4m_{H_d}^2 \operatorname{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) - 4m_{L_4}^2 \operatorname{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger}\Big) \\
&- 4 \operatorname{Tr}\Big(h^E h^{E\dagger} T^E T^{E\dagger}\Big) - 4 \operatorname{Tr}\Big(h^E T^{h^E\dagger} T^E y^{E\dagger}\Big) \\
&- 36m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{D\dagger} y^D y^{D\dagger}\Big) - 36 \operatorname{Tr}\Big(y^D y^{D\dagger} T^D T^{D\dagger}\Big) \\
&- 6m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) - 6m_{H_u}^2 \operatorname{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) \\
&- 6 \operatorname{Tr}\Big(y^D y^{U\dagger} T^U T^{D\dagger}\Big) - 36 \operatorname{Tr}\Big(y^D T^{D\dagger} T^D y^{D\dagger}\Big) \\
&- 6 \operatorname{Tr}\Big(y^D T^{U\dagger} T^U y^{D\dagger}\Big) - 6 \operatorname{Tr}\Big(y^D T^{g^D*} T^{g^DT} y^{D\dagger}\Big) \\
&- 12m_{H_d}^2 \operatorname{Tr}\Big(y^E y^{E\dagger} y^E y^{E\dagger}\Big) - 4 \operatorname{Tr}\Big(y^E y^{E\dagger} T^{h^E} T^{h^E\dagger}\Big) \\
&- 12 \operatorname{Tr}\Big(y^E y^{E\dagger} T^E T^{E\dagger}\Big) - 4 \operatorname{Tr}\Big(y^E T^{E\dagger} T^{h^E} h^{E\dagger}\Big) - 12 \operatorname{Tr}\Big(y^E T^{E\dagger} T^E y^{E\dagger}\Big) \\
&- 6 \operatorname{Tr}\Big(y^U y^{D\dagger} T^D T^{U\dagger}\Big) - 6 \operatorname{Tr}\Big(y^U T^{D\dagger} T^D y^{U\dagger}\Big) - 2m_{H_d}^2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big) \\
&- 2m_S^2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\Big) - 2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger T^{fT} T^{f*}\Big) - 2 \operatorname{Tr}\Big(f^\dagger T^f T^{\tilde{\lambda}*} \tilde{\lambda}^T\Big) \\
&- 6 \operatorname{Tr}\Big(g^{D\dagger} y^{DT} T^{D*} T^{g^D}\Big) - 6 \operatorname{Tr}\Big(y^{D\dagger} T^D T^{g^D*} g^{DT}\Big) - 2 \operatorname{Tr}\Big(\tilde{\lambda}^\dagger f^T T^{f*} T^{\tilde{\lambda}}\Big) \\
&- 6 \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger f f^\dagger\Big) - 4 \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger \tilde{f}\tilde{f}^\dagger\Big) - 6 \operatorname{Tr}\Big(f f^\dagger f m_{H_2}^2 f^\dagger\Big) \\
&- 4 \operatorname{Tr}\Big(f f^\dagger \tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) - 4 \operatorname{Tr}\Big(f f^\dagger \tilde{f}\tilde{f}^\dagger m_\Sigma^{2*}\Big) - 12 \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*} f f^\dagger\Big) \\
&- 4 \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*} \tilde{f}\tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(f \tilde{\lambda}^* \tilde{\lambda}^T f^\dagger m_\Sigma^{2*}\Big) - 6 \operatorname{Tr}\Big(g^D g^{D\dagger} m_Q^{2*} y^{DT} y^{D*}\Big) \\
&- 6 \operatorname{Tr}\Big(g^D g^{D\dagger} y^{DT} m_{d^c}^{2*} y^{D*}\Big) - 6 \operatorname{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*} m_Q^{2*}\Big) \\
&- 6 \operatorname{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger} y^{DT} y^{D*}\Big) - 4 \operatorname{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger} y^E y^{E\dagger}\Big) \\
&- 4 \operatorname{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2 y^E y^{E\dagger}\Big) - 4 \operatorname{Tr}\Big(h^E h^{E\dagger} y^E m_L^2 y^{E\dagger}\Big) \\
&- 4 \operatorname{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger} m_{e^c}^2\Big) - 36 \operatorname{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger} y^D y^{D\dagger}\Big) \\
&- 6 \operatorname{Tr}\Big(m_{d^c}^2 y^D y^{U\dagger} y^U y^{D\dagger}\Big) - 12 \operatorname{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger} y^E y^{E\dagger}\Big) \\
&- 2 \operatorname{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger f^T f^* \tilde{\lambda}\Big) - 12 \operatorname{Tr}\Big(m_L^2 y^{E\dagger} y^E y^{E\dagger} y^E\Big) \\
&- 36 \operatorname{Tr}\Big(m_Q^2 y^{D\dagger} y^D y^{D\dagger} y^D\Big) - 6 \operatorname{Tr}\Big(m_Q^2 y^{D\dagger} y^D y^{U\dagger} y^U\Big) \\
&- 6 \operatorname{Tr}\Big(m_Q^2 y^{U\dagger} y^U y^{D\dagger} y^D\Big) - 6 \operatorname{Tr}\Big(m_{u^c}^2 y^U y^{D\dagger} y^D y^{U\dagger}\Big) \\
&- 2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*} f^T f^*\Big) - 2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^* m_{H_2}^{2*}\Big),
\end{aligned}
\tag{A.99}
$$

$$
\begin{aligned}
\beta_{m_{H_u}^2}^{(1)} = &-\frac{6}{5} g_1^2 |M_1|^2 - \frac{4}{5} g_1'^2 |M_1'|^2 - 6 g_2^2 |M_2|^2 + 2m_{H_d}^2 |\lambda|^2 + 2m_{H_u}^2 |\lambda|^2 \\
&+ 2m_S^2 |\lambda|^2 + 2|T_\lambda|^2 + \sqrt{\frac{3}{5}} g_1 \Sigma_{1,1} - \sqrt{\frac{2}{5}} g_1' \Sigma_{1,4} + 2m_{H_u}^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+ 6m_{H_u}^2 \operatorname{Tr}\Big(y^U y^{U\dagger}\Big) + 2\operatorname{Tr}\Big(T^{\tilde{f}*}T^{\tilde{f}T}\Big) + 6\operatorname{Tr}\Big(T^{U*}T^{UT}\Big) \\
&+ 2\operatorname{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) + 2\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger m_{\Sigma}^{2*}\Big) + 6\operatorname{Tr}\Big(m_Q^2 y^{U\dagger} y^U\Big) \\
&+ 6\operatorname{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big),
\end{aligned} \tag{A.100}
$$

$$
\begin{aligned}
\beta_{m_{H_u}^2}^{(2)} =\; & \frac{18}{5}g_1^2 g_2^2|M_2|^2 + \frac{12}{5}g_1'^2 g_2^2|M_2|^2 + 87g_2^4|M_2|^2 + 3g_1'^2 m_{H_d}^2|\lambda|^2 \\
&+ 3g_1'^2 m_{H_u}^2|\lambda|^2 + 3g_1'^2 m_S^2|\lambda|^2 + 3g_1'^2|T_\lambda|^2 + \frac{9}{5}g_1^2 g_2^2 M_1 M_2^* + \frac{6}{5}g_1'^2 g_2^2 M_1' M_2^* \\
&- 12m_{H_d}^2|\lambda|^4 - 12m_{H_u}^2|\lambda|^4 - 12m_S^2|\lambda|^4 - 2m_{H_d}^2|\sigma|^2|\lambda|^2 - 2m_{H_u}^2|\sigma|^2|\lambda|^2 \\
&- 2m_\phi^2|\sigma|^2|\lambda|^2 - 4m_S^2|\sigma|^2|\lambda|^2 - 2m_{\bar{S}}^2|\sigma|^2|\lambda|^2 - 3g_1'^2 M_1'\lambda T_\lambda^* - 24|\lambda|^2|T_\lambda|^2 \\
&- 2|\sigma|^2|T_\lambda|^2 - 2\sigma\lambda^* T_\sigma^* T_\lambda - 2\lambda\sigma^* T_\lambda^* T_\sigma - 2|\lambda|^2|T_\sigma|^2 + 6g_2^4\Sigma_{2,2} + \frac{6}{5}g_1^2\Sigma_{2,11} \\
&- \frac{2}{5}\sqrt{6}g_1 g_1'\Sigma_{2,14} - \frac{2}{5}\sqrt{6}g_1 g_1'\Sigma_{2,41} + \frac{4}{5}g_1'^2\Sigma_{2,44} + 4\sqrt{\frac{3}{5}}g_1\Sigma_{3,1} - 4\sqrt{\frac{2}{5}}g_1'\Sigma_{3,4} \\
&- 4m_{H_d}^2|\lambda|^2\operatorname{Tr}\Big(ff^\dagger\Big) - 2m_{H_u}^2|\lambda|^2\operatorname{Tr}\Big(ff^\dagger\Big) - 2m_S^2|\lambda|^2\operatorname{Tr}\Big(ff^\dagger\Big) \\
&- 2|T_\lambda|^2\operatorname{Tr}\Big(ff^\dagger\Big) + 3g_1'^2 m_{H_u}^2\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 12m_{H_d}^2|\lambda|^2\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6m_{H_u}^2|\lambda|^2\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 6m_S^2|\lambda|^2\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 6|T_\lambda|^2\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 4m_{H_d}^2|\lambda|^2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2m_{H_u}^2|\lambda|^2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2m_S^2|\lambda|^2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 2|T_\lambda|^2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) + \frac{8}{5}g_1^2 m_{H_u}^2\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - \frac{3}{5}g_1'^2 m_{H_u}^2\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&+ 32g_3^2 m_{H_u}^2\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) + 64g_3^2|M_3|^2\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 6m_{H_d}^2|\lambda|^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 6m_{H_u}^2|\lambda|^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 12m_S^2|\lambda|^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 6|T_\lambda|^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 4m_{H_d}^2|\lambda|^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 4m_{H_u}^2|\lambda|^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 8m_S^2|\lambda|^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 4|T_\lambda|^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\lambda T_\lambda^*\operatorname{Tr}\Big(f^\dagger T^f\Big) - 6\lambda T_\lambda^*\operatorname{Tr}\Big(y^{D\dagger}T^D\Big) \\
&- 2\lambda T_\lambda^*\operatorname{Tr}\Big(y^{E\dagger}T^E\Big) + \frac{1}{25}g_1^2 M_1^*\Big\{-40\operatorname{Tr}\Big(y^{U\dagger}T^U\Big) + 80M_1\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&+ 9\Big[2g_1'^2\Big(2M_1 + M_1'\Big) + 5g_2^2\Big(2M_1 + M_2\Big) + 99g_1^2 M_1\Big]\Big\} \\
&- 32g_3^2 M_3^*\operatorname{Tr}\Big(y^{U\dagger}T^U\Big) + \frac{3}{25}g_1'^2 M_1'^*\Big[6g_1^2 M_1 + 12g_1^2 M_1' + 217g_1'^2 M_1' \\
&+ 20g_2^2 M_1' + 10g_2^2 M_2 + 25\lambda^*\Big(2M_1'\lambda - T_\lambda\Big) + 50M_1'\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&- 10M_1'\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) - 25\operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) + 5\operatorname{Tr}\Big(y^{U\dagger}T^U\Big)\Big] \\
&- 6\lambda T_\lambda^*\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) - 4\lambda T_\lambda^*\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - 2\lambda^* T_\lambda\operatorname{Tr}\Big(T^{f*}f^T\Big) \\
&- 2|\lambda|^2\operatorname{Tr}\Big(T^{f*}T^{fT}\Big) - 3g_1'^2 M_1'\operatorname{Tr}\Big(T^{\tilde{f}*}\tilde{f}^T\Big) + 3g_1'^2\operatorname{Tr}\Big(T^{\tilde{f}*}T^{\tilde{f}T}\Big) \\
&- 6\lambda^* T_\lambda\operatorname{Tr}\Big(T^{D*}y^{DT}\Big) - 6|\lambda|^2\operatorname{Tr}\Big(T^{D*}T^{DT}\Big) - 2\lambda^* T_\lambda\operatorname{Tr}\Big(T^{E*}y^{ET}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 2|\lambda|^2 \operatorname{Tr}\Big(T^{E*}T^{ET}\Big) - \frac{8}{5}g_1^2 M_1 \operatorname{Tr}\Big(T^{U*}y^{UT}\Big) + \frac{3}{5}g_1'^2 M_1' \operatorname{Tr}\Big(T^{U*}y^{UT}\Big) \\
&- 32 g_3^2 M_3 \operatorname{Tr}\Big(T^{U*}y^{UT}\Big) + \frac{8}{5}g_1^2 \operatorname{Tr}\Big(T^{U*}T^{UT}\Big) - \frac{3}{5}g_1'^2 \operatorname{Tr}\Big(T^{U*}T^{UT}\Big) \\
&+ 32 g_3^2 \operatorname{Tr}\Big(T^{U*}T^{UT}\Big) - 6\lambda^* T_\lambda \operatorname{Tr}\Big(T^{\kappa*}\kappa^T\Big) - 6|\lambda|^2 \operatorname{Tr}\Big(T^{\kappa*}T^{\kappa T}\Big) \\
&- 4\lambda^* T_\lambda \operatorname{Tr}\Big(T^{\tilde{\lambda}*}\tilde{\lambda}^T\Big) - 4|\lambda|^2 \operatorname{Tr}\Big(T^{\tilde{\lambda}*}T^{\tilde{\lambda}T}\Big) - 2|\lambda|^2 \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) \\
&- 2|\lambda|^2 \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big) + 3g_1'^2 \operatorname{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) + 3g_1'^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger m_\Sigma^{2*}\Big) \\
&- 6|\lambda|^2 \operatorname{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) - 2|\lambda|^2 \operatorname{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big) - 4|\lambda|^2 \operatorname{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\Big) \\
&- 2|\lambda|^2 \operatorname{Tr}\Big(m_L^2 y^{E\dagger} y^E\Big) - 6|\lambda|^2 \operatorname{Tr}\Big(m_Q^2 y^{D\dagger} y^D\Big) + \frac{8}{5}g_1^2 \operatorname{Tr}\Big(m_Q^2 y^{U\dagger} y^U\Big) \\
&- \frac{3}{5}g_1'^2 \operatorname{Tr}\Big(m_Q^2 y^{U\dagger} y^U\Big) + 32 g_3^2 \operatorname{Tr}\Big(m_Q^2 y^{U\dagger} y^U\Big) + \frac{8}{5}g_1^2 \operatorname{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big) \\
&- \frac{3}{5}g_1'^2 \operatorname{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big) + 32 g_3^2 \operatorname{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big) - 6|\lambda|^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger m_D^{2*}\Big) \\
&- 6|\lambda|^2 \operatorname{Tr}\Big(\kappa m_{\bar{D}}^{2*} \kappa^\dagger\Big) - 4|\lambda|^2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\Big) - 4 m_{H_d}^2 \operatorname{Tr}\Big(f f^\dagger \tilde{f}\tilde{f}^\dagger\Big) \\
&- 4 m_{H_u}^2 \operatorname{Tr}\Big(f f^\dagger \tilde{f}\tilde{f}^\dagger\Big) - 4 \operatorname{Tr}\Big(f f^\dagger T^{\tilde{f}} T^{\tilde{f}\dagger}\Big) - 4 \operatorname{Tr}\Big(f T^{f\dagger} T^{\tilde{f}} \tilde{f}^\dagger\Big) \\
&- 12 m_{H_u}^2 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger \tilde{f}\tilde{f}^\dagger\Big) - 4 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^f T^{f\dagger}\Big) - 12 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} T^{\tilde{f}\dagger}\Big) \\
&- 2 m_{L_4}^2 \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger\Big) - 2 m_{H_u}^2 \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} h^E \tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} T^{h^E} T^{\tilde{f}\dagger}\Big) \\
&- 2 m_{H_u}^2 \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger\Big) - 2 m_S^2 \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{f}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} T^{\tilde{f}\dagger}\Big) \\
&- 4 \operatorname{Tr}\Big(\tilde{f} T^{\tilde{f}\dagger} T^f f^\dagger\Big) - 12 \operatorname{Tr}\Big(\tilde{f} T^{\tilde{f}\dagger} T^{\tilde{f}} \tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{f} T^{h^E\dagger} T^{h^E} \tilde{f}^\dagger\Big) \\
&- 2 \operatorname{Tr}\Big(\tilde{f} T^{\tilde{\lambda}\dagger} T^{\tilde{\lambda}} \tilde{f}^\dagger\Big) - 6 m_{L_4}^2 \operatorname{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) \\
&- 6 m_{H_u}^2 \operatorname{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*}\Big) - 6 \operatorname{Tr}\Big(g^D g^{D\dagger} T^{UT} T^{U*}\Big) \\
&- 2 \operatorname{Tr}\Big(h^E \tilde{f}^\dagger T^{\tilde{f}} T^{h^E\dagger}\Big) - 2 \operatorname{Tr}\Big(h^E T^{\tilde{f}\dagger} T^{\tilde{f}} h^{E\dagger}\Big) - 6 m_{H_d}^2 \operatorname{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) \\
&- 6 m_{H_u}^2 \operatorname{Tr}\Big(y^D y^{U\dagger} y^U y^{D\dagger}\Big) - 6 \operatorname{Tr}\Big(y^D y^{U\dagger} T^U T^{D\dagger}\Big) - 6 \operatorname{Tr}\Big(y^D T^{U\dagger} T^U y^{D\dagger}\Big) \\
&- 6 \operatorname{Tr}\Big(y^U y^{D\dagger} T^D T^{U\dagger}\Big) - 36 m_{H_u}^2 \operatorname{Tr}\Big(y^U y^{U\dagger} y^U y^{U\dagger}\Big) \\
&- 36 \operatorname{Tr}\Big(y^U y^{U\dagger} T^U T^{U\dagger}\Big) - 6 \operatorname{Tr}\Big(y^U T^{D\dagger} T^D y^{U\dagger}\Big) - 36 \operatorname{Tr}\Big(y^U T^{U\dagger} T^U y^{U\dagger}\Big) \\
&- 6 \operatorname{Tr}\Big(y^U T^{g^D*} T^{g^D T} y^{U\dagger}\Big) - 2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}} T^{\tilde{\lambda}\dagger}\Big) - 2 \operatorname{Tr}\Big(\tilde{\lambda} T^{\tilde{f}\dagger} T^{\tilde{f}} \tilde{\lambda}^\dagger\Big) \\
&- 6 \operatorname{Tr}\Big(g^{D\dagger} y^{UT} T^{U*} T^{g^D}\Big) - 6 \operatorname{Tr}\Big(y^{U\dagger} T^U T^{g^D*} g^{DT}\Big) - 4 \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger \tilde{f}\tilde{f}^\dagger\Big) \\
&- 4 \operatorname{Tr}\Big(f f^\dagger \tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) - 4 \operatorname{Tr}\Big(f f^\dagger \tilde{f}\tilde{f}^\dagger m_\Sigma^{2*}\Big) - 4 \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*} \tilde{f}\tilde{f}^\dagger\Big) \\
&- 6 \operatorname{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger \tilde{f}\tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{f} m_{H_1}^2 h^{E\dagger} h^E \tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\tilde{f}^\dagger\Big) \\
&- 6 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger \tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) - 12 \operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger m_\Sigma^{2*} \tilde{f}\tilde{f}^\dagger\Big) - 2 \operatorname{Tr}\Big(\tilde{f} h^{E\dagger} h^E m_{H_1}^2 \tilde{f}^\dagger\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 2\,\mathrm{Tr}\Big(\tilde{f} h^{E\dagger} h^{E} \tilde{f}^{\dagger} m_{\Sigma}^{2*}\Big) - 2\,\mathrm{Tr}\Big(\tilde{f} h^{E\dagger} m_{e^c}^{2} h^{E} \tilde{f}^{\dagger}\Big) - 2\,\mathrm{Tr}\Big(\tilde{f} \tilde{\lambda}^{\dagger} \tilde{\lambda} m_{H_1}^{2} \tilde{f}^{\dagger}\Big) \\
&- 2\,\mathrm{Tr}\Big(\tilde{f} \tilde{\lambda}^{\dagger} \tilde{\lambda} \tilde{f}^{\dagger} m_{\Sigma}^{2*}\Big) - 2\,\mathrm{Tr}\Big(\tilde{f} \tilde{\lambda}^{\dagger} m_{H_2}^{2*} \tilde{\lambda} \tilde{f}^{\dagger}\Big) - 6\,\mathrm{Tr}\Big(g^{D} g^{D\dagger} m_{Q}^{2*} y^{UT} y^{U*}\Big) \\
&- 6\,\mathrm{Tr}\Big(g^{D} g^{D\dagger} y^{UT} m_{u^c}^{2*} y^{U*}\Big) - 6\,\mathrm{Tr}\Big(g^{D} g^{D\dagger} y^{UT} y^{U*} m_{Q}^{2*}\Big) \\
&- 6\,\mathrm{Tr}\Big(g^{D} m_{\bar{D}}^{2*} g^{D\dagger} y^{UT} y^{U*}\Big) - 6\,\mathrm{Tr}\Big(m_{d^c}^{2} y^{D} y^{U\dagger} y^{U} y^{D\dagger}\Big) \\
&- 6\,\mathrm{Tr}\Big(m_{Q}^{2} y^{D\dagger} y^{D} y^{U\dagger} y^{U}\Big) - 6\,\mathrm{Tr}\Big(m_{Q}^{2} y^{U\dagger} y^{U} y^{D\dagger} y^{D}\Big) \\
&- 36\,\mathrm{Tr}\Big(m_{Q}^{2} y^{U\dagger} y^{U} y^{U\dagger} y^{U}\Big) - 6\,\mathrm{Tr}\Big(m_{u^c}^{2} y^{U} y^{D\dagger} y^{D} y^{U\dagger}\Big) \\
&- 36\,\mathrm{Tr}\Big(m_{u^c}^{2} y^{U} y^{U\dagger} y^{U} y^{U\dagger}\Big), \qquad \text{(A.101)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_{d^c}^2}^{(1)} =& -\frac{8}{15} g_1^2 \mathbf{1} |M_1|^2 - \frac{4}{5} g_1'^2 \mathbf{1} |M_1'|^2 - \frac{32}{3} g_3^2 \mathbf{1} |M_3|^2 + 4 m_{H_d}^2 y^{D} y^{D\dagger} + 4 T^{D} T^{D\dagger} \\
&+ 2 m_{d^c}^2 y^{D} y^{D\dagger} + 4 y^{D} m_{Q}^2 y^{D\dagger} + 2 y^{D} y^{D\dagger} m_{d^c}^2 + \frac{2}{\sqrt{15}} g_1 \mathbf{1} \Sigma_{1,1} \\
&+ \sqrt{\frac{2}{5}} g_1' \mathbf{1} \Sigma_{1,4}, \qquad \text{(A.102)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_{d^c}^2}^{(2)} =& \frac{128}{45} g_1^2 g_3^2 \mathbf{1} |M_3|^2 + \frac{64}{15} g_1'^2 g_3^2 \mathbf{1} |M_3|^2 + \frac{160}{3} g_3^4 \mathbf{1} |M_3|^2 + \frac{64}{45} g_1^2 g_3^2 M_1 \mathbf{1} M_3^* \\
&+ \frac{32}{15} g_1'^2 g_3^2 M_1' \mathbf{1} M_3^* + \frac{4}{5} g_1^2 m_{H_d}^2 y^{D} y^{D\dagger} + \frac{6}{5} g_1'^2 m_{H_d}^2 y^{D} y^{D\dagger} + 12 g_2^2 m_{H_d}^2 y^{D} y^{D\dagger} \\
&+ 24 g_2^2 |M_2|^2 y^{D} y^{D\dagger} - 8 m_{H_d}^2 |\lambda|^2 y^{D} y^{D\dagger} - 4 m_{H_u}^2 |\lambda|^2 y^{D} y^{D\dagger} \\
&- 4 m_{S}^2 |\lambda|^2 y^{D} y^{D\dagger} - 4 |T_\lambda|^2 y^{D} y^{D\dagger} - \frac{4}{5} g_1^2 M_1 y^{D} T^{D\dagger} - \frac{6}{5} g_1'^2 M_1' y^{D} T^{D\dagger} \\
&- 12 g_2^2 M_2 y^{D} T^{D\dagger} + \frac{1}{75} g_1'^2 M_1'^* \bigg\{ \Big[160 g_3^2 \Big(2 M_1' + M_3\Big) - 16 g_1^2 \Big(2 M_1' + M_1\Big) \\
&+ 1953 g_1'^2 M_1' \Big] \mathbf{1} + 90 \Big(2 M_1' y^{D} y^{D\dagger} - T^{D} y^{D\dagger}\Big) \bigg\} \\
&+ \frac{4}{225} g_1^2 M_1^* \bigg\{ 4 \Big[20 g_3^2 \Big(2 M_1 + M_3\Big) + 219 g_1^2 M_1 - 3 g_1'^2 \Big(2 M_1 + M_1'\Big) \Big] \mathbf{1} \\
&- 45 T^{D} y^{D\dagger} + 90 M_1 y^{D} y^{D\dagger} \bigg\} - 12 g_2^2 M_2^* T^{D} y^{D\dagger} - 4 \lambda T_\lambda^* T^{D} y^{D\dagger} \\
&+ \frac{4}{5} g_1^2 T^{D} T^{D\dagger} + \frac{6}{5} g_1'^2 T^{D} T^{D\dagger} + 12 g_2^2 T^{D} T^{D\dagger} - 4 |\lambda|^2 T^{D} T^{D\dagger} \\
&+ \frac{2}{5} g_1^2 m_{d^c}^2 y^{D} y^{D\dagger} + \frac{3}{5} g_1'^2 m_{d^c}^2 y^{D} y^{D\dagger} + 6 g_2^2 m_{d^c}^2 y^{D} y^{D\dagger} - 2 |\lambda|^2 m_{d^c}^2 y^{D} y^{D\dagger} \\
&+ \frac{4}{5} g_1^2 y^{D} m_{Q}^2 y^{D\dagger} + \frac{6}{5} g_1'^2 y^{D} m_{Q}^2 y^{D\dagger} + 12 g_2^2 y^{D} m_{Q}^2 y^{D\dagger} - 4 |\lambda|^2 y^{D} m_{Q}^2 y^{D\dagger} \\
&+ \frac{2}{5} g_1^2 y^{D} y^{D\dagger} m_{d^c}^2 + \frac{3}{5} g_1'^2 y^{D} y^{D\dagger} m_{d^c}^2 + 6 g_2^2 y^{D} y^{D\dagger} m_{d^c}^2 - 2 |\lambda|^2 y^{D} y^{D\dagger} m_{d^c}^2 \\
&- 8 m_{H_d}^2 y^{D} y^{D\dagger} y^{D} y^{D\dagger} - 4 y^{D} y^{D\dagger} T^{D} T^{D\dagger} - 4 m_{H_d}^2 y^{D} y^{U\dagger} y^{U} y^{D\dagger} \\
&- 4 m_{H_u}^2 y^{D} y^{U\dagger} y^{U} y^{D\dagger} - 4 y^{D} y^{U\dagger} T^{U} T^{D\dagger} - 4 y^{D} T^{D\dagger} T^{D} y^{D\dagger} \\
&- 4 y^{D} T^{U\dagger} T^{U} y^{D\dagger} - 4 m_{H_d}^2 y^{D} g^{D*} g^{DT} y^{D\dagger} - 4 m_{L_4}^2 y^{D} g^{D*} g^{DT} y^{D\dagger} \\
&- 4 y^{D} g^{D*} T^{g^{D}T} T^{D\dagger} - 4 y^{D} T^{g^{D}*} T^{g^{D}T} y^{D\dagger} - 4 T^{D} y^{D\dagger} y^{D} T^{D\dagger} \\
&- 4 T^{D} y^{U\dagger} y^{U} T^{D\dagger} - 4 T^{D} T^{D\dagger} y^{D} y^{D\dagger} - 4 T^{D} T^{U\dagger} y^{U} y^{D\dagger} - 4 T^{D} g^{D*} g^{DT} T^{D\dagger}
\end{aligned}
$$

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$$
\begin{aligned}
& - 4T^D T^{g^D*} g^{DT} y^{D\dagger} - 2m_{d^c}^2 y^D y^{D\dagger} y^D y^{D\dagger} - 2m_{d^c}^2 y^D y^{U\dagger} y^U y^{D\dagger} \\
& - 2m_{d^c}^2 y^D g^{D*} g^{DT} y^{D\dagger} - 4y^D m_Q^2 y^{D\dagger} y^D y^{D\dagger} - 4y^D m_Q^2 y^{U\dagger} y^U y^{D\dagger} \\
& - 4y^D m_Q^2 g^{D*} g^{DT} y^{D\dagger} - 4y^D y^{D\dagger} m_{d^c}^2 y^D y^{D\dagger} - 4y^D y^{D\dagger} y^D m_Q^2 y^{D\dagger} \\
& - 2y^D y^{D\dagger} y^D y^{D\dagger} m_{d^c}^2 - 4y^D y^{U\dagger} m_{u^c}^2 y^U y^{D\dagger} - 4y^D y^{U\dagger} y^U m_Q^2 y^{D\dagger} \\
& - 2y^D y^{U\dagger} y^U y^{D\dagger} m_{d^c}^2 - 4y^D g^{D*} m_{\bar{D}}^2 g^{DT} y^{D\dagger} - 4y^D g^{D*} g^{DT} m_Q^2 y^{D\dagger} \\
& - 2y^D g^{D*} g^{DT} y^{D\dagger} m_{d^c}^2 - 4\lambda^* y^D T^{D\dagger} T_\lambda + \frac{32}{3} g_3^4 \mathbf{1} \Sigma_{2,3} + \frac{8}{15} g_1^2 \mathbf{1} \Sigma_{2,11} \\
& + \frac{4}{5}\sqrt{\frac{2}{3}} g_1 g_1' \mathbf{1} \Sigma_{2,14} + \frac{4}{5}\sqrt{\frac{2}{3}} g_1 g_1' \mathbf{1} \Sigma_{2,41} + \frac{4}{5} g_1'^2 \mathbf{1} \Sigma_{2,44} + \frac{8}{\sqrt{15}} g_1 \mathbf{1} \Sigma_{3,1} \\
& + 4\sqrt{\frac{2}{5}} g_1' \mathbf{1} \Sigma_{3,4} - 8m_{H_d}^2 y^D y^{D\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) - 4T^D T^{D\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) \\
& - 2m_{d^c}^2 y^D y^{D\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) - 4y^D m_Q^2 y^{D\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) - 2y^D y^{D\dagger} m_{d^c}^2 \operatorname{Tr}\Big(f f^\dagger\Big) \\
& - 24m_{H_d}^2 y^D y^{D\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12T^D T^{D\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
& - 6m_{d^c}^2 y^D y^{D\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12y^D m_Q^2 y^{D\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
& - 6y^D y^{D\dagger} m_{d^c}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 8m_{H_d}^2 y^D y^{D\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
& - 4T^D T^{D\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2m_{d^c}^2 y^D y^{D\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
& - 4y^D m_Q^2 y^{D\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2y^D y^{D\dagger} m_{d^c}^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 4y^D T^{D\dagger} \operatorname{Tr}\Big(f^\dagger T^f\Big) \\
& - 12y^D T^{D\dagger} \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) - 4y^D T^{D\dagger} \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) - 4T^D y^{D\dagger} \operatorname{Tr}\Big(T^{f*} f^T\Big) \\
& - 4y^D y^{D\dagger} \operatorname{Tr}\Big(T^{f*} T^{fT}\Big) - 12T^D y^{D\dagger} \operatorname{Tr}\Big(T^{D*} y^{DT}\Big) \\
& - 12y^D y^{D\dagger} \operatorname{Tr}\Big(T^{D*} T^{DT}\Big) - 4T^D y^{D\dagger} \operatorname{Tr}\Big(T^{E*} y^{ET}\Big) \\
& - 4y^D y^{D\dagger} \operatorname{Tr}\Big(T^{E*} T^{ET}\Big) - 4y^D y^{D\dagger} \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) - 4y^D y^{D\dagger} \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big) \\
& - 12y^D y^{D\dagger} \operatorname{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) - 4y^D y^{D\dagger} \operatorname{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big) \\
& - 4y^D y^{D\dagger} \operatorname{Tr}\Big(m_L^2 y^{E\dagger} y^E\Big) - 12y^D y^{D\dagger} \operatorname{Tr}\Big(m_Q^2 y^{D\dagger} y^D\Big),
\end{aligned}
\tag{A.103}
$$

$$
\begin{aligned}
\beta_{m_{u^c}^2}^{(1)} = & -\frac{32}{15} g_1^2 \mathbf{1} |M_1|^2 - \frac{1}{5} g_1'^2 \mathbf{1} |M_1'|^2 - \frac{32}{3} g_3^2 \mathbf{1} |M_3|^2 + 4m_{H_u}^2 y^U y^{U\dagger} + 4T^U T^{U\dagger} \\
& + 2m_{u^c}^2 y^U y^{U\dagger} + 4y^U m_Q^2 y^{U\dagger} + 2y^U y^{U\dagger} m_{u^c}^2 - \frac{4}{\sqrt{15}} g_1 \mathbf{1} \Sigma_{1,1} \\
& + \frac{1}{\sqrt{10}} g_1' \mathbf{1} \Sigma_{1,4},
\end{aligned}
\tag{A.104}
$$

$$
\begin{aligned}
\beta_{m_{u^c}^2}^{(2)} = & \frac{512}{45} g_1^2 g_3^2 \mathbf{1} |M_3|^2 + \frac{16}{15} g_1'^2 g_3^2 \mathbf{1} |M_3|^2 + \frac{160}{3} g_3^4 \mathbf{1} |M_3|^2 + \frac{256}{45} g_1^2 g_3^2 M_1 \mathbf{1} M_3^* \\
& + \frac{8}{15} g_1'^2 g_3^2 M_1' \mathbf{1} M_3^* - \frac{4}{5} g_1^2 m_{H_u}^2 y^U y^{U\dagger} + \frac{4}{5} g_1'^2 m_{H_u}^2 y^U y^{U\dagger} + 12 g_2^2 m_{H_u}^2 y^U y^{U\dagger} \\
& + 24 g_2^2 |M_2|^2 y^U y^{U\dagger} - 4m_{H_d}^2 |\lambda|^2 y^U y^{U\dagger} - 8m_{H_u}^2 |\lambda|^2 y^U y^{U\dagger} \\
& - 4m_S^2 |\lambda|^2 y^U y^{U\dagger} - 4|T_\lambda|^2 y^U y^{U\dagger} + \frac{4}{5} g_1^2 M_1 y^U T^{U\dagger} - \frac{4}{5} g_1'^2 M_1' y^U T^{U\dagger}
\end{aligned}
$$

$$
\begin{aligned}
&- 12 g_2^2 M_2 y^U T^{U\dagger} + \frac{1}{150} g_1'^2 M_1'^* \Big\{ 120 \Big(2M_1' y^U y^{U\dagger} - T^U y^{U\dagger} \Big) \\
&+ \Big[64 g_1^2 \Big(2M_1' + M_1 \Big) + 80 g_3^2 \Big(2M_1' + M_3 \Big) + 963 g_1'^2 M_1' \Big] \mathbf{1} \Big\} \\
&- 12 g_2^2 M_2^* T^U y^{U\dagger} - 4\lambda T_\lambda^* T^U y^{U\dagger} + \frac{4}{225} g_1^2 M_1^* \Big\{ 45 \Big(-2M_1 y^U y^{U\dagger} \\
&+ T^U y^{U\dagger} \Big) + 8 \Big[3 g_1'^2 \Big(2M_1 + M_1' \Big) + 40 g_3^2 \Big(2M_1 + M_3 \Big) + 456 g_1^2 M_1 \Big] \mathbf{1} \Big\} \\
&- \frac{4}{5} g_1^2 T^U T^{U\dagger} + \frac{4}{5} g_1'^2 T^U T^{U\dagger} + 12 g_2^2 T^U T^{U\dagger} - 4|\lambda|^2 T^U T^{U\dagger} \\
&- \frac{2}{5} g_1^2 m_{u^c}^2 y^U y^{U\dagger} + \frac{2}{5} g_1'^2 m_{u^c}^2 y^U y^{U\dagger} + 6 g_2^2 m_{u^c}^2 y^U y^{U\dagger} - 2|\lambda|^2 m_{u^c}^2 y^U y^{U\dagger} \\
&- \frac{4}{5} g_1^2 y^U m_Q^2 y^{U\dagger} + \frac{4}{5} g_1'^2 y^U m_Q^2 y^{U\dagger} + 12 g_2^2 y^U m_Q^2 y^{U\dagger} - 4|\lambda|^2 y^U m_Q^2 y^{U\dagger} \\
&- \frac{2}{5} g_1^2 y^U y^{U\dagger} m_{u^c}^2 + \frac{2}{5} g_1'^2 y^U y^{U\dagger} m_{u^c}^2 + 6 g_2^2 y^U y^{U\dagger} m_{u^c}^2 - 2|\lambda|^2 y^U y^{U\dagger} m_{u^c}^2 \\
&- 4 m_{H_d}^2 y^U y^{D\dagger} y^D y^{U\dagger} - 4 m_{H_u}^2 y^U y^{D\dagger} y^D y^{U\dagger} - 4 y^U y^{D\dagger} T^D T^{U\dagger} \\
&- 8 m_{H_u}^2 y^U y^{U\dagger} y^U y^{U\dagger} - 4 y^U y^{U\dagger} T^U T^{U\dagger} - 4 y^U T^{D\dagger} T^D y^{U\dagger} - 4 y^U T^{U\dagger} T^U y^{U\dagger} \\
&- 4 m_{L_4}^2 y^U g^{D*} g^{DT} y^{U\dagger} - 4 m_{H_u}^2 y^U g^{D*} g^{DT} y^{U\dagger} - 4 y^U g^{D*} T^{g^D T} T^{U\dagger} \\
&- 4 y^U T^{g^D *} T^{g^D T} y^{U\dagger} - 4 T^U y^{D\dagger} y^D T^{U\dagger} - 4 T^U y^{U\dagger} y^U T^{U\dagger} - 4 T^U T^{D\dagger} y^D y^{U\dagger} \\
&- 4 T^U T^{U\dagger} y^U y^{U\dagger} - 4 T^U g^{D*} g^{DT} T^{U\dagger} - 4 T^U T^{g^D *} g^{DT} y^{U\dagger} \\
&- 2 m_{u^c}^2 y^U y^{D\dagger} y^D y^{U\dagger} - 2 m_{u^c}^2 y^U y^{U\dagger} y^U y^{U\dagger} - 2 m_{u^c}^2 y^U g^{D*} g^{DT} y^{U\dagger} \\
&- 4 y^U m_Q^2 y^{D\dagger} y^D y^{U\dagger} - 4 y^U m_Q^2 y^{U\dagger} y^U y^{U\dagger} - 4 y^U m_Q^2 g^{D*} g^{DT} y^{U\dagger} \\
&- 4 y^U y^{D\dagger} m_{d^c}^2 y^D y^{U\dagger} - 4 y^U y^{D\dagger} y^D m_Q^2 y^{U\dagger} - 2 y^U y^{D\dagger} y^D y^{U\dagger} m_{u^c}^2 \\
&- 4 y^U y^{U\dagger} m_{u^c}^2 y^U y^{U\dagger} - 4 y^U y^{U\dagger} y^U m_Q^2 y^{U\dagger} - 2 y^U y^{U\dagger} y^U y^{U\dagger} m_{u^c}^2 \\
&- 4 y^U g^{D*} m_{\bar{D}}^2 g^{DT} y^{U\dagger} - 4 y^U g^{D*} g^{DT} m_Q^2 y^{U\dagger} - 2 y^U g^{D*} g^{DT} y^{U\dagger} m_{u^c}^2 \\
&- 4\lambda^* y^U T^{U\dagger} T_\lambda + \frac{32}{3} g_3^4 \mathbf{1} \Sigma_{2,3} + \frac{32}{15} g_1^2 \mathbf{1} \Sigma_{2,11} - \frac{4}{5} \sqrt{\frac{2}{3}} g_1 g_1' \mathbf{1} \Sigma_{2,14} \\
&- \frac{4}{5} \sqrt{\frac{2}{3}} g_1 g_1' \mathbf{1} \Sigma_{2,41} + \frac{1}{5} g_1'^2 \mathbf{1} \Sigma_{2,44} - \frac{16}{\sqrt{15}} g_1 \mathbf{1} \Sigma_{3,1} + 2\sqrt{\frac{2}{5}} g_1' \mathbf{1} \Sigma_{3,4} \\
&- 8 m_{H_u}^2 y^U y^{U\dagger} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger \Big) - 4 T^U T^{U\dagger} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger \Big) - 2 m_{u^c}^2 y^U y^{U\dagger} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger \Big) \\
&- 4 y^U m_Q^2 y^{U\dagger} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger \Big) - 2 y^U y^{U\dagger} m_{u^c}^2 \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger \Big) - 24 m_{H_u}^2 y^U y^{U\dagger} \operatorname{Tr}\Big(y^U y^{U\dagger} \Big) \\
&- 12 T^U T^{U\dagger} \operatorname{Tr}\Big(y^U y^{U\dagger} \Big) - 6 m_{u^c}^2 y^U y^{U\dagger} \operatorname{Tr}\Big(y^U y^{U\dagger} \Big) \\
&- 12 y^U m_Q^2 y^{U\dagger} \operatorname{Tr}\Big(y^U y^{U\dagger} \Big) - 6 y^U y^{U\dagger} m_{u^c}^2 \operatorname{Tr}\Big(y^U y^{U\dagger} \Big) \\
&- 4 y^U T^{U\dagger} \operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}} \Big) - 12 y^U T^{U\dagger} \operatorname{Tr}\Big(y^{U\dagger} T^U \Big) \\
&- 4 T^U y^{U\dagger} \operatorname{Tr}\Big(T^{\tilde{f}*} \tilde{f}^T \Big) - 4 y^U y^{U\dagger} \operatorname{Tr}\Big(T^{\tilde{f}*} T^{\tilde{f}T} \Big) - 12 T^U y^{U\dagger} \operatorname{Tr}\Big(T^{U*} y^{UT} \Big) \\
&- 12 y^U y^{U\dagger} \operatorname{Tr}\Big(T^{U*} T^{UT} \Big) - 4 y^U y^{U\dagger} \operatorname{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger \Big) - 4 y^U y^{U\dagger} \operatorname{Tr}\Big(\tilde{f} \tilde{f}^\dagger m_\Sigma^{2*} \Big) \\
&- 12 y^U y^{U\dagger} \operatorname{Tr}\Big(m_Q^2 y^{U\dagger} y^U \Big) - 12 y^U y^{U\dagger} \operatorname{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger} \Big),
\end{aligned} \tag{A.105}
$$

$$
\begin{aligned}
\beta_{m_{e^c}^2}^{(1)} = & -\frac{24}{5} g_1^2 \mathbf{1} |M_1|^2 - \frac{1}{5} g_1'^2 \mathbf{1} |M_1'|^2 + 4 m_{L_4}^2 h^E h^{E\dagger} + 4 m_{H_d}^2 y^E y^{E\dagger} + 4 T^{h^E} T^{h^E\dagger} \\
& + 4 T^E T^{E\dagger} + 4 h^E m_{H_1}^2 h^{E\dagger} + 2 h^E h^{E\dagger} m_{e^c}^2 + 2 m_{e^c}^2 h^E h^{E\dagger} + 2 m_{e^c}^2 y^E y^{E\dagger} \\
& + 4 y^E m_L^2 y^{E\dagger} + 2 y^E y^{E\dagger} m_{e^c}^2 + 2\sqrt{\frac{3}{5}} g_1 \mathbf{1} \Sigma_{1,1} + \frac{1}{\sqrt{10}} g_1' \mathbf{1} \Sigma_{1,4}, \qquad \text{(A.106)} \\
\beta_{m_{e^c}^2}^{(2)} = & -\frac{12}{5} g_1^2 m_{L_4}^2 h^E h^{E\dagger} + \frac{12}{5} g_1'^2 m_{L_4}^2 h^E h^{E\dagger} + 12 g_2^2 m_{L_4}^2 h^E h^{E\dagger} \\
& + 24 g_2^2 |M_2|^2 h^E h^{E\dagger} - 8 m_{L_4}^2 |\tilde{\sigma}|^2 h^E h^{E\dagger} - 4 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 h^E h^{E\dagger} \\
& - 4 m_\phi^2 |\tilde{\sigma}|^2 h^E h^{E\dagger} - 4 |T_{\tilde{\sigma}}|^2 h^E h^{E\dagger} + \frac{12}{5} g_1^2 M_1 h^E T^{h^E\dagger} - \frac{12}{5} g_1'^2 M_1' h^E T^{h^E\dagger} \\
& - 12 g_2^2 M_2 h^E T^{h^E\dagger} - \frac{12}{5} g_1^2 m_{H_d}^2 y^E y^{E\dagger} + \frac{12}{5} g_1'^2 m_{H_d}^2 y^E y^{E\dagger} \\
& + 12 g_2^2 m_{H_d}^2 y^E y^{E\dagger} + 24 g_2^2 |M_2|^2 y^E y^{E\dagger} - 8 m_{H_d}^2 |\lambda|^2 y^E y^{E\dagger} - 4 m_{H_u}^2 |\lambda|^2 y^E y^{E\dagger} \\
& - 4 m_S^2 |\lambda|^2 y^E y^{E\dagger} - 4 |T_\lambda|^2 y^E y^{E\dagger} + \frac{12}{5} g_1^2 M_1 y^E T^{E\dagger} - \frac{12}{5} g_1'^2 M_1' y^E T^{E\dagger} \\
& - 12 g_2^2 M_2 y^E T^{E\dagger} - 12 g_2^2 M_2^* T^{h^E} h^{E\dagger} - 4 \tilde{\sigma} T_{\tilde{\sigma}}^* T^{h^E} h^{E\dagger} - \frac{12}{5} g_1^2 T^{h^E} T^{h^E\dagger} \\
& + \frac{12}{5} g_1'^2 T^{h^E} T^{h^E\dagger} + 12 g_2^2 T^{h^E} T^{h^E\dagger} - 4 |\tilde{\sigma}|^2 T^{h^E} T^{h^E\dagger} \\
& + \frac{3}{50} g_1'^2 M_1'^* \Big\{ \Big[107 g_1'^2 M_1' - 4 g_1^2 \Big(2 M_1' + M_1 \Big) \Big] \mathbf{1} + 40 \Big(2 M_1' h^E h^{E\dagger} \\
& + 2 M_1' y^E y^{E\dagger} - T^E y^{E\dagger} - T^{h^E} h^{E\dagger} \Big) \Big\} - 12 g_2^2 M_2^* T^E y^{E\dagger} - 4 \lambda T_\lambda^* T^E y^{E\dagger} \\
& + \frac{6}{25} g_1^2 M_1^* \Big\{ 10 \Big(-2 M_1 h^E h^{E\dagger} - 2 M_1 y^E y^{E\dagger} + T^{h^E} h^{E\dagger} + T^E y^{E\dagger} \Big) \\
& + \Big[648 g_1^2 M_1 - g_1'^2 \Big(2 M_1 + M_1' \Big) \Big] \mathbf{1} \Big\} - \frac{12}{5} g_1^2 T^E T^{E\dagger} + \frac{12}{5} g_1'^2 T^E T^{E\dagger} \\
& + 12 g_2^2 T^E T^{E\dagger} - 4 |\lambda|^2 T^E T^{E\dagger} - \frac{12}{5} g_1^2 h^E m_{H_1}^2 h^{E\dagger} + \frac{12}{5} g_1'^2 h^E m_{H_1}^2 h^{E\dagger} \\
& + 12 g_2^2 h^E m_{H_1}^2 h^{E\dagger} - 4 |\tilde{\sigma}|^2 h^E m_{H_1}^2 h^{E\dagger} - \frac{6}{5} g_1^2 h^E h^{E\dagger} m_{e^c}^2 + \frac{6}{5} g_1'^2 h^E h^{E\dagger} m_{e^c}^2 \\
& + 6 g_2^2 h^E h^{E\dagger} m_{e^c}^2 - 2 |\tilde{\sigma}|^2 h^E h^{E\dagger} m_{e^c}^2 - \frac{6}{5} g_1^2 m_{e^c}^2 h^E h^{E\dagger} + \frac{6}{5} g_1'^2 m_{e^c}^2 h^E h^{E\dagger} \\
& + 6 g_2^2 m_{e^c}^2 h^E h^{E\dagger} - 2 |\tilde{\sigma}|^2 m_{e^c}^2 h^E h^{E\dagger} - \frac{6}{5} g_1^2 m_{e^c}^2 y^E y^{E\dagger} + \frac{6}{5} g_1'^2 m_{e^c}^2 y^E y^{E\dagger} \\
& + 6 g_2^2 m_{e^c}^2 y^E y^{E\dagger} - 2 |\lambda|^2 m_{e^c}^2 y^E y^{E\dagger} - \frac{12}{5} g_1^2 y^E m_L^2 y^{E\dagger} + \frac{12}{5} g_1'^2 y^E m_L^2 y^{E\dagger} \\
& + 12 g_2^2 y^E m_L^2 y^{E\dagger} - 4 |\lambda|^2 y^E m_L^2 y^{E\dagger} - \frac{6}{5} g_1^2 y^E y^{E\dagger} m_{e^c}^2 + \frac{6}{5} g_1'^2 y^E y^{E\dagger} m_{e^c}^2 \\
& + 6 g_2^2 y^E y^{E\dagger} m_{e^c}^2 - 2 |\lambda|^2 y^E y^{E\dagger} m_{e^c}^2 - 4 m_{L_4}^2 h^E \tilde{f}^\dagger \tilde{f} h^{E\dagger} - 4 m_{H_u}^2 h^E \tilde{f}^\dagger \tilde{f} h^{E\dagger} \\
& - 4 h^E \tilde{f}^\dagger T^{\tilde{f}} T^{h^E\dagger} - 8 m_{L_4}^2 h^E h^{E\dagger} h^E h^{E\dagger} - 4 h^E h^{E\dagger} T^{h^E} T^{h^E\dagger} \\
& - 4 m_{L_4}^2 h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} - 4 m_S^2 h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} - 4 h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} T^{h^E\dagger} - 4 h^E T^{\tilde{f}\dagger} T^{\tilde{f}} h^{E\dagger} \\
& - 4 h^E T^{h^E\dagger} T^{h^E} h^{E\dagger} - 4 h^E T^{\tilde{\lambda}\dagger} T^{\tilde{\lambda}} h^{E\dagger} - 8 m_{H_d}^2 y^E y^{E\dagger} y^E y^{E\dagger} \\
& - 4 y^E y^{E\dagger} T^E T^{E\dagger} - 4 y^E T^{E\dagger} T^E y^{E\dagger} - 4 T^{h^E} \tilde{f}^\dagger \tilde{f} T^{h^E\dagger}
\end{aligned}
$$

$$
\begin{aligned}
&- 4T^{h^E} h^{E\dagger} h^E T^{h^E\dagger} - 4T^{h^E} \tilde{\lambda}^\dagger \tilde{\lambda} T^{h^E\dagger} - 4T^{h^E} T^{\tilde{f}\dagger} \tilde{f} h^{E\dagger} - 4T^{h^E} T^{h^E\dagger} h^E h^{E\dagger} \\
&- 4T^{h^E} T^{\tilde{\lambda}\dagger} \tilde{\lambda} h^{E\dagger} - 4T^E y^{E\dagger} y^E T^{E\dagger} - 4T^E T^{E\dagger} y^E y^{E\dagger} - 4h^E m_{H_1}^2 \tilde{f}^\dagger \tilde{f} h^{E\dagger} \\
&- 4h^E m_{H_1}^2 h^{E\dagger} h^E h^{E\dagger} - 4h^E m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} - 4h^E \tilde{f}^\dagger \tilde{f} m_{H_1}^2 h^{E\dagger} \\
&- 2h^E \tilde{f}^\dagger \tilde{f} h^{E\dagger} m_{e^c}^2 - 4h^E \tilde{f}^\dagger m_\Sigma^{2*} \tilde{f} h^{E\dagger} - 4h^E h^{E\dagger} h^E m_{H_1}^2 h^{E\dagger} \\
&- 2h^E h^{E\dagger} h^E h^{E\dagger} m_{e^c}^2 - 4h^E h^{E\dagger} m_{e^c}^2 h^E h^{E\dagger} - 4h^E \tilde{\lambda}^\dagger \tilde{\lambda} m_{H_1}^2 h^{E\dagger} \\
&- 2h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} m_{e^c}^2 - 4h^E \tilde{\lambda}^\dagger m_{H_2}^{2*} \tilde{\lambda} h^{E\dagger} - 2m_{e^c}^2 h^E \tilde{f}^\dagger \tilde{f} h^{E\dagger} \\
&- 2m_{e^c}^2 h^E h^{E\dagger} h^E h^{E\dagger} - 2m_{e^c}^2 h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} - 2m_{e^c}^2 y^E y^{E\dagger} y^E y^{E\dagger} \\
&- 4y^E m_L^2 y^{E\dagger} y^E y^{E\dagger} - 4y^E y^{E\dagger} m_{e^c}^2 y^E y^{E\dagger} - 4y^E y^{E\dagger} y^E m_L^2 y^{E\dagger} \\
&- 2y^E y^{E\dagger} y^E y^{E\dagger} m_{e^c}^2 - 4\lambda^* y^E T^{E\dagger} T_\lambda - 4\tilde{\sigma}^* h^E T^{h^E\dagger} T_{\tilde{\sigma}} + \frac{24}{5} g_1^2 \mathbf{1}\Sigma_{2,11} \\
&+ \frac{2}{5}\sqrt{6} g_1 g_1' \mathbf{1}\Sigma_{2,14} + \frac{2}{5}\sqrt{6} g_1 g_1' \mathbf{1}\Sigma_{2,41} + \frac{1}{5} g_1'^2 \mathbf{1}\Sigma_{2,44} + 8\sqrt{\frac{3}{5}} g_1 \mathbf{1}\Sigma_{3,1} \\
&+ 2\sqrt{\frac{2}{5}} g_1' \mathbf{1}\Sigma_{3,4} - 8m_{H_d}^2 y^E y^{E\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) - 4T^E T^{E\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) \\
&- 2m_{e^c}^2 y^E y^{E\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) - 4y^E m_L^2 y^{E\dagger} \operatorname{Tr}\Big(f f^\dagger\Big) - 2y^E y^{E\dagger} m_{e^c}^2 \operatorname{Tr}\Big(f f^\dagger\Big) \\
&- 24m_{L_4}^2 h^E h^{E\dagger} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 12T^{h^E} T^{h^E\dagger} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 12h^E m_{H_1}^2 h^{E\dagger} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 6h^E h^{E\dagger} m_{e^c}^2 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 6m_{e^c}^2 h^E h^{E\dagger} \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 8m_{L_4}^2 h^E h^{E\dagger} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 4T^{h^E} T^{h^E\dagger} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 4h^E m_{H_1}^2 h^{E\dagger} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 2h^E h^{E\dagger} m_{e^c}^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 2m_{e^c}^2 h^E h^{E\dagger} \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 24m_{H_d}^2 y^E y^{E\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12T^E T^{E\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6m_{e^c}^2 y^E y^{E\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 12y^E m_L^2 y^{E\dagger} \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6y^E y^{E\dagger} m_{e^c}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 8m_{H_d}^2 y^E y^{E\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 4T^E T^{E\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2m_{e^c}^2 y^E y^{E\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 4y^E m_L^2 y^{E\dagger} \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2y^E y^{E\dagger} m_{e^c}^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 4y^E T^{E\dagger} \operatorname{Tr}\Big(f^\dagger T^f\Big) - 12h^E T^{h^E\dagger} \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) \\
&- 4h^E T^{h^E\dagger} \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) - 12y^E T^{E\dagger} \operatorname{Tr}\Big(y^{D\dagger} T^D\Big) - 4y^E T^{E\dagger} \operatorname{Tr}\Big(y^{E\dagger} T^E\Big) \\
&- 4T^E y^{E\dagger} \operatorname{Tr}\Big(T^{f*} f^T\Big) - 4y^E y^{E\dagger} \operatorname{Tr}\Big(T^{f*} T^{fT}\Big) - 12T^{h^E} h^{E\dagger} \operatorname{Tr}\Big(T^{g^D*} g^{DT}\Big) \\
&- 12h^E h^{E\dagger} \operatorname{Tr}\Big(T^{g^D*} T^{g^D T}\Big) - 4T^{h^E} h^{E\dagger} \operatorname{Tr}\Big(T^{h^E*} h^{ET}\Big) \\
&- 4h^E h^{E\dagger} \operatorname{Tr}\Big(T^{h^E*} T^{h^E T}\Big) - 12T^E y^{E\dagger} \operatorname{Tr}\Big(T^{D*} y^{DT}\Big) \\
&- 12y^E y^{E\dagger} \operatorname{Tr}\Big(T^{D*} T^{DT}\Big) - 4T^E y^{E\dagger} \operatorname{Tr}\Big(T^{E*} y^{ET}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 4y^E y^{E\dagger} \operatorname{Tr}\Big(T^{E*}T^{ET}\Big) - 4y^E y^{E\dagger} \operatorname{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) \\
&- 4y^E y^{E\dagger} \operatorname{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big) - 12h^E h^{E\dagger} \operatorname{Tr}\Big(g^D g^{D\dagger} m_Q^{2*}\Big) \\
&- 12h^E h^{E\dagger} \operatorname{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger}\Big) - 4h^E h^{E\dagger} \operatorname{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) \\
&- 4h^E h^{E\dagger} \operatorname{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2\Big) - 12y^E y^{E\dagger} \operatorname{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) \\
&- 4y^E y^{E\dagger} \operatorname{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big) - 4y^E y^{E\dagger} \operatorname{Tr}\Big(m_L^2 y^{E\dagger} y^E\Big) \\
&- 12y^E y^{E\dagger} \operatorname{Tr}\Big(m_Q^2 y^{D\dagger} y^D\Big), \qquad \text{(A.107)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_S^2}^{(1)} &= -5g_1'^2|M_1'|^2 + 4\Big(m_{H_d}^2 + m_{H_u}^2 + m_S^2\Big)|\lambda|^2 + 2m_\phi^2|\sigma|^2 + 2m_S^2|\sigma|^2 \\
&+ 2m_{\bar{S}}^2|\sigma|^2 + 4|T_\lambda|^2 + 2|T_\sigma|^2 + \sqrt{\frac{5}{2}}g_1'\Sigma_{1,4} + 6m_S^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&+ 4m_S^2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 6\operatorname{Tr}\Big(T^{\kappa*}T^{\kappa T}\Big) + 4\operatorname{Tr}\Big(T^{\tilde{\lambda}*}T^{\tilde{\lambda}T}\Big) + 4\operatorname{Tr}\Big(m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\Big) \\
&+ 6\operatorname{Tr}\Big(\kappa\kappa^\dagger m_D^{2*}\Big) + 6\operatorname{Tr}\Big(\kappa m_{\bar{D}}^{2*}\kappa^\dagger\Big) + 4\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\Big), \qquad \text{(A.108)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_S^2}^{(2)} &= \frac{12}{5}g_1^2|T_\lambda|^2 - \frac{12}{5}g_1'^2|T_\lambda|^2 + 12g_2^2|T_\lambda|^2 - 16\Big(m_{H_d}^2 + m_{H_u}^2 + m_S^2\Big)|\lambda|^4 \\
&- 16m_\phi^2|\sigma|^2|\kappa_\phi|^2 - 4m_S^2|\sigma|^2|\kappa_\phi|^2 - 4m_{\bar{S}}^2|\sigma|^2|\kappa_\phi|^2 - 8m_\phi^2|\sigma|^4 \\
&- 8m_S^2|\sigma|^4 - 8m_{\bar{S}}^2|\sigma|^4 - 4m_{L_4}^2|\tilde{\sigma}|^2|\sigma|^2 - 4m_{\bar{L}_4}^2|\tilde{\sigma}|^2|\sigma|^2 - 8m_\phi^2|\tilde{\sigma}|^2|\sigma|^2 \\
&- 4m_S^2|\tilde{\sigma}|^2|\sigma|^2 - 4m_{\bar{S}}^2|\tilde{\sigma}|^2|\sigma|^2 - \frac{12}{5}g_1^2 M_1\lambda T_\lambda^* + \frac{12}{5}g_1'^2 M_1'\lambda T_\lambda^* \\
&- 12g_2^2 M_2\lambda T_\lambda^* - 4|\sigma|^2|T_{\kappa_\phi}|^2 - 4\sigma\kappa_\phi^* T_\sigma^* T_{\kappa_\phi} - 4\kappa_\phi\sigma^* T_{\kappa_\phi}^* T_\sigma - 4|\kappa_\phi|^2|T_\sigma|^2 \\
&- 16|\sigma|^2|T_\sigma|^2 - 4|\tilde{\sigma}|^2|T_\sigma|^2 - 4\tilde{\sigma}\sigma^* T_{\tilde{\sigma}}^* T_\sigma - 4\sigma\tilde{\sigma}^* T_\sigma^* T_{\tilde{\sigma}} - 4|\sigma|^2|T_{\tilde{\sigma}}|^2 \\
&+ 5g_1'^2\Sigma_{2,44} + 2\sqrt{10}g_1'\Sigma_{3,4} - 4|T_\lambda|^2\operatorname{Tr}\Big(ff^\dagger\Big) - 4|T_\lambda|^2\operatorname{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&- 12|T_\lambda|^2\operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 4|T_\lambda|^2\operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 12|T_\lambda|^2\operatorname{Tr}\Big(y^U y^{U\dagger}\Big) \\
&+ \frac{8}{5}g_1^2 m_S^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - \frac{18}{5}g_1'^2 m_S^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 32g_3^2 m_S^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&+ \frac{16}{5}g_1^2|M_1|^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 64g_3^2|M_3|^2\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + \frac{12}{5}g_1^2 m_S^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- \frac{12}{5}g_1'^2 m_S^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + 12g_2^2 m_S^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) + \frac{24}{5}g_1^2|M_1|^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&+ 24g_2^2|M_2|^2\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 4\lambda T_\lambda^*\operatorname{Tr}\Big(f^\dagger T^f\Big) - 4\lambda T_\lambda^*\operatorname{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) \\
&- 12\lambda T_\lambda^*\operatorname{Tr}\Big(y^{D\dagger}T^D\Big) - 4\lambda T_\lambda^*\operatorname{Tr}\Big(y^{E\dagger}T^E\Big) - 12\lambda T_\lambda^*\operatorname{Tr}\Big(y^{U\dagger}T^U\Big) \\
&- \frac{8}{5}g_1^2 M_1^*\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) - 32g_3^2 M_3^*\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) - \frac{12}{5}g_1^2 M_1^*\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) \\
&- 12g_2^2 M_2^*\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + \frac{3}{10}g_1'^2 M_1'^*\Big[12\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big) - 16M_1'\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 24M_1'\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) + 595g_1'^2 M_1' + 8\lambda^*\Big(-2M_1'\lambda + T_\lambda\Big) + 8\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big)\Big] \\
&- \frac{8}{5}g_1^2 M_1\operatorname{Tr}\Big(T^{\kappa*}\kappa^T\Big) + \frac{18}{5}g_1'^2 M_1'\operatorname{Tr}\Big(T^{\kappa*}\kappa^T\Big) - 32g_3^2 M_3\operatorname{Tr}\Big(T^{\kappa*}\kappa^T\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{8}{5} g_1^2 \,\mathrm{Tr}\Big(T^{\kappa *} T^{\kappa T}\Big) - \frac{18}{5} g_1'^2 \,\mathrm{Tr}\Big(T^{\kappa *} T^{\kappa T}\Big) + 32 g_3^2 \,\mathrm{Tr}\Big(T^{\kappa *} T^{\kappa T}\Big) \\
&- \frac{12}{5} g_1^2 M_1 \,\mathrm{Tr}\Big(T^{\tilde{\lambda} *} \tilde{\lambda}^T\Big) + \frac{12}{5} g_1'^2 M_1' \,\mathrm{Tr}\Big(T^{\tilde{\lambda} *} \tilde{\lambda}^T\Big) - 12 g_2^2 M_2 \,\mathrm{Tr}\Big(T^{\tilde{\lambda} *} \tilde{\lambda}^T\Big) \\
&+ \frac{12}{5} g_1^2 \,\mathrm{Tr}\Big(T^{\tilde{\lambda} *} T^{\tilde{\lambda} T}\Big) - \frac{12}{5} g_1'^2 \,\mathrm{Tr}\Big(T^{\tilde{\lambda} *} T^{\tilde{\lambda} T}\Big) + 12 g_2^2 \,\mathrm{Tr}\Big(T^{\tilde{\lambda} *} T^{\tilde{\lambda} T}\Big) \\
&+ \frac{12}{5} g_1^2 \,\mathrm{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\Big) - \frac{12}{5} g_1'^2 \,\mathrm{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\Big) + 12 g_2^2 \,\mathrm{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\Big) \\
&+ \frac{4}{5} \lambda^* \Big[3 g_1^2 m_{H_d}^2 \lambda - 3 g_1'^2 m_{H_d}^2 \lambda + 15 g_2^2 m_{H_d}^2 \lambda + 3 g_1^2 m_{H_u}^2 \lambda - 3 g_1'^2 m_{H_u}^2 \lambda \\
&+ 15 g_2^2 m_{H_u}^2 \lambda + 3 g_1^2 m_S^2 \lambda - 3 g_1'^2 m_S^2 \lambda + 15 g_2^2 m_S^2 \lambda - 40 \lambda |T_\lambda|^2 \\
&+ 3 g_1^2 M_1^* \Big(2 M_1 \lambda - T_\lambda\Big) + 15 g_2^2 M_2^* \Big(2 M_2 \lambda - T_\lambda\Big) - 10 m_{H_d}^2 \lambda \,\mathrm{Tr}\Big(f f^\dagger\Big) \\
&- 5 m_{H_u}^2 \lambda \,\mathrm{Tr}\Big(f f^\dagger\Big) - 5 m_S^2 \lambda \,\mathrm{Tr}\Big(f f^\dagger\Big) - 5 m_{H_d}^2 \lambda \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) \\
&- 10 m_{H_u}^2 \lambda \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) - 5 m_S^2 \lambda \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger\Big) - 30 m_{H_d}^2 \lambda \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 15 m_{H_u}^2 \lambda \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 15 m_S^2 \lambda \,\mathrm{Tr}\Big(y^D y^{D\dagger}\Big) - 10 m_{H_d}^2 \lambda \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 5 m_{H_u}^2 \lambda \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 5 m_S^2 \lambda \,\mathrm{Tr}\Big(y^E y^{E\dagger}\Big) - 15 m_{H_d}^2 \lambda \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 30 m_{H_u}^2 \lambda \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 15 m_S^2 \lambda \,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 5 T_\lambda \,\mathrm{Tr}\Big(T^{f *} f^T\Big) \\
&- 5 \lambda \,\mathrm{Tr}\Big(T^{f *} T^{f T}\Big) - 5 T_\lambda \,\mathrm{Tr}\Big(T^{\tilde{f} *} \tilde{f}^T\Big) - 5 \lambda \,\mathrm{Tr}\Big(T^{\tilde{f} *} T^{\tilde{f} T}\Big) \\
&- 15 T_\lambda \,\mathrm{Tr}\Big(T^{D *} y^{D T}\Big) - 15 \lambda \,\mathrm{Tr}\Big(T^{D *} T^{D T}\Big) - 5 T_\lambda \,\mathrm{Tr}\Big(T^{E *} y^{E T}\Big) \\
&- 5 \lambda \,\mathrm{Tr}\Big(T^{E *} T^{E T}\Big) - 15 T_\lambda \,\mathrm{Tr}\Big(T^{U *} y^{U T}\Big) - 15 \lambda \,\mathrm{Tr}\Big(T^{U *} T^{U T}\Big) \\
&- 5 \lambda \,\mathrm{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) - 5 \lambda \,\mathrm{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big) - 5 \lambda \,\mathrm{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) \\
&- 5 \lambda \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger m_\Sigma^{2*}\Big) - 15 \lambda \,\mathrm{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) - 5 \lambda \,\mathrm{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big) \\
&- 5 \lambda \,\mathrm{Tr}\Big(m_L^2 y^{E\dagger} y^E\Big) - 15 \lambda \,\mathrm{Tr}\Big(m_Q^2 y^{D\dagger} y^D\Big) - 15 \lambda \,\mathrm{Tr}\Big(m_Q^2 y^{U\dagger} y^U\Big) \\
&- 15 \lambda \,\mathrm{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big) \Big] + \frac{8}{5} g_1^2 \,\mathrm{Tr}\Big(\kappa \kappa^\dagger m_D^{2*}\Big) - \frac{18}{5} g_1'^2 \,\mathrm{Tr}\Big(\kappa \kappa^\dagger m_D^{2*}\Big) \\
&+ 32 g_3^2 \,\mathrm{Tr}\Big(\kappa \kappa^\dagger m_D^{2*}\Big) + \frac{8}{5} g_1^2 \,\mathrm{Tr}\Big(\kappa m_{\bar{D}}^{2*} \kappa^\dagger\Big) - \frac{18}{5} g_1'^2 \,\mathrm{Tr}\Big(\kappa m_{\bar{D}}^{2*} \kappa^\dagger\Big) \\
&+ 32 g_3^2 \,\mathrm{Tr}\Big(\kappa m_{\bar{D}}^{2*} \kappa^\dagger\Big) + \frac{12}{5} g_1^2 \,\mathrm{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^\dagger m_{H_2}^{2*}\Big) - \frac{12}{5} g_1'^2 \,\mathrm{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^\dagger m_{H_2}^{2*}\Big) \\
&+ 12 g_2^2 \,\mathrm{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^\dagger m_{H_2}^{2*}\Big) - 4 \,\mathrm{Tr}\Big(f T^{\tilde{\lambda} *} T^{\tilde{\lambda} T} f^\dagger\Big) - 4 m_{H_u}^2 \,\mathrm{Tr}\Big(\tilde{f} \tilde{\lambda}^\dagger \tilde{\lambda} \tilde{f}^\dagger\Big) \\
&- 4 m_S^2 \,\mathrm{Tr}\Big(\tilde{f} \tilde{\lambda}^\dagger \tilde{\lambda} \tilde{f}^\dagger\Big) - 4 \,\mathrm{Tr}\Big(\tilde{f} \tilde{\lambda}^\dagger T^{\tilde{\lambda}} T^{\tilde{f}\dagger}\Big) - 4 \,\mathrm{Tr}\Big(\tilde{f} T^{\tilde{\lambda}\dagger} T^{\tilde{\lambda}} \tilde{f}^\dagger\Big) \\
&- 12 m_{L_4}^2 \,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 12 m_S^2 \,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger}\Big) - 12 \,\mathrm{Tr}\Big(g^D \kappa^\dagger T^\kappa T^{g^D \dagger}\Big) \\
&- 12 \,\mathrm{Tr}\Big(g^D T^{\kappa\dagger} T^\kappa g^{D\dagger}\Big) - 4 m_{L_4}^2 \,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big) - 4 m_S^2 \,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big) \\
&- 4 \,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger T^{\tilde{\lambda}} T^{h^E \dagger}\Big) - 4 \,\mathrm{Tr}\Big(h^E T^{\tilde{\lambda}\dagger} T^{\tilde{\lambda}} h^{E\dagger}\Big) - 12 \,\mathrm{Tr}\Big(\kappa g^{D\dagger} T^{g^D} T^{\kappa\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 24 m_S^2 \operatorname{Tr}\left(\kappa\kappa^\dagger\kappa\kappa^\dagger\right) - 24 \operatorname{Tr}\left(\kappa\kappa^\dagger T^\kappa T^{\kappa\dagger}\right) - 12 \operatorname{Tr}\left(\kappa T^{g^D\dagger} T^{g^D}\kappa^\dagger\right) \\
&- 24 \operatorname{Tr}\left(\kappa T^{\kappa\dagger} T^\kappa \kappa^\dagger\right) - 4 \operatorname{Tr}\left(\tilde{\lambda}\tilde{f}^\dagger T^{\tilde{f}} T^{\tilde{\lambda}\dagger}\right) - 4 \operatorname{Tr}\left(\tilde{\lambda} h^{E\dagger} T^{h^E} T^{\tilde{\lambda}\dagger}\right) \\
&- 16 m_S^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\right) - 16 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} T^{\tilde{\lambda}\dagger}\right) - 4 m_{H_d}^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\right) \\
&- 4 m_S^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^*\right) - 4 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger T^{fT} T^{f*}\right) - 4 \operatorname{Tr}\left(\tilde{\lambda} T^{\tilde{f}\dagger} T^{\tilde{f}} \tilde{\lambda}^\dagger\right) \\
&- 4 \operatorname{Tr}\left(\tilde{\lambda} T^{h^E\dagger} T^{h^E} \tilde{\lambda}^\dagger\right) - 16 \operatorname{Tr}\left(\tilde{\lambda} T^{\tilde{\lambda}\dagger} T^{\tilde{\lambda}} \tilde{\lambda}^\dagger\right) - 4 \operatorname{Tr}\left(f^\dagger T^f T^{\tilde{\lambda}*} \tilde{\lambda}^T\right) \\
&- 4 \operatorname{Tr}\left(\tilde{\lambda}^\dagger f^T T^{f*} T^{\tilde{\lambda}}\right) - 4 \operatorname{Tr}\left(f \tilde{\lambda}^* \tilde{\lambda}^T f^\dagger m_\Sigma^{2*}\right) - 4 \operatorname{Tr}\left(\tilde{f} m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda} \tilde{f}^\dagger\right) \\
&- 4 \operatorname{Tr}\left(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda} m_{H_1}^2 \tilde{f}^\dagger\right) - 4 \operatorname{Tr}\left(\tilde{f}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{f}^\dagger m_\Sigma^{2*}\right) - 4 \operatorname{Tr}\left(\tilde{f}\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda}\tilde{f}^\dagger\right) \\
&- 12 \operatorname{Tr}\left(g^D\kappa^\dagger\kappa g^{D\dagger} m_Q^{2*}\right) - 12 \operatorname{Tr}\left(g^D\kappa^\dagger\kappa m_{\bar{D}}^{2*} g^{D\dagger}\right) - 12 \operatorname{Tr}\left(g^D\kappa^\dagger m_D^{2*}\kappa g^{D\dagger}\right) \\
&- 12 \operatorname{Tr}\left(g^D m_{\bar{D}}^{2*}\kappa^\dagger\kappa g^{D\dagger}\right) - 4 \operatorname{Tr}\left(h^E m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda} h^{E\dagger}\right) - 4 \operatorname{Tr}\left(h^E\tilde{\lambda}^\dagger\tilde{\lambda} m_{H_1}^2 h^{E\dagger}\right) \\
&- 4 \operatorname{Tr}\left(h^E\tilde{\lambda}^\dagger\tilde{\lambda} h^{E\dagger} m_{e^c}^2\right) - 4 \operatorname{Tr}\left(h^E\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda} h^{E\dagger}\right) - 16 \operatorname{Tr}\left(m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\right) \\
&- 4 \operatorname{Tr}\left(m_{H_1}^2\tilde{\lambda}^\dagger f^T f^*\tilde{\lambda}\right) - 12 \operatorname{Tr}\left(\kappa\kappa^\dagger\kappa\kappa^\dagger m_D^{2*}\right) - 12 \operatorname{Tr}\left(\kappa\kappa^\dagger\kappa m_{\bar{D}}^{2*}\kappa^\dagger\right) \\
&- 12 \operatorname{Tr}\left(\kappa\kappa^\dagger m_D^{2*}\kappa\kappa^\dagger\right) - 12 \operatorname{Tr}\left(\kappa m_{\bar{D}}^{2*}\kappa^\dagger\kappa\kappa^\dagger\right) - 8 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\right) \\
&- 8 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda}\tilde{\lambda}^\dagger\right) - 4 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*} f^T f^*\right) \\
&- 4 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger f^T f^* m_{H_2}^{2*}\right),
\end{aligned}
\tag{A.109}
$$

$$
\beta_{m_{\bar{S}}^2}^{(1)} = 2\left(m_\phi^2 + m_S^2 + m_{\bar{S}}^2\right)|\sigma|^2 + 2|T_\sigma|^2 - 5 g_1'^2 |M_1'|^2 - \sqrt{\frac{5}{2}} g_1' \Sigma_{1,4}, \tag{A.110}
$$

$$
\begin{aligned}
\beta_{m_{\bar{S}}^2}^{(2)} = {} & \frac{357}{2} g_1'^4 |M_1'|^2 - 4 m_{H_d}^2 |\sigma|^2 |\lambda|^2 - 4 m_{H_u}^2 |\sigma|^2 |\lambda|^2 - 4 m_\phi^2 |\sigma|^2 |\lambda|^2 \\
&- 8 m_S^2 |\sigma|^2 |\lambda|^2 - 4 m_{\bar{S}}^2 |\sigma|^2 |\lambda|^2 - 8 m_\phi^2 |\sigma|^4 - 8 m_S^2 |\sigma|^4 - 8 m_{\bar{S}}^2 |\sigma|^4 \\
&- 4 m_{L_4}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 4 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 8 m_\phi^2 |\tilde{\sigma}|^2 |\sigma|^2 - 4 m_S^2 |\tilde{\sigma}|^2 |\sigma|^2 \\
&- 4 m_{\bar{S}}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 4 |\sigma|^2 |T_{\kappa_\phi}|^2 - 4 |\sigma|^2 |T_\lambda|^2 - 4 \sigma\lambda^* T_\sigma^* T_\lambda - 4 \kappa_\phi \sigma^* T_{\kappa_\phi}^* T_\sigma \\
&- 4 \lambda\sigma^* T_\lambda^* T_\sigma - 4 |\lambda|^2 |T_\sigma|^2 - 16 |\sigma|^2 |T_\sigma|^2 - 4 |\tilde{\sigma}|^2 |T_\sigma|^2 - 4 \tilde{\sigma}\sigma^* T_{\tilde{\sigma}}^* T_\sigma \\
&- 4 \kappa_\phi^* \left[\left(4 m_\phi^2 + m_S^2 + m_{\bar{S}}^2\right)\kappa_\phi |\sigma|^2 + T_\sigma^* \left(\kappa_\phi T_\sigma + \sigma T_{\kappa_\phi}\right)\right] - 4 \sigma\tilde{\sigma}^* T_\sigma^* T_{\tilde{\sigma}} \\
&- 4 |\sigma|^2 |T_{\tilde{\sigma}}|^2 + 5 g_1'^2 \Sigma_{2,44} - 2\sqrt{10} g_1' \Sigma_{3,4} - 6 m_\phi^2 |\sigma|^2 \operatorname{Tr}\left(\kappa\kappa^\dagger\right) \\
&- 12 m_S^2 |\sigma|^2 \operatorname{Tr}\left(\kappa\kappa^\dagger\right) - 6 m_{\bar{S}}^2 |\sigma|^2 \operatorname{Tr}\left(\kappa\kappa^\dagger\right) - 6 |T_\sigma|^2 \operatorname{Tr}\left(\kappa\kappa^\dagger\right) \\
&- 4 m_\phi^2 |\sigma|^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) - 8 m_S^2 |\sigma|^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) - 4 m_{\bar{S}}^2 |\sigma|^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) \\
&- 4 |T_\sigma|^2 \operatorname{Tr}\left(\tilde{\lambda}\tilde{\lambda}^\dagger\right) - 6 \sigma T_\sigma^* \operatorname{Tr}\left(\kappa^\dagger T^\kappa\right) - 4 \sigma T_\sigma^* \operatorname{Tr}\left(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\right) \\
&- 6 \sigma^* T_\sigma \operatorname{Tr}\left(T^{\kappa*}\kappa^T\right) - 6 |\sigma|^2 \operatorname{Tr}\left(T^{\kappa*} T^{\kappa T}\right) - 4 \sigma^* T_\sigma \operatorname{Tr}\left(T^{\tilde{\lambda}*}\tilde{\lambda}^T\right)
\end{aligned}
$$

$$
\begin{aligned}
&- 4|\sigma|^2 \operatorname{Tr}\Big(T^{\tilde\lambda *}T^{\tilde\lambda T}\Big) - 4|\sigma|^2 \operatorname{Tr}\Big(m_{H_1}^2 \tilde\lambda^\dagger \tilde\lambda\Big) - 6|\sigma|^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger m_{\overline{D}}^{2*}\Big) \\
&- 6|\sigma|^2 \operatorname{Tr}\Big(\kappa m_{\overline{D}}^{2*}\kappa^\dagger\Big) - 4|\sigma|^2 \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger m_{H_2}^{2*}\Big),
\end{aligned} \tag{A.111}
$$

$$
\begin{aligned}
\beta_{m_{H_1}^2}^{(1)} =& -\frac{6}{5}g_1^2\mathbf{1}|M_1|^2 - \frac{9}{5}g_1'^2\mathbf{1}|M_1'|^2 - 6g_2^2\mathbf{1}|M_2|^2 + 2m_{H_u}^2\tilde f^\dagger\tilde f + 2m_{L_4}^2 h^{E\dagger}h^E \\
&+ 2m_S^2\tilde\lambda^\dagger\tilde\lambda + 2T^{\tilde f\dagger}T^{\tilde f} + 2T^{h^E\dagger}T^{h^E} + 2T^{\tilde\lambda\dagger}T^{\tilde\lambda} + m_{H_1}^2\tilde f^\dagger\tilde f + m_{H_1}^2 h^{E\dagger}h^E \\
&+ m_{H_1}^2\tilde\lambda^\dagger\tilde\lambda + \tilde f^\dagger\tilde f m_{H_1}^2 + 2\tilde f^\dagger m_\Sigma^{2*}\tilde f + h^{E\dagger}h^E m_{H_1}^2 + 2h^{E\dagger}m_{e^c}^2 h^E + \tilde\lambda^\dagger\tilde\lambda m_{H_1}^2 \\
&+ 2\tilde\lambda^\dagger m_{H_2}^{2*}\tilde\lambda - \sqrt{\frac{3}{5}}g_1\mathbf{1}\Sigma_{1,1} - \frac{3}{\sqrt{10}}g_1'\mathbf{1}\Sigma_{1,4},
\end{aligned} \tag{A.112}
$$

$$
\begin{aligned}
\beta_{m_{H_1}^2}^{(2)} =& \frac{18}{5}g_1^2g_2^2\mathbf{1}|M_2|^2 + \frac{27}{5}g_1'^2g_2^2\mathbf{1}|M_2|^2 + 87g_2^4\mathbf{1}|M_2|^2 + \frac{9}{5}g_1^2g_2^2M_1\mathbf{1}M_2^* \\
&+ \frac{27}{10}g_1'^2g_2^2M_1'\mathbf{1}M_2^* + 2g_1'^2m_{H_u}^2\tilde f^\dagger\tilde f - 2m_{H_d}^2|\lambda|^2\tilde f^\dagger\tilde f - 4m_{H_u}^2|\lambda|^2\tilde f^\dagger\tilde f \\
&- 2m_S^2|\lambda|^2\tilde f^\dagger\tilde f - 2|T_\lambda|^2\tilde f^\dagger\tilde f - 2\lambda T_\lambda^*\tilde f^\dagger T^{\tilde f} + \frac{12}{5}g_1^2m_{L_4}^2h^{E\dagger}h^E \\
&- \frac{2}{5}g_1'^2m_{L_4}^2h^{E\dagger}h^E - 4m_{L_4}^2|\tilde\sigma|^2h^{E\dagger}h^E - 2m_{\overline{L}_4}^2|\tilde\sigma|^2h^{E\dagger}h^E - 2m_\phi^2|\tilde\sigma|^2h^{E\dagger}h^E \\
&- 2|T_{\tilde\sigma}|^2h^{E\dagger}h^E + \frac{3}{50}g_1^2M_1^*\Big\{\Big[30g_2^2\Big(2M_1+M_2\Big) - 3g_1'^2\Big(2M_1+M_1'\Big) \\
&+ 594g_1^2M_1\Big]\mathbf{1} - 40h^{E\dagger}T^{h^E} + 80M_1h^{E\dagger}h^E\Big\} - 2\tilde\sigma T_{\tilde\sigma}^*h^{E\dagger}T^{h^E} \\
&+ 2g_1'^2m_S^2\tilde\lambda^\dagger\tilde\lambda - 4m_{H_d}^2|\lambda|^2\tilde\lambda^\dagger\tilde\lambda - 4m_{H_u}^2|\lambda|^2\tilde\lambda^\dagger\tilde\lambda - 8m_S^2|\lambda|^2\tilde\lambda^\dagger\tilde\lambda \\
&- 2m_\phi^2|\sigma|^2\tilde\lambda^\dagger\tilde\lambda - 4m_S^2|\sigma|^2\tilde\lambda^\dagger\tilde\lambda - 2m_{\overline{S}}^2|\sigma|^2\tilde\lambda^\dagger\tilde\lambda - 4|T_\lambda|^2\tilde\lambda^\dagger\tilde\lambda - 2|T_\sigma|^2\tilde\lambda^\dagger\tilde\lambda \\
&+ \frac{1}{50}g_1'^2M_1'^*\Big(-9\Big\{-3\Big[111g_1'^2M_1' + 5g_2^2\Big(2M_1'+M_2\Big)\Big] + g_1^2\Big(2M_1' \\
&+ M_1\Big)\Big\}\mathbf{1} + 20\Big(10M_1'\tilde f^\dagger\tilde f + 10M_1'\tilde\lambda^\dagger\tilde\lambda - 2M_1'h^{E\dagger}h^E - 5\tilde f^\dagger T^{\tilde f} \\
&- 5\tilde\lambda^\dagger T^{\tilde\lambda} + h^{E\dagger}T^{h^E}\Big)\Big) - 4\lambda T_\lambda^*\tilde\lambda^\dagger T^{\tilde\lambda} - 2\sigma T_\sigma^*\tilde\lambda^\dagger T^{\tilde\lambda} - 2g_1'^2M_1'T^{\tilde f\dagger}\tilde f \\
&+ 2g_1'^2T^{\tilde f\dagger}T^{\tilde f} - 2|\lambda|^2T^{\tilde f\dagger}T^{\tilde f} - \frac{12}{5}g_1^2M_1T^{h^E\dagger}h^E + \frac{2}{5}g_1'^2M_1'T^{h^E\dagger}h^E \\
&+ \frac{12}{5}g_1^2T^{h^E\dagger}T^{h^E} - \frac{2}{5}g_1'^2T^{h^E\dagger}T^{h^E} - 2|\tilde\sigma|^2T^{h^E\dagger}T^{h^E} - 2g_1'^2M_1'T^{\tilde\lambda\dagger}\tilde\lambda \\
&+ 2g_1'^2T^{\tilde\lambda\dagger}T^{\tilde\lambda} - 4|\lambda|^2T^{\tilde\lambda\dagger}T^{\tilde\lambda} - 2|\sigma|^2T^{\tilde\lambda\dagger}T^{\tilde\lambda} + g_1'^2m_{H_1}^2\tilde f^\dagger\tilde f - |\lambda|^2m_{H_1}^2\tilde f^\dagger\tilde f \\
&+ \frac{6}{5}g_1^2m_{H_1}^2h^{E\dagger}h^E - \frac{1}{5}g_1'^2m_{H_1}^2h^{E\dagger}h^E - |\tilde\sigma|^2m_{H_1}^2h^{E\dagger}h^E + g_1'^2m_{H_1}^2\tilde\lambda^\dagger\tilde\lambda \\
&- 2|\lambda|^2m_{H_1}^2\tilde\lambda^\dagger\tilde\lambda - |\sigma|^2m_{H_1}^2\tilde\lambda^\dagger\tilde\lambda + g_1'^2\tilde f^\dagger\tilde f m_{H_1}^2 - |\lambda|^2\tilde f^\dagger\tilde f m_{H_1}^2 \\
&+ 2g_1'^2\tilde f^\dagger m_\Sigma^{2*}\tilde f - 2|\lambda|^2\tilde f^\dagger m_\Sigma^{2*}\tilde f + \frac{6}{5}g_1^2h^{E\dagger}h^Em_{H_1}^2 - \frac{1}{5}g_1'^2h^{E\dagger}h^Em_{H_1}^2 \\
&- |\tilde\sigma|^2h^{E\dagger}h^Em_{H_1}^2 + \frac{12}{5}g_1^2h^{E\dagger}m_{e^c}^2h^E - \frac{2}{5}g_1'^2h^{E\dagger}m_{e^c}^2h^E \\
&- 2|\tilde\sigma|^2h^{E\dagger}m_{e^c}^2h^E + g_1'^2\tilde\lambda^\dagger\tilde\lambda m_{H_1}^2 - 2|\lambda|^2\tilde\lambda^\dagger\tilde\lambda m_{H_1}^2 - |\sigma|^2\tilde\lambda^\dagger\tilde\lambda m_{H_1}^2 \\
&+ 2g_1'^2\tilde\lambda^\dagger m_{H_2}^{2*}\tilde\lambda - 4|\lambda|^2\tilde\lambda^\dagger m_{H_2}^{2*}\tilde\lambda - 2|\sigma|^2\tilde\lambda^\dagger m_{H_2}^{2*}\tilde\lambda - 4m_{H_d}^2\tilde f^\dagger f f^\dagger\tilde f \\
&- 4m_{H_u}^2\tilde f^\dagger f f^\dagger\tilde f - 4\tilde f^\dagger f T^{f\dagger}T^{\tilde f} - 8m_{H_u}^2\tilde f^\dagger\tilde f\tilde f^\dagger\tilde f - 4\tilde f^\dagger\tilde f T^{\tilde f\dagger}T^{\tilde f}
\end{aligned}
$$

$$
\begin{aligned}
&- 4\tilde{f}^\dagger T^f T^{f\dagger}\tilde{f} - 4\tilde{f}^\dagger T^{\tilde{f}} T^{\tilde{f}\dagger}\tilde{f} - 8m_{L_4}^2 h^{E\dagger}h^E h^{E\dagger}h^E - 4h^{E\dagger}h^E T^{h^E\dagger}T^{h^E} \\
&- 4m_{H_d}^2 h^{E\dagger}y^E y^{E\dagger}h^E - 4m_{L_4}^2 h^{E\dagger}y^E y^{E\dagger}h^E - 4h^{E\dagger}y^E T^{E\dagger}T^{h^E} \\
&- 4h^{E\dagger}T^{h^E}T^{h^E\dagger}h^E - 4h^{E\dagger}T^E T^{E\dagger}h^E - 4m_S^2\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda} - 2\tilde{\lambda}^\dagger\tilde{\lambda}T^{\tilde{\lambda}\dagger}T^{\tilde{\lambda}} \\
&- 2\tilde{\lambda}^\dagger T^{\tilde{\lambda}}T^{\tilde{\lambda}\dagger}\tilde{\lambda} - 2m_{H_d}^2\tilde{\lambda}^\dagger f^T f^*\tilde{\lambda} - 2m_S^2\tilde{\lambda}^\dagger f^T f^*\tilde{\lambda} - 2\tilde{\lambda}^\dagger f^T T^{f*}T^{\tilde{\lambda}} \\
&- 2\tilde{\lambda}^\dagger T^{fT}T^{f*}\tilde{\lambda} - 4T^{\tilde{f}\dagger}ff^\dagger T^{\tilde{f}} - 4T^{\tilde{f}\dagger}\tilde{f}\tilde{f}^\dagger T^{\tilde{f}} - 4T^{\tilde{f}\dagger}T^f f^\dagger\tilde{f} \\
&- 4T^{\tilde{f}\dagger}T^{\tilde{f}}\tilde{f}^\dagger\tilde{f} - 4T^{h^E\dagger}h^E h^{E\dagger}T^{h^E} - 4T^{h^E\dagger}y^E y^{E\dagger}T^{h^E} - 4T^{h^E\dagger}T^{h^E}h^{E\dagger}h^E \\
&- 4T^{h^E\dagger}T^E y^{E\dagger}h^E - 2T^{\tilde{\lambda}\dagger}\tilde{\lambda}\tilde{\lambda}^\dagger T^{\tilde{\lambda}} - 2T^{\tilde{\lambda}\dagger}T^{\tilde{\lambda}}\tilde{\lambda}^\dagger\tilde{\lambda} - 2T^{\tilde{\lambda}\dagger}f^T f^*T^{\tilde{\lambda}} \\
&- 2T^{\tilde{\lambda}\dagger}T^{fT}f^*\tilde{\lambda} - 2m_{H_1}^2\tilde{f}^\dagger ff^\dagger\tilde{f} - 2m_{H_1}^2\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger\tilde{f} - 2m_{H_1}^2 h^{E\dagger}h^E h^{E\dagger}h^E \\
&- 2m_{H_1}^2 h^{E\dagger}y^E y^{E\dagger}h^E - m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda} - m_{H_1}^2\tilde{\lambda}^\dagger f^T f^*\tilde{\lambda} - 4\tilde{f}^\dagger f m_{H_2}^2 f^\dagger\tilde{f} \\
&- 2\tilde{f}^\dagger ff^\dagger\tilde{f}m_{H_1}^2 - 4\tilde{f}^\dagger ff^\dagger m_\Sigma^{2*}\tilde{f} - 4\tilde{f}^\dagger\tilde{f}m_{H_1}^2\tilde{f}^\dagger\tilde{f} - 2\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger\tilde{f}m_{H_1}^2 \\
&- 4\tilde{f}^\dagger\tilde{f}\tilde{f}^\dagger m_\Sigma^{2*}\tilde{f} - 4\tilde{f}^\dagger m_\Sigma^{2*}ff^\dagger\tilde{f} - 4\tilde{f}^\dagger m_\Sigma^{2*}\tilde{f}\tilde{f}^\dagger\tilde{f} - 4h^{E\dagger}h^E m_{H_1}^2 h^{E\dagger}h^E \\
&- 2h^{E\dagger}h^E h^{E\dagger}h^E m_{H_1}^2 - 4h^{E\dagger}h^E h^{E\dagger}m_{e^c}^2 h^E - 4h^{E\dagger}m_{e^c}^2 h^E h^{E\dagger}h^E \\
&- 4h^{E\dagger}m_{e^c}^2 y^E y^{E\dagger}h^E - 4h^{E\dagger}y^E m_L^2 y^{E\dagger}h^E - 2h^{E\dagger}y^E y^{E\dagger}h^E m_{H_1}^2 \\
&- 4h^{E\dagger}y^E y^{E\dagger}m_{e^c}^2 h^E - 2\tilde{\lambda}^\dagger\tilde{\lambda}m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda} - \tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda}m_{H_1}^2 - 2\tilde{\lambda}^\dagger\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda} \\
&- 2\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda}\tilde{\lambda}^\dagger\tilde{\lambda} - 2\tilde{\lambda}^\dagger m_{H_2}^{2*}f^T f^*\tilde{\lambda} - 2\tilde{\lambda}^\dagger f^T m_\Sigma^2 f^*\tilde{\lambda} - \tilde{\lambda}^\dagger f^T f^*\tilde{\lambda}m_{H_1}^2 \\
&- 2\tilde{\lambda}^\dagger f^T f^* m_{H_2}^{2*}\tilde{\lambda} - 2\lambda^* T^{\tilde{f}\dagger}\tilde{f}T_\lambda - 4\lambda^* T^{\tilde{\lambda}\dagger}\tilde{\lambda}T_\lambda - 2\sigma^* T^{\tilde{\lambda}\dagger}\tilde{\lambda}T_\sigma \\
&- 2\tilde{\sigma}^* T^{h^E\dagger}h^E T_{\tilde{\sigma}} + 6g_2^4\mathbf{1}\Sigma_{2,2} + \frac{6}{5}g_1^2\mathbf{1}\Sigma_{2,11} + \frac{3}{5}\sqrt{6}g_1 g_1'\mathbf{1}\Sigma_{2,14} \\
&+ \frac{3}{5}\sqrt{6}g_1 g_1'\mathbf{1}\Sigma_{2,41} + \frac{9}{5}g_1'^2\mathbf{1}\Sigma_{2,44} - 4\sqrt{\frac{3}{5}}g_1\mathbf{1}\Sigma_{3,1} - 6\sqrt{\frac{2}{5}}g_1'\mathbf{1}\Sigma_{3,4} \\
&- 4m_{H_u}^2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 2T^{\tilde{f}\dagger}T^{\tilde{f}}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - m_{H_1}^2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) \\
&- \tilde{f}^\dagger\tilde{f}m_{H_1}^2\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 2\tilde{f}^\dagger m_\Sigma^{2*}\tilde{f}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big) - 12m_{L_4}^2 h^{E\dagger}h^E\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 6T^{h^E\dagger}T^{h^E}\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 3m_{H_1}^2 h^{E\dagger}h^E\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 3h^{E\dagger}h^E m_{H_1}^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 6h^{E\dagger}m_{e^c}^2 h^E\,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 4m_{L_4}^2 h^{E\dagger}h^E\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 2T^{h^E\dagger}T^{h^E}\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- m_{H_1}^2 h^{E\dagger}h^E\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - h^{E\dagger}h^E m_{H_1}^2\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 2h^{E\dagger}m_{e^c}^2 h^E\,\mathrm{Tr}\Big(h^E h^{E\dagger}\Big) - 12m_{H_u}^2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 6T^{\tilde{f}\dagger}T^{\tilde{f}}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 3m_{H_1}^2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 3\tilde{f}^\dagger\tilde{f}m_{H_1}^2\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) - 6\tilde{f}^\dagger m_\Sigma^{2*}\tilde{f}\,\mathrm{Tr}\Big(y^U y^{U\dagger}\Big) \\
&- 12m_S^2\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 6T^{\tilde{\lambda}\dagger}T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 3m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 3\tilde{\lambda}^\dagger\tilde{\lambda}m_{H_1}^2\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 6\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger\Big) - 8m_S^2\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 4T^{\tilde{\lambda}\dagger}T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\tilde{\lambda}^\dagger\tilde{\lambda}m_{H_1}^2\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&- 4\tilde{\lambda}^\dagger m_{H_2}^{2*}\tilde{\lambda}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2T^{\tilde{f}\dagger}\tilde{f}\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big) - 6T^{h^E\dagger}h^E\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big)\\
&- 2T^{h^E\dagger}h^E\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big) - 6T^{\tilde{f}\dagger}\tilde{f}\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big) - 6T^{\tilde{\lambda}\dagger}\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa^\dagger T^\kappa\Big)\\
&- 4T^{\tilde{\lambda}\dagger}\tilde{\lambda}\,\mathrm{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - 2\tilde{f}^\dagger T^{\tilde{f}}\,\mathrm{Tr}\Big(T^{\tilde{f}*}\tilde{f}^T\Big) - 2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(T^{\tilde{f}*}T^{\tilde{f}T}\Big)\\
&- 6h^{E\dagger}T^{h^E}\,\mathrm{Tr}\Big(T^{g^D*}g^{DT}\Big) - 6h^{E\dagger}h^E\,\mathrm{Tr}\Big(T^{g^D*}T^{g^DT}\Big)\\
&- 2h^{E\dagger}T^{h^E}\,\mathrm{Tr}\Big(T^{h^E*}h^{ET}\Big) - 2h^{E\dagger}h^E\,\mathrm{Tr}\Big(T^{h^E*}T^{h^ET}\Big)\\
&- 6\tilde{f}^\dagger T^{\tilde{f}}\,\mathrm{Tr}\Big(T^{U*}y^{UT}\Big) - 6\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(T^{U*}T^{UT}\Big) - 6\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(T^{\kappa*}\kappa^T\Big)\\
&- 6\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(T^{\kappa*}T^{\kappa T}\Big) - 4\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\,\mathrm{Tr}\Big(T^{\tilde{\lambda}*}\tilde{\lambda}^T\Big) - 4\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(T^{\tilde{\lambda}*}T^{\tilde{\lambda}T}\Big)\\
&- 2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(\tilde{f}m_{H_1}^2\tilde{f}^\dagger\Big) - 2\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger m_\Sigma^{2*}\Big) - 6h^{E\dagger}h^E\,\mathrm{Tr}\Big(g^Dg^{D\dagger}m_Q^{2*}\Big)\\
&- 6h^{E\dagger}h^E\,\mathrm{Tr}\Big(g^Dm_{\bar{D}}^{2*}g^{D\dagger}\Big) - 2h^{E\dagger}h^E\,\mathrm{Tr}\Big(h^Em_{H_1}^2h^{E\dagger}\Big)\\
&- 2h^{E\dagger}h^E\,\mathrm{Tr}\Big(h^Eh^{E\dagger}m_{e^c}^2\Big) - 4\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\Big) - 6\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(m_Q^2y^{U\dagger}y^U\Big)\\
&- 6\tilde{f}^\dagger\tilde{f}\,\mathrm{Tr}\Big(m_{u^c}^2y^Uy^{U\dagger}\Big) - 6\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa\kappa^\dagger m_D^{2*}\Big) - 6\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\kappa m_{\bar{D}}^{2*}\kappa^\dagger\Big)\\
&- 4\tilde{\lambda}^\dagger\tilde{\lambda}\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\Big),
\end{aligned} \tag{A.113}
$$

$$
\begin{aligned}
\beta_{m_{H_2}^2}^{(1)} ={}& -\frac{6}{5}g_1^2\mathbf{1}|M_1|^2 - \frac{4}{5}g_1'^2\mathbf{1}|M_1'|^2 - 6g_2^2\mathbf{1}|M_2|^2 + 2m_{H_d}^2f^\dagger f + 2T^{f\dagger}T^f\\
&+ 2m_S^2\tilde{\lambda}^*\tilde{\lambda}^T + 2T^{\tilde{\lambda}*}T^{\tilde{\lambda}T} + m_{H_2}^2f^\dagger f + m_{H_2}^2\tilde{\lambda}^*\tilde{\lambda}^T + f^\dagger fm_{H_2}^2 + 2f^\dagger m_\Sigma^{2*}f\\
&+ 2\tilde{\lambda}^*m_{H_1}^{2*}\tilde{\lambda}^T + \tilde{\lambda}^*\tilde{\lambda}^Tm_{H_2}^2 + \sqrt{\frac{3}{5}}g_1\mathbf{1}\Sigma_{1,1} - \sqrt{\frac{2}{5}}g_1'\mathbf{1}\Sigma_{1,4},
\end{aligned} \tag{A.114}
$$

$$
\begin{aligned}
\beta_{m_{H_2}^2}^{(2)} ={}& \frac{18}{5}g_1^2g_2^2\mathbf{1}|M_2|^2 + \frac{12}{5}g_1'^2g_2^2\mathbf{1}|M_2|^2 + 87g_2^4\mathbf{1}|M_2|^2\\
&+ \frac{9}{25}g_1^2\Big[2g_1'^2\Big(2M_1 + M_1'\Big) + 5g_2^2\Big(2M_1 + M_2\Big) + 99g_1^2M_1\Big]\mathbf{1}M_1^*\\
&+ \frac{9}{5}g_1^2g_2^2M_1\mathbf{1}M_2^* + \frac{6}{5}g_1'^2g_2^2M_1'\mathbf{1}M_2^* + 3g_1'^2m_{H_d}^2f^\dagger f - 4m_{H_d}^2|\lambda|^2f^\dagger f\\
&- 2m_{H_u}^2|\lambda|^2f^\dagger f - 2m_S^2|\lambda|^2f^\dagger f - 2|T_\lambda|^2f^\dagger f - 2\lambda T_\lambda^*f^\dagger T^f\\
&- 3g_1'^2M_1'T^{f\dagger}f + 3g_1'^2T^{f\dagger}T^f - 2|\lambda|^2T^{f\dagger}T^f + 3g_1'^2m_S^2\tilde{\lambda}^*\tilde{\lambda}^T\\
&- 4m_{H_d}^2|\lambda|^2\tilde{\lambda}^*\tilde{\lambda}^T - 4m_{H_u}^2|\lambda|^2\tilde{\lambda}^*\tilde{\lambda}^T - 8m_S^2|\lambda|^2\tilde{\lambda}^*\tilde{\lambda}^T - 2m_\phi^2|\sigma|^2\tilde{\lambda}^*\tilde{\lambda}^T\\
&- 4m_S^2|\sigma|^2\tilde{\lambda}^*\tilde{\lambda}^T - 2m_{\bar{S}}^2|\sigma|^2\tilde{\lambda}^*\tilde{\lambda}^T - 4|T_\lambda|^2\tilde{\lambda}^*\tilde{\lambda}^T - 2|T_\sigma|^2\tilde{\lambda}^*\tilde{\lambda}^T\\
&+ \frac{3}{25}g_1'^2M_1'^*\Big\{\Big[10g_2^2\Big(2M_1' + M_2\Big) + 217g_1'^2M_1' + 6g_1^2\Big(2M_1' + M_1\Big)\Big]\mathbf{1}\\
&+ 25\Big(2M_1'f^\dagger f + 2M_1'\tilde{\lambda}^*\tilde{\lambda}^T - f^\dagger T^f - \tilde{\lambda}^*T^{\tilde{\lambda}T}\Big)\Big\} - 4\lambda T_\lambda^*\tilde{\lambda}^*T^{\tilde{\lambda}T}\\
&- 2\sigma T_\sigma^*\tilde{\lambda}^*T^{\tilde{\lambda}T} - 3g_1'^2M_1'T^{\tilde{\lambda}*}\tilde{\lambda}^T + 3g_1'^2T^{\tilde{\lambda}*}T^{\tilde{\lambda}T} - 4|\lambda|^2T^{\tilde{\lambda}*}T^{\tilde{\lambda}T}\\
&- 2|\sigma|^2T^{\tilde{\lambda}*}T^{\tilde{\lambda}T} + \frac{3}{2}g_1'^2m_{H_2}^2f^\dagger f - |\lambda|^2m_{H_2}^2f^\dagger f + \frac{3}{2}g_1'^2m_{H_2}^2\tilde{\lambda}^*\tilde{\lambda}^T\\
&- 2|\lambda|^2m_{H_2}^2\tilde{\lambda}^*\tilde{\lambda}^T - |\sigma|^2m_{H_2}^2\tilde{\lambda}^*\tilde{\lambda}^T + \frac{3}{2}g_1'^2f^\dagger fm_{H_2}^2 - |\lambda|^2f^\dagger fm_{H_2}^2
\end{aligned}
$$

$$
\begin{aligned}
&+3g_1'^2 f^\dagger m_\Sigma^{2*} f - 2|\lambda|^2 f^\dagger m_\Sigma^{2*} f + 3g_1'^2 \tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T - 4|\lambda|^2 \tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T \\
&- 2|\sigma|^2 \tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T + \frac{3}{2} g_1'^2 \tilde\lambda^* \tilde\lambda^T m_{H_2}^2 - 2|\lambda|^2 \tilde\lambda^* \tilde\lambda^T m_{H_2}^2 \\
&- |\sigma|^2 \tilde\lambda^* \tilde\lambda^T m_{H_2}^2 - 8m_{H_d}^2 f^\dagger f f^\dagger f - 4f^\dagger f T^{f\dagger} T^f - 4m_{H_d}^2 f^\dagger \tilde f \tilde f^\dagger f \\
&- 4m_{H_u}^2 f^\dagger \tilde f \tilde f^\dagger f - 4f^\dagger \tilde f T^{\tilde f\dagger} T^f - 4f^\dagger T^f T^{f\dagger} f - 4f^\dagger T^{\tilde f} T^{\tilde f\dagger} f \\
&- 4T^{f\dagger} f f^\dagger T^f - 4T^{f\dagger} \tilde f \tilde f^\dagger T^f - 4T^{f\dagger} T^f f^\dagger f - 4T^{f\dagger} T^{\tilde f} \tilde f^\dagger f \\
&- 2m_{H_u}^2 \tilde\lambda^* \tilde f^T \tilde f^* \tilde\lambda^T - 2m_S^2 \tilde\lambda^* \tilde f^T \tilde f^* \tilde\lambda^T - 2\tilde\lambda^* \tilde f^T T^{\tilde f *} T^{\tilde\lambda T} \\
&- 2m_{L_4}^2 \tilde\lambda^* h^{ET} h^{E*} \tilde\lambda^T - 2m_S^2 \tilde\lambda^* h^{ET} h^{E*} \tilde\lambda^T - 2\tilde\lambda^* h^{ET} T^{h^E *} T^{\tilde\lambda T} \\
&- 4m_S^2 \tilde\lambda^* \tilde\lambda^T \tilde\lambda^* \tilde\lambda^T - 2\tilde\lambda^* \tilde\lambda^T T^{\tilde\lambda *} T^{\tilde\lambda T} - 2\tilde\lambda^* T^{\tilde f T} T^{\tilde f *} \tilde\lambda^T \\
&- 2\tilde\lambda^* T^{h^E T} T^{h^E *} \tilde\lambda^T - 2\tilde\lambda^* T^{\tilde\lambda T} T^{\tilde\lambda *} \tilde\lambda^T - 2T^{\tilde\lambda *} \tilde f^T \tilde f^* T^{\tilde\lambda T} \\
&- 2T^{\tilde\lambda *} h^{ET} h^{E*} T^{\tilde\lambda T} - 2T^{\tilde\lambda *} \tilde\lambda^T \tilde\lambda^* T^{\tilde\lambda T} - 2T^{\tilde\lambda *} T^{\tilde f T} \tilde f^* \tilde\lambda^T - 2T^{\tilde\lambda *} T^{h^E T} h^{E*} \tilde\lambda^T \\
&- 2T^{\tilde\lambda *} T^{\tilde\lambda T} \tilde\lambda^* \tilde\lambda^T - 2m_{H_2}^2 f^\dagger f f^\dagger f - 2m_{H_2}^2 f^\dagger \tilde f \tilde f^\dagger f - m_{H_2}^2 \tilde\lambda^* \tilde f^T \tilde f^* \tilde\lambda^T \\
&- m_{H_2}^2 \tilde\lambda^* h^{ET} h^{E*} \tilde\lambda^T - m_{H_2}^2 \tilde\lambda^* \tilde\lambda^T \tilde\lambda^* \tilde\lambda^T - 4f^\dagger f m_{H_2}^2 f^\dagger f - 2f^\dagger f f^\dagger f m_{H_2}^2 \\
&- 4f^\dagger f f^\dagger m_\Sigma^{2*} f - 4f^\dagger \tilde f m_{H_1}^2 \tilde f^\dagger f - 2f^\dagger \tilde f \tilde f^\dagger f m_{H_2}^2 - 4f^\dagger \tilde f \tilde f^\dagger m_\Sigma^{2*} f \\
&- 4f^\dagger m_\Sigma^{2*} f f^\dagger f - 4f^\dagger m_\Sigma^{2*} \tilde f \tilde f^\dagger f - 2\tilde\lambda^* m_{H_1}^{2*} \tilde f^T \tilde f^* \tilde\lambda^T \\
&- 2\tilde\lambda^* m_{H_1}^{2*} h^{ET} h^{E*} \tilde\lambda^T - 2\tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T \tilde\lambda^* \tilde\lambda^T - 2\tilde\lambda^* \tilde f^T m_\Sigma^2 \tilde f^* \tilde\lambda^T \\
&- 2\tilde\lambda^* \tilde f^T \tilde f^* m_{H_1}^{2*} \tilde\lambda^T - \tilde\lambda^* \tilde f^T \tilde f^* \tilde\lambda^T m_{H_2}^2 - 2\tilde\lambda^* h^{ET} h^{E*} m_{H_1}^{2*} \tilde\lambda^T \\
&- \tilde\lambda^* h^{ET} h^{E*} \tilde\lambda^T m_{H_2}^2 - 2\tilde\lambda^* h^{ET} m_{e^c}^{2*} h^{E*} \tilde\lambda^T - 2\tilde\lambda^* \tilde\lambda^T m_{H_2}^2 \tilde\lambda^* \tilde\lambda^T \\
&- 2\tilde\lambda^* \tilde\lambda^T \tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T - \tilde\lambda^* \tilde\lambda^T \tilde\lambda^* \tilde\lambda^T m_{H_2}^2 - 2\lambda^* T^{f\dagger} f T_\lambda - 4\lambda^* T^{\tilde\lambda *} \tilde\lambda^T T_\lambda \\
&- 2\sigma^* T^{\tilde\lambda *} \tilde\lambda^T T_\sigma + 6g_2^4 \mathbf{1}\Sigma_{2,2} + \frac{6}{5} g_1^2 \mathbf{1}\Sigma_{2,11} - \frac{2}{5}\sqrt{6} g_1 g_1' \mathbf{1}\Sigma_{2,14} \\
&- \frac{2}{5}\sqrt{6} g_1 g_1' \mathbf{1}\Sigma_{2,41} + \frac{4}{5} g_1'^2 \mathbf{1}\Sigma_{2,44} + 4\sqrt{\frac{3}{5}} g_1 \mathbf{1}\Sigma_{3,1} - 4\sqrt{\frac{2}{5}} g_1' \mathbf{1}\Sigma_{3,4} \\
&- 4m_{H_d}^2 f^\dagger f \operatorname{Tr}\Big(f f^\dagger\Big) - 2T^{f\dagger} T^f \operatorname{Tr}\Big(f f^\dagger\Big) - m_{H_2}^2 f^\dagger f \operatorname{Tr}\Big(f f^\dagger\Big) \\
&- f^\dagger f m_{H_2}^2 \operatorname{Tr}\Big(f f^\dagger\Big) - 2f^\dagger m_\Sigma^{2*} f \operatorname{Tr}\Big(f f^\dagger\Big) - 12m_{H_d}^2 f^\dagger f \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6T^{f\dagger} T^f \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 3m_{H_2}^2 f^\dagger f \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 3f^\dagger f m_{H_2}^2 \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) \\
&- 6f^\dagger m_\Sigma^{2*} f \operatorname{Tr}\Big(y^D y^{D\dagger}\Big) - 4m_{H_d}^2 f^\dagger f \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2T^{f\dagger} T^f \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- m_{H_2}^2 f^\dagger f \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - f^\dagger f m_{H_2}^2 \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) - 2f^\dagger m_\Sigma^{2*} f \operatorname{Tr}\Big(y^E y^{E\dagger}\Big) \\
&- 12m_S^2 \tilde\lambda^* \tilde\lambda^T \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 6T^{\tilde\lambda *} T^{\tilde\lambda T} \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 3m_{H_2}^2 \tilde\lambda^* \tilde\lambda^T \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 6\tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 3\tilde\lambda^* \tilde\lambda^T m_{H_2}^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 8m_S^2 \tilde\lambda^* \tilde\lambda^T \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) \\
&- 4T^{\tilde\lambda *} T^{\tilde\lambda T} \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 2m_{H_2}^2 \tilde\lambda^* \tilde\lambda^T \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 4\tilde\lambda^* m_{H_1}^{2*} \tilde\lambda^T \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) \\
&- 2\tilde\lambda^* \tilde\lambda^T m_{H_2}^2 \operatorname{Tr}\Big(\tilde\lambda\tilde\lambda^\dagger\Big) - 2T^{f\dagger} f \operatorname{Tr}\Big(f^\dagger T^f\Big) - 6T^{f\dagger} f \operatorname{Tr}\Big(y^{D\dagger} T^D\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 2T^{f\dagger} f \,\mathrm{Tr}\Big(y^{E\dagger} T^{E}\Big) - 6T^{\tilde{\lambda}*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(\kappa^{\dagger} T^{\kappa}\Big) - 4T^{\tilde{\lambda}*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(\tilde{\lambda}^{\dagger} T^{\tilde{\lambda}}\Big) \\
&- 2f^{\dagger} T^{f} \,\mathrm{Tr}\Big(T^{f*} f^{T}\Big) - 2f^{\dagger} f \,\mathrm{Tr}\Big(T^{f*} T^{fT}\Big) - 6f^{\dagger} T^{f} \,\mathrm{Tr}\Big(T^{D*} y^{DT}\Big) \\
&- 6f^{\dagger} f \,\mathrm{Tr}\Big(T^{D*} T^{DT}\Big) - 2f^{\dagger} T^{f} \,\mathrm{Tr}\Big(T^{E*} y^{ET}\Big) - 2f^{\dagger} f \,\mathrm{Tr}\Big(T^{E*} T^{ET}\Big) \\
&- 6\tilde{\lambda}^{*} T^{\tilde{\lambda}T} \,\mathrm{Tr}\Big(T^{\kappa*} \kappa^{T}\Big) - 6\tilde{\lambda}^{*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(T^{\kappa*} T^{\kappa T}\Big) - 4\tilde{\lambda}^{*} T^{\tilde{\lambda}T} \,\mathrm{Tr}\Big(T^{\tilde{\lambda}*} \tilde{\lambda}^{T}\Big) \\
&- 4\tilde{\lambda}^{*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(T^{\tilde{\lambda}*} T^{\tilde{\lambda}T}\Big) - 2f^{\dagger} f \,\mathrm{Tr}\Big(f m_{H_2}^2 f^{\dagger}\Big) - 2f^{\dagger} f \,\mathrm{Tr}\Big(f f^{\dagger} m_{\Sigma}^{2*}\Big) \\
&- 6f^{\dagger} f \,\mathrm{Tr}\Big(m_{d^c}^2 y^{D} y^{D\dagger}\Big) - 2f^{\dagger} f \,\mathrm{Tr}\Big(m_{e^c}^2 y^{E} y^{E\dagger}\Big) - 4\tilde{\lambda}^{*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^{\dagger} \tilde{\lambda}\Big) \\
&- 2f^{\dagger} f \,\mathrm{Tr}\Big(m_{L}^2 y^{E\dagger} y^{E}\Big) - 6f^{\dagger} f \,\mathrm{Tr}\Big(m_{Q}^2 y^{D\dagger} y^{D}\Big) - 6\tilde{\lambda}^{*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(\kappa \kappa^{\dagger} m_{D}^{2*}\Big) \\
&- 6\tilde{\lambda}^{*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(\kappa m_{\bar{D}}^{2*} \kappa^{\dagger}\Big) - 4\tilde{\lambda}^{*} \tilde{\lambda}^{T} \,\mathrm{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^{\dagger} m_{H_2}^{2*}\Big),
\end{aligned}
\tag{A.115}
$$

$$
\begin{aligned}
\beta_{m_\Sigma^2}^{(1)} = &-5g_1'^2 \mathbf{1} |M_1'|^2 + 2\Big(2m_{H_d}^2 f^{*} f^{T} + 2m_{H_u}^2 \tilde{f}^{*} \tilde{f}^{T} + 2T^{f*} T^{fT} + 2T^{\tilde{f}*} T^{\tilde{f}T} \\
&+ m_{\Sigma}^2 f^{*} f^{T} + m_{\Sigma}^2 \tilde{f}^{*} \tilde{f}^{T} + 2f^{*} m_{H_2}^{2*} f^{T} + f^{*} f^{T} m_{\Sigma}^2 + 2\tilde{f}^{*} m_{H_1}^{2*} \tilde{f}^{T} \\
&+ \tilde{f}^{*} \tilde{f}^{T} m_{\Sigma}^2\Big) + \sqrt{\frac{5}{2}} g_1' \mathbf{1} \Sigma_{1,4},
\end{aligned}
\tag{A.116}
$$

$$
\begin{aligned}
\beta_{m_\Sigma^2}^{(2)} = &\frac{12}{5} g_1^2 m_{H_d}^2 f^{*} f^{T} - \frac{12}{5} g_1'^2 m_{H_d}^2 f^{*} f^{T} + 12 g_2^2 m_{H_d}^2 f^{*} f^{T} + \frac{24}{5} g_1^2 |M_1|^2 f^{*} f^{T} \\
&+ 24 g_2^2 |M_2|^2 f^{*} f^{T} - 8 m_{H_d}^2 |\lambda|^2 f^{*} f^{T} - 4 m_{H_u}^2 |\lambda|^2 f^{*} f^{T} - 4 m_{S}^2 |\lambda|^2 f^{*} f^{T} \\
&- 4|T_\lambda|^2 f^{*} f^{T} - \frac{12}{5} g_1^2 M_1^{*} f^{*} T^{fT} - 12 g_2^2 M_2^{*} f^{*} T^{fT} - 4\lambda T_\lambda^{*} f^{*} T^{fT} \\
&+ \frac{12}{5} g_1^2 m_{H_u}^2 \tilde{f}^{*} \tilde{f}^{T} - \frac{12}{5} g_1'^2 m_{H_u}^2 \tilde{f}^{*} \tilde{f}^{T} + 12 g_2^2 m_{H_u}^2 \tilde{f}^{*} \tilde{f}^{T} + \frac{24}{5} g_1^2 |M_1|^2 \tilde{f}^{*} \tilde{f}^{T} \\
&+ 24 g_2^2 |M_2|^2 \tilde{f}^{*} \tilde{f}^{T} - 4 m_{H_d}^2 |\lambda|^2 \tilde{f}^{*} \tilde{f}^{T} - 8 m_{H_u}^2 |\lambda|^2 \tilde{f}^{*} \tilde{f}^{T} - 4 m_{S}^2 |\lambda|^2 \tilde{f}^{*} \tilde{f}^{T} \\
&- 4|T_\lambda|^2 \tilde{f}^{*} \tilde{f}^{T} - \frac{12}{5} g_1^2 M_1^{*} \tilde{f}^{*} T^{\tilde{f}T} - 12 g_2^2 M_2^{*} \tilde{f}^{*} T^{\tilde{f}T} - 4\lambda T_\lambda^{*} \tilde{f}^{*} T^{\tilde{f}T} \\
&+ \frac{3}{10} g_1'^2 M_1'^{*} \Big[595 g_1'^2 M_1' \mathbf{1} + 8\Big(-2M_1' f^{*} f^{T} - 2M_1' \tilde{f}^{*} \tilde{f}^{T} + f^{*} T^{fT} \\
&+ \tilde{f}^{*} T^{\tilde{f}T}\Big)\Big] - \frac{12}{5} g_1^2 M_1 T^{f*} f^{T} + \frac{12}{5} g_1'^2 M_1' T^{f*} f^{T} - 12 g_2^2 M_2 T^{f*} f^{T} \\
&+ \frac{12}{5} g_1^2 T^{f*} T^{fT} - \frac{12}{5} g_1'^2 T^{f*} T^{fT} + 12 g_2^2 T^{f*} T^{fT} - 4|\lambda|^2 T^{f*} T^{fT} \\
&- \frac{12}{5} g_1^2 M_1 T^{\tilde{f}*} \tilde{f}^{T} + \frac{12}{5} g_1'^2 M_1' T^{\tilde{f}*} \tilde{f}^{T} - 12 g_2^2 M_2 T^{\tilde{f}*} \tilde{f}^{T} + \frac{12}{5} g_1^2 T^{\tilde{f}*} T^{\tilde{f}T} \\
&- \frac{12}{5} g_1'^2 T^{\tilde{f}*} T^{\tilde{f}T} + 12 g_2^2 T^{\tilde{f}*} T^{\tilde{f}T} - 4|\lambda|^2 T^{\tilde{f}*} T^{\tilde{f}T} + \frac{6}{5} g_1^2 m_{\Sigma}^2 f^{*} f^{T} \\
&- \frac{6}{5} g_1'^2 m_{\Sigma}^2 f^{*} f^{T} + 6 g_2^2 m_{\Sigma}^2 f^{*} f^{T} - 2|\lambda|^2 m_{\Sigma}^2 f^{*} f^{T} + \frac{6}{5} g_1^2 m_{\Sigma}^2 \tilde{f}^{*} \tilde{f}^{T} \\
&- \frac{6}{5} g_1'^2 m_{\Sigma}^2 \tilde{f}^{*} \tilde{f}^{T} + 6 g_2^2 m_{\Sigma}^2 \tilde{f}^{*} \tilde{f}^{T} - 2|\lambda|^2 m_{\Sigma}^2 \tilde{f}^{*} \tilde{f}^{T} + \frac{12}{5} g_1^2 f^{*} m_{H_2}^{2*} f^{T} \\
&- \frac{12}{5} g_1'^2 f^{*} m_{H_2}^{2*} f^{T} + 12 g_2^2 f^{*} m_{H_2}^{2*} f^{T} - 4|\lambda|^2 f^{*} m_{H_2}^{2*} f^{T} + \frac{6}{5} g_1^2 f^{*} f^{T} m_{\Sigma}^2 \\
&- \frac{6}{5} g_1'^2 f^{*} f^{T} m_{\Sigma}^2 + 6 g_2^2 f^{*} f^{T} m_{\Sigma}^2 - 2|\lambda|^2 f^{*} f^{T} m_{\Sigma}^2 + \frac{12}{5} g_1^2 \tilde{f}^{*} m_{H_1}^{2*} \tilde{f}^{T}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{12}{5}g_1'^2\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T+12g_2^2\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T-4|\lambda|^2\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T+\frac{6}{5}g_1^2\tilde{f}^*\tilde{f}^Tm_\Sigma^2\\
&-\frac{6}{5}g_1'^2\tilde{f}^*\tilde{f}^Tm_\Sigma^2+6g_2^2\tilde{f}^*\tilde{f}^Tm_\Sigma^2-2|\lambda|^2\tilde{f}^*\tilde{f}^Tm_\Sigma^2-4m_{H_d}^2f^*\tilde{\lambda}\tilde{\lambda}^\dagger f^T\\
&-4m_S^2f^*\tilde{\lambda}\tilde{\lambda}^\dagger f^T-4f^*\tilde{\lambda}T^{\tilde{\lambda}\dagger}T^{fT}-4f^*T^{\tilde{\lambda}}T^{\tilde{\lambda}\dagger}f^T-8m_{H_d}^2f^*f^Tf^*f^T\\
&-4f^*f^TT^{f*}T^{fT}-4f^*T^{fT}T^{f*}f^T-8m_{H_u}^2\tilde{f}^*\tilde{f}^T\tilde{f}^*\tilde{f}^T-4\tilde{f}^*\tilde{f}^TT^{\tilde{f}*}T^{\tilde{f}T}\\
&-4m_{L_4}^2\tilde{f}^*h^{ET}h^{E*}\tilde{f}^T-4m_{H_u}^2\tilde{f}^*h^{ET}h^{E*}\tilde{f}^T-4\tilde{f}^*h^{ET}T^{h^E*}T^{\tilde{f}T}\\
&-4m_{H_u}^2\tilde{f}^*\tilde{\lambda}^T\tilde{\lambda}^*\tilde{f}^T-4m_S^2\tilde{f}^*\tilde{\lambda}^T\tilde{\lambda}^*\tilde{f}^T-4\tilde{f}^*\tilde{\lambda}^TT^{\tilde{\lambda}*}T^{\tilde{f}T}-4\tilde{f}^*T^{\tilde{f}T}T^{\tilde{f}*}\tilde{f}^T\\
&-4\tilde{f}^*T^{h^ET}T^{h^E*}\tilde{f}^T-4\tilde{f}^*T^{\tilde{\lambda}T}T^{\tilde{\lambda}*}\tilde{f}^T-4T^{f*}\tilde{\lambda}\tilde{\lambda}^\dagger T^{fT}-4T^{f*}T^{\tilde{\lambda}}\tilde{\lambda}^\dagger f^T\\
&-4T^{f*}f^Tf^*T^{fT}-4T^{f*}T^{fT}f^*f^T-4T^{\tilde{f}*}\tilde{f}^T\tilde{f}^*T^{\tilde{f}T}-4T^{\tilde{f}*}h^{ET}h^{E*}T^{\tilde{f}T}\\
&-4T^{\tilde{f}*}\tilde{\lambda}^T\tilde{\lambda}^*T^{\tilde{f}T}-4T^{\tilde{f}*}T^{\tilde{f}T}\tilde{f}^*\tilde{f}^T-4T^{\tilde{f}*}T^{h^ET}h^{E*}\tilde{f}^T-4T^{\tilde{f}*}T^{\tilde{\lambda}T}\tilde{\lambda}^*\tilde{f}^T\\
&-2m_\Sigma^2f^*\tilde{\lambda}\tilde{\lambda}^\dagger f^T-2m_\Sigma^2f^*f^Tf^*f^T-2m_\Sigma^2\tilde{f}^*\tilde{f}^T\tilde{f}^*\tilde{f}^T-2m_\Sigma^2\tilde{f}^*h^{ET}h^{E*}\tilde{f}^T\\
&-2m_\Sigma^2\tilde{f}^*\tilde{\lambda}^T\tilde{\lambda}^*\tilde{f}^T-4f^*\tilde{\lambda}m_{H_1}^2\tilde{\lambda}^\dagger f^T-4f^*\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}f^T-2f^*\tilde{\lambda}\tilde{\lambda}^\dagger f^Tm_\Sigma^2\\
&-4f^*m_{H_2}^{2*}\tilde{\lambda}\tilde{\lambda}^\dagger f^T-4f^*m_{H_2}^{2*}f^Tf^*f^T-4f^*f^Tm_\Sigma^2f^*f^T-4f^*f^Tf^*m_{H_2}^{2*}f^T\\
&-2f^*f^Tf^*f^Tm_\Sigma^2-4\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T\tilde{f}^*\tilde{f}^T-4\tilde{f}^*m_{H_1}^{2*}h^{ET}h^{E*}\tilde{f}^T\\
&-4\tilde{f}^*m_{H_1}^{2*}\tilde{\lambda}^T\tilde{\lambda}^*\tilde{f}^T-4\tilde{f}^*\tilde{f}^Tm_\Sigma^2\tilde{f}^*\tilde{f}^T-4\tilde{f}^*\tilde{f}^T\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T\\
&-2\tilde{f}^*\tilde{f}^T\tilde{f}^*\tilde{f}^Tm_\Sigma^2-4\tilde{f}^*h^{ET}h^{E*}m_{H_1}^{2*}\tilde{f}^T-2\tilde{f}^*h^{ET}h^{E*}\tilde{f}^Tm_\Sigma^2\\
&-4\tilde{f}^*h^{ET}m_{e^c}^{2*}h^{E*}\tilde{f}^T-4\tilde{f}^*\tilde{\lambda}^Tm_{H_2}^2\tilde{\lambda}^*\tilde{f}^T-4\tilde{f}^*\tilde{\lambda}^T\tilde{\lambda}^*m_{H_1}^{2*}\tilde{f}^T\\
&-2\tilde{f}^*\tilde{\lambda}^T\tilde{\lambda}^*\tilde{f}^Tm_\Sigma^2-4\lambda^*T^{f*}f^TT_\lambda-4\lambda^*T^{\tilde{f}*}\tilde{f}^TT_\lambda+g_1'\mathbf{1}\Big(2\sqrt{10}\Sigma_{3,4}\\
&+5g_1'\Sigma_{2,44}\Big)-8m_{H_d}^2f^*f^T\,\mathrm{Tr}\Big(ff^\dagger\Big)-4T^{f*}T^{fT}\,\mathrm{Tr}\Big(ff^\dagger\Big)\\
&-2m_\Sigma^2f^*f^T\,\mathrm{Tr}\Big(ff^\dagger\Big)-4f^*m_{H_2}^{2*}f^T\,\mathrm{Tr}\Big(ff^\dagger\Big)-2f^*f^Tm_\Sigma^2\,\mathrm{Tr}\Big(ff^\dagger\Big)\\
&-8m_{H_u}^2\tilde{f}^*\tilde{f}^T\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-4T^{\tilde{f}*}T^{\tilde{f}T}\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-2m_\Sigma^2\tilde{f}^*\tilde{f}^T\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)\\
&-4\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-2\tilde{f}^*\tilde{f}^Tm_\Sigma^2\,\mathrm{Tr}\Big(\tilde{f}\tilde{f}^\dagger\Big)-24m_{H_d}^2f^*f^T\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&-12T^{f*}T^{fT}\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-6m_\Sigma^2f^*f^T\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-12f^*m_{H_2}^{2*}f^T\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)\\
&-6f^*f^Tm_\Sigma^2\,\mathrm{Tr}\Big(y^Dy^{D\dagger}\Big)-8m_{H_d}^2f^*f^T\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-4T^{f*}T^{fT}\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)\\
&-2m_\Sigma^2f^*f^T\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-4f^*m_{H_2}^{2*}f^T\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)-2f^*f^Tm_\Sigma^2\,\mathrm{Tr}\Big(y^Ey^{E\dagger}\Big)\\
&-24m_{H_u}^2\tilde{f}^*\tilde{f}^T\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-12T^{\tilde{f}*}T^{\tilde{f}T}\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-6m_\Sigma^2\tilde{f}^*\tilde{f}^T\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)\\
&-12\tilde{f}^*m_{H_1}^{2*}\tilde{f}^T\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-6\tilde{f}^*\tilde{f}^Tm_\Sigma^2\,\mathrm{Tr}\Big(y^Uy^{U\dagger}\Big)-4T^{f*}f^T\,\mathrm{Tr}\Big(f^\dagger T^f\Big)\\
&-4T^{\tilde{f}*}\tilde{f}^T\,\mathrm{Tr}\Big(\tilde{f}^\dagger T^{\tilde{f}}\Big)-12T^{f*}f^T\,\mathrm{Tr}\Big(y^{D\dagger}T^D\Big)-4T^{f*}f^T\,\mathrm{Tr}\Big(y^{E\dagger}T^E\Big)\\
&-12T^{\tilde{f}*}\tilde{f}^T\,\mathrm{Tr}\Big(y^{U\dagger}T^U\Big)-4f^*T^{fT}\,\mathrm{Tr}\Big(T^{f*}f^T\Big)-4f^*f^T\,\mathrm{Tr}\Big(T^{f*}T^{fT}\Big)\\
&-4\tilde{f}^*T^{\tilde{f}T}\,\mathrm{Tr}\Big(T^{\tilde{f}*}\tilde{f}^T\Big)-4\tilde{f}^*\tilde{f}^T\,\mathrm{Tr}\Big(T^{\tilde{f}*}T^{\tilde{f}T}\Big)-12f^*T^{fT}\,\mathrm{Tr}\Big(T^{D*}y^{DT}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 12f^* f^T \,\mathrm{Tr}\Big(T^{D*} T^{DT}\Big) - 4 f^* T^{fT} \,\mathrm{Tr}\Big(T^{E*} y^{ET}\Big) - 4 f^* f^T \,\mathrm{Tr}\Big(T^{E*} T^{ET}\Big) \\
&- 12 \tilde{f}^* T^{\tilde{f}T} \,\mathrm{Tr}\Big(T^{U*} y^{UT}\Big) - 12 \tilde{f}^* \tilde{f}^T \,\mathrm{Tr}\Big(T^{U*} T^{UT}\Big) - 4 f^* f^T \,\mathrm{Tr}\Big(f m_{H_2}^2 f^\dagger\Big) \\
&- 4 f^* f^T \,\mathrm{Tr}\Big(f f^\dagger m_\Sigma^{2*}\Big) - 4 \tilde{f}^* \tilde{f}^T \,\mathrm{Tr}\Big(\tilde{f} m_{H_1}^2 \tilde{f}^\dagger\Big) - 4 \tilde{f}^* \tilde{f}^T \,\mathrm{Tr}\Big(\tilde{f} \tilde{f}^\dagger m_\Sigma^{2*}\Big) \\
&- 12 f^* f^T \,\mathrm{Tr}\Big(m_{d^c}^2 y^D y^{D\dagger}\Big) - 4 f^* f^T \,\mathrm{Tr}\Big(m_{e^c}^2 y^E y^{E\dagger}\Big) \\
&- 4 f^* f^T \,\mathrm{Tr}\Big(m_L^2 y^{E\dagger} y^E\Big) - 12 f^* f^T \,\mathrm{Tr}\Big(m_Q^2 y^{D\dagger} y^D\Big) \\
&- 12 \tilde{f}^* \tilde{f}^T \,\mathrm{Tr}\Big(m_Q^2 y^{U\dagger} y^U\Big) - 12 \tilde{f}^* \tilde{f}^T \,\mathrm{Tr}\Big(m_{u^c}^2 y^U y^{U\dagger}\Big), \qquad \text{(A.117)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_D^2}^{(1)} = &- \frac{8}{15} g_1^2 \mathbf{1} |M_1|^2 - \frac{4}{5} g_1'^2 \mathbf{1} |M_1'|^2 - \frac{32}{3} g_3^2 \mathbf{1} |M_3|^2 + 2 m_S^2 \kappa^* \kappa^T + 2 T^{\kappa *} T^{\kappa T} \\
&+ m_D^2 \kappa^* \kappa^T + 2 \kappa^* m_{\bar{D}}^2 \kappa^T + \kappa^* \kappa^T m_D^2 - \frac{2}{\sqrt{15}} g_1 \mathbf{1} \Sigma_{1,1} - \sqrt{\frac{2}{5}} g_1' \mathbf{1} \Sigma_{1,4}, \qquad \text{(A.118)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_D^2}^{(2)} = &\frac{128}{45} g_1^2 g_3^2 \mathbf{1} |M_3|^2 + \frac{64}{15} g_1'^2 g_3^2 \mathbf{1} |M_3|^2 + \frac{160}{3} g_3^4 \mathbf{1} |M_3|^2 \\
&+ \frac{16}{225} g_1^2 \Big[20 g_3^2 \Big(2M_1 + M_3\Big) + 219 g_1^2 M_1 - 3 g_1'^2 \Big(2M_1 + M_1'\Big)\Big] \mathbf{1} M_1^* \\
&+ \frac{64}{45} g_1^2 g_3^2 M_1 \mathbf{1} M_3^* + \frac{32}{15} g_1'^2 g_3^2 M_1' \mathbf{1} M_3^* + 3 g_1'^2 m_S^2 \kappa^* \kappa^T - 4 m_{H_d}^2 |\lambda|^2 \kappa^* \kappa^T \\
&- 4 m_{H_u}^2 |\lambda|^2 \kappa^* \kappa^T - 8 m_S^2 |\lambda|^2 \kappa^* \kappa^T - 2 m_\phi^2 |\sigma|^2 \kappa^* \kappa^T - 4 m_S^2 |\sigma|^2 \kappa^* \kappa^T \\
&- 2 m_{\bar{S}}^2 |\sigma|^2 \kappa^* \kappa^T - 4 |T_\lambda|^2 \kappa^* \kappa^T - 2 |T_\sigma|^2 \kappa^* \kappa^T \\
&+ \frac{1}{75} g_1'^2 M_1'^* \Big\{ \Big[160 g_3^2 \Big(2M_1' + M_3\Big) - 16 g_1^2 \Big(2M_1' + M_1\Big) + 1953 g_1'^2 M_1' \Big] \mathbf{1} \\
&+ 225 \Big(2 M_1' \kappa^* \kappa^T - \kappa^* T^{\kappa T}\Big)\Big\} - 4 \lambda T_\lambda^* \kappa^* T^{\kappa T} - 2 \sigma T_\sigma^* \kappa^* T^{\kappa T} \\
&- 3 g_1'^2 M_1' T^{\kappa *} \kappa^T + 3 g_1'^2 T^{\kappa *} T^{\kappa T} - 4 |\lambda|^2 T^{\kappa *} T^{\kappa T} - 2 |\sigma|^2 T^{\kappa *} T^{\kappa T} \\
&+ \frac{3}{2} g_1'^2 m_D^2 \kappa^* \kappa^T - 2 |\lambda|^2 m_D^2 \kappa^* \kappa^T - |\sigma|^2 m_D^2 \kappa^* \kappa^T + 3 g_1'^2 \kappa^* m_{\bar{D}}^2 \kappa^T \\
&- 4 |\lambda|^2 \kappa^* m_{\bar{D}}^2 \kappa^T - 2 |\sigma|^2 \kappa^* m_{\bar{D}}^2 \kappa^T + \frac{3}{2} g_1'^2 \kappa^* \kappa^T m_D^2 - 2 |\lambda|^2 \kappa^* \kappa^T m_D^2 \\
&- |\sigma|^2 \kappa^* \kappa^T m_D^2 - 4 m_{L_4}^2 \kappa^* g^{DT} g^{D*} \kappa^T - 4 m_S^2 \kappa^* g^{DT} g^{D*} \kappa^T \\
&- 4 \kappa^* g^{DT} T^{g^D *} T^{\kappa T} - 4 m_S^2 \kappa^* \kappa^T \kappa^* \kappa^T - 2 \kappa^* \kappa^T T^{\kappa *} T^{\kappa T} \\
&- 4 \kappa^* T^{g^D T} T^{g^D *} \kappa^T - 2 \kappa^* T^{\kappa T} T^{\kappa *} \kappa^T - 4 T^{\kappa *} g^{DT} g^{D*} T^{\kappa T} \\
&- 2 T^{\kappa *} \kappa^T \kappa^* T^{\kappa T} - 4 T^{\kappa *} T^{g^D T} g^{D*} \kappa^T - 2 T^{\kappa *} T^{\kappa T} \kappa^* \kappa^T \\
&- 2 m_D^2 \kappa^* g^{DT} g^{D*} \kappa^T - m_D^2 \kappa^* \kappa^T \kappa^* \kappa^T - 4 \kappa^* m_{\bar{D}}^2 g^{DT} g^{D*} \kappa^T \\
&- 2 \kappa^* m_{\bar{D}}^2 \kappa^T \kappa^* \kappa^T - 4 \kappa^* g^{DT} m_Q^2 g^{D*} \kappa^T - 4 \kappa^* g^{DT} g^{D*} m_{\bar{D}}^2 \kappa^T \\
&- 2 \kappa^* g^{DT} g^{D*} \kappa^T m_D^2 - 2 \kappa^* \kappa^T m_D^2 \kappa^* \kappa^T - 2 \kappa^* \kappa^T \kappa^* m_{\bar{D}}^2 \kappa^T \\
&- \kappa^* \kappa^T \kappa^* \kappa^T m_D^2 - 4 \lambda^* T^{\kappa *} \kappa^T T_\lambda - 2 \sigma^* T^{\kappa *} \kappa^T T_\sigma + \frac{32}{3} g_3^4 \mathbf{1} \Sigma_{2,3} \\
&+ \frac{8}{15} g_1^2 \mathbf{1} \Sigma_{2,11} + \frac{4}{5} \sqrt{\frac{2}{3}} g_1 g_1' \mathbf{1} \Sigma_{2,14} + \frac{4}{5} \sqrt{\frac{2}{3}} g_1 g_1' \mathbf{1} \Sigma_{2,41} + \frac{4}{5} g_1'^2 \mathbf{1} \Sigma_{2,44}
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{8}{\sqrt{15}} g_1 \mathbf{1}\Sigma_{3,1} - 4\sqrt{\frac{2}{5}} g_1' \mathbf{1}\Sigma_{3,4} - 12 m_S^2 \kappa^* \kappa^T \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 6 T^{\kappa *} T^{\kappa T} \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 3 m_D^2 \kappa^* \kappa^T \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 6\kappa^* m_{\bar{D}}^2 \kappa^T \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 3\kappa^*\kappa^T m_D^2 \operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 8 m_S^2 \kappa^*\kappa^T \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 4 T^{\kappa *} T^{\kappa T} \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 2 m_D^2 \kappa^*\kappa^T \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 4\kappa^* m_{\bar{D}}^2 \kappa^T \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 2\kappa^*\kappa^T m_D^2 \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 6 T^{\kappa *}\kappa^T \operatorname{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) - 4 T^{\kappa *}\kappa^T \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - 6\kappa^* T^{\kappa T} \operatorname{Tr}\Big(T^{\kappa *}\kappa^T\Big) \\
&- 6\kappa^*\kappa^T \operatorname{Tr}\Big(T^{\kappa *} T^{\kappa T}\Big) - 4\kappa^* T^{\kappa T} \operatorname{Tr}\Big(T^{\tilde{\lambda} *}\tilde{\lambda}^T\Big) - 4\kappa^*\kappa^T \operatorname{Tr}\Big(T^{\tilde{\lambda} *} T^{\tilde{\lambda} T}\Big) \\
&- 4\kappa^*\kappa^T \operatorname{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\Big) - 6\kappa^*\kappa^T \operatorname{Tr}\Big(\kappa\kappa^\dagger m_D^{2*}\Big) - 6\kappa^*\kappa^T \operatorname{Tr}\Big(\kappa m_{\bar{D}}^{2*}\kappa^\dagger\Big) \\
&- 4\kappa^*\kappa^T \operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\Big),
\end{aligned}
\tag{A.119}
$$

$$
\begin{aligned}
\beta_{m_{\bar{D}}^2}^{(1)} = &- \frac{8}{15} g_1^2 \mathbf{1}|M_1|^2 - \frac{9}{5} g_1'^2 \mathbf{1}|M_1'|^2 - \frac{32}{3} g_3^2 \mathbf{1}|M_3|^2 + 4 m_{L_4}^2 g^{DT} g^{D*} \\
&+ 2 m_S^2 \kappa^T \kappa^* + 4 T^{g^D T} T^{g^D *} + 2 T^{\kappa T} T^{\kappa *} + 2 m_{\bar{D}}^2 g^{DT} g^{D*} + m_{\bar{D}}^2 \kappa^T \kappa^* \\
&+ 4 g^{DT} m_Q^2 g^{D*} + 2 g^{DT} g^{D*} m_{\bar{D}}^2 + 2\kappa^T m_D^2 \kappa^* + \kappa^T \kappa^* m_{\bar{D}}^2 \\
&+ \frac{2}{\sqrt{15}} g_1 \mathbf{1}\Sigma_{1,1} - \frac{3}{\sqrt{10}} g_1' \mathbf{1}\Sigma_{1,4},
\end{aligned}
\tag{A.120}
$$

$$
\begin{aligned}
\beta_{m_{\bar{D}}^2}^{(2)} = &\frac{128}{45} g_1^2 g_3^2 \mathbf{1}|M_3|^2 + \frac{48}{5} g_1'^2 g_3^2 \mathbf{1}|M_3|^2 + \frac{160}{3} g_3^4 \mathbf{1}|M_3|^2 + \frac{64}{45} g_1^2 g_3^2 M_1 \mathbf{1} M_3^* \\
&+ \frac{24}{5} g_1'^2 g_3^2 M_1' \mathbf{1} M_3^* + \frac{4}{5} g_1^2 m_{L_4}^2 g^{DT} g^{D*} - \frac{4}{5} g_1'^2 m_{L_4}^2 g^{DT} g^{D*} \\
&+ 12 g_2^2 m_{L_4}^2 g^{DT} g^{D*} + 24 g_2^2 |M_2|^2 g^{DT} g^{D*} - 8 m_{L_4}^2 |\tilde{\sigma}|^2 g^{DT} g^{D*} \\
&- 4 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 g^{DT} g^{D*} - 4 m_\phi^2 |\tilde{\sigma}|^2 g^{DT} g^{D*} - 4|T_{\tilde{\sigma}}|^2 g^{DT} g^{D*} - \frac{4}{5} g_1^2 M_1 g^{DT} T^{g^D *} \\
&+ \frac{4}{5} g_1'^2 M_1' g^{DT} T^{g^D *} - 12 g_2^2 M_2 g^{DT} T^{g^D *} + 2 g_1'^2 m_S^2 \kappa^T \kappa^* - 4 m_{H_d}^2 |\lambda|^2 \kappa^T \kappa^* \\
&- 4 m_{H_u}^2 |\lambda|^2 \kappa^T \kappa^* - 8 m_S^2 |\lambda|^2 \kappa^T \kappa^* - 2 m_\phi^2 |\sigma|^2 \kappa^T \kappa^* - 4 m_S^2 |\sigma|^2 \kappa^T \kappa^* \\
&- 2 m_{\bar{S}}^2 |\sigma|^2 \kappa^T \kappa^* - 4|T_\lambda|^2 \kappa^T \kappa^* - 2|T_\sigma|^2 \kappa^T \kappa^* - 2 g_1'^2 M_1' \kappa^T T^{\kappa *} \\
&+ \frac{2}{225} g_1^2 M_1^* \Big\{ \Big[160 g_3^2 \Big(2M_1 + M_3\Big) + 1752 g_1^2 M_1 + 81 g_1'^2 \Big(2M_1 + M_1'\Big) \Big] \mathbf{1} \\
&+ 90 \Big(2M_1 g^{DT} g^{D*} - T^{g^D T} g^{D*} \Big) \Big\} - 12 g_2^2 M_2^* T^{g^D T} g^{D*} - 4\tilde{\sigma} T_{\tilde{\sigma}}^* T^{g^D T} g^{D*} \\
&+ \frac{4}{5} g_1^2 T^{g^D T} T^{g^D *} - \frac{4}{5} g_1'^2 T^{g^D T} T^{g^D *} + 12 g_2^2 T^{g^D T} T^{g^D *} - 4|\tilde{\sigma}|^2 T^{g^D T} T^{g^D *} \\
&- 4\lambda T_\lambda^* T^{\kappa T} \kappa^* - 2\sigma T_\sigma^* T^{\kappa T} \kappa^* + \frac{1}{50} g_1'^2 M_1'^* \Big\{ 3 \Big[12 g_1^2 \Big(2M_1' + M_1\Big) \\
&+ 80 g_3^2 \Big(2M_1' + M_3\Big) + 999 g_1'^2 M_1' \Big] \mathbf{1} - 20 \Big(-10 M_1' \kappa^T \kappa^* - 2 T^{g^D T} g^{D*} \\
&+ 4 M_1' g^{DT} g^{D*} + 5 T^{\kappa T} \kappa^* \Big) \Big\} + 2 g_1'^2 T^{\kappa T} T^{\kappa *} - 4|\lambda|^2 T^{\kappa T} T^{\kappa *} \\
&- 2|\sigma|^2 T^{\kappa T} T^{\kappa *} + \frac{2}{5} g_1^2 m_{\bar{D}}^2 g^{DT} g^{D*} - \frac{2}{5} g_1'^2 m_{\bar{D}}^2 g^{DT} g^{D*} + 6 g_2^2 m_{\bar{D}}^2 g^{DT} g^{D*} \\
&- 2|\tilde{\sigma}|^2 m_{\bar{D}}^2 g^{DT} g^{D*} + g_1'^2 m_{\bar{D}}^2 \kappa^T \kappa^* - 2|\lambda|^2 m_{\bar{D}}^2 \kappa^T \kappa^* - |\sigma|^2 m_{\bar{D}}^2 \kappa^T \kappa^*
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{4}{5}g_1^2 g^{DT} m_Q^2 g^{D*} - \frac{4}{5}g_1'^2 g^{DT} m_Q^2 g^{D*} + 12 g_2^2 g^{DT} m_Q^2 g^{D*} - 4|\tilde{\sigma}|^2 g^{DT} m_Q^2 g^{D*} \\
&+\frac{2}{5}g_1^2 g^{DT} g^{D*} m_{\bar{D}}^2 - \frac{2}{5}g_1'^2 g^{DT} g^{D*} m_{\bar{D}}^2 + 6 g_2^2 g^{DT} g^{D*} m_{\bar{D}}^2 - 2|\tilde{\sigma}|^2 g^{DT} g^{D*} m_{\bar{D}}^2 \\
&+ 2 g_1'^2 \kappa^T m_D^2 \kappa^* - 4|\lambda|^2 \kappa^T m_D^2 \kappa^* - 2|\sigma|^2 \kappa^T m_D^2 \kappa^* + g_1'^2 \kappa^T \kappa^* m_{\bar{D}}^2 \\
&- 2|\lambda|^2 \kappa^T \kappa^* m_{\bar{D}}^2 - |\sigma|^2 \kappa^T \kappa^* m_{\bar{D}}^2 - 4 m_{H_d}^2 g^{DT} y^{D\dagger} y^D g^{D*} \\
&- 4 m_{L_4}^2 g^{DT} y^{D\dagger} y^D g^{D*} - 4 g^{DT} y^{D\dagger} T^D T^{g^D *} - 4 m_{L_4}^2 g^{DT} y^{U\dagger} y^U g^{D*} \\
&- 4 m_{H_u}^2 g^{DT} y^{U\dagger} y^U g^{D*} - 4 g^{DT} y^{U\dagger} T^U T^{g^D *} - 4 g^{DT} T^{D\dagger} T^D g^{D*} \\
&- 4 g^{DT} T^{U\dagger} T^U g^{D*} - 8 m_{L_4}^2 g^{DT} g^{D*} g^{DT} g^{D*} - 4 g^{DT} g^{D*} T^{g^D T} T^{g^D *} \\
&- 4 g^{DT} T^{g^D *} T^{g^D T} g^{D*} - 4 m_S^2 \kappa^T \kappa^* \kappa^T \kappa^* - 2 \kappa^T \kappa^* T^{\kappa T} T^{\kappa *} - 2 \kappa^T T^{\kappa *} T^{\kappa T} \kappa^* \\
&- 4 T^{g^D T} y^{D\dagger} y^D T^{g^D *} - 4 T^{g^D T} y^{U\dagger} y^U T^{g^D *} - 4 T^{g^D T} T^{D\dagger} y^D g^{D*} \\
&- 4 T^{g^D T} T^{U\dagger} y^U g^{D*} - 4 T^{g^D T} g^{D*} g^{DT} T^{g^D *} - 4 T^{g^D T} T^{g^D *} g^{DT} g^{D*} \\
&- 2 T^{\kappa T} \kappa^* \kappa^T T^{\kappa *} - 2 T^{\kappa T} T^{\kappa *} \kappa^T \kappa^* - 2 m_{\bar{D}}^2 g^{DT} y^{D\dagger} y^D g^{D*} \\
&- 2 m_{\bar{D}}^2 g^{DT} y^{U\dagger} y^U g^{D*} - 2 m_{\bar{D}}^2 g^{DT} g^{D*} g^{DT} g^{D*} - m_{\bar{D}}^2 \kappa^T \kappa^* \kappa^T \kappa^* \\
&- 4 g^{DT} m_Q^2 y^{D\dagger} y^D g^{D*} - 4 g^{DT} m_Q^2 y^{U\dagger} y^U g^{D*} - 4 g^{DT} m_Q^2 g^{D*} g^{DT} g^{D*} \\
&- 4 g^{DT} y^{D\dagger} m_{d^c}^2 y^D g^{D*} - 4 g^{DT} y^{D\dagger} y^D m_Q^2 g^{D*} - 2 g^{DT} y^{D\dagger} y^D g^{D*} m_{\bar{D}}^2 \\
&- 4 g^{DT} y^{U\dagger} m_{u^c}^2 y^U g^{D*} - 4 g^{DT} y^{U\dagger} y^U m_Q^2 g^{D*} - 2 g^{DT} y^{U\dagger} y^U g^{D*} m_{\bar{D}}^2 \\
&- 4 g^{DT} g^{D*} m_{\bar{D}}^2 g^{DT} g^{D*} - 4 g^{DT} g^{D*} g^{DT} m_Q^2 g^{D*} - 2 g^{DT} g^{D*} g^{DT} g^{D*} m_{\bar{D}}^2 \\
&- 2 \kappa^T m_D^2 \kappa^* \kappa^T \kappa^* - 2 \kappa^T \kappa^* m_{\bar{D}}^2 \kappa^T \kappa^* - 2 \kappa^T \kappa^* \kappa^T m_D^2 \kappa^* - \kappa^T \kappa^* \kappa^T \kappa^* m_{\bar{D}}^2 \\
&- 4 \lambda^* \kappa^T T^{\kappa *} T_\lambda - 2 \sigma^* \kappa^T T^{\kappa *} T_\sigma - 4 \tilde{\sigma}^* g^{DT} T^{g^D *} T_{\tilde{\sigma}} + \frac{32}{3} g_3^4 \mathbf{1} \Sigma_{2,3} \\
&+ \frac{8}{15} g_1^2 \mathbf{1} \Sigma_{2,11} - \frac{2}{5}\sqrt{6} g_1 g_1' \mathbf{1} \Sigma_{2,14} - \frac{2}{5}\sqrt{6} g_1 g_1' \mathbf{1} \Sigma_{2,41} + \frac{9}{5} g_1'^2 \mathbf{1} \Sigma_{2,44} \\
&+ \frac{8}{\sqrt{15}} g_1 \mathbf{1} \Sigma_{3,1} - 6\sqrt{\frac{2}{5}} g_1' \mathbf{1} \Sigma_{3,4} - 24 m_{L_4}^2 g^{DT} g^{D*} \operatorname{Tr}\left(g^D g^{D\dagger}\right) \\
&- 12 T^{g^D T} T^{g^D *} \operatorname{Tr}\left(g^D g^{D\dagger}\right) - 6 m_{\bar{D}}^2 g^{DT} g^{D*} \operatorname{Tr}\left(g^D g^{D\dagger}\right) \\
&- 12 g^{DT} m_Q^2 g^{D*} \operatorname{Tr}\left(g^D g^{D\dagger}\right) - 6 g^{DT} g^{D*} m_{\bar{D}}^2 \operatorname{Tr}\left(g^D g^{D\dagger}\right) \\
&- 8 m_{L_4}^2 g^{DT} g^{D*} \operatorname{Tr}\left(h^E h^{E\dagger}\right) - 4 T^{g^D T} T^{g^D *} \operatorname{Tr}\left(h^E h^{E\dagger}\right) \\
&- 2 m_{\bar{D}}^2 g^{DT} g^{D*} \operatorname{Tr}\left(h^E h^{E\dagger}\right) - 4 g^{DT} m_Q^2 g^{D*} \operatorname{Tr}\left(h^E h^{E\dagger}\right) \\
&- 2 g^{DT} g^{D*} m_{\bar{D}}^2 \operatorname{Tr}\left(h^E h^{E\dagger}\right) - 12 m_S^2 \kappa^T \kappa^* \operatorname{Tr}\left(\kappa \kappa^\dagger\right) \\
&- 6 T^{\kappa T} T^{\kappa *} \operatorname{Tr}\left(\kappa \kappa^\dagger\right) - 3 m_{\bar{D}}^2 \kappa^T \kappa^* \operatorname{Tr}\left(\kappa \kappa^\dagger\right) - 6 \kappa^T m_D^2 \kappa^* \operatorname{Tr}\left(\kappa \kappa^\dagger\right) \\
&- 3 \kappa^T \kappa^* m_{\bar{D}}^2 \operatorname{Tr}\left(\kappa \kappa^\dagger\right) - 8 m_S^2 \kappa^T \kappa^* \operatorname{Tr}\left(\tilde{\lambda} \tilde{\lambda}^\dagger\right) - 4 T^{\kappa T} T^{\kappa *} \operatorname{Tr}\left(\tilde{\lambda} \tilde{\lambda}^\dagger\right) \\
&- 2 m_{\bar{D}}^2 \kappa^T \kappa^* \operatorname{Tr}\left(\tilde{\lambda} \tilde{\lambda}^\dagger\right) - 4 \kappa^T m_D^2 \kappa^* \operatorname{Tr}\left(\tilde{\lambda} \tilde{\lambda}^\dagger\right) - 2 \kappa^T \kappa^* m_{\bar{D}}^2 \operatorname{Tr}\left(\tilde{\lambda} \tilde{\lambda}^\dagger\right) \\
&- 12 g^{DT} T^{g^D *} \operatorname{Tr}\left(g^{D\dagger} T^{g^D}\right) - 4 g^{DT} T^{g^D *} \operatorname{Tr}\left(h^{E\dagger} T^{h^E}\right)
\end{aligned}
$$

$$
\begin{aligned}
&- 6\kappa^T T^{\kappa *} \operatorname{Tr}\Big(\kappa^\dagger T^{\kappa}\Big) - 4\kappa^T T^{\kappa *} \operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) - 12 T^{g^D T} g^{D*} \operatorname{Tr}\Big(T^{g^D *} g^{DT}\Big) \\
&- 12 g^{DT} g^{D*} \operatorname{Tr}\Big(T^{g^D *} T^{g^D T}\Big) - 4 T^{g^D T} g^{D*} \operatorname{Tr}\Big(T^{h^E *} h^{ET}\Big) \\
&- 4 g^{DT} g^{D*} \operatorname{Tr}\Big(T^{h^E *} T^{h^E T}\Big) - 6 T^{\kappa T} \kappa^* \operatorname{Tr}\Big(T^{\kappa *} \kappa^T\Big) \\
&- 6\kappa^T \kappa^* \operatorname{Tr}\Big(T^{\kappa *} T^{\kappa T}\Big) - 4 T^{\kappa T} \kappa^* \operatorname{Tr}\Big(T^{\tilde{\lambda} *} \tilde{\lambda}^T\Big) - 4\kappa^T \kappa^* \operatorname{Tr}\Big(T^{\tilde{\lambda} *} T^{\tilde{\lambda} T}\Big) \\
&- 12 g^{DT} g^{D*} \operatorname{Tr}\Big(g^D g^{D\dagger} m_Q^{2*}\Big) - 12 g^{DT} g^{D*} \operatorname{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger}\Big) \\
&- 4 g^{DT} g^{D*} \operatorname{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) - 4 g^{DT} g^{D*} \operatorname{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2\Big) \\
&- 4\kappa^T \kappa^* \operatorname{Tr}\Big(m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda}\Big) - 6\kappa^T \kappa^* \operatorname{Tr}\Big(\kappa \kappa^\dagger m_D^{2*}\Big) \\
&- 6\kappa^T \kappa^* \operatorname{Tr}\Big(\kappa m_{\bar{D}}^{2*} \kappa^\dagger\Big) - 4\kappa^T \kappa^* \operatorname{Tr}\Big(\tilde{\lambda} \tilde{\lambda}^\dagger m_{H_2}^{2*}\Big),
\end{aligned}
\tag{A.121}
$$

$$
\begin{aligned}
\beta_{m_{L_4}^2}^{(1)} ={}& -\frac{6}{5} g_1^2 |M_1|^2 - \frac{4}{5} g_1'^2 |M_1'|^2 - 6 g_2^2 |M_2|^2 + 2 m_{L_4}^2 |\tilde{\sigma}|^2 + 2 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 \\
&+ 2 m_\phi^2 |\tilde{\sigma}|^2 + 2 |T_{\tilde{\sigma}}|^2 - \sqrt{\frac{3}{5}} g_1 \Sigma_{1,1} + \sqrt{\frac{2}{5}} g_1' \Sigma_{1,4} + 6 m_{L_4}^2 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ 2 m_{L_4}^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + 6 \operatorname{Tr}\Big(T^{g^D *} T^{g^D T}\Big) + 2 \operatorname{Tr}\Big(T^{h^E *} T^{h^E T}\Big) \\
&+ 6 \operatorname{Tr}\Big(g^D g^{D\dagger} m_Q^{2*}\Big) + 6 \operatorname{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger}\Big) + 2 \operatorname{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) \\
&+ 2 \operatorname{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2\Big),
\end{aligned}
\tag{A.122}
$$

$$
\begin{aligned}
\beta_{m_{L_4}^2}^{(2)} ={}& \frac{18}{5} g_1^2 g_2^2 |M_2|^2 + \frac{12}{5} g_1'^2 g_2^2 |M_2|^2 + 87 g_2^4 |M_2|^2 + \frac{9}{5} g_1^2 g_2^2 M_1 M_2^* \\
&+ \frac{6}{5} g_1'^2 g_2^2 M_1' M_2^* - 4 m_{L_4}^2 |\tilde{\sigma}|^2 |\kappa_\phi|^2 - 4 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 |\kappa_\phi|^2 - 16 m_\phi^2 |\tilde{\sigma}|^2 |\kappa_\phi|^2 \\
&- 2 m_{L_4}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 2 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 4 m_\phi^2 |\tilde{\sigma}|^2 |\sigma|^2 - 2 m_S^2 |\tilde{\sigma}|^2 |\sigma|^2 \\
&- 2 m_{\bar{S}}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 12 m_{L_4}^2 |\tilde{\sigma}|^4 - 12 m_{\bar{L}_4}^2 |\tilde{\sigma}|^4 - 12 m_\phi^2 |\tilde{\sigma}|^4 - 4 |\tilde{\sigma}|^2 |T_{\kappa_\phi}|^2 \\
&- 4 \tilde{\sigma} \kappa_\phi^* T_{\tilde{\sigma}}^* T_{\kappa_\phi} - 2 |\tilde{\sigma}|^2 |T_\sigma|^2 - 2 \tilde{\sigma} \sigma^* T_{\tilde{\sigma}}^* T_\sigma - 4 \kappa_\phi \tilde{\sigma}^* T_{\kappa_\phi}^* T_{\tilde{\sigma}} - 2 \sigma \tilde{\sigma}^* T_\sigma^* T_{\tilde{\sigma}} \\
&- 4 |\kappa_\phi|^2 |T_{\tilde{\sigma}}|^2 - 2 |\sigma|^2 |T_{\tilde{\sigma}}|^2 - 24 |\tilde{\sigma}|^2 |T_{\tilde{\sigma}}|^2 + 6 g_2^4 \Sigma_{2,2} + \frac{6}{5} g_1^2 \Sigma_{2,11} \\
&- \frac{2}{5} \sqrt{6} g_1 g_1' \Sigma_{2,14} - \frac{2}{5} \sqrt{6} g_1 g_1' \Sigma_{2,41} + \frac{4}{5} g_1'^2 \Sigma_{2,44} - 4 \sqrt{\frac{3}{5}} g_1 \Sigma_{3,1} + 4 \sqrt{\frac{2}{5}} g_1' \Sigma_{3,4} \\
&- \frac{4}{5} g_1^2 m_{L_4}^2 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + \frac{9}{5} g_1'^2 m_{L_4}^2 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 32 g_3^2 m_{L_4}^2 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&+ 64 g_3^2 |M_3|^2 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + \frac{12}{5} g_1^2 m_{L_4}^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) + \frac{3}{5} g_1'^2 m_{L_4}^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 32 g_3^2 M_3^* \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + \frac{1}{25} g_1^2 M_1^* \Big[891 g_1^2 M_1 + 36 g_1'^2 M_1 + 90 g_2^2 M_1 \\
&+ 18 g_1'^2 M_1' + 45 g_2^2 M_2 - 40 M_1 \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 120 M_1 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&+ 20 \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 60 \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big)\Big] + \frac{3}{25} g_1'^2 M_1'^* \Big[6 g_1^2 M_1 + 12 g_1^2 M_1' \\
&+ 217 g_1'^2 M_1' + 20 g_2^2 M_1' + 10 g_2^2 M_2 + 30 M_1' \operatorname{Tr}\Big(g^D g^{D\dagger}\Big) + 10 M_1' \operatorname{Tr}\Big(h^E h^{E\dagger}\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&- 15\,\mathrm{Tr}\Big(g^{D\dagger}T^{g^D}\Big) - 5\,\mathrm{Tr}\Big(h^{E\dagger}T^{h^E}\Big)\Big] + \frac{4}{5}g_1^2 M_1\,\mathrm{Tr}\Big(T^{g^D*}g^{DT}\Big) \\
&- \frac{9}{5}g_1'^2 M_1'\,\mathrm{Tr}\Big(T^{g^D*}g^{DT}\Big) - 32g_3^2 M_3\,\mathrm{Tr}\Big(T^{g^D*}g^{DT}\Big) - \frac{4}{5}g_1^2\,\mathrm{Tr}\Big(T^{g^D*}T^{g^D T}\Big) \\
&+ \frac{9}{5}g_1'^2\,\mathrm{Tr}\Big(T^{g^D*}T^{g^D T}\Big) + 32g_3^2\,\mathrm{Tr}\Big(T^{g^D*}T^{g^D T}\Big) - \frac{12}{5}g_1^2 M_1\,\mathrm{Tr}\Big(T^{h^E*}h^{ET}\Big) \\
&- \frac{3}{5}g_1'^2 M_1'\,\mathrm{Tr}\Big(T^{h^E*}h^{ET}\Big) + \frac{12}{5}g_1^2\,\mathrm{Tr}\Big(T^{h^E*}T^{h^E T}\Big) + \frac{3}{5}g_1'^2\,\mathrm{Tr}\Big(T^{h^E*}T^{h^E T}\Big) \\
&- \frac{4}{5}g_1^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}m_Q^{2*}\Big) + \frac{9}{5}g_1'^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}m_Q^{2*}\Big) + 32g_3^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}m_Q^{2*}\Big) \\
&- \frac{4}{5}g_1^2\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*}g^{D\dagger}\Big) + \frac{9}{5}g_1'^2\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*}g^{D\dagger}\Big) + 32g_3^2\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*}g^{D\dagger}\Big) \\
&+ \frac{12}{5}g_1^2\,\mathrm{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) + \frac{3}{5}g_1'^2\,\mathrm{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) + \frac{12}{5}g_1^2\,\mathrm{Tr}\Big(h^E h^{E\dagger}m_{e^c}^2\Big) \\
&+ \frac{3}{5}g_1'^2\,\mathrm{Tr}\Big(h^E h^{E\dagger}m_{e^c}^2\Big) - 2m_{L_4}^2\,\mathrm{Tr}\Big(\tilde{f}h^{E\dagger}h^E\tilde{f}^\dagger\Big) - 2m_{H_u}^2\,\mathrm{Tr}\Big(\tilde{f}h^{E\dagger}h^E\tilde{f}^\dagger\Big) \\
&- 2\,\mathrm{Tr}\Big(\tilde{f}h^{E\dagger}T^{h^E}T^{\tilde{f}\dagger}\Big) - 2\,\mathrm{Tr}\Big(\tilde{f}T^{h^E\dagger}T^{h^E}\tilde{f}^\dagger\Big) - 36m_{L_4}^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}g^D g^{D\dagger}\Big) \\
&- 36\,\mathrm{Tr}\Big(g^D g^{D\dagger}T^{g^D}T^{g^D\dagger}\Big) - 6m_{H_d}^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{DT}y^{D*}\Big) \\
&- 6m_{L_4}^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{DT}y^{D*}\Big) - 6m_{L_4}^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big) \\
&- 6m_{H_u}^2\,\mathrm{Tr}\Big(g^D g^{D\dagger}y^{UT}y^{U*}\Big) - 6\,\mathrm{Tr}\Big(g^D g^{D\dagger}T^{DT}T^{D*}\Big) \\
&- 6\,\mathrm{Tr}\Big(g^D g^{D\dagger}T^{UT}T^{U*}\Big) - 6m_{L_4}^2\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) - 6m_S^2\,\mathrm{Tr}\Big(g^D\kappa^\dagger\kappa g^{D\dagger}\Big) \\
&- 6\,\mathrm{Tr}\Big(g^D\kappa^\dagger T^\kappa T^{g^D\dagger}\Big) - 36\,\mathrm{Tr}\Big(g^D T^{g^D\dagger}T^{g^D}g^{D\dagger}\Big) - 6\,\mathrm{Tr}\Big(g^D T^{\kappa\dagger}T^\kappa g^{D\dagger}\Big) \\
&- 2\,\mathrm{Tr}\Big(h^E\tilde{f}^\dagger T^{\tilde{f}}T^{h^E\dagger}\Big) - 12m_{L_4}^2\,\mathrm{Tr}\Big(h^E h^{E\dagger}h^E h^{E\dagger}\Big) \\
&- 4m_{H_d}^2\,\mathrm{Tr}\Big(h^E h^{E\dagger}y^E y^{E\dagger}\Big) - 4m_{L_4}^2\,\mathrm{Tr}\Big(h^E h^{E\dagger}y^E y^{E\dagger}\Big) \\
&- 12\,\mathrm{Tr}\Big(h^E h^{E\dagger}T^{h^E}T^{h^E\dagger}\Big) - 4\,\mathrm{Tr}\Big(h^E h^{E\dagger}T^E T^{E\dagger}\Big) \\
&- 2m_{L_4}^2\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big) - 2m_S^2\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger\tilde{\lambda}h^{E\dagger}\Big) - 2\,\mathrm{Tr}\Big(h^E\tilde{\lambda}^\dagger T^{\tilde{\lambda}}T^{h^E\dagger}\Big) \\
&- 2\,\mathrm{Tr}\Big(h^E T^{\tilde{f}\dagger}T^{\tilde{f}}h^{E\dagger}\Big) - 12\,\mathrm{Tr}\Big(h^E T^{h^E\dagger}T^{h^E}h^{E\dagger}\Big) - 4\,\mathrm{Tr}\Big(h^E T^{h^E\dagger}T^E y^{E\dagger}\Big) \\
&- 2\,\mathrm{Tr}\Big(h^E T^{\tilde{\lambda}\dagger}T^{\tilde{\lambda}}h^{E\dagger}\Big) - 6\,\mathrm{Tr}\Big(y^D T^{g^D*}T^{g^D T}y^{D\dagger}\Big) - 4\,\mathrm{Tr}\Big(y^E y^{E\dagger}T^{h^E}T^{h^E\dagger}\Big) \\
&- 4\,\mathrm{Tr}\Big(y^E T^{E\dagger}T^{h^E}h^{E\dagger}\Big) - 6\,\mathrm{Tr}\Big(y^U T^{g^D*}T^{g^D T}y^{U\dagger}\Big) - 6\,\mathrm{Tr}\Big(\kappa g^{D\dagger}T^{g^D}T^{\kappa\dagger}\Big) \\
&- 6\,\mathrm{Tr}\Big(\kappa T^{g^D\dagger}T^{g^D}\kappa^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{\lambda}h^{E\dagger}T^{h^E}T^{\tilde{\lambda}\dagger}\Big) - 2\,\mathrm{Tr}\Big(\tilde{\lambda}T^{h^E\dagger}T^{h^E}\tilde{\lambda}^\dagger\Big) \\
&- 6\,\mathrm{Tr}\Big(g^{D\dagger}y^{DT}T^{D*}T^{g^D}\Big) - 6\,\mathrm{Tr}\Big(g^{D\dagger}y^{UT}T^{U*}T^{g^D}\Big) \\
&- 6\,\mathrm{Tr}\Big(y^{D\dagger}T^D T^{g^D*}g^{DT}\Big) - 6\,\mathrm{Tr}\Big(y^{U\dagger}T^U T^{g^D*}g^{DT}\Big) \\
&- 2\,\mathrm{Tr}\Big(\tilde{f}m_{H_1}^2 h^{E\dagger}h^E\tilde{f}^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{f}h^{E\dagger}h^E m_{H_1}^2\tilde{f}^\dagger\Big) - 2\,\mathrm{Tr}\Big(\tilde{f}h^{E\dagger}h^E\tilde{f}^\dagger m_\Sigma^{2*}\Big) \\
&- 2\,\mathrm{Tr}\Big(\tilde{f}h^{E\dagger}m_{e^c}^2 h^E\tilde{f}^\dagger\Big) - 18\,\mathrm{Tr}\Big(g^D g^{D\dagger}g^D m_{\bar{D}}^{2*}g^{D\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 36\,\mathrm{Tr}\Big(g^D g^{D\dagger} m_Q^{2*} g^D g^{D\dagger}\Big) - 6\,\mathrm{Tr}\Big(g^D g^{D\dagger} m_Q^{2*} y^{DT} y^{D*}\Big)\\
&- 6\,\mathrm{Tr}\Big(g^D g^{D\dagger} m_Q^{2*} y^{UT} y^{U*}\Big) - 6\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} m_{d^c}^{2*} y^{D*}\Big)\\
&- 6\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{DT} y^{D*} m_Q^{2*}\Big) - 6\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} m_{u^c}^{2*} y^{U*}\Big)\\
&- 6\,\mathrm{Tr}\Big(g^D g^{D\dagger} y^{UT} y^{U*} m_Q^{2*}\Big) - 6\,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa g^{D\dagger} m_Q^{2*}\Big)\\
&- 6\,\mathrm{Tr}\Big(g^D \kappa^\dagger \kappa m_{\bar{D}}^{2*} g^{D\dagger}\Big) - 6\,\mathrm{Tr}\Big(g^D \kappa^\dagger m_D^{2*} \kappa g^{D\dagger}\Big)\\
&- 18\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger} g^D g^{D\dagger}\Big) - 6\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger} y^{DT} y^{D*}\Big)\\
&- 6\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger} y^{UT} y^{U*}\Big) - 6\,\mathrm{Tr}\Big(g^D m_{\bar{D}}^{2*} \kappa^\dagger \kappa g^{D\dagger}\Big)\\
&- 6\,\mathrm{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger} h^E h^{E\dagger}\Big) - 4\,\mathrm{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger} y^E y^{E\dagger}\Big)\\
&- 2\,\mathrm{Tr}\Big(h^E m_{H_1}^2 \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger}\Big) - 6\,\mathrm{Tr}\Big(h^E h^{E\dagger} h^E m_{H_1}^2 h^{E\dagger}\Big)\\
&- 6\,\mathrm{Tr}\Big(h^E h^{E\dagger} h^E h^{E\dagger} m_{e^c}^2\Big) - 6\,\mathrm{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2 h^E h^{E\dagger}\Big)\\
&- 4\,\mathrm{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2 y^E y^{E\dagger}\Big) - 4\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E m_L^2 y^{E\dagger}\Big)\\
&- 4\,\mathrm{Tr}\Big(h^E h^{E\dagger} y^E y^{E\dagger} m_{e^c}^2\Big) - 2\,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} m_{H_1}^2 h^{E\dagger}\Big)\\
&- 2\,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger \tilde{\lambda} h^{E\dagger} m_{e^c}^2\Big) - 2\,\mathrm{Tr}\Big(h^E \tilde{\lambda}^\dagger m_{H_2}^{2*} \tilde{\lambda} h^{E\dagger}\Big), \qquad \text{(A.123)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_{\bar{L}_4}^2}^{(1)} &= -\frac{6}{5} g_1^2 |M_1|^2 - \frac{4}{5} g_1'^2 |M_1'|^2 - 6 g_2^2 |M_2|^2 + 2 m_{L_4}^2 |\tilde{\sigma}|^2 + 2 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2\\
&\quad + 2 m_\phi^2 |\tilde{\sigma}|^2 + 2 |T_{\tilde{\sigma}}|^2 + \sqrt{\frac{3}{5}} g_1 \Sigma_{1,1} - \sqrt{\frac{2}{5}} g_1' \Sigma_{1,4}, \qquad \text{(A.124)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_{\bar{L}_4}^2}^{(2)} &= \frac{18}{5} g_1^2 g_2^2 |M_2|^2 + \frac{12}{5} g_1'^2 g_2^2 |M_2|^2 + 87 g_2^4 |M_2|^2\\
&\quad + \frac{9}{25} g_1^2 \Big[2 g_1'^2 \Big(2M_1 + M_1'\Big) + 5 g_2^2 \Big(2M_1 + M_2\Big) + 99 g_1^2 M_1\Big] M_1^*\\
&\quad + \frac{3}{25} g_1'^2 \Big[10 g_2^2 \Big(2M_1' + M_2\Big) + 217 g_1'^2 M_1' + 6 g_1^2 \Big(2M_1' + M_1\Big)\Big] M_1'^*\\
&\quad + \frac{9}{5} g_1^2 g_2^2 M_1 M_2^* + \frac{6}{5} g_1'^2 g_2^2 M_1' M_2^* - 4 m_{L_4}^2 |\tilde{\sigma}|^2 |\kappa_\phi|^2 - 4 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 |\kappa_\phi|^2\\
&\quad - 16 m_\phi^2 |\tilde{\sigma}|^2 |\kappa_\phi|^2 - 2 m_{L_4}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 2 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 4 m_\phi^2 |\tilde{\sigma}|^2 |\sigma|^2\\
&\quad - 2 m_S^2 |\tilde{\sigma}|^2 |\sigma|^2 - 2 m_{\bar{S}}^2 |\tilde{\sigma}|^2 |\sigma|^2 - 12 m_{L_4}^2 |\tilde{\sigma}|^4 - 12 m_{\bar{L}_4}^2 |\tilde{\sigma}|^4\\
&\quad - 12 m_\phi^2 |\tilde{\sigma}|^4 - 4 |\tilde{\sigma}|^2 |T_{\kappa_\phi}|^2 - 4 \tilde{\sigma} \kappa_\phi^* T_{\tilde{\sigma}}^* T_{\kappa_\phi} - 2 |\tilde{\sigma}|^2 |T_\sigma|^2 - 2 \tilde{\sigma} \sigma^* T_{\tilde{\sigma}}^* T_\sigma\\
&\quad - 4 \kappa_\phi \tilde{\sigma}^* T_{\kappa_\phi}^* T_{\tilde{\sigma}} - 2 \sigma \tilde{\sigma}^* T_\sigma^* T_{\tilde{\sigma}} - 4 |\kappa_\phi|^2 |T_{\tilde{\sigma}}|^2 - 2 |\sigma|^2 |T_{\tilde{\sigma}}|^2 - 24 |\tilde{\sigma}|^2 |T_{\tilde{\sigma}}|^2\\
&\quad + 6 g_2^4 \Sigma_{2,2} + \frac{6}{5} g_1^2 \Sigma_{2,11} - \frac{2}{5} \sqrt{6} g_1 g_1' \Sigma_{2,14} - \frac{2}{5} \sqrt{6} g_1 g_1' \Sigma_{2,41}\\
&\quad + \frac{4}{5} g_1'^2 \Sigma_{2,44} + 4 \sqrt{\frac{3}{5}} g_1 \Sigma_{3,1} - 4 \sqrt{\frac{2}{5}} g_1' \Sigma_{3,4} - 12 m_{L_4}^2 |\tilde{\sigma}|^2 \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big)\\
&\quad - 6 m_{\bar{L}_4}^2 |\tilde{\sigma}|^2 \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 6 m_\phi^2 |\tilde{\sigma}|^2 \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big) - 6 |T_{\tilde{\sigma}}|^2 \,\mathrm{Tr}\Big(g^D g^{D\dagger}\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 4m_{L_4}^2|\tilde{\sigma}|^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 2m_{\bar{L}_4}^2|\tilde{\sigma}|^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 2m_\phi^2|\tilde{\sigma}|^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 2|T_{\tilde{\sigma}}|^2 \operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 6\tilde{\sigma} T_{\tilde{\sigma}}^* \operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) - 2\tilde{\sigma} T_{\tilde{\sigma}}^* \operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big) \\
&- 6\tilde{\sigma}^* T_{\tilde{\sigma}} \operatorname{Tr}\Big(T^{g^D *} g^{DT}\Big) - 6|\tilde{\sigma}|^2 \operatorname{Tr}\Big(T^{g^D *} T^{g^D T}\Big) - 2\tilde{\sigma}^* T_{\tilde{\sigma}} \operatorname{Tr}\Big(T^{h^E *} h^{ET}\Big) \\
&- 2|\tilde{\sigma}|^2 \operatorname{Tr}\Big(T^{h^E *} T^{h^E T}\Big) - 6|\tilde{\sigma}|^2 \operatorname{Tr}\Big(g^D g^{D\dagger} m_Q^{2*}\Big) - 6|\tilde{\sigma}|^2 \operatorname{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger}\Big) \\
&- 2|\tilde{\sigma}|^2 \operatorname{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) - 2|\tilde{\sigma}|^2 \operatorname{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2\Big), \qquad \text{(A.125)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_\phi^2}^{(1)} = 2\Big[&2m_{\bar{L}_4}^2|\tilde{\sigma}|^2 + 2m_{L_4}^2|\tilde{\sigma}|^2 + 2m_\phi^2|\tilde{\sigma}|^2 + 2|T_{\kappa_\phi}|^2 + 2|T_{\tilde{\sigma}}|^2 + 6m_\phi^2|\kappa_\phi|^2 \\
&+ \Big(m_\phi^2 + m_S^2 + m_{\bar{S}}^2\Big)|\sigma|^2 + |T_\sigma|^2\Big], \qquad \text{(A.126)}
\end{aligned}
$$

$$
\begin{aligned}
\beta_{m_\phi^2}^{(2)} = &-96m_\phi^2|\kappa_\phi|^4 - 8\Big(m_\phi^2 + m_S^2 + m_{\bar{S}}^2\Big)|\sigma|^4 - 8\kappa_\phi^*\Big[\Big(4m_\phi^2 + m_S^2 + m_{\bar{S}}^2\Big)\kappa_\phi|\sigma|^2 \\
&+ 2\Big(4m_\phi^2 + m_{L_4}^2 + m_{\bar{L}_4}^2\Big)\kappa_\phi|\tilde{\sigma}|^2 + 8\kappa_\phi|T_{\kappa_\phi}|^2 + \kappa_\phi|T_\sigma|^2 + 2\kappa_\phi|T_{\tilde{\sigma}}|^2 \\
&+ \sigma T_\sigma^* T_{\kappa_\phi} + 2\tilde{\sigma} T_{\tilde{\sigma}}^* T_{\kappa_\phi}\Big] + \frac{1}{5}\Big(-80\Big(m_{L_4}^2 + m_{\bar{L}_4}^2 + m_\phi^2\Big)|\tilde{\sigma}|^4 \\
&- 4T_{\tilde{\sigma}}^*\Big\{T_{\tilde{\sigma}}\Big[-15g_2^2 + 15\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 2g_1'^2 - 3g_1^2 + 5\operatorname{Tr}\Big(h^E h^{E\dagger}\Big)\Big] \\
&+ \tilde{\sigma}\Big[15g_2^2 M_2 + 15\operatorname{Tr}\Big(g^{D\dagger} T^{g^D}\Big) + 2g_1'^2 M_1' + 3g_1^2 M_1 + 5\operatorname{Tr}\Big(h^{E\dagger} T^{h^E}\Big)\Big]\Big\} \\
&- 5T_\sigma^*\Big\{4\lambda^*\Big(\lambda T_\sigma + \sigma T_\lambda\Big) + \sigma\Big[4\operatorname{Tr}\Big(\tilde{\lambda}^\dagger T^{\tilde{\lambda}}\Big) + 5g_1'^2 M_1' + 6\operatorname{Tr}\Big(\kappa^\dagger T^\kappa\Big)\Big] \\
&+ T_\sigma\Big[4\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 5g_1'^2 + 6\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big)\Big]\Big\} + 4\tilde{\sigma}^*\Big[3g_1^2 m_{L_4}^2\tilde{\sigma} + 2g_1'^2 m_{L_4}^2\tilde{\sigma} \\
&+ 15g_2^2 m_{L_4}^2\tilde{\sigma} + 3g_1^2 m_{\bar{L}_4}^2\tilde{\sigma} + 2g_1'^2 m_{\bar{L}_4}^2\tilde{\sigma} + 15g_2^2 m_{\bar{L}_4}^2\tilde{\sigma} + 3g_1^2 m_\phi^2\tilde{\sigma} \\
&+ 2g_1'^2 m_\phi^2\tilde{\sigma} + 15g_2^2 m_\phi^2\tilde{\sigma} + 30g_2^2\tilde{\sigma}|M_2|^2 - 20\tilde{\sigma}|T_{\kappa_\phi}|^2 - 40\tilde{\sigma}|T_{\tilde{\sigma}}|^2 \\
&+ 3g_1^2 M_1^*\Big(2M_1\tilde{\sigma} - T_{\tilde{\sigma}}\Big) + 2g_1'^2 M_1'^*\Big(2M_1'\tilde{\sigma} - T_{\tilde{\sigma}}\Big) - 15g_2^2 M_2^* T_{\tilde{\sigma}} \\
&- 20\kappa_\phi T_{\kappa_\phi}^* T_{\tilde{\sigma}} - 30m_{L_4}^2\tilde{\sigma}\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 15m_{\bar{L}_4}^2\tilde{\sigma}\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) \\
&- 15m_\phi^2\tilde{\sigma}\operatorname{Tr}\Big(g^D g^{D\dagger}\Big) - 10m_{L_4}^2\tilde{\sigma}\operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 5m_{\bar{L}_4}^2\tilde{\sigma}\operatorname{Tr}\Big(h^E h^{E\dagger}\Big) \\
&- 5m_\phi^2\tilde{\sigma}\operatorname{Tr}\Big(h^E h^{E\dagger}\Big) - 15T_{\tilde{\sigma}}\operatorname{Tr}\Big(T^{g^D *} g^{DT}\Big) - 15\tilde{\sigma}\operatorname{Tr}\Big(T^{g^D *} T^{g^D T}\Big) \\
&- 5T_{\tilde{\sigma}}\operatorname{Tr}\Big(T^{h^E *} h^{ET}\Big) - 5\tilde{\sigma}\operatorname{Tr}\Big(T^{h^E *} T^{h^E T}\Big) - 15\tilde{\sigma}\operatorname{Tr}\Big(g^D g^{D\dagger} m_Q^{2*}\Big) \\
&- 15\tilde{\sigma}\operatorname{Tr}\Big(g^D m_{\bar{D}}^{2*} g^{D\dagger}\Big) - 5\tilde{\sigma}\operatorname{Tr}\Big(h^E m_{H_1}^2 h^{E\dagger}\Big) - 5\tilde{\sigma}\operatorname{Tr}\Big(h^E h^{E\dagger} m_{e^c}^2\Big)\Big]\Big) \\
&+ \sigma^*\Big[5g_1'^2 m_\phi^2\sigma + 5g_1'^2 m_S^2\sigma + 5g_1'^2 m_{\bar{S}}^2\sigma - 4\Big(2m_S^2 + m_{H_d}^2 + m_{H_u}^2 \\
&+ m_\phi^2 + m_{\bar{S}}^2\Big)\sigma|\lambda|^2 - 8\sigma|T_{\kappa_\phi}|^2 - 4\sigma|T_\lambda|^2 - 16\sigma|T_\sigma|^2 \\
&+ 5g_1'^2 M_1'^*\Big(2M_1'\sigma - T_\sigma\Big) - 8\kappa_\phi T_{\kappa_\phi}^* T_\sigma - 4\lambda T_\lambda^* T_\sigma - 6m_\phi^2\sigma\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) \\
&- 12m_S^2\sigma\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 6m_{\bar{S}}^2\sigma\operatorname{Tr}\Big(\kappa\kappa^\dagger\Big) - 4m_\phi^2\sigma\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) \\
&- 8m_S^2\sigma\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 4m_{\bar{S}}^2\sigma\operatorname{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger\Big) - 6T_\sigma\operatorname{Tr}\Big(T^{\kappa *}\kappa^T\Big)
\end{aligned}
$$

$$
\begin{aligned}
&- 6\sigma\,\mathrm{Tr}\Big(T^{\kappa *}T^{\kappa T}\Big) - 4T_\sigma\,\mathrm{Tr}\Big(T^{\tilde{\lambda}*}\tilde{\lambda}^T\Big) - 4\sigma\,\mathrm{Tr}\Big(T^{\tilde{\lambda}*}T^{\tilde{\lambda}T}\Big) \\
&- 4\sigma\,\mathrm{Tr}\Big(m_{H_1}^2\tilde{\lambda}^\dagger\tilde{\lambda}\Big) - 6\sigma\,\mathrm{Tr}\Big(\kappa\kappa^\dagger m_D^{2*}\Big) - 6\sigma\,\mathrm{Tr}\Big(\kappa m_{\bar{D}}^{2*}\kappa^\dagger\Big) \\
&- 4\sigma\,\mathrm{Tr}\Big(\tilde{\lambda}\tilde{\lambda}^\dagger m_{H_2}^{2*}\Big)\Big]. \qquad \text{(A.127)}
\end{aligned}
$$

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