

# Signatures of the genuine and matter-induced components of the CP violation asymmetry in neutrino oscillations

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ABSTRACT: CP asymmetries for neutrino oscillations in matter can be disentangled into the matter-induced CPT-odd (T-invariant) component and the genuine T-odd (CPT-invariant) component. For their understanding in terms of the relevant ingredients, we develop a new perturbative expansion in both  $\Delta m_{21}^2$ ,  $|a| \ll |\Delta m_{31}^2|$  without any assumptions between  $\Delta m_{21}^2$  and  $a$ , and study the subtleties of the vacuum limit in the two terms of the CP asymmetry, moving from the CPT-invariant vacuum limit  $a \rightarrow 0$  to the T-invariant limit  $\Delta m_{21}^2 \rightarrow 0$ . In the experimental region of terrestrial accelerator neutrinos, we calculate their approximate expressions from which we prove that, at medium baselines, the CPT-odd component is small and nearly  $\delta$ -independent, so it can be subtracted from the experimental CP asymmetry as a theoretical background, provided the hierarchy is known. At long baselines, on the other hand, we find that (i) a Hierarchy-odd term in the CPT-odd component dominates the CP asymmetry for energies above the first oscillation node, and (ii) the CPT-odd term vanishes, independent of the CP phase  $\delta$ , at  $E = 0.92$  GeV ( $L/1300$  km) near the second oscillation maximum, where the T-odd term is almost maximal and proportional to  $\sin \delta$ . A measurement of the CP asymmetry in these energy regions would thus provide separate information on (i) the neutrino mass ordering, and (ii) direct evidence of genuine CP violation in the lepton sector.

KEYWORDS: CP violation, Discrete Symmetries, Neutrino Physics

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## 1 Introduction

One of the most important open questions in fundamental physics is the existence of CP symmetry breaking in the lepton sector. A positive answer could open the door to understand the matter-antimatter asymmetry of the Universe through Leptogenesis [1] at higher energy scales. Next generation neutrino flavor oscillation experiments, like T2HK [2] and DUNE [3], have this challenge as their priority aim. But the neutrino propagation from terrestrial accelerator facilities is not taking place in vacuum and there are matter effects [4, 5] induced by the CP-asymmetric interaction with Earth. In the quest for a direct evidence of CP violation in the lepton sector, we have recently derived a theorem [6] for the observable CP asymmetry in neutrino oscillations propagating in matter

$$\mathcal{A}_{\alpha\beta}^{\text{CP}} \equiv P_{\alpha\beta}(\nu) - P_{\alpha\beta}(\bar{\nu}) = \mathcal{A}_{\alpha\beta}^{\text{CPT}} + \mathcal{A}_{\alpha\beta}^{\text{T}}, \quad (1.1)$$

where  $P_{\alpha\beta}$  is the probability for any flavor transition  $\alpha \rightarrow \beta$ . Its achievement consists in providing the disentanglement of the genuine  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  and matter-induced  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  components of CP-violating (CPV)  $\mathcal{A}_{\alpha\beta}^{\text{CP}}$  based on the concept that they have different properties under the other discrete symmetries Time Reversal T and CPT:  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is CPT-odd T-invariant, whereas  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is T-odd CPT-invariant. These two components are distinctly identified by their behavior as functions of the baseline  $L$ : whereas the matter-induced component  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is an even function of  $L$ , the genuine component  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is odd in  $L$ . For a description of the effective Hamiltonian in which the matter-induced term is generated by the parameter

$a = 2EV$  of the matter potential  $V$  due to charged current interactions between electron neutrinos and electrons and the genuine CPV term is generated by a phase  $\delta$  in the three-family PMNS mixing matrix, the  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  components of the CP asymmetry have the correct behavior, for any neutrino energy  $E$  and baseline  $L$ , in these parameters: whereas  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is odd in  $a \forall \delta$ ,  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is odd in  $\sin \delta \forall a$ . In addition, the change from a Normal Hierarchy in the ordering of the neutrino mass spectrum to an Inverted Hierarchy leads to the result that  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  remains invariant and, for energies above the first oscillation node,  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  changes its sign.

The planned experiments T2HK and DUNE consider the golden transition  $\nu_\mu \xrightarrow{L} \nu_e$  for neutrinos propagating in the Earth mantle with fixed  $L$  and a continuum energy spectrum  $E$ . In the search of interesting experimental signatures able to separate the  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  components of  $\mathcal{A}_{\mu e}^{\text{CP}}$ , we study in this paper the characteristic energy dependencies of the two terms. In doing so, we develop appropriate analytical relations between the quantities in matter and the quantities in vacuum in order to provide guiding paths in our scrutiny of the peculiar behavior of the separate  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  components with energy.

The paper is organized as follows. Section 2 identifies the two components, matter-induced  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  and genuine  $\mathcal{A}_{\alpha\beta}^{\text{T}}$ , of the CP asymmetry  $\mathcal{A}_{\alpha\beta}^{\text{CP}}$  by means of rephasing-invariant mixings and neutrino masses in matter. The conceptual basis of the Disentanglement Theorem is provided by the different behavior under T and CPT symmetries. In section 3 we build a consistent perturbative expansion of the relevant ingredients in terms of the vacuum parameters with  $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$  and  $|U_{e3}|^2 \ll 1$ , with the interaction parameter  $a$  moving from above to below  $\Delta m_{21}^2$ . This approach will allow us to understand the intricacies of the ordering of the two limits  $a \rightarrow 0$  and  $\Delta m_{21}^2 \rightarrow 0$ . In section 4 we check that our analytic expansions are an excellent approximation to the exact unique rephasing-invariant CP-odd mixing appearing in the genuine  $\mathcal{A}_{\mu e}^{\text{T}}$  component of the CP asymmetry, leading in fact to  $\mathcal{A}_{\mu e}^{\text{T}}$  as in vacuum by an interesting compensation of matter effects between the mixing and oscillation factors. In section 5 we discuss the dependence of these observables on the neutrino mass hierarchy, proving that it is determined by the sign of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  at high energies, whereas  $\mathcal{A}_{\mu e}^{\text{T}}$  is blind to it. Section 6 makes a scan of the energy dependencies of the two  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  components, identifying zeros and extremal values of these functions. The occurrence of magic energies, in which  $\mathcal{A}_{\mu e}^{\text{CPT}}$  vanishes — independent of  $\delta$  — and  $|\mathcal{A}_{\mu e}^{\text{T}}|$  is maximal, will be understood. In section 7 we discuss our conclusions and outlook.

## 2 The CP asymmetry disentanglement theorem

Neutrino oscillations in matter are described through the effective Hamiltonian in the flavor basis [4, 7–11]

$$H = \frac{1}{2E} \left\{ U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} = \frac{1}{2E} \tilde{U} \tilde{M}^2 \tilde{U}^\dagger, \quad (2.1)$$

where the first term describes neutrino oscillations in vacuum and the second one accounts for matter effects. The  $a$  parameter is given by  $a = 2EV$ , with  $V$  the interaction potential with matter and  $E$  the relativistic neutrino energy. For antineutrinos,  $U \rightarrow U^*$ , originating a genuine CP violation effect through a CP phase  $\delta$  in  $U_{\text{PMNS}}$ , as well as  $a \rightarrow -a$ , originating matter-induced CP violation. In this description, the genuine CPV observable has to be odd in  $\sin \delta$ , whereas the matter-induced CPV effect has to be odd in  $a$ . All neutrino masses ( $\tilde{M}^2$ ) and mixings ( $\tilde{U}$ ) in matter, i.e. eigenvalues and eigenstates of  $H$ , can be calculated in terms of the parameters in the vacuum Hamiltonian ( $M^2, U$ ) and  $a$ , as studied in section 3.

The exact Hamiltonian leads to the flavor oscillation probabilities for any  $\alpha \rightarrow \beta$  transition

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{j<i} \text{Re } \tilde{J}_{\alpha\beta}^{ij} \sin^2 \tilde{\Delta}_{ij} - 2 \sum_{j<i} \text{Im } \tilde{J}_{\alpha\beta}^{ij} \sin 2\tilde{\Delta}_{ij}, \quad (2.2)$$

where  $\tilde{J}_{\alpha\beta}^{ij} \equiv \tilde{U}_{\alpha i} \tilde{U}_{\alpha j}^* \tilde{U}_{\beta i}^* \tilde{U}_{\beta j}$  are the rephasing-invariant mixings and  $\tilde{\Delta}_{ij} \equiv \frac{\Delta \tilde{m}_{ij}^2 L}{4E}$ . Notice that both  $\tilde{J}_{\alpha\beta}^{ij}$  and  $\Delta \tilde{m}_{ij}^2$  are energy dependent in matter. Antineutrino oscillations are given in general by the same expression with different masses ( $\Delta \tilde{m}_{ij}^2$ ) and mixings ( $\tilde{J}_{\alpha\beta}^{ij}$ ), so one can explicitly write the CP asymmetry  $\mathcal{A}_{\alpha\beta}^{\text{CP}}$  defined in eq. (1.1).

If CPT holds, as assumed in vacuum, necessarily  $\Delta \tilde{m}_{ij}^2 = \Delta m_{ij}^2$  and  $\tilde{J}_{\alpha\beta}^{ij} = (J_{\alpha\beta}^{ij})^*$ . Therefore, all  $L$ -even terms will cancel out in  $\mathcal{A}_{\alpha\beta}^{\text{CP}}$ , proving they are CPT-violating. On the other hand, the absence of genuine CP violation leads to real  $\tilde{J}_{\alpha\beta}^{ij}$  and  $\tilde{J}_{\alpha\beta}^{ij}$ , so all transition probabilities  $P_{\alpha\beta}$  are  $L$ -even functions. This result shows that  $L$ -odd terms in  $\mathcal{A}_{\alpha\beta}^{\text{CP}}$  are T-violating.

From the different behavior of each of these terms under the discrete T and CPT symmetry transformations, one derives the *Asymmetry Disentanglement Theorem* [6] by separating the observable CP asymmetry in any flavor transition into  $L$ -even (CPT-violating) and  $L$ -odd (T-violating) functions,  $\mathcal{A}_{\alpha\beta}^{\text{CP}} = \mathcal{A}_{\alpha\beta}^{\text{CPT}} + \mathcal{A}_{\alpha\beta}^{\text{T}}$ ,

$$\mathcal{A}_{\alpha\beta}^{\text{CPT}} = -4 \sum_{j<i} \left[ \text{Re } \tilde{J}_{\alpha\beta}^{ij} \sin^2 \tilde{\Delta}_{ij} - \text{Re } \tilde{J}_{\alpha\beta}^{ji} \sin^2 \tilde{\Delta}_{ij} \right], \quad (2.3a)$$

$$\mathcal{A}_{\alpha\beta}^{\text{T}} = -2 \sum_{j<i} \left[ \text{Im } \tilde{J}_{\alpha\beta}^{ij} \sin 2\tilde{\Delta}_{ij} - \text{Im } \tilde{J}_{\alpha\beta}^{ji} \sin 2\tilde{\Delta}_{ij} \right]. \quad (2.3b)$$

Let us emphasize that not only  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is CPT-violating and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is T-violating, we also find that  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is T-invariant and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is CPT-invariant. In this sense the two terms are truly disentangled. To prove these properties, we analyze both CPT and T transformations.

Under CPT:  $\{\Delta \tilde{m}_{ij}^2 \leftrightarrow \Delta \tilde{m}_{ij}^2, \tilde{J}_{\alpha\beta}^{ij} \leftrightarrow (\tilde{J}_{\alpha\beta}^{ij})^*\}$ , neutrino and antineutrino terms in  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  are interchanged, so  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  changes its sign. This sign in  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is compensated by the change of sign in both  $\text{Im } \tilde{J}_{\alpha\beta}^{ij}$  and  $\text{Im } \tilde{J}_{\alpha\beta}^{ji}$ , leaving  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  invariant. Under T:  $\{\tilde{J}_{\alpha\beta}^{ij} \rightarrow (\tilde{J}_{\alpha\beta}^{ij})^*, \tilde{J}_{\alpha\beta}^{ji} \rightarrow (\tilde{J}_{\alpha\beta}^{ji})^*\}$ , the only change in the asymmetries is a change of sign in all imaginary parts, changing the sign of  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  and leaving  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  invariant.

These properties lead cleanly to the disentanglement of  $\mathcal{A}_{\alpha\beta}^{\text{CP}} = \mathcal{A}_{\alpha\beta}^{\text{CPT}} + \mathcal{A}_{\alpha\beta}^{\text{T}}$ , where  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is T-invariant (even in  $\sin \delta$ ) and CPT-odd in  $a$ , whereas  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is CPT-invariant

(even in  $a$ ) and T-odd in  $\sin\delta$ . As a consequence,  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  vanishes for  $a = 0 \forall\delta$  and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  vanishes for  $\delta = 0, \pi \forall a$ . These complementary behaviors of the two components of the experimental CP asymmetry identify the CPV component  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  as CPT-invariant and thus a fully genuine CPV observable, whereas the CPV component  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is T-invariant and thus a fully fake CPV observable.

### 3 Analytic perturbation expansions

To understand the behavior of  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  in eqs. (2.3) required by the CPT and T symmetries, as proved in the previous section, we proceed to their analytic study for neutrino oscillations in matter of constant density. Notice that the formal description of the system is equivalent to neutrino oscillations in vacuum, if one parametrizes matter effects as a redefinition of neutrino masses and mixings. However, this redefinition is strongly dependent on the neutrino energy, so it does not provide a clear insight into the intrinsic properties of the system. A useful description should write all observables in matter, relevant to our two components of the experimentally accessible CP asymmetry, as functions of the vacuum parameters and the matter potential  $a$ . A similar methodology is being applied to calculations of the T [12] and CPT [13] asymmetries in matter.

The search of these formulae unavoidably finds the same issue: an exact description of the matter effects in neutrino oscillations leads to cumbersome expressions which do not provide a clear understanding [14]. The way to simplify the results is to treat perturbatively the small parameters of the system, namely  $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$  and  $|U_{e3}|^2 \ll 1$ . The most important drawback of this procedure is that, in perturbing in  $\Delta m_{21}^2$ , the implicit relation  $\Delta m_{21}^2 \ll |a|$  is also assumed, so one should not expect to reproduce the right vacuum limit  $a \rightarrow 0$  for all matter ingredients. Even so, this perturbation theory leads to compact and percent-level precise expressions for neutrino oscillation probabilities written in terms of vacuum parameters only [15], as well as more precise relations mapping the mixings in matter to the quantities in vacuum [16, 17].

We develop a new perturbative expansion in both  $\Delta m_{21}^2, |a| \ll |\Delta m_{31}^2|$  without assumptions between  $\Delta m_{21}^2$  and  $a$ , similar to ref. [18], oriented to the understanding of masses, mixings and the separate behavior of  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  as functions of the different variables. In doing so, we can check from our analytic expressions both the vacuum limit  $a \rightarrow 0$  and the T-invariant limit  $\Delta m_{21}^2 \rightarrow 0$ . The expansion in  $|a| \ll |\Delta m_{31}^2|$  holds for energies below a few GeV taking into account the definite  $a$ -parity of each component of the CP asymmetry. This way, we find the most simple expressions for  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  at the energies accessible by accelerator experiments, which are accurate enough to let the reader clearly understand their behavior.

We emphasize that any desired precision can be achieved using a numerical computation of the neutrino propagation. Our aim is not finding very precise expansions, but precise enough to identify and understand the distinct characteristic patterns of the energy behavior of the two components  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  and  $\mathcal{A}_{\alpha\beta}^{\text{T}}$ , with the objective of serving as a guide for experimental signatures.

### 3.1 The crucial role of the reference $\tilde{m}_0^2$ in matter

Since a diagonal  $m_1^2 \mathbf{1}$  in the Hamiltonian  $H$  in eq. (2.1) leads to a global phase in time evolution, which is unobservable, the equivalent Hamiltonian  $2E \Delta H \equiv \Delta H' = U \Delta M^2 U^\dagger + a P_e$ , where  $\Delta M^2 = \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$  and  $P_e = \text{diag}(1, 0, 0)$  is the  $e$ -flavor projector, is widely used.

Analogously, one could argue that  $\tilde{m}_1^2$  is unobservable in neutrino oscillations in matter. Even though this is true, one must take into account that either  $m_1^2$  or  $\tilde{m}_1^2$  can be chosen as origin of phases, but not both of them at the same time when connecting the parameters in matter to those in vacuum. Indeed, one can easily check that the Hamiltonian  $\Delta H'$  has three non-vanishing eigenvalues, so choosing  $m_1^2 = 0$  automatically leads to all  $\tilde{m}_i \neq 0$ , despite one of them being unobservable.

On the following, we call  $\tilde{m}_0^2$  the mass squared in matter leading to the relative phase shift between the unobservable global phases in vacuum and matter, writing the Hamiltonian as  $\Delta H' = U \Delta M^2 U^\dagger + a P_e = \tilde{m}_0^2 \mathbf{1} + \tilde{U} \Delta \tilde{M}^2 \tilde{U}^\dagger$ . In this notation, the three eigenvalues of  $\Delta H'$  will be the reference scale in matter  $\tilde{m}_0^2$  and the two observable mass squared differences in matter  $\Delta \tilde{m}_{ij}^2$ .

As proposed in ref. [14], we choose to diagonalize the Hamiltonian in the vacuum eigenbasis,  $\Delta H'_{ij} = \Delta M^2 + a U^\dagger P_e U = \tilde{m}_0^2 \mathbf{1} + V \Delta \tilde{M}^2 V^\dagger$ . Since the eigenvalues are basis independent and real, this cleanly shows that  $\tilde{m}_0^2$  and  $\Delta \tilde{m}_{ij}^2$  can only depend on  $(\Delta m_{ij}^2, |U_{ei}|, a)$ . Moreover, this factorizes the mixing matrix in matter into  $\tilde{U} = UV$ , where  $U$  is the (vacuum) PMNS matrix and  $V$  is the change of basis between vacuum and matter eigenstates, which must go to the identity when  $a \rightarrow 0$ .

The Hamiltonian being a  $3 \times 3$  matrix leads to the characteristic equation

$$p(\lambda) \equiv -\lambda^3 + \lambda^2 \text{tr}(\Delta H') + \frac{1}{2} \lambda [\text{tr}[(\Delta H')^2] - \text{tr}^2(\Delta H')] + \det(\Delta H') = 0, \quad (3.1)$$

where  $p(\lambda)$  is the characteristic polynomial of  $\Delta H'$  and its roots provide the three neutrino squared masses in matter  $\lambda_i$ , and the observable  $\Delta \tilde{m}_{ij}^2 = \lambda_i - \lambda_j$ . The three invariants can be easily calculated,

$$\text{tr}(\Delta H') = \Delta m_{21}^2 + \Delta m_{31}^2 + a, \quad (3.2a)$$

$$\text{tr}[(\Delta H')^2] = (\Delta m_{21}^2)^2 + (\Delta m_{31}^2)^2 + a^2 + 2a \left[ \Delta m_{21}^2 |U_{e2}|^2 + \Delta m_{31}^2 |U_{e3}|^2 \right], \quad (3.2b)$$

$$\det(\Delta H') = a |U_{e1}|^2 \Delta m_{21}^2 \Delta m_{31}^2. \quad (3.2c)$$

From this straightforward setup of the problem we find a fundamental result. Since

$$p(0) = a |U_{e1}|^2 \Delta m_{21}^2 \Delta m_{31}^2 \geq 0, \quad (3.3a)$$

$$p(\Delta m_{21}^2) = a |U_{e2}|^2 \Delta m_{21}^2 (\Delta m_{21}^2 - \Delta m_{31}^2) \leq 0, \quad (3.3b)$$

at least one of the eigenvalues of  $\Delta H'$  will always lie in the range  $[0, \Delta m_{21}^2]$ . All  $\Delta \tilde{m}_{ij}^2$  are known to be nonbound by  $\Delta m_{21}^2$ , as will be shown in figure 1, so we find that  $0 \leq \tilde{m}_0^2 \leq \Delta m_{21}^2$ . Although the inequalities in (3.3a) and (3.3b) have been written for Normal Hierarchy neutrinos, the reader may check that the argument is also valid for the Inverted Hierarchy and antineutrinos.

Given that physically  $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$ , this result shows that  $\tilde{m}_0^2$  is also a good perturbative parameter. Therefore, we focus in this subsection on writing the two observable  $\Delta\tilde{m}_{ij}^2$  exactly as functions of  $(\Delta m_{ij}^2, \tilde{m}_0^2, |U_{ei}|, a)$ , which gives enough information to calculate all observables of neutrino oscillations in matter.

A simple way to calculate the physical  $\Delta\tilde{m}_{ij}^2$  is the diagonalization of the displaced Hamiltonian  $\Delta H'_{ij} - \tilde{m}_0^2 \mathbf{1} = \Delta M^2 + a U^\dagger P_e U - \tilde{m}_0^2 \mathbf{1} = V \Delta \tilde{M}^2 V^\dagger$ . By construction, one of its eigenvalues is zero, so  $\det(\Delta H' - \tilde{m}_0^2 \mathbf{1}) = 0$  and its two non-vanishing eigenvalues are given by a quadratic equation,

$$\Delta\tilde{m}_\pm^2 = \frac{1}{2} (\Delta m_{21}^2 + \Delta m_{31}^2 + a - 3\tilde{m}_0^2) \pm \frac{1}{2} \sqrt{l^2 + 2\tilde{m}_0^2(\Delta m_{21}^2 + \Delta m_{31}^2 + a) - 3(\tilde{m}_0^2)^2}, \quad (3.4)$$

$$l^2 \equiv (\Delta m_{31}^2 + \Delta m_{21}^2 - a)^2 - 4\Delta m_{21}^2 \Delta m_{31}^2 + 4a\Delta m_{21}^2 |U_{e2}|^2 + 4a\Delta m_{31}^2 |U_{e3}|^2.$$

From this definition it is clear that  $\Delta\tilde{m}_+^2 > \Delta m_-^2$ , but notice that  $|\Delta\tilde{m}_+^2| > |\Delta\tilde{m}_-^2|$  for Normal Hierarchy, whereas  $|\Delta\tilde{m}_-^2| > |\Delta\tilde{m}_+^2|$  for Inverted Hierarchy. This expression for  $\Delta\tilde{m}_\pm^2$  is a good starting point from which one can easily derive approximate formulae in the limit  $\tilde{m}_0^2 \leq \Delta m_{21}^2 \ll |\Delta m_{31}^2|$ . In order to write  $\Delta\tilde{m}_\pm^2$  as functions of vacuum parameters only, this same limit can be used directly in eq. (3.1) to find  $\tilde{m}_0^2$  perturbatively, as we will do in the following subsection.

We finish this subsection writing explicitly the eigenstates of  $\Delta H'_{ij}$  in the canonical basis of mass eigenstates in vacuum,

$$|\tilde{\nu}_i\rangle = \frac{1}{N_i} \begin{bmatrix} a \left[ (\lambda_i - \Delta m_{31}^2) |U_{e2}|^2 + (\lambda_i - \Delta m_{21}^2) |U_{e3}|^2 \right] U_{e1}^* \\ \left[ \lambda_i - a|U_{e1}|^2 \right] \left[ \lambda_i - \Delta m_{31}^2 \right] U_{e2}^* \\ \left[ \lambda_i - a|U_{e1}|^2 \right] \left[ \lambda_i - \Delta m_{21}^2 \right] U_{e3}^* \end{bmatrix}, \quad (3.5)$$

where the normalization factor  $N_i$  is needed to ensure  $\langle \tilde{\nu}_i | \tilde{\nu}_i \rangle = 1$ , and its phase must be chosen so that  $\lim_{a \rightarrow 0} |\tilde{\nu}_i\rangle = |\nu_i\rangle$ .

The eigenvalues  $\lambda_i$ , labeled according to  $\lambda_1 < \lambda_2 < \lambda_3$  ( $\lambda_3 < \lambda_1 < \lambda_2$ ) if the hierarchy is normal (inverted), are given by  $\tilde{m}_0^2$  and  $\Delta\tilde{m}_\pm^2$  as shown in table 1 for neutrinos and antineutrinos. The reason why  $\lambda_1 = \tilde{m}_0^2$  whereas  $\bar{\lambda}_2 = \tilde{m}_0^2$  will be explained analytically when exploring the vacuum limit. This fact is also shown in figure 1 by a numerical determination of the evolution of the eigenvalues of  $\Delta H'$  with the matter parameter  $a$ . All results are produced using the best-fit values in ref. [19] for Normal Hierarchy. To show the theoretical implications of a change in mass hierarchy, we compute the Inverted Hierarchy case using  $\Delta m_{31}^2|_{\text{IH}} = -\Delta m_{32}^2|_{\text{NH}}$ , which is physically consistent since it keeps the absolute value of the largest mass splitting unchanged.

As seen, the eigenvalues that fulfill  $0 \leq \tilde{m}_0^2 \leq \Delta m_{21}^2$  are  $\lambda_1$  and  $\bar{\lambda}_2$ , independently of whether the hierarchy is normal or inverted. To distinguish these two functions, from now on we will call them  $\lambda_1 \equiv \tilde{m}_0^2$  and  $\bar{\lambda}_2 \equiv \tilde{m}_0^2$ . Notice that, even if both  $\tilde{m}_0^2$  and  $\tilde{m}_0^2$  are bounded by  $\Delta m_{21}^2$ , they are necessarily different functions, as seen by their different vacuum limits  $\lim_{a \rightarrow 0} \tilde{m}_0^2 = 0$  and  $\lim_{a \rightarrow 0} \tilde{m}_0^2 = \Delta m_{21}^2$ .

	Neutrinos ( $a > 0$ )	Antineutrinos ( $a < 0$ )
NH ( $\Delta m_{31}^2 > 0$ )	$\lambda_3 = \tilde{m}_0^2 + \Delta \tilde{m}_+^2$	$\bar{\lambda}_3 = \tilde{m}_0^2 + \Delta \tilde{m}_+^2$
	$\lambda_2 = \tilde{m}_0^2 + \Delta \tilde{m}_-^2$	$\bar{\lambda}_2 = \tilde{m}_0^2$
	$\lambda_1 = \tilde{m}_0^2$	$\bar{\lambda}_1 = \tilde{m}_0^2 + \Delta \tilde{m}_-^2$
IH ( $\Delta m_{31}^2 < 0$ )	$\lambda_2 = \tilde{m}_0^2 + \Delta \tilde{m}_+^2$	$\bar{\lambda}_2 = \tilde{m}_0^2$
	$\lambda_1 = \tilde{m}_0^2$	$\bar{\lambda}_1 = \tilde{m}_0^2 + \Delta \tilde{m}_+^2$
	$\lambda_3 = \tilde{m}_0^2 + \Delta \tilde{m}_-^2$	$\bar{\lambda}_3 = \tilde{m}_0^2 + \Delta \tilde{m}_-^2$

**Table 1.** Relation between the eigenvalues  $\lambda_i$ , with the convention  $\lambda_i \xrightarrow{a \rightarrow 0} \Delta m_{i1}^2$ , and the quantities  $\tilde{m}_0^2$  and  $\Delta \tilde{m}_\pm^2$  as calculated from eq. (3.4) with the corresponding sign of  $a$  for  $\nu/\bar{\nu}$  and the sign and value of  $\Delta m_{31}^2$  for NH/IH. According to hierarchy, the eigenvalues are ordered from larger to smaller. The observable  $\Delta \tilde{m}_{ij}^2 = \lambda_i - \lambda_j$  can be read from the table.

### 3.2 The way to the vacuum limit at fixed ( $E, L$ )

The perturbation theory used in the literature to make profit of the experimental relation  $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$  also assumes  $\Delta m_{21}^2 \ll |a|$ . In order to ensure that all our expressions will reproduce the right vacuum limit, which is crucial to study the CPT-invariant limit on  $\mathcal{A}_{\alpha\beta}^{\text{CP}}$ , we expand  $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$  without any assumption between  $\Delta m_{21}^2$  and  $a$ . Up to first order in this regime, eqs. (3.1) and (3.4) reduce to

$$(\tilde{m}_0^2)^2(\Delta m_{31}^2 + a) - \tilde{m}_0^2[\Delta m_{21}^2 \Delta m_{31}^2 + a \Delta m_{21}^2(1 - |U_{e2}|^2) + a \Delta m_{31}^2(1 - |U_{e3}|^2)] + a|U_{e1}|^2 \Delta m_{21}^2 \Delta m_{31}^2 = 0, \quad (3.6a)$$

$$\Delta \tilde{m}_\pm^2 = \frac{1}{2}(\Delta m_{31}^2 + a + \Delta m_{21}^2 - 3\tilde{m}_0^2) \pm \frac{1}{2}\sqrt{(\Delta m_{31}^2 - a)^2 + 4|U_{e3}|^2 a \Delta m_{31}^2 - 2(\Delta m_{21}^2 + \tilde{m}_0^2)(\Delta m_{31}^2 + a) + 4|U_{e2}|^2 a \Delta m_{21}^2}. \quad (3.6b)$$

These analytical results are shown in figure 1 for neutrinos ( $a > 0$ ) and antineutrinos ( $a < 0$ ), as well as Normal ( $\Delta m_{31}^2 > 0$ ) and Inverted ( $\Delta m_{31}^2 < 0$ ) Hierarchies, and they match perfectly the exact numerical values. Notice that the two solutions of eq. (3.6a) in vacuum are 0,  $\Delta m_{21}^2$ . From the discussion in the previous subsection we know that the first one corresponds to  $\tilde{m}_0^2$ , which is bound by  $\Delta m_{21}^2$  when  $a > 0$ , whereas the second one,  $\tilde{m}_0^2$ , is bound by  $\Delta m_{21}^2$  when  $a < 0$ .

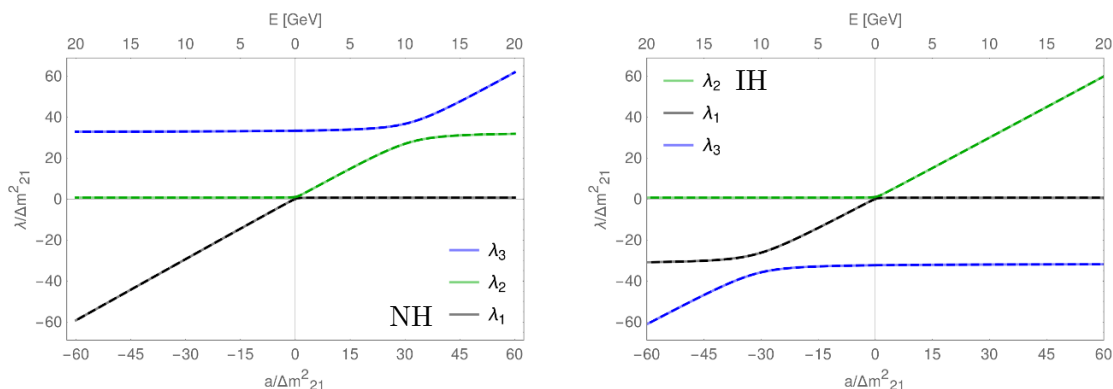
The appropriate expansion for  $\Delta \tilde{m}_\pm^2$  in both  $\tilde{m}_0^2 \leq \Delta m_{21}^2 \ll |\Delta m_{31}^2|$  and  $|a| \ll |\Delta m_{31}^2|$  is

$$\Delta \tilde{m}_{+\text{sign}(\Delta m_{31}^2)}^2 = \Delta m_{31}^2 + a|U_{e3}|^2 - \tilde{m}_0^2. \quad (3.7a)$$

$$\Delta \tilde{m}_{-\text{sign}(\Delta m_{31}^2)}^2 = \Delta m_{21}^2 + a(1 - |U_{e3}|^2) - 2\tilde{m}_0^2. \quad (3.7b)$$

The same expressions apply to antineutrinos changing  $a \rightarrow -a$ ,  $\tilde{m}_0^2 \rightarrow \tilde{m}_0^2$ , and the dependence on the Hierarchy is implicit in  $\text{sign}(\Delta m_{31}^2)$ , that accounts for the interchange of the expressions  $\Delta \tilde{m}_\pm^2|_{\text{NH}} \leftrightarrow \Delta \tilde{m}_\mp^2|_{\text{IH}}$ .





**Figure 1.** Eigenvalues  $\lambda_i$  of the mass matrix in matter in units of  $\Delta m_{21}^2$ , for both neutrinos ( $a > 0$ ) and antineutrinos ( $a < 0$ ). The horizontal axis shows both the evolution of the matter parameter  $a$  at fixed energy (lower labels), i.e. changing the matter density, and as function of the energy if the constant density is chosen as that of the Earth mantle (upper labels). Both the exact (dashed) and the analytical (solid) results from eqs. (3.6) are shown to illustrate the excellence of the analytic approximation. Normal Hierarchy ( $\lambda_1 < \lambda_2 < \lambda_3$ ) in the left panel, Inverted Hierarchy ( $\lambda_3 < \lambda_1 < \lambda_2$ ) in the right panel.

The approximation of  $\tilde{m}_0^2$  and  $\tilde{\tilde{m}}_0^2$ , on the other hand, comes from neglecting  $\Delta m_{31}^2$ -independent terms in eq. (3.6a),

$$\tilde{m}_0^2 = \frac{1}{2} \left[ \Delta m_{21}^2 + a(1 - |U_{e3}|^2) \right] \pm \frac{1}{2} \sqrt{\left[ \Delta m_{21}^2 + a(1 - |U_{e3}|^2) \right]^2 - 4|U_{e1}|^2 a \Delta m_{21}^2}, \quad (3.8)$$

where the  $-(+)$  sign corresponds to  $\tilde{m}_0^2$  ( $\tilde{\tilde{m}}_0^2$ ). In order to compare their behavior above and below  $\Delta m_{21}^2$ , one can further expand eq. (3.8) in the two regions

$$\tilde{m}_0^2 = \begin{cases} a|U_{e1}|^2 \left[ 1 - \frac{a|U_{e2}|^2}{\Delta m_{21}^2} + \dots \right], & a \ll \Delta m_{21}^2 \quad (3.9a) \\ \frac{\Delta m_{21}^2 |U_{e1}|^2}{1 - |U_{e3}|^2} \left[ 1 - \frac{\Delta m_{21}^2 |U_{e2}|^2}{a(1 - |U_{e3}|^2)^2} + \dots \right], & \Delta m_{21}^2 \ll a \quad (3.9b) \end{cases}$$

$$\tilde{\tilde{m}}_0^2 = \begin{cases} \Delta m_{21}^2 - |a||U_{e2}|^2 \left[ 1 - \frac{|a||U_{e1}|^2}{\Delta m_{21}^2} + \dots \right], & |a| \ll \Delta m_{21}^2 \quad (3.10a) \\ \frac{\Delta m_{21}^2 |U_{e1}|^2}{1 - |U_{e3}|^2} \left[ 1 + \frac{\Delta m_{21}^2 |U_{e2}|^2}{|a|(1 - |U_{e3}|^2)^2} + \dots \right], & \Delta m_{21}^2 \ll |a| \quad (3.10b) \end{cases}$$

adequate when looking at the CPT-invariant (vacuum) limit or the T-invariant limit, respectively. Notice that both  $\tilde{m}_0^2$  and  $\tilde{\tilde{m}}_0^2$  converge to the same asymptotic limit in  $|a|$  above  $\Delta m_{21}^2$ .

At the expense of losing precision, the evolution of these parameters between the two limits is illustrated by the approximate interpolations

$$\tilde{m}_0^2 \approx |U_{e1}|^2 \frac{a \Delta m_{21}^2}{a + \Delta m_{21}^2}, \quad \tilde{m}_0^2 \approx \Delta m_{21}^2 - |U_{e2}|^2 \frac{|a| \Delta m_{21}^2}{|a| + \Delta m_{21}^2}, \quad (3.11)$$

with errors  $\sim 20\%$ , that roughly describes their behavior: their vacuum limits are  $\tilde{m}_0^2 \rightarrow 0$  and  $\tilde{m}_0^2 \rightarrow \Delta m_{21}^2$ , both vanish when  $\Delta m_{21}^2$  goes to zero (since  $\tilde{m}_0^2 \leq \Delta m_{21}^2$ ) and go to  $\approx |U_{e1}|^2 \Delta m_{21}^2$  above  $\Delta m_{21}^2$ . Notice that first-order approximations in eqs. (3.11) reproduce all four limits in eqs. (3.9), (3.10) if one neglects  $|U_{e3}|^2$  terms.

The eigenstates in eq. (3.5) up to leading order, together with the eigenvalues in eqs. (3.7), reduce to the simple expressions

$$|\tilde{\nu}_1\rangle = \frac{1}{N_1} \begin{bmatrix} 1 \\ \frac{\tilde{m}_0^2 - a|U_{e1}|^2}{aU_{e1}^*U_{e2}} \\ 0 \end{bmatrix}, \quad |\tilde{\nu}_2\rangle = \frac{1}{N_2} \begin{bmatrix} \frac{aU_{e1}^*U_{e2}}{\Delta m_{21}^2 - \tilde{m}_0^2 + a|U_{e2}|^2} \\ 1 \\ 0 \end{bmatrix}, \quad |\tilde{\nu}_3\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (3.12)$$

valid for both hierarchies. Antineutrino eigenstates are given by

$$|\tilde{\bar{\nu}}_1\rangle = \frac{1}{N_1} \begin{bmatrix} 1 \\ -\frac{\Delta m_{21}^2 - \tilde{m}_0^2 - a|U_{e2}|^2}{aU_{e1}U_{e2}^*} \\ 0 \end{bmatrix}, \quad |\tilde{\bar{\nu}}_2\rangle = \frac{1}{N_2} \begin{bmatrix} -\frac{aU_{e1}U_{e2}^*}{\tilde{m}_0^2 + a|U_{e1}|^2} \\ 1 \\ 0 \end{bmatrix}, \quad |\tilde{\bar{\nu}}_3\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (3.13)$$

These eigenstates determine the columns of the  $V$  mixing matrix between matter and vacuum mass eigenstates, which allows us to write  $\tilde{U} = UV$  as

$$\begin{aligned} \tilde{U}_{\alpha 1} &= \frac{1}{N_1} \left[ U_{\alpha 1} + \frac{\tilde{m}_0^2 - a|U_{e1}|^2}{aU_{e1}^*U_{e2}} U_{\alpha 2} \right], & \tilde{U}_{\alpha 1} &= \frac{1}{N_1} \left[ U_{\alpha 1}^* - \frac{\Delta m_{21}^2 - \tilde{m}_0^2 - a|U_{e2}|^2}{aU_{e1}U_{e2}^*} U_{\alpha 2}^* \right], \\ \tilde{U}_{\alpha 2} &= \frac{1}{N_2} \left[ U_{\alpha 2} + \frac{aU_{e1}^*U_{e2}}{\Delta m_{21}^2 - \tilde{m}_0^2 + a|U_{e2}|^2} U_{\alpha 1} \right], & \tilde{U}_{\alpha 2} &= \frac{1}{N_2} \left[ U_{\alpha 2}^* - \frac{aU_{e1}U_{e2}^*}{\tilde{m}_0^2 + a|U_{e1}|^2} U_{\alpha 1}^* \right], \\ \tilde{U}_{\alpha 3} &= U_{\alpha 3}, & \tilde{U}_{\alpha 3} &= U_{\alpha 3}^*. \end{aligned} \quad (3.14)$$

Since matter effects do not depend on all elements of  $U_{\text{PMNS}}$  but only  $U_{ei}$ , the above expressions are simpler in the  $\alpha = e$  case. In particular, notice that both

$$\tilde{U}_{e1} = \frac{U_{e1}}{N_1} \frac{\tilde{m}_0^2}{a|U_{e1}|^2}, \quad \tilde{U}_{e2} = \frac{U_{e2}^*}{N_2} \frac{\tilde{m}_0^2}{\tilde{m}_0^2 + a|U_{e2}|^2}, \quad (3.15)$$

vanish if  $\Delta m_{21}^2 = 0$  for all  $a$ . This fact originates in the transmutation [20] of masses in vacuum to mixings in matter, leading to the absence of genuine CP violation in matter if  $\Delta m_{21}^2 = 0$ , even though there are three non-degenerate neutrino masses.

These expressions reproduce the right vacuum limit, as seen by developing  $|a| \ll \Delta m_{21}^2$ ,

$$\tilde{U}_{\alpha 1} = U_{\alpha 1} - \frac{a}{\Delta m_{21}^2} U_{e1} U_{e2}^* U_{\alpha 2}, \quad \tilde{U}_{\alpha 2} = U_{\alpha 2} + \frac{a}{\Delta m_{21}^2} U_{e2} U_{e1}^* U_{\alpha 1}. \quad (3.16)$$

A surprising result, however, appears when assuming  $\Delta m_{21}^2 \ll |a|$ ,

$$\tilde{U}_{\alpha 1} = \frac{|U_{e2}|}{\sqrt{1 - |U_{e3}|^2}} \left[ U_{\alpha 1} - \frac{U_{e1}}{U_{e2}} U_{\alpha 2} + \frac{\Delta m_{21}^2 U_{e1}}{a (1 - |U_{e3}|^2)^2} (U_{e1}^* U_{\alpha 1} + U_{e2}^* U_{\alpha 2}) \right], \quad (3.17a)$$

$$\tilde{U}_{\alpha 2} = \frac{|U_{e2}|}{\sqrt{1 - |U_{e3}|^2}} \left[ U_{\alpha 2} + \frac{U_{e1}^*}{U_{e2}^*} U_{\alpha 1} - \frac{\Delta m_{21}^2 U_{e1}^*}{a (1 - |U_{e3}|^2)^2} (U_{e2} U_{\alpha 1} - U_{e1} U_{\alpha 2}) \right], \quad (3.17b)$$

showing that  $\lim_{a \rightarrow 0} \lim_{\Delta m_{21}^2 \rightarrow 0} \tilde{U} \neq U_{\text{PMNS}} = \lim_{\Delta m_{21}^2 \rightarrow 0} \lim_{a \rightarrow 0} \tilde{U}$  ! This is strongly illustrated in the case  $\lim_{\Delta m_{21}^2 \rightarrow 0} \tilde{U}_{e1} = \lim_{\Delta m_{21}^2 \rightarrow 0} \tilde{U}_{e2} = 0 \forall a$ . The reason behind this subtlety is the following. Setting  $\Delta m_{21}^2 = 0$  in vacuum means that  $\nu_1$  and  $\nu_2$  are degenerate. Therefore, any two independent linear combinations of them can be chosen as basis states, which in the language of the standard parametrization would mean that  $\theta_{12}$  is nonphysical. Adding the matter potential to this system breaks the degeneracy: the arbitrariness in  $\theta_{12}$  is lost in favor of the eigenstates of the perturbation. Since the matter term in the neutrino Hamiltonian adds  $a > 0$  to the  $e$ -flavor component, this fact results in  $\tilde{\nu}_1$  and  $\tilde{\nu}_2$  such that  $\tilde{\nu}_2$  is mainly  $\nu_e$ , forcing the  $\tilde{U}_{e1} = 0$  we obtained. The change of sign in  $a$  for the antineutrino case forces analogously  $\tilde{\bar{\nu}}_1$  to be mainly  $\bar{\nu}_e$ , explaining the limit  $\tilde{\bar{U}}_{e2} = 0$ .

This behavior shows that the vacuum connection should be analyzed in the regime where  $|a| \ll \Delta m_{21}^2 \ll |\Delta m_{31}^2|$ . The definite  $a$ -parity of the two components of the CP asymmetry defined in the previous section forces the leading-order term in  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  to be independent of  $a$ , whereas  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is linear. To provide a precise description of  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  in this region, we keep  $|U_{e3}|^2$  terms in the leading order, as well as all linear terms in  $a/\Delta m_{21}^2$  and  $a/\Delta m_{31}^2$  in both the mass squared differences,

$$\Delta \tilde{m}_{21}^2 \approx \Delta m_{21}^2 - a(|U_{e1}|^2 - |U_{e2}|^2), \quad \Delta \tilde{m}_{31}^2 \approx \Delta m_{31}^2 - a|U_{e1}|^2, \quad \Delta \tilde{m}_{32}^2 \approx \Delta m_{32}^2 - a|U_{e2}|^2, \quad (3.18)$$

and the mixings,

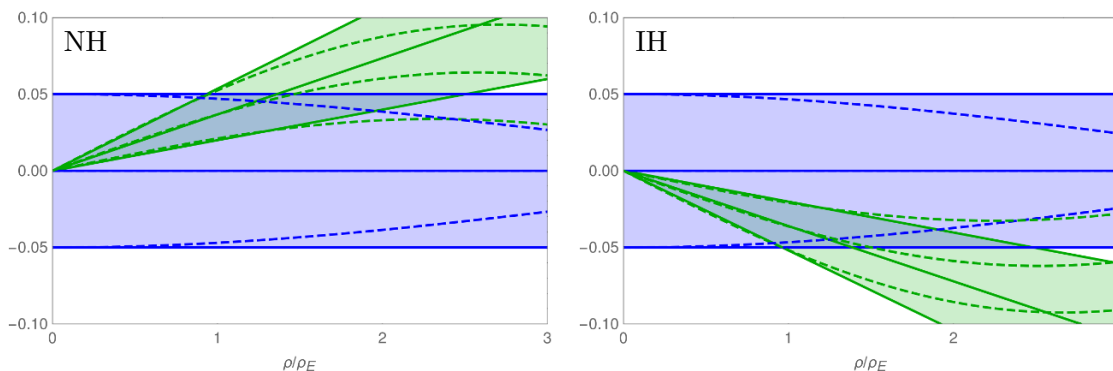
$$\tilde{U}_{\alpha 1} = U_{\alpha 1} - \frac{a U_{e1}}{\Delta m_{21}^2} U_{e2}^* U_{\alpha 2} - \frac{a U_{e1}}{\Delta m_{31}^2} U_{e3}^* U_{\alpha 3}, \quad (3.19a)$$

$$\tilde{U}_{\alpha 2} = U_{\alpha 2} + \frac{a U_{e2}}{\Delta m_{21}^2} U_{e1}^* U_{\alpha 1} - \frac{a U_{e2}}{\Delta m_{31}^2} U_{e3}^* U_{\alpha 3}, \quad (3.19b)$$

$$\tilde{U}_{\alpha 3} = U_{\alpha 3} + \frac{a U_{e3}}{\Delta m_{31}^2} U_{e1}^* U_{\alpha 1} + \frac{a U_{e3}}{\Delta m_{31}^2} U_{e2}^* U_{\alpha 2}. \quad (3.19c)$$

The CP asymmetry components  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  computed using these expressions are represented in figure 2, compared with the exact results, for both hierarchies at fixed  $E$  and  $L$  as functions of the matter potential.

The analytic approximations for constant  $\mathcal{A}_{\mu e}^{\text{T}}$  and linear  $\mathcal{A}_{\mu e}^{\text{CPT}}$  work well at low matter densities, as they should, but their range of validity is much larger than expected. For the values used in the figure, the point  $a = \Delta m_{21}^2$  corresponds to  $\rho = 0.44\rho_E$ , so the previous expansions should only work for  $\rho \ll 0.44\rho_E$ . The fact that they work reasonably well even above  $\rho_E$  hints that higher-order corrections are dominated by  $(a/\Delta m_{31}^2)^2$ .



**Figure 2.** CPT-odd (green) and T-odd (blue) components of  $\mathcal{A}_{\mu e}^{\text{CP}}$  as functions of the matter density  $\rho$  in units of that of the Earth mantle, at fixed  $(E, L) = (0.75 \text{ GeV}, 1300 \text{ km})$ . Both the exact (dashed) and the analytical (solid) results from eqs. (3.18), (3.19) are shown. Normal Hierarchy in the left panel, Inverted Hierarchy in the right panel. The bands correspond to all possible values changing  $\delta$  in  $(0, 2\pi)$ ; the upper/central/lower lines for  $\mathcal{A}_{\mu e}^{\text{CPT}}(\mathcal{A}_{\mu e}^{\text{T}})$  correspond to  $\cos \delta(\sin \delta) = -1, 0, 1$ .

This surprising feature stems from the fact that corrections  $(a/\Delta m_{21}^2)^2$  are inoperative in the region  $|a| \ll \Delta m_{21}^2 \ll |\Delta m_{31}^2|$  for the  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  observables. This behavior is explained by peculiar dependence on the mixings and masses of the oscillation probabilities, as can be understood from the matter-vacuum invariants we will exploit in the following sections for both  $\text{Im} \tilde{J}_{\alpha\beta}^{ij}$  (section 4) and  $\text{Re} \tilde{J}_{\alpha\beta}^{ij}$  (section 5). As will be discussed, they lead to dependencies in the oscillation probabilities in eq. (2.2) on the phases associated to the small quantities  $a$  and  $\Delta m_{21}^2$  of the form  $\frac{1}{\Delta} \sin \Delta$ , which cancel out if both of them are small, independently of whether  $|a| \ll \Delta m_{21}^2$  or  $\Delta m_{21}^2 \ll |a|$ . This cancellation will happen as long as  $\Delta = \frac{\epsilon L}{4E} \ll 1$ , for  $\epsilon = a, \Delta m_{21}^2$ . This peculiar dependence in the oscillation probabilities is responsible for the restoration of the commutability of the limits  $a \rightarrow 0$  and  $\Delta m_{21}^2 \rightarrow 0$  at this level, even though they do not commute at the mixings level.

### 3.3 Actual experiments: fixed $L$ in the Earth mantle and variable $E$

In the previous subsection we discussed the way to obtain analytic approximated expressions for neutrino oscillations in matter that reproduce the right vacuum limit, i.e. the limit when the matter parameter  $a \rightarrow 0$  at fixed energy due to the matter density going to zero.

In the following we consider the constant value of the matter density in the Earth mantle [21], and discuss dependencies in  $a$  as dependencies in the neutrino energy in  $\nu_\mu \rightarrow \nu_e$  transitions. In fact, the actual best-fit value [19] for  $\Delta m_{21}^2$  shows that the relation between  $a$  and  $\Delta m_{21}^2$  is given by  $|a| \approx 3(E/\text{GeV})\Delta m_{21}^2$ , so we can use eqs. (3.14) from the previous subsection expanding up to second order in  $\Delta m_{21}^2/a$ , with errors only  $\sim 3\%$  around 1 GeV.

As in eq. (2.2), all observable quantities can be written in terms of the rephasing-invariant mixings  $\tilde{J}_{\alpha\beta}^{ij}$ . Since  $\tilde{U}_{e1}$  in eq. (3.15) is a first order quantity, as we discussed, and expanding up to second order also in  $|U_{e3}| \ll 1$ , which is of the same size than  $\Delta m_{21}^2/a$ , we find that all  $\tilde{J}_{e\alpha}^{ij}$  can be calculated at second order in these two quantities using our

first-order  $\tilde{U}_{\alpha i}$  in eqs. (3.17),

$$\tilde{J}_{e\alpha}^{13} = \frac{\Delta m_{21}^2}{a} \left( |U_{e2}|^2 J_{e\alpha}^{13} - |U_{e1}|^2 J_{e\alpha}^{23} \right), \quad (3.20a)$$

$$\tilde{J}_{e\alpha}^{23} = J_{e\alpha}^{23} + J_{e\alpha}^{13} - \frac{\Delta m_{21}^2}{a} \left( |U_{e2}|^2 J_{e\alpha}^{13} - |U_{e1}|^2 J_{e\alpha}^{23} \right), \quad (3.20b)$$

$$\begin{aligned} \tilde{J}_{e\alpha}^{12} = & \frac{\Delta m_{21}^2}{a} \left[ |U_{e2}|^2 J_{e\alpha}^{12} - |U_{e1}|^2 J_{e\alpha}^{21} + |U_{e1}|^2 |U_{e2}|^2 \left( |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2 \right) \right] \\ & - \left[ \frac{\Delta m_{21}^2}{a} \right]^2 |U_{e1}|^2 |U_{e2}|^2 (1 - |U_{\alpha 3}|^2). \end{aligned} \quad (3.20c)$$

However, as discussed in the previous subsection, the definite odd  $a$ -parity of the CPT component of the CP asymmetry implies that linear terms in  $a/\Delta m_{31}^2$  are relevant to describe  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$ , so we must keep them as well. These linear terms can be easily calculated setting  $\Delta m_{21}^2 \rightarrow 0$  in the eigenstates in eq. (3.5). Analogously, we obtain linear corrections in  $\Delta m_{21}^2/\Delta m_{31}^2$  to the previous  $\tilde{J}_{\alpha\beta}^{ij}$  setting  $a \rightarrow 0$  in the eigenstates. The resulting rephasing-invariant mixings, written in the standard parametrization for  $\alpha = \mu$ , which is the relevant transition for accelerator experiments, are

$$\tilde{J}_{e\mu}^{13} = - \left( \frac{\Delta m_{21}^2}{a} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right) c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} e^{i\delta}, \quad (3.21a)$$

$$\tilde{J}_{e\mu}^{23} = -c_{13}^2 s_{13}^2 s_{23}^2 \left( 1 + \frac{2a}{\Delta m_{31}^2} \right) + \left( \frac{\Delta m_{21}^2}{a} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right) c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} e^{i\delta}, \quad (3.21b)$$

$$\tilde{J}_{e\mu}^{12} = \left( \frac{\Delta m_{21}^2}{a} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \right) c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} e^{i\delta} - \left[ \frac{\Delta m_{21}^2}{a} \right]^2 c_{12}^2 s_{12}^2 c_{23}^2. \quad (3.21c)$$

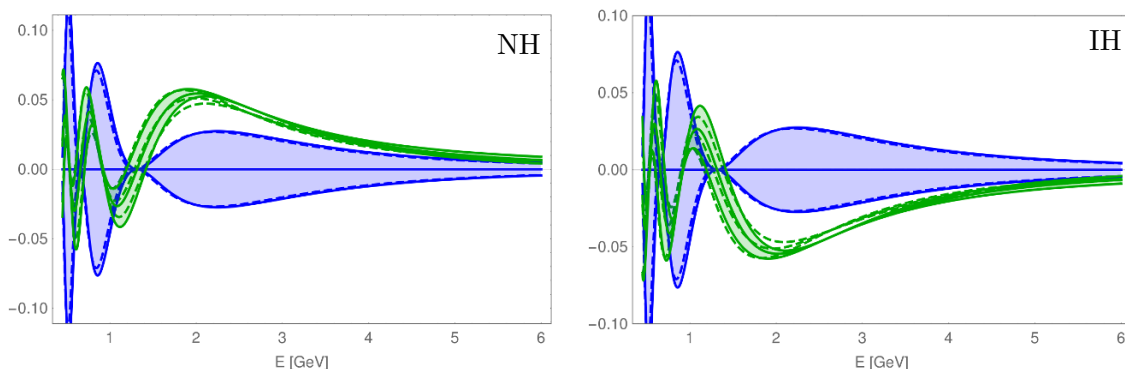
We find in the rephasing-invariant mixings of eqs. (3.21) the four observable reparametrization invariants  $\mathcal{J} \equiv J_r \sin \delta = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin \delta$ ,  $R \equiv J_r \cos \delta$ ,  $S \equiv c_{13}^2 s_{13}^2 s_{23}^2$  and  $T \equiv c_{12}^2 s_{12}^2 c_{23}^2$ . Notice that these are the same results found in ref. [15] after further expanding in  $|a| \ll |\Delta m_{31}^2|$ , as expected. Since  $|\Delta m_{31}^2| \approx 33\Delta m_{21}^2$ , it turns out that  $|\Delta m_{31}^2| \approx 11|a|/(E/\text{GeV})$ , so expanding in  $|a| \ll |\Delta m_{31}^2|$  around the  $E \sim \text{GeV}$  region is as reasonable as expanding in  $|U_{e3}| \ll 1$ . All  $\tilde{J}_{\mu e}^{ij}$  are already second order in  $\Delta m_{21}^2$  and  $|U_{e3}|$ , so we can neglect them in the oscillation arguments,

$$\begin{aligned} \Delta \tilde{m}_{21}^2 &\approx a, & \Delta \tilde{m}_{31}^2 &\approx \Delta m_{31}^2, & \Delta \tilde{m}_{32}^2 &\approx \Delta m_{31}^2 - a, \\ \Delta \tilde{m}_{21}^2 &\approx |a|, & \Delta \tilde{m}_{31}^2 &\approx \Delta m_{31}^2 + |a|, & \Delta \tilde{m}_{32}^2 &\approx \Delta m_{31}^2. \end{aligned} \quad (3.22)$$

In this regime, the only oscillation phases are the vacuum phase  $\Delta_{31} \propto L/E$  and the constant (for a given baseline through the Earth mantle)

$$A \equiv \frac{aL}{4E} = 3.8 \Delta m_{21}^2 (\text{eV}^2) L(\text{km}) = 0.29 \frac{L}{1000 \text{ km}}. \quad (3.23)$$

This value is not particularly small at long baselines, but we remind the reader that both  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  have definite parity in  $a$ , the first one being odd and the second one even, as we proved in section 2. This means that corrections to the leading order in each component of the CP asymmetry will be quadratic in  $a$ , and so we can also expand up to leading order.



**Figure 3.** CPT-odd (green) and T-odd (blue) components of  $\mathcal{A}_{\mu e}^{\text{CP}}$  as functions of the neutrino energy  $E$  through the Earth mantle (of constant density) and a baseline of  $L = 1300$  km. Both the exact (dashed) and the analytical (solid) results from eqs. (3.25) are shown. Normal/Inverted Hierarchy in the left/right panel. The bands correspond to all possible values changing  $\delta$  in  $(0, 2\pi)$ ; the upper/central/lower lines for  $\mathcal{A}_{\mu e}^{\text{CPT}}(\mathcal{A}_{\mu e}^{\text{T}})$  correspond to  $\cos \delta(\sin \delta) = -1, 0, 1$ .

In summary, the expansion quantities used are the phase  $A$  and, up to second order,

$$\frac{\Delta m_{21}^2}{\Delta m_{31}^2} \sim 0.030, \quad \frac{\Delta m_{21}^2}{a} \sim \frac{0.34}{E/\text{GeV}}, \quad \frac{a}{\Delta m_{31}^2} \sim 0.091 E/\text{GeV}, \quad |U_{e3}| \sim 0.15. \quad (3.24)$$

Taking into account the rephasing-invariant mixings (3.21), with the symmetry property  $\tilde{J}_{\mu e}^{ij} = \tilde{J}_{e\mu}^{ji}$ , and the mass differences in matter (3.22), we find

$$\mathcal{A}_{\mu e}^{\text{CPT}} = 16 A \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] (S \sin \Delta_{31} + J_r \cos \delta \Delta_{21} \cos \Delta_{31}) + \mathcal{O}(A^3), \quad (3.25a)$$

$$\mathcal{A}_{\mu e}^{\text{T}} = -16 J_r \sin \delta \Delta_{21} \sin^2 \Delta_{31} + \mathcal{O}(A^2), \quad (3.25b)$$

where  $S \equiv c_{13}^2 s_{13}^2 s_{23}^2$ ,  $J_r \equiv c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23}$ ,  $A \equiv \frac{aL}{4E} \propto L$  and the two  $\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E} \propto L/E$ . From these expressions, which are precise enough to provide understanding of the physics behind these observables, we find that  $\mathcal{A}_{\mu e}^{\text{T}}$  in matter is well described by its vacuum value. Since  $\Delta_{21}$  is small, this means that  $\mathcal{A}_{\mu e}^{\text{T}}$  oscillates as  $\frac{1}{E} \sin^2 \Delta_{31}$ .  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , which vanishes when  $a \rightarrow 0$ , is very well described by its leading (first) order in  $a$ .

The agreement of eqs. (3.25) with the exact result is shown in figure 3, which makes clear that, even if the value of the asymmetries in the maxima are a bit off, their position and the general behavior are well reproduced. Therefore, eqs. (3.25) are the perfect tool to understand the energy dependencies of the two disentangled components  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  of the CP asymmetry and search for their actual experimental separation.

#### 4 A closer look at the genuine CPV component

The last term in eq. (2.1) indicates that the Hamiltonian of our problem in the flavor basis is proportional to the hermitian mass matrix squared in matter

$$2E H \equiv H' = \tilde{M}_\nu \tilde{M}_\nu^\dagger. \quad (4.1)$$

In such a basis, the necessary and sufficient condition for CP invariance is [22]

$$\text{Im}[H'_{e\mu}H'_{\mu\tau}H'_{\tau e}] = 0. \quad (4.2)$$

For any flavor-diagonal interaction of neutrinos with matter, the last condition is equal to that for neutrino mass matrices in vacuum. This invariance [23] of the left-hand side of the last equation (4.2) between the CP behavior of neutrinos in vacuum and in matter has far-reaching consequences for the observable rephasing-invariant mixings of neutrinos  $\tilde{J}_{\alpha\beta}^{ij}$  and antineutrinos  $\tilde{\tilde{J}}_{\alpha\beta}^{ij}$  in matter.

The explicit calculation of the matter-vacuum invariant genuine CP violation expression for neutrinos, antineutrinos and in vacuum leads to

$$\Delta\tilde{m}_{12}^2\Delta\tilde{m}_{23}^2\Delta\tilde{m}_{31}^2\tilde{\mathcal{J}} = \Delta\tilde{m}_{12}^2\Delta\tilde{m}_{23}^2\Delta\tilde{m}_{31}^2\tilde{\tilde{\mathcal{J}}} = \Delta m_{12}^2\Delta m_{23}^2\Delta m_{31}^2\mathcal{J}, \quad (4.3)$$

where  $\mathcal{J} = c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}\sin\delta$  is the rephasing-invariant CPV quantity in vacuum [24]. The proportionality of  $\tilde{\mathcal{J}}$  and  $\tilde{\tilde{\mathcal{J}}}$  to  $\Delta m_{21}^2$  explains the absence of genuine CP violation in matter in the limit of vanishing  $\Delta m_{21}^2$ , even in the presence of three non-degenerate neutrinos and antineutrinos in matter. The vanishing of  $\tilde{\mathcal{J}}$  and  $\tilde{\tilde{\mathcal{J}}}$  in this limit comes from the transmutation of masses in vacuum to mixings in matter calculated in section 3.2, leading to  $\tilde{U}_{e1} = 0$  and  $\tilde{U}_{e2} = 0$ . To leading order in  $\Delta m_{21}^2$ , the non-vanishing  $\tilde{\mathcal{J}}$  and  $\tilde{\tilde{\mathcal{J}}}$  differ by linear terms in the matter potential  $a$  present in the neutrino masses in matter.

Using the analytic perturbation expansion of section 3 for the connection between quantities in matter and in vacuum, we can write

$$\tilde{\mathcal{J}} = \frac{\Delta m_{21}^2 [\Delta m_{31}^2 + a]}{\Delta m_{31}^2 [\Delta m_{21}^2 - 2\tilde{m}_0^2 + a(1 - |U_{e3}|^2)]} \mathcal{J}, \quad a \geq 0, \quad (4.4a)$$

$$\tilde{\tilde{\mathcal{J}}} = \frac{\Delta m_{21}^2 [\Delta m_{31}^2 + a]}{\Delta m_{31}^2 [2\tilde{m}_0^2 - \Delta m_{21}^2 - a(1 - |U_{e3}|^2)]} \mathcal{J}, \quad a \leq 0, \quad (4.4b)$$

Notice that the proportionality factors in eqs. (4.4) are neutrino energy dependent through  $a$ , as shown in figure 4. The behavior at low/high energies can be easily understood using the expansions at leading order of  $\tilde{m}_0^2$  and  $\tilde{m}_0^2$  in eqs. (3.9), (3.10). Indeed, at low energies

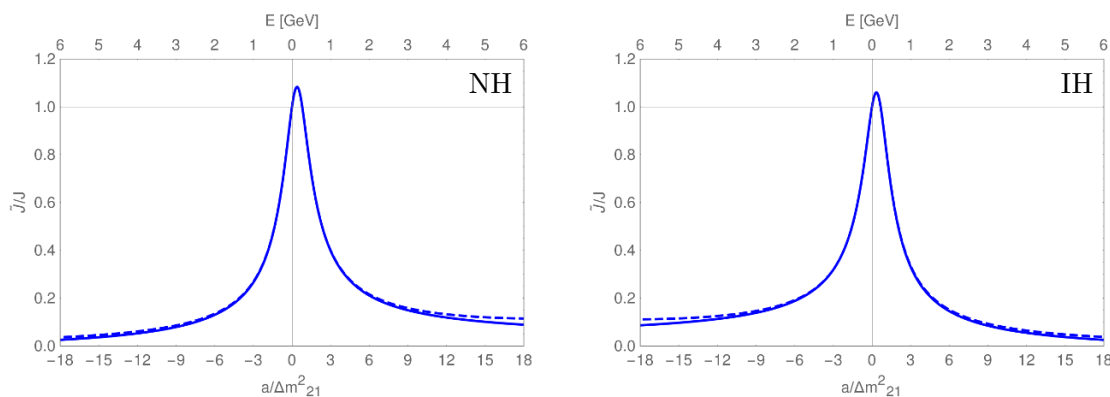
$$\tilde{\mathcal{J}} \approx \mathcal{J} \left[ 1 + \frac{a(|U_{e1}|^2 - |U_{e2}|^2)}{\Delta m_{21}^2} + \frac{a}{\Delta m_{31}^2} \right] > \mathcal{J}, \quad (4.5a)$$

$$\tilde{\tilde{\mathcal{J}}} \approx \mathcal{J} \left[ 1 - \frac{|a|(1 - |U_{e2}|^2)}{\Delta m_{21}^2} - \frac{|a|}{\Delta m_{31}^2} \right] < \mathcal{J}, \quad (4.5b)$$

the ratio increases (decreases) with respect to 1 for (anti)neutrinos independently of  $\text{sign}(\Delta m_{31}^2)$  due to  $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$ , whereas at high energies

$$\tilde{\mathcal{J}} \approx \mathcal{J} \frac{\Delta m_{21}^2 (\Delta m_{31}^2 + a)}{a \Delta m_{31}^2}, \quad (4.6a)$$

$$\tilde{\tilde{\mathcal{J}}} \approx \mathcal{J} \frac{\Delta m_{21}^2 (\Delta m_{31}^2 - |a|)}{|a| \Delta m_{31}^2}, \quad (4.6b)$$



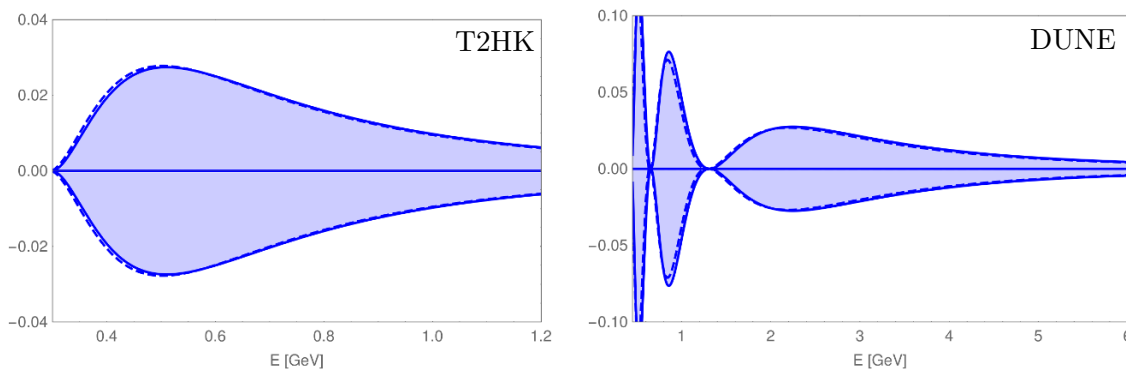
**Figure 4.**  $\tilde{\mathcal{J}}/\mathcal{J}$  ratio for both neutrinos ( $a > 0$ ) and antineutrinos ( $a < 0$ ). The horizontal axis shows both the evolution of the matter parameter  $a$  at fixed energy (lower labels), i.e. changing the matter density, and as function of the energy if the constant density is chosen as that of the Earth mantle (upper labels). Both the exact (dashed) and the analytical (solid) results from eqs. (4.4) are shown. Normal Hierarchy in the left panel, Inverted Hierarchy in the right panel.

both of them decrease roughly as  $1/a$ , and changing the sign( $\Delta m_{31}^2$ ) is equivalent to changing the sign( $a$ ), which explains why the two plots in figure 4 seem to be symmetrical.

The decreasing value of the  $\tilde{\mathcal{J}}/\mathcal{J}$  ratio with higher energies described by eqs. (4.6), i.e. when  $|a| \gg \Delta m_{21}^2$ , is a consequence of the absence of genuine CP violation in matter in the limit  $\Delta m_{21}^2 = 0$  even if there are three non-vanishing neutrino masses in matter. The transmutation of masses in vacuum to mixings in matter forces the smallness of the imaginary part of the rephasing-invariant mixing in matter at high energies. However, this fact does not necessarily mean that genuine CP violation is unobservable at these energies, since the genuine CPV component of the CP asymmetry contains this energy-dependent factor together with the matter-dependent oscillation function —odd in  $L$ — that depends on both energy and baseline. The effects of the baseline are shown in figure 5, comparing the whole  $\mathcal{A}_{\mu e}^T$  as function of the energy for T2HK  $L = 295$  km and DUNE  $L = 1300$  km, where it is seen that the oscillation amplitude of each of them (fixed  $L$ ) decreases as  $1/E$ , as expected, and a higher baseline (at fixed  $E$ ) enhances the values of  $\mathcal{A}_{\mu e}^T$ .

This behavior is understood with the perturbation expansion in  $|U_{e3}|^2 \ll 1$  in the energy regime between the two MSW resonances,  $\Delta m_{21}^2 \ll |a| \ll |\Delta m_{31}^2|$ , that we performed in the previous section. The  $1/E$  dependence in  $\tilde{\mathcal{J}}$  is changed by the approximated oscillating functions into  $L/E$ , producing genuine CPV components of the same size at the spectrum peak of both experiments. In fact, the matter effects in  $\tilde{\mathcal{J}}$  and oscillating phases just compensate to generate in this approximation a genuine CPV asymmetry equal to that in vacuum, i.e. eq. (3.25b). As such, it is odd in  $L/E$ , independent of  $a$  and the Hierarchy, and proportional to  $\sin \delta$ .





**Figure 5.** Energy distribution of the T-odd component of  $\mathcal{A}_{\mu e}^{\text{CP}}$  at T2HK  $L = 295$  km (left) and DUNE  $L = 1300$  km (right), which is Hierarchy independent. Both the exact (dashed) and the analytical (solid) results from eq. (3.25b) are shown. The bands correspond to all possible values changing  $\delta$  in  $(0, 2\pi)$ ; the upper/central/lower lines correspond to  $\sin \delta = -1, 0, 1$ . Notice the different energy range and scale of the asymmetry between T2HK and DUNE plots.

## 5 Neutrino mass ordering discrimination

Last section has demonstrated that the genuine  $\mathcal{A}_{\mu e}^{\text{T}}$  component of the CP asymmetry in matter is, to a good approximation for energies —as planned in accelerator facilities— between the two resonances  $\Delta m_{21}^2 \ll |a| \ll |\Delta m_{31}^2|$ , given by the vacuum CP asymmetry. Its information content is then crucial to identify experimental signatures of genuine CPV. On the other hand, it has nothing to say about the neutrino mass ordering: it is invariant under the change of sign in  $\Delta m_{31}^2$ . This simple change of sign, without changing the absolute value  $|\Delta m_{31}^2|$ , is in fact the only effect of changing the hierarchy under the approximations leading to eqs. (3.25).

This section discusses the information on the neutrino mass ordering contained in  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , which is even in  $L$  and  $\sin \delta$  and odd in  $a$ . Propagation in matter is needed to generate effects of the change of hierarchy and our  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is able to separate out this information, going beyond studies of its influence on transition probabilities [25].

There is no simple matter-vacuum relation such as eq. (4.3) to easily write  $\text{Re} \tilde{J}_{\alpha\beta}^{ij}$  as function of the vacuum  $\text{Re} J_{\alpha\beta}^{ij}$  —the most compact result following this idea is [26, 27]

$$\Delta \tilde{m}_{12}^2 \Delta \tilde{m}_{23}^2 \Delta \tilde{m}_{31}^2 \Delta \tilde{m}_{ij}^2 \text{Re} \tilde{J}_{\alpha\beta}^{ij} = K_{\alpha\beta}^{ij} + \Delta m_{12}^2 \Delta m_{23}^2 \Delta m_{31}^2 \Delta m_{ij}^2 \text{Re} J_{\alpha\beta}^{ij}, \quad (5.1)$$

where all  $K_{\alpha\beta}^{ij}$  vanish in vacuum. This relation explains the dependence of all  $L$ -even terms in the oscillation probabilities in each of the  $\tilde{\Delta}_{ij}$  phases as  $\frac{1}{\tilde{\Delta}_{ij}^2} \sin^2 \tilde{\Delta}_{ij}$ , which is the reason why the vacuum limit  $a \rightarrow 0$  is restored in these observables even after taking  $\Delta m_{21}^2 \ll |a|$ , as discussed in section 3. However, the  $K_{\alpha\beta}^{ij}$  are complicated functions of the vacuum quantities, and do not provide a clear insight into the behavior of  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$ , so we will use eq. (3.25a) instead.

In general, this matter-induced component of the CP asymmetry has no definite transformation properties under the change of sign in  $\Delta m_{31}^2$ . Under the approximations made in

section 3, there are two distinct terms in  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , a first one  $\mathcal{A}_-^{\text{CPT}}$  which is an odd function of  $\Delta m_{31}^2$  and a second one  $\mathcal{A}_+^{\text{CPT}}$  which is an even function of  $\Delta m_{31}^2$ ,

$$\mathcal{A}_{\mu e}^{\text{CPT}} = \mathcal{A}_-^{\text{CPT}} + \mathcal{A}_+^{\text{CPT}},$$

$$\mathcal{A}_-^{\text{CPT}} = 16 A \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] S \sin \Delta_{31}, \quad (5.2a)$$

$$\mathcal{A}_+^{\text{CPT}} = 16 A \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] J_r \cos \delta \Delta_{21} \cos \Delta_{31}. \quad (5.2b)$$

Notice that both terms, as well as the whole  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , vanish simultaneously when the  $\delta$ -independent common prefactor vanishes. Alternatively,  $\mathcal{A}_{\mu e}^{\text{CPT}}$  vanishes  $\delta$ -dependently when these  $\mathcal{A}_-^{\text{CPT}}$  and  $\mathcal{A}_+^{\text{CPT}}$  terms compensate each other.

As seen, the information content in  $\mathcal{A}_{\mu e}^{\text{CPT}}$  on the neutrino mass hierarchy is due to  $\mathcal{A}_-^{\text{CPT}}$ , its dominant zeroth-order term in  $\Delta m_{21}^2$ , independent of the phase  $\delta$ . In the limit  $\Delta m_{21}^2 \rightarrow 0$ , our results from eq. (3.22) in section 3.3 show that the mass spectrum in matter changes under a change of hierarchy from neutrinos to antineutrinos as

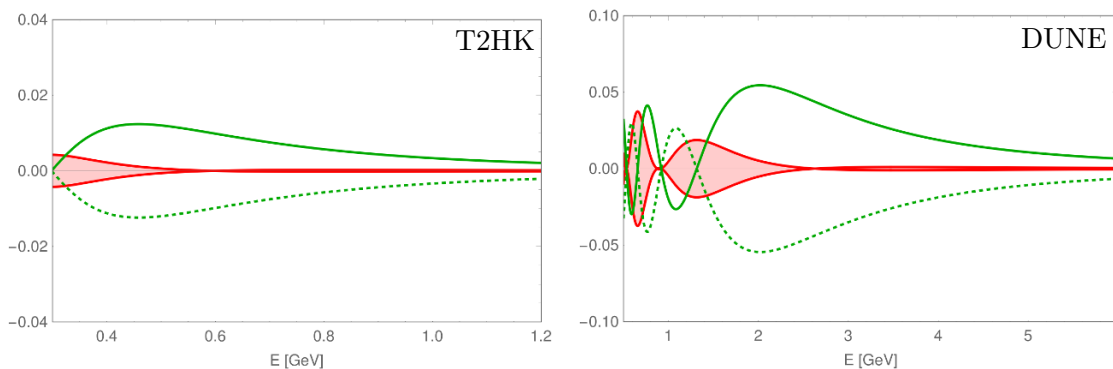
$$\Delta \tilde{m}_{21}^2 \leftrightarrow \Delta \tilde{m}_{21}^2, \quad \Delta \tilde{m}_{31}^2 \leftrightarrow -\Delta \tilde{m}_{32}^2, \quad \Delta \tilde{m}_{32}^2 \leftrightarrow -\Delta \tilde{m}_{31}^2, \quad (5.3)$$

whereas the  $\tilde{J}_{\alpha\beta}^{ij}$  do not change sign, so all  $L$ -even terms in the oscillation probabilities — which are blind to the sign change in eq. (5.3) — are simply interchanged between neutrinos and antineutrinos. As the CP asymmetry is a difference between neutrino and antineutrino oscillation probabilities, we discover that  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is only changing its sign under a change of hierarchy in the vanishing limit of  $\Delta m_{21}^2$ .

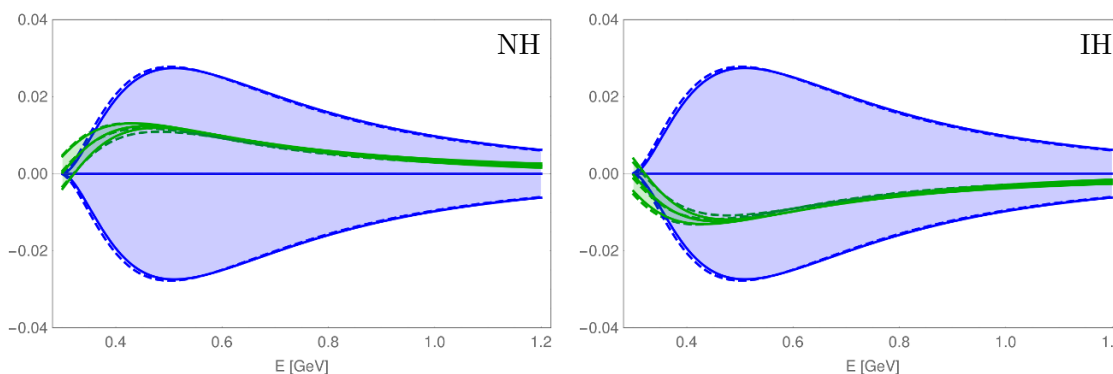
The  $\mathcal{A}_+^{\text{CPT}}$  term in eqs. (5.2) is appreciable only at low energies, needing a non-vanishing  $\Delta m_{21}^2$  and then sensitive to the  $\delta$  phase as a CP conserving  $\cos \delta$  factor. In figure 6 we represent these two components of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  as function of  $E$  for the baselines of T2HK and DUNE for Normal and Inverted Hierarchies. To test the neutrino mass ordering from  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , we find that imposing the condition  $|\mathcal{A}_-^{\text{CPT}}| > |\mathcal{A}_+^{\text{CPT}}|$  in the non-oscillating (high energy) region leads to  $E > 1.1 E_{1\text{st node}}$ . For these energies above the first node of the vacuum oscillation probability, the whole effect of the change of sign in  $\Delta m_{31}^2$  is an almost odd  $\mathcal{A}_{\mu e}^{\text{CPT}}$ . In addition, the  $\mathcal{A}_-^{\text{CPT}}$  term in  $\mathcal{A}_{\mu e}^{\text{CPT}}$  dominates the whole CP asymmetry at long baselines, as seen in figure 3, so the measurement of the sign of  $\mathcal{A}_{\mu e}^{\text{CP}}$  at these energies fixes the hierarchy.

## 6 Signatures of the peculiar energy dependencies

In this section we identify those aspects of the energy distribution of the CP asymmetry that can offer an experimental signature for the separation of its genuine and matter-induced components. With experiments in which the fingerprint of the baseline dependence,  $L$ -odd and  $L$ -even functions, cannot be used, the peculiar patterns of the energy distribution provide precious information. The general trend of this dependence for  $L = 1300$  km is given in figure 3, showing the appearance of oscillations in the low and medium energy regions of the spectrum with different behavior for the two components  $\mathcal{A}_{\mu e}^{\text{T}}$  and  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , where nodes and extremal values are at different energies.



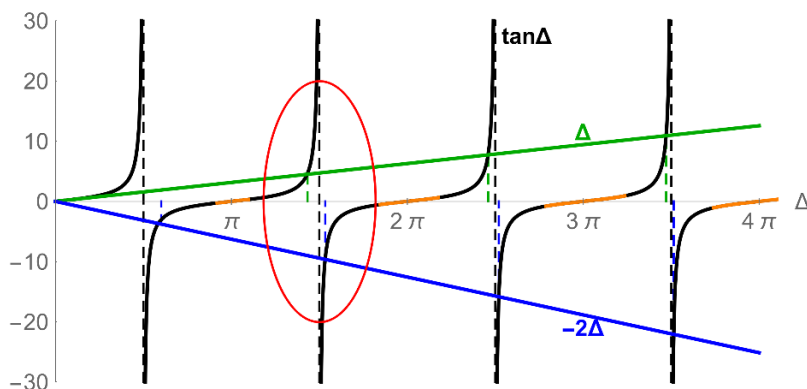
**Figure 6.** Energy distribution of the two distinct terms of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  as defined in eqs. (5.2),  $\mathcal{A}^{\text{CPT}}$  (green,  $\delta$ -independent, hierarchy-odd) and  $\mathcal{A}_+^{\text{CPT}}$  (red,  $\cos \delta$ -odd, hierarchy-invariant), at T2HK  $L = 295$  km (left) and DUNE  $L = 1300$  km (right). Both Normal Hierarchy (solid) and Inverted Hierarchy (dashed) shown. The bands correspond to all possible values changing  $\delta$  in  $(0, 2\pi)$ . Notice the different energy range and scale of the asymmetry between T2HK and DUNE plots.



**Figure 7.** CPT-odd (green) and T-odd (blue) components of  $\mathcal{A}_{\mu e}^{\text{CP}}$  as functions of the neutrino energy  $E$  through the Earth mantle (of constant density) at T2HK baseline  $L = 295$  km. Both the exact (dashed) and the analytical (solid) results from eqs. (3.25) are shown. Normal/Inverted Hierarchy in the left/right panel. The bands correspond to all possible values changing  $\delta$  in  $(0, 2\pi)$ ; the upper/central/lower lines for  $\mathcal{A}_{\mu e}^{\text{CPT}}(\mathcal{A}_{\mu e}^{\text{T}})$  correspond to  $\cos \delta(\sin \delta) = -1, 0, 1$ .

However, this rich structure is lost when the baseline is decreased to  $L = 295$  km and a threshold energy of 300 MeV is imposed. The emerging picture under these conditions is given in figure 7 and the main conclusion is the relative suppression of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  with respect to  $\mathcal{A}_{\mu e}^{\text{T}}$ , due to its proportionality to  $A \propto L$ . In addition, this small  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is mainly the  $\delta$ -independent  $\mathcal{A}_-^{\text{CPT}}$  in eq. (5.2a) and so it can be subtracted away from the experimental  $\mathcal{A}_{\mu e}^{\text{CP}}$ , if the neutrino mass hierarchy is previously known, as a theoretical background. This would allow to separate the genuine  $\mathcal{A}_{\mu e}^{\text{T}}$  component.

Using the analytical approximate expressions of the observable components given in eqs. (3.25), we perform a detailed study of the position of extremal values and zeros of each of them, as well as their behavior around the zeros. The energy dependence is controlled by the phase  $\Delta_{31} \propto 1/E$  and we will take as reference the functional form of the CP-conserving transition probability  $f(\Delta) = \sin^2 \Delta$ .



**Figure 8.** Illustration of the position of the relevant zeros of  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , given by  $\tan \Delta_{31} = \Delta_{31}$ , and the maxima of  $|\mathcal{A}_{\mu e}^{\text{T}}|$ , given by  $\tan \Delta_{31} = -2\Delta_{31}$ . The vertical dashed lines are the asymptotes of  $\tan \Delta$  (black), corresponding to oscillation maxima, and the perturbative solutions of the previous equations,  $\Delta_0^{\text{CPT}} = (2n + 1)\frac{\pi}{2} - [(2n + 1)\frac{\pi}{2}]^{-1}$  (green) and  $\Delta_{\text{max}}^{\text{T}} = (2n + 1)\frac{\pi}{2} + \frac{1}{2} [(2n + 1)\frac{\pi}{2}]^{-1}$  (blue). As calculated in eq. (6.5), the  $\delta$ -dependent zeros of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  around  $\Delta_0 = n\pi$  are bounded within the orange regions of  $\tan \Delta$ . The highest-energy point, i.e. smallest  $\Delta$ , where  $\mathcal{A}_{\mu e}^{\text{CPT}}$  vanishes  $\delta$ -independently is emphasized by the red ellipse.

For the genuine component  $\mathcal{A}_{\mu e}^{\text{T}}$ , the energy distribution is

$$f_{\text{T}}(\Delta) = \Delta \sin^2 \Delta. \quad (6.1)$$

Contrary to  $f(\Delta)$ , the amplitude of the oscillations of  $f_{\text{T}}(\Delta)$  decreases as  $1/E$ , but the zeros are the same  $\Delta_0 = 0, \pi, 2\pi \dots$  as for  $f(\Delta)$ . There are, however, two series of extremal values. The first kind, those corresponding to solutions of  $\sin \Delta = 0$  as for the zeros, are double zeros, which indicates that  $\mathcal{A}_{\mu e}^{\text{T}}$  keeps the same sign around the zeros, and so in the whole energy spectrum. From eq. (3.25b) it is clear that the sign is given by  $\text{sign}(\mathcal{A}_{\mu e}^{\text{T}}) = -\text{sign}(\sin \delta)$ .

The additional extremal values appear for

$$\tan \Delta + 2\Delta = 0. \quad (6.2)$$

In figure 8 we identify graphically the solutions to eq. (6.2), which appear slightly above the oscillation maxima  $\Delta_{\text{max}}^{\text{osc}} = (2n + 1)\frac{\pi}{2}$ . This is a first fortunate fact, implying that the experimental configurations with maximal  $|\mathcal{A}_{\mu e}^{\text{T}}|$  are close to those with highest statistics. A perturbative expansion of  $\cot \Delta$  around  $\Delta_{\text{max}}^{\text{osc}}$  leads to the approximate solutions

$$\begin{aligned} \Delta_{\text{max}}^{\text{T}} &= (2n + 1)\frac{\pi}{2} + \frac{1}{2} \left[ (2n + 1)\frac{\pi}{2} \right]^{-1} + \dots, \quad n \geq 0 \\ &\approx \frac{\pi}{2} + \frac{1}{\pi}, \frac{3\pi}{2} + \frac{1}{3\pi}, \frac{5\pi}{2} + \frac{1}{5\pi} \dots, \end{aligned} \quad (6.3)$$

which show that the interesting (see below) second and higher maxima in  $|\mathcal{A}_{\mu e}^{\text{T}}|$  are within a 3% interval above the oscillation maxima.

In the case of the matter-induced  $\mathcal{A}_{\mu e}^{\text{CPT}}$  component of the CP asymmetry, the energy distribution in eq. (3.25a) is

$$f_{\text{CPT}}(\Delta) = \left( \frac{\sin \Delta}{\Delta} - \cos \Delta \right) \left( S \sin \Delta + J_r \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \cos \delta \Delta \cos \Delta \right), \quad (6.4)$$

which has two kinds of zeros with distinct implications. The vanishing condition for the second factor are the  $\delta$ -dependent solutions of

$$\tan \Delta = -\frac{J_r}{S} \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \cos \delta \Delta = -0.09 \cos \delta \Delta, \quad (6.5)$$

that reduce to vacuum nodes  $\sin \Delta = 0$  if  $\cos \delta = 0$ , where  $\mathcal{A}_{\mu e}^{\text{T}}$  also vanishes. The actual position of these zeros is strongly dependent on  $\cos \delta$ , and the set of solutions is illustrated in figure 8 by the region around  $\Delta_0 = n\pi$  where  $\tan \Delta$  is orange.

The second kind of zeros in eq. (6.4) are solutions of the equation

$$\tan \Delta = \Delta, \quad (6.6)$$

and are graphically depicted in figure 8 too. As seen, they appear slightly below the oscillation maxima in  $f(\Delta)$  starting from the second one, with approximate values

$$\begin{aligned} \Delta_0^{\text{CPT}} &= (2n+1)\frac{\pi}{2} - \left[ (2n+1)\frac{\pi}{2} \right]^{-1} + \dots, \quad n \geq 1 \\ &\approx \frac{3\pi}{2} - \frac{2}{3\pi}, \quad \frac{5\pi}{2} - \frac{2}{5\pi}, \dots, \end{aligned} \quad (6.7)$$

which almost coincide with the maxima  $\Delta_{\text{max}}^{\text{T}}$  in eq. (6.3) of  $|\mathcal{A}_{\mu e}^{\text{T}}|$ . Not only that: these zeros  $\Delta_0^{\text{CPT}}$  of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  are again near the oscillation maxima  $\Delta_{\text{max}}^{\text{osc}} = (2n+1)\frac{\pi}{2}$ , so we conclude that there are “magic energies” at these (6.7) phase values, within a 5% interval below the corresponding oscillation maximum, in which  $\mathcal{A}_{\mu e}^{\text{CPT}}$  vanishes and  $|\mathcal{A}_{\mu e}^{\text{T}}|$  is close to a maximum. These magic points have additional bonuses: i) the zero of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is independent of  $\cos \delta$ , providing no ambiguity in its position; ii) these are simple zeros, in such a way that the sign of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is changing around them; iii) although  $|\mathcal{A}_{\mu e}^{\text{T}}|$  is not exactly at its maximum value when  $\mathcal{A}_{\mu e}^{\text{CPT}} = 0$ , the leading order deviations from  $\Delta_{\text{max}}^{\text{osc}}$  we calculated show that its value is above 90%  $|\mathcal{A}_{\mu e}^{\text{T}}|_{\text{max}}$ . A look into the derivative of  $f_{\text{CPT}}(\Delta)$  shows that the sign-change of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  around these zeros is such that  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is always decreasing (increasing) around the relevant  $\delta$ -independent zeros for Normal (Inverted) Hierarchy, and opposite around  $\delta$ -dependent zeros.

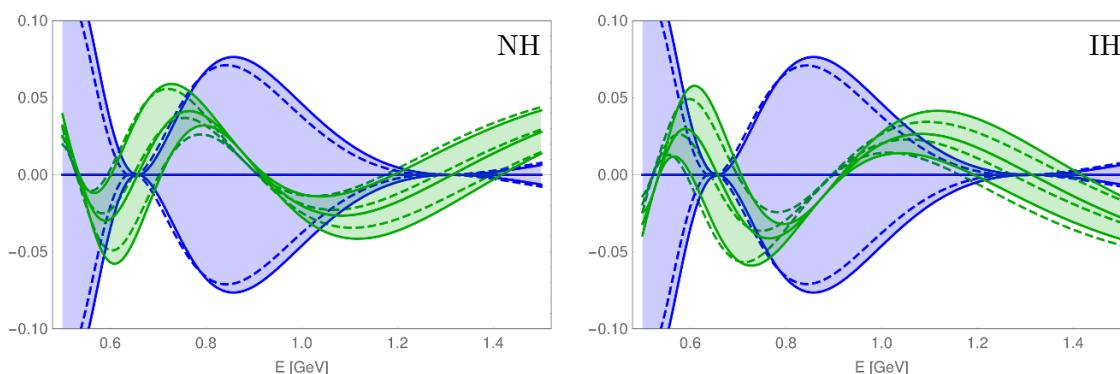
Taking into account the dependence in  $L/E$  of these remarkable values of the phases, we give in table 2 the relevant energies around the second oscillation maximum for both the baselines of the T2HK and DUNE experiments. The precise position of this energy, which is slightly above the second oscillation maximum, is proportional to  $L|\Delta m_{31}^2|$  as

$$E = 0.92 \text{ GeV} \frac{L}{1300 \text{ km}} \frac{|\Delta m_{31}^2|}{2.5 \times 10^{-3} \text{ eV}^2}, \quad (6.8)$$

which explains the absence of this rich oscillatory structure in figure 7: at the short baseline of T2HK, all interesting points lie below the threshold energy of 300 MeV.

	$\Delta_{31}$	$\frac{L}{E} \left( \frac{\text{km}}{\text{GeV}} \right)$	$E$ (GeV)	
			T2HK	DUNE
Vanishing $\mathcal{A}_{\mu e}^{\text{CPT}}$	4.50	1420	0.21	0.92
2 <sup>nd</sup> Oscillation Maximum	4.71	1480	0.20	0.88
Maximum $ \mathcal{A}_{\mu e}^{\text{T}} $	4.82	1520	0.19	0.86

**Table 2.** Specific position of the first zero  $\Delta_0^{\text{CPT}}$  in eq. (6.7), the second vacuum oscillation maximum and the second maximum  $\Delta_{\text{max}}^{\text{T}}$  in eq. (6.3), corresponding to the highest-energy zero of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  independent of  $\delta$ . For each of these three points, we show the value of the oscillation phase, which is independent of any experimental parameter; the  $L/E$ , whose value depends linearly on the inverse of  $|\Delta m_{31}^2|$ ; and the particular energy associated to this  $L/E$  for T2HK  $L = 295$  km and DUNE  $L = 1300$  km. Notice that these three values of the phase  $\Delta_{31}$  correspond to the position of the green/black/blue dashed lines within the red ellipse in figure 8.

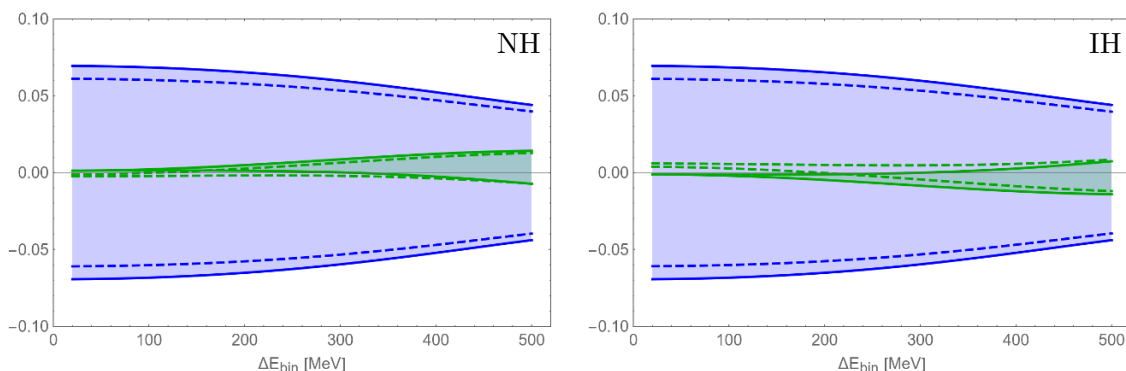


**Figure 9.** Zoom of figure 3 at low  $E$ , showing  $\mathcal{A}_{\mu e}^{\text{CPT}}$  (green) and  $\mathcal{A}_{\mu e}^{\text{T}}$  (blue) at DUNE  $L = 1300$  km.

This magic configuration around the second oscillation maximum is well apparent in the results presented in figure 9 for<sup>1</sup>  $L = 1300$  km. One can observe that the uninteresting (increasing/decreasing for NH/IH) zeros in  $\mathcal{A}_{\mu e}^{\text{CPT}}$  are strongly dependent on  $\cos \delta$ , as seen in eq. (6.5), and their position when  $\cos \delta = 0$  is that of the  $\delta$ -independent zeros in  $\mathcal{A}_{\mu e}^{\text{T}}$ . As understood from the previous discussion, we have identified the most relevant  $\delta$ -independent zeros (6.7) of  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , decreasing/increasing for NH/IH, correlated to near maximal  $|\mathcal{A}_{\mu e}^{\text{T}}|$  proportional to  $\sin \delta$ . Due to the first-order character of this  $\delta$ -independent (and hierarchy-independent too) zero,  $\mathcal{A}_{\mu e}^{\text{CPT}}$  is changing sign around it.

Integrating statistics in an energy bin around this point would still result in a vanishing fake CPV term in the experimental CP asymmetry, providing a direct test of CP violation in the lepton sector as clean as in vacuum. As shown in figure 10, we have checked that this is the case for an energy bin width up to 0.15 – 0.20 GeV, which keeps an almost vanishing  $\mathcal{A}_{\mu e}^{\text{CPT}} \forall \delta$  and an almost maximal  $\mathcal{A}_{\mu e}^{\text{T}} \propto \sin \delta$ . Such an energy resolution appears to be

<sup>1</sup>Notice that an equivalent figure could be obtained at  $L = 295$  km for energies between 100 and 350 MeV, with the same energy-dependence for both components of the CP asymmetry, but a relatively smaller  $\mathcal{A}_{\mu e}^{\text{CPT}}$  due to its proportionality to  $A \propto L$ .



**Figure 10.** Average value of the CPT-odd (green) and T-odd (blue) components of  $\mathcal{A}_{\mu e}^{\text{CP}}$ , at DUNE baseline  $L = 1300$  km, in an energy bin width  $\Delta E_{\text{bin}}$  centered on the magic energy (6.8). Both the exact (dashed) and the analytical (solid) results from eqs. (3.25) are shown. Normal/Inverted Hierarchy in the left/right panel. The bands correspond to all possible values changing  $\delta$  in  $(0, 2\pi)$ ; the upper/lower lines for  $\mathcal{A}_{\mu e}^{\text{CPT}}(\mathcal{A}_{\mu e}^{\text{T}})$  correspond to  $\cos \delta(\sin \delta) = -1, 1$ .

feasible at DUNE [28] around the second oscillation maximum, and the accumulated events would provide enough statistical significance to the transition probability distribution.

The whole discussion in this section, which stems from the analytical expressions (3.25), allows the reader to understand the peculiar energy distributions of the two components of the experimental CP asymmetry. In particular, the value of the magic energy (6.8), as well as its (in)dependence on the different oscillation parameters, is explained. This result in the energy distribution of the experimental CP asymmetry provides a positive response to our search of observable signatures able to separate out the genuine and matter-induced components.

## 7 Conclusions

A direct evidence of genuine CP violation means the measurement of an observable odd under the symmetry. The CP asymmetry for long baseline neutrino oscillation experiments suffers from fake effects induced by the interaction with matter. This matter effect is, however, welcome as a source of information for the ordering of the neutrino mass spectrum. Based on the different transformation properties under T and CPT of the genuine and matter-induced CP violation we have proved a Disentanglement Theorem for these two components. In order to raise this disentanglement to a phenomenological separation of the two components we have identified in this work their peculiar signatures from a detailed study in terms of the experimentally accessible variables.

For a precise-enough understanding of the problem, we have developed a new analytical perturbative expansion in both  $\Delta m_{21}^2, |a| \ll |\Delta m_{31}^2|$  without any assumption between  $\Delta m_{21}^2$  and  $a$ , which we use to analyze each of the disentangled components of the CP asymmetry,  $\mathcal{A}_{\alpha\beta}^{\text{CP}} = \mathcal{A}_{\alpha\beta}^{\text{CPT}} + \mathcal{A}_{\alpha\beta}^{\text{T}}$ , the first one ( $L$ -even) accounting for matter effects, the second one ( $L$ -odd) being genuine.

The two components of the CP violation asymmetry for the  $\nu_\mu \rightarrow \nu_e$  transition are shown in figure 2 as function of the interaction parameter  $a$ . They fulfill all the T and CPT symmetry requirements proved in section 2: the CPT-odd component  $\mathcal{A}_{\alpha\beta}^{\text{CPT}}$  is an odd function of  $a$  and vanishes linearly in the limit  $a \rightarrow 0$  for any value of the CP phase  $\delta$ , as well as being an even function of  $\sin \delta$  due to T-invariance. The T-odd component  $\mathcal{A}_{\alpha\beta}^{\text{T}}$  is an odd function of  $\sin \delta$  that vanishes, even in matter, if there is no genuine CP violation, as well as being even in  $a$  due to CPT-invariance, which means that its value is that of the CP asymmetry in vacuum up to small quadratic corrections  $\mathcal{O}(a/\Delta m_{31}^2)^2$ .

By analyzing the vacuum limit  $a \rightarrow 0$  both above and below the T-invariant limit  $\Delta m_{21}^2 \rightarrow 0$ , some intricacies for the mixings in matter appear. If one assumes  $|a| \ll \Delta m_{21}^2$ , the vacuum limit of the mixing matrix in matter will be the free PMNS matrix. On the other hand,  $\Delta m_{21}^2 \ll |a|$  will force  $U_{e1} = 0$  in the vacuum limit. This different behavior stems from the fact that setting  $\Delta m_{21}^2 = 0$  in the vacuum Hamiltonian leads to degenerate  $\nu_1, \nu_2$  mass eigenstates. The two limits mentioned above correspond to breaking this degeneracy in favor of  $\Delta m_{21}^2$  or  $a$ , respectively, projecting onto different bases in the 12 subspace. At the level of oscillation probabilities and asymmetries, the matter-vacuum invariant relations studied in sections 4 and 5, which involve both mixings and masses, show that the dependence on the phases associated to the small quantities  $\epsilon = a, \Delta m_{21}^2$  are of the form  $\frac{1}{\epsilon} \sin \frac{\epsilon L}{4E}$ , which cancel out if both of them are small, independently of whether  $|a| \ll \Delta m_{21}^2$  or  $\Delta m_{21}^2 \ll |a|$ . Therefore, the commutability of the two limits  $a \rightarrow 0$  and  $\Delta m_{21}^2 \rightarrow 0$  is restored for the final observables.

We have searched for experimental signatures in the  $\nu_\mu \rightarrow \nu_e$  oscillation channel assuming  $\Delta m_{21}^2 \ll |a|$ , valid for actual accelerator neutrino energies through the Earth mantle. The definite  $a$ -parity of each component of the CP asymmetry allows us to expand in  $|a| \ll |\Delta m_{31}^2|$  to leading (linear in  $\mathcal{A}_{\mu e}^{\text{CPT}}$ , constant in  $\mathcal{A}_{\mu e}^{\text{T}}$ ) order, since corrections are quadratic. For baselines and energies such that both  $\epsilon = \Delta m_{21}^2, a$  lead to  $\epsilon L/4E \ll 1$ , and taking  $|U_{e3}| \ll 1$ , we find compact expressions that faithfully reproduce the exact results,

$$\begin{aligned} \mathcal{A}_{\mu e}^{\text{CPT}} &= 16 A \left[ \frac{\sin \Delta_{31}}{\Delta_{31}} - \cos \Delta_{31} \right] (S \sin \Delta_{31} + J_r \cos \delta \Delta_{21} \cos \Delta_{31}) + \mathcal{O}(A^3), \\ \mathcal{A}_{\mu e}^{\text{T}} &= -16 J_r \sin \delta \Delta_{21} \sin^2 \Delta_{31} + \mathcal{O}(A^2), \end{aligned}$$

where  $S \equiv c_{13}^2 s_{13}^2 s_{23}^2$ ,  $J_r \equiv c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23}$ ,  $A \equiv \frac{aL}{4E} \propto L$  and  $\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E} \propto L/E$ . Equipped with such precise-enough analytical results, we have performed a detailed study of the different features of these quantities, focusing especially on signatures of genuine CP violation and hierarchy effects.

Since  $\mathcal{A}_{\mu e}^{\text{T}}$  is blind to  $\text{sign}(\Delta m_{31}^2)$ , a determination of the neutrino mass ordering must come from regions where the hierarchy-odd (and  $\delta$ -independent) term of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  dominates, which can only happen at long baselines due to the proportionality of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  to  $A \propto L$ . Our analysis at DUNE  $L = 1300$  km shows that this is the case for energies above the first node of the vacuum oscillation, where the sign of the experimental  $\mathcal{A}_{\mu e}^{\text{CP}}$  determines the hierarchy.

The strategy towards the measurement of genuine CP violation depends on the baseline. At medium baselines such as T2HK  $L = 295$  km, the CPT-odd component  $\mathcal{A}_{\mu e}^{\text{CPT}}$



is small and, for energies above the first oscillation node, dominated by its  $\delta$ -independent term. Therefore, it can be theoretically subtracted from the experimental  $\mathcal{A}_{\mu e}^{\text{CP}}$ , if the hierarchy is previously known, in order to obtain the genuine component  $\mathcal{A}_{\mu e}^{\text{T}}$ .

At long baselines, both  $\mathcal{A}_{\mu e}^{\text{CPT}}$  and  $\mathcal{A}_{\mu e}^{\text{T}}$  are of the same order, so  $\mathcal{A}_{\mu e}^{\text{CP}}$  will directly test genuine CP violation only when the CPT-odd component vanishes. We find a family of simple zeros of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  with decreasing/increasing slope for Normal/Inverted Hierarchy corresponding to the solutions of  $\tan \Delta_{31} = \Delta_{31}$ . These zeros are close to the second and higher vacuum oscillation maxima  $\sin^2 \Delta_{31} = 1$ , implying that their position is independent of  $\delta$  and corresponds to a nearly maximal  $|\mathcal{A}_{\mu e}^{\text{T}}|$  proportional to  $\sin \delta$ .

The main conclusion is thus that the magic energy around the second oscillation maximum is the ideal choice to find a direct evidence of genuine CP violation in the lepton sector. This vanishing of  $\mathcal{A}_{\mu e}^{\text{CPT}}$  occurs at

$$E = 0.92 \text{ GeV} \frac{L}{1300 \text{ km}} \frac{|\Delta m_{31}^2|}{2.5 \times 10^{-3} \text{ eV}^2}.$$

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