A(m, 12)	A(m, 11)	A(m, 10)	A(m, 9)	A(m, 8)	A(m, 7)	A(m, 6)	A(m, 5)	A(m, 4)	A(m, 3)	A(m, 2)	A(m, 1)	A(m, 0)	m	power as sum
												1	0	1
											6	1	1	3
										30	0	1	2	5
									140	0	-14	1	3	7
								630	0	0	-120	1	4	9
							2772	0	0	660	-1386	1	5	11
						12012	0	0	0	18018	-21840	1	6	13
					51480	0	0	0	-60060	491400	-450054	1	7	15
				218790	0	0	0	0	-3712800	15506040	-11880960	1	8	17
			923780	0	0	0	0	8817900	-196409840	581981400	-394788954	1	9	19
		3879876	0	0	0	0	0	1031151660	-10863652800	26003271294	-16172552880	1	10	21
	16224936	0	0	0	0	0	-1897319054.4	93699005400	-664528044180	1373080177128	-800361655623.6	1	11	23
67603900	0	0	0	0	0	0	-374796021600	8306600552250	-45784397325333.3	84902331848880	-47049773103666.7	1	12	25

Table 1. List of coefficients of polynomial $A_{0,m}(n-k)^0k^0 + A_{1,m}(n-k)^1k^1 + \cdots + A_{m,m}(n-k)^mk^m$ such that

$$\sum_{k=0}^{n-1}\sum_{j=0}^{m} A_{j,m}(n-k)^jk^j = n^{2m+1}, \qquad m = 0,1,2,\dots$$

For example, consider the second ($m = 2$) row, that is set of coefficients $\{30, 0, 1\}$, then

$$\sum_{k=0}^{n-1}\sum_{j=0}^{2} A_{j,2}(n-k)^jk^j = \sum_{k=0}^{n-1} 30(n-k)^2k^2 + 1 = n^5$$

Note that blue-marked cells are items of OEIS sequence A002457 and $A_{j,m};\ j = 0,..,m;\ \ m = 1,2,3$ are items of in definitions of sequences A287326, A300656, A300785. Present in Table 1 coefficients $A_{j,m};\ j = 0,..,m;\ \ m = 1,\dots,12$ are reached as solution of system of equations, to verify it refer to Mathematica code here. Also, the items of Table 1 are close related to coefficients β_{mv} (see C. Jordan, Calculus of Finite Differences, pp. 448-450). Note that sum of m-th row of Table 1 equals to $2^{(2m+1)} - 1$. Excel version of Table 1 available at this link.

Question 1:

- Is it exist any generating formula $F(j,m) = A_{j,m},\ m = 0,1,2,3,\dots$? – Yes, the sequences A302971 and A304042 are nominators and denominators of $A_{j,m},\ m = 0,1,2,\dots,\ 0 \le j \le m$. Results concerning $F(j,m)$ could be verified via Mathematica code.