Nested Refinement Domains for Tetrahedral and Diamond Hierarchies

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Hierarchical domain decomposition

Hierarchical domain decompositions play a fundamental role in the analysis and visualization of scientific and mathematical problems since they enable adaptive decompositions driven by application-dependent constraints.

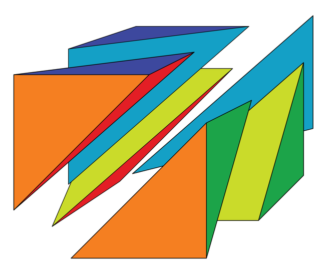
Conforming, i.e. crack-free, refinement is important since cracks can lead to discontinuities in functions defined over the domain.

Nested hierarchies enable a localized top-down refinement without needing to propagate refinements back up the hierarchy.

Regular Simplex Bisection Scheme

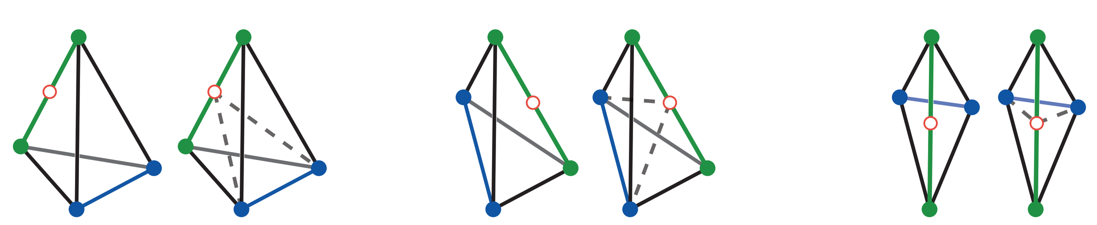
1. Kuhn's triangulation

Cubic domain subdivided along diagonal into six tetrahedra.



2. Cyclic tetrahedral bisection

Bisection edge implicitly determined by the refinement step.
Generates only three similarity classes of tetrahedra.



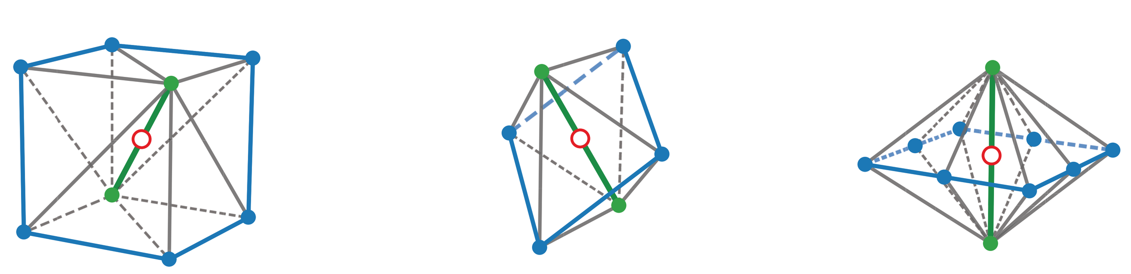
0-refinement
Bisect along diagonal of cube

1-refinement
Bisect along face diagonal of cube

2-refinement
Bisect along edge of cube

Diamond

Set of tetrahedra sharing the same bisection edge.
Concurrent bisection of all tetrahedra ensures conforming meshes.

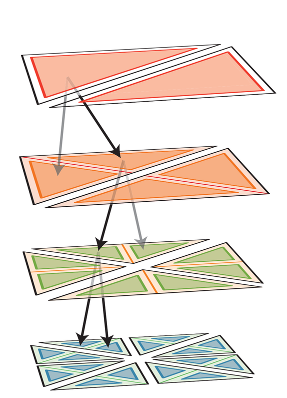


0-diamond ***1-diamond*** ***2-diamond***

Hierarchy of Tetrahedra

Bisection induces a *parent-child* relation on tetrahedra.
Hierarchy encoded as a forest of six rooted binary trees.

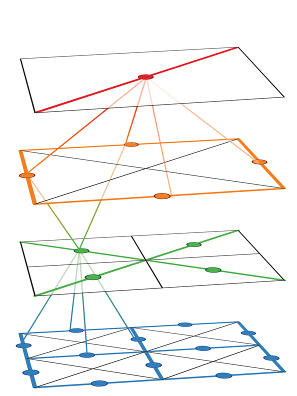
Encodes all possible variable-resolution meshes that can be generated via regular simplex bisection.
The hierarchy is nested, but not conforming.



Hierarchy of Diamonds

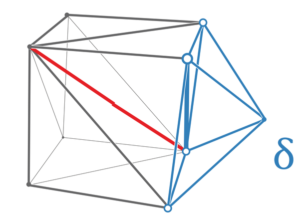
Conforming bisections induce a *direct dependency relation* on the diamonds. A *child* diamond δ_c directly depends on a *parent* diamond δ_p if refinement of δ_p generates a tetrahedron belonging to δ_c.

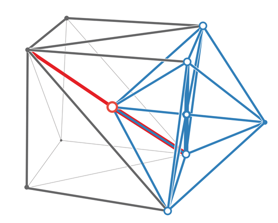
Hierarchy encoded as a Directed Acyclic Graph of diamonds.
Encodes all possible conforming variable-resolution meshes.
The hierarchy is conforming, but not nested.



Nested Refinement Domains

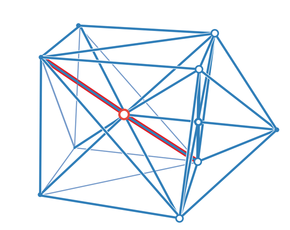
Consider the refinement of the tetrahedra belonging to a diamond δ within a mesh.



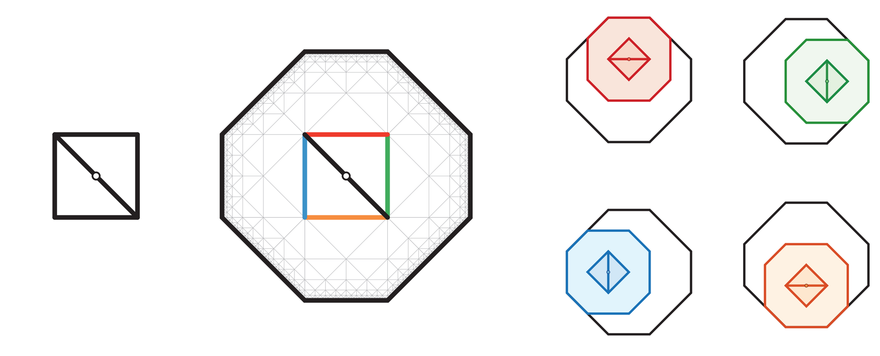


Non-conforming bisection refinements have a nested domain but can introduce cracks between tetrahedra belonging to ancestors of δ in the hierarchy.

Conforming refinements prevent cracks, but do not have a nested relationship. Thus, we must ensure that all ancestors have refined before we can refine δ.

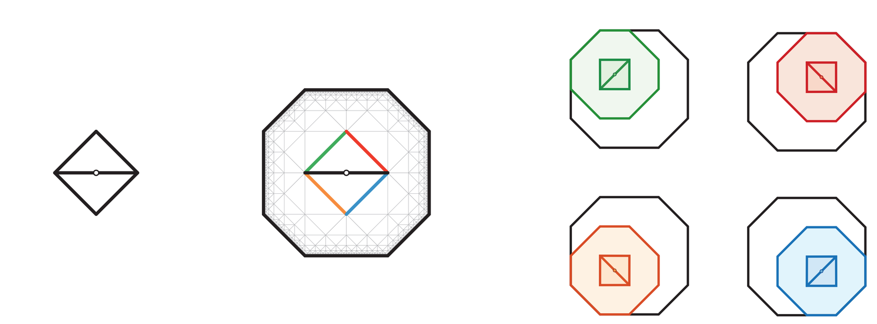


In 2D, researchers have proposed several nested refinement domains for triangle and (2D) diamond hierarchies by associating with each triangle or diamond a polygon covering the domain of its descendants within the hierarchy.
The limit shape of this process is an octagonal region centered at the midpoint of the bisection edge.



2D diamonds, their octagonal refinement domains and the nested relationship between the octagons.

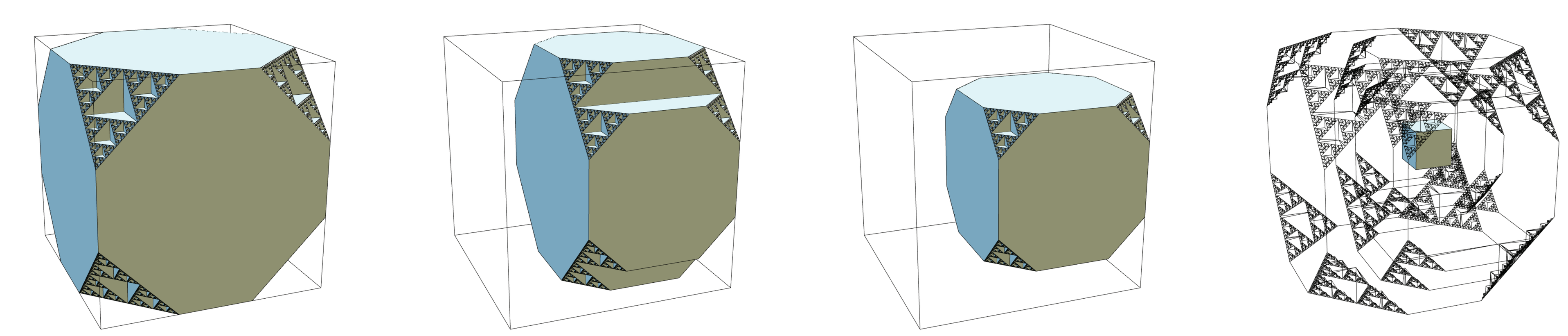
We generalize this notion to 3D by introducing three families of polyhedra that form nested refinement domains for conforming refinements of hierarchies of tetrahedra and diamonds.



Descendant domain

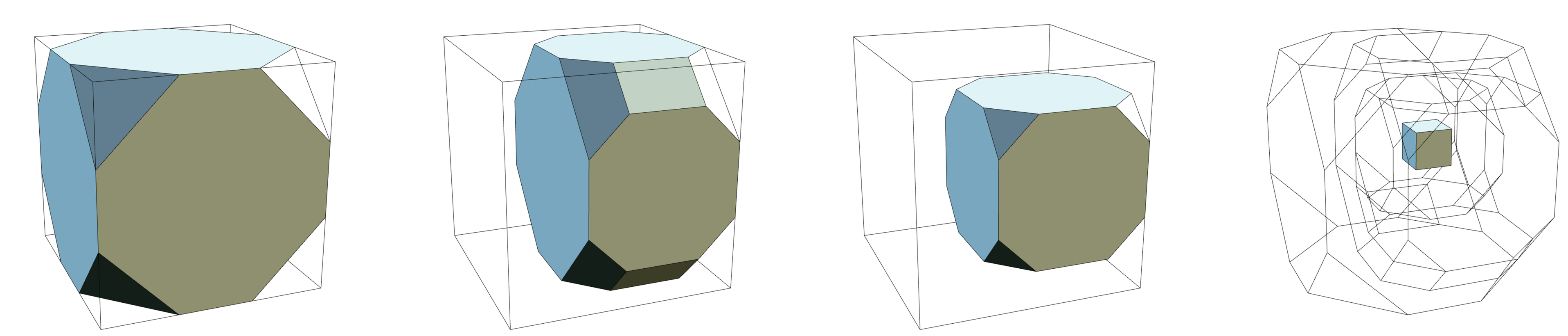
The limit shape of the domain covered by all descendants of a diamond.

These shapes have a fractal boundary which can be seen as a quaternary refinement of the triangular faces.
The middle triangles are then trisected once and the midpoint is offset along its normal by a factor of $\sqrt{6}/6$.



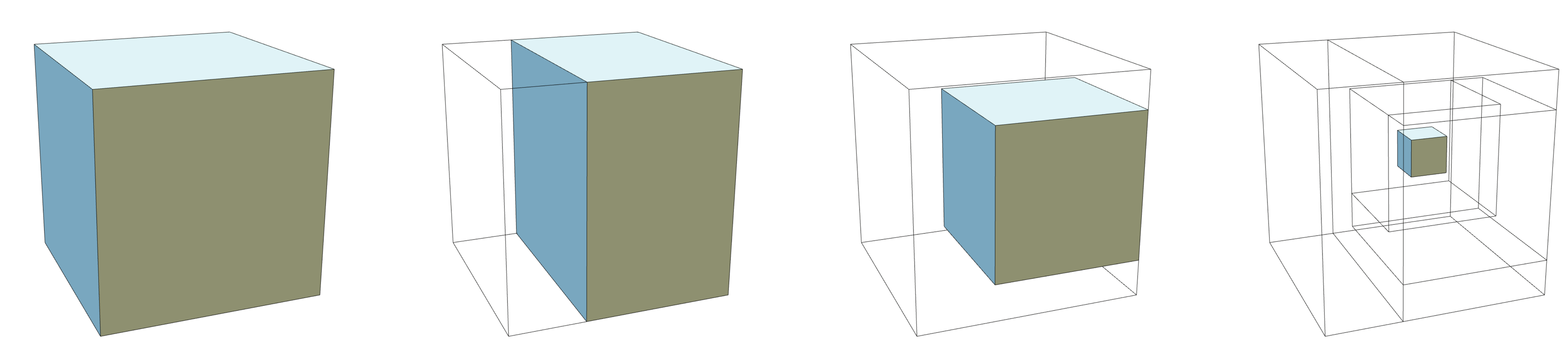
Convex descendant domain

To simplify the computation, we define a second hierarchy based on the convex hulls of the descendant domain.
Combinatorially, these shapes are related to truncated cuboids.



Bounding box descendant domain

We define a third hierarchy based on the axis-aligned bounding boxes of the descendant domain.
This hierarchy is not as tight as the convex hull domain, but can be considerably easier to compute with.
Relative to a unit sized 0-diamond, the dimensions of these boxes are (3·3·3), (3·3·2) and (2·2·2), respectively.



Discussion

All three nested refinement domains extend the domain under consideration by at most a factor of three.

These hierarchies have several potential applications to interactive volume visualization including:

Domain-based — As in the 2D case, these refinement domains can be easily incorporated into a hierarchical frustum culling algorithm.
If a node is more than three scaled units away from the view frustum its descendants no longer require cull testing.

Range-based — The bounding box refinement domain implies that we can convert simple non-nested metrics, such as the Min/Max Octree to a (slightly conservative) nested one by considering the isovalue range of only a constant number of cubes.

To find out more about this project and related work, please visit: www.kennyweiss.com This work has been partially supported by NSF grant CCF-0541032.