High-order, Mesh-free Numerical Quadrature for Trimmed Curved Parametric Domains

University of Colorado Boulder

Lawrence Livermore National Laboratory

D. Gunderman (CU Boulder, LLNL), K. Weiss (LLNL), J.A. Evans (CU Boulder)

Motivation

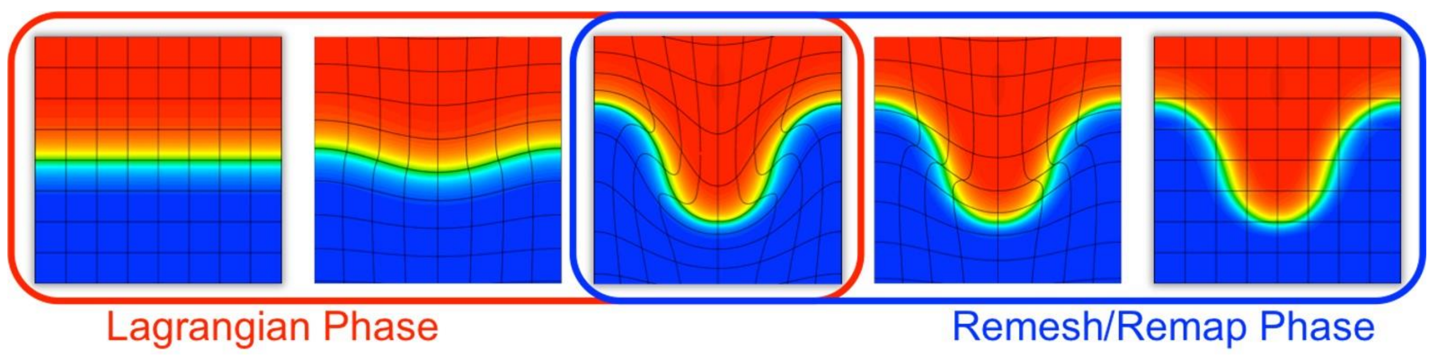

Figure 1: In Lagrange finite element methods (FEM), fields are remapped from a source mesh to a target mesh, requiring integration over intersections of source and target elements. (Image courtesy of [2])

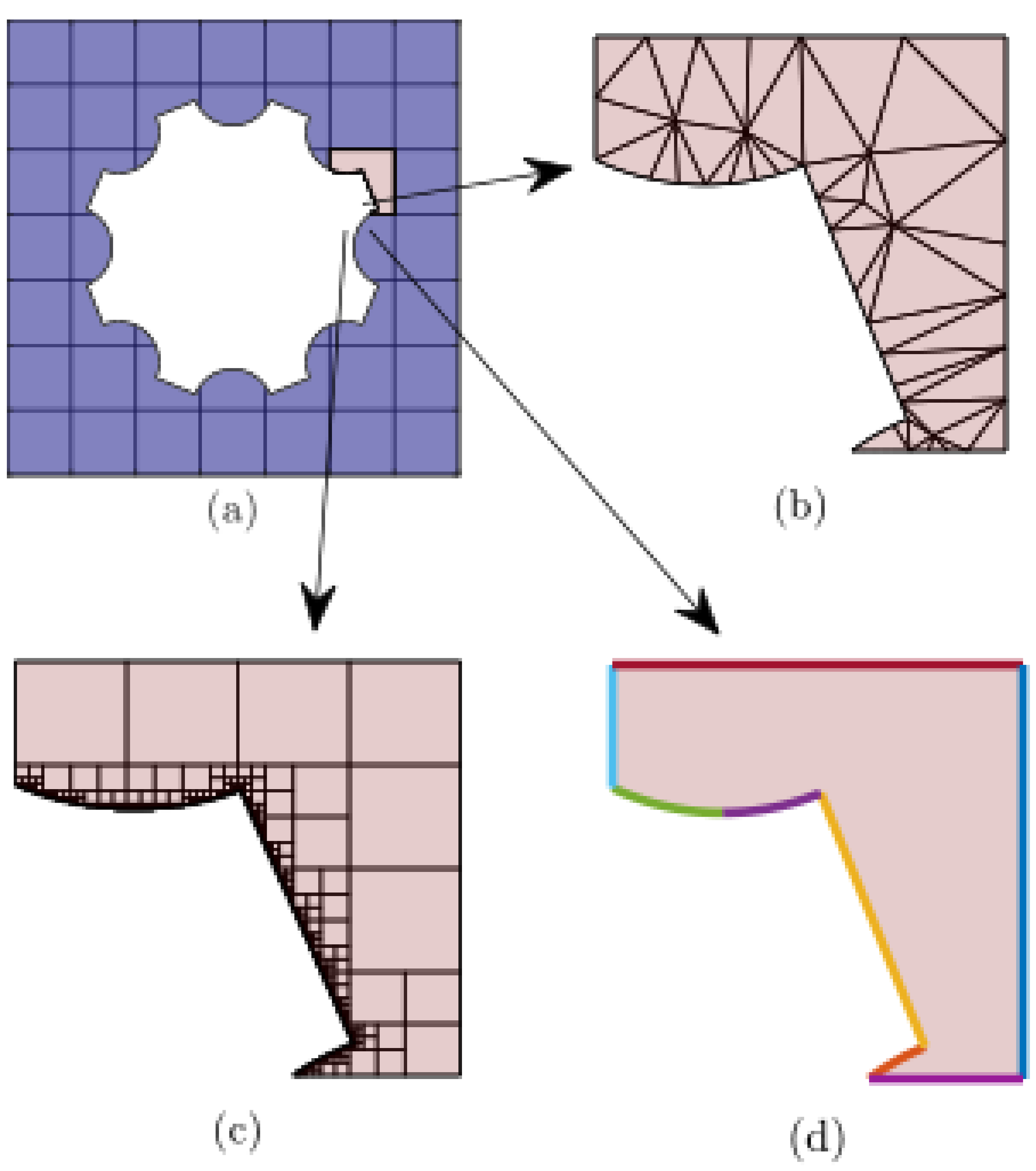

Figure 2: (a) In immersed boundary FEM, integration is performed over each cut cell. (b) and (c) Traditional techniques are computationally expensive due to low-order convergence. (d) Our method integrates over the boundaries of regions, achieving meshless high-order convergence.

Preliminary Results

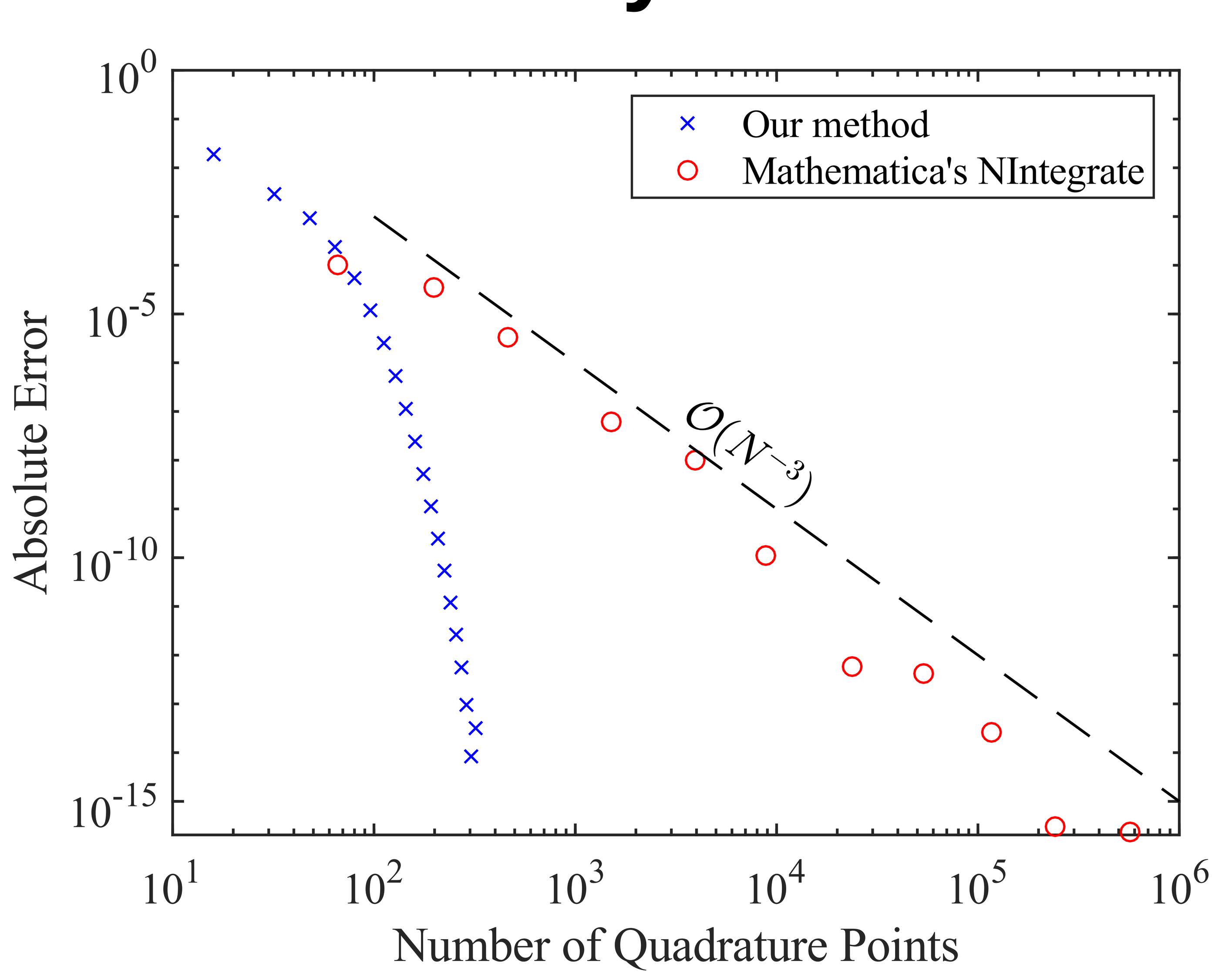

Figure 5: Our method achieves exponential convergence when trimming curves are given exactly, as is the case in this preliminary experiment. With approximate trimming curves, we expect high-order convergence. We integrate *f(x,y,z)=1* over the region R given in Figure 4 in this example.

Summary

- High-order trimmed parametric geometries (e.g. NURBS) are popular in design.
- Efficient integration over trimmed geometries is important in many analysis paradigms.
- We are developing an algorithm which achieves high-order convergence by incorporating geometric information without the need for 2D or 3D meshing.

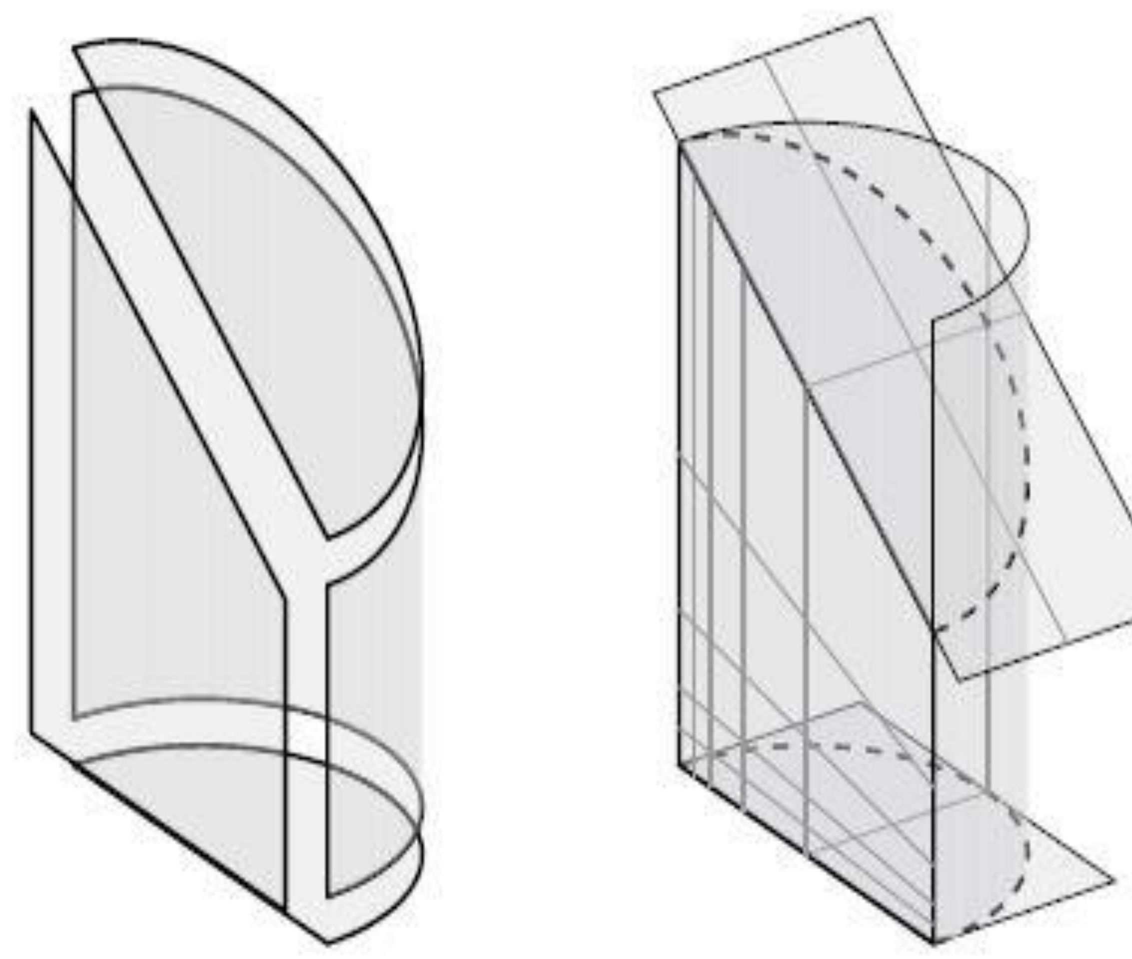

Figure 3: An example of a region formed by trimmed surface patches. The region is difficult to mesh. (Image courtesy of [3])

Method

Our algorithm proceeds in three steps:

1. Approximate true trimming curves with high-order polynomial curves using surface/surface intersection [1].
2. Convert volume integrals to surface integrals over the boundary surfaces using Stokes' theorem.
3. Convert surface integrals to line integrals along the approximated trimming curves using Green's theorem.

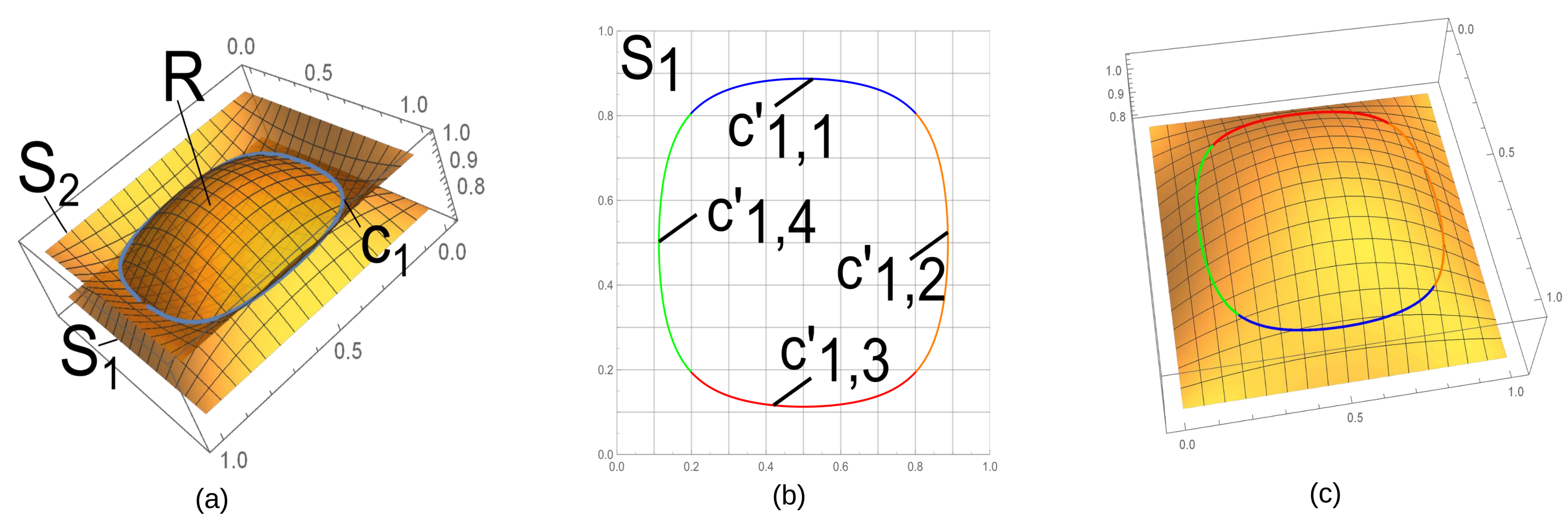

Figure 4: (a) An example of a region R formed by the intersection of two surfaces S_1 and S_2, forming a trimming curve c_1. (b) Our algorithm transforms the volume integral over R into integrals over the approximated trimming curves $c'_{1,1}$, $c'_{1,2}$,… (c) The approximated trimming curves map back approximately onto the original surface S_1.

Next Steps

1. Find an efficient, robust surface/surface intersection algorithm to approximate trimming curves.
2. Apply method to Lagrange-remap and immersed boundary FEM and compare to existing methods.

References

[1] G. E. Farin. *Curves and Surfaces for CAGD: A Practical Guide.* Morgan Kaufmann, 2002.

[2] Lawrence Livermore National Laboratory. BLAST. https://computing.llnl.gov/projects/blast. Accessed: 2020-5-27.

[3] B. Marussig, and T.J.R. Hughes. A review of trimming in isogeometric analysis: Challenges, data exchange and simulation aspects. *Archives of Computational Methods in Engineering,* 25(4):1059—1127, 2018.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344