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Price dynamics and volatility jumps in bitcoin options

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Abstract

In the FinTech era, we contribute to the literature by studying the pricing of Bitcoin options, which is timely and important given that both Nasdaq and the CME Group have started to launch a variety of Bitcoin derivatives. We find pricing errors in the presence of market smiles in Bitcoin options, especially for short-maturity ones. Long-maturity options display more of a "smirk" than a smile. Additionally, the ARJI-EGARCH model provides a better overall fit for the pricing of Bitcoin options than the other ARJI-GARCH type models. We also demonstrate that the ARJI-GARCH model can provide more precise pricing of Bitcoin and its options than the SVCJ model in term of the goodness-of-fit in forecasting. Allowing for jumps is crucial for modeling Bitcoin options as we find evidence of time-varying jumps. Our empirical results demonstrate that the realized jump variation can describe the volatility behavior and capture the jump risk dynamics in Bitcoin and its options.

Keywords: ARJI-GARCH models, Blockchain, Bitcoin options, FinTech

Introduction

Bitcoin (BTC) has drawn extraordinary global attention and investors' interest over the recent years.[1] The "black swan" crypto event of Terra's collapse and the FTX catastrophe in 2022 clearly catch regulators' eyes around the world. The increased popularity of BTC is also evidenced from numerous studies on its key features such as jump dynamics (e.g., Dyhrberg 2016; Gronwald 2019; Hou et al. 2020) and unique properties of its derivative pricing (e.g., Cretarola et al. 2020; Hoang and Baur 2020; Chi and Hao 2021; Chen and Huang 2021). Due to BTC's unique characters, it is not clear whether existing option pricing models can be applied to Bitcoin options. From the option pricing and risk management perspectives, modeling volatility dynamics is crucial for option valuation (e.g., Hoang and Baur 2020; Siu and Elliott 2021; Kapetanios et al. 2019). Unfortunately, studies on Bitcoin options are scarce and our understanding of Bitcoin options is limited. This study aims to investigate the volatility dynamics of BTC and its options as well as the information content of Bitcoin option pricing.

[1] A major innovation in the financial technology (FinTech), namely, blockchain, enables the existence of cryptocurrency that has a potential to dramatically impact the world economy.

Although Bitcoin futures are available on major exchanges,[2] Bitcoin options are relatively new and have yet to be offered on organized exchanges. The Deribit Company, based in Netherlands, has provided Bitcoin futures and options trading platform since the summer of 2016, presenting a good opportunity to study Bitcoin option trading and pricing. On November 12, 2019, the CME Group announced its plan to launch Bitcoin options, which have become available for trading since January 13, 2020.[3] In addition, Nasdaq plans to launch a variety of Bitcoin derivatives.[4] Deribit crypto options and futures exchange has been consistently ranked the top crypto option market. As depicted in Table A, approximately 88.3% of the open interest (OI) were from the Deribit exchange, while the second-ranked CME had only 8%, as of July 12, 2022. The collapse of FTX Trading Ltd. led to subsequent sale of its four main business divisions, including Ledger X. Consequently, the Deribit exchange has reached 93.82% of the OI on April 4, 2023 and experienced an all-time high OI of $ 20 Billion. According to the Deribit, over 80% of their trading volume comes from large professional and institutional traders, implying that the Deribit exchange has the best liquidity and order book in crypto options with a wide range of contracts. As a result, our investigation of Bitcoin option pricing in Deribit can provide emerging insights and policy implications for exchanges and regulators.

Realizing the importance of incorporating market information from sudden shocks into option pricing models, we use a jump detection test to characterize the dynamic jump for Bitcoin options. Specifically, we adopt the extended Autoregressive Jump Intensity (ARJI)-GARCH models for the diffusive volatility of Bitcoin returns to allow for easy valuation of European options. We assume that the Blockchain progress is equivalent to the Poisson process. The price manipulations in the Bitcoin ecosystem, news impact, affright sentiment, and hash war of Bitcoin all potentially lead to the Poisson distribution.[5]

From the price discovery perspective, the purpose of this paper is to determine whether the GARCH option pricing models can provide reasonable price discovery when applied to the volatile Bitcoin options market in the existence of jumps. To capture the evolution of the price discovery process, we consider some benchmark GARCH models in pricing options and show that the ARJI-EGARCH model produces more accurate option prices as reflected from lower pricing error than those of other competing ARJI-GARCH forecasting models.

We contribute to the literature in several ways. First, this is the pioneer study attempting to empirically investigate the information content in Bitcoin options pricing. We find extractable mis-pricings on the Deribit options exchange. Although Hou et al. (2020) apply the stochastic volatility with a correlated jump (SVCJ) model to the pricing of cryptocurrency options, their approach is limited to an experimental simulation. Our analysis is based on the option trading data from the Deribit Company. Second, we

[2] Given the extreme volatility and strong interest in Bitcoin, the Chicago Board Options Exchange (CBOE) officially launched standardized futures products on Bitcoin in December 2017, giving rise to a new era for Bitcoin trades and opening the door for other standardized derivatives.

[3] https://www.cmegroup.com/media-room/press-releases/2019/11/12/cme_group_announcesjan132020launchforbitcoinoptions.html

[4] https://www.cnbc.com/2018/11/28/nasdaq-to-launch-bitcoin-futures-despite-cryptocurrencies-bear-market.html

[5] Concerned about the relative power of the attackers, Grunspan et al. (2017) compute the probability of success of the attackers using a Poisson distribution.

show that the ARJI-GARCH model can achieve better performance in forecasting volatility than the competing SVCJ model. This finding implies that specifying asymmetric jumps in conditional jump intensity on Bitcoin and its options contains more unique information than the traditional jump-diffusion process. Third, from the price discovery perspective, we emphasize three ARJI-GARCH-type models of realized jump variation prediction and their implications on the Bitcoin and its options markets. Our empirical results indicate that the ARJI-EGARCH model performs well in out-of-sample tests. We also show that the ARJI-EGARCH model's robust outperformance is partly due to its improved ability to describe the smirk and its distinctive ability to capture the volatility term structure. Our findings may provide reference for the newly introduced Bitcoin options at the CME Group and the planned Bitcoin derivatives at Nasdaq.

The remainder of the paper proceeds as follows. Section "Related literature" reviews the related literature. Section "Models for bitcoin prices and option valuation" discusses our models and develops the theoretical framework combined with jump detection methods from physical measure to risk neutralization for option valuation. Section "Empirical analyses and implications" presents empirical results on option valuations using parameters estimated on BTC returns. Section "Performance evaluation" performs the performance evaluation on option pricing models. Finally, Sect. "Conclusion and suggestions for future work" concludes with suggestions for future work.

Related literature

The literature related to our work can generally be classified into three streams. The first stream of studies has shown that Bitcoin prices are highly volatile (see e.g., Yermack 2014). Furthermore, a steadily increasing number of studies also investigate Bitcoin prices and shed new light on the GARCH volatility dynamics of this cryptocurrency (see Ardia et al. 2019) and its bubble behavior (see Cheah and Fry 2015; Baek and Elbeck 2014). These papers demonstrate that speculative motivation is the main driver of the Bitcoin prices and it is less likely to be demanded by risk-averse investors. Given the high volatility of Bitcoin prices, it is important to appropriately estimate its risk metrics,[6] which are critical for evaluating margin requirements, developing hedging strategies, and pricing derivatives. To appropriately estimate the risk metrics of Bitcoin, it is thus crucial to model jumps in a volatility model that incorporates returns.

The second stream of the literature considers jump dynamics. Several models have been proposed to model jumps (Aysan et al. 2024; Zhang et al. 2022; 2023a; 2023b). Popular ones include the jump-diffusion model pioneered by Merton (1976) and Bates (1991), the double exponential jump model by Kou (2002), and the GARCH-Jump Model (Duan et al. (2006). A volatility jump model assumes that the logarithm return of a risky asset is driven by the Brownian motion plus a compound Poisson process with jump sizes distributed in the model. Christoffersen et al. (2016) and Li (2019) discuss the key factors on pricing derivatives in crude oil and natural gas futures markets.

[6] In this regard, a number of studies, such as Rogers and Satchell (1991) and Yang and Zhang (2000), have introduced alternative measures of volatility to achieve a better understanding of the nature of ranges and their significance in predicting volatility. Gillaizeau et al. (2019), Sheraz et al. (2022), and Chen and Yang (2023) specifically apply those volatility measures for their study of Bitcoin and cryptocurrencies.

These models have two major advantages. First, they lead to analytical solutions for solving options valuation issues. Second, they help explain the two empirical phenomena, namely the asymmetric leptokurtic feature and the volatility smile. In general, the presence of price jumps provides a better explanation for the skewness and leptokurtosis of return distributions as well as implied volatility smiles. In addition, Siu and Elliott (2021) adopt both the GARCH model and the self-exciting threshold autoregressive (SETAR) model for modeling Bitcoin return dynamics and find Bitcoin to be extremely volatile. Walther et al. (2018) use the BEKK-GARCH model to estimate time-varying correlations between Bitcoin and the S&P 500 returns. They find that the correlation is highly volatile and characterize Bitcoin as extremely unstable.

The third stream of the literature takes nonparametric jump tests into consideration and primarily focuses on issues that are relevant to estimate ex post realized jumps. Several studies document that effective modeling of asset price jumps in realized volatility is a critical procedure for predicting index option prices (Andersen et al. 2007; Qiao et al. 2020; Feunou & Okou 2019). Specifically, Duan et al. (2007) show that the GARCH option pricing model with jumps significantly improves the fit of historical time series of the S&P 500 index returns and the benefits of incorporating these jumps extend to option pricing by capturing the volatility smile in option prices. However, only scant research considers the applications of realized jumps for Bitcoin options. Thus, our study fills the gap in the aforementioned related but different streams of research by considering realized jumps in Bitcoin options.

Models for bitcoin prices and option valuation

Volatility jump detection in BTC prices

Consider a market with a risky asset (like Bitcoin in this study) whose price at time t is denoted by $S(t)$. In a jump detection (JD) model, the stochastic differential equation for the Bitcoin price is given:

$$d\log S(t) = \mu(t)dt + \sigma(t)dW(t) + Y(t)dN(t), \tag{1}$$

where μ is the expected return; σ is volatility; $\{W(\mathrm{t})\colon t \in [0, \tau]\}$ is a standard Brownian motion; $Y(t)$ denotes the jump size of $S(t)$; $N(t)$ is a jump process. The postulated process differs from the process seen in the jump-diffusion stream finance literature (e.g., Merton (1976) and Kapetanios et al. (2019)) in several significant aspects. First, jumps are allowed to be asymmetric, and possibly with nonzero means. In addition, the jump risk is systematic and nondiversifiable.[7] More importantly, the BTC option valuation depends heavily on not only the spot BTC price but also the underlying value.

Jump detection methodology

In this study, our analysis focuses on daily returns and volatilities. Therefore, for notational simplicity, we recall the corresponding daily returns, and normalize the daily time interval to unity as follows:

[7] The authors are thankful to an anonymous reviewer for suggesting that the continuous model used is quite common in the finance literature and the jump risk is nondiversifiable for all jump-diffusion models.

$$r_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{2}$$

r_t denotes the logarithm (log) returns at time t. Also, to formally postulate our empirical volatility measures for trading day t, RV can then be computed from the squared j-th intraday returns as follows:

$$RV_t = \sum\nolimits_{j=1}^{M} r_{t,j}^2, \tag{3}$$

where M denotes the number of intraday returns that can be formed on one day. Following the literature, the realized Bipower variation (BV) can be described as:

$$BV_t \equiv \mu_1^{-2} \frac{M}{M-1} \sum\nolimits_{j=2}^{M} \left|r_{t,j}\right|\left|r_{t,j-1}\right|, \tag{4}$$

where $\mu_1^{-2} = \frac{\pi}{2}$, $\mu \sim \mathcal{N}(0,1)$.

To precisely distinguish the continuous variation from the jump specification, based on Barndorff-Nielsen and Shephard (2006), we define the realized variance as follows.

$$RV_t \equiv \sum\nolimits_{j=1}^{M} r_{t,j}^2 \rightarrow \int_{t-1}^{t} \sigma_s^2 ds + \sum\nolimits_{s=0}^{n_t} J_s^2, \tag{5}$$

where J_s is the jump size and n_t denotes the number of jumps within day t. In other words, the RV_t is a consistent estimator of the integrated variance $\int_{t-1}^{t} \sigma_s^2 ds$ and the discontinuous jump $\sum_{s=0}^{n_t} J_s^2$.

Based on the Eq. (4), the realized bi-power variation can also be described as:

$$BV_t \equiv \frac{\pi}{2} \frac{M}{M-1} \sum\nolimits_{j=2}^{M} \left|r_{t,j}\right|\left|r_{t,j-1}\right| \rightarrow \int_{t-1}^{t} \sigma_s^2 ds, \tag{6}$$

where the $\frac{M}{M-1}$ denotes a finite sample bias correction term, which provides unbiased estimates in the Brownian motion case.

Jump detection test

The difference between the realized variance process and the Bipower variation process represents 'significant' jumps and implies a simple and consistent nonparametric estimator where the pure jump component contributes to the total return variance. Therefore, following the suggestion of Huang and Tauchen (2005), we develop an empirically more robust measure by the following relative jump statistic:

$$RJ_t = \frac{RV_t - BV_t}{RV_t}, \tag{7}$$

or the corresponding (approximate) logarithmic form as:

$$RJ_t = \log RV_t - \log BV_t \tag{8}$$

Furthermore, we can rely on a joint model approach for both BV_t and RJ_t to capture the distinct components that account for the total price variation. Lee and Mykland

(2008) propose that the volatility estimated by employing the realized Bipower variation preceding BV_t is robust to jumps. Hence, the jump detection statistic can be described as:

$$\mathcal{L}(i) = \frac{r_{t,i}}{\widehat{\sigma_{t,i}}} \tag{9}$$

where $\widehat{\sigma_{t,i}}^2 = \frac{1}{K-2}\sum_{j=i-K+2}^{i-1}|r_{t,j}||r_{t,j-1}|$,

K denotes the window size. To examine the null hypothesis test of no jump at $(t_{i-1}, t_i]$, if the test statistics locate in the rejection region at a given significance level α. The corresponding observation is detected a jump if the following condition is met.

$$\frac{|\mathcal{L}(i)| - C_n}{S_n} > -\log\left(-\log(1-\alpha)\right), \tag{10}$$

where $C_n = \frac{\sqrt{2\log n}}{0.7979} - \frac{\log\pi + \log(\log n)}{1.5958\sqrt{2\log n}}$ and $S_n = \frac{1}{1.5958\sqrt{2\log n}}$.

In practice, Lee and Mykland (2008) suggest the jump statistic under a significance level of α= 0.05, one can reject the null of no jump if $\frac{\mathcal{L}(i)-C_n}{S_n} > \beta^*$, with β^* such that $exp(-e^{-\boldsymbol{\beta}^*}) = 1-\alpha$ =0.95, i.e. $\beta^* = -\log\left(-\log(0.95)\right) = 2.9702$. This stabilizing procedure can identify only a spurious jump in a given sample of data observations.

The extended ARJI- GARCH-type model specifications

To determine whether including jump components in modelling Bitcoin volatility can improve the forecasting accuracy, we follow Andersen et al. (2007) to incorporate the sudden discontinuous jump variation to our model. Specifically, to capture the volatility risk and jump dynamics of BTC, the extended ARJI-GARCH model can be expressed as:

$$R_t = ln\left(\frac{S_t}{S_{t-1}}\right) = \mu + mR_{t-1} + \sqrt{h_t}\mathbb{Y}_t \tag{11}$$

$$\mathbb{Y}_t = Z_t + \sum_{l=1}^{n_t} J_t^l,$$

$$Z_t \sim N^Q(0, h_t), \mathbb{Y}_t \sim N^Q\left(\theta, \delta^2\right), dn_t \sim \text{Poission}(\lambda, dt)$$

In the above postulation, μ and m are the expected return and the coefficient of AR, respectively. h_t represents the conditional variance of the Bitcoin price process over the interval (t-1, t), and $\mathbb{Y}_t$ denotes a compound Poisson process, which is expressed as the combination of the diffusion process Z_t and the Poisson jump process. n_t is expressed as the number of jumps at Poisson events occurring at (t-1, t) and the independent l and time t determine the normally distributed jump innovation magnitude J_t^l. To describe the time-varying jump on BTC returns, we incorporate the framework in Chan and Maheu (2002) and Christoffersen et al. (2015) to specify the jump-arrival intensity of Bitcoin return as an exogenous variable in an autoregressive moving average form, which is expressed as:

$$R_t = \mu + mR_{t-1} - d\lambda_t + \epsilon_t, \tag{12}$$

where the parameter d captures the effect of jump intensity on the conditional mean of Bitcoin returns. Then, the conditional jump intensity λ_t is assumed time-varying and distributed with an autoregressive conditional jump intensity as follows:

$$\lambda_t = \lambda_0 + \rho_1\lambda_{t-1} + \gamma\xi_{t-1} \tag{13}$$

Given the observed R_t and the Bayes' rule, the ex-post probability of the arrival of j jumps at time t with the filter is expressed as:

$$P(n_t = j|\,\Omega_t) = \frac{f(R_t|\,n_t = j, \Omega_{t-1})P(n_t = j|\,\Omega_t)}{P(R_t|\,\Omega_{t-1})}, \mathrm{j} = 0, 1, 2, \ldots\ldots \tag{14}$$

where the filter in Eq. (14) has a crucial component of time-varying jump intensity**.** The conditional density of returns can be obtained by integrating the number of jumps n_t, which is given as:

$$P(R_t|\,\Omega_{t-1}) = \sum\nolimits_{j=0}^{n} f(R_t|\,n_t = j, \Omega_t)P(n_t = j|\,\Omega_{t-1}), \tag{15}$$

where $P(n_t = j|\,\Omega_{t-1})= \frac{e^{-\lambda_t}(\lambda_t)^j}{j!}$ denotes the probability density of counting j jumps, which is assumed to be governed by the Poisson distribution. Consider the comprising j jumps during the unit interval (t-1, t), the ex-post conditional probability density function is therefore written as:

$$f(R_t, n_t = j|\,\Omega_t) = \frac{1}{\sqrt{2\pi(h_t + j\delta_t^2)}} exp\left[-\frac{(R_t - \widetilde{\mathrm{u}} - j\theta_t)^2}{2\pi(h_t + j\delta_t^2)}\right], \tag{16}$$

where $R_t = ln\left(\frac{S_t}{S_{t-1}}\right)$ and $\widetilde{\mathrm{u}} = \mu_t + mR_{t-1} - d\lambda_t$.

The log-likelihood function of the ARJI-EGARCH model is then calculated as the product of the conditional distributions across the sample as follows:

$$L(\boldsymbol{\Psi}) = \sum_{t=1}^{T} f(R_t|\,\Omega_{t-1}, \boldsymbol{\Psi}), \tag{17}$$

where $\boldsymbol{\Psi}=$ $(\alpha, \beta, d, \gamma, \mu, \delta, \phi, \rho, m, \omega)$ denotes all the parameters to be estimated. We then employ the maximum likelihood estimation technique to obtain the estimates.

Variance decomposition and dynamic jump components

To improve the precision in measuring volatility and accuracy in forecasting the jump variation, we develop an extended specification of the GARCH model in Eq. (18) that considers the realized jump variation parameter (*RJ* in Eq. 8) as an exogenous variable into the variance equation of GARCH(1,1) model to construct a GARCH-RJ model, which is expressed as:

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} + \delta RJ, \tag{18}$$

where *RJ* describes the impact of realized jump variation on the conditional variance of Bitcoin returns. This step to separate realized jump variation from the realized variance is crucial to obtain an accurate Bitcoin option pricing. A similar extension is applied to the CGARCH(1,1) and EGARCH(1,1) process.

To identify the conditional variance of the BTC returns, we use selected GARCH-type models of the form $h_t = F\,(\epsilon_{t-1}, h_{t-1})$, where h_t refers to the conditional volatility. Basically, the GARCH of Engle (1982), the exponential GARCH (EGARCH) of Nelson (1991) and the component GARCH (CGARCH) of Engle and Lee (1999) are widely applied and considered special cases of GARCH-family models. For the following GARCH-type models, we assume returns follow a GARCH process with Gaussian innovations under martingale measure $\mathbb{P}$. Duan (1995) proposes an alternative option pricing model which utilizes the locally risk-neutral valuation relationship (LRNVR) and then applies the LRNVR method under the risk-neutral measure $\mathbb{Q}$ to the GARCH option pricing model. The conditional volatility can be modeled by those GARCH models with and without LRNVR as follows[8]:

GARCH model:

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta h_{t-1} \tag{19}$$

GARCH model under LRNVR can be expressed as:

$$h_t = \omega + \alpha(\xi_{t-1} - \overline{\lambda}\sqrt{h_{t-1}})^2 + \beta h_{t-1} \tag{20}$$

CGARCH model:

$$h_t = q_t + \alpha(\epsilon_{t-1}^2 - q_{t-1}) + \beta(h_{t-1} - q_{t-1})$$

$$q_{t+1} = \omega + \rho q_t + \varphi(\epsilon_{t-1}^2 - h_{t-1}) \tag{21}$$

CGARCH under LRNVR can be expressed as:

$$h_t = q_t + \alpha\left[\left(\epsilon_{t-1} - \overline{\boldsymbol{\lambda}}\sqrt{h_{t-1}}\right)^2 - q_{t-1}\right] + \beta(h_{t-1} - q_{t-1})$$

$$q_{t+1} = \omega + \rho q_t + \varphi\left[\left(\epsilon_t - \overline{\boldsymbol{\lambda}}\sqrt{h_t}\right)^2 - h_t\right] \tag{22}$$

EGARCH model:

$$\log(h_t) = \omega + \varphi|z_{t-1}| + \gamma z_{t-1} + \beta\log(h_{t-1}), \tag{23}$$

where $z_{t-1} = \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}$

EGARCH under LRNVR is expressed as:

$$\ln(h_t) = \omega + \alpha(|\epsilon_{t-1} - \overline{\lambda}| + \gamma|\epsilon_{t-1} - \overline{\lambda}|) + \beta\ln(h_{t-1}), \tag{24}$$

where $\epsilon_{t-1} = z_{t-1} + \overline{\boldsymbol{\lambda}}$ is the standard normal random variable under the LRNVR $\mathbb{Q}$.

[8] For detailed explanations of those models, please refer to Engle (1982), Nelson (1991), Engle and Lee (1999), and Duan (1995).

The parameter γ determines how the EGARCH specification model incorporates the leverage effect. In this study, we also apply jump detection applications to address the BTC price jump issue in the pricing of BTC options.

Option pricing simulation and its closed-form valuation

Our jump detection approach is to conduct the model-free estimation of jump component and then fit the specific parametric models by the maximum likelihood estimation (MLE) method. In this subsection, we refer to Christoffersen et al. (2016) and introduce the jump-diffusion model with continuous-time jump risk measures. We now proceed to the alternative option valuation on an underlying asset with jump-diffusion process. European call options are priced as the discounted expected value of their terminal payoffs, assuming that the terminal distribution is determined under the risk-neutral world. The dependence of BTC option valuation on both spot BTC price and underlying value can be expressed as:

$$\boldsymbol{C}(S, V, \tau; X, \theta) = e^{-r\tau} E_{\mathbb{Q}} \max(S_\tau - X, 0) \tag{25}$$

In conjunction with a strike price X, spot BTC price S_τ, time-to-maturity τ, and a risk-free interest rate r, the price of a European call option that expires within time τ can then be calculated. As in the work of Bates (1991), suppose the Bitcoin price follows the dynamics expressed in Eq. (1), the corresponding model for the European call option price $\boldsymbol{C}$ is given by:

$$\boldsymbol{C}(S, \tau; X) = \sum_{j=0}^{\infty} \frac{e^{-\lambda^* \tau} (\lambda^* \tau)^j}{j!} \left[S_t e^{-\overline{b}_j \tau} N\left(d_{1j}\right) - X e^{-r\tau} N\left(d_{2j}\right) \right], \tag{26}$$

where $d_{1j} = \frac{ln \frac{S_t}{X} + (\overline{b}_j - \lambda^* k^* + \frac{\sigma_j^2}{2})\tau}{\sigma^2 \tau + j\sigma_S^2}$ and $d_{2j} = d_{1j} - \sigma_i \sqrt{\tau}$,

$\lambda^* = \lambda^*(1 + k^*)$ is defined as before.

$k^* = exp\left(\overline{\mu} + \frac{\sigma_s^2}{2}\right) - 1$ is jump size.

$\overline{\mu} \equiv \mu + \gamma \sigma_s^2$ i.e., unconditional mean,

$\overline{b}_j = (b - \lambda k)\tau + (\mu - \frac{\sigma_s^2}{2}) j$ i.e., cost of carry,

$\sigma_j^2 = \sigma^2 \tau + j\sigma_s^2$ or $\sigma_j = \sqrt{\sigma^2 \tau + j\sigma_s^2}$ i.e., standard deviation,

$\tau \equiv T - t$ is time to expiration.

In addition to the above pricing formula, we provide more detailed procedure of numerical technique for the closed-form valuation in Appendix A.

4. Empirical analyses and implications

Data description and Deribit crypto options exchange

The historical Bitcoin data cover the periods of 01/01/2014 to 6/30/2021. The daily log-returns of the Bitcoin are calculated as $R_t = \log(S_t/S_{t-1})$ with 10,932 observations. We obtain the data from https://www.coinbase.com/price/data which provides data on a number of liquid BTC exchanges. For option prices, this paper applies the data from the Deribit[9] website at https://deribit.com. Jalan et al. (2021) also employ near-the-money

[9] The Deribit Company provides Bitcoin futures and options trading platform and has started operation since summer 2016, presenting a good opportunity to study Bitcoin option trading and pricing. The company is based in the Netherlands and operating under that country's laws to ensure reliable transactions.

call and put options on the Deribit traded platform as of 27 January 2020 and analyze the risk inherent in Bitcoin options by computing their Greeks. The BTC options data contain 2,252 observations during the sample period from September 14, 2018 to November 27, 2021.

Deribit crypto options and futures exchange is currently the largest in terms of daily volume and offers about 93.82% open interest in crypto options market. More than 9 out of 10 BTC and ETH options are actively traded at Deribit, with over 80% of their trading volume coming from large professional and institutional traders. Among all option exchanges, Deribit has the best liquidity and offers a wide range of contracts, such as dated, perpetual, linear, and inverse. In addition, the Deribit exchange remains anonymous for traders and allows settlements in BTC to prevent frequent conversion between crypto and fiat currencies.

The evolution process of bitcoin returns

After Bitcoin launched in January 2009, the Bitcoin price stayed under $1,200 until February 22, 2017, as shown in Fig. 1. The price continued to fall in 2018, reaching the $3,500 level. BTC price has been very volatile, with several small or large jumps and experienced the peak in December 2017. Consistent with this observation, Scaillet et al. (2020) show that jumps are an indispensable component of the Bitcoin price dynamics. The properties of Bitcoin returns display extremely movement and volatility clustering. In addition, considerable evidence of persistence of volatility exists. To depict Bitcoin volatility, the middle Panel of Fig. 1 shows that Bitcoin is very volatile, with the log-return ranging from 21% to −48%.

Empirical stylized results

We report the first four moments (mean, variance, skewness, and kurtosis) of Bitcoin and its option returns, realized variance, Bipower variance, and realized jump variations in Table 1. Those variance and variation figures are computed based on high frequency intra-day five-minute returns. The logarithmic returns of Bitcoin exhibit a substantial amount of variation. Each of the series of the volatility measures (BV_t, RV_t,RJ_t) are highly skewed with leptokurtic distribution or "fat-tails". The values of skewness show evidence of asymmetry in the distribution of variances, especially the BV_t and RV_t series of 8.74 and 6.83, respectively. The Jarque–Bera test confirms the departure from the normal distribution.

We also empirically examine the null hypothesis of a white-noise process for Bitcoin returns by employing the Liung-Box test Q(21) for the logarithmic returns and jump components. Our results show that we can reject the null hypothesis of no serial correlation at the 1% significance level. Accordingly, this empirical evidence implies significant serial dependence in the return and its jump component. In addition, Table 1 depicts the results of two unit root tests: augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests of nonstationary, for the sample returns. The null hypothesis of a unit root is rejected and thus we conclude that the Bitcoin return series is stationary.

Figures 2 and 3 provide a bird's eye view on the time series of the Bipower variation BV, the daily realized volatility RV, the relative jump component RJ, and the jump intensity λ on the volatile Bitcoin and its options. The RV, BV, and RJ display the existence

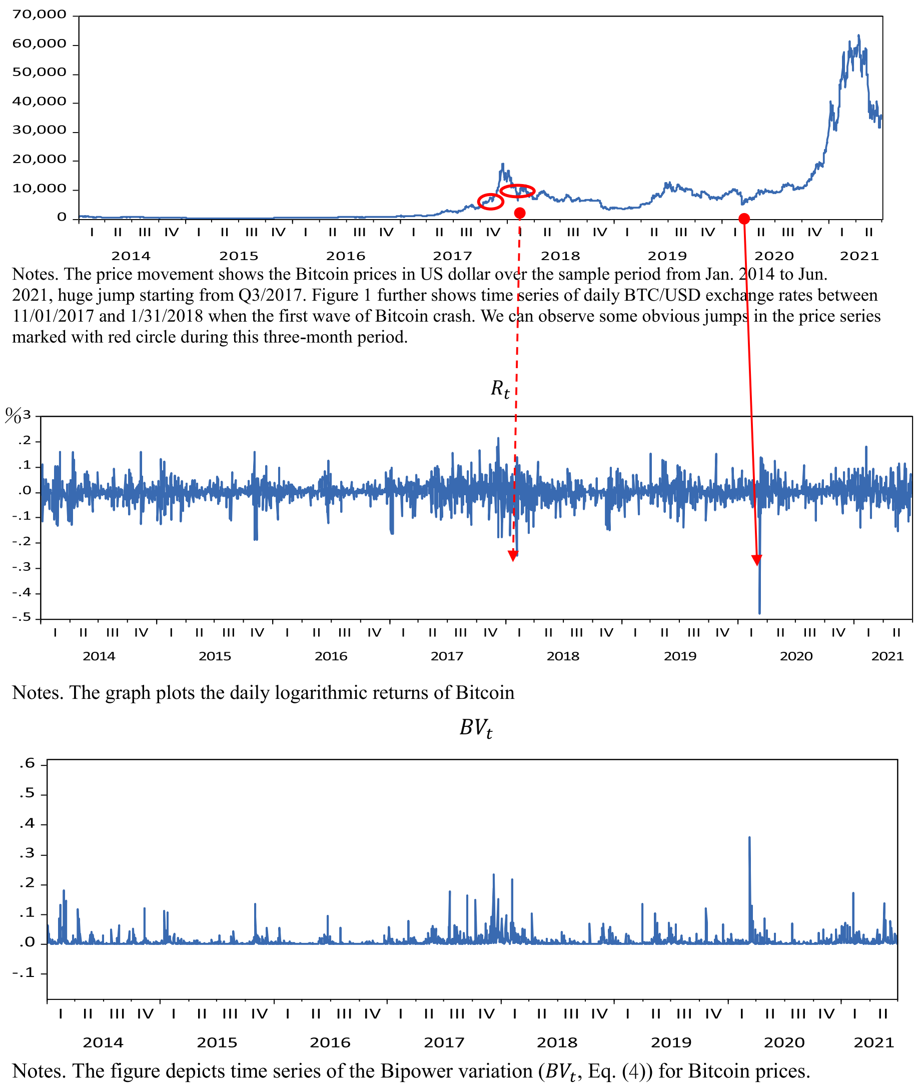


Fig. 1 The Trajectory, Log-Returns and the Bipower Variation of Bitcoin

of volatility persistence and asymmetry features, while the jump intensity λ depicts the phenomenon of time-varying diffusion. The GARCH volatility form is appropriate for modelling those phenomena. Those volatility estimators are also robust in identifying sudden jump arrivals and market structure noise, which are both desirable properties in volatility proxies based on the semiparametric model of Barndorff-Nielsen & Shephard (2006). Some of these volatility estimators are also robust to sudden jump arrivals, but they are not robust to market structure noise. The mean and variance of the realized variance exceeds that of the Bipower variation series. Andersen et al. (2012) also show that the Bipower variation is preferred to RV_t. The RJ_t depicted in the second Panel of Fig. 2 displays mostly positive and small values, but Table 1 shows that the minimum is −0.164. These small negative values can be due to discretization error measurement, errors owing to the usage of finitely many returns in the establishment of the underlying measurements (Bollerslev et al. 2008). The series also displays several extreme

Table 1 Summary statistics of log-returns, realized variance, Bipower variation and jumps

	R_t	BV_t	RV_t	RJ_t	λ	$\widehat{\sigma}^2_{PK}$
Panel A: Bitcoin spot prices						
Mean	0.001399	0.009479	0.008117	0.268404	0.106606	0.027852
Median	0.001539	0.003024	0.002235	0.186838	0.104839	0.021601
Maximum	0.214508	0.504477	0.358519	5.566198	0.228571	0.293944
Minimum	−0.479934	7.18E−08	4.55E−08	−0.163900	0.008333	0.001509
Std. Dev	0.040836	0.021800	0.019138	0.499181	0.039105	0.024004
Skewness	−0.792997	8.741236	6.834358	2.475781	0.269519	2.629541
Kurtosis	13.33273	139.3435	76.22375	13.59514	3.039272	15.66039
Jarque–Bera	12,467.11	2,155,625	632,997.2	15,603.74	33.23901	21,824.92
Probability	(0.000***)	(0.000***)	(0.000***)	(0.000***)	(0.000***)	0.0000***
Q (21)	35.34 (0.026**)	717.05 (0.000***)	1186.9 (0.000***)	39.869 (0.008***)	66.457 (0.000***)	
Unit-Root Test ADF	−55.33 (0.000***)					
PP	−55.25 (0.000***)					
Panel B: Call Bitcoin options prices in the derivatives market						
Mean	−0.004768	0.271628	0.326609	0.222383	0.174599	0.005365
Median	−0.002811	0.225902	0.312368	0.130638	0.175000	0.002472
Maximum	0.567365	0.795598	0.805863	1.506230	0.250000	0.116571
Minimum	−0.629462	0.025867	0.023120	−1.134348	0.075000	0.00000
Std. Dev	0.180631	0.163849	0.167181	0.346220	0.037401	0.007848
Skewness	−0.048997	0.953270	0.561982	0.934208	−0.329271	5.831186
Kurtosis	3.846065	3.418426	2.728009	4.714722	2.437947	65.26500
Jarque–Bera	15.23401	80.00946	28.08275	135.0563	15.74121	147,141.2
Probability	(0.00049***)	(0.000***)	(0.0000***)	(0.000***)	(0.00038***)	0.0000***

1. ** and *** denote statistical significance levels of 5%, and 1%

2. R_t denotes the log- returns computed from daily closing prices of the BTC. Q (21) denotes Ljung–Box statistics and lags 21 in parentheses

3. Both realized variance and realized jump variations are computed based on high frequency intra-day five-minute returns. The realized jumps (RJ_t) identification mechanism is based on the Bipower variation (BV_t) method

4. These realized measures of BTC daily variance are calculated as the sum of 5-min intraday squared returns. The market prices of European-style Deribit options written on BTC futures traded on the CME Bitcoin Futures are obtained from the Risk Lab (provided through Booth School of Business, University of Chicago) https://dachxiu.chicagobooth.edu/#risklab

5. Range-based volatility estimators of Parkinson (1980) are measured as following:

$\sigma^2_{Parkinson} = \widehat{\sigma}^2_{PK} = \frac{1}{4\ln(2)n}\sum_{i=1}^{n} ln\left(\frac{H_i}{L_i}\right)^2$

where H_i and L_i represent the high and low prices, respectively, on the trading day i

observations (outliers), indicative of realized large-sized jumps on those trading days. Bollerslev et al. (2009), among others, have provided similar results for the volatility jump from other markets. We contribute to the literature by providing strong evidence for the presence of jumps in the daily Bitcoin and its option price series.

Inference of number of jumps and jump intensity

The bottom Panels in Figs. 2 and 3 depict the jump-arrival intensity of Bitcoin and its options, respectively. As expected, they both exhibit time-varying jump dynamics. Specifically, Fig. 3 illustrates the stylized characteristics of leptokurtosis that arises from a pattern of time-varying volatility and clustering nature in the Bitcoin options market where periods of high (low) volatility are followed by periods of high (low) volatility. Apart from the realized variance, the Bipower variation of Bitcoin options exhibits time-varying behavior, suggesting the presence of stylized feature depicted by the dependence structure of Bitcoin markets. To further investigate the jump dynamics, we provide more information on quarterly estimates of the significant jump components with a significance level of $\alpha = 0.05$ in Table 2. We refer to the work of Novotný et al. (2015) on the selection of $K = 10$ in the jump test. The proportion of observations with a jump ranges from 0.049 in Q4 of 2020 to 0.151 in Q2 of 2018 during the sample period, with an average of 0.107. On an annual basis, the jump intensity appears to be the highest in 2018, relative to any other year of the sample period. More specifically, there are 54, 55, 43, and 51 jumps in Q1, Q2, Q3, and Q4 of 2018, respectively. Hence, this finding sheds new light on jump intensity in Bitcoin prices and highlights the importance of incorporating jumps in empirical models for Bitcoin returns and volatility.

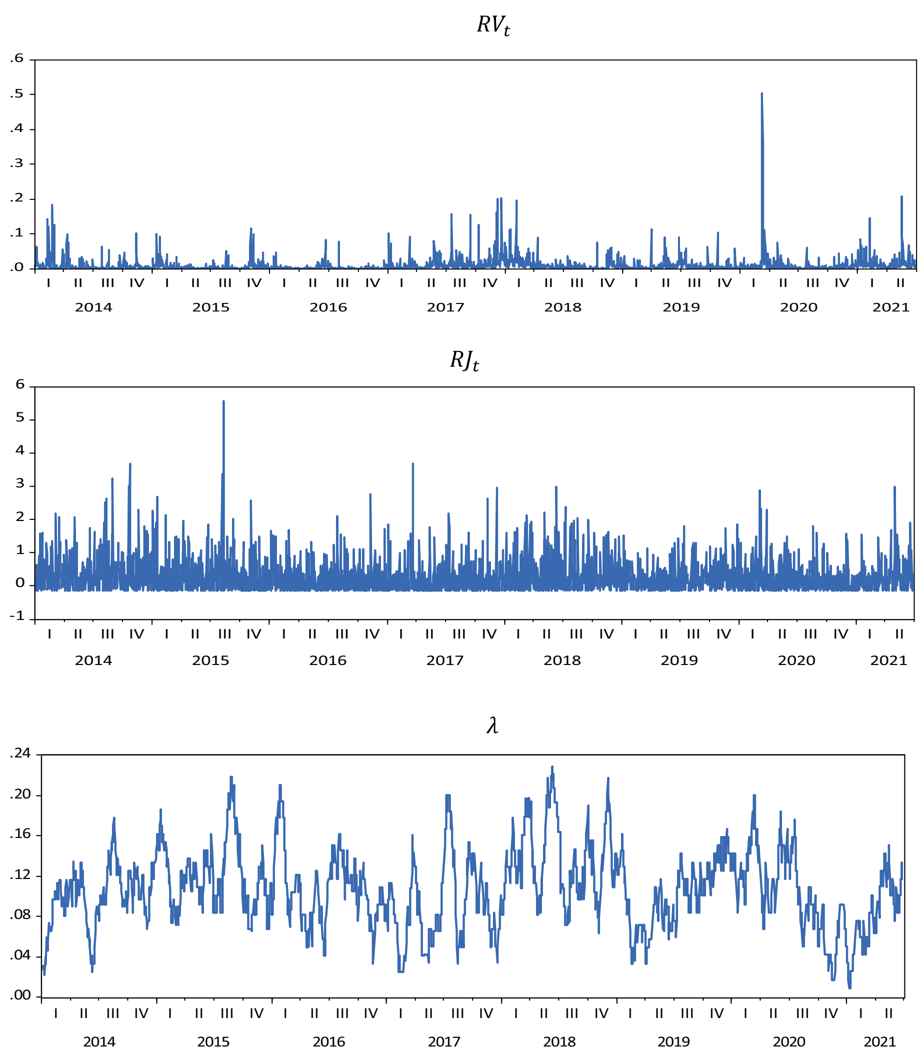


Fig. 2 Time series of the realized volatility, relative jump and jump intensity component for Bitcoin prices

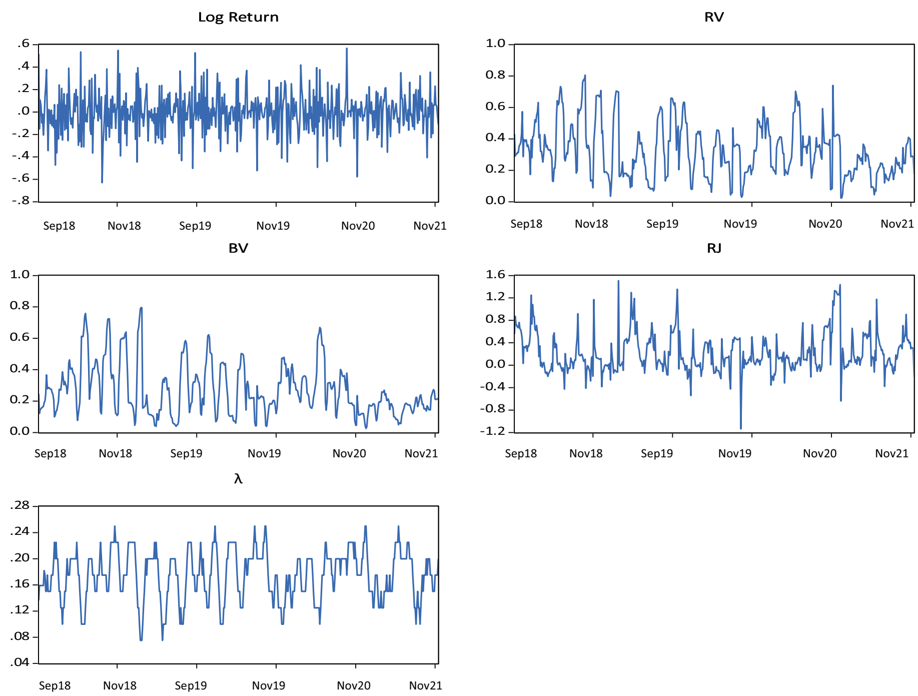


Fig. 3 Log-Returns and the Realized Variation Measures of Bitcoin Options

Bitcoin option empirics

We classify the options data into six distinct levels based on the moneyness and days-to-maturity (DTM). Table 3 presents descriptive statistics of Bitcoin options for both bid and ask prices as well as comparisons across various moneyness. The average call (put) option prices range from \$186.55 (\$317.27) for the shortest maturity (within 7 days) to \$1,263.35 (\$1,493.70) for the longest maturity (greater than195 days). For each given scenario of maturities, the smile across moneyness is evident. When the time to maturity increases, the bid-ask spreads tend to widen. For the calls, the mean bid-ask spread is \$31.58, near 5.24% of the average premium for all maturities. For the puts, the bid-ask spread is \$39.79, about 4.92% of the average premium. We observe that the skewness of option prices appears positive and the distribution is skewed to the right. It is thus important for a pricing model to capture the asymmetric distribution of Bitcoin option prices.

As depicted in Fig. 4, market implied volatilities for options with the shortest maturities display a *smile* shape on average, similar to that found at different moneyness. For the longer maturities, the shape is closer to the *smirk*. These patterns suggest the relative importance of return skewness and kurtosis at different maturities for characterizing the option data because excess kurtosis is sufficient to generate a smile but not a smirk. Moreover, in the previous literature, several studies have argued that volatility jumps are useful in explaining option volatility smiles and smirks. See for instance Eraker et al. (2003) and Eraker (2004). Therefore, we incorporate jumps into Bitcoin option pricing models to capture the volatility smile and smirk in option prices.

Building on the findings in Panels A and B of Fig. 4, we analyze the source of differences in the observed implied volatility and plot the smiles/smirks across moneyness

Table 2 Descriptive statistics on jumps detected by LM statistics

	Q1		Q2		Q3		Q4	
	No. of Jumps	P(Jump fre)	No. of Jumps	P(Jump fre.)	No. of Jumps	P(Jump fre.)	No. of Jumps	P(Jump fre.)
2014	33	0.0916	30	0.083	42	0.117	41	0.114
# Observations	360		360		360		360	
015	41	0.1138	44	0.122	53	0.144	36	0.0978
# Observations	360		364		368		368	
2016	46	0.1263	33	0.091	43	0.117	33	0.089
# Observations	364		364		368		368	
2017	29	0.081	27	0.074	43	0.117	32	0.087
# Observations	360		364		368		368	
2018	54	0.150	55	0.151	43	0.118	51	0.137
# Observations	360		364		364		372	
2019	24	0.066	30	0.083	42	0.114	49	0.133
# Observations	360		364		368		368	
2020	49	0.135	44	0.122	34	0.092	18	0.049
# Observations	364		364		368		368	
2021	22	0.061	42	0.115				
# Observations	360		364					
# Jumps		1,163						
Mean		0.107						
Total # Obs		10,932						

1. Regarding the jump intensity (λ), it is computed from the total number of jumps over the number of sample observations, their proportion (%) as shown in Eq. (2a), and their mean of full sample observations

$$\mathrm{P(jump\ fre.)} = \frac{\text{Number of Jumps}}{\text{Number of sample observations}} \quad (2a)$$

2. Quarterly estimates for no. of jumps for BTC is the number of detected jumps, and P (jump freq.) is evaluated jumps with a significant level at α = 0.05

3. Lee and Mykland (2008) jump test statistics is described as LM statistics with the selected window width K, which is set to 10

for different days-to-maturity buckets. In Panels C and D of Fig. 4, we repeat the same analysis for two maturity scenarios, 7 days and 36 days. The major conclusion from Fig. 4 is that whereas the models differ greatly with regard to the level of the smile, the slope of the smile does not seem to differ much. In some cases, the slope is a little steeper.

As can be seen from the preliminary results of the foregoing content in Section "Jump detection test"–Section "Variance decomposition and dynamic jump components", it is clear that the benefits of incorporating these jumps flow into option pricing so those models can capture the volatility smile in option prices. Accordingly, building an extension class of ARJI-GARCH-type models for predicting realized jump variation is our next task.

Model estimations

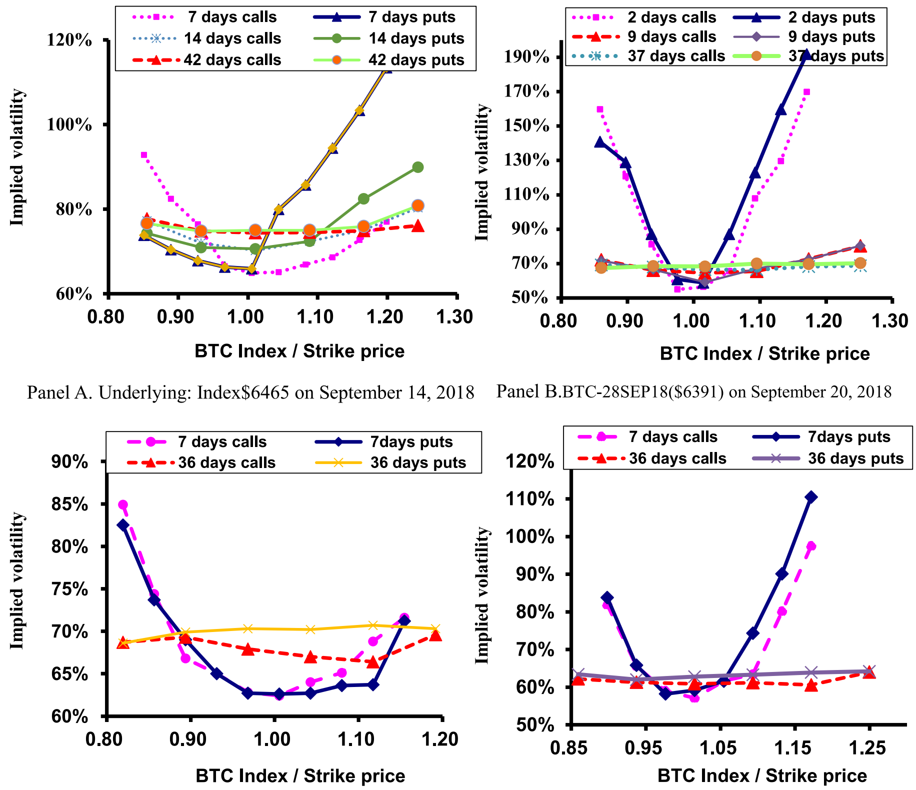
This subsection presents our model estimations based on historical time series of BTC returns under measure $\mathbb{P}$ and emphasizes some important stylized facts. We use daily returns of the BTC from January 1, 2014 to June 30, 2021 to estimate the ARJI-GARCH model parameters and report the outcomes among the models from the perspective of option valuation under measure $\mathbb{Q}$. The calculation of the GARCH α, β and γ parameters

Table 3 Descriptive statistics of Bitcoin options prices

Call	≦7 days		8–44 days		45–104 days		105–195 days		> 195 days	
Moneyness	**Bid**	**Ask**	**Bid**	**Ask**	**Bid**	**Ask**	**Bid**	**Ask**	**Bid**	**Ask**
$m > 1.07$	$508.27	$547.06	$579.74	$604.37	$758.75	$787.67	$1,139.12	$1,186.56	$1,502.45	$1,558.29
$1.03 < m \leqq 1.07$	$298.72	$327.67	$374.31	$412.17	615.74	636.22	$952.39	$1,002.31	$1,396.30	$1,450.82
$0.99 < m \leqq 1.03$	$143.52	$165.13	$261.82	$274.66	$502.89	$531.49	$866.55	$909.61	$1,290.16	$1,343.34
$0.96 < m \leqq 0.99$	$63.77	$79.65	$211.71	$226.55	$401.93	$421.85	$780.71	$824.61	$1,184.01	$1,235.87
$0.92 < m \leqq 0.96$	$30.95	$40.36	$101.81	$130.67	$322.10	$345.27	$694.87	$735.76	$1,077.87	$1,128.39
$0.89 < m \leqq 0.92$	$13.26	$20.22	$29.38	$38.52	$253.74	$267.96	$294.98	$317.04	$971.72	$1,020.92
Mean		$186.55		$270.48		$487.13		$808.71		$1,263.35
Mean Full sample	$603.24									
Std. Dev		190.23		191.49		182.5259		$277.75		192.4919
Skewness		$1.02		0.566		0.381		-0.788		0.006
Bid-ask spreads	**(31.58)**	$20.27		$21.36		$22.56		$41.21		$52.52
Average Bid-ask Premium	**(5.24)**									
Put										
$m > 1.07$	$17.00	$26.95	$97.55	$119.73	$342.73	$368.75	$714.44	$751.69	$1,108.35	$1,169.58
$1.03 < m \leqq 1.07$	$48.63	$62.22	$207.49	$231.58	$464.51	$493.11	$818.74	$858.98	$1,248.47	$1,313.25
$0.99 < m \leqq 1.03$	$201.58	$225.18	$312.00	$338.42	$586.30	$617.47	$978.71	$1,016.65	$1,388.59	$1,456.92
$0.96 < m \leqq 0.99$	$300.69	$336.78	$460.87	$488.84	$743.38	$777.28	$1,138.68	$1,178.42	$1,528.71	$1,600.59
$0.92 < m \leqq 0.96$	$509.10	$551.59	$635.09	$665.31	$900.45	$937.09	$1,298.65	$1,340.20	$1,668.83	$1,744.27
$0.89 < m \leqq 0.92$	$740.70	$786.85	$871.29	$920.76	$1,086.56	$1,122.74	$1,458.61	$1,490.12	$1,808.95	$1,887.94
Mean		$317.27		$445.74		$703.36		$1,086.99		$1,493.70
Mean Full sample	$809.42									
Std. Dev		273.975		278.37		267.585		271.76		255.76
Skewness		0.581		0.495		0.2356		0.115		0.012
Bid-ask spreads	**(39.79)**	$28.64		$30.06		$32.09		$38.04		$70.11
Average Bid- ask Premium	**(4.92)**									

1. Average bid-ask spreads and premium at full sample in parentheses, respectively

2. The values are reported respectively the average option prices, bid-ask spread, standard dev. and skewness for each moneyness- maturity category for a total of 2,252 calls and puts



Panel A. Underlying: Index$6465 on September 14, 2018 Panel B.BTC-28SEP18($6391) on September 20, 2018

Panel C. Underlying: BTC-28SEP18($6713) on September 22, 2018 Panel D. Underlying: BTC-28SEP18($6402) on September 25, 2018

Fig. 4 IV (implied volatility) comparison by Moneyness and Maturity

requires the maximum likelihood estimation on historical data under measure $\mathbb{P}$ and the option valuation by changing measure[10] from $\mathbb{P}$ to $\mathbb{Q}$.

Table 4 presents the results obtained with the historical parameter estimates $\omega = 5.59 \times 10^{-5}$, $\alpha = 0.171$, and $\beta = 0.783$ in the ARJI-GARCH (1, 1) model. These parameter values indicate a volatility persistence of 0.954 (i.e., $\alpha + \beta$), a level consistent with empirical findings documented in the previous literature, under measure $\mathbb{P}$. The Ljung-Box-Q statistic for testing autocorrelations in Table 4 shows that those GARCH models are adequate for the data at the 5% significance level. The p-values with the ARCH-LM test implemented to the residuals of the GARCH, C-GARCH and EGARCH models are 0.882, 0.888 and 0.821, respectively. Results in Table 4 rejects the null hypothesis of the existence of ARCH effect at the 5% significance level. It is thus apparent that the ARCH effect no longer exists, and our models can fit well the daily returns of BTC.

As reported in Table 4, coefficients d (0.1136, 0.1138 and 0.076) of the jump intensity (λ) among the three ARJI-GARCH type models are very close to the mean of jump intensity in Table 1 ($\lambda = 0.1066$). The coefficient of the jump intensity is significant in each of our models for Bitcoin prices. Indeed, compared to jumps with constant intensity, time-varying jumps on the occurrence of sudden shocks can better capture Bitcoin's

[10] The authors are thankful to an anonymous reviewer for suggesting the estimated model for the change of measure. The proof of the physical measure $\mathbb{P}$ transformed into risk-neutral measure $\mathbb{Q}$ is shown in Appendix B.

Table 4 MLE of ARJI-GARCH-type models on Daily Bitcoin Returns

	GARCH (1,1)		CGARCH (1,1)		EGARCH (1,1)	
	coeff	**Pr(>|t|)**	**coeff**	**Pr(>|t|)**	**coeff**	**Pr(>|t|)**
Mean equation						
$\widehat{\mu}$	0.0142*** [7.967]	0.000	0.0143*** [7.979]	0.0001	0.0121*** [8.203]	0.000
m	−0.052** [−2.313]	0.021	−0.053*** [−2.247]	0.000	−0.075*** [−3.614]	0.000
d	0.1136*** [7.119]	0.000	0.1138*** [7.967]	0.000	0.076*** [5.267]	0.000
Variation equation						
ω	5.59E−05*** [10.558]	0.000	0.0012 [7.378]	0.000	−0.946*** [−20.838]	0.000
ρ	–	–	0.954*** [146.977]	0.000	–	–
φ CGARCH	–	–	0.169*** [11.789]	0.000	–	–
γ EGARCH	–	–	0.012 [0.587]	0.55–7	0.093** [7.583]	0.000
α	0.171*** [13.393]	0.000	0.012 [0.587]	0.557	0.312*** [17.451]	0.000
β	0.783*** [63.339]	0.000	−0.206 [−0.137]	0.891	0.906*** [174.463]	0.000
δ	1.52E−04*** [15.094]	0.000	1.53E−04*** [14.740]	0.000	0.376*** [28.837]	0.000
AIC		−3.822		−3.820		−3.837
SIC		−3.806		−3.801		−3.856
DW-stat		2.023		2.021		1.969
Log likelihood		5,225.14		5,225.24		5,297.26
ARCH-LM (5) Test		0.882		0.888		0.821
Out of sample model performance metrics						
RMSE		0.043		0.043		0.0429
MAE		0.027		0.028		0.0265
MAPE		139.87		139.097		137.891

1. *, ** and *** denote the significance levels of 10%, 5%, and 1%, respectively

2. The numbers in parentheses are z-statistic.AIC and BIC are the Akaike and Bayesian information criteria, respectively. Heteroskedasticity Test is the ARCH -LM test in p-value of ChiSq

3. Where the estimated parameter $\widehat{\mu}$ is approximated to the term i n Eq. (12)

price behavior. On the contrary, as shown in Table 5, the jump intensity is insignificant for Bitcoin options. This finding suggests that the Bitcoin spot market contains relatively more information than the Bitcoin option market in related to the jump intensity. That is, Bitcoin options quoted in the Deribit reflect relatively less information for predicting time-varying jumps on the occurrence of price shocks.

As shown in Tables 4 and 5, the lower AIC and BIC values as well as the higher log-likelihood indicate that the ARJI-EGARCH model is a better fitting model than the other two models, in support of findings from previous studies (see, e.g., Zhang et al. 2011; Dyhrberg 2016). Overall, although the data display substantial richness and variation, our proposed models appear to capture them reasonably well.

Variance decomposition and jump components

Regarding the impact of realized jump variation on the variance decomposition in Eq. (18), we find that the coefficients (δ=1.52E−04, 1.53E−04, 0.376) shown in Table 4 as well as those coefficients (δ=0.021, 0.02, 0.761) depicted in Table 5 are statistically different from zero at any conventional levels. This empirical evidence provides support for the explanatory capacity of realized jump variation on the conditional volatility of both Bitcoin and Bitcoin options.

In sum, the results in Tables 4 and 5 indicate that allowing for realized jump variation is crucial for jump models in both Bitcoin and its option markets. The

Table 5 Maximum Likelihood Estimation on Daily Bitcoin Returns, Realized Measures, and Options

	GARCH (1,1)		CGARCH (1,1)		EGARCH (1,1)	
	coeff	**Pr(>|t|)**	**coeff**	**Pr(>|t|))**	**coeff**	**Pr(>|t|)**
Mean equation						
$\hat{\mu}$	− 0.006 [− 0.463]	0.644	− 0.008 [− 0.648]	0.517	− 0.009 [− 0.332]	0.740
m	0.231*** [4.829]	0.000	0.234*** [5.037]	0.000	0.144*** [2.867]	0.004
d	0.008 [0.116]	0.908	0.026 [0.332]	0.740	0.025 [0.148]	0.883
Variation equation						
ω	0.011*** [3.528]	0.000	0.026 [7.418]	0.000	− 2.324*** [− 5.905]	0.000
ρ	–	–	0.434*** [2.575]	0.01	–	–
φ CGARCH	–	–	0.517 [0.694]	0.488	–	–
γ EGARCH	–	–	–	–	0.047 [0.735]	0.462
α	0.202*** [3.344]	0.000	− 0.342 [− 0.459]	0.646	0.524*** [5.259]	0.000
β	0.323*** [2.739]	0.006	0.536 [0.489]	0.624	0.512*** [5.238]	0.000
δ	0.021*** [5.319]	0.000	0.02*** [5.919]	0.000	0.761*** [4.771]	0.000
AIC		− 0.578		− 0.575		− 0.736
SIC		− 0.519		− 0.498		− 0.669
DW-stat		1.886		1.902		1.808
Log likelihood		152.4		153.48		193.18
ARCH-LM (5) Test		0.151		0.242		0.394
Out of sample Model Performance Metrics						
RMSE		0.196		0.198		0.194
MAE		0.141		0.142		0.139
MAPE		165.035		192.869		140.184

To evaluate the option pricing performance of jump models, we select the out-of-the-money (OTM) Bitcoin call option data for the sample period of September 14, 2018 to November 27, 2021. The Bitcoin option data are retrieved from Deribit with maturities within two weeks. We eliminate quotes with zero trading volume. The data is filtered only OTM option quotes since they tend to have high liquidity

discontinuous jump variation can describe the volatility behavior of Bitcoin prices and capture the jump risk dynamics**.** In addition, these estimates are statistically significant and consistent across models. Bates (1996) argues that the most rigorous test of an option pricing model is determined by its fitting capacity to the option data and the underlying returns. Our findings suggest that the ARJI-GARCH type models are appropriate in the Bitcoin spot and options markets because they fit the data reasonably well.

Performance evaluation

Out-of-sample prediction

We now proceed to assess the performance of various models on BTC option pricing, with special attention to the out-of-sample predictive capacity. In order to appropriately evaluate the out-of-sample predictive performance, we divide the sample into two subsamples separated by the COVID-19 Pandemic outbreak in Dec. 2019. The first subsample covering the pre-COVID-19 pandemic period is adopted for model estimation, while the second subsample of the post-COVID-19 pandemic period is applied to model prediction. From the model performance metrics depicted in the bottom of Tables 4 and 5, we can assess the forecast performance in out-of-sample predictions for the three ARJI-GARCH models. The metrics of RMSE, MAE, and MAPE show that the ARJI-EGARCH model outperforms in the out-of-sample performance among the estimated ARJI-GARCH models. We further examine the out-of-sample performance of all models by plotting the forecasts with plus and minus two standard error bands as well as the predicted variance in Fig. 5. Overall, ARJI-EGARCH model in the Panel C of Fig. 5 exhibits the smallest value of standard error bands and predicted variance.

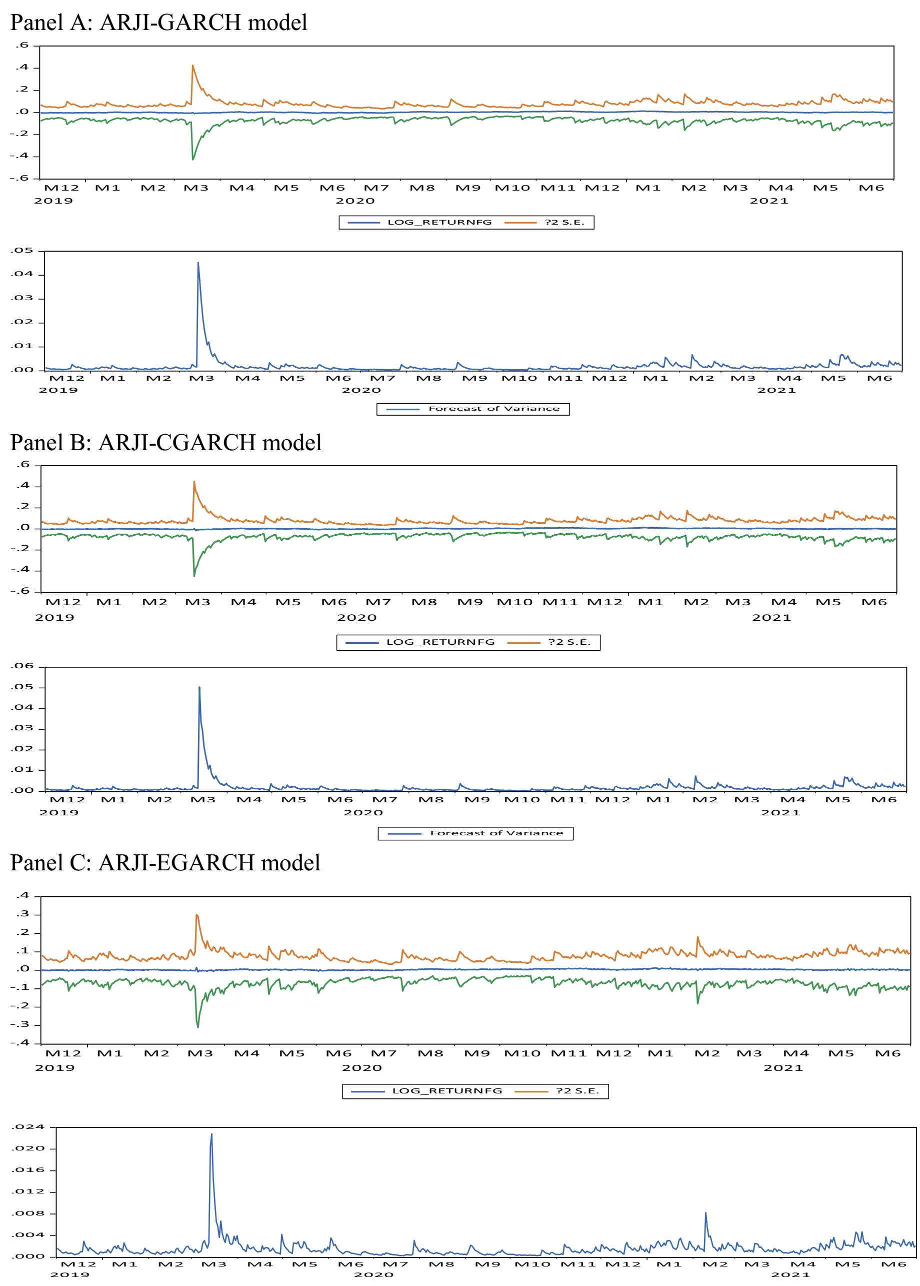


Fig. 5 Forecast ± 2 standard errors and predicted variance from the models

Robustness check

Table B of Appendix C reports the parameter estimates and the goodness-of-fit results with the SVCJ model for Bitcoin and its options. The empirical results clearly show the superiority of ARJI-GARCH model over existing BTC models in the pricing ability (see the lower AIC and BIC, and higher log-likelihood results in Tables 4 and 5). Therefore, the ARJI-GARCH model can achieve better performance than the SVCJ model in terms of the AIC, BIC, and log-likelihood.

Apart from classical performance evaluation criteria, this study further uses the known Diebold–Mariano (1995) DM test to compare the forecasting performance among the competing ARJI -GARCH models.[11] The results are displayed in Table C of Appendix D. The observed differences in forecasting performances are significant at the 5% level. Therefore, we conclude that the ARJI -EGARCH model outperforms other ARJI -GARCH type models.

This subsection also examines whether our empirical findings are robust. Table 6 demonstrates overall pricing errors for European call options. The overall RMSEs are 0.1814, 0.1012, and 0.2519 for the GARCH, EGARCH and CGARCH, respectively. Perhaps surprisingly, on average, the EGARCH stands out as having a superior performance than the GARCH and CGARCH specifications. A closer look at the RMSE by moneyness and DTM categories would provide further insight as it ranges from 0.034 to 0.6686. We observe some interesting empirical results. First, compared to the GARCH and CGARCH models, the EGARCH model appears to have the smallest valuation error for each DTM category. Second, all three models exhibit the lowest RMSE from the longest DTM options. This evidence is similar to the findings from Bitcoin's volatility prediction in Naimy et al. (2018).

The behavior of RMSE results on the BTC put options regarding different moneyness and maturity is illustrated in Table 7. Similar to the results for the call options, the EGARCH model also provides a better overall fit for put options than the GARCH and CGARCH models with Poisson jumps. Unlike the call option results, all three models seem to perform better for short DTM put options. In addition, in terms of moneyness, all three models perform better for deep in-the-money put options within each DTM category. A critical determinant of model performance is the ability to describe some important stylized aspects about option prices, including volatility clusters and the jump effect. The overall low RMSE from the EGARCH model confirms its ability.

In sum, we find important stylized empirical features as follows. The EGARCH models adequately capture the stylized feature of fat tails or term structure of volatility than do the GARCH (1,1) and CGARCH models in the sample period.

A close look at some facts on option pricing bias

Figure 6 exhibits the plots of the percentage errors and illustrates the bias in percentage errors. The plots in Panels A-C of Fig. 6 reveal large smile patterns associated with the GARCH-type models in shorter-maturity options. The biases for this model are particularly large on average and increase as the contract moves out of the

[11] We thank an anonymous reviewer for suggesting this additional test.

Table 6 The RMSE of pricing call options by moneyness and maturity for the three GARCH Jump models

Moneyness	GARCH					EGARCH					CGARCH				
	τ≦7 days	7 < τ≦45	45 < τ≦105	105 < τ≦195	τ > 195	τ≦7	7 < τ≦45	45 < τ≦105	105 < τ≦195	τ > 195	τ≦7	7 < τ≦45	45 < τ≦105	105 < τ≦195	τ > 195
$m > 1.07$	0.0839	0.0751	0.0720	0.1751	0.1117	0.0893	0.0599	0.0340	0.1443	0.0484	0.0713	0.0658	0.0811	0.2021	0.1560
$1.03 < m \leq 1.07$	0.1041	0.1606	0.1085	0.0624	0.1168	0.1030	0.1477	0.0676	0.0454	0.0578	0.0734	0.1516	0.1221	0.1580	0.1899
$0.99 < m \leq 1.03$	0.1642	0.1880	0.1342	0.0967	0.1305	0.1786	0.0578	0.0393	0.0421	0.0643	0.1450	0.2063	0.1798	0.1849	0.2127
$0.96 < m \leq 0.99$	0.1592	0.2016	0.1210	0.1307	0.1396	0.2261	0.1245	0.0728	0.0544	0.0673	0.2687	0.3156	0.2414	0.2366	0.2398
$0.92 < m \leq 0.96$	0.4710	0.3528	0.2072	0.2040	0.1605	0.2287	0.1154	0.0599	0.0667	0.0665	0.6686	0.4298	0.3064	0.2870	0.2398
$0.89 < m \leq 0.92$	0.3776	0.4953	0.2095	0.1737	0.1416	0.2727	0.2664	0.0991	0.0724	0.0645	0.5915	0.5894	0.3870	0.3254	0.2532
Average	0.2262	0.2650 0.1421	0.1404	0.1334	0.1831	0.1286	0.0621	0.0709	0.0615		0.3031	0.2931	0.2196	0.2323	0.2116
	All:0.1814					All:0.1012					All:0.25194				

1. Deribit offer a wide range of option contracts with strikes ranging from 2500 to 45,000 (USD) corresponding to the option quote in the sample period. In this study, we use the moneyness filters adopted by Bakshi et al. (1997) to the data. Therefore, moneyness (m) is determined by S/X, where S represents the spot BTC price index level and X is the strike price. Thus, a call option is then said to be Deep OTM (DOTM) if $m \leq 0.92$, and out-of-the-money (OTM hereafter) if $0.92 < m \leq 0.96$; at-the-money (ATM) if $0.96 < m \leq 1.03$; and in-the-money (ITM) if $m > 1.03$

2. European options prices were computed using Eq. (26) and **τ** represents days to maturity (DTM)

3. The minimum quote size of the strike price is 250 market price quotes in Deribit trading platform. The some strikes of Tables 6, 7 are interpolated between the expiries to obtain an option market valuation. Interpolation of market strike prices are more art than science. In these research designs the market effective parameters are linearly interpolated between every two expiries. The pricing errors are expressed as the difference between fitted (forecasted) and actual (market) option prices and reported in percentages

Table 7 The RMSE of pricing puts option by moneyness and maturity for the three GARCH Jump models

Moneyness	GARCH					EGARCH					CGARCH				
	$\tau \leqq 7$	$7 < \tau \leqq 45$	$45 < \tau \leqq 105$	$105 < \tau \leqq 195$	$\tau > 195$	$\tau \leqq 7$	$7 < \tau \leqq 45$	$45 < \tau \leqq 105$	$105 < \tau \leqq 195$	$\tau > 195$	$\tau \leqq 7$	$7 < \tau \leqq 45$	$45 < \tau \leqq 105$	$105 < \tau \leqq 195$	$\tau > 195$
$\boldsymbol{m} > 1.07$	0.2223	0.1816	0.1965	0.2251	0.2686	0.3931	0.1509	0.0752	0.1378	0.1976	0.2329	0.2982	0.2800	0.3068	0.3471
$1.03 < m \leq 1.07$	0.0892	0.1960	0.1767	0.1794	0.2578	0.1803	0.0740	0.0822	0.1007	0.1833	0.1232	0.2674	0.2567	0.2721	0.3413
$0.99 < m \leq 1.03$	0.0854	0.1503	0.1260	0.1784	0.2350	0.0808	0.0311	0.0606	0.1088	0.1660	0.1125	0.1599	0.2005	0.2599	0.3125
$0.96 < m \leq 0.99$	0.0433	0.1056	0.1171	0.1674	0.2109	0.0478	0.0494	0.0633	0.1060	0.1469	0.0649	0.1275	0.1870	0.2397	0.2828
$0.92 < m \leq 0.96$	0.0314	0.0523	0.0870	0.1453	0.1754	0.0262	0.0221	0.0472	0.0961	0.1270	0.0355	0.0656	0.1279	0.1959	0.2326
$0.89 < m \leq 0.92$	0.0217	0.0305	0.0804	0.1300	0.1607	0.0170	0.0226	0.0462	0.0820	0.1063	0.0250	0.0444	0.1236	0.1857	0.2227
Average	0.0822	0.1194	0.1306	0.1709	0.2181	0.1247	0.0584	0.0624	0.1052	0.1545	0.0990	0.1605	0.1959	0.2434	0.2898
	All:0.1442					All:0.101					All:0.1977				

1. To perform our empirical application, we divide the options into several categories, according to either time to maturity or moneyness. A put option is said to be DOTM if $m > 1.07$ and OTM if $1.03 < m \leq 1.07$; if $0.96 < m \leq 1.03$ ATM; if $0.92 < m \leq 0.96$ Near ITM put; Deep ITM put if $m \leq 0.92$

2. Using the Eq. (7a), we compute RMSEs for all three models, RMSE refers to the square root of the mean-squared valuation errors

$$RMSE_\theta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(C_i^{market} - \widehat{C}_i^{model}(\theta)\right)^2}, \text{(7a)}$$

$$\%RMSE_\theta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{C_i^{market} - \widehat{C}_i^{model}(\theta)}{C_i^{market}}\right)^2}, \text{(7b)}$$

where N is the number of option contracts, C_i^{market} is the option price observed from the market, $\widehat{C}_i^{model}$ is the estimated price from the model considered, and θ represents the parameter set of our models. For robustness, we also use the following mean absolute percentage error ($MAPE$) to evaluate the performance of the alternative option models. $MAPE_\theta = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{C_i^{market} - \widehat{C}_i^{model}(\theta)}{C_i^{market}}\right|$. (7c) money. Our results are analogous to findings from other studies, such as Hsieh and Ritchken (2005), and consistent with classical market volatility that forms a “smile” pattern. For instance, on average, pricing errors measured by RMSE for deep out-the-money call options are almost 30%. Indeed, more than one in four contracts (25th quantile (Q25)) in this category exhibit pricing errors of at least 19%, and the 75th quantile (Q75) extends to 44%. In longer-maturity options, as depicted in

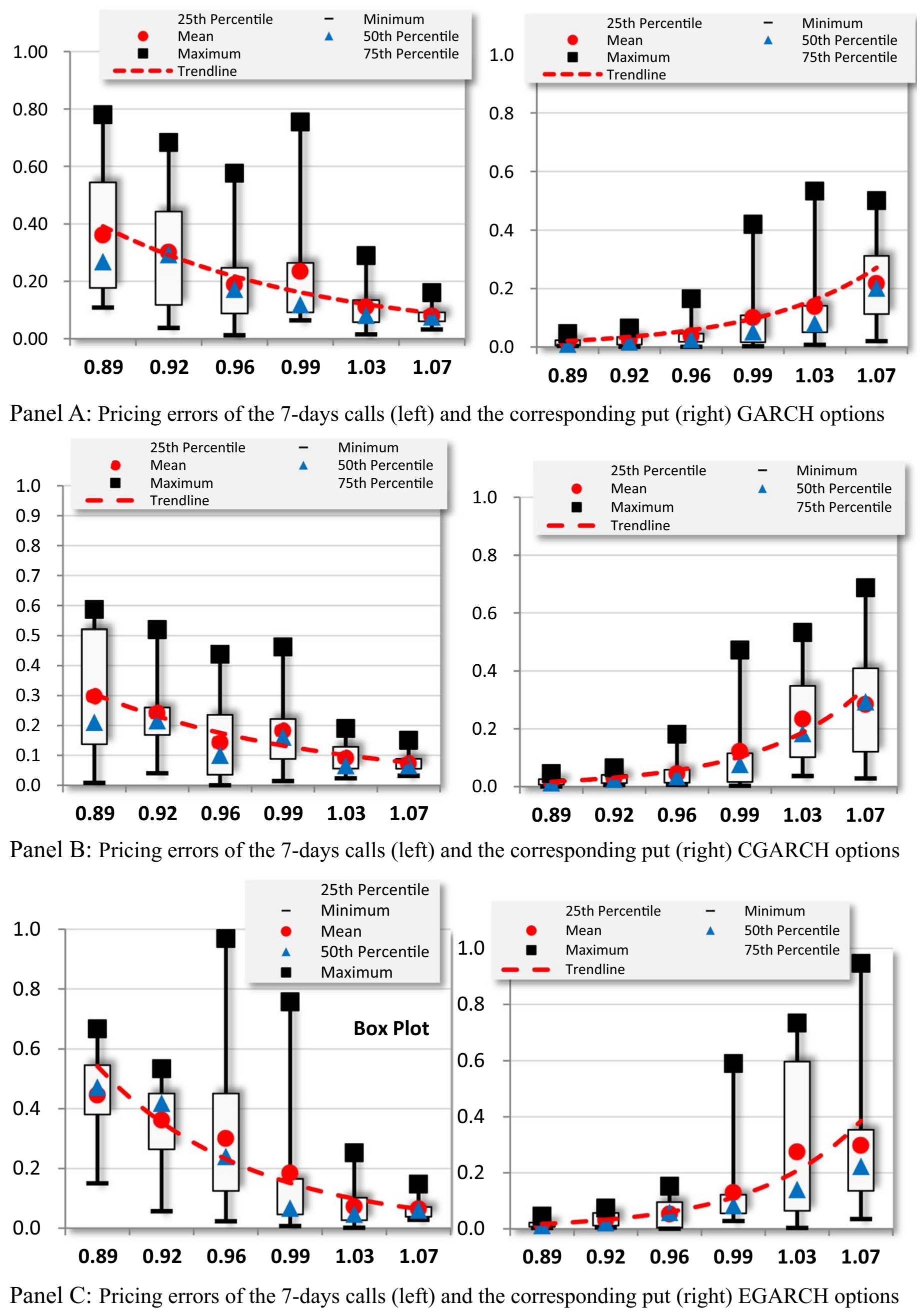
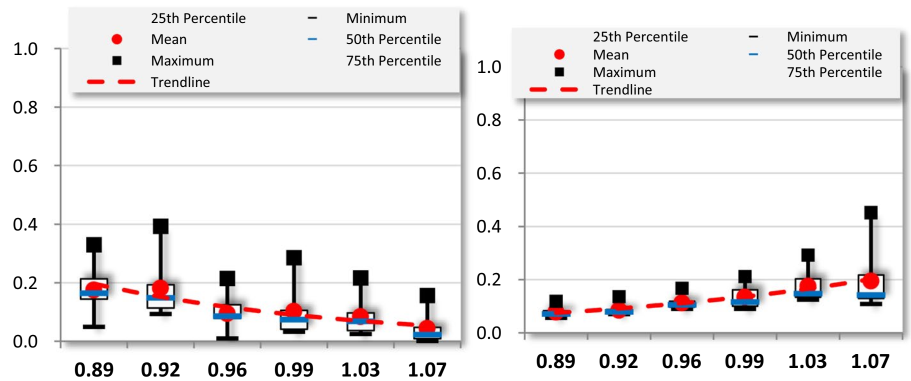
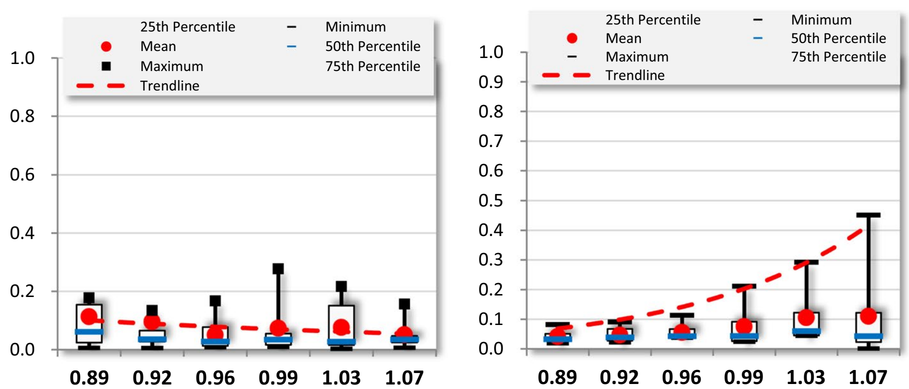


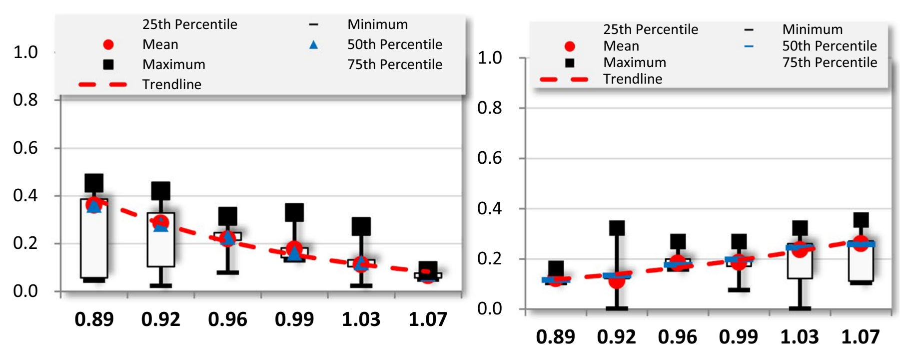
Fig. 6 The residuals for the GARCH type models in the form of box-whisker plots for each moneyness-maturity bucket



Panel D: Pricing errors of the 42-days calls (left) and the corresponding put (right) GARCH options



Panel E: Pricing errors of the 42-days calls (left) and the corresponding put (right) CGARCH options



Panel F: Pricing errors of the 42-days calls (left) and the corresponding put (right) EGARCH options

Fig. 6 continued

Panels D-F of Fig. 6, the volatility smile does not change. In contrast, the plots reveal volatility semi-smile patterns associated with the GARCH-type models.

Empirical implications

Bitcoin has gradually been recognized as an asset class. Gronwald (2019) shows that Bitcoin shares some similar features with commodities such as gold. In May 2021, Goldman Sachs officially states that Bitcoin is now considered an investable asset. Exchange-traded funds on Bitcoin are being introduced in some exchanges. As a result, Bitcoin options will become more popular given the growing need for hedging

Bitcoin risk. Our results are thus relevant to investors, corporations, and policy makers. In this section, we provide specific implications from our key findings.

First of all, from the pricing error perspective, our empirical findings specifically suggest that it is quite adequate to capture stylized facts using the GARCH option pricing model for price discovery in the presence of a volatile Bitcoin options market. In addition, we show that the Bitcoin spot market contains relatively more information than the Bitcoin options market in relation to the jump intensity. Perhaps surprisingly, risk premia may lead to Bitcoin options quoted in the Deribit reflecting relatively less information for predicting time-varying jumps in the occurrence of price shocks. The presence of pricing inefficiencies in the Bitcoin options market suggests that cryptocurrency enthusiasts, arbitragers, corporate hedgers, and other investors operating in the BTC option markets should be aware of the unique BTC market microstructure created by a set of decentralized, unregulated (such as the Deribit), and highly speculation-driven markets.

Second, theoretically, Bitcoin prices can show highly volatile behavior due to government interventions, speculative interests, and the potential contemporaneous news-driven shocks in the market. Our empirical results suggest that the realized jump variation can describe the volatility behavior and capture the jump risk dynamics in Bitcoin and its options. Although Bitcoin behaves as an extremely volatile asset, the model-based forecasting evaluation outlined above has shown that ARJI-GARCH type models could help substantially to pin down the option pricing errors and achieve better performance in forecasting volatility than the SVCJ model. This finding implies that specifying asymmetric jumps in conditional jump intensity on Bitcoin and its options contain more unique information than the traditional jump-diffusion process. Sophisticated investors of Bitcoin and Bitcoin options should consider ARJI-GARCH type models to better capture relevant pricing information.

Third, considering the Bitcoin price evolution over the past decade, our results also provide important policy implications. It's plausible that Bitcoin's volatility is what makes it valuable and attractive to investors. Although it can be unstable at times, Bitcoin is emerging as an asset class and can offer investors an opportunity to diversify their portfolios. Government policies and regulations should be directed to make sure that investors are aware of the risk involved and the potential benefit associated with Bitcoin and its options. Our empirical analyses provide investors and policy makers tools to appropriately assess risks and pricing errors of Bitcoin options. Specifically, the GARCH option pricing model with jumps significantly improves the fit of historical time series of Bitcoin prices and captures the volatility smile and smirk in Bitcoin option prices.

Conclusion and suggestions for future work

In the FinTech era, the national motto "In God We Trust" printed on the U.S. paper and coin currencies could lead to the new phrase "In Bitcoin We trust?" We borrow the idea that Bitcoin prices are driven by jump dynamics on the Bitcoin system and underlying technology (Kristoufek 2015; Gronwald 2019). Given the fact that both Nasdaq and the CME Group plan to launch a variety of Bitcoin derivatives, this study provides researchers and practitioners appropriate models to help them price Bitcoin options when they become available for trading. Recognizing the difficulty of estimating jump parameters, we consider models with jump detection to capture the dynamic jump condition.

Based on the jump diffusion framework in Lee and Mykland (2008) via jump detection test, we conduct an easy-to-implement practical approach for detecting "significant" jumps in BTC prices and study its application on option pricing. We provide plausible results for the presence of jumps in the BTC options market. When the market risk is high, jumps occur more frequently, especially during the period of cryptocurrency bubbles in late 2017. The primary benefit of modeling jumps in BTC option markets is that the negative skewness (or equivalently volatility clustering) of the distribution can be better modeled.

Our empirical evidence has several major implications for market participants in the Bitcoin markets. First, how much cryptocurrency should be included in an investor's portfolio ultimately relies on the investor's risk tolerance and beliefs about crypto assets. Moreover, for risk managers, they must be innovators and notice that the intensity and magnitude of jumps are time varying, which is attributed to the jump-arrival intensity. We emphasize the stylized fact that the presence of jump risk should not be ignored and report the mean absolute hedging error for hedging options in the Bitcoin market. The substantial improvements can be achieved by utilizing robust procedures.

We suggest several interesting extensions. First, this study uses the jump detection statistics and corresponding nonparametric tests to detect irregular jump arrivals of Bitcoin prices. It would be interesting to examine the Bitcoin market microstructure in high-frequency observations. Second, market sentiments about Bitcoin or, more importantly, on cryptocurrencies or IT finance are not directly observed, so some indicator variables can be considered in the future work. Lastly, Bitcoin options are now available for trading on the CME Group, so our model can be used to test its performance when sufficient trading data become available.

Abbreviations

FinTech	Financial Technology
CME	Chicago Mercantile Exchange
ARJI-GARCH	Autoregressive Jump Intensity Generalized Autoregressive Conditional Heteroskedasticity
SVCJ	Stochastic Volatility Correlated Jump
BTC	Bitcoin
BEKK	Baba, Engle, Kraft and Kroner
JD	Jump Detection
BV	Bipower Variation
RV	Realized Volatility
RJ	Relative Jump
HAR-RV-J	Heterogeneous Autoregressive Realized Volatility Jump
ARMA	Autoregressive Moving Average
BOV	Bitcoin Option Volatility
MLE	Maximum Likelihood Estimation
ADF	Augmented Dickey–Fuller
PP	Phillips–Perron
LM	Lee and Mykland
DTM	Days-to-Maturity
OTM	Out-of-the-Money
ATM	At-the-Money
ITM	In-the-Money
RMSE	Root-Mean-Square Error
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
DM	Diebold-Mariano

Supplementary Information

The online version contains supplementary material available at https://doi.org/10.1186/s40854-024-00653-z.

Additional file 1

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Author contributions
KC: Conceptualization, Models, Analysis, Writing - original draft. JY: Conceptualization, Analysis, Writing - review & editing. All authors read and approved the final manuscript.

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