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Examining time–frequency quantile dependence between green bond and green equity markets

Md. Bokhtiar Hasan[1], Gazi Salah Uddin[2], Md. Sumon Ali[1], Md. Mamunur Rashid[1], Donghyun Park[3] and Sang Hoon Kang[4*]

*Correspondence: sanghoonkang@pusan.ac.kr

[1] Department of Finance and Banking, Islamic University, Kushtia 7003, Bangladesh
[2] Department of Management and Engineering, Linköping University, Linköping, Sweden
[3] Economic Research and Development Impact Department, Asian Development Bank, Mandaluyong, Philippines
[4] School of Business, Pusan National University, Busan, Republic of Korea

Abstract

In the context of the rapidly growing demand for green investments and the need to combat climate change, this study contributes to the emerging literature on green investments by exploring the time–frequency connectedness between green bonds (GBs) and green equities. Specifically, we examine the degree of connection between GBs and green equities, the extent to which these markets influence each other, and which one is the primary net transmitter versus the net receiver of shocks under diverse market conditions. To accomplish these objectives, we use the wavelet-based Quantile-on-Quantile (QQ), dynamic conditional correlation (DCC), portfolio implications, and Quantile VAR approaches. The results show that GBs and green equities have a strong positive connection, depending on time and frequency domains. However, a negative association between GBs and green equities is observed during periods of crisis, highlighting GBs' ability to hedge green equity portfolios. The portfolio strategies demonstrate that investors require to invest in the Green Economy equity and S&P GB portfolio to reach the highest level of hedging effectiveness. The findings further imply that the Global Water Equity Index transmits the highest spillover to other green assets, while the Green Economy Equity Index receives the most spillover from other assets. The pairwise volatility connectivity reveals that most pairs have minimal quantile dependence, indicating the potential for diversification across the GB and green equity pairs. These findings have significant implications for investors and policymakers concerned with green investments and climate change mitigation.

Keywords: Green bond, Green equity, Frequency connectedness, Quantile dependency, Diversification

JEL Classification: C58, G12, G15, Q01

Introduction

Climate change, undoubtedly one of the most formidable obstacles facing humankind, is being driven by the escalating rate of greenhouse gas emissions that are heating the earth's atmosphere and will, if not reversed, lead to irreversible and catastrophic consequences for the planet's ecosystems (Naeem et al. 2021b). Therefore, it is critical that we undertake immediate and large-scale efforts to diminish carbon emissions by reducing the use of fossil fuels in an attempt to avoid the adverse effects of climate change on the planet's ecosystems and human existence (Rasoulinezhad 2020). Developing clean, renewable power is critical to decarbonizing the energy system to help achieve environmental sustainability (Van Hoang et al. 2019; Hasan et al. 2022c). The amount of CO_2 emissions has grown significantly since 1970, from around 14,312 million tons to over 34,169 million tons in 2019, posing a significant worldwide concern (Yoshino et al. 2021). However, a large amount of capital will be required to finance transformational initiatives that will speed up the transition to clean energy and a low-carbon economy. Green bonds (GBs), initially developed by the European Investment Bank in 2007, have emerged as a critical green financial instrument for raising funds for low-carbon, environmentally-friendly initiatives (Russo et al. 2021; Khalfaoui et al. 2023). GBs are a class of fixed-income instruments that differ from traditional bonds in that the proceeds of GBs are committed to financing environmental-friendly projects, i.e., developing renewable energy sources, clean technologies, alternative fuels, and clean transportation (Hille et al. 2020; Tolliver et al. 2020).

Thus, GB markets have been established to encourage sustainable finance with the explicit purpose of improving environmental quality. The International Capital Markets Association published the Green Bond Principles (GBPs) in 2014 to promote GB markets. The GBPs provide standardized guidelines for designating bonds as green, increasing the transparency and credibility of GBs (Reboredo 2018). Since GBs can meet both financial and environmental needs, with the potential to function as a source of portfolio diversification, their appeal has expanded in recent years (Huynh et al. 2020). Furthermore, many stock markets around the world, including those in Shenzhen, Shanghai, Luxembourg, Mexico, London, Oslo, and Italy, have recognized the importance of green bonds as an alternative asset class and have introduced them to enhance their liquidity and integrity in the financial markets (Reboredo and Ugolini 2020; Mensi et al. 2023a). As a result, GBs are increasingly attracting investor attention and support from policymakers, particularly from those who are concerned about the environment (Reboredo and Ugolini 2020; Tolliver et al. 2020; Lee et al. 2021). The market capitalization of GBs grew from US$37 billion in 2013 to US$280 billion in 2020 (Climate Bonds Initiative 2020), and GB issuance reached US$354.2 billion as of the end of Q3 2021 and is now anticipated to reach half a trillion US$ by the end of 2021.[1]

Despite GBs' rising popularity, little attention has been paid to their relative position as a portfolio investment. Some studies show a link between GBs and other financial assets such as traditional stocks, other fixed-income securities, and commodities, with varying results (e.g., Reboredo 2018; Reboredo et al. 2020; Reboredo and Ugolini 2020; Naeem et al. 2021a; Nguyen et al. 2021; Liu et al. 2021). Some of these studies find GBs offer diversification opportunities in portfolios containing other financial assets. However, these studies do not explore the interconnectedness of green financial markets, especially between GBs and green equities.

Like GB, green-related stocks, i.e., green equities (green equities), are stocks of environmentally-focused companies whose primary business is to support the environment by investing in clean-energy projects, water and waste management, natural resources,

[1] https://www.climatebonds.net/resources/reports/sustainable-debt-summary-q3-2021.

and pollution mitigation (Arif et al. 2021). Such financial instruments have become popular with environmentally-conscious investors as these companies pursue environmentally-friendly endeavors. Exploring the interdependencies between GBs and green equities has gained importance for multiple reasons. Although both asset types aim to provide capital to environmentally or climatically-favorable projects, they each possess distinct characteristics. Consequently, their association might be heterogeneous, presenting hedging opportunities while promoting environmental sustainability through green-related investments. Furthermore, such an analysis could facilitate the creation of "pure" green portfolios rather than hybrid ones that blend green and non-green assets. Conversely, being highly correlated would make green assets a homogeneous asset class and would have significant policy implications, as any policy alteration for one green market would have spillover effects on all other green markets.

Furthermore, as the scope and size of green financial markets continue to expand, examining the interplay among them is becoming increasingly relevant. This analysis can help investors and market participants identify the key factors that contribute to the efficacy of GBs and other environmentally-friendly instruments such as green equities, as portfolio diversifiers (Chatziantoniou et al. 2022). For investors with an environmental focus, green equities, aside from GBs, may serve as a viable tool for hedging or may offer safe-haven benefits, and they may be interested in constructing bond-equity portfolios with strong green credentials. However, to the best of our knowledge, no previous research has examined the interrelationships between green financial assets, specifically GBs and green equities, by considering time–frequency quantile dependencies and portfolio implications. We investigate these unexplored issues by accounting for diverse market conditions and considering time and frequency domains to facilitate informed decision-making by investors (both conventional and environmentally-conscious) and regulators who oversee the green financial markets.

Another impetus for this study stems from a methodological perspective. Prior research typically uses various connectedness methodologies discussed in the literature to scrutinize the interconnection between green and traditional financial markets. Nonetheless, extant studies lack empirical evidence regarding the time and frequency connectedness between GBs and green equities under diverse market regimes. While earlier studies have examined various aspects of GBs, they have not investigated the quantile dependencies between GBs and green equities at various frequencies and across various quantile levels. Policymakers and investors would benefit from an understanding of the linkages between green-labeled markets over different times and frequencies, accounting for diverse quantiles, to assess the extent to which green markets offer diversification and hedge opportunities.

In light of various market circumstances and varied investment horizons, this study offers new empirical evidence on the degree of quantile interconnectivity and directional spillovers among the GB and GE markets, across various market circumstances and investment horizons. To do so, we use the S&P Dow Jones Green Bond (GB) index and five NASDAQ OMX Green equity indices over the period from July 28, 2011 to July 28, 2021. The results of a wavelet-based Quantile-on-Quantile (QQ) regression indicate a strong positive interconnection between GBs and green equities across various market conditions and investment horizons. The time-varying approach reveals that

GBs can be used as a hedging instrument for green equities during market turbulence, as the two assets are decoupled in those times. The portfolio strategies show that to obtain the highest level of hedging effectiveness investors should invest in the GE–S&P GB portfolio. Furthermore, under all market conditions, we find that the global water green equity index transmits the greatest spillover to other green assets, followed by US water, whereas green economy (GE) index is the highest recipient of spillover from other green assets. A pairwise directional volatility connectedness analysis shows little quantile dependency among most pairs, suggesting diversification opportunities between GB and green equity pairs. These findings have many implications for risk management and portfolio choices for international investors with a significant stake in the GB market.

This study contributes to the emerging literature on green investments in several unique ways. First, previous studies investigate the association between GBs and traditional markets; however, other facets of GB markets have not been sufficiently investigated. To the best of our knowledge, this is the first attempt to explore the time–frequency interconnectedness between the GB and green equities markets under several market circumstances to explore their hedging and diversification properties. Second, from a methodological viewpoint, the wavelet-based QQ model captures different investment horizons (short, medium, and long-term) and market conditions (bullish, normal, and bearish). QVAR is used to estimate the dynamic quantile dependency between variables, which can help market participants make informed decisions. We also use the dynamic conditional correlation (DCC) method to capture the time-varying connection between green markets. Third, our findings suggest a significant positive connection between GBs and green equities. However, they are likely to be decoupled, with a low quantile dependence, during crisis periods, offering hedging opportunities. Finally, our findings have several important implications for investors and policymakers concerned about environmental degradation.

The rest of this study is organized as follows. In "Review of related literature" section reviews the related literature. In "Data" section presents the data and preliminary statistics. In "Econometric methodology" section explains our methodology. In "Empirical results and discussion" section discusses the empirical results. Finally, "Conclusions and policy implications" section offers conclusions with some prominent policy implications.

Review of related literature

Although the GB market has existed for almost a decade and has been growing exponentially since 2014, academic research on GBs is still scarce (Ferrer et al. 2021). Previous studies examine the relationship between GBs and other financial markets such as traditional stocks and bonds, clean energy index, commodities, etc. Still, in this segment, we review some of the latest empirical studies related to green financial markets.

Hammoudeh et al. (2020) analyze the relationship between GBs and other assets using a time-varying causality method. They show a unidirectional causal relationship flowing from US 10-year government bonds to GBs across the whole time horizon. They also find a link that runs from clean energy (CE) to GBs, but only after 2019. Using copulas and CoVaR models, Liu et al. (2021) study the tail dependence between the GB and CE markets, revealing strong co-movements between them. Furthermore, downward and upward movements in CE markets indicate the presence of a spillover effect on GB markets; however, the spillover effect for downside moves is greater than for upside moves. Nguyen et al. (2021) also look at the correlation between the GB and other assets such as stocks, commodities, traditional bonds, and clean energy index. They reveal that most correlations began and peaked in the aftermath of the 2008 global financial crisis. They also find that GBs may provide diversification benefits, as they have a low or negative correlation with stocks and commodities. Similar findings are unveiled by Reboredo (2018).

Furthermore, Reboredo and Ugolini (2020) examine connections between GBs and other financial markets, including Treasuries, corporate bonds, currencies, stocks, and energy commodities. They find the GB market is more closely linked to the fixed-income and currency markets than to stocks, energy commodities, and high-yield corporate bonds and show that the GB market is only tangentially related to the stock market. Similarly, Reboredo et al. (2020) investigate the network connections between GBs and various other asset classes (Treasuries, corporate debt, high-yield corporate debt, stock, and energy markets) across different timeframes. In the short and long run, they observe strong connections between GBs and Treasury and corporate bonds, with GBs receiving considerable spillovers from Treasury and corporate bond prices with minimal impacts from others. They also demonstrate that GBs are not particularly related to high-yield corporate bonds, stocks, or energy assets over various time horizons.

Arif et al. (2021) study how GBs and other financial assets (e.g., stocks, bonds, and energy commodities) are affected by the COVID-19 pandemic and revealed that financial assets might have had a skewed connection with GBs during that period. Pham and Nguyen (2021) examine tail dependencies between GBs and other asset classes (e.g., energy commodities, stocks, and conventional bonds) using the cross-quantilogram technique. They conclude that spillovers between GBs and other asset classes vary across timeframes, such as between normal and extreme market horizons. Naeem et al. (2021a) investigate the nonlinear links between GBs and commodities and show that GBs may offer diversification and hedging benefits for certain commodities, i.e., natural gas, industrial metals, and agricultural commodities.

Chatziantoniou et al. (2022) examine the dynamic association and return transmission between four widely recognized environmentally-focused financial indices. Using the quantile frequency connectedness approach, they observe that both short-term and long-term shocks tend to flow into GBs and clean energy, while a global environmental index and the Dow Jones sustainability index serve as short-term and long-term spreaders of shocks. Furthermore, they establish that the total connectedness indices fluctuate over time and depend on economic events. Elsayed et al. (2022) investigate the interdependence between GBs and various financial markets across different time horizons using the multivariate wavelet and dynamic connectedness approaches. They find the benefits of diversification opportunities are more evident in the short run, and that GBs and financial markets are highly connected in the long run. Their rolling window approach indicates that the interconnection between GBs and financial markets varies significantly over time. Pham (2021) examines the frequency connectedness and cross-quantile dependence between GBs and green equity markets. They find limited dependence between GB and green equity under average market conditions but show they are more interconnected in volatile markets when they rise and fall together.

An interesting question addressed in the literature is whether GBs should be considered a distinct asset class and how they compare in terms of efficiency to conventional bonds. For example, Baker et al. (2018) examine the US GB market and identify a premium for municipal GBs compared to other bonds with similar characteristics, externally certified. Naeem et al. (2021b) investigate the relative price efficiency of GBs and conventional bonds and find that GBs exhibited high efficiency during the COVID-19 pandemic and greater efficiency throughout their entire sample period. They suggest that GBs could potentially serve as a diversifier for investors during times of crisis. Ferrer et al. (2021) investigate whether GBs are a distinct asset class compared to various mainstream financial and energy markets. They present evidence indicating that GBs are closely linked to global Treasury and corporate bond markets, suggesting that it is not a distinct asset class, based on similarities in coupon rates, issuers, maturity, credit rating, currency, and other characteristics of such assets.

This literature review, summarized in Table 1, highlights a few gaps in GB research. The extant literature on GBs primarily focuses on examining their relationship with other financial markets, such as treasury, stocks, conventional bonds, clean energy stocks, currency, etc. Also, these studies examine GBs' relative efficiency and determine whether GBs are a distinct asset class. However, a research gap exists with respect to the interconnection between GBs and green equities. It is important to understand the dependence structure between these two markets, for both policymakers and investors. Policymakers need to understand the interdependence between GBs and GEs to ensure the integrity of the green market, while investors need to understand the hedging possibilities available within the market for green assets. This study endeavors to address this research gap by employing wavelet-based QQ, QVAR, and DCC-GJR-GARCH models to investigate the dependence structure between GBs and green equities over short-, medium- and long-run investment horizons under different market circumstances, and their dynamic connectedness. We include the COVID-19 pandemic period to explore whether the interconnectedness between the two markets differs during market crises, to help investors and policymakers make informed investment decisions under turbulent market conditions.

Data

Our dataset includes the S&P Dow Jones GB (S&P GB) index and the five Nasdaq OMX green equity indices: Solar, Global Water (GW), US Water (UW), Wind, and Green Economy (GE). We use daily index values to analyze the interdependence between GBs and green equities for the period from July 28, 2011, to July 28, 2021. The data for the S&P GB index and the Nasdaq OMX green equity indices are sourced from DataStream and www.investing.com, respectively. The S&P GB index tracks global GB markets that fund environmentally-focused projects (Liu et al. 2021). The GE index tracks stocks of firms that seek to enhance economic development by reducing carbon emissions and through other environmentally-focused activities.[2] Figure 1 depicts the price evolutions of the S&P GB and GE index, showing an upward trend for the green equity indices until

[2] https://www.nasdaq.com/solutions/green-equity-indexes.

Table 1 Summary of the previous literature

Author	Variables and Period	Methodology	Findings
Hammoudeh et al. (2020)	Green Bond (GB), US 10-year Treasury bond, the WilderHill clean energy index, and CO2 Emission Allowances (30 July 2014–10 February 2020)	Granger causality test	Unidirectional causal relationship flowing from US 10-year government bonds to GB across the whole-time horizon
Liu et al. (2021)	GB, Global and Sectoral Clean Energy indices (5 July 2011–24 February 2020)	Conditional value-at-risk (CoVaR) and delta CoVaR	Downward or upward movements in CE markets indicate the presence of a spillover effect on GB markets. The spillover effect is greater in the downside risks than the upside risks
Nguyen et al. (2021)	GB, S&P 500 Composite Index, the S&P GSCI Commodity Index, S&P Clean Energy Index, and Barclays Bloomberg Global Treasury Index (2008–2019)	Rolling-window wavelet correlation, wavelet coherence	Most correlations began and peaked in the aftermath of the 2008 global financial crisis (GFC). GB has a low or negative correlation with stocks and commodities
Reboredo and Ugolini (2020)	Barclays MSCI GB Index, S&P GB Index, Solactive GB Index and Bank of America Merrill Lynch GB Index (14 October 2014–25 June 2019)	Structural vector autoregressive (VAR)	GB market is closely linked to the fixed-income and currency markets than stock, energy, and high-yield corporate bond markets. GB market is only tangentially related to the stock market
Reboredo et al. (2020)	Bloomberg Barclays Euro Aggregate Treasury Total Return Index, Bloomberg Barclays Euro Aggregate Corporate Total Return Index, Bloomberg Barclays Pan-European High Yield Total Return Index, MSCI Europe Index, Thomson Reuters Europe Energy Index (12 October 2014–20 December 2018)	Wavelet and VAR	GBs and treasury and corporate bonds are strongly associated, with GBs receiving considerable spillovers from treasury and corporate bond prices while having minimal impacts on others in the short and long run. GBs are poorly related to high-yield corporate bonds, stocks, and energy assets
Arif et al. (2021)	Green Investment: S&P GB Index, S&P Global Clean Energy Index, and Dow Jones Sustainability Index; Conventional investment: Barclays Bloomberg Global Treasury Index, S&P GSCI Energy Index, and MSCI World Index (January 01, 2008–July 31, 2020)	Spillover models of Diebold and Yilmaz (2012) and Baruník and Křehlík (2018)	financial assets might have a skewed connection with GB during COVID-19
Pham and Nguyen (2021)	GB: Bloomberg Barclays MSCI USD GB Index (GBUS) and the Bloomberg Barclays MSCI EUR GB Index; Other Assets: Bloomberg Barclays Euro Aggregate Treasury Total Return Index, Bloomberg Barclays Euro Aggregate Corporate Total Return Index, Bloomberg Barclays Pan-European High Yield Total Return Index, MSCI Europe Index, MSCI US Index, MSCI Energy Europe, and the MSCI Energy U.S. (October 12, 2014–February 12, 2021)	Cross-quantilogram	Spillovers between GB and asset classes vary across timeframes, such as between the normal and extreme market horizons

Table 1 (continued)

Author	Variables and Period	Methodology	Findings
Naeem et al. (2021a)	GB: Barclays MSCI GB Index, S&P GB Index, Solactive GB Index and Bank of America Merrill Lynch GB Index; Commodities: energy, metals, and agriculture (2008–2019)	Cross-quantilogram	GB's diversification and hedging benefits are possible against commodities
Chatziantoniou et al. (2022)	S&P Green Bond Index, MSCI Global Environment, Dow Jones Sustainability Index World, and S&P Global Clean Energy Index (November 28, 2008–January 12, 2022)	Quantile frequency connectedness	GB and clean Energy appear to be both short-term and long-term net receivers of shocks, while global environment and DJ sustainability indices are both short-term and long-term spreaders of shocks
Elsayed et al. (2022)	GB: Barclays MSCI GB Index, S&P GB Index, Solactive GB Index and Bank of America Merrill Lynch GB Index; Other financial markets: Bloomberg Barclays Global Treasury Index, Bloomberg Barclays Global Corporate Index, MSCI World Stock Price Index, MSCI World Energy Price Index, and WilderHill's clean energy price index (30/9/2014–30/6/2020)	Multivariate wavelet and dynamic connectedness combining Ensemble Empirical Mode Decomposition (EEMD)	The benefits of diversification opportunities are more evident in the short run, and GBs and financial markets are highly connected in the long run
Pham (2021)	GB: S&P GB index; Green Equity: Clean Energy Focused Index, Green Building Index, Green Transportation Index, Global Water Index; Other Financial Markets: Bloomberg Barclays Global Treasury and Corporate bond, MSCI World and MSCI Energy (2014–2020)	Spillover models of Diebold and Yilmaz (2014) and Baruník and Křehlík (2018)	Dependence between GB and green equity amid average market states is comparatively little. The variables are more connected throughout extreme market changes
Naeem et al. (2021b)	GB: S&P GB Index, Solactive GB Index, and Bloomberg Barclays MSCI GB Index; Conventional Bond: S&P Global Developed Aggregate Ex-Collateralized Bond Index, Solactive Global Developed Government Bond TR EU Index, and Bloomberg Barclays Global Aggregate Total Return Index (November 3, 2014–September 3, 2020)	Asymmetric multifractal detrended fluctuation analysis (A-MF-DFA)	GB market reveals high efficient behavior during COVID-19, while greater efficiency is noticed during their entire sample period. GB shows potential diversifiers for investors in extreme crisis episodes
Ferrer et al. (2021)	GB: Bloomberg Barclays MSCI GB Index; Other financial markets: Bloomberg Barclays Global Treasury total return index, Bloomberg Barclays Global Aggregate Corporate Index, MSCI world index, Trade Weighted U.S. dollar index, RENIXX world index, and Brent oil price (October 14, 2014–December 19, 2019)	Baruník and Křehlík (2018)'s Model	GB and the global treasury and corporate bond markets are strongly coupled, suggesting that GB is not a different asset class

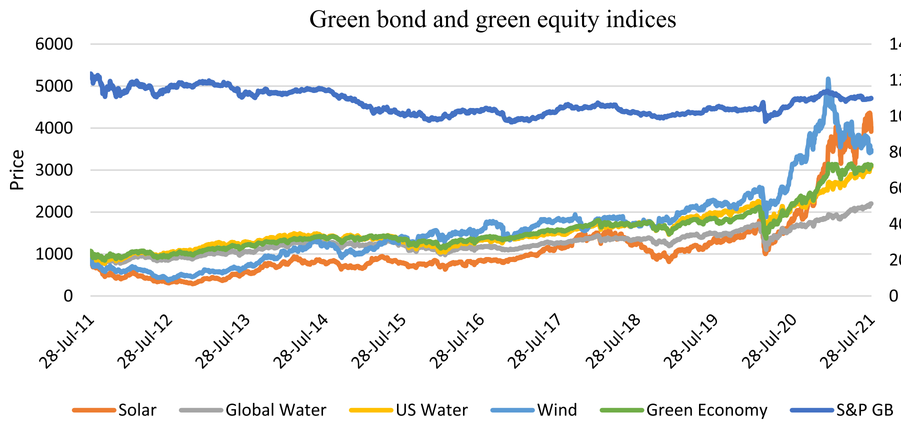

Fig. 1 Price dynamics of GB and Green equities

Table 2 Descriptive statistics. *Source*: Authors' own calculations

Variables	Mean	SD	Skewness	Kurtosis	Jarque–Bera	ADF	PP
S&P GB	− 0.005	0.371	− 0.372	10.800	6591.8***	− 49.925***	− 49.922***
Solar	0.060	2.051	− 0.551	9.910	5256.6***	− 32.609***	− 50.425***
GW	0.032	1.010	− 0.834	16.358	1945.0***	− 16.489***	− 48.194***
UW	0.045	1.250	− 0.524	13.973	1304.5***	− 21.214***	− 57.188***
Wind	0.054	1.628	− 0.447	7.430	2193.1***	− 47.967***	− 47.950***
GE	0.041	1.080	− 1.015	18.177	2517.9***	− 16.614***	− 50.957***

The table contains summary statistics and unit root tests for the return series of the GB and green equity indices. S&P GB, and Solar, GW, UW, Wind, and GE correspond to the S&P Dow Jones Green Bond, NASDAQ OMX Solar, NASDAQ OMX Global Water, NASDAQ OMX US Water, NASDAQ OMX Wind, and NASDAQ OMX Green Economy indices. ADF and PP are the Augmented Dickey–Fuller and Phillips–Perron test statistics considering a constant and trend. Std. Dev. indicates the standard deviation. *** indicates significance at the 1% level

they declined considerably in early 2020, corresponding to the onset of the COVID-19 pandemic.

The summary statistics in Table 2 show the GB index delivered a negative return over the study period, whereas all green equity indices have positive returns. Solar power equities exhibit the highest mean returns and the largest volatility. All asset return series have negative skewness, indicating a longer or fatter tail on the left side of the distribution. Positive excess kurtosis values indicate that all return series are leptokurtic, implying larger tails or outliers. As a result, we conclude the returns series are not normally distributed, as confirmed by the Jarque–Bera statistics that reject the test statistics at the 1% significance level. Therefore, the returns series are likely to have tail dependence, and using a simple, static model may lead to biased findings, underscoring our decision to employ a quantile-based approach.

The presence of a unit root or stationarity is a critical issue for any time-series econometric analysis. Although a number of tests to check for the unit root of data series, we use the Augmented Dickey-Fuller (ADF) (Dickey and Fuller 1981) and Phillips–Perron (PP) (Phillips and Perron 1988) unit root tests, which offer several benefits over other tests, as

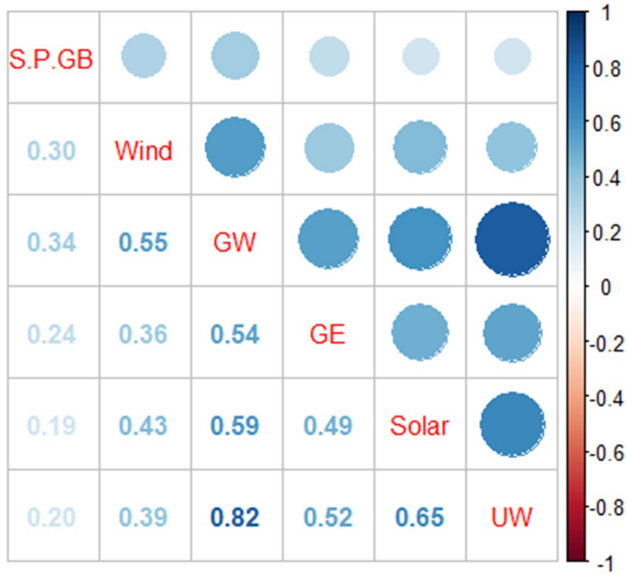

Fig. 2 Correlation plot

suggested by Hasan et al. (2022a). The results indicate that no stationarity issues persist in our time-series data.

Figure 2 shows the graphical correlation matrix between the GB and Green equity indices, revealing the positive connections with each other. All asset pairs have a positive correlation, implying that they are likely to be associated.

Econometric methodology

Discrete wavelet transforms

Using discrete wavelet transforms (DWT), this study, following Hasan et al. (2022b), decomposes the return series into multiple frequencies (e.g., short-, medium-, and long-term). The specification is as follows:

$$m(t) = \sum_k R_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t), \tag{1}$$

where two essential functions of wavelets, ϕ and ψ, highlight the low (smooth) and high (detailed) frequency elements of the series, respectively. The contribution of the wavelet functions to the overall signal is denoted by the wavelet transform's coefficients ($R_{J,k}$, $d_{J,k}$, ...,$d_{1,k}$).

Therefore, a time series $m(t)$ can be represented in terms of those signals using a J -level multi-resolution decomposition analysis:

$$m(t) = R_j(t) + D_j(t) + D_{j-1}(t) + \cdots + D_1(t), \tag{2}$$

where D_j exhibits the three frequency scales derived from 2^j time bands. After eliminating $D_1,\ldots D_j$ from the time series, R_j is generated as a residual. We select $J = 3$ to decompose the multi-resolution level J.

Quantile-on-quantile

Then, using the decomposed datasets, we employ the QQ regression framework. For several grounds, this technique is preferable over other approaches such as simple linear regression and quantile regression (QR). Linear regression and QR (developed by Koenker and Bassett 1978) cannot detect the full extent of data dependence in a time series. Furthermore, these methods fail to account for quantile interdependence between the independent and dependent variables, as the quantiles of the independent variable may exert varying effects on the quantiles of the dependent variable (Koenker and Bassett 1978), given the non-normality and presence of heavier tails in our dataset. Results obtained using a QQ regression may help stakeholders to make better decisions regarding a variety of market conditions.

In addition, the returns series for all the variables are nonlinear (see the results in Table 2) and the QQ method is well-suited to capture this, as shown in recent studies that use this method (e.g., Mishra et al. 2019; Chang et al. 2020; Hasan et al. 2022a; inter alia). Finally, the influence of the Green equity indices' return quantiles on the conditional quantiles of GB returns is determined at three-time scales (e.g., short-, medium-, and long run), which forms the wavelet-based quantile-on-quantile framework. The econometric specification is as follows:

$$GB_t = \beta^{\theta} GEs_t + \varepsilon_t^{\theta}, \tag{3}$$

where GB_t and ε_t^{θ} denote the logarithmic returns of GB and the error term (containing a zero θ-quantile), respectively. $\beta^{\theta}(.)$ is an unknown factor as there is no prior knowledge relating GB returns to changes in green equities. The bandwidth (k) choice is crucial for non-parametric QQ estimates as it controls the smoothness of the predicted coefficients (Razzaq et al. 2020). Based on Sim and Zhou (2015), we use a 5% (h = 0.05) bandwidth as the density function for optimal parameters. The parameter $\beta^{\theta}(.)$ is linearized by taking a first-order Taylor expansion of $\beta^{\theta}(.)$ around GE^{τ} to examine the dependency between the variables. The presentation is as follows:

$$\beta^{\theta}(GEs_t) \approx \beta^{\theta}\left(GEs^{\tau}\right) + \beta^{\theta\prime}\left(GEs^{\tau}\right)\left(GEs_t - GEs^{\tau}\right) \tag{4}$$

Sim and Zhou (2015) show that $\beta^{\theta}(GEs^{\tau})$ and $\beta^{\theta\prime}(GEs^{\tau})$ can be expressed as $\beta_0(\theta,\tau)$ and $\beta_1(\theta,\tau)$, respectively. Based on this, Eq. 5 is a rewrite of Eq. 4:

$$\beta^{\theta}(GEs_t) \approx \beta_0(\theta,\tau) + \beta_1(\theta,\tau)\left(GEs_t - GEs^{\tau}\right) \tag{5}$$

We then substitute Eq. 5 into Eq. 3 to develop Eq. 6:

$$GB_t = \beta_0(\theta,\tau) + \beta_1(\theta,\tau)\left(GEs_t - GEs^{\tau}\right) + \varepsilon_t^{\theta}. \tag{6}$$

Finally, a Gaussian kernel is used by weighing the observations in the experiential quantile area of green equities, as is common in financial economics.

Dynamic conditional correlation

Engle's (2002) dynamic conditional correlation (DCC) technique is used to estimate the time-varying conditional correlation between GBs and green equities. In addition, we integrate the DCC approach with a form of the generalized autoregressive conditional heteroscedasticity (GARCH)-based method known as Glosten et al. (1993) (GJR), forming the DCC-GJR-GARCH model.[3] By responding to bad news with high volatility and good news with low volatility, the GJR-GARCH model can capture asymmetric effects. Recent studies (e.g., Hassan et al. 2021; Hasan et al. 2023; Menis et al. 2021) use this technique for this reason. The equation is as follows:

$$r_t = \mu + \psi r_{t-1} + \varepsilon_t, \quad \varepsilon_t = z_t h_t, z_t \sim N(0,1), \tag{7}$$

where $r_t = [r_{i,t}, ..., r_{n,t}]$ is the $n \times 1$ vector of financial asset returns. The vector of constants with length n and the vector of the autoregresve terms' coefficients are denoted by μ and ψ, respectively. $\varepsilon_t = [\varepsilon_{i,t}, \ldots, \varepsilon_{n,t}]$ represents the residual vector. The GJR-GARCH (1, 1) model's conditional volatility is calculated as follows:

$$h_{i,t}^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1}, \tag{8}$$

where I_{t-1}=1 if $\varepsilon_{t-1} < 0$, otherwise $I_{t-1} = 0$. The leverage term γ captures the asymmetric effects of positive and negative shocks. When the parameters $\omega, \alpha, \beta,$ and γ satisfy the constraints $\omega > 0, \alpha, \beta,$ and $\gamma \geq 0$, and $\gamma + (\alpha + \beta)/2 < 1$, the volatility mechanism in Eq. 8 is assumed to be positive and stationary.

The diagnostic test results on the standardized squared residuals show that the chosen GJR-GARCH method using the Student's-t distribution is correctly specified, given there are no autocorrelation effects in the estimated residuals. The model also identifies the probability of the GJR's second or higher-order moments' existence. Finally, the marginal models' residuals effectively capture the return distributions.

Assume that $\in_{t-1}[\varepsilon_t] = 0$ and $\in_{t-1}\left[\varepsilon_t, \varepsilon_t'\right] = H_t$, where the conditional projection using the available information is illustrated by $\in[\cdot]$ at time t. H_t can be reinterpreted as:

$$H_t = D_t^{1/2} R_t D_t^{1/2}, \tag{9}$$

where R_t indicates the matrix of $n \times n$ time-varying correlations, and $D_t = diag\left(h_{i,t}, \ldots, h_{n,t}\right)$ gives the diagonal conditional variances. Engle (2002) proposes the following time-varying correlation structure to model the right-hand side of Eq. 10, as an alternative to H_t:

$$R_t = diag(K_t)^{-1/2} K_t diag(K_t)^{-1/2}, \tag{10}$$

$$\mathrm{K_t = (1 - a - b)V + adiag(K_{t-1})^{1/2}\hat{\varepsilon}_{i,t-1}\hat{\varepsilon}_{i,t-1}' diag(K_{t-1})^{1/2} + bK_{t-1}}, \tag{11}$$

where V represents the $n \times n$ unconditional covariance matrix of the standardized residuals $\hat{\varepsilon}_{i,t-1}$, and x and y are non-negative scalars satisfying $x + y < 1$.

[3] Based on the Akaike Information Criterion (AIC) and Schwarz Criterion, the GJR-GARCH (1, 1) model is chosen. Details are available upon request.

Quantile VAR specification

This study also employs the Quantile VAR (QVAR) approach, which offers superior accuracy and depth in measuring connectedness between variables owing to its lack of sensitivity to outliers (Chatziantoniou et al. 2021). This technique allows for a comprehensive exploration of time-domain connections, with three quantiles offering additional insights into tail dependencies (Ramham et al. 2021; Alomari et al. 2022; Jain et al. 2023; Mensi et al. 2023b, c, d). The analysis begins with a quantile regression, following the method used in Koenker and Ng (2005), Furno and Vistocco (2018), and Jena et al. (2021), where the dependence of variable y_t on x_t at each quantile (τ) of the conditional distribution of $y_t|x_t$ as follows:

$$Q_\tau(y_t|x_t) = x_t\beta(\tau), \tag{12}$$

where Q_τ represents the τth conditional quantile function of y_t where the range of every quantile between 0 and 1 is signified by τ. The x_t and $\beta(\tau)$ denotes the explanatory variables' vector and the affiliation between x_t and the τth conditional quantile function of y_t, respectively. The vector of the parameter $(\beta(\tau))$ at the τth quartile (τ) is measured as follows:

$$\hat{\beta}_{(\tau)} = argmin\sum_{t=1}^{T}(\tau - 1_{\{y_t<x_t\beta(\tau)\}})\left|y_t < x_t\beta(\tau)\right| \tag{13}$$

Then, the p^{th} order of the n-variable for the quantile VAR method is evaluated as follows:

$$y_t = c(\tau) + \sum_{i=1}^{p} Bi(\tau)\boldsymbol{y}_{t-1} + \boldsymbol{e}_t(\tau), t = 1, \ldots, T \tag{14}$$

where y_t depicts the n-vector of the dependent variable, and $c(\tau)$ and $\boldsymbol{e}_t(\tau)$ represent the n-vector of constants and residuals at quantile (τ), respectively. The dependent variable's matrix of lagged coefficients at τ is denoted by $Bi(\tau)$, where $i = 1, \ldots, P$. Then, $\hat{\beta}_{(\tau)}$ and $\hat{c}(\tau)$ are estimated by simulating the residuals, based on the number of quantile constraints, $Q_\tau\left(e_t(\tau)|y_{t-1}, \ldots y_{t-p}\right) = 0$. Then, the repercussion y for the population of the τth conditional quantile is represented in Eq. (15). Finally, it allows approximating an equation-by-equation at each quantile τ subsequently.

$$Q_\tau\left(y_t|y_{t-1}, \ldots y_{t-p}\right) = c(\tau) + \sum_{i=1}^{p} Bi(\tau)y_{t-1} \tag{15}$$

Measuring connectedness at each quantile

In this section, we compute multiple estimates of return interconnectivity in each quantile (τ) based on Ando et al. (2018), which is a modified form of Diebold and Yilmaz's (2012, 2014) mean-based estimates. This approach, which allows stakeholders to make informed policy and investment decisions based on different market conditions, is used in Su (2020) and Jena et al. (2021). To measure connectedness at each quantile, Eq. (14) is modified as an unspecified order vector moving average model:

$$y_t = \mu(\tau) + \sum_{s=0}^{\infty} A_s(\tau)\boldsymbol{e}_{t-s}(\tau), t = 1, \ldots, T \tag{16}$$

here,

$$\mu(\tau) = \left(\mathrm{I}_n - B_1(\tau) - \ldots - B_p(\tau)\right)^{-1} c(\tau),\ \mathrm{A}_s(\tau) = \begin{cases} 0, s < 0 : I_n, s = 0 \\ B_1(\tau)\mathrm{A}_{s-1}(\tau) + \cdots + B_p(\tau)\mathrm{A}_{S-p}(\tau), s = 0 \end{cases}$$

where y_t is the summation of residuals $\boldsymbol{e}_t(\tau)$.

In contrast to Su (2020), we use strategies from Koop et al. (1996) and Pesaran and Shin (1998) that appear to be robust to variable rearrangement. For a given forecast horizon H, the generalized forecast error variance decomposition (GFEVD) of a variable due to shocks to other variables is:

$$\theta_{ij}^{g}(H) = \frac{\sigma_{jj}^{-1} \Sigma_{h=0}^{H-1} \left(e_i' \mathrm{A}_S \Sigma e_j\right)^2}{\Sigma_{h=0}^{H-1} \left(e_i' \mathrm{A}_S \Sigma e_j\right)} \tag{17}$$

where $\theta_{ij}^{g}(H)$ represents the impact of the jth variable on the variance of the forecast error of variable i to horizon H. Then, Σ is used for the vector of error variance matrix in the equation, and the jth diagonal component of the Σ matrix is symbolized by σ_{jj}. e_i presents a vector that contains a value of 1 for the ith component and 0 otherwise.

Each component of the variance decomposition matrix then is standardized as follows:

$$\theta_{ij}^{\sim g}(H) = \frac{\theta_{ij}^{g}(H)}{\Sigma_{j=1}^{N} \theta_{ij}^{g}(H)} \tag{18}$$

We then consider the GFEVD, allowing the formulation of four connectivity estimates at each quantile. Thus, the total spillover index at quantile τ is signified as follows:

$$TSI(\tau) = \frac{\Sigma_{i=1}^{N} \Sigma_{j=1, i \neq j}^{N} \theta_{ji}^{\sim g}(\tau)}{\Sigma_{i=1}^{N} \Sigma_{j=1,}^{N} \theta_{ji}^{\sim g}(\tau)} \times 100 \tag{19}$$

We use "*TO*" to designate the total directional spillover index from index i to the group of indices j in quantile τ as follows:

$$SI_{i \to j}(\tau) = \frac{\Sigma_{j=1, i \neq j}^{N} \theta_{ji}^{\sim g}(\tau)}{\Sigma_{j=1}^{N} \theta_{ji}^{\sim g}(\tau)} \times 100 = TO \tag{20}$$

The term *"FROM"* is used to represent the total directional spillover index from indices j to index i in quantile τ, as follows:

$$SI_{i \leftarrow j}(\tau) = \frac{\Sigma_{j=1, i \neq j}^{N} \theta_{ji}^{\sim g}(\tau)}{\Sigma_{j=1}^{N} \theta_{ji}^{\sim g}(\tau)} \times 100 = FROM \tag{21}$$

Finally, the net total directional spillover index for quantile τ is as follows:

$$NTDSI_I(\tau) = SI_{i\to j}(\tau) - SI_{i\leftarrow j}(\tau) = NSI \tag{22}$$

According to the AIC criteria, the lag duration factors suggest one lag for the empirical research, with a forecast range of 10 days. Using a 200-day rolling window, we calculate the time-varying spillover of various return spillover measures.

Empirical results and discussion

Quantile-on-quantile estimations

Figure 3 illustrates the empirical results of the QQ regression between the GB index and Green equity indices. The estimated QQ slope coefficient $\beta_1(\theta, \tau)$ captures the effects of the τth quantile of green equities on θth quantiles of GBs, and vice-versa. The datasets are decomposed into three frequencies, e.g., short-term (1–16 days), medium-term (16–64 days), and long-term (64–256 days). The QQ results reveal that the GE index returns are strongly and positively associated with the GB index's returns across all quantiles and frequency domains. These interconnections are not surprising, given that both GBs and green equities have the same purpose, i.e., to raise funds for low-carbon and emission-reducing projects. This finding suggests that investors cannot hedge their GE exposures using GBs under all market conditions and frequencies, which is consistent with Liu et al. (2021) who find a positive linkage between GBs and clean-energy markets, suggesting no hedging benefits between the two under different market conditions. Our results suggest that policymakers should be careful when changing any policy related to both GBs and green equities, as these markets are highly interconnected.

To assess the robustness of our QQ technique, we combine the average QQ coefficients and match them to the estimated QR coefficients following the methodology outlined in Lin and Su (2020). The results are reported in Fig. 6 of the "Appendix".[4] The analysis reveals that the QQ slope coefficients follow the identical trend as the QR coefficients for all GB and GE pairs over short, medium, and long-term frequencies, suggesting that the findings of the QQ approach are robust and reliable.

DCC-GJR-GARCH (1, 1) estimations

In this section, we use Engle's (2002) dynamic conditional correlation (DCC) approach, in combination with the GJR-GARCH method in Glosten et al. (1993), to capture the asymmetric time-varying dependency between the GB and Green equity indices. Table 3 presents the DCC-GJR-GARCH (1, 1) estimations for the S&P GB index returns, and the five green equity indices' returns. The parameters of the ARCH term (α) and GARCH term (β) are positive and significant in all cases except for the ARCH parameter for the UW index, and the sum of αandβ is ≤ 1. Furthermore, the GJR (gamma) parameters are

[4] In order to assess the robustness of our QQ estimations, we compare our findings with the quantile regression (QR) coefficients based on the study by Sim and Zhou (2015). The QQ methodology, which can be viewed as a refinement of basic quantile regression, allows for the examination of individual estimates at different quantiles of the independent variable (Iqbal et al. 2021). This is particularly valuable when the explanatory variable exhibits heterogeneity across quantiles, as the QQ technique provides more precise insights into the relationship between green bonds and green equities returns compared to QR. By accounting for the inherent decomposition aspects in the QQ technique, our QQ estimates uphold the fundamental properties of QR (Sim and Zhou 2015). This ensures methodological accuracy and reliability of the results (Lin and Su 2020).

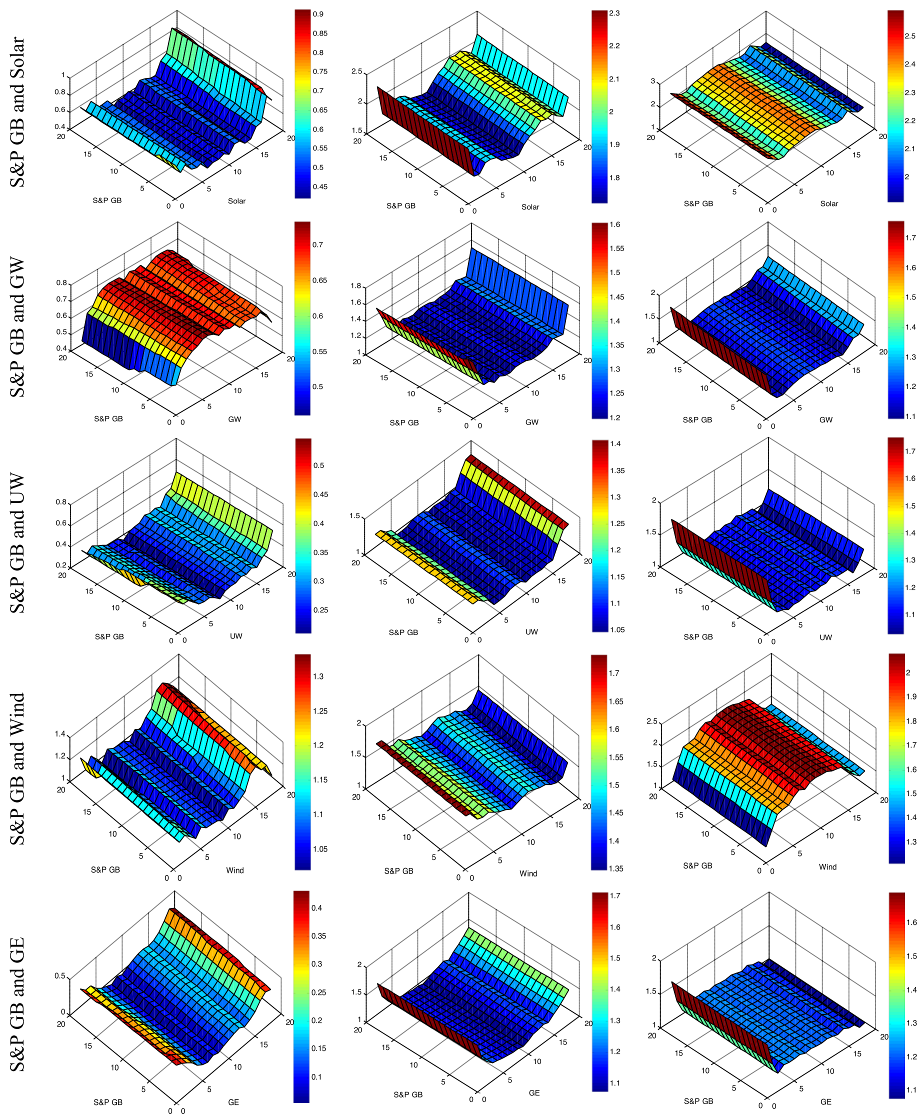

Fig. 3 Quantile-on-Quantile (QQ) plots. *Notes*: The figure portrays the QQ estimation between GB and green equities. The numbers 1 to 19 are shown on the x- and y-axis, from 0.05 to 0.95 quantiles. The slope coefficient $\beta_1(\theta, \tau)$ in the *z*-axis is estimated against the *x*-axis (green equities) and *y*-axis (GB) quantiles

positive and statistically significant for all green equity indices other than UW, suggesting that negative shocks cause more volatility than positive shock. The results of these DCC-GJR-GARCH (1, 1) estimations are robust and reliable based on several diagnostic tests (e.g., Li-McLeod, standardized squared residuals, and multivariate Hosking) and information criteria (e.g., Akaike, Shibata, Schwarz, and Hannan-Quin) with Student's-*t* distribution, as presented in Panels C and D.

The S&P GB index shows a positive dynamic correlation with all of the green equity indices considered in this study, over both the short and long term; however, the magnitude of the effect is more significant in the long run. Therefore, we conclude that green equity markets have a high volatility spillover impact on the GB market in the long run

Table 3 Estimations of the DCC-GJR-GARCH model. *Source*: Authors' own estimation

	Solar	S&P GB	GW	S&PGB	UW	S&P GB	Wind	S&P GB	GE	S&P GB
Panel A: AR (1)-GARCH (1, 1) estimation										
Const. (M)	0.001 (0.000)	−0.000 (0.000)	0.000** (0.000)	−0.000 (0.000)	0.000** (0.000)	−0.000 (0.000)	0.001** (0.000)	−0.000 (0.000)	0.000** (0.000)	−0.000 (0.000)
AR (1)	0.416* (0.154)	0.392*** (0.149)	0.189*** (0.113)	0.392*** (0.149)	−0.955* (0.018)	0.392*** (0.149)	0.910* (0.046)	0.392*** (0.149)	0.351* (0.125)	0.392*** (0.149)
Const. (V)	0.104 (0.064)	0.103** (0.041)	0.024** (0.010)	0.103** (0.041)	0.045* (0.016)	0.103** (0.041)	0.116*** (0.062)	0.103** (0.041)	0.024* (0.007)	0.103** (0.041)
α (ARCH 1)	0.042* (0.015)	0.053* (0.018)	0.037*** (0.021)	0.053* (0.018)	0.023 (0.024)	0.053* (0.018)	0.044* (0.016)	0.053* (0.018)	0.052* (0.020)	0.053* (0.018)
β (GARCH 1)	0.904* (0.037)	0.935* (0.015)	0.862* (0.042)	0.935* (0.015)	0.853* (0.042)	0.935* (0.015)	0.872* (0.044)	0.935* (0.015)	0.853* (0.025)	0.935* (0.015)
$(\alpha + \beta)$	0.947	0.988	0.899	0.988	0.877	0.988	0.916	0.988	0.906	0.988
GJR(Gamma)	0.049*** (0.027)	0.008 (0.015)	0.135* (0.041)	0.008 (0.015)	0.165 (0.036)	0.008 (0.015)	0.081** (0.037)	0.008 (0.015)	0.138* (0.038)	0.008 (0.015)
Panel B: DCC estimates										
Avr. Corr	0.482** (0.262)		0.271* (0.051)		0.069 (0.072)		0.236* (0.049)		0.177 (0.159)	
Dcc1 (*a*)	0.017* (0.006)		0.062* (0.009)		0.028* (0.010)		0.031* (0.008)		0.028* (0.005)	
Dcc2 (*b*)	0.983* (0.007)		0.908* (0.014)		0.963* (0.015)		0.952* (0.014)		0.967* (0.008)	
Panel C: diagnostic tests										
Qs (20)	23.781 [0.252]	18.186 [0.575]	29.910 [0.071]	16.309 [0.697]	27.462 [0.123]	18.159 [0.576]	25.451 [0.184]	14.708 [0.792]	29.949 [0.070]	17.785 [0.601]
Hosking (20)	103.783 [0.027]		110.046 [0.010]		121.711 [0.001]		112.650 [0.006]		150.935 [0.000]	
Li-McLeod(20)	103.717 [0.027]		110.011 [0.010]		121.713 [0.001]		112.544 [0.006]		150.926 [0.000]	

Table 3 (continued)

	Solar	S&P GB	GW	S&PGB	UW	S&P GB	Wind	S&P GB	GE	S&P GB
Panel D: information criteria										
Akaike	− 13.988		− 15.718		− 15.177		− 14.420		− 15.538	
Shibata	− 13.988		− 15.718		− 15.177		− 14.420		− 15.538	
SIC	− 13.947		15.677		− 15.136		− 14.379		− 15.497	
Hannan-Quin	− 13.973		− 15.704		− 15.162		− 14.405		− 15.523	

The table presents the estimation of DCC-GJR-GARCH results. Qs (20) are the Ljung–Box test statistics applied to the squared standardized residuals and 20 lags. Hosking (Hosking 1980) and McLeod and Li (McLeod and Li 1983) multivariate Portmanteau statistics check for the null hypothesis of no serial correlation (using 20 lags). The asterisks, *, **, and *** represent the significance at the 10%, 5%, and 1% levels, respectively. The p-values are in brackets, and the standard errors are in parentheses

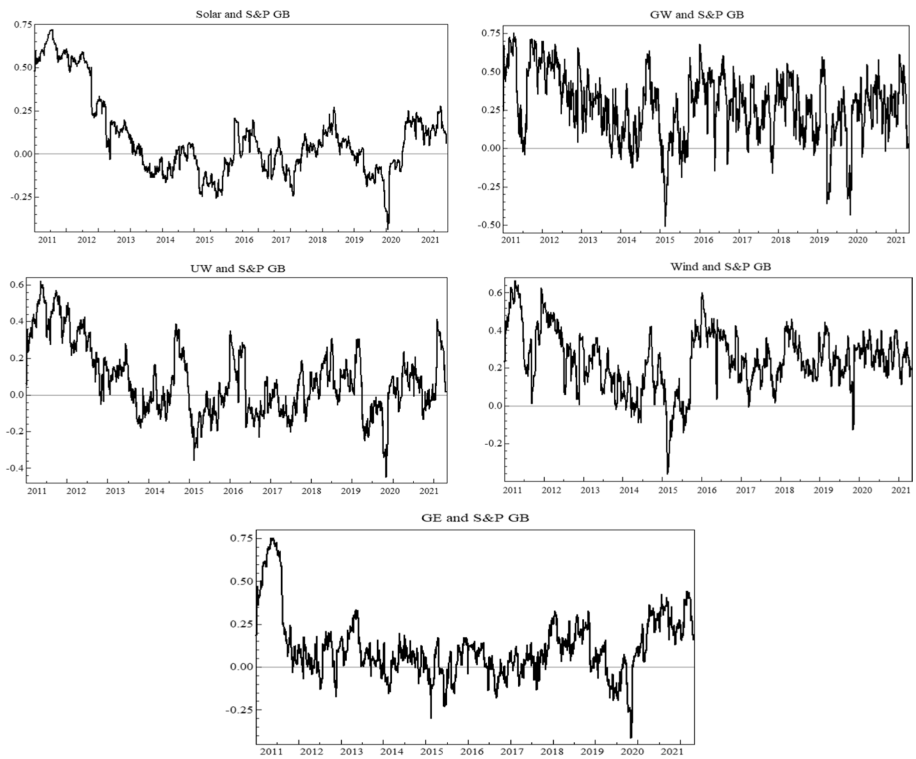

Fig. 4 Dynamic conditional correlation plots for GB and green equities

than in the short run. Moreover, the sum of the DCC parameters αandβ is ≤ 1, signifying that the S&P GB and green equity indices exhibit high volatility clustering.

The DCC plots shown in Fig. 4 suggest that the relationship between GBs and green equities is time-varying and positive over most of the sample period. The graphs do show that GB was inversely linked with green equities during 2015, a period, in which commodity markets sparked a banking crisis.[5] During the height of the COVID-19 pandemic in 2020, GBs were also negatively correlated with Green equity indices (excluding Wind). However, the time-varying outcomes signify that GBs may provide hedging benefits for green equity portfolios during crises.

Many studies (e.g., Zhang et al. 2020; Sharif et al. 2020; Hasan et al. 2021a, b; Akhtaruzzaman et al. 2021; Mensi et al. 2022, 2023d; Naeem et al. 2023, among others) show the COVID-19 pandemic adversely affected the global economy and financial systems. Accordingly, a study by Smith et al. (2021) conveys evidence that fossil fuel use and CO_2 emissions are adversely affected by the global pandemic. As a result, green energy project growth may be slowed, finding it challenging to meet the Paris Agreement goals and recuperate the world economy from the COVID-19 crisis. To address the challenges posed by crises, time-varying estimations yield interesting insights for investors who may wish to use GBs and GEs as diversifiable and safe-haven assets for safeguarding green energy projects against the negative impacts of crises. Arif et al. (2021), Nguyen et al. (2021), and Naeem et al. (2021b) support this observation. In light of these findings,

[5] https://www.ft.com/content/1756dc45-bb66-341e-b965-4c327a6e2be0.

Table 4 Portfolio design and hedging effectiveness. *Source*: Authors' own calculations

	Optimal weights	Hedge ratios	Hedging effectiveness
Solar/S&P GB	0.9745	0.3426	5.1382
GW/S&P GB	0.9255	0.6460	2.7249
UW/S&P GB	0.9153	0.1715	3.6163
Wind/S&P GB	0.9874	1.1523	0.3500
GE/S&P GB	0.8871	0.3559	**9.4396**

This table shows the optimal weights and hedge ratios for portfolios consisting of the GB index and one of the Green equity indices. The numbers in bold indicate the hedged portfolio with the lowest and highest variance reductions

governments should consider issuing GBs during times of crisis, as investors in Green equities may benefit from holding GBs during periods of extreme market turbulence. Additionally, policymakers could encourage investors to maintain their GB and GE holdings during tumultuous times by implementing measures aimed at reducing financial contagion within green financial markets. Early interventions such as those used to mitigate the economic effects of the pandemic could stabilize market sentiment and mitigate cross-market spillovers (Pham 2021).

Table 4 reports the optimal portfolio weights, average hedge ratio (long/short), and hedging effectiveness of portfolios containing both the S&P GB index and Green equity indices. The table reports the hedge ratio for each long/short pair, with a one-dollar long position in each green equity indices and a one-dollar short position in the S&P GB index. Apparently, the results in the table reveal that the amount of investment in the S&P GB is strongly relative to Green equity indices. For example, for the Solar-S&P GB, the study shows that the optimal weight is 0.9745, suggesting that the optimal weight of S&P GB holding in a one-dollar should be 97% and the remainder of 3% in the Solar equity.

By evaluating the cost of hedging, our findings reveal that the optimal hedge ratio ranges from a maximum value of 1.1523 for Wind to a minimum of 0.1715 for UW. As shown in Table 4, Wind (GE) provides the high (low) value of the hedge ratio, referring to an expensive (cheaper) hedge. Specifically, Table 4 shows the need to invest in a combined GE–S&P GB portfolio to reach the highest level of hedging effectiveness, indicating that GE offers a better hedging strategy to reduce the portfolio risk.

Network pairwise connectedness

To gain deeper insights into volatility connectedness across different market conditions, we employ the network connectedness approach of Diebold and Yilmaz (2014) using the Quantile VAR (QVAR) procedure. The quantile static and dynamic net directional pairwise directional connectedness results are presented in "Appendix" (see Table 5 and Fig. 7). The QVAR connectedness map provides important information concerning senders or receivers and the magnitude of the connectedness in the three market circumstances, which will offer the relevant stakeholders to undertake prolific decisions considering diverse market conditions. A range of colors is also referred to distinguish between the interrelations within the network. Specifically, the node's color reflects a given market's role in the system. For example, the blue (yellow)

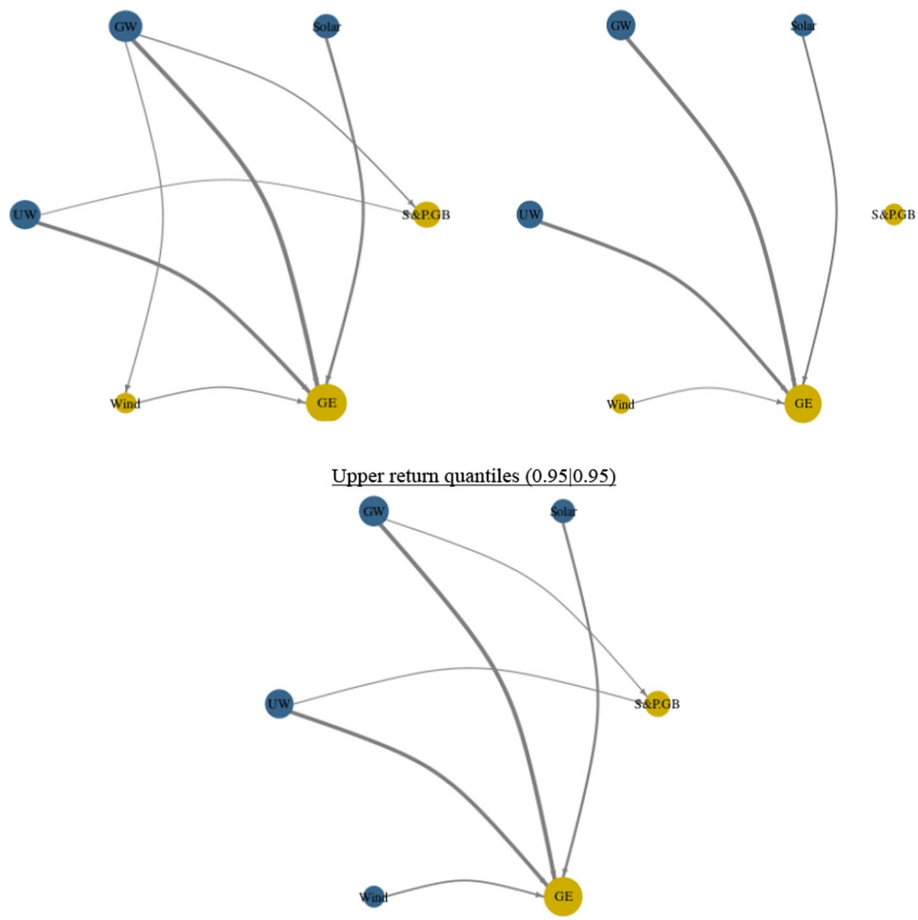

Fig. 5 Network Connectedness between GB and Green equities for three return quantiles. *Notes*: The figure depicts pairwise network connectedness between variables in the three market conditions. Blue (yellow) nodes represent the net shock transmitter (receiver). Averaged net pairwise directional connectedness measurements are used to weigh vertices. Weighted average net total directional connectivity is represented by the size of nodes. Arrows indicate positive net directional connectivity from a source to the arrow's edge. More arrows mean stronger connectedness

color nodes represent the main transmitter (recipient). The node's size regarding different market states also shows the economic magnitude of the interlinkage among the assets under consideration. Lastly, the arrow thickness represents the amount of directional connectedness.

The findings, as reported in Fig. 5, indicate that the degree of connectivity in the lower and upper quantiles is slightly higher when compared to the normal quantile plot. Moreover, in all market conditions, GW exhibits the greatest spillover towards other green equity and bond assets, followed by UW, while GE is the foremost net recipient of spillover from other assets. In the normal market condition, S&P GB does not observe any transmission or reception of volatility within the system. In addition, the results of pairwise direction volatility connectedness, depicted in Fig. 6 of the "Appendix", reveal minimal quantile dependence among most pairs, which varies across time and market conditions. This observation suggests that diversification opportunities are apparent among the green bond and green equity pairs, partially aligning with the findings of Pham (2021), Elsayed et al. (2022), and Chatziantoniou et al. (2022).

Conclusions and policy implications

The transition to a low-carbon economy will require considerable investment in long-term financial resources, and adopting appropriate financial instruments is essential to meeting this need for capital. GBs and green-related stocks are examples of securities that can be used to raise funds for projects that reduce carbon emissions. Hence, these financial instruments are now regarded as effective sustainable financing mechanisms that have gained traction among environmentally aware investors, resulting in the rapid growth of green financial markets over the past decade. This might allow investors to diversify their environmentally-friendly investments within the green assets framework. This study examines the relationship between GBs and green equity assets using a wavelet-based Quantile-on-Quantile regression, QVAR, and DCC techniques with data spanning the period from July 28, 2011, to July 28, 2021.

The findings reveal that GB and green equities are strongly and positively linked in all market conditions and at various frequencies, suggesting that GB has no diversification opportunity for green equity investors. Like the baseline model, the time-varying approach findings show a positive relationship between GB and green equities for most sample periods. Interestingly, a negative association is witnessed during times of crisis, indicating that GB can be used as a hedging mechanism against green equity portfolios, especially during market downturns. To attain optimal hedging effectiveness, investors should allocate their investments towards the GE–S&P GB portfolio based on the recommended portfolio strategies. Additionally, regardless of market conditions, GW exhibits the highest spillover to other green assets, whereas GE experiences the greatest spillover from other assets. The pairwise directional volatility connectedness reveals low quantile dependence among the majority of pairs, indicating the possibility of diversification across GB and GE pairs.

Investors and policymakers can benefit greatly from the findings in this study. First, we show that investing in GBs can help investors diversify their green equity portfolios during times of crisis. Crisis episodes may adversely impact environmentally-friendly projects, and investors may be able to protect the value of these projects by investing in GBs during those periods. In the post-COVID era, such investments may serve as a catalyst for bridging the gap between the need for economic restoration and the move to a green economy. Moreover, the low degree of volatility connectedness between GBs and green equities across quantile levels shows that investors and portfolio managers can construct a green bond-stock portfolio to diversify portfolio risk. Second, because these two markets are intertwined, policy changes in one can have a ripple effect on the other, especially among the GW, UW, GE, and GB markets, as GW and UW have the greatest volatility spillover to other green assets. As a result, policymakers should proceed with prudence when making changes that could affect both GBs and green equities. Policymakers should seek to increase environmental awareness among all investors, not only those concerned about the environment, to establish a cleaner, zero-carbon economy and achieve a sustainable recovery in the post-COVID era. Policymakers can also use the findings of this study on the transfer and receipt of volatility to and from green assets to develop methods to promote a smooth recovery under volatile market conditions.

This study has some limitations that pave the way for more research in the future. Further research could be conducted on GBs and other asset classes to gain a deeper understanding of these markets, thus enhancing green investing. Future research might expand the sample used here to incorporate regional, sectoral, and country-specific indices. Such research could lead to more precise conclusions about the relationship between green and conventional investments.

Appendix

See Table 5, Figs. 6 and 7.

Table 5 Quantile spillover connectedness. *Source*: Authors' own estimation

Variables	S&P GB	Solar	GW	UW	Wind	GE	FROM
Panel A: lower return quantile (0.05)							
S&P GB	24.80	14.73	16.44	14.79	15.59	13.65	75.20
Solar	13.32	22.31	17.69	17.94	15.40	13.34	77.69
GW	13.90	16.79	21.78	19.60	15.26	12.66	78.22
UW	13.00	17.26	19.77	22.13	14.75	13.09	77.87
Wind	14.43	16.26	17.39	16.08	22.75	13.10	77.25
GE	13.71	17.71	19.46	19.33	15.43	14.36	85.64
TO	68.37	82.75	90.75	87.75	76.42	65.84	471.87
NET	− 6.83	5.06	12.52	9.88	− 0.83	− 19.80	TCI
							78.65
Panel B: normal return quantile (0.5)							
S&P GB	75.37	3.66	8.21	4.32	5.90	2.54	24.63
Solar	2.23	51.70	16.27	18.46	7.44	3.90	48.30
GW	4.44	12.74	41.32	27.54	10.34	3.63	58.68
UW	2.07	15.33	28.96	44.14	5.66	3.83	55.86
Wind	4.76	9.37	15.45	8.93	58.74	2.74	41.26
GE	3.46	18.34	30.24	27.81	10.24	9.91	90.09
TO	16.95	59.44	99.14	87.06	39.57	16.64	318.82
NET	− 7.68	11.14	40.46	31.21	− 1.68	− 73.45	TCI
							53.14
Panel C: upper return quantile (0.95)							
S&P GB	25.71	14.40	16.04	14.69	15.79	13.37	74.29
Solar	12.81	23.43	17.29	17.96	15.51	13.00	76.57
GW	13.93	16.42	21.88	19.57	15.95	12.24	78.12
UW	12.57	17.06	19.88	22.74	15.00	12.74	77.26
Wind	14.58	15.86	17.08	15.91	23.57	12.99	76.43
GE	13.64	17.36	19.36	19.26	16.20	14.19	85.81
TO	67.54	81.12	89.64	87.40	78.45	64.34	468.48
NET	− 6.76	4.55	11.52	10.14	2.02	− 21.47	TCI
							78.08

The table reports the pairwise directional connectedness between the green bond and green equities for lower, normal, and upper return quantiles (0.05, 0.5, and 0.95, respectively). The total directional connectedness to and from others are represented by the "TO Others" row and the "FROM Others" column, respectively. NET and TCI refer to the net directional connectedness and total connectedness index, respectively

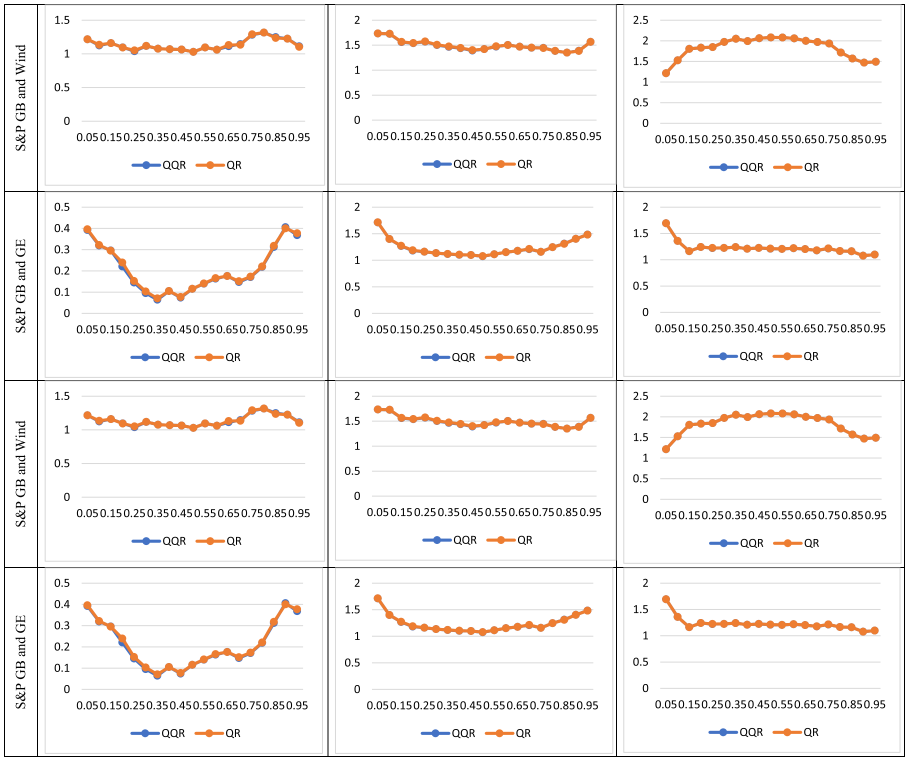

Fig. 6 Comparison of the average QQ and QR coefficients at different frequencies of green bond and green equity pairs

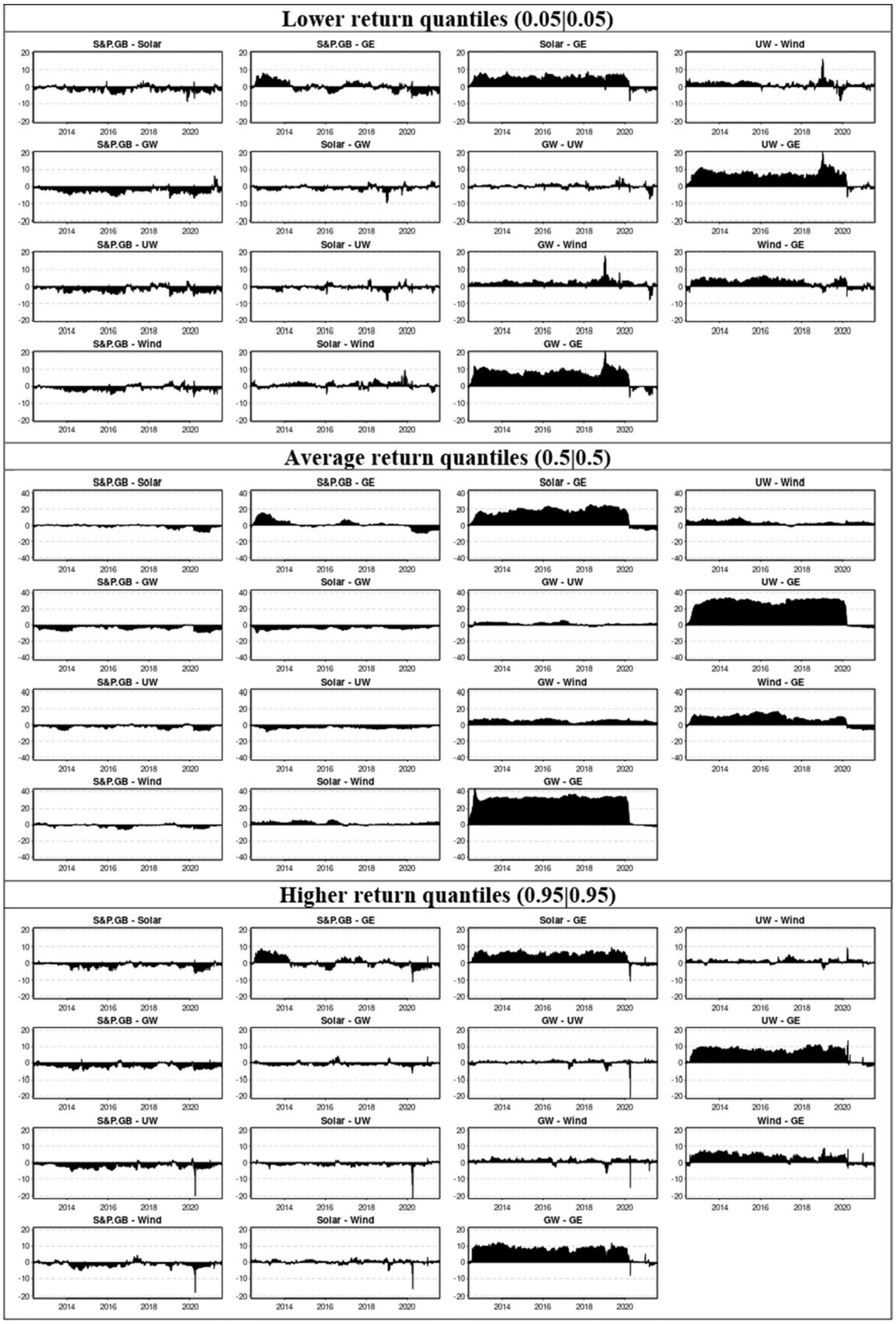

Fig. 7 Net pairwise directional volatility spillovers for three return quantiles (0.05, 0.5, and 0.95)

Abbreviations

CE	Clean energy
DCC	Dynamic conditional correlation
DJ	Dow Jones
DWT	Discrete wavelet transform
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GBs	Green bonds
GE	Green economy
GFC	Global financial crisis
GW	Global water
UW	U.S. water
QQ	Quantile-on-Quantile
S&P GB	S&P Dow Jones GB

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Author contributions
MBH: Conceptualization, Methodology, Final draft, and Supervision. GSU: Conceptualization, Supervision, and Analysis. MSA: Data curation, Data analysis, Visualization, and Formal analysis and investigation. MMR: Methodology, Data analysis, Software, and Writing- Original draft preparation. DP: Writing-review and editing. SHK: Writing-review and editing, Investigation, Funding acquisition.

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Availability of data and materials
The datasets generated and/or analysed during the current study are not publicly available due to data security but are available from the corresponding author on reasonable request.

Declarations

Competing interests
There are no conflicts of interest to declare.

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