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A firm-specific Malmquist productivity index model for stochastic data envelopment analysis: an application to commercial banks

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Abstract

In the data envelopment analysis (DEA) literature, productivity change captured by the Malmquist productivity index, especially in terms of a deterministic environment and stochastic variability in inputs and outputs, has been somewhat ignored. Therefore, this study developed a firm-specific, DEA-based Malmquist index model to examine the efficiency and productivity change of banks in a stochastic environment. First, in order to estimate bank-specific efficiency, we employed a two-stage double bootstrap DEA procedure. Specifically, in the first stage, the technical efficiency scores of banks were calculated by the classic DEA model, while in the second stage, the double bootstrap DEA model was applied to determine the effect of the contextual variables on bank efficiency. Second, we applied a two-stage procedure for measuring productivity change in which the first stage included the estimation of stochastic technical efficiency and the second stage included the regression of the estimated efficiency scores on a set of explanatory variables that influence relative performance. Finally, an empirical investigation of the Iranian banking sector, consisting of 120 bank-year observations of 15 banks from 2014 to 2021, was performed to measure their efficiency and productivity change. Based on the findings, the explanatory variables (i.e., the nonperforming loan ratio and the number of branches) indicated an inverse relationship with stochastic technical efficiency and productivity change. The implication of the findings is that, in order to improve the efficiency and productivity of banks, it is important to optimize these factors.

Keywords: Stochastic data envelopment analysis, Stochastic Malmquist productivity index, Double bootstrap procedure, Technical efficiency, Banking

Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. (1978; also known as the CCR model) and extended by Banker et al. (1984; also known as the BCC model), is a robust methodology for evaluating the relative efficiency of a set of homogeneous firms,[1] especially when they consume multiple incommensurate inputs to generate multiple incommensurate outputs. During the last three decades, DEA has been

[1] Throughout this study, the terms "firm," "decision-making unit (DMU)," and "production unit" are used synonymously.

applied in various areas, including the banking industry, the insurance sector, educational systems, healthcare units, agricultural production, military logistics, etc. This has enabled decision-makers to examine a significant number of real applications in various sectors. One of these widely studied applications is measuring productivity change over time. Specifically, the performance of a firm tends to change over time, which can lead to progress or regress in productivity. In this regard, the Malmquist productivity index (Malmquist 1953) is a well-known measure for determining the total factor productivity (TFP) of firms.

In traditional DEA models (and in subsequent extensions), production sets are assumed to be constructed by deterministic data. Consequently, existing TFP measures are presented in a deterministic environment. However, this ignores the variability and uncertainty of such data. To the best of our knowledge, few attempts have been made to examine productivity change in a stochastic environment. As is well-known, real-life problems are often stochastic and, in the presence of stochastic data variations, the concepts of technical efficiency and productivity change in a deterministic DEA setting may become sensitive to such variations. In this regard, it is important to modify classic DEA models in such a way that they can be applied in a stochastic environment. Meanwhile, when using classic DEA models to evaluate performance, it is important to account for all inputs and outputs. Based on the systemic view of the production process in Fig. 1, in addition to firm-specific inputs and outputs, there are other exogenous factors (e.g., nondiscretionary factors and managerial effort and ability) that can influence the efficiency of firms.

In order to obtain a firm-specific measure of efficiency and productivity change, it is important to remove the impact of these contextual variables on the efficiency and productivity of firms. Thus, this study developed a firm-specific, DEA-based Malmquist index model to examine the efficiency and productivity change of firms in a stochastic environment. In this case, we assumed that, in addition to firm-specific inputs and outputs, there are other factors (e.g., contextual and explanatory variables as well as managerial effort and ability) that may have a significant impact on the performance and productivity of firms. The question of how to handle such factors when analyzing the efficiency and productivity of firms in a stochastic environment is particularly important in this study.

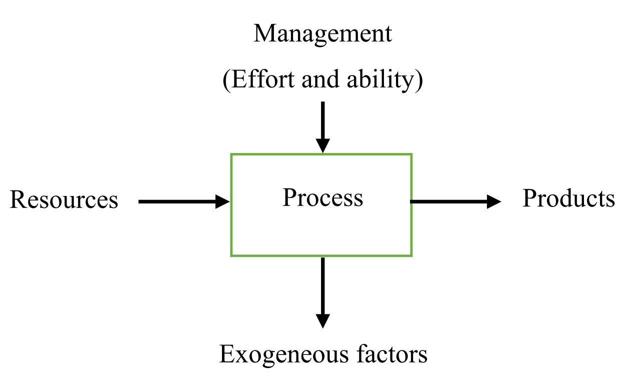

Fig. 1 Systemic view of the production process

To achieve a firm-specific technical efficiency and productivity measure, it is important to remove the impact of contextual variables on the efficiency of firms. Banker and Natarajan (2008), Banker et al. (2019) found that when input and output data are generated by a monotone increasing and concave production function, the application of a two-stage procedure, in which the first stage involves the estimation of technical efficiency through DEA and the second stage involves the regression of the estimated efficiency scores on the contextual variables through ordinary least squares, can generate consistent estimators of the parameters of such variables. In this case, to estimate firm-specific efficiency, we employed a two-stage double bootstrap DEA procedure. Specifically, we first estimated the technical efficiency scores of the banks by the BCC model, after which the double bootstrap DEA model was applied to determine the impact of the contextual variables on bank efficiency.

In order to set up the firm-specific stochastic Malmquist productivity index, we first used the stochastic BCC model to calculate the stochastic technical efficiency of each firm based on specific inputs and outputs. Then, the logarithm of the stochastic technical efficiency score obtained from the first stage was regressed on a set of contextual variables. In order to demonstrate the real applicability of our procedure, we analyzed the data for 15 Iranian banks from 2014 to 2021.

The Iranian banking sector started in 1888, but by 2010, many of the banks moved into the private sector. Therefore, it is important to examine the progress (or regression) in the banking industry in general, and in the unstable banking industry (such as Iran) in particular. In this empirical application, we analyzed the technical efficiency and productivity change in the nongovernmental banking sector in Iran. Although our proposed application is illustrative, the firm-specific stochastic Malmquist productivity index model can potentially be applied to evaluate productivity change in many real-life situations in which the underlying production processes are stochastic. Figure 2 presents the conceptual framework of our procedure, including both the methodological and applied framework.

The remainder of this study is organized as follows. Section "Literature review" provides a brief literature review of related works on deterministic and stochastic environments, while Section "Methodology" discusses the proposed approach for estimating productivity change in a stochastic environment. Section "Analysis and results in the banking sector" applies the proposed approach to evaluate productivity growth in the sample of Iranian banks, while Section "Conclusion" presents the conclusion.

Literature review

Previously, the evaluation of technical efficiency in the classic DEA framework was performed by using deterministic inputs and outputs. However, to account for stochastic inputs and outputs, several scholars have extended the deterministic DEA framework to the stochastic environment. In this regard, Sengupta (1982, 1987) was the first to propose the chance-constrained theory in the DEA framework to evaluate the technical efficiency of firms. The research of Sengupta has since been extended by numerous scholars (Sengupta 2000; Banker 1993; Cooper et al.

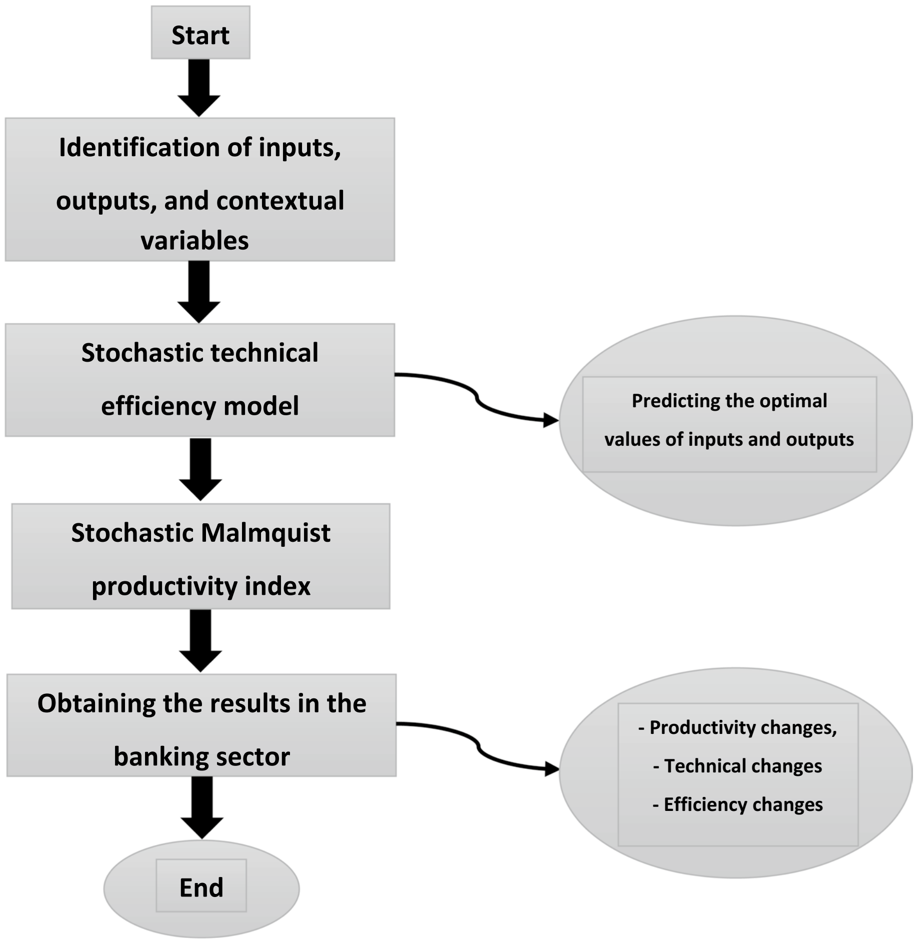

Fig. 2 Conceptual framework

1996, 1998; Land et al. 1993; Olesen and Petersen 1995; Horrace and Schmidt 1996; Grosskopf 1996; Simar 1996; Simar and Wilson 1998; Cooper et al. 2011; Shiraz et al. 2020; Simar and Wilson 2015; Olesen and Petersen 2016; Kao and Liu 2014; Kao and Liu 2019; Wei et al. 2014).

During the last three decades, several authors have used the DEA-based Malmquist productivity index to capture productivity change over time. For example, using the true fixed-effects model of trans-log stochastic production frontier, Chou et al. (2012) evaluated the performance of information technology industries for 19 Organization for Economic Cooperation and Development countries from 2000 to 2009, while Falavigna et al. (2018) applied the DEA-based Malmquist productivity index to understand court reforms. In related research, Odeck and Schøyen (2020) used the SFA-based Malmquist productivity index to evaluate the productivity and convergence of Norwegian container seaports, while Khoshroo et al. (2022) proposed an alternative double frontier-based Malmquist productivity index to calculate the TFP of the energy sector in the presence of undesirable pollutants.

Additionally, Giacalone et al. (2020) used the DEA-based Malmquist productivity index to evaluate the dynamic efficiency of the Italian judicial system, while Yu and Nguyen (2023) used the Malmquist productivity index and two-stage dynamic production structure to examine productivity change in Asia–Pacific airlines. Finally, Pourmahmoud and Bagheri (2023) applied the uncertain (imprecise) Malmquist productivity index to evaluate healthcare systems during the COVID-19 pandemic. For more related references, see Färe et al. (1994), Chen and Ali (2004), Kao (2010), Ma et al. (2017), Fernandez et al. (2018), Cao et al. (2019), Liu et al. (2021), Cho and Chen (2021), Zhao et al. (2022), and Bansal et al. (2022).

Although many studies have assessed the relative efficiency of production processes based on stochastic data, to the best of our knowledge, research on the measurement of productivity change under the stochastic DEA framework has been somewhat limited. However, Raayatpanah and Ghasvari (2011) applied the Malmquist productivity index in a stochastic environment and proposed a quadratic programming problem to calculate the TFP of firms. In this case, since the technical efficiency and subsequent productivity indexes were calculated by this nonlinear programming problem, their computational effort was relatively high. More recently, Arhin et al. (2023) used the double bootstrap DEA model to evaluate overall malaria spending efficiency in Sub-Saharan Africa.

To date, the evaluation of the Malmquist productivity index under the DEA framework has been mainly conducted by using firm-specific inputs and outputs. However, to the best of our knowledge, other factors, such as explanatory and contextual variables, have yet to be considered. Therefore, in the following section, we present our methodology for developing a firm-specific, DEA-based Malmquist productivity index in both deterministic and stochastic environments, especially in the presence of contextual variables.

Methodology

Stochastic BCC model

Suppose that there are J decision-making units (DMU), and each $DMU_j : j = 1, \ldots, J$ uses random inputs $\widetilde{x}_j = (\widetilde{x}_{1j}, \ldots, \widetilde{x}_{Ij})$ to produce random outputs$\widetilde{y}_j = (\widetilde{y}_{1j}, \ldots, \widetilde{y}_{Rj})$. In this case, all of the random input and output variables are assumed to be normally distributed with known mean and variance.

In the performance evaluation of real-world situations, the selection of an underlying model is important. In our real application in the Iranian banking sector, we found that the majority of the banks did not perform at an optimal level. In this sense, we believe that the variable returns to scale model of Banker et al. (1984; i.e., the BCC model) is more appropriate for analyzing the performance of banks than the constant returns to scale model of Charnes et al. (1978; i.e., the CCR model).

In related research, Land et al. (1993) established the following DEA model (Model (1)) for the variable returns to scale in order to estimate the input-oriented relative efficiency of a specific DMU_o:

$$
\begin{aligned}
& Min\,\theta \\
& \text{s.t.} \\
& P\left\{\sum_{j=1}^{J}\lambda_j\tilde{x}_j \le \theta\tilde{x}_o\right\} \ge 1-\alpha, \\
& P\left\{\sum_{j=1}^{J}\lambda_j\tilde{y}_j \ge \tilde{y}_o\right\} \ge 1-\alpha, \\
& \sum_{j=1}^{J}\lambda_j = 1, \\
& \lambda_j \ge 0, \forall j.
\end{aligned}
\tag{1}
$$

where $\alpha \in [0,1]$ is the user-defined parameter that reflects the confidence level and θ is the abatement factor that reduces the level of inputs (radially). In addition, suppose that $x_{ij} : i = 1, \ldots, I$ and $y_{rj} : r = 1, \ldots, R$ are the mean values of the inputs and outputs for jth DMU, respectively, while a_{ij} and b_{rj} are their corresponding standard deviations of the inputs and outputs, respectively. Meanwhile, function ϕ is the cumulative standard normal distribution function and ϕ^{-1} is its inverse. Thus, according to the central limit theorem, the deterministic form of Model (1) can be represented as follows:

$$
\begin{aligned}
& Min\theta \\
& \text{s.t.} \\
& \sum_{j=1}^{J}\lambda_j x_{ij} - \phi^{-1}(\alpha)\sigma_i^I(\lambda,\theta) \le \theta x_{io}, \quad i = 1,\ldots,I, \\
& \sum_{j=1}^{J}\lambda_j y_{rj} + \phi^{-1}(\alpha)\sigma_r^O(\lambda) \ge y_{ro}, \quad r = 1,\ldots,R, \\
& \sum_{j=1}^{J}\lambda_j = 1, \\
& \lambda_j \ge 0, \quad \forall j.
\end{aligned}
\tag{2}
$$

where:

$$
\left(\sigma_i^I\right)^2 = \sum_{j=1}^{J}\sum_{k=1}^{J}\lambda_j\lambda_k \mathrm{cov}\left(\tilde{x}_{ij},\tilde{x}_{ik}\right) + \theta^2 \mathrm{var}\left(\tilde{x}_{io}\right) - 2\theta\sum_{j=1}^{J}\lambda_j \mathrm{cov}\left(\tilde{x}_{ij},\tilde{x}_{io}\right), \quad i = 1,\ldots,I.
$$

$$
\left(\sigma_r^O\right)^2 = \sum_{j=1}^{J}\sum_{k=1}^{J}\lambda_j\lambda_k \mathrm{cov}\left(\tilde{y}_{rj},\tilde{y}_{rk}\right) + \mathrm{var}\left(\tilde{y}_{ro}\right) - 2\sum_{j=1}^{J}\lambda_j \mathrm{cov}\left(\tilde{y}_{rj},\tilde{y}_{ro}\right), \quad r = 1,\ldots,R.
$$

If we use the single-factor assumption of random variables in economics and finance (Sharpe 1963; Kahane 1977), Model (2) can be transformed into the following linear form:

$$
\begin{aligned}
&Min\theta \\
&\text{s.t.} \\
&\sum_{j=1}^{J} \lambda_j x_{ij} - \phi^{-1}(\alpha)\left(p_i^{+} + p_i^{-}\right) \leq \theta x_{io}, \quad i = 1, \ldots, I, \\
&\sum_{j=1}^{J} \lambda_j a_{ij} - \theta a_{io} = p_i^{+} - p_i^{-}, \quad i = 1, \ldots, I, \\
&\sum_{j=1}^{J} \lambda_j y_{rj} + \phi^{-1}(\alpha)\left(q_r^{+} + q_r^{-}\right) \geq y_{ro}, \quad r = 1, \ldots, R, \\
&\sum_{j=1}^{J} \lambda_j b_{rj} - b_{ro} = q_r^{+} - q_r^{-}, \quad r = 1, \ldots, R, \\
&\sum_{j=1}^{J} \lambda_j = 1, \\
&\lambda_j, q_i^{+}, q_i^{-}, q_r^{+}, q_r^{-} \geq 0, \quad \forall j, i, r.
\end{aligned}
\tag{3}
$$

In Model (3), $p_i^{+}, p_i^{-}, q_r^{+}, \text{and} q_r^{-}$ are deviation variables. In this case, it is not difficult to show that this model is feasible under all confidence levels of α. In related research, Banker (1993) developed a statistical foundation for DEA and claimed that DEA estimators are not only consistent, but that they also maximize likelihood. He also indicated that DEA estimators of the best practice monotone increasing, and concave production function are maximum likelihood estimators. Thus, we present the following theorem:

Theorem 1 *For any predetermined level of* $\alpha \leq 0.5$, *the stochastic efficiency score calculated from Model* (3) *ranges from* 0 *to* 1.

Proof See Cooper et al. (2011).

Definition 1 The unit under evaluation, DMU_o, is stochastically efficient at confidence level α, if and only if $\theta_o^{*} = 1$.

It should be noted that Model (3) uses firm-specific stochastic inputs and outputs. However, as stated earlier, in many real-life processes, three types of variables (inputs, outputs, and contextual variables) can affect the performance of firms. In order to obtain a firm-specific technical efficiency measure, we must remove the impact of contextual variables on firm efficiency. In this regard, we employed a two-stage double bootstrap DEA procedure. Specifically, in the first stage, the technical efficiency scores of banks were calculated by the classic DEA model, while in the second stage, we used truncated regression analysis to determine the impact of the contextual variables on bank efficiency. In the latter stage, we also applied the following regression model (Model (4)):

$$
Log\left(E_p\right) = \beta_0 + \beta_1 z_1 + \beta_2 z_2 + \cdots + \beta_N z_N + U \tag{4}
$$

where $Log(E_p)$ is the logarithm of Euler's number ($e = 2.71828$) for the stochastic BCC efficiency score of DMU_j obtained in this model. Moreover, $z_1, z_2, \ldots, z_N$ are the aforementioned contextual variables, while U is the error term. Based on the regression results, the refined measure of technical efficiency was computed as follows:

$$E_p^{New} = E_p - \left(Log\left(E_p\right) - \beta_0 - \beta_1 z_1 - \beta_2 z_2 - \cdots - \beta_N z_N\right) \tag{5}$$

In this case, after removing the impact of the contextual variables on bank efficiency, E_p^{New} is the bank-specific efficiency score.

Stochastic productivity index

Measuring the productivity growth or productivity change of firms over time can play a key role in economic and business analytics. In this regard, the two main factors that can capture such growth or change include: general technological progress (or regress); and special initiatives within a firm. As stated earlier, the Malmquist productivity index is a useful and suitable index for measuring the growth and productivity change of a firm in a deterministic environment. However, since the classic DEA-based Malmquist productivity index was mainly formulated for deterministic technologies, contextual/explanatory variables are basically ignored in productivity analyses. Specifically, production technology and production frontier are based on the assumption that inputs and outputs are certain and deterministic. Meanwhile, the systemic view of production can make evaluations more complicated, since it considers multiple dimensions, including contextual/explanatory variables that can influence the performance of firms.

In related research, Banker and Natarajan (2008) found that when input and output data are generated by a monotone increasing and concave production function, the application of a two-step DEA approach in which technical efficiency is calculated in the first stage and a regression of the estimated efficiency scores on the explanatory variables is performed in the second stage can generate a consistent estimate of the parameters of the contextual variables. They also showed that DEA in the first stage followed by maximum likelihood estimation in the second stage can yield consistent estimators of the impact of the contextual variables. The only exception is that the contextual variables must be independent from the input variables. Additionally, they used Monte Carlo simulations to compare the performance of their two-stage approach with the one- and two-stage parametric approaches. Following Banker and Natarajan (2008), after calculating the technical efficiency in the first stage, we regressed the log of our new productivity index on the stochastic contextual variables in the second stage. Considering that the explanatory variables are uncertain in stochastic form, we used a stochastic regressors model, which is described as follows.

Suppose that we have $J \times T$ observations on $j = 1, \ldots, J$ DMUs and $t = 1, \ldots, T$ years as $DMU_j^{(t)}$, and that $x_j^{(t)} = (x_{1j}^{(t)}, \ldots, x_{Ij}^{(t)})$ and $y_j^{(t)} = (y_{1j}^{(t)}, \ldots, y_{Rj}^{(t)})$ are the means of the inputs and outputs of $DMU_j^{(t)}$ (DMU_j in period t), respectively. Moreover, suppose that $SE_o(s,t)$ is the technical efficiency of DMU_o in period s, against the technology in period t. In order to estimate the relative efficiency of a specific $DMU_o^{(t)}$ in the first stage of our analysis, we solved the following stochastic BCC model:

$$
\begin{aligned}
& E_o^{(t)} = Min\theta \\
& \text{s.t.} \\
& \sum_{t=1}^{T}\sum_{j=1}^{J}\lambda_{jt}x_{ij}^{(t)} - \phi^{-1}(\alpha)\left(p_i^{+} + p_i^{-}\right) \le \theta x_{io}^{(t)}, \quad i = 1,\ldots,I, \\
& \sum_{t=1}^{T}\sum_{j=1}^{J}\lambda_{jt}a_{ij}^{(t)} - \theta a_{io}^{(t)} = \left(p_i^{+} - p_i^{-}\right), \quad i = 1,\ldots,I, \\
& \sum_{t=1}^{T}\sum_{j=1}^{J}\lambda_{jt}y_{rj}^{(t)} - \phi^{-1}(\alpha)\left(q_r^{+} + q_r^{-}\right) \ge y_{ro}^{(t)}, \quad r = 1,\ldots,R, \\
& \sum_{t=1}^{T}\sum_{j=1}^{J}\lambda_{jt}b_{rj}^{(t)} - b_{ro}^{(t)} = \left(q_r^{+} - q_r^{-}\right), \quad r = 1,\ldots,R, \\
& \sum_{t=1}^{T}\sum_{j=1}^{J}\lambda_{jt} = 1, \\
& \lambda_{jt},\, p_i^{+},\, p_i^{-},\ q_r^{+}, q_r^{-} \ \ge 0, \quad \forall j,i,r,t.
\end{aligned}
\tag{6}
$$

Suppose that $SE_o(s,t)$ is the measure of the stochastic efficiency of DMU_o in period s, against the technology in period t. Here, $SE_o(s,t)$ is estimated by solving the following model (Model (7)):

$$
\begin{aligned}
& SE_o(s,t) = Min\theta \\
& \text{s.t.} \\
& \sum_{j=1}^{J}\lambda_{jt}x_{ij}^{(t)} - \phi^{-1}(\alpha)\sigma\left(p_i^{+} + p_i^{-}\right) \le \theta x_{io}^{(s)}, \quad i = 1,\ldots,I, \\
& \sum_{j=1}^{J}\lambda_{jt}a_{ij}^{(t)} - \theta a_{io}^{(s)} = \left(p_i^{+} - p_i^{-}\right), \quad i = 1,\ldots,I, \\
& \sum_{j=1}^{J}\lambda_{jt}y_{rj}^{(t)} - \phi^{-1}(\alpha)\sigma\left(q_r^{+} + q_r^{-}\right) \ge y_{ro}^{(s)}, \quad r = 1,\ldots,R, \\
& \sum_{j=1}^{J}\lambda_{jt}b_{rj}^{(t)} - b_{ro}^{(s)} = \left(q_r^{+} - q_r^{-}\right), \quad r = 1,\ldots,R, \\
& \sum_{j=1}^{J}\lambda_{jt} = 1, \\
& \lambda_{jt},\, p_i^{+},\, p_i^{-},\ q_r^{+}, q_r^{-} \ \ge 0, \quad \forall j,i,r,t.
\end{aligned}
\tag{7}
$$

Similarly, $SE_o(t,s)$ is the measure of the stochastic efficiency of DMU_o in period t, against the technology in period s. It is important to note that in $SE_o(s,t)$, the input and output data are taken from period s, while the technology is from period t. In this study, we first applied Model (7) to calculate stochastic efficiency measures $SE_o(s,t)$, $SE_o(t,t)$, $SE_o(t,s)$, and $SE_o(s,s)$.

As stated earlier, Banker and Natarajan (2008) showed that when the input/output data is generated by a concave and monotone increasing production function, a two-stage procedure can determine the firm-specific efficiency score. In this regard, after calculating the stochastic BCC efficiency of DMU_o, the following regression model (Model (8)) was used to determine the impact of the contextual variables on efficiency when the DMU has no control:

$$Log(SE_o(s,t)) = \beta_0^{(s)} + \beta_1^{(s)} z_{1o}^{(s)} + \beta_2^{(s)} z_{2o}^{(s)} + \cdots + \beta_N^{(s)} z_{No}^{(s)} + \varepsilon_o^{(s)} \tag{8}$$

where $z_{nj}^{(s)} : n = 1, \ldots, N, j = 1, \ldots, J$ are the contextual variables in period s, $\varepsilon_o^{(s)}$ is the error term, and $\beta_n^{(s)} : n = 1, \ldots, N$ are the weights corresponding to the contextual variables in period s. Using Model 8, we also examined the contextual variables that affect efficiency score $SE_o(s,t)$ in order to remove their impact. It is important to note that the logarithm of the stochastic efficiency can be regressed on the contextual variables taken from the same period as the input/output data. Meanwhile, the assumption that "the explanatory variables are stochastic" poses no issue in the ordinary least squares estimation of $\beta_n^{(s)} : n = 1, \ldots, N$ and $\varepsilon_o^{(s)}$. In order to remove the impact of the contextual variables on efficiency when banks have no control, we refined stochastic technical efficiencies $SE_o(s,t)$, $SE_o(t,t)$, $SE_o(t,s)$, and $SE_o(s,s)$ by removing the effect of the contextual variables, as shown in Model (9):

$$\overline{SE_o}(.,.) = SE_o(.,.) - \left(Log(SE_o(.,.)) - \beta_0 - \beta_1 z_1 - \beta_2 z_2 - \ldots - \beta_N z_N\right) \tag{9}$$

In Model (10), the refined stochastic measures of efficiency in different time periods are denoted as $\overline{SE_o}(s,s)$, $\overline{SE_o}(t,s)$, $\overline{SE_o}(t,t)$, and $\overline{SE_o}(s,t)$. Here, the stochastic Malmquist productivity index for DMU_o is as follows:

$$SM_o(s,t) = \sqrt{\frac{\overline{SE_o}(t,s)}{\overline{SE_o}(t,t)} \times \frac{\overline{SE_o}(s,s)}{\overline{SE_o}(s,t)}} \times \frac{\overline{SE_o}(t,t)}{\overline{SE_o}(s,s)} = STC(s,t) \times SEC(s,t) \tag{10}$$

where $STC(s,t) = \sqrt{\frac{\overline{SE_o}(t,s)}{\overline{SE_o}(t,t)} \times \frac{\overline{SE_o}(s,s)}{\overline{SE_o}(s,t)}}$ is the technological change and $SEC(s,t) = \frac{\overline{SE_o}(t,t)}{\overline{SE_o}(s,s)}$ is the efficiency change.

Analysis and results in the banking sector

Due to the daily challenges faced by banks around the world, especially those in Iran, it is becoming increasingly clear that banking executives must optimize their performance and productivity. During the last two decades, the performance analysis of banks and other financial institutions has attracted significant attention among economists and managers. As for the Iranian banking sector, it operated under the supervision of the government until 2008, after which it moved into the private sector. In this section, we present an illustrative empirical application of our proposed stochastic productivity growth model based on a sample of banks in Iran.

Description of the variables

First, an evaluation of bank efficiency requires the identification of inputs and outputs. In this regard, Banker et al. (2010) determined three types of variables in the banking system: input, output, and explanatory (or contextual) variables. Inspired by their research, we used four indicators as inputs and outputs: interest expenses and

Table 1 Input, outputs, contextual variables

Type of variables	Description
Input variables	Interest expense (IE), Other operating expenses (OE)
Output Variables	Net interest revenue (NIR), Operating revenue (OR)
Contextual Variables	Total capital adequacy ratio (TCAR), NPL ratio (NPLR), Total assets (TA), Number of branches (NB), Time series dummy (TSD)

Table 2 Summary statistics of mean of the data

	Indicator	Min	Median	Max	Mean	Std	Q1	Q3
Inputs	IE	12,463.7	41,186.14	2,255,442.54	403,791.01	641,022.21	20,365.60	637,010.15
	OE	10,921.12	38,082.90	750,163.72	142,981.49	195,146.99	29,317.12	178,365.52
Outputs	NIR	102,980.39	150,876.39	2,209,283.69	411,561.99	537,132.9	130,221.03	338,595.24
	OR	101,447.87	178,312.73	2,918,290.36	568,105.98	770,128.64	126,946.89	786,784.70
Contextual variables	TCAR	13.48	19.45	88.14	23.556833	15.503543	16.775	22.53
	NPLR	0.3	1.32	1.8	1.0530833	1.4729465	0.8725	2.20
	TA	1,199,890.57	5,409,334.59	123,761,924.4	24,285,919.69	35,568,940.13	1,868,356.17	27,948,449.27
	NB	339	1055	3126	1241	761.89	581.25	1688

other operating expenses (as inputs); and net interest revenue and operating revenue (as outputs). We also employed five explanatory variables: the total capital adequacy ratio; the nonperforming loan ratio; total assets; and the number of branches. In order to take the role of time into account, we used a time series dummy (1 for each specific year, or 0 otherwise). Table 1 summarizes the inputs, outputs, and contextual variables.

In banks, annual data is generally kept as an average of 12 months, while monthly data is kept as a daily aggregation. Additionally, the mean and standard deviation of the data for each 12-month period are calculated. Considering the behavior and performance of banks, it is possible that all inputs and outputs are random variables. Table 2 present the descriptive statistics of the input, output, and contextual variables in this study. Overall, the data is based on 120 bank-year observations of 15 banks from 2014 to 2021.

Efficiency evaluation of the banks

In order to evaluate the stochastic efficiency of the sample of banks in each year, we used the two-stage procedure proposed by Banker and Natarajan (2008). In the first stage, we used Model (4) to calculate the stochastic BCC technical efficiency of the banks based on their specific input consumption and output generation. In this case, three different confidence levels were used: $\alpha = 0.1, 0.3, 0.5$. The results of the stochastic BCC efficiency at the three different confidence levels are given in Tables 3, 4, and 5, respectively. Meanwhile, the trends of the average stochastic BCC efficiency scores over time at the three different confidence levels are shown in Fig. 3. Based on the findings, the average stochastic technical efficiency decreased from 2014 to 2018 and then increased. Additionally, the maximum average stochastic efficiency occurred in 2021, while the minimum was in 2018.

Table 3 Stochastic BCC efficiency scores in different periods $\alpha = 0.1$

Bank	2014	2015	2016	2017	2018	2019	2020	2021
1	0.8061	0.9189	0.9394	1	0.9211	1	0.8207	1
2	0.9173	0.9028	0.88	1	0.6216	0.7495	0.9494	1
3	0.5934	0.6593	0.658	0.8517	0.8831	0.7476	0.8471	0.8531
4	0.4546	0.5538	0.5531	0.5316	0.4659	1	0.5416	0.6174
5	0.1282	0.1254	0.1279	0.1399	0.1613	0.154	0.14	0.1377
6	0.8098	0.9856	1	1	1	0.6779	1	0.9451
7	0.1833	0.1692	0.1855	0.1663	0.1703	0.1976	0.1866	0.1735
8	0.955	0.8819	0.8561	0.7295	0.4769	0.38	0.3879	0.3723
9	1	0.9483	0.9279	0.876	0.8959	1	1	0.9748
10	1	0.9307	0.9004	1	0.8518	0.7957	1	1
11	0.4661	0.4792	0.4765	0.4624	0.5352	0.523	0.5946	0.6786
12	0.9462	0.8647	0.7825	0.7838	0.5479	0.4868	0.8424	0.9328
13	0.6944	0.6712	0.6501	0.6332	0.6022	0.5906	0.6693	1
14	0.7993	0.7057	0.6636	0.6281	0.6261	0.6222	0.696	0.6626
15	0.9336	0.7847	0.7535	0.733	0.7318	0.7196	0.77	0.7293
Min	0.1282	0.1254	0.1279	0.1399	0.1613	0.154	0.14	0.1377
Median	0.8061	0.7847	0.7535	0.733	0.6216	0.6779	0.77	0.8531
Max	1	0.9856	1	1	1	1	1	1
Mean	0.7125	0.7054	0.6903	0.7024	0.6327	0.6429	0.6964	0.7384
Std	0.2779	0.2629	0.2548	0.2727	0.2468	0.2561	0.2714	0.2899

Table 4 Stochastic BCC efficiency scores in different periods $\alpha = 0.3$

Bank	2014	2015	2016	2017	2018	2019	2020	2021
1	0.7945	0.9119	0.9352	1	0.9139	1	0.8186	1
2	0.9019	0.8942	0.8786	1	0.616	0.745	0.9482	1
3	0.5838	0.6452	0.6447	0.8345	0.8784	0.7403	0.8429	0.8499
4	0.4269	0.5307	0.5307	0.5116	0.4449	1	0.521	0.601
5	0.1031	0.1012	0.1037	0.1187	0.1269	0.1323	0.1183	0.1163
6	0.8098	0.9856	1	1	0.9685	0.6779	1	0.9429
7	0.1465	0.1353	0.1411	0.1257	0.1308	0.1688	0.1614	0.1486
8	0.955	0.8819	0.8561	0.7239	0.4769	0.3639	0.3755	0.3578
9	1	0.9398	0.9225	0.876	0.8959	1	1	0.9697
10	1	0.9307	0.9004	1	0.8121	0.7543	1	1
11	0.4611	0.4772	0.4765	0.4624	0.5352	0.523	0.5946	0.6602
12	0.8791	0.8107	0.7309	0.7529	0.5479	0.4717	0.8424	0.9212
13	0.6944	0.6698	0.6501	0.6332	0.6022	0.5906	0.6267	1
14	0.7435	0.7057	0.6636	0.6281	0.6261	0.6222	0.6534	0.6222
15	0.8568	0.7847	0.7535	0.733	0.7318	0.7196	0.7407	0.7009
Min	0.1031	0.1012	0.1037	0.1187	0.1269	0.1323	0.1183	0.1163
Median	0.7945	0.7847	0.7309	0.733	0.616	0.6779	0.7407	0.8499
Max	1	0.9856	1	1	0.9685	1	1	1
Mean	0.6904	0.6936	0.6792	0.6933	0.6205	0.6339	0.6829	0.7260
Std	0.2795	0.2691	0.2636	0.2807	0.2512	0.2623	0.2791	0.2980

Table 6 presents the descriptive statistics (mean and standard deviation) of the projection points corresponding to the inputs and outputs at a confidence level of $\alpha = 0.5$ (deterministic case). According to the findings, the first two inputs (interest expense

Table 5 Stochastic BCC efficiency scores in different periods $\alpha = 0.5$

Bank	2014	2015	2016	2017	2018	2019	2020	2021
1	0.7846	0.9096	0.9316	1.0000	0.9069	1.0000	0.8171	1.0000
2	0.8829	0.8829	0.8763	1.0000	0.6117	0.7418	0.9473	1.0000
3	0.5743	0.6346	0.6348	0.8180	0.8748	0.7349	0.8397	0.8472
4	0.4065	0.5138	0.5141	0.4969	0.4294	1.0000	0.5054	0.5887
5	0.0792	0.0780	0.0802	0.0970	0.0971	0.1096	0.0960	0.0947
6	0.8055	0.9846	1.0000	1.0000	0.8049	0.6712	1.0000	0.9373
7	0.1108	0.1031	0.1049	0.0941	0.0992	0.1399	0.1331	0.1218
8	0.9550	0.8819	0.8561	0.7156	0.4769	0.3423	0.3589	0.3373
9	1.0000	0.9261	0.9138	0.8760	0.8959	1.0000	1.0000	0.9579
10	1.0000	0.9307	0.9004	1.0000	0.7611	0.7018	1.0000	1.0000
11	0.4527	0.4732	0.4765	0.4623	0.5352	0.5230	0.5946	0.5697
12	0.7957	0.7422	0.6661	0.7116	0.5479	0.4524	0.8424	0.8208
13	0.6315	0.6287	0.6231	0.6202	0.5907	0.5895	0.5901	0.9303
14	0.6998	0.6921	0.6593	0.6281	0.6261	0.6222	0.6194	0.5901
15	0.7879	0.7468	0.7377	0.7330	0.7318	0.7196	0.7163	0.6774
Min	0.0792	0.0780	0.0802	0.0941	0.0971	0.1096	0.0960	0.0947
Median	0.7846	0.7422	0.6661	0.7156	0.6117	0.6712	0.7163	0.8208
Max	1.0000	0.9846	1.0000	1.0000	0.9069	1.0000	1.0000	1.0000
Mean	0.6644	0.6752	0.6650	0.6835	0.5993	0.6232	0.6707	0.6982
Std	0.2823	0.2744	0.2713	0.2878	0.2447	0.2692	0.2878	0.3007

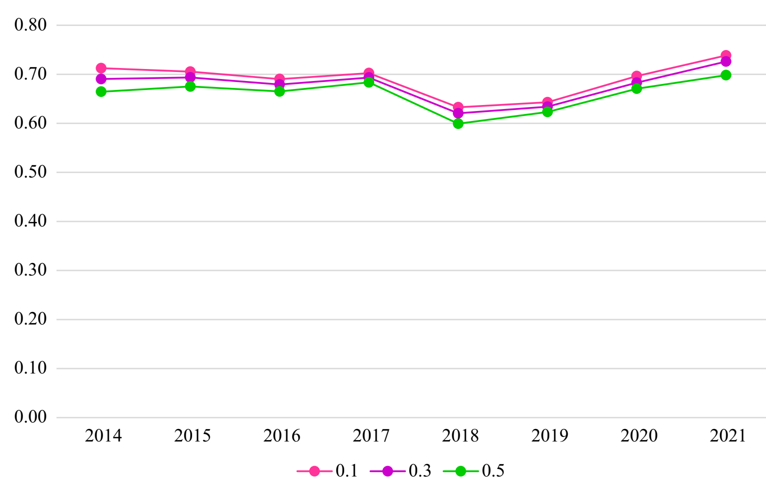

Fig. 3 Trends of the average stochastic BCC efficiency scores

Table 6 Summary statistics of the projection points of the data

	IE	OE	NIR	OR
Mean	340,107.4743	78,079.51062	461,163.6623	574,039.1894
STD	597,764.5414	125,250.5949	547,123.5875	774,703.203

and other operating expenses) must be reduced by 16% and 45%, respectively. In contrast, the net interest revenue and operating revenue must be increased by 12% and 1%, respectively.

Table 7 Stochastic BCC efficiency scores in different periods $\alpha = 0.5$

Bank	2014	2015	2016	2017	2018	2019	2020	2021
1	0.7846	0.9096	0.9316	1.0000	0.9069	1.0000	0.8171	1.0000
2	0.8829	0.8829	0.8763	1.0000	0.6117	0.7418	0.9473	1.0000
3	0.5743	0.6346	0.6348	0.8180	0.8748	0.7349	0.8397	0.8472
4	0.4065	0.5138	0.5141	0.4969	0.4294	1.0000	0.5054	0.5887
5	0.0792	0.0780	0.0802	0.0970	0.0971	0.1096	0.0960	0.0947
6	0.8055	0.9846	1.0000	1.0000	0.8049	0.6712	1.0000	0.9373
7	0.1108	0.1031	0.1049	0.0941	0.0992	0.1399	0.1331	0.1218
8	0.9550	0.8819	0.8561	0.7156	0.4769	0.3423	0.3589	0.3373
9	1.0000	0.9261	0.9138	0.8760	0.8959	1.0000	1.0000	0.9579
10	1.0000	0.9307	0.9004	1.0000	0.7611	0.7018	1.0000	1.0000
11	0.4527	0.4732	0.4765	0.4623	0.5352	0.5230	0.5946	0.5697
12	0.7957	0.7422	0.6661	0.7116	0.5479	0.4524	0.8424	0.8208
13	0.6315	0.6287	0.6231	0.6202	0.5907	0.5895	0.5901	0.9303
14	0.6998	0.6921	0.6593	0.6281	0.6261	0.6222	0.6194	0.5901
15	0.7879	0.7468	0.7377	0.7330	0.7318	0.7196	0.7163	0.6774
Min	0.0792	0.0780	0.0802	0.0941	0.0971	0.1096	0.0960	0.0947
Max	1.0000	0.9846	1.0000	1.0000	0.9069	1.0000	1.0000	1.0000
Mean	0.6644	0.6752	0.6650	0.6835	0.5993	0.6232	0.6707	0.6982
Std	0.2922	0.2840	0.2808	0.2979	0.2533	0.2786	0.2979	0.3113
Median	0.7846	0.7422	0.6661	0.7156	0.6117	0.6712	0.7163	0.8208

Results of the bootstrap DEA procedure for $\alpha = 0.5$ (deterministic case)

After calculating the original DEA efficiency scores in different years, we used the bootstrap DEA procedure to calculate the bias and bias-corrected efficiency scores. The technical efficiency scores of the banks were computed by using General Algebraic Modeling System software on a personal computer (with an Intel Core i7 processor). As stated earlier, we applied the input-oriented BCC model to calculate the original efficiency scores. Table 7 presents the stochastic BCC efficiency scores in different periods for $\alpha = 0.5$ (deterministic case).

The last four rows of the table include the statistical indicators. Since the original efficiency scores might be biased, we used the bootstrap DEA procedure to improve these scores. Tables 8 and 9 present the bias and bias-corrected efficiency scores, respectively. For example, in 2021, the average original stochastic BCC efficiency score was 0.6982, while the average bias-corrected efficiency score was 0.6291.

The trends of the average bias and bias-corrected efficiency scores are depicted in Fig. 4. Overall, before applying the bootstrap DEA procedure, there were 16 efficient banks, whereas after correcting the bias, the discrimination power significantly increased. In this case, the bias-corrected efficiency scores were used to rank all of the banks (given in parentheses in Table 9), with Banks 5 and 7 occupying the last two ranks. Moreover, the maximum average efficiency occurred in 2015, while the minimum was in 2018.

Table 8 Bias for $\alpha = 0.5$

Bank	2014	2015	2016	2017	2018	2019	2020	2021
1	0.0443	0.0212	0.0908	0.0943	0.0572	0.0689	0.0197	0.0969
2	0.0639	0.0566	0.0810	0.0994	0.0214	0.0178	0.0083	0.0464
3	0.0077	0.0345	0.0601	0.0004	0.0724	0.0953	0.0066	0.0922
4	0.0490	0.0730	0.0604	0.0414	0.0550	0.0360	0.0038	0.0885
5	0.0585	0.0608	0.0144	0.0286	0.0398	0.0600	0.0444	0.0872
6	0.0260	0.0909	0.0549	0.0296	0.0262	0.0629	0.0592	0.0430
7	0.0459	0.0057	0.0781	0.0842	0.0088	0.0175	0.0598	0.0309
8	0.0639	0.0558	0.0689	0.0289	0.0059	0.0884	0.0952	0.0761
9	0.0873	0.0701	0.0888	0.0323	0.0503	0.0588	0.0708	0.0970
10	0.0565	0.0397	0.0264	0.0449	0.0428	0.0700	0.0302	0.0601
11	0.0807	0.0129	0.0101	0.0598	0.0687	0.0735	0.0995	0.0475
12	0.0344	0.0041	0.0838	0.0867	0.0756	0.0950	0.0933	0.0367
13	0.0174	0.0138	0.0444	0.0384	0.0149	0.0120	0.0846	0.0940
14	0.0774	0.0622	0.0423	0.0292	0.0452	0.0428	0.0360	0.0787
15	0.0943	0.0350	0.0447	0.0866	0.0985	0.0487	0.0940	0.0612
Min	0.0077	0.0041	0.0101	0.0004	0.0059	0.0120	0.0038	0.0309
Max	0.0943	0.0909	0.0908	0.0994	0.0985	0.0953	0.0995	0.0970
Mean	0.0538	0.0424	0.0566	0.0523	0.0455	0.0565	0.0537	0.0691
Std	0.0254	0.0270	0.0261	0.0305	0.0268	0.0272	0.0351	0.0237
Median	0.0565	0.0397	0.0601	0.0414	0.0452	0.0600	0.0592	0.0761

Table 9 Bias-corrected efficiency scores for $\alpha = 0.5$

Bank	2014	2015	2016	2017	2018	2019	2020	2021
1	0.7403(7)	0.8884(3)	0.8408(3)	0.9057(3)	0.8497(1)	0.9311(3)	0.7974(6)	0.9031(3)
2	0.819(4)	0.8263(5)	0.7953(5)	0.9006(4)	0.5903(7)	0.724(4)	0.939(3)	0.9536(1)
3	0.5666(11)	0.6001(11)	0.5747(11)	0.8176(6)	0.8024(3)	0.6396(6)	0.8331(5)	0.755(8)
4	0.3575(12)	0.4408(13)	0.4537(13)	0.4555(12)	0.3744(13)	0.964(1)	0.5016(11)	0.5002(12)
5	0.0207(15)	0.0172(15)	0.0658(14)	0.0684(14)	0.0573(15)	0.0496(15)	0.0516(15)	0.0075(15)
6	0.7795(5)	0.8937(1)	0.9451(1)	0.9704(1)	0.7787(4)	0.6083(8)	0.9408(2)	0.8943(4)
7	0.0649(14)	0.0974(14)	0.0268(15)	0.0099(15)	0.0904(14)	0.1224(14)	0.0733(14)	0.0909(14)
8	0.8911(3)	0.8261(6)	0.7872(6)	0.6867(7)	0.471(11)	0.2539(13)	0.2637(13)	0.2612(13)
9	0.9127(2)	0.856(4)	0.825(4)	0.8437(5)	0.8456(2)	0.9412(2)	0.9292(4)	0.8609(5)
10	0.9435(1)	0.891(2)	0.874(2)	0.9551(2)	0.7183(5)	0.6318(7)	0.9698(1)	0.9399(2)
11	0.372(13)	0.4603(12)	0.4664(12)	0.4025(13)	0.4665(12)	0.4495(11)	0.4951(12)	0.5222(10)
12	0.7613(6)	0.7381(7)	0.5823(9)	0.6249(9)	0.4723(10)	0.3574(12)	0.7491(7)	0.7841(7)
13	0.6141(10)	0.6149(10)	0.5787(10)	0.5818(11)	0.5758(9)	0.5775(10)	0.5055(10)	0.8363(6)
14	0.6224(9)	0.6299(9)	0.617(8)	0.5989(10)	0.5809(8)	0.5794(9)	0.5834(9)	0.5114(11)
15	0.6936(8)	0.7118(8)	0.693(7)	0.6464(8)	0.6333(6)	0.6709(5)	0.6223(8)	0.6162(9)
Min	0.0207	0.0172	0.0268	0.0099	0.0573	0.0496	0.0516	0.0075
Max	0.9435	0.8937	0.9451	0.9704	0.8497	0.9640	0.9698	0.9536
Mean	0.6106	0.6328	0.6084	0.6312	0.5538	0.5667	0.6170	0.6291
Std	0.2888	0.2772	0.2723	0.2977	0.2443	0.2801	0.3058	0.3091
Median	0.6936	0.7118	0.6170	0.6464	0.5809	0.6083	0.6223	0.7550

Numbers in parentheses show the ranks of the banks

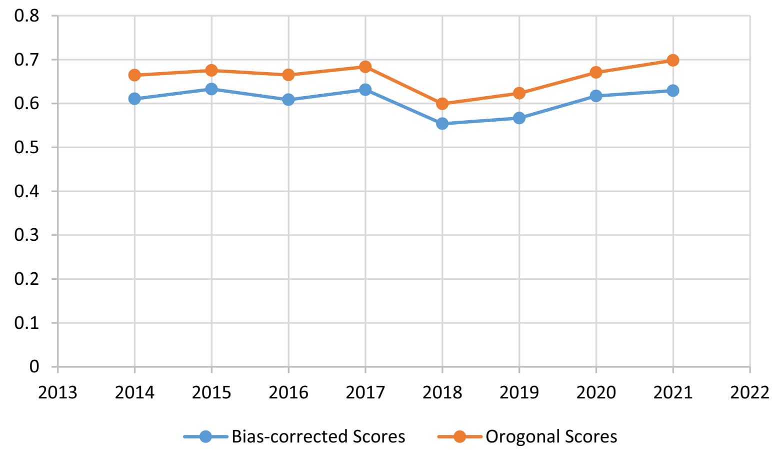

Fig. 4 Trends of the average bias and bias-corrected efficiency scores

Table 10 Pearson correlation coefficients

	Log θ	TCAR	NPLR	TASS	NBRNCH
Log θ		− 0.01309	− 0.111546	0.20756	0.012313
TCAR			0.310011	− 0.27033	− 0.26794
NPLR				− 0.51014	− 0.40821
TASS					0.911628
NBRNCH					

The effect of the contextual variables for $\alpha = 0.5$

After calculating the stochastic technical efficiency of the banks, we first paired all of the variables in this application (the log of technical efficiency and the explanatory variables) and then employed Pearson's correlation test to measure the relations between the different pairs of variables. The correlation coefficients are presented in Table 10. Based on the findings, there is a positive correlation between *Logθ* (the logarithm of technical efficiency), total assets, and the number of branches. However, the correlation between θ, the nonperforming loan ratio, and the total capital adequacy ratio is significantly negative. Meanwhile, the maximum correlation of *Logθ* is related to total assets.

At this point, we removed the impact of the contextual variables on bank efficiency in order to calculate the refined (bank-specific) efficiency scores and to determine the impact of the contextual variables on efficiency. We also examined the contextual variables that impact the technical efficiency of banks to remove their effects over which the banks have no control. In this regard, we focused on a deterministic case ($\alpha = 0.5$). Additionally, we assumed that the bias-corrected efficiency score is the dependent variable, while the four contextual variables (the total capital adequacy ratio, the nonperforming loan ratio, total assets, and the number of branches) are independent variables.

Table 11 Regression results on the contextual variables ($\alpha = 0.5$)

Contextual variable	Coefficient	Std	95% bootstrap confidence Interval	
			Lower	Upper
TCAR	− 0.0018	0.112103	− 0.098123	0.311325
NPLR	− 0.06818	0.013108	− 0.093181	0.043402
TA	1.03E−7	0.004411	0	0.000103
NB	− 0.00038	0.006813	− 0.000013	0.001393

The results of our regression Model (4) in the meta-frontier approach are listed in Table 11. Based on the findings, the nonperforming loan ratio variable is statistically significant. The positive values of the coefficients also indicate that there is a direct relationship between the efficiency value and the corresponding contextual variable, while the negative value shows that there is an inverse relationship between them. Moreover, there is a negative relationship between the nonperforming loan ratio and technical efficiency as well as a negative relationship between the total capital adequacy ratio and technical efficiency. Regarding the coefficient -0.06818 for the nonperforming loan ratio, it indicates that the change in the BCC technical efficiency associated with a one-unit increase in this variable is $100 \times \left(e^{-0.06818} - 1\right) = -6.59$. In other words, a one-unit increase in the nonperforming loan ratio results in a 6.59% reduction in productivity.

It is important to note that the majority of nonperforming loans are due to incorrect policies of the central bank. Meanwhile, banks are forced to accept borrowers introduced by upstream institutions (typically the government and the central bank), with many of these borrowers defaulting on their loans. Thus, since there are no strong control mechanisms, the majority of these loans are nonperforming. In this regard, banks should be more careful in identifying the creditworthiness of borrowers, regardless of the influence of the government and the central bank.

Furthermore, there is a negative relationship (-0.00038) between the number of branches and technical efficiency. This indicates that as the number of branches increases, the technical efficiency decreases. This may be due to the fact that an increase in the number of branches requires an increase in the number of staff (and related costs), which ultimately reduces technical efficiency. Therefore, bank managers should conduct research on developing and regulating the networks of each bank to deactivate inefficient banks in the system.

Productivity change of the banks

In this study, we used our proposed stochastic productivity measurement model to assess productivity change of the banks in our sample. For this purpose, we applied Models (5), (6), and (7). Specifically, we first used the stochastic input-oriented Model (5) to evaluate stochastic efficiency measures $SE_o(s,t)$, $SE_o(t,t)$, $SE_o(t,s)$, and $SE_o(s,s)$. In order to calculate the firm-specific technical efficiency and productivity index, we

Table 12 Statistical description of stochastic Malmquist productivity index (SM)

Alpha	Indicators	2014–2015	2015–2016	2016–2017	2017–2018	2018–2019	2019–2020	2020–2021
0.1	Min	0.7501	0.7257	0.7782	0.7360	0.1979	0.9239	0.3967
	Median	0.9717	0.9751	1.1152	0.9259	0.9810	1.3353	1.0976
	Max	1.5087	1.1227	2.7271	1.2416	2.4239	2.0255	1.6751
	Mean	0.9588	0.9686	1.2203	0.9466	0.9789	1.3999	1.0319
	Std	0.1789	0.1060	0.4328	0.1320	0.4538	0.2539	0.2892
0.3	Min	0.7727	0.7228	0.7813	0.7172	0.2081	0.9010	0.4113
	Median	0.9742	0.9848	1.1078	0.9354	0.9788	1.3299	1.0594
	Max	1.5098	1.1427	2.7470	1.2013	2.6389	1.8013	1.5980
	Mean	0.9829	0.9686	1.2278	0.9354	1.0061	1.3374	1.0159
	Std	0.1621	0.1043	0.4385	0.1380	0.4997	0.2285	0.2714
0.5	Min	0.7536	0.6476	0.9080	0.6237	0.2240	0.7956	0.4316
	Median	1.0327	0.9981	1.1283	0.9430	0.9940	1.2176	1.0358
	Max	1.1767	1.1492	1.5909	1.1708	2.8217	1.7291	1.4149
	Mean	1.0019	0.9668	1.1542	0.9272	1.0357	1.2569	0.9801
	Std	0.1123	0.1428	0.1966	0.1560	0.5476	0.2702	0.2395

removed the impact of the contextual variables on firm efficiency. By using the regression Model (6), the previously calculated measures were regressed on their period-related contextual variables, after which the refined measures were used in Model (7) to calculate the stochastic Malmquist productivity index. To better understand the calculation of $\overline{SE_o}(s,t)$, the detailed procedure and results for the first period (2014–2015) at a confidence level of $\alpha = 0.5$ is presented in "Appendix 3".

The results for the three different confidence levels ($\alpha = 0.1, 0.3, 0.5$) are listed in Table 12. Based on the findings, the minimum and maximum growth (at confidence levels of 0.1 and 0.3) occurred in 2018–2019 and 2016–2017, respectively. However, at a confidence level of 0.5, both of these minimum and maximum values occurred in 2018–2019.

Figure 4 presents the average stochastic productivity change for each period from 2014 to 2021. According to this figure, the stochastic TFP trend from 2014 to 2021 significantly fluctuated. Examples are as follows. First, at a confidence level of 0.1, it increased from 2014 to 2017, but decreased in 2017–2018. Then, it increased in 2019–2020, but decreased in 2020–2021. Despite such fluctuations, from 2014 to 2020, the overall stochastic TFP increased by 7.62%. Second, at a confidence level of 0.3, the TFP trend decreased in 2014–2015 and 2015–2016, but increased in 2015–2016. Then, it decreased in 2017–2018, but increased in 2018–2019 and 2019–2020. Again, despite such fluctuations, from 2014 to 2020, the overall stochastic TFP increased by 3.36%. Finally, the TFP trend at a confidence level of 0.5 was similar to that of $\alpha = 0.3$. Specifically, the TFP trend from 2014 to 2021 was 3.7%. An interesting finding is that at all three confidence levels, the TFP trend in 2016–2017 was at the maximum.

Table 13 Statistical description of stochastic technical change (STC)

Alpha	Indicators	2014–2015	2015–2016	2016–2017	2017–2018	2018–2019	2019–2020	2020–2021
0.1	Min	0.7436	0.7039	0.7508	0.7273	0.2001	1.3405	0.3989
	Median	0.9026	1.0455	1.0743	0.9124	0.9151	1.5918	1.0562
	Max	1.5289	1.1456	2.8250	1.1824	2.1276	2.2714	1.3487
	Mean	0.9519	0.9965	1.2875	0.9137	0.9182	1.6297	0.9998
	Std	0.1919	0.1159	0.4740	0.0983	0.3815	0.2448	0.2455
0.3	Min	0.7866	0.6996	0.7508	0.7258	0.2107	1.3072	0.4142
	Median	0.9335	1.0560	1.1289	0.9045	0.9050	1.5161	1.0436
	Max	1.5277	1.1159	2.8937	1.1745	2.2414	2.0305	1.3064
	Mean	0.9791	0.9978	1.3122	0.8988	0.9343	1.5594	0.9805
	Std	0.1728	0.1170	0.4928	0.1083	0.4014	0.1853	0.2260
0.5	Min	0.6576	0.5891	0.6483	0.6180	0.2219	0.5449	0.4360
	Median	0.9516	1.0519	1.1597	0.8983	0.9278	1.4801	1.0079
	Max	2.0794	1.1192	7.4377	1.1695	2.8283	2.2847	1.2924
	Mean	1.0415	1.0079	1.6229	0.8887	0.9894	1.4462	0.9427
	Std	0.3066	0.1310	1.5857	0.1249	0.5332	0.5274	0.2000

Table 14 Statistical description of stochastic efficiency change (SEC)

Alpha	Indicators	2014–2015	2015–2016	2016–2017	2017–2018	2018–2019	2019–2020	2020–2021
0.1	Min	0.9250	0.8832	0.6790	0.8528	0.9021	0.6378	0.9875
	Median	0.9962	0.9785	1.0241	1.0120	0.9973	0.8793	1.0055
	Max	1.1381	1.0826	1.1207	1.4938	1.5339	1.0291	1.2420
	Mean	1.0109	0.9745	0.9640	1.0403	1.0591	0.8600	1.0257
	Std	0.0520	0.0527	0.1263	0.1401	0.1599	0.0986	0.0606
0.3	Min	0.9228	0.8702	0.6310	0.8691	0.9202	0.6085	0.9752
	Median	0.9957	0.9798	1.0221	1.0133	0.9984	0.8763	1.0072
	Max	1.1097	1.0984	1.1280	1.4269	1.6133	1.0293	1.2836
	Mean	1.0064	0.9739	0.9553	1.0439	1.0657	0.8579	1.0299
	Std	0.0447	0.0585	0.1352	0.1284	0.1777	0.1076	0.0725
0.5	Min	0.5590	0.6244	0.1498	0.8197	0.7056	0.4074	0.9516
	Median	1.0887	0.9684	0.9612	1.0165	0.9977	0.8720	1.0091
	Max	1.3864	1.4419	1.4004	1.3501	2.1358	1.9014	1.3199
	Mean	1.0143	0.9791	0.9562	1.0452	1.0710	1.0058	1.0350
	Std	0.2206	0.2021	0.3222	0.1196	0.3076	0.4388	0.0828

Tables 13 and 14 present the results of the stochastic technical change (STC) and the stochastic efficiency change (SEC), respectively. The STC and SEC trends are also depicted in Figs. 4 and 5 in "Appendix 1", respectively. As for the STC, from 2014 to 2021, there was a 5% growth in productivity at a confidence level of 0.1. However, at a confidence level of 0.3, there was a growth of 0.1%. Meanwhile, at a confidence level of 0.5, there was a decrease of 6.6% in productivity. Similarly, at confidence levels of 0.1, 0.3, and 0.5, there was a 1.5%, 2.33%, and 2.35% growth in productivity in the SEC, respectively. This indicates that as the confidence level increases, the STC increases, whereas the SEC decreases.

An interesting finding is that at confidence levels of 0.1 and 0.3, the mean of the STC in 2019–2020 was at the maximum, whereas the mean SEC was at the minimum. The trends of the SEC as well the technological and productivity changes are depicted in Figs. 5, 6 and 7 in "Appendix 1". Based on the results of our regression analysis, the most statistically significant contextual variable in Iranian banks was the nonperforming loan ratio, followed by the total capital adequacy ratio.

Conclusion

In this study, we applied a two-stage double bootstrap DEA procedure to evaluate the technical efficiency of 15 Iranian banks from 2014 to 2021. In the first stage, the stochastic BCC model was used to calculate the technical efficiency of the banks. Then, in the second stage, ordinary least squares were used to determine the impact of the contextual variables on the efficiency scores obtained in the first stage. Subsequently, we developed a firm-specific, DEA-based Malmquist index model for a stochastic environment in which other factors, such as contextual and explanatory variables, can have a significant impact on the performance and productivity of firms.

In order to refine firm-specific relative efficiency and productivity growth, we first applied a two-stage procedure in which the first stage included the estimation of stochastic efficiency by using the stochastic BCC model and the second stage included the regression of estimated efficiency scores on the contextual variables by using ordinary least squares. Then, we applied the proposed theoretical framework to analyze the productivity growth of the sample of Iranian banks. Based on the 120 bank-year observations of 15 banks from 2014 to 2021, the explanatory variables (i.e., the nonperforming loan ratio and the number of branches) were negatively related to the stochastic technical efficiency of the banks. Moreover, from 2014 to 2021, there was a growth in productivity in terms of both technology and efficiency.

As a practical suggestion for improving the technical efficiency and productivity of banks, we recommend optimizing bank-specific inputs and outputs. Specifically, increasing the nominal interest rate and decreasing operational expenses may help improve the technical efficiency and productivity of banks. Furthermore, since the nonperforming loan ratio and the number of branches have a significant negative relationship with technical efficiency, we recommend decreasing the nonperforming loan ratio and the number of branches in order to the improve the efficiency and productivity of banks.

At this point, some suggestions for future research are as follows. First, the proposed productivity change measurement procedure can be developed for other types of uncertainty such as cardinal, fuzzy, and interval data. Second, in order to examine the stochastic technical efficiency of firms, it is important to compare the stochastic DEA model with a stochastic frontier model. Third, the proposed approach in this study can be used to compare the managerial ability of top managers over time. Finally, the proposed stochastic productivity index measurement model and two-stage procedure can be applied to analyze the productivity change of firms on a wider scale.

Appendix 1

See Figs. 5, 6 and 7.

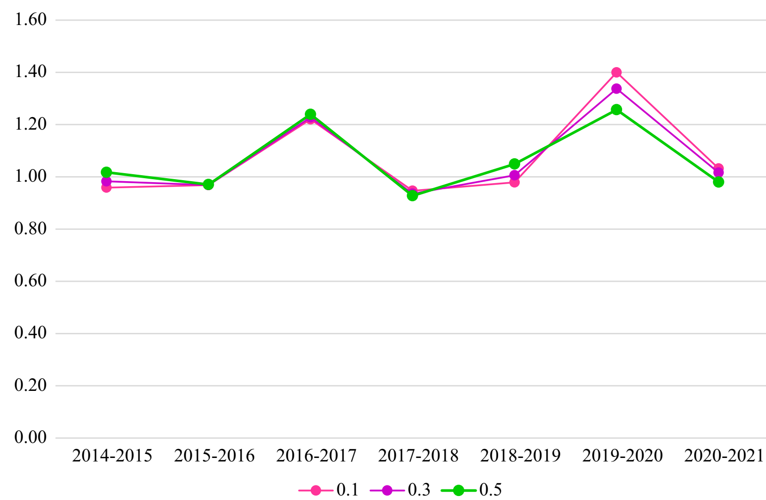

Fig. 5 Average of stochastic productivity changes

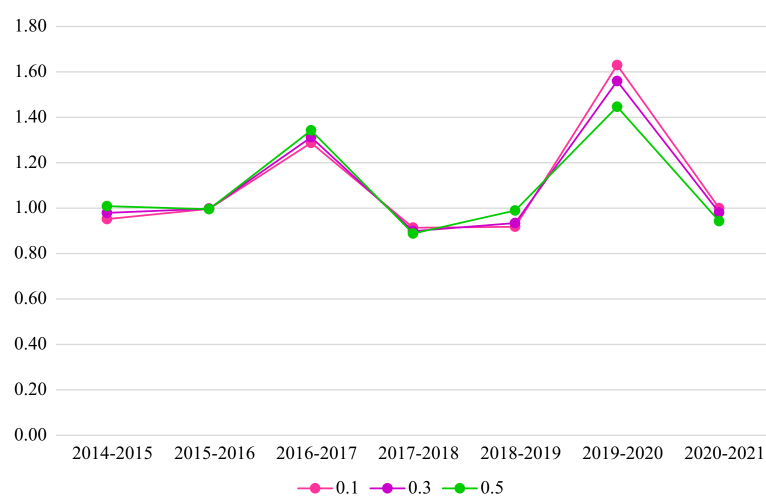

Fig. 6 Average of stochastic technical changes

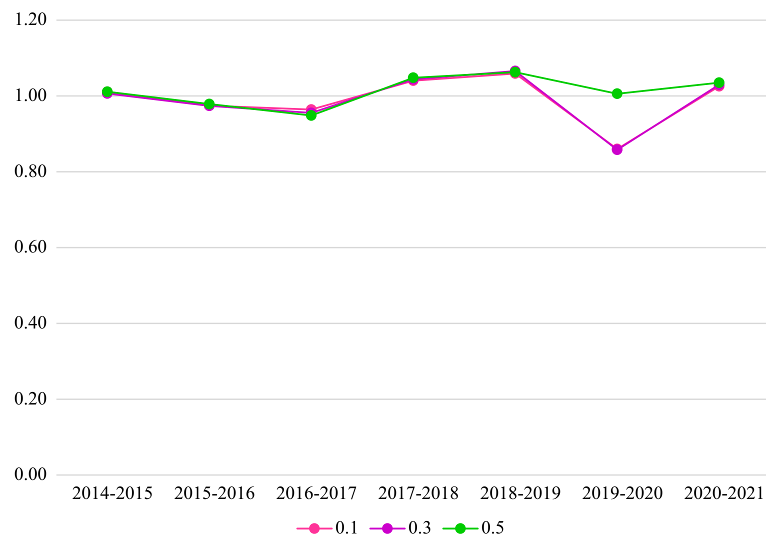

Fig. 7 Average of stochastic efficiency changes

Appendix 2

To show how Model 5 works, we now give the empirical form of this model. Suppose we have two DMUs with a single input and single output in two periods. The mean and STD of the data are given in the following two Tables 15 and 16.

Suppose DMU1 is under evaluation and we are interested in computing $SE_1(1,2)$ at confidence level $\alpha = 0.5$. Model (5) is written as follows:

Table 15 Mean of the input and output in two periods

	Period 1		Period 2	
	$x_1^{(1)}$	$y_1^{(1)}$	$x_1^{(2)}$	$y_1^{(2)}$
DMU1	2	3	1	2
DMU2	4	1	3	2

Table 16 STD of the input and output in two periods

	Period 1		Period 2	
	$x_1^{(1)}$	$y_1^{(1)}$	$x_1^{(2)}$	$y_1^{(2)}$
DMU1	0.2	0.3	0.1	0.2
DMU2	0.4	0.1	0.3	0.2

$$\begin{aligned}
&SE_o(1,2) = Min\theta \\
&\text{s.t.} \\
&\lambda_{12} + 3\lambda_{22} - \phi^{-1}(0.5)\left(p_1^+ - p_1^-\right) \leq 2\theta, \\
&0.1\lambda_{12} + 0.3\lambda_{22} - 0.2\theta = \left(p_1^+ + p_1^-\right), \\
&2\lambda_{12} + 2\lambda_{22} - \phi^{-1}(0.5)\left(q_1^+ + q_1^-\right) \geq 3, \\
&0.2\lambda_{12} + 0.2\lambda_{22} - 0.2 = \left(q_1^+ - q_1^-\right), \\
&\lambda_{12} + \lambda_{22} = 1, \\
&\lambda_{12}, \lambda_{22}, p_1^+, p_1^-, \ \ q_1^+, q_1^- \ \geq 0.
\end{aligned}$$

Similarly, $SE_o(2,1)$ is formulated as follows:

$$\begin{aligned}
&SE_o(1,2) = Min\theta \\
&\text{s.t.} \\
&2\lambda_{12} + 4\lambda_{22} - \phi^{-1}(0.5)\left(p_1^+ - p_1^-\right) \leq \theta, \\
&0.2\lambda_{12} + 0.4\lambda_{22} - 0.1\theta = \left(p_1^+ + p_1^-\right), \\
&3\lambda_{12} + \lambda_{22} - \phi^{-1}(0.5)\left(q_1^+ + q_1^-\right) \geq 2, \\
&0.3\lambda_{12} + 0.1\lambda_{22} - 0.2 = \left(q_1^+ - q_1^-\right), \\
&\lambda_{12} + \lambda_{22} = 1, \\
&\lambda_{12}, \lambda_{22}, p_1^+, p_1^-, \ \ q_1^+, q_1^- \ \geq 0.
\end{aligned}$$

We then applied GAMS software to solve our models.

Appendix 3

To better understan the process of calculating $\overline{SE_o}(1,1)$, the detailed procedure and results for the first period (2014–2015) at confidence level $\alpha = 0.5$ is given. Tables 17 and 18 give the inputs and outputs of the banks in two successive periods (2014 to 2015). The numerical values of the four contextual variables in two successive periods are given in

Table 17 Mean and std of inputs and outputs in 2014

	IE		OE		NIR		OR	
	Mean	Std	Mean	std	Mean	std	Mean	Std
1	1,817,822	45,800.06	150,948.1	41,131.63	1,319,232	43,645.27	2,123,132	35,294.81
2	1,606,116	35,651.15	64,386.12	31,782.72	1,233,526	21,096.37	1,343,218	22,745.91
3	890,012.7	36,079.01	316,013.5	30,410.57	663,879.2	33,624.22	1,020,246	25,273.76
4	631,195.9	32,295.27	568,944.6	28,526.83	202,196.2	19,840.48	694,750.3	21,490.02
5	253,019.9	42,416.48	176,865.5	36,548.05	116,781	29,061.7	208,356.2	30,711.24
6	31,291.24	16,149.4	30,193.43	27,430.36	148,321.8	38,144.01	265,172.7	19,793.55
7	121,514.1	25,910.36	137,239.7	22,941.92	177,851.2	11,455.57	154,423.3	13,105.11
8	41,009.28	29,324.76	11,861.19	10,978.17	149,829.1	26,869.98	144,002.8	18,519.52
9	14,411.78	11,033.48	11,151.32	9649.265	127,051.3	7112.177	179,853.9	9861.715
10	15,941.78	2299.053	10,921.12	9761.051	127,331.2	9935.75	110,013.2	11,585.29
11	37,671.14	24,265.59	28,005.39	22,397.16	137,899.3	11,810.81	190,983.1	13,460.35
12	36,306.11	17,707.87	13,804.82	8569.717	117,579.1	16,253.08	129,174.8	7902.619
13	19,857.42	18,038.77	34,108.41	15,970.33	128,865.4	10,283.98	129,803.3	9333.518
14	17,809.32	1804.099	307,959.9	17,172.56	124,781.1	10,386.2	121,087.1	7535.741
15	15,819.33	1097.835	28,659.31	14,088.26	121,329.8	36,301.91	120,110.5	9551.449

Table 18 Mean and std of inputs and outputs in 2015

	IE		OE		NIR		OR	
	Mean	Std	Mean	std		Mean	std	Mean
1	1,827,822	46,426.05	163,948.1	43,221.91	1,351,232	45,591.17	2,553,133	37,405.05
2	1,616,116	36,277.14	65,386.7	33,873	1,234,326	23,042.26	1,443,218	24,856.14
3	900,012.7	36,704.99	317,013.8	32,500.85	664,879.2	35,570.12	1,120,237	27,383.99
4	641,195.9	32,921.25	569,944.7	30,617.11	205,197.1	22,886.38	794,740.6	23,600.25
5	263,019.9	43,042.47	178,865.5	38,638.33	117,891	31,007.6	208,346.1	32,821.47
6	41,091.14	32,924.78	31,093.36	29,520.64	149,321.8	40,089.9	365,182.6	21,903.78
7	131,514.1	26,536.34	138,239.7	25,032.2	178,901.2	13,401.47	156,423.3	15,215.34
8	51,009.28	29,950.75	12,861.17	12,023.31	150,919.1	29,815.88	144,113	20,629.75
9	15,811.78	11,346.47	12,151.72	10,694.41	129,051.3	9058.073	180,053.9	10,871.95
10	16,041.78	11,808.26	11,921.72	10,806.19	128,431.3	11,881.65	110,053.2	13,695.52
11	38,671.14	24,891.58	29,005.39	24,487.44	138,909.7	13,756.7	200,983.1	15,570.58
12	37,306.11	18,333.85	14,804.9	9614.857	118,979.1	18,198.98	130,175	10,012.85
13	19,957.42	9332.376	35,108.41	18,060.61	129,965.6	13,061.92	130,003.5	9743.751
14	18,009.32	9333.487	31,999.9	19,262.84	125,981	12,332.1	122,087.2	8945.974
15	16,689.33	11,291.34	29,889.31	16,178.54	122,329.8	38,247.81	120,235.5	10,061.68

Table 19. We first applied Model 5 to calculate $SE_o(1,1)$, $SE_o(2,1)$, $SE_o(1,2)$ and $SE_o(2,2)$. The results are listed in Table 20.

Table 19 Contextual variables in two successive periods (2014 and 2015)

	2014				2015			
	NPLR	TCAR	TA	NB	NPLR	TCAR	TA	NB
1	13.48	0.44	112,919,756.41	3099	14.38	0.53	113,919,756.50	3103
2	14.25	0.32	84,875,973.23	2297	15.15	0.41	85,875,973.32	2301
3	15.81	0.74	66,084,600.81	1849	16.71	0.83	67,084,600.90	1853
4	19.62	1.41	27,391,242.43	1684	20.52	1.5	28,391,242.52	1688
5	14.85	0.56	19,009,781.50	1601	15.75	0.65	20,009,781.59	1605
6	19.31	2.93	11,874,512.71	1457	20.21	3.02	12,874,512.80	1461
7	15.99	1.92	6,565,485.67	1229	16.89	2.01	7,565,485.76	1233
8	27.42	4.35	5,276,504.52	1044	28.32	4.44	5,376,504.61	1048
9	19.08	0.93	2,923,209.15	1031	19.98	1.02	3,023,209.24	1035
10	19.09	1.8	3,089,804.69	784	19.99	1.89	3,189,804.78	788
11	73.64	1.78	1,914,307.12	709	74.54	1.87	2,014,307.21	713
12	23.24	1.74	1,682,139.09	562	24.14	1.83	1,782,139.18	566
13	17.61	1.99	1,509,843.13	387	18.51	2.08	1,609,843.22	391
14	15.2	2.04	14,990,876.24	365	16.1	2.13	15,090,876.33	369
15	21.55	1.54	1,199,890.57	339	22.45	1.63	1,259,890.66	343

Table 20 Different efficiency scores in 2014 to 2015 at $\alpha = 0.5$

	$SE_o(1,1)$	$SE_o(2,1)$	$SE_o(1,2)$	$SE_o(2,2)$
1	1	0.2025	0.9736	1
2	1	1.1544	1.0149	1
3	0.9101	0.9483	0.8923	0.8886
4	0.704	0.843	0.4915	0.6112
5	0.0945	0.0928	0.0845	0.0834
6	1	3.1018	1.2733	1
7	0.6102	0.5754	0.679	0.6389
8	1	0.9264	1.1185	1
9	1	1.0251	1.0971	1
10	1	0.9843	1.0916	1
11	0.6437	0.649	0.715	0.7282
12	0.7957	0.7422	0.8681	0.8097
13	0.7983	0.838	0.7963	0.8494
14	0.8092	0.8002	0.8878	0.878
15	0.911	0.8635	0.9995	0.9474

The results of our regression analysis are listed in the following:

2014 against 2014:

Regression Statistics	
Multiple R	0.593342
R Square	0.352055
Adjusted R Square	0.092877
Standard Error	0.245798
Observations	15

ANOVA	***Df***	***SS***	***MS***	***F***	***Significance F***
Regression	4	0.32827	0.082067	1.358352	0.315241
Residual	10	0.604169	0.060417		
Total	14	0.932438			

Abbreviations

DEA	Data envelopment analysis
DMU	Decision-making unit
TFP	Total factor productivity
BCC	Banker, Charnes and Cooper
SE	Stochastic efficiency
SM	Stochastic Malmquist
STC	Stochastic technical change
SEC	Stochastic efficiency change
Log	Logarithm
NPLR	Non-performing loan ratio
TCAR	Total capital adequacy ratio

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Author contributions
In this research, AA and TA wrote the body of the manuscript. MN wrote the codes and prepared the first draft of the paper. AA finalized the manuscript.

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Declarations

Competing interests
On behalf of my co-authors, I declare that we have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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