# The black hole interior from non-isometric codes protected by complexity

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July 18, 2022

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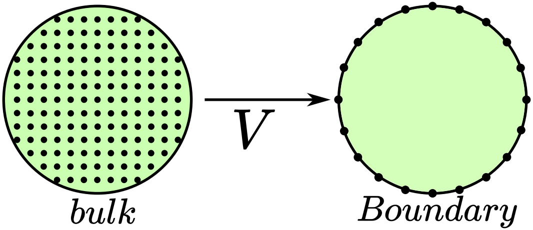


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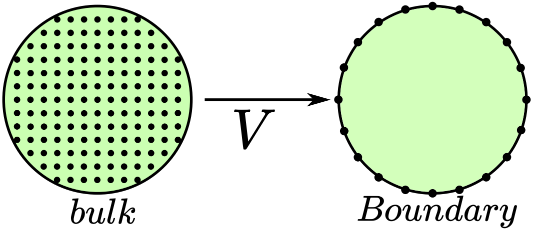


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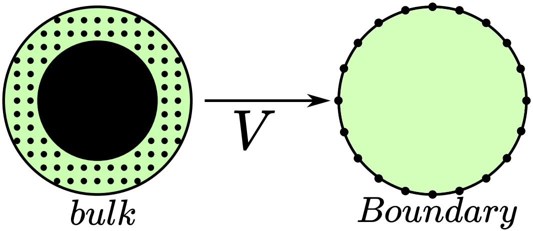
This idea can be extended to states with black holes, but with the encoding map now only acting on exterior degrees of freedom:



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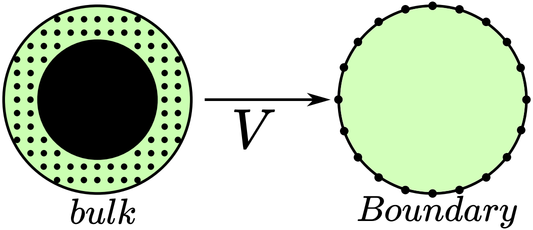


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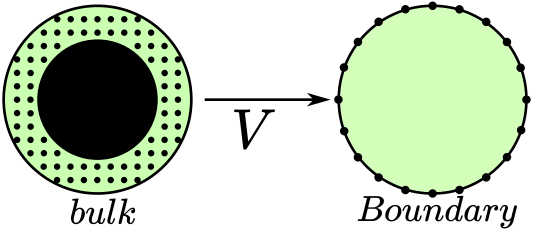


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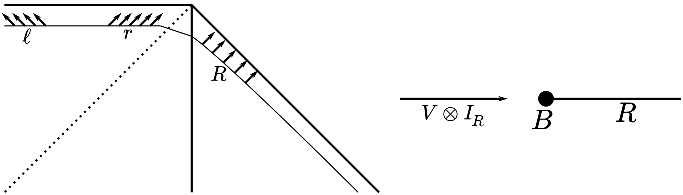
Today I will present a proposal for how it can be reconstructed, with applications to the black hole information problem.

Akers/Engelhardt/Harlow/Penington/Vardhan, 2022

# A holographic map for the BH interior?




We'd like a holographic map $V : \mathcal{H}_\ell \otimes \mathcal{H}_r \to \mathcal{H}_B$ mapping interior left and right moving modes $\ell, r$ to some microstate degrees of freedom $B$, and we'll introduce a "reservoir" (or "radiation") system $R$ into which the black hole can evaporate. The full encoding map is $V \otimes I_R$, since $R$ could be any auxiliary system.

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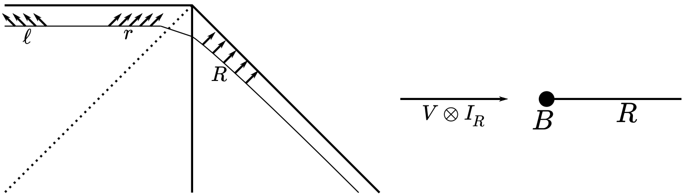


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We will refer to the left side of this diagram as the "effective description" and the right side as the "fundamental description".

- The basic problem is that as the black hole evaporates we eventually reach a situation where
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- This sounds dangerous, and we will indeed need to handle it carefully, but we will see that it also comes with a benefit: when $V$ is non-isometric it is not necessarily the case that

$$\mathrm{tr}_B \left( (V \otimes I_R) \rho_{\ell r R} (V^\dagger \otimes I_R) \right) = \rho_R,$$

since information about the interior can be teleported out into the radiation. We will see that this is the basic mechanism behind the quantum extremal surface calculations of the Page curve. Penington, Almheiri/Engelhardt/Marolf/Maxfield 2019

Our basic proposal is the following:

**There is a large set of "null states" in the Hilbert space of effective field theory inside a black hole, each of which is annihilated by the holographic map to the fundamental degrees of freedom. This however cannot be detected by any observer who does not perform an operation of exponential complexity.**

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- $V \otimes I_R$ preserves the overlaps of all states of sub-exponential complexity
- In sub-exponential states the entropy of $R$ can either be computed directly in the fundamental description or in the effective description using the QES formula Engelhardt/Wall 2014, with the same results up to $O(e^{-\gamma S_{BH}})$ for some $\gamma > 0$.

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- In sub-exponential states the entropy of $R$ can either be computed directly in the fundamental description or in the effective description using the QES formula Engelhardt/Wall 2014, with the same results up to $O(e^{-\gamma S_{BH}})$ for some $\gamma > 0$.
- Sub-exponential interior observables can be (non-linearly but unambiguously) reconstructed in the fundamental description, in agreement with the effective description up to $O(e^{-\gamma S_{BH}})$.

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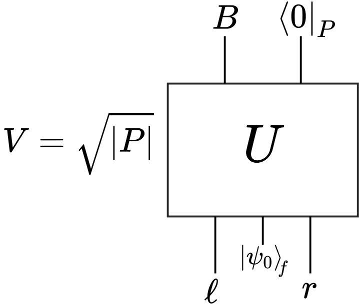
Thus (1), (2), and (3*) are not incompatible, and so within this model one can say that the information problem is resolved.

# Static model

Our basic model works likes this:

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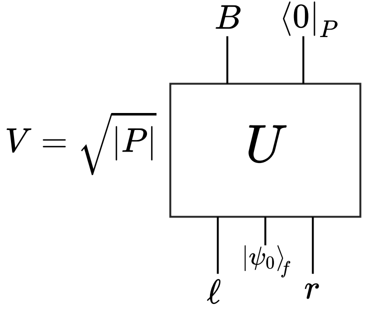


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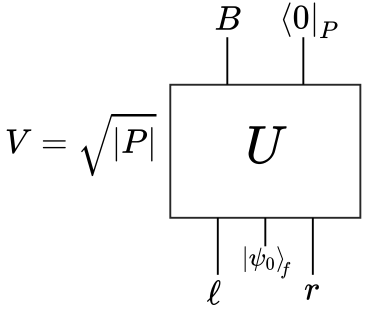


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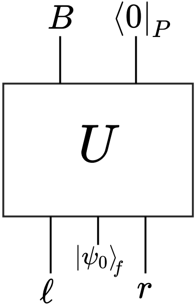
$V = \sqrt{|P|}$ 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$V$ is not necessarily an isometry due to the post-selection onto $\langle 0|_P$.
I emphasize that $U$ is drawn *only once*; we are not doing "fundamental averaging".
We will still use averaging over $U$ to learn about its typical features though.

# Overlap calculations

We can first observe that for any $|\psi_1\rangle, |\psi_2\rangle$ we have

$$\int dU \langle\psi_1|(V^\dagger V \otimes I_R)|\psi_2\rangle = \langle\psi_1|\psi_2\rangle$$

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Note in particular that the suppression of the fluctuations is exponential in $\log|B| \sim S_{BH}$, so this works up to $O(e^{-S_{BH}/2})$.

# An even simpler model

How is this possible? We can illustrate the basic mechanism using an even simpler model:

$$V_{phase}|n\rangle_r = \frac{1}{\sqrt{|B|}} \sum_b e^{i\theta(n,b)} |b\rangle_B .$$

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You can fit quite a large number of "nearly orthogonal" states into a Hilbert space, many more than the dimensionality would suggest!

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- The proof uses "measure concentration", which is a theory which produces results such as the following: for any $\kappa$-Lipschitz function $F : U(N) \to \mathbb{R}$ we have Meckes

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- Applying this formula to the problem at hand, for any $\alpha > 0$ we have

$$\Pr\left[\sup_{|\psi\rangle,|\phi\rangle\ \mathrm{sub-exp}} \Big|\langle\psi|V^\dagger V \otimes I_{LR}|\phi\rangle - \langle\psi|\phi\rangle\Big| > \sqrt{18}|B|^{-\gamma}\right]$$
$$\lesssim \exp\left(|B|^\alpha \log\log(|l||r|) - \frac{|B|^{1-2\gamma}}{24}\right).$$

# QES formula from the static model

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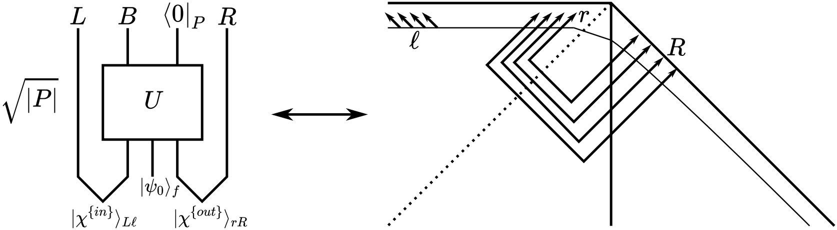


Let's compute the entropy of the radiation system $R$ in the encoded state

$$\Psi_{LBR}(U) \equiv (V \otimes I_{LR})|\psi_{\text{Hawk}}\rangle\langle\psi_{\text{Hawk}}|(V^\dagger \otimes I_{LR}).$$

We can compute the second Renyi entropy of $\Psi_R(U)$ using Haar technology: at large $|B|$ we have

$$\int dU e^{-S_2(\Psi_R(U))} \approx e^{-S_2\left(\chi_R^{\{out\}}\right)} + \frac{e^{-S_2\left(\chi_\ell^{\{in\}}\right)}}{|B|},$$

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The same result with $2 \to n$ turns out to hold, although the calculation is a bit more elaborate.

Taking $n \to 1$, the von Neumann entropy is thus

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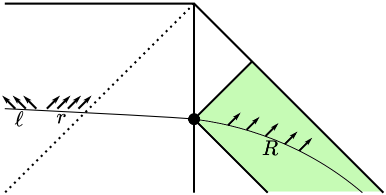


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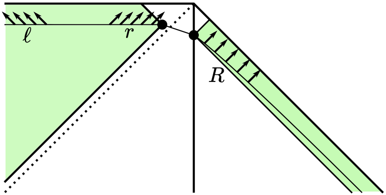


$S(\chi_R^{out}) \gg \log|B| + S(\chi_\ell^{in})$

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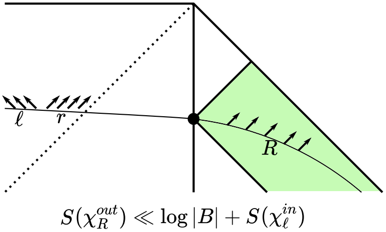


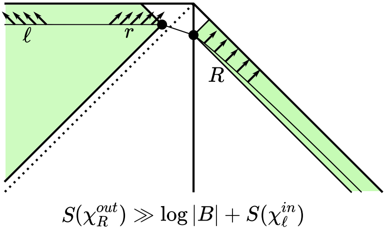


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Note however that we have *not* taken the QES formula as input or "derived" it from Euclidean gravity, we have instead obtained it as output from a "microscopic" calculation of the entropy in a non-isometric code.

# Reconstruction and measurement theory

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The key fact is the following: for any sub-exponential states $|\psi_1\rangle$ and $|\psi_2\rangle$, and for any $0 < \gamma < 1/2$, with high probability we have

$$\Big|\Big| |\psi_1\rangle - |\psi_2\rangle \Big|\Big| \leq \Big|\Big| (V \otimes I_{LR})(|\psi_1\rangle - |\psi_2\rangle) \Big|\Big| + |B|^{-\gamma}.$$

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On the other hand this inverse is *not* a linear operator, since the set of sub-exponential states does not form a subspace.

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- The non-linearity is reminiscent of the proposal of Papadodimas/Raju 2014, but the invertibility just mentioned avoids the ambiguities which arise there.
- This reconstruction is compatible with expectations from "entanglement wedge reconstruction", and in particular at late times we can reconstruct the interior using the radiation alone.

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- We would like to model the full dynamical process proposed by Hawking: collapse some matter to form a black hole, and then watch it evaporate and see if the process is unitary.

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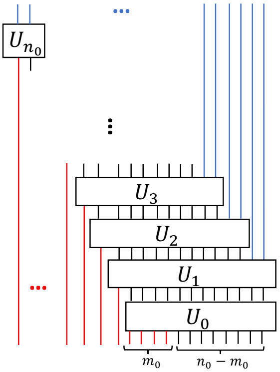


- We model the collapsing shell as $m_0$ qudits whose state we control and $n_0 - m_0$ qudits in a fixed state $|\psi_0\rangle_f$, with the collapse being implemented by a random unitary $U_0$.

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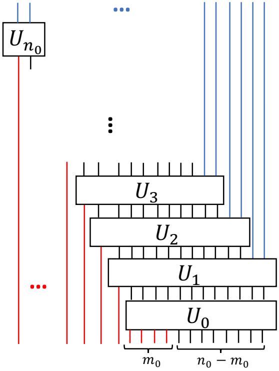


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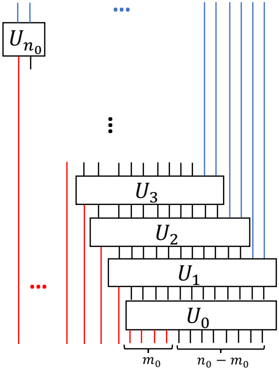


- We model the collapsing shell as $m_0$ qudits whose state we control and $n_0 - m_0$ qudits in a fixed state $|\psi_0\rangle_f$, with the collapse being implemented by a random unitary $U_0$.
- At each time $t$ we then act with a random unitary $U_{t+1}$ which absorbs one ingoing qudit from the reservoir and radiates two outgoing qudits.
- This model clearly has a finite BH entropy and a unitary S-matrix.

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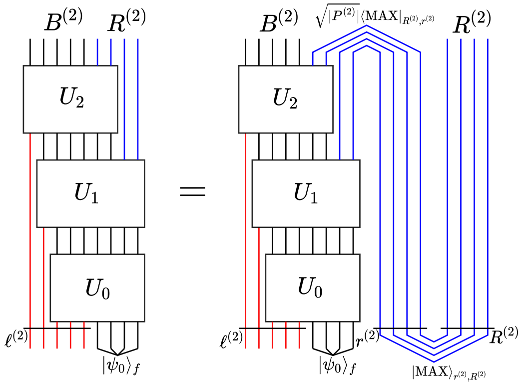
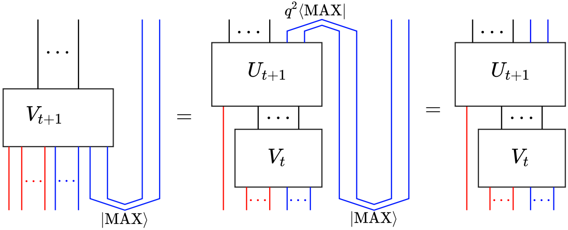



By "bending around" the outgoing modes using post-selection, we can re-interpret that fundamental dynamics as a non-isometric holographic map $V \otimes I_R$ acting on a (maximally-entangled) Hawking state in the effective description!

This map is easily shown to be equivariant:



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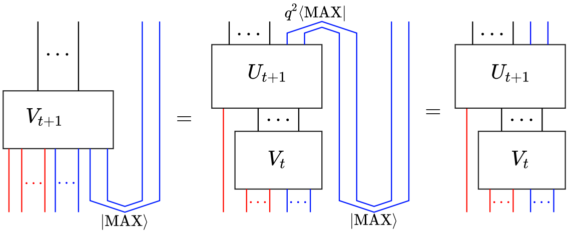


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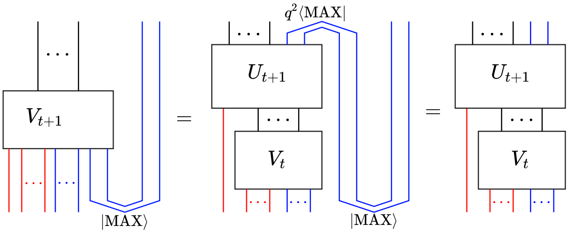


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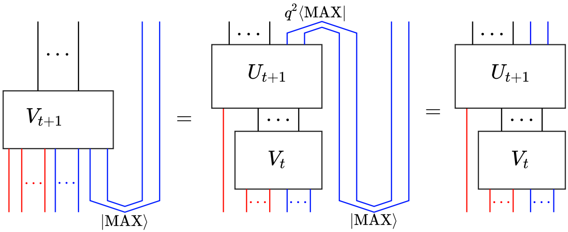


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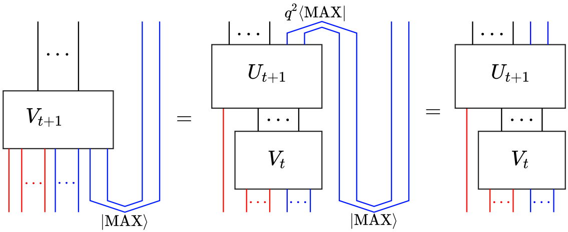


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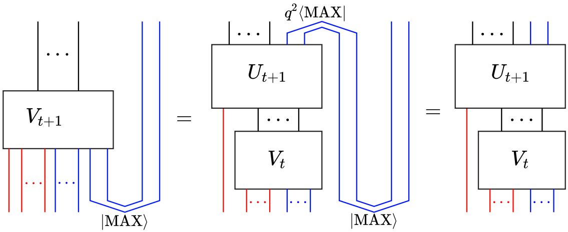


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The Hayden-Preskill scrambling argument can also be checked in a more refined version of the model where the $U_t$ are less random.

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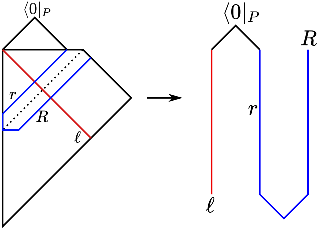
The mathematical mechanism for how information escapes here is the same as in the "black hole final state" proposal of Horowitz/Maldacena 2003.



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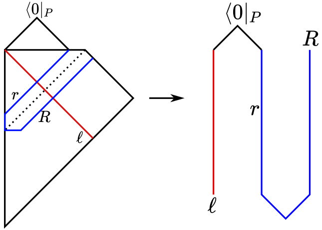


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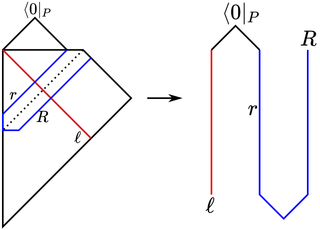


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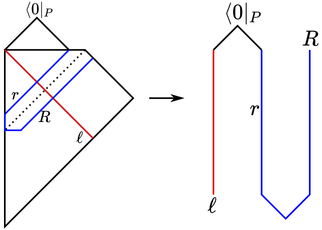


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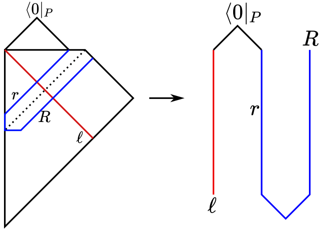


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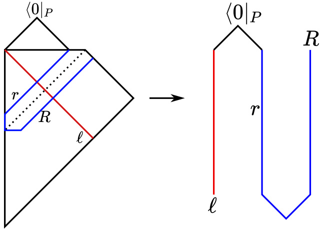


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- In HM post-selection is "at the singularity", while for us there is a post-selection in defining each $V_t$.
- The HM proposal is both non-linear and acausal, while our measurement theory is non-linear but perfectly causal.

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There is clearly lots more to think about, in particular we'd like to understand what this all means for cosmology!

Thanks for listening!