

Finnish High School Mathematical Contest 1997

Final Round

Varkaus, January 25, 1997

1. Find all numbers a for which the equation $a3^x + 3^{-x} = 3$ has a unique solution x .
2. Two circles of radii R and $r < R$ are externally tangent. Determine the maximal radius of a circle drawn in the domain bounded by the circles and their external common tangent.
3. Twelve knights sit at a round table. Every knight hates the two knights sitting next to him, but none of the other knights. A task group of five knights is to be sent to save a princess in trouble. No two knights who hate each other can be included in this group. In how many ways can the group be selected?
4. Find the sum of all four-digit numbers whose all digits are odd.
5. Given $n \geq 3$, find a configuration of n points in the plane such that the mutual distance of no pair of points exceeds 1 and exactly n pairs of points have a mutual distance equal to 1.