Promoting Mathematics[1]

Ronnie Brown

Why promote mathematics?

The following was an Introduction to a booklet produced at Bangor in World Mathematical Year 2000 as part of an EC Contract led by Mireille Chaleyat-Maurel, and which was distributed at the European Mathematical Congress, 2000, with a CDRom of John Robinson sculptures.

"Raising Public Awareness of Mathematics is probably the most important goal originally set for the World Mathematics Year 2000. And there are good reasons for that. The role of mathematics in society is subtle and not generally recognised in the needs of people in everyday life and most often it remains totally hidden in scientific and technological advancements. The old saying "The one who lives hidden lives best" is not true in present day society. If a subject becomes invisible, it may soon be forgotten and eventually it may even disappear. Mathematics has such a prominent place in school curricula all over the world that probably nobody can imagine such a fate for this subject. But if we do not constantly care about the image of mathematics, we will see continuing pressures to lower the amount of mathematics at primary schools, secondary schools and at the university level. Mathematics is exciting to many people but at the same time is considered difficult and somewhat inaccessible by many more. Since mathematics is the fundamental cornerstone in many diverse areas of society, it is important for civilisation as a whole that mathematicians do their utmost to help explaining and clarifying the role of mathematics."

Vagn Lundsgaard Hansen,
Chairman of the World Mathematics Year 2000
Committee of the European Mathematical Society.

Promoting mathematics to students?

In this article I discuss some roles of mathematics, and explore how trying to use the mathematics degree to promote mathematics to students might affect the emphasis of courses.

There are over 10,000 students doing honours degrees in mathematics in the UK. It would help to promote mathematics if they acquire at University the tools of language, background and preparation to act as ambassadors for the subject, to be able to argue the case for mathematics, and even for particular courses. They should be able to describe the role of a course, and to explain why mathematics is important not just for its applications, but for itself; and how the investigations by many, the development of technique, the following through of concepts, ideas and explanation in mathematics have led to the opening of new worlds and in the end to many applications. This means students being assessed not only on the technical skills which we expect from a mathematics degree, but also on having some idea of what judgement in mathematics entails, what is professionalism, what is the context. This is perhaps analogous to what in music is called 'musicality'.

In the UK there are 60 Royal Institution Mathematics Masterclasses with over 3,000 participants which get young people excited about mathematics, the ideas, free discussion, and co-operation rather than competition. Every effort is made by presenters to get over the key ideas, and if the children are bored, or unhappy, as evidenced by questionnaires, it is regarded as the fault of the presenter. The approach is non authoritative. Will those who, inspired by these courses, come to study mathematics for a degree expect these features to be continued, especially at a 'top' University?

Dr Brian Stewart of Oxford, an ex Chairman of the London Mathematical Society Education Committee, wrote to me: "The theme that interests me most is: 'how do we educate them while they are with us'. My own experience is that mathematicians do almost always discuss this in terms of *content*; how much can we pack in. (There's a 19th C Punch verse: Ram it in, Cram it in, Children's heads are hollow! which I like to quote.) This is very odd, because it doesn't match how we speak to each other about how we learn and develop new ideas/understandings."

An emphasis on content and assessment may lead to difficult and inaccessible courses, to 'mind-forg'd manacles' (Blake), from which some students emerge scarred, and which may give students the impression that the highest achievement for a mathematician is to write many neat answers to examination questions.

But what is such a training for? (see [12,10,4])

Training for research?

There is the old question: "How much do you need to know to do mathematical research?" and the old answer: "Everything, or nothing"!

Is the standard degree and assessment structure the best possible preparation for research which in any area requires independence and creativity, and the developing of expansive accounts of areas? Writing a thesis will

[1] This is an updated and modified version of an article published in MSOR Connections 7 no 2 July 2007, 21–25. Republished with permission.

surely involve finding and formulating problems, and solving some of them, but the key is determining the overall aims, and developing and evaluating the background to those aims, on a 'need to know' basis. Research students at Bangor have found that writing up this background has proved very helpful to the progress of their research.

Training for employment?

Most graduates go on to some form of employment. The 1974 McLone Report [12] wrote:

"A description of the employers' view of the average Mathematics graduate might be summarized thus: Good at solving problems, not so good at formulating them, the graduate has a reasonable knowledge of mathematical literature and technique; he has some ingenuity and is capable of seeking out further knowledge. On the other hand the graduate is not particularly good at planning his work, nor at making a critical evaluation of it when completed; and in any event he has to keep his work to himself as he has apparently little idea of how to communicate it to others."

Perhaps the situation has changed since 1974. Can the education and assessment in mathematics degrees be designed and to train in matters such as 'formulating problems'? 'communicating to others'? And then evaluated in quality assurance crtiteria?

On the other hand, one of our graduates went into a software firm, and I asked him when he visited us what course was most useful. To my surprise he said: 'Your course in analysis, as it gave me an idea of rigour.'

A Bangor PhD in Pure Mathematics, Keith Dakin, wrote to us in 1976 from Marconi: "We can get as many computer scientists as we want, but a mathematician who can see what mathematics is relevant to the problem at hand, is worth his weight in gold."

Are such skills developed in a mathematics degree? Compare [10].

Mathematics in context?

Conveying to students something about professionalism in mathematics was part of the idea behind a course in 'Mathematics in Context' at Bangor, where the frank discussions were stimulating and led to some great work, [5]. One student was a poor examinee, but wanted to be a teacher. He decided as a Maths in Context Project to write an assessment of the first year linear algebra course, comparing the syllabus, the lectures, the example classes, the text and the examinations! It was written in a very mature way, and he wrote: "Doing this project enabled me to come to terms with my own attitudes towards mathematics." I know many others have experimented similarly, particularly in the USA, and the debate on this needs extending, to tackle the questions raised in [4]. There is a tendency in the UK to evaluate the 'quality' of a degree course in terms of the proportion of top grade degrees; and it can then be that for the bottom 40% 'the devil take the hindmost'.

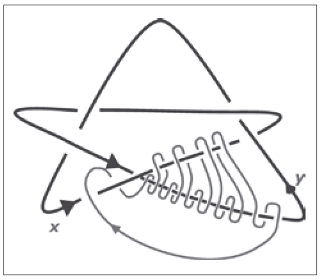
An old debating society tag is: "Text without context is merely pretext." What about "maths without context"?

My argument is that the dough of content needs leavening with the yeast of context to form digestible and flavoursome bread. The notion of 'context project' allows for a wide range of topics and a wide range of interest and abilities.

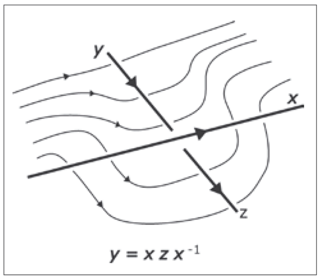
A relevant aspect is the experience of communication, in which the student wholly owns the result by determining the topic, the information, and the mode of presentation.

Communication of the essence of mathematical ideas

Graduates in mathematics will need to communicate their subject and ideas to lay people. Hassler Whitney remarked that a major problem with mathematics teaching is to make mathematical objects real. One of our demonstrations in the spirit of this and of 'advanced mathematics from an elementary viewpoint' relates to the general themes of: (i) local-to-global problems, (ii) non commutativity.



Looping the pentoil



Relation at a crossing

As shown in the picture on the left, we tie string onto a pentoil knot according to the left hand side of the rule

$$xyxyxy^{-1}x^{-1}y^{-1}x^{-1}y^{-1} = 1,$$

Amazingly, the string can be slipped off the knot, showing how algebra can structure space! The pentoil is defined by the global interrelationship of the five crossings, which are each local, at a place.

The above global rule is deduced from five relations, one at each crossing of the type shown on the right (for more details see [3]). (There is a bit of a cheat here since the mathematics of the fundamental group is about loops which can pass through each other, unlike the string!)

Notice here that if we had commutativity, i.e. $xy = yx$, the above formula degenerates. So to study knots we need a form of non commutative algebra.

We can see a further problem from this demonstration: how do you classify the ways of taking the loop off the knot? This 'higher dimensional problem' requires extending group theory as usually understood to a theory extending to all dimensions, not just dimension 1 as for loops, [3]. In some sense, this theory is 'more non-commutative' than group theory! The intuition for this is the start of what is called higher dimensional algebra, which combines many themes sketched in [1], namely 'commutative to non-commutative', 'local-to-global', 'higher dimensions', 'unifying mathematics', and could become a grand area of mathematics for the 21st century.

The above demonstration and others have been given to a wide range of audiences, including [7]. After that talk a senior neuroscientist came up to me and said: 'That was the first time I have heard a lecture by a mathematician which made any sense.' That this was 'the first time' illustrates the serious problems there are in promoting mathematics to other scientists! It also suggests that scientists in general are very interested in what new ideas and concepts have recently come into mathematcs.

Such demonstrations were developed over many years including for Royal Institution Mathematics Masterclasses for Young People in North West Wales, which have been running supported by Anglesey Aluminium since 1985.

Our exhibition 'Mathematics and Knots' was developed over four years for the 1989 Pop Maths Roadshow at Leeds University. In its development, it was very easy to say: 'I think we should use this picture or graphics', but then another member of the team will say: 'What are you trying to say about mathematics, and what is the relation of this picture to all the other pictures?' In this way we developed a philosophy of the exhibition, [6], and also learned a lot about presentation from four graphic designers!

So when one asks 'For whom does one promote mathematics?' a partial answer has to be: 'For oneself, as a mathematician and scientist.'

Applications of mathematics

A major role of mathematics is its wide range of applications. However there is a puzzlement, put to me by a young woman, Bree, at a party in Montana in 2004, where I was giving a seminar to the Department of Computational Biology. She wanted to know **why** mathematics has lots of applications. I replied: 'Mathematics is a developing language for expression, description, deduction, verification and calculation.' Bree seemed very happy with this.

Students, and Governments, need to appreciate what have been the contributions of mathematics to society and that some of these contributions are long term, unpredictable, possibly of tortuous route. Many advances have come from trying to improve understanding and exposition, following the lead of Euclid, Galileo, Euler, Faraday, Klein, Poincaré, Einstein, Hilbert, Feynman, and many others.

The urge to understand is often a motive for studying mathematics. Writing and rewriting mathematics to make things clear has been for me a stimulus to new ideas and approaches. If you write something out five times, you may see that it can be expressed a little differently, and then a bit more differently, and so on. You may even get the insight: What is really going on is …! This can be the glimmerings of a new concept. The rigour and aura of certainty in mathematics comes from the structure of interlocking concepts and proofs, each of which has been tested by thousands of people. Mathematics is by no means 'absolute truth' as some have suggested; that would not give room for development. Again, to suggest mathematics reduces to logic is like trying to describe a path to the station in terms of the paving stones. But the interlocking concepts give a landscape in which to find new paths, even repair old ones, and this landscape changes over time.

Often advances have been made by the less clever simply in order to make things clear. Indeed it is a lot of fun to take a standard subject and attempt to rewrite it in one's own, or someone else's, language or vision, formulating one's own problems. Whatever else, one will be sure to learn a lot.

My efforts to clarify to myself and obtain a nice exposition in the first edition of [2] led me to entirely new research areas, [3,7], though some high-ups have called all these 'nonsense' or 'completely irrelevant to the mainstream'. However, as Grothendieck wrote to me in 1982: 'The introduction of the cipher 0 or the group concept was general nonsense too, and mathematics was more or less stagnating for thousands of years because nobody was around to take such childish steps…', [11].

Mathematics and famous problems

An impression is sometimes given by mathematicians that the most important aspect is tackling famous problems, for example the 'million dollar problems'. Yet often their solutions, in the common view, or even in the view of scientific colleagues, are not of great moment. Mathematics is not only about doing difficult things, but also providing the framework to make difficult things easy (thus giving new opportunities for difficult tasks!).

Certainly such problems have been a challenge and have led to the development of great new methods in mathematics. Often this has involved developing notation and concepts which explain why things are true.

As G.-C. Rota writes in [13, p.48]:

"What can you prove with exterior algebra that you cannot prove without it?" Whenever you hear this question raised about some new piece of mathematics, be assured that you are likely to be in the presence of something important. In my time, I have heard it repeated for random variables, Laurent Schwartz' theory of distributions, ideles and Grothendieck's schemes, to mention only a few. A proper retort might be: "You are right. There is nothing in yesterday's mathematics that could not also be proved without it. Exterior algebra is not meant to prove old facts, it is meant to disclose a new world. Disclosing new worlds is as worthwhile a mathematical enterprise as proving old conjectures."

I met S. Ulam in Syracuse, Sicily, in1964, and he told me: 'A young person may think the most ambitious thing to do is to tackle some famous problem; but that may be a distraction from developing the mathematics most appropriate to him or her.' It was interesting to me that this should be said by someone as good as Ulam!

Mathematics and concepts

Mathematics deals with and defines concepts, by combining them into mathematical structures. These structures, these patterns, show the relations between concepts and their structural behaviour. The objects of study of mathematics are patterns and structures. These patterns and structures are abstract; the power of abstraction is that it allows for far reaching analogies.

An advantage to this theme is the democratisation of mathematics. There is a tendency in the social structure of mathematics to assume that the only interesting mathematics is that done by acknowledged geniuses, and the rest is a kind of 'fringe mathematics'. A (once!) young researcher told me the advice he was given by a top topologist was to look at what the top people do, and then find some little thing they have not done. This is the 'crumbs from the table' approach. But what young person with gumption wants to do that? In what way will that attract young people to the subject?

A related question is that of historiography, the 'history of history'. There is a considerable literature on this, and a lot on the historiography of science (the discussion in Wikipedia on these topics is informative). In contrast, the 'historiography of mathematics' is quite limited, and the 'history of mathematics' is largely concerned with the works of 'great men' (and a few women). Can this really give a fair assessment of the contributions to the progress of mathematics of the tens of thousands who have worked in the subject? What should be the methodology of making such an assessment?

For me the interest is not so much in problems already formulated, but in the ideas and intuitions, the aims, the concepts behind these formulations. It is these intuitions and concepts, the development of language, which fuel the next generation of discoveries and which, it can be argued, have been the major contribution of mathematics to science, technology and culture over the last two and a half millennia.

The theme of concepts is confirmed by the famous physicist, E. Wigner [14]

"Mathematics is the science of skilful operations with concepts and rules invented just for this purpose. [this purpose being the skilful operation …]
The principal emphasis is on the invention of concepts.
The depth of thought which goes into the formation of mathematical concepts is later justified by the skill with which these concepts are used."

Just some of the great concepts to which mathematics has given rigorous treatment are: number, length, area, volume, rate of change, randomness, proof, computation and computability, symmetry, motion, force, energy, curvature, space, continuity, infinity, deduction.

Often the route to solving problems is, in the words of a master of new concepts, Alexander Grothendieck, "to bring new concepts out of the dark", [11]. Is it possible to help students to see for themselves, in even a small way, how this comes about, and how concepts come to be invented both for applications and to disclose new worlds?

Conclusion

Einstein wrote in 1916 [9] with regard to the theory of knowledge:

"…the following questions must burningly interest me as a disciple of science: What goal will be reached by the science to which I am dedicating myself? To what extent are its general results 'true'? What is essential and what is based only on the accidents of development?… Concepts which have proved useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts. … … It is therefore not just an idle game to exercise our ability to analyse familiar concepts, and to <u>demonstrate the conditions on which their justification and usefulness depend, and the way in which these developed, little by little</u>. [my emphasis]"

I hope this article will provoke discussion, for example in the pages of the EMS Newsletter, of the challenge to departments to help to promote their subject through wide activities; and also by helping and encouraging students to explain and clarify to themselves and others the justification and usefulness of mathematics, as an important part of an education in mathematics.

Acknowledgements
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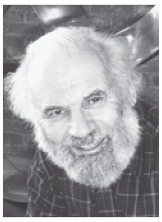

R. Brown [ronnie.profbrown@btinternet.com] was born in 1935. His undergraduate and postgraduate work was at Oxford, and he held University posts successively at Liverpool, Hull and Bangor, where he was appointed Professor of Pure Mathematics in 1970. He has currently 119 items on MathSciNet, and this has involved much national and international collaboration. One theme of his research since the 1970s has been the use of groupoids and higher dimensional versions of groupoids in mathematics, leading to his coining in 1987 the term 'higher dimensional algebra' for the study and application of algebraic systems with partial operations defined under geometric conditions. He has also written extensively on teaching and popularisation, and has given many presentations at various levels and to various audiences on knots and on the Symbolic Sculptures of John Robinson.

Clay Millenium Prize

The Clay Mathematics Institute (CMI) announces that

Dr. Grigoriy Perelman
(St. Petersburg, Russia)

is the recipient of the Millennium Prize for resolution of the Poincaré conjecture.

For more, see www.claymath.org/millennium/

International Centre for Mathematical Sciences (Edinburgh, UK)

New Scientific Director

The Board of ICMS is pleased to announce that

Professor Keith Ball

of University College London has been appointed to the post of Scientific Director of ICMS.

Later this year he will succeed John Toland who was appointed in 2002.

The 2010 Wolf Foundation Prize in Mathematics

The Prize Committee for Mathematics has unanimously decided that the 2010 Wolf Prize will be jointly awarded to:

Shing-Tung Yau (Harvard University, USA)
for his work in geometric analysis that has had a profound and dramatic impact on many areas of geometry and physics; together with

Dennis Sullivan (Stony Brook University and CUNY Graduate School and University Center, USA)
for his innovative contributions to algebraic topology and conformal dynamics.