Axiomathes (2007) 17:223–352
DOI 10.1007/s10516-007-9011-2

ORIGINAL PAPER

Categorical Ontology of Complex Spacetime Structures: The Emergence of Life and Human Consciousness

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Received: 16 June 2007 / Accepted: 25 June 2007 / Published online: 22 November 2007

Abstract A categorical ontology of space and time is presented for emergent biosystems, super-complex dynamics, evolution and human consciousness. Relational structures of organisms and the human mind are naturally represented in non-abelian categories and higher dimensional algebra. The ascent of man and other organisms through adaptation, evolution and social co-evolution is viewed in categorical terms as variable biogroupoid representations of evolving species. The unifying theme of local-to-global approaches to organismic development, evolution and human consciousness leads to novel patterns of relations that emerge in super- and ultra- complex systems in terms of colimits of biogroupoids, and more generally, as compositions of local procedures to be defined in terms of locally Lie groupoids. Solutions to such local-to-global problems in highly complex systems with 'broken symmetry' may be found with the help of generalized van Kampen theorems in algebraic topology such as the Higher Homotopy van Kampen theorem (HHvKT). Primordial organism structures are predicted from the simplest metabolic-repair systems extended to self-replication through autocatalytic reactions. The intrinsic dynamic 'asymmetry' of genetic networks in organismic development and evolution is investigated in terms of categories of many-valued, Łukasiewicz–Moisil logic algebras and then compared with those obtained for (non-commutative) quantum

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logics. The claim is defended in this essay that human consciousness is *unique* and should be viewed as an ultra-complex, global process of processes. The emergence of consciousness and its existence seem dependent upon an extremely complex structural and functional unit with an asymmetric network topology and connectivities—the human brain—that developed through societal co-evolution, elaborate language/symbolic communication and 'virtual', higher dimensional, non-commutative processes involving separate space and time perceptions. Philosophical theories of the mind are approached from the theory of levels and ultra-complexity viewpoints which throw new light on previous representational hypotheses and proposed semantic models in cognitive science. Anticipatory systems and complex causality at the top levels of reality are also discussed in the context of the ontological theory of levels with its complex/entangled/intertwined ramifications in psychology, sociology and ecology. The presence of strange attractors in modern society dynamics gives rise to very serious concerns for the future of mankind and the continued persistence of a multi-stable biosphere. A paradigm shift towards *non-commutative*, or non-Abelian, theories of highly complex dynamics is suggested to unfold now in physics, mathematics, life and cognitive sciences, thus leading to the realizations of higher dimensional algebras in neurosciences and psychology, as well as in human genomics, bioinformatics and interactomics.

Keywords Space, time, chronotopoids and spacetime, ST ·
ST in automata vs. quantum automata and organisms ·
Categorical ontology and the theory of levels · Relational biology principles ·
What is life and life's multiple logics, LM- and Q-logic ·
Organismic categories and relational patterns · Abelian vs. non-Abelian theories ·
Commutativity limitations in logics, mathematics, physics and emergent systems ·
Łukasiewicz–Moisil logic algebras of genetic networks and interactomes ·
Homo erectus, habilis and sapiens · *Australopithecus* and chimpanzees (*Pan*) ·
The emergence of hominins and hominoides · Cognitive science ·
Mental representations and intentionality ·
Brentano, Harman, Dennett, Field and Fodor's philosophy of the mind ·
Higher dimensional algebra of brain functions ·
Higher Homotopy-General Van Kampen Theorems (HHvKT) and Non-Abelian Algebraic Topology (NAAT) ·
Non-commutativity of diagrams and non-Abelian theories ·
Non-Abelian categorical ontology ·
Non-commutative topological invariants of complex dynamic state spaces ·
Double groupoids and quantum double groupoids ·
Natural transformations in molecular and relational biology ·
Molecular class variables (mcv) ·
Natural transformations and the Yoneda-Grothendieck Lemma/construction ·
The Primordial MR and *Archea* unicellular organisms ·
Evolution and memory evolutive systems (MES) ·
The Thalamocortical model, categorical limits, colimits and MES ·
Biogroupoids ·
Variable groupoids, variable categories, variable topology and atlas structures ·

Irreversibility and open systems · Selective boundaries vs. horizons ·
Universal temporality · Occam'razor and reductionist approaches ·
Super-complex systems and brain dynamics ·
Global and local aspects of biological evolution in terms of colimits of variable biogroupoids, or as chains and compositions of local procedures (COLPs) of locally Lie groupoids in the evolution and co-evolution of species ·
What is consciousness and synaesthesia? ·
The human mind, human consciousness and brain dynamics viewed as non-Abelian ultra-complex processes ·
Emergence of human consciousness through co-evolution/social interactions and symbolic communication ·
Rosetta biogroupoids as models of human social interactions ·
Objectivation and memes · Anticipation and feedforward ·
Systems of internal representations, propositional attitudes and sentence-analogs ·
Tarskian compositional semantics ·
Moral duality and strange attractors of modern society dynamics

1 Introduction

The overall time scale of our Universe is on the order of 20 billion years—the estimated age of the known Universe—with the last 4 billion years on Earth including the evolution of living organisms and species. In this essay we shall focus on the latter 4 billion years, the emergence of life, the ascent of man, the emergence of consciousness, and the dynamic links between the biological, mental and social levels of reality. The 'known' Universe space scale estimates are still increasing, and the Universe itself expands with time, totalling some 20 billion light years, or 6.5×10^{22} km, or metric units of space.

As the intended readers are both philosophers and scientists interested in Ontology—in part philosophy and hopefully, also science—we do not lay claim to 'solve' any major ontological problem in this essay, such as the question of existence of an essence for every ontological item, or indeed how highly complex systems, processes or 'items', in general, have come into existence. Neither do we aim to provide a complete (ontological) taxonomy of highly complex systems which would take far more space and time than available to us here, if it were at all possible to accomplish it rigorously. Instead, we are enquiring here if new methodological tools may be brought to bear, and indeed further developed, with the hope of being able to help one understand the spacetime ontology, and indeed highly complex dynamics, of both 'natural systems'—such as various biosystems or organisms, the human brain, the mind and society. This 'methodological' ontology task of providing the 'right kind of tools' is indeed monumental in itself, and one may be surprised at first that it has not been seriously attempted before throughout ages; then, it may be that we are now at a stage of our development in philosophy and science that such tools are beginning to emerge or are indeed in the process of being developed by the 'exact sciences', including logics, mathematics, physics, genetics, molecular biology, relational biology, etc. as a result of trends towards unity in logics, mathematics, physics, biology and psychology. Having divided to conquer their corresponding ontological systems, one might be inclined to consider them merely as fields of knowledge, or 'epistemology' without direct relevance to the essential levels of 'objective reality', if that indeed exists. Surprisingly enough, there seem to be many philosophers who consider life itself as an 'epiphenomenon', or some kind of delusion if not a human illusion. Without adequate tools, we feel that one cannot either defend or reject such extreme claims, either to the 'right or left' of 'epiphenomena' or epistemology of highly complex systems and the human mind without falling into the error of being both 'outgunned' and/or 'outlived', as philosophical arguments based on words and Boolean logic alone seem to be in many instances misguided, very lengthy, and often not even wrong. The challenges that one must face, we feel, are so great that one cannot accept on an a priori (e.g., Platonic, Aristotelian, Kantian, Wittgensteinian, etc.) basis, theses, axioms or even assumptions that may not be decidable 'rationally' (whatever that means)—without choosing first one's 'weapons to do battle' with any deep ontological, or in general, philosophical problems. Perhaps, such was also Newton's injunction : "I don't make any hypotheses" (even though he made several).

Such is one's predicament in deciding to be rigorous especially when approaching highly complex dynamic problems, or even the ontological question of spacetime itself. As 'great Will' said: "To be or not to be, that is the question…" Indeed, as the very existence of human society may now depend on understanding highly complex systems and the mind, one cannot engage such difficult and deep philosophical/ontological problems without adequate tools or 'conceptual weapons to do battle with complexity'; this is because such highly complex problems cannot be 'reduced' in any conceivable manner to simplicity (as sometimes incorrectly claimed), or even to many simpler subproblems. We shall indeed endeavor to show on a strictly methodological/rigorous basis that problems of truly complex systems require new tools far more potent than language, rather than beginning with mere speculation, 'self-evident truths', or some such intended short-cuts that are often some philosophers' strategy invoking the claim that "words" will suffice for any philosopher, and especially to one interested in the philosophy of science. It is only the latter claim that we reject or simply do not choose, pragmatically from the outset, not because it seems 'self-evident' to us and also to most scientists, but because we would like to make as few assumptions as one can for a rigorous foundation of ontology, and indeed philosophy of science; it may be that such tools that we consider might have value not only to the sciences of complexity and ontology but, more generally also, to all philosophers seriously interested in keeping on the rigorous side of the fence in their arguments. In the not too distant past, similar methodological and indeed theoretical arguments may have been made for the theory of sets, rather promptly disposed of by Whitehead, Bertrand Russell and many others, followed in a different vein by Zermelo, Carnap, Tarski, Chomsky, Turing, von Neumann, Gödel, Eilenberg, Mac Lane, Charles Ehresmann, Lawvere and Alexander Grothendieck to mention just the most prominent ones who contributed, besides the 'anonymous Nicolas Bourbaki', and also Wittgenstein who found fault with *set-like problems* in philosophers' language; the latter had problems especially when language is used to carry on certain deep philosophical arguments—that Wittgenstein himself has used in his two published, monumental/philosophical treatments that he afterwards labeled as "non-sense". His point is well taken—there seems to be a long-standing, methodological problem in philosophy related to language, and perhaps to all other extensions or forms of 'languages' that are set-theoretical in their essence. Consistent with this standpoint, we use throughout this essay the attribute 'categorial' for philosophical-linguistic arguments only; on the other hand, the rigorous term *'categorical'* shall be utilized only in conjunction with applications of concepts and results from the general, mathematical *Theory of Categories, Functors and Natural Transformations* (TC-FNT) whose conceptual framework is concisely presented in the accompanying paper by Brown, Glazebrook and Baianu (2007, in this volume) in this issue along with current, fundamental concepts and results from algebraic topology.

Thus, the authors aim in this original report at a self-contained, but concise, presentation of novel methodologies for studying the difficult, as well as the controversial, ontological problem of space and time in complex, super-complex and ultra-complex systems, ranging from biological organisms to societies, but excluding computer-simulated systems that are recursively computable. As stated above, we neither claim nor offer solutions to complexity problems but only provide both a conceptual framework and advanced methodology for studying complexity that we think is both internally consistent and open to further developments.

This introduction and the first four sections will provide the essential concepts and also define the approach required for a self-contained presentation of the subsequent six sections. The underlying, precise conceptual framework for the proposed categorical ontology of spacetime structures is stated explicitly in the fourth paper in this issue (Brown, Glazebrook and Baianu 2007, in this volume).

Therefore, the reader is not here required to have either a mathematical or physical background, although a background in biology, neuroscience and especially in logics or psychology/cognitive sciences might be helpful for a critical evaluation and understanding of the fundamental problems in the space/time categorical ontology of (super) complex systems, such as Life, the functional human brain, living organisms, and also the ultra-complex human mind and societies.

Our essay also introduces a novel higher-dimensional algebra approach to space/time ontology that is uniquely characteristic to the human brain and the mind. The human brain is perhaps one of the most complex systems—a part of the human organism which has evolved earlier than 2 million years ago forming a separate species from those of earlier *hominins/hominides*. Linked to this apparently unique evolutionary step—the evolution of the *H. sapiens* species—human consciousness emerged and co-evolved through *social* interactions, elaborate *speech*, *symbolic communication/language* somewhere between the last 2.2 million and 60,000 years ago. The term *ultra-complexity level* is here proposed to stand for the mind, or the mental level, that is a certain dynamic pattern of layered processes emerging to the most complex level of reality based upon super-complex activities and higher-level processes in special, super-complex systems of the human brain coupled through certain synergistic and/or mimetic interactions in human societies. In this sense, we are proposing a non-reductionist, categorical ontology that possesses both universal attributes and a top level of complexity encompassed by the human consciousness, *v. infra*—Sects. 2, and 10 to 14.

The focus in this essay is therefore on the emergence of highly complex systems as categorical, universal and dynamic patterns/structures in spacetime, followed by the even more complex—and also harder to understand, or precisely represent—the emergence of the *unique* human consciousness. The claim is defended here that the emergence of ultra-complexity requires the occurrence of '*symmetry breaking*' at several levels of underlying organization, thus leading to the *asymmetry* of the human brain—both functional and anatomical; such recurring symmetry breaking may also require a sharp complexity increase in our representations of mathematical-relational structure of the human brain and also human consciousness. Arguably, such repeated symmetry breaking does result in *layered complexity dynamic patterns* (Baianu and Poli 2008; Poli 2006c) in the human mind that appear to be organized in a hierarchical manner. Thus, 'conscious planes' and the focus of attention in the human mind are linked to an emergent *context-dependent variable topology* of the human brain, which is most evident during the brain's developmental stages guided by environmental stimuli such as human/social interactions; the earliest stages of a child's brain development would be thus greatly influenced by its mother.

The human mind is then represented for the first time in this essay as an *ultra-complex* 'system of processes' based on, *but not necessarily reducible to*, the human brain's highly complex activities enabling and entailing the emergence of mind's own consciousness; thus, an attempt is made here to both define and represent in categorical ontology terms the human consciousness as an *emergent/global, ultra-complex process* of mental activities as distinct from—but correlated with—a multitude of integrated local super-complex processes that occur in the human brain. Following a more detailed analysis, the claim is defended that the human mind is more like a '*multiverse with a horizon, or horizons*' rather than merely a '*super-complex system with a finite boundary*'. The mind has thus freed itself of the real constraints of spacetime by separating, and also 'evading', through virtual constructs the concepts of time and space that are being divided in order to be conquered by the human free will. Among such powerful, 'virtual' constructs of the human mind(s) are: symbolic representations, the infinity concept, continuity, evolution, multi-dimensional spaces, universal objects, mathematical categories and abstract structures of relations among relations, to still higher dimensions, many-valued logics, local-to-global procedures, colimits/limits, Fourier transforms, and so on, it would appear without end.

On the other hand, alternative, Eastern philosophical ontology approaches are not based on a duality of concepts such as: mind and body, system and environment, objective and subjective, etc. In this essay, we shall follow the Western philosophy 'tradition' and recognize such dual concepts as essentially distinctive items. The possible impact of Eastern philosophies on psychological theories—alongside the Western philosophy of the mind—is then also considered in the concluding sections.

We shall also consider briefly how the space and time concepts evolved, resulting in the joint concept of an objective *'spacetime'* in the physical Relativity theory, in spite of the distinct, (human) perception of space and time dimensions. Then, we shall proceed to define the role(s) played by the space, time and spacetime concepts in the broader context(s) of categorical ontology.

Ontology has acquired over time several meanings and has been approached in many different ways, however mostly connected to the concept of an *'objective existence'*; we shall consider here the noun 'existence' as a basic, or primitive, concept not definable in more fundamental terms. The attribute 'objective' will be assumed with the same meaning as in *'objective reality'*, and reality is understood as whatever has an existence which can be rationally or empirically verified independently by human observers in a manner which is neither arbitrary nor counter-factual. Furthermore, the *meaningful classification of items* that belong to such an 'objective reality' is one of the major tasks of ontology.

Here, we are in harmony with the theme and approach of the ontological theory of levels of reality (Poli 1998, 2001a,b) by considering categorical models of complex systems in terms of an evolutionary dynamic viewpoint. Thus our main descriptive mechanism involves the mathematical techniques of category theory which afford describing the characteristics and binding of levels, besides the links with other theories. Whereas Hartmann (1952) stratified levels in terms of the four frameworks: physical, 'organic'/biological, mental and spiritual, we restrict mainly to the first three. The categorical techniques which we introduce provide a means of describing levels in both a linear and interwoven fashion thus leading to the necessary bill of fare: emergence, complexity and open non-equilibrium/irreversible systems. Furthermore, as shown by Baianu and Poli (2008), an effective approach to Philosophical Ontology is concerned with universal items assembled in categories of objects and relations, transformations and/or processes in general. Thus, Categorical Ontology is fundamentally dependent upon both space and time considerations. The formation, changes and indeed the evolution of the key concepts of space, time and spacetime will be therefore considered first in Sect. 2. Basic aspects of Categorical Ontology are then introduced in Sect. 3, whereas precise formal definitions are relegated to the Brown, Glazebrook and Baianu (2007, in this volume) paper in order to maintain an uninterrupted flow of discourse.

Our viewpoint is that models constructed from category theory and higher dimensional algebra have potential applications towards creating a higher science of analogies which, in a descriptive sense, is capable of mapping imaginative subjectivity beyond conventional relations of complex systems. Of these, one may strongly consider a *generalized chronoidal-topos* notion that transcends the concepts of spatial–temporal geometry by incorporating *non-commutative multi-valued logic*. Current trends in the fundamentally new areas of quantum-gravity theories appear to endorse taking such a direction. We aim further to discuss some prerequisite algebraic-topological and categorical ontology tools for this endeavor, again relegating all rigorous mathematical definitions to the Brown, Glazebrook and Baianu (2007, in this volume) paper.

It is interesting that Abelian categorical ontology (ACO) is also acquiring several new meanings and practical usefulness in the recent literature related to computer-aided (ontic/ontologic) classification, as in the case of: neural network categorical ontology (Baianu 1972; Ehresmann and Vanbremeersch 1987; Healy and Caudell

2006), genetic ontology, biological ontology, environmental representations by categories and functors (Levich and Solovy'ov 1999), or ultra-complex societies.

We propose a dynamic classification of systems at different levels of reality, beginning with the physical levels (including the fundamental quantum level) and continuing in an increasing order of complexity to the chemical/molecular levels, and then higher, towards the biological, psychological, societal and environmental levels. Indeed, it is the principal tenet in the theory of levels that "*there is a two-way interaction between social and mental systems that impinges upon the material realm for which the latter is the bearer of both*" (Poli 2001a, b).

The evolution in our universe is thus seen to proceed from the level of 'elementary' quantum 'wave–particles', their interactions via quantized fields (photons, bosons, gluons, etc.), also including the quantum gravitation level, towards aggregates or categories of increasing complexity. In this sense, the classical macroscopic systems are defined as 'simple' dynamical systems, computable recursively as numerical solutions of mathematical systems of either ordinary or partial differential equations. Underlying such mathematical systems is always the Boolean, or crysippian, logic, namely, the logic of sets, Venn diagrams, digital computers and perhaps automatic reflex movements/motor actions of animals. The simple dynamical systems are always recursively computable, and in a certain specific sense, both degenerate and *non-generic*, consequently also *structurally unstable* to small perturbations. The next higher order of systems is then exemplified by 'systems with chaotic dynamics' that are conventionally called 'complex' by physicists and computer scientists/modelers even though such physical, dynamical systems are still completely deterministic. It can be formally proven that such systems are *recursively non-computable* (see for example, Baianu (1987a, b) for a 2-page, rigorous mathematical proof and relevant references), and therefore they cannot be completely and correctly simulated by digital computers, even though some are often expressed mathematically in terms of iterated maps or algorithmic-style formulas. In Sect. 9 we proceed to introduce the next higher-level systems above the chaotic ones, which we shall call *Super-Complex Biological systems* (SCBS, or 'organisms'), followed at still higher levels by the *Utra-Complex 'systems'* (UCS) of the human mind and human societies that will be discussed in the last two sections. With an increasing level of complexity generated through billions of years of evolution in the beginning, followed by millions of years for the ascent of man (and perhaps 10,000 more years for human societies and their civilizations), there is an increasing degree of *genericity* for the dynamic states of the evolving systems (Thom 1980; Rosen 2001). The evolution to the next higher order of complexity- the ultra-complex 'system' of processes—the human mind—may have become possible, and indeed accelerated, only through human societal interactions and effective, elaborate/rational and symbolic communication through speech (rather than screech—as in the case of chimpanzees, gorillas.

The most important claim defended here is, however, that the *ultra-complex* process of processes called *human consciousness* involves fundamentally *asymmetric* structures and their corresponding, recursively non-computable dynamics/ psychological processes and functions. Such *non-commutative* dynamic patterns of structure–function are therefore represented by a Higher Dimensional Algebra of neurons, neuronal (both intra- and inter-) signaling pathways, and especially high-level psychological processes viewed as *non-computable patterns* of linked-super-aggregate processes of processes,..., of still further sub-processes. The latter, at the biochemical/molecular-quantum level, are likely to include quantum 'machines' or quantum automata (Baianu 1971a, b), such as related to essential quantum-tunnelling enzymes and certain RNAs that are known to exist, and that are implicated in biochemical/quantum signalling pathways in the human brain.

Moreover, as it will be shown in Sect. 7, any complete Categorical Ontology theory is *a fortiori non-Abelian*, and thus recursively non-computable, on account of both the quantum level (which is generally accepted as being non-commutative), and the top ontological level of the human mind—which also operates in a non-commutative manner, albeit with a different, *multi-valued* logic than Quantum Logic. To sum it up, the operating/operational logics at both the top and the fundamental levels are *non-commutative* (the 'invisible' actor (s) who—behind the visible scene—make(s) both the action and play possible!). At the fundamental level, spacetime events occur according to a quantum logic (QL), or *Q-logic*, whereas at the top level of human consciousness, a different, non-commutative Higher Dimensional Logic Algebra prevails, in a manner akin to the many-valued (Łukasiewicz–Moisil, or LM) logics of genetic networks which were shown previously to exhibit non-linear, and also non-commutative/non-computable, biodynamics (Baianu 1977, 1987a, b; Baianu et al. 2006b).

2 The Fundamental Concepts of Space, Time and SpaceTime. Observers and Reference Frames. The Paradigm Shifts of Expanding Universe and Contingent Universes

Whereas Newton, Riemann, Einstein, Weyl, Hawking, Penrose, Weinberg and many other exceptionally creative theoreticians regarded physical space as represented by a *continuum*, there is an increasing number of proponents for a *discrete, 'quantized'* structure of space–time, since space itself is considered as discrete on the Planck scale. Like most radical theories, the latter view carries its own set of problems. The biggest problem arises from the fact that any discrete, 'point-set' (or discrete topology), view of physical space–time is not only in immediate conflict with Einstein's General Relativity representation of space–time as a *continuous Riemann* space, but it also conflicts with the fundamental impossibility of carrying out quantum measurements that would localize precisely either quantum events or masses at singular (in the sense of disconnected, or isolated), sharply defined, geometric points in space–time.

Let us mention some attempts at this problem. Differential structures in a non-commutative setting are replaced by such objects as quantized differential forms, Fredholm modules and *quantum groups* (Connes 1994; Majid 1995, 2002). Again, since GR breaks down at the Planck scale, space–time may no longer be describable by a smooth manifold structure. While not neglecting the large scale classical model, one may propose the structure of 'ideal observations' as manifest in a limit, in some sense, of 'discrete', or at least separable, measurements, where such a limit encompasses the classical event. Then the latter is represented as a *'point'* which is not influenced by quantum interference; nevertheless, the idea is to admit *coherent* quantum superpositions of events. Thus, at the quantum level, the events can decohere (in the large-scale limit) to the classical events, somewhat in accordance with the correspondence principle. Algebraic developments of the Sorkin (1991) model can be seen in Raptis and Zapatrin (2000), and quantum causal sets were considered in Raptis (2000). A main framework is Abstract Differential Geometry (ADG) which employs sheaf–theoretic methods enabling one to avoid point-based smooth manifolds, unusual gyroscopic frames and the chimera of 'classical, mathematical singularities' (see for instance Mallios and Raptis 2003).

Another proposed resolution of the problem is through *non-commutative* Geometry (NCG), or 'Quantum Geometry', where QST has 'no points', in the sense of visualizing a 'geometrical space' as some kind of a distributive and non-commutative lattice of space–time 'points'. The quantum 'metric' of QST in NCG would be related to a certain, fundamental quantum field operator, or to a 'fundamental triplet (or quintet)' construction (Connes 1994).

2.1 Current Status of Quantum Theory vs. General Relativity. The Changing Roles of the Observer. Irreversibility and Microscopic Entropy

A notable feature of current 21-st century physical thought involves questioning the validity of the classical model of space–time as a 4-dimensional manifold equipped with a Lorentz metric. The extension of the earlier approaches to quantum gravity (QG) was to cope with microscopic length scales where a traditional manifold structure (in the conventional sense) needs to be forsaken (for instance, at the Planck length $L_p = (\frac{G\hbar}{c^3})^{\frac{1}{2}} \approx 10^{-35}$m). On the other hand, one needs to reconcile the *discrete versus continuum* views of space–time diffeomorphisms with the possibility that space–time may be suitably modeled as some type of 'combinatorial space' such as a simplicial complex, a poset, or a spin foam (i.e., *a cluster of spin networks*). The monumental difficulty is that to the present day, apart from a dire absence of experimental evidence, there is no general consensus on the actual nature of the data necessary, or the actual conceptual framework required for obtaining the data in the first place. This difficulty equates with how one can gear the approach to QG to run the gauntlet of conceptual problems from non-Abelian Quantum Field Theory (NA-QFT) to General Relativity (GR).

2.1.1 Deterministic Time–Reversible vs. Probabilistic Time–Irreversibility and Its Laws. Unitary or General Transformations?

A significant part of the scientific-philosophical work of Ilya Prigogine (see e.g., Prigogine 1980) has been devoted to the dynamical meaning of *irreversibility* expressed in terms of the second law of thermodynamics. For systems with strong enough instability the concept of phase space trajectories is no longer meaningful and the dynamical description has to be replaced by the motion of distribution functions on the phase space. The viewpoint is that quantum theory produces a more coherent type of motion than in the classical setting and the quantum effects induce correlations between neighbouring classical trajectories in phase space (which can be compared with the Bohr–Sommerfeld postulate of the image of phase cells having area $\hbar$). The idea of Prigogine (1980) is to associate a macroscopic entropy (or Lyapounov function) with a microscopic entropy super-operator. Here the time-parametrized distribution functions ρ_t are regarded as densities in phase space such that the inner product $\langle \rho_t, M\rho_t \rangle$ varies monotonously with t as the functions ρ_t evolve in accordance with Liouville's equation

$$\imath \frac{\partial \rho_t}{\partial t} = L\rho_t, \tag{2.1}$$

where L denotes *the Liouville (super) operator* (Prigogine 1980; Misra et al. 1979). In order to show that there are well defined systems for which the super-operators M exist, a time operator T ('age' or 'internal time') is introduced such that we have the 'uncertainty' relation

$$\imath[L, T] = \imath(LT - TL) = I. \tag{2.2}$$

The super-operators M may then be obtained as monotone positive operator functions of T, and under certain conditions may engender similarity transformations $\Lambda = M^{\frac{1}{2}}$ which convert the original deterministic evolution described by the Liouville equation into the stochastic evolution of a certain Markov process, and in this way the second law of thermodynamics can be expressed via the M super-operators (Misra et al. 1979). Furthermore, the equations of motion with randomness on the microscopic level then emerge as irreversibility on the macroscopic level. Unlike the usual quantum operators representing observables, the super-operators are non-Hermitian operators.

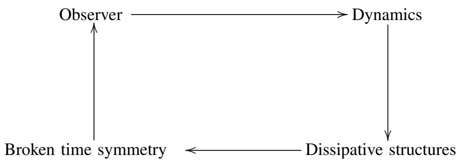
One also notes the possibility of 'contingent universes' with this 'probabilistic time' paradigm.

Now the requirement that the super-operator M increases monotonically with time is given by the following relation with the Hamiltonian of the system

$$\imath[H, M] = D \geq 0, \tag{2.3}$$

where D denotes an *(micro)-entropy operator* whose measurement is compatible with M, which implies the further (commutativity) equation $[M, D] = 0$. However, there are certain provisions which have to be made in terms of the spectrum of the Hamiltonian H: if H has a pure point spectrum, then M does not exist, and likewise, if H has a continuous but bounded spectrum then M cannot exist. Thus, the super-operator M cannot exist in the case of only finitely extended systems containing only a finite number of particles. Furthermore, M does not admit a factorization in terms of self-adjoint operators A_1, A_2, or in other words $M\rho \neq A_1\rho A_2$.Thus the super-operator M cannot preserve the class of 'pure states' since it is non-factorizable. The distinction between pure states (represented by vectors in a Hilbert space) and mixed states (represented by density operators) is thus lost in the process of measurement. In other words, the distinction between pure and mixed states is lost in a quantum system for which the algebra of observables can be extended to include a new dynamical variable representing the non-equilibrium entropy. In this way, one may formulate the second law of thermodynamics in terms of M for quantum mechanical systems.

Let us mention that the time super-operator T represents 'internal time' and the usual, 'secondary' time in quantum dynamics is regarded as an average over T. When T reduces to a trivial operator the usual concept of time is recovered $T\rho(x, v, t) = t\rho(x, v, t)$, and thus time in the usual sense is conceived as an average of the individual times as registered by the observer. Given the latter's ability to distinguish between future and past, a self-consistent scheme may be summarized in the following diagram (Prigogine 1980):



(2.4)

for which 'irreversibility' occurs as the intermediary stage in the following sequence

$$\text{Dynamics} \Longrightarrow \text{Irreversibility} \Longrightarrow \text{Dissipative structures}$$

Note however, that certain quantum theorists, as well as Einstein, regarded irreversibility of time as an 'illusion'. Others—operating with minimal representations in quantum logic for finite quantum systems—go further still by denying that there is any need for real time to appear in the formulation of quantum theory.

2.2 Quantum Fields, General Relativity and Symmetries

As the experimental findings in high-energy physics—coupled with theoretical studies—have revealed the presence of new fields and symmetries, there appeared the need in modern physics to develop systematic procedures for generalizing space–time and Quantum State Space (QSS) representations in order to reflect these new concepts.

In the General Relativity (GR) formulation, the local structure of space–time, characterized by its various tensors (of energy–momentum, torsion, curvature, etc.), incorporates the gravitational fields surrounding various masses. In Einstein's own representation, the physical space–time of GR has the structure of a Riemannian R^4 space over large distances, although the detailed local structure of space–time—as Einstein perceived it—is likely to be significantly different.

On the other hand, there is a growing consensus in theoretical physics that a valid theory of Quantum Gravity requires a much deeper understanding of the small(est)-scale structure of Quantum Space–Time (QST) than currently developed. In Einstein's GR theory and his subsequent attempts at developing a unified field theory (as in the space concept advocated by Leibnitz), space–time does *not* have an *independent existence* from objects, matter or fields, but is instead an entity generated by the *continuous* transformations of fields. Hence, the continuous nature of space–time was adopted in GR and Einstein's subsequent field theoretical developments. Furthermore, the quantum, or 'quantized', versions of space–time, QST, are operationally defined through local quantum measurements in general reference frames that are prescribed by GR theory. Such a definition is therefore subject to the postulates of both GR theory and the axioms of Local Quantum Physics. We must emphasize, however, that this is *not* the usual definition of position and time observables in 'standard' QM. The general reference frame positioning in QST is itself subject to the Heisenberg uncertainty principle, and therefore it acquires through quantum measurements, a certain 'fuzziness' at the Planck scale which is intrinsic to all microphysical quantum systems.

2.3 Measurement Theories

2.3.1 Measurements and Phase–Space

We have already mentioned the issue of quantum measurement and now we offer a sketch of the background to its origins and where it may lead. Firstly, the question of measurement in quantum mechanics (QM) and quantum field theory (QFT) has flourished for about 75 years. The intellectual stakes have been dramatically high, and the problem rattled the development of 20th (and 21st) century physics at the foundations. Up to 1955, Bohr's Copenhagen school dominated the terms and practice of quantum mechanics having reached (partially) eye-to-eye with Heisenberg on empirical grounds, although not the case with Einstein who was firmly opposed on grounds on incompleteness with respect to physical reality. Even to the present day, the hard philosophy of this school is respected throughout most of theoretical physics. On the other hand, post 1955, the measurement problem adopted a new lease of life when von Neumann's beautifully formulated QM in the mathematically rigorous context of Hilbert spaces of states. As Birkhoff and von Neumann (1936) remark:

> There is one concept which quantum theory shares alike with classical mechanics and classical electrodynamics. This is the concept of a mathematical "phase–space". According to this concept, any physical system $\mathfrak{C}$ is at each instant hypothetically associated with a "point" in a fixed phase–space Σ; this point is supposed to represent mathematically, the "state" of $\mathfrak{C}$, and the "state" of $\mathfrak{C}$ is supposed to be ascertained by "maximal" observations.

In this respect, *pure states* are considered as maximal amounts of information about the system, such as in standard representations using *position–momenta* coordinates (Dalla Chiara et al. 2004).

The concept of 'measurement' has been argued to involve the influence of the Schrödinger equation for time evolution of the wave function ψ, so leading to the notion of entanglement of states and the indeterministic reduction of the wave packet. Once ψ is determined it is possible to compute the probability of measurable outcomes, at the same time modifying ψ relative to the probabilities of outcomes and observations eventually causes its collapse. The well-known paradox of Schrödinger's cat and the Einstein–Podolsky–Rosen (EPR) experiment are questions mooted once dependence on reduction of the wave packet is jettisoned, but then other interesting paradoxes have shown their faces. Consequently, QM opened the door to other interpretations such as 'the hidden variables' and the Everett–Wheeler assigned measurement within different worlds, theories not without their respective shortcomings. In recent years some countenance has been shown towards Cramer's 'advanced–retarded waves' transactional formulation (Cramer 1980) where $\psi\psi^*$ corresponds to a probability that a wave transaction has been finalized ('the quantum handshake').

Let us now turn to another facet of quantum measurement. Note firstly that QFT pure states resist description in terms of field configurations since the former are not always physically interpretable. Algebraic quantum field theory (AQFT) as expounded by Roberts (2004) points to various questions raised by considering theories of (unbounded) operator-valued distributions and nets of von Neumann algebras. Using in part a gauge theoretic approach, the idea is to regard two field theories as equivalent when their associated nets of observables are isomorphic. More specifically, AQFT considers taking (additive) nets of field algebras $\mathcal{O} \longrightarrow \mathcal{F}(\mathcal{O})$ over subsets of Minkowski space, which among other properties, enjoy Bose–Fermi commutation relations. Although at first glances there may be analogues with sheaf theory, these analogues are severely limited. The typical net does not give rise to a presheaf because the relevant morphisms are in reverse. Closer then is to regard a net as a precosheaf, but then the additivity does not allow proceeding to a cosheaf structure. This may reflect upon some incompatibility of AQFT with those aspects of quantum gravity (QG) where for example sheaf–theoretic/topos approaches are advocated (as in e.g., Butterfield and Isham 1998, 1999, 2000–2002).

2.3.2 The Kochen-Specker (KS) Theorem

Arm-in-arm with the measurement problem goes a problem of 'the right logic', for quantum mechanical/complex biological systems and quantum gravity. It is well-known that classical Boolean truth-valued logics are patently inadequate for quantum theory. Logical theories founded on projections and self-adjoint operators on Hilbert space H do run into certain problems. One 'no–go' theorem is that of *Kochen-Specker* (KS) which for dim $H > 2$, does not permit an evaluation (global) on a Boolean system of 'truth values'. In Butterfield and Isham (1998, 1999, 2000–2002), self-adjoint operators on H with purely discrete spectrum were considered. The KS theorem is then interpreted as saying that a particular presheaf does not admit a global section. Partial valuations corresponding to local sections of this presheaf are introduced, and then generalized evaluations are defined. The latter enjoy the structure of a Heyting algebra and so comprise an intuitionistic logic. Truth values are describable in terms of sieve-valued maps, and the *generalized evaluations* are identified as *subobjects in a topos*. The further relationship with interval valuations motivates associating to the presheaf a von Neumann algebra where the supports of states on the algebra determines this relationship.

The above considerations lead directly to the next subsections which proceeds from linking quantum measurements with *Quantum Logics*, and then to the *construction* of spacetime structures on the basis of Quantum Algebra/Algebraic Quantum Field Theory (AQFT) concepts; such constructions of QST representations as those presented in Sects. 4 and 9 of Baianu et al. (2007) are based on the existing QA, AQFT and Algebraic Topology concepts, as well as several new QAT concepts that are being developed in this paper. For the QSS detailed properties, and also the rigorous proofs of such properties, the reader is referred to the recent book by Alfsen and Schultz 2003). We utilized in Sects. 6 and 9 of *loc.cit*. a significant amount of recently developed results in Algebraic Topology (AT), such as for example, the *Higher Homotopy van Kampen theorem* (see the relevant subsection in the Brown, Glazebrook and Baianu (2007, in this volume). for further mathematical details) to illustrate how constructions of QSS and QST, *non-Abelian* representations can be either generalized or extended on the basis of *GvKT*. We also employ the categorical form of the *CW–complex Approximation* (CWA) theorem) to both systematically construct such generalized representations of quantum space–time and provide, together with GvKT, the principal methods for determining the general form of the fundamental *algebraic invariants* of their *local or global*, topological structures. The algebraic invariant of Quantum Loop (such as, the graviton) Topology in QST is defined in Sect. 5 as the *Quantum Fundamental Groupoid (QFG)* of QST which can be then calculated—at least in principle—with the help of AT fundamental theorems, such as *GvKT*, especially for the relevant case of spacetime representations in *non-commutative* algebraic topology.

Several competing, tentative but promising, frameworks were recently proposed in terms of categories and the 'standard' topos for Quantum, Classical and Relativistic observation processes. These represent important steps towards developing a Unified Theory of Quantum Gravity, especially in the context-dependent measurement approach to Quantum Gravity (Isham 1998, 1999, 2000–2002; Isham and Butterfield 1999). The possibility of a unified theory of measurement was suggested in the context of both classical, Newtonian systems and quantum gravity (Isham 1998; Isham and Butterfield 1999; Butterfield and Isham 1999). From this standpoint, Butterfield and Isham (2001) proposed to utilize the concept of 'standard' topos (Mac Lane and Moerdijk 1992) for further development of an unified measurement theory and quantum gravity (see also, Butterfield and Isham 1999 for the broader aspects of this approach). Previous and current approaches to quantum gravity in terms of categories and higher dimensional algebra (especially, 2-categories) by Baez (2001) and Baez and Dolan (1995) should also be mentioned in this context. Furthermore, time—as in Minkowski 'spacetime'—is not included in this mathematical concept of "most general space" and, therefore, from the beginning such quantum gravity theories appear to be heavily skewed in favor of the quantum aspects, at the expense of time as considered in the space–time of general relativity theory.

The first choice of logic in such a general framework for quantum gravity and context-dependent measurement theories was intuitionistic related to the set-theoretic and presheaf constructions utilized for a context-dependent valuation theory (Isham 1998, 1999, 2000–2002). The attraction, of course, comes from the fact that a topos is arguably a very general, mathematical model of a 'generalized space' that involves an intuitionistic logic algebra in the form of a special distributive lattice called a *Heyting Logic Algebra*, as discussed Sect. 3.3.1.

2.3.3 The Basic Principle of Quantization

At the microscopic/indeterministic level certain physical quantities assume only discrete values. The means of quantization describes the passage from a classical to an associated quantum theory where, at the probabilistic level, Bayesian rules are replaced by theorems on the composition of amplitudes. The classical situation is considered as 'commutative': one considers a pair (A, Π) where typically A is a commutative algebra of a class of continuous functions on some topological space and Π is a state on A. Quantization involves the transference to a 'non-commutative' situation via an integral transform: $(\mathsf{A}, \Pi) \longrightarrow (\mathcal{A}^{\mathrm{ad}}, \psi)$ where $\mathcal{A}^{\mathrm{ad}}$ denotes the self-adjoint part of the non-commutative Banach algebra $\mathcal{A} = \mathcal{L}(H)$, the bounded linear operators (observables) on a Hilbert space H. In this case, the state ψ can be specified as $\psi(T) = \mathrm{Tr}\,(\rho \mathrm{T})$, for T in $\mathcal{L}(H)$ and where ρ is a density operator. Alternative structures may involve a Poisson manifold (with Hamiltonian) and $(\mathcal{A}^{\mathrm{ad}}, \psi)$ possibly with time evolution. Such quantization procedures are realized by the transforms of Weyl–Heisenberg, Berezin, Wigner–Weyl–Moyal, along with certain variants of these. Problematic can be the requirements that the adopted quantum theory should converge to the classical limit, as $\hbar \longrightarrow 0$, meaning that in the Planck limit, $\hbar$ is small in relationship to other relevant quantities of the same dimension (Landsman 1998).

2.4 Quantum Effects

Let $\mathcal{H}$ be a (complex) Hilbert space (with inner product denoted $\langle , \rangle$) and $\mathcal{L}(\mathcal{H})$ the bounded linear operators on $\mathcal{H}$. We place a natural *partial ordering* "≤" on $\mathcal{L}(\mathcal{H})$ by $S \leq T$ if

$$\langle S\psi, \psi \rangle \leq \langle T\psi, \psi \rangle \quad \text{for all } \psi \in \mathcal{H}.$$

In the terminology of Gudder (2004), an operator $A \in \mathcal{H}$ is said to represent a *quantum effect* if $0 \leq A \leq 1$. Let $\mathcal{E}(\mathcal{H})$ denote the set of quantum effects on $\mathcal{H}$. Next, let

$$P(\mathcal{H}) = \{P \in \mathcal{L}(\mathcal{H}) : P^2 = P, P = P^*\},$$

denote the space of projection operators on $\mathcal{H}$. The space $P(\mathcal{H}) \subseteq \mathcal{E}(\mathcal{H})$ constitutes the *sharp quantum effects* on $\mathcal{H}$. Likewise a natural partial ordering "≤" can be placed on $P(\mathcal{H})$ by defining $P \leq Q$ if $PQ = P$.

A *quantum state* is specified in terms of a probability measure $m : P(\mathcal{H}) \longrightarrow [0, 1]$, where $m(I) = 1$ and if P_i are mutually orthogonal, then $m(\sum P_i) = \sum m(P_i)$. The corresponding quantum probabilities and stochastic processes, may be either "sharp" or "fuzzy". A brief mathematical formulation following Gudder (2004) accounts for these distinctions as will be explained next.

Let $\mathcal{A}(\mathcal{H})$ be a σ-algebra generated by open sets and consider the *pure states* as denoted by $\Omega(\mathcal{H}) = \{\omega \in \mathcal{H} : \|\omega\| = 1\}$. We have then relative to the latter an *effects space* $\mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H}))$ less "sharp" than the space of projections $P(\mathcal{H})$ and thus comprising an entity which is "*fuzzy*" in nature. For a given *unitary operator* $U : \mathcal{H} \longrightarrow \mathcal{H}$, a *sharp observable* X_U is expressed abstractly by a map

$$X_U : \mathcal{A}(\mathcal{H}) \longrightarrow \mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H})),$$

for which $X_U(A) = I_{U^{-1}(A)}$.

Suppose then we have a *dynamical group* ($t \in \mathbb{R}$) satisfying $U(s + t) = U(s)\,U(t)$, such as in the case $U(t) = \exp(-it\,H)$ where H denotes the energy operator of Schrödinger's equation. Such a group of operators extends X_U as above to a *fuzzy (quantum) stochastic process*

$$\widetilde{X}_{U(t)} : \mathcal{A}(\mathcal{H}) \longrightarrow \mathcal{E}(\Omega(\mathcal{H}), \mathcal{A}(\mathcal{H})).$$

One can thus define classes of *analogous* quantum processes with 'similar' dynamic behavior (see also our discussion in the following Sect. 9.9) by employing dynamical group *isomorphisms*, whereas comparisons between dissimilar quantum processes could be represented by dynamical group *homomorphisms*.

3 Immanent Logic Structures in Quantum Theory

Quantum Logics (QL) and Logical Algebras. Von Neumann-Birkhoff (VNB) Quantum Logic. Operational Quantum Logic (OQL) and Łukasiewicz Quantum Logic (LQL)

3.1 Quantum Logics (QL) and Logical Algebras (LA)

As pointed out by Birkhoff and von Neumann (1936), a logical foundation of quantum mechanics consistent with quantum algebra is essential for both the completeness and mathematical validity of the theory. With the exception of a non-commutative geometry approach to unified quantum field theories (Connes 1994), the Isham and Butterfield framework in terms of the 'standard' Topos (Mac Lane and Moerdijk 2002), and the 2-category approach by John Baez (2001); other quantum algebra and topological approaches are ultimately based on set-theoretical concepts and differentiable spaces (manifolds). Since it has been shown that standard set theory which is subject to the axiom of choice relies on Boolean logic (Diaconescu 1976; Mac Lane and Moerdijk 2002), there appears to exist a basic logical inconsistency between the quantum logic—which is not Boolean—and the Boolean logic underlying all differentiable manifold approaches that rely on continuous spaces of points, or certain specialized sets of elements. A possible solution to such inconsistencies is the definition of a generalized Topos concept, and more specifically, of a Quantum Topos concept which is consistent with both Quantum Logic and Quantum Algebras, being thus suitable as a framework for unifying quantum field theories and physical modeling of complex systems and systems biology.

The problem of logical consistency between the quantum algebra and the Heyting logic algebra as a candidate for quantum logic is here discussed next. The development of Quantum Mechanics from its very beginnings both inspired and required the consideration of specialized logics compatible with a new theory of measurements for microphysical systems. Such a specialized logic was initially formulated by Birkhoff and von Neumann (1936) and called 'Quantum Logic'. Subsequent research on Quantum Logics (Chang 1960/61; Genoutti 1968; Dalla Chiara 1968, 2004) resulted in several approaches that involve several types of non-distributive lattice (algebra) for n-valued quantum logics. Thus, modifications of the Łukasiewicz Logic Algebras that were introduced in the context of algebraic categories by Georgescu and Vraciu (1970), can provide an appropriate framework for representing quantum systems, or—in their unmodified form—for describing the activities of complex networks in categories of Łukasiewicz Logic Algebras (Baianu 1977).

3.2 Lattices and Von Neumann-Birkhoff (VNB) Quantum Logic: Definitions and Logical Properties

We commence here by giving the *set-based Definition of a Lattice*. An *s-lattice* **L**, or a 'set-based' lattice, is defined as a *partially ordered set* that has all binary products (defined by the s-lattice operation "$\bigwedge$)") and coproducts (defined by the s-lattice operation "$\bigvee$"), with the "partial ordering" between two elements X and Y belonging to the s-lattice being written as "$X \preceq Y$". The partial order defined by $\preceq$ holds in **L** as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ Eq. (3.1) (p. 49 of Mac Lane and Moerdijk 1992).

3.3 Categorical Definition of a Lattice

Utilizing the category theory concepts defined in Brown, Glazebrook and Baianu (2007, in this volume), one needs to introduce a categorical definition of the concept of lattice that can be '*set–free*' in order to maintain logical consistency with the algebraic foundation of Quantum Logics and relativistic spacetime geometry. Such category–theoretical concepts unavoidably appear also in several sections of this paper as they provide the tools for deriving very important, general results that link Quantum Logics and Classical (Boolean) Logic, as well as pave the way towards a universal theory applicable also to semi-classical, or mixed, systems. Furthermore, such concepts are indeed applicable to measurements in complex biological networks, as it will be shown in considerable detail in a subsequent paper in this volume (Baianu and Poli 2008).

A *lattice* is defined as a category (see, for example: Lawvere, 1966; Baianu 1970; Baianu et al. 2004b) subject to all ETAC axioms, (but not subject, in general, to the Axiom of Choice usually encountered with sets relying on (distributive) Boolean Logic), that has all binary products and all binary coproducts, as well as the following 'partial ordering' properties:

(i) when unique arrows $X \longrightarrow Y$ exist between objects X and Y in **L** such arrows will be labeled by "$\preceq$", as in "$X \preceq Y$";
(ii) *the coproduct* of X and Y, written as "$X \bigvee Y$" will be called the "*sup object*", or "*the least upper bound*", whereas the product of X and Y will be written as "$X \bigwedge Y$", and it will be called an *inf object*, or "*the greatest lower bound*";
(iii) *the partial order* defined by $\preceq$ holds in **L**, as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ (p. 49 of Mac Lane and Moerdijk 1992).

If a lattice **L** has **0** and **1** as objects, such that $0 \longrightarrow X \longrightarrow 1$ (or equivalently, such that $0 \preceq X \preceq 1$) for all objects X in the lattice **L** viewed as a category, then **0** and **1** are the unique, initial, and respectively, terminal objects of this concrete category **L**. Therefore, **L** has all finite limits and all finite colimits (p. 49 of Mac Lane and Moerdijk 1992), and is said to be *finitely complete and co-complete*. Alternatively, the lattice 'operations' can be defined via functors in a 2-category (for definitions of functors and 2-categories see, for example, p. 21 of Mac Lane 2000, p. xx of Brown 1998, or Section 9 of Baianu et al. 2004b), as follows:

$$\bigwedge : L \times L \longrightarrow L, \qquad \bigvee : L \times L \rightarrow L \tag{3.1}$$

and 0,1: $1 \rightarrow L$ as a "lattice object" in a 2-category with finite products.

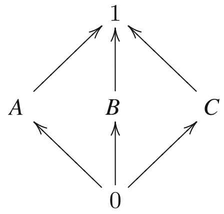
A lattice is called *distributive* if the following identity :

$$X \bigwedge (Y \bigvee Z) = (X \bigwedge Y) \bigvee (X \bigwedge Z). \tag{3.2}$$

holds for all X, Y, and Z objects in **L**. Such an identity also implies the dual distributive lattice law:

$$X \bigvee (Y \bigwedge Z) = (X \bigvee Y) \bigwedge (X \bigvee Z). \tag{3.3}$$

(Note how the lattice operators are 'distributed' symmetrically around each other when they appear in front of a parenthesis.) A *non-distributive* lattice is not subject to either restriction (3.2) or (3.3). An example of a non-distributive lattice is (cf. Pedicchio and Tholen 2004):



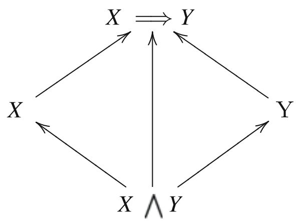
(3.4)

3.3.1 *Definitions of an Intuitionistic Logic Lattice*

A *Heyting algebra*, or *Brouwerian lattice*, H, is a *distributive lattice* with all finite products and coproducts, and which is also *cartesian closed*. Equivalently, a Heyting algebra can be defined as a distributive lattice with both initial (0) and terminal (1) objects which has an "exponential" object defined for each pair of objects X, Y, written as: "$X \Rightarrow Y$" or X^Y, such that:

$$Z = (X \Rightarrow Y) \Longleftarrow Z = X^Y, \tag{3.5}$$

In the Heyting algebra, $X \Rightarrow Y$ is a least upper bound for all objects Z that satisfy the condition $Z = X^Y$. Thus, in terms of a categorical diagram, the partial order in a Heyting algebra can be represented as



(3.6)

A lattice will be called complete when it has all small limits and small colimits (e.g., small products and coproducts, respectively). It can be shown (p. 51 of Mac Lane and Moerdijk 1992) that any complete and infinitely distributive lattice is a Heyting algebra.

3.4 Łukasiewicz Quantum Logic (LQL)

With all assertions of the type *system A* is excitable to the *i*-th level and system B is excitable to the *j*-th level" on *e* can form a distributive lattice, **L** (as defined above in subsection 3.3). The composition laws for the lattice will be denoted by $\bigcup$ and $\bigcap$. The symbol $\bigcup$ will stand for the logical non-exclusive 'or', and $\bigcap$ will stand for the logical conjunction 'and'. Another symbol "$\preceq$" allows for the ordering of the levels and is defined as *the canonical ordering* of the lattice. Then, one is able to give a symbolic characterization of the system dynamics with respect to each 'energy', or truth level *i*. This is achieved by means of the maps δ_t: $L \rightarrow L$ and N: $L \rightarrow L$, (with N being the negation). The necessary logical restrictions on the actions of these maps lead to an *n-valued Łukasiewicz* Algebra of logical 'truth values' or nuances and operands:

(I) There is a map $N : L \longrightarrow L$, so that

$$N(N(X)) = X, \tag{3.7}$$

$$N(X \bigcup Y) = N(X) \bigcap N(Y) \tag{3.8}$$

and

$$N(X \bigcap Y) = N(X) \bigcup N(Y), \tag{3.9}$$

for any $X, Y \in \mathbf{L}$.

(II) There are (n–1) maps $\delta_i : L \longrightarrow L$ which have the following properties:
- (a) δ_i (0) = 0, δ_i (1) = 1, for any $1 \leq i \leq n$–1, where indices i represent 'truth levels' or nuances;
- (b) $\delta_i\ (X \bigcup Y) = \delta_i\ (X) \bigcup \delta_i\ (Y), \delta_i\ (X \bigcap Y) = \delta_i\ (X) \bigcap \delta_i\ (Y)$, for any $X, Y \in \mathbf{L}$, and $1 \leq i \leq n$–1;
- (c) $\delta_i\ (X) \bigcup N(\delta_i\ (X)) = 1, \delta_i\ (X) \bigcap N(\delta_i\ (X)) = 0$, for any $X \in \mathbf{L}$;
- (d) $\delta_i\ (X) \subset \delta_2\ (X) \subset \cdots \subset \delta(n-1)(X)$, for any $X \in \mathbf{L}$;
- (e) $\delta_i * \delta_j = \delta_i$ for any $1 \leq i, j \leq n$–1;
- (f) If δ_i (X) = δ_i (Y) for any $1 \leq i \leq n$–1, then $X = Y$;
- (g) δ_i ($N(X)$) = $N(\delta_j$ (X)), for $i + j = n$. (Georgescu and Vraciu 1970).

The first axiom states that the double negation has no effect on any assertion concerning any level, and that a simple negation changes the disjunction into conjunction and conversely. The second axiom presents ten sub-cases that are summarized in equations (a)–(g). Sub-case (IIa) states that the dynamics of the system is such that it maintains the structural integrity of the system. It does not allow for structural changes that would alter the lowest and the highest energy levels of the system. Thus, maps $\delta : L \longrightarrow L$ are chosen to represent the dynamic behavior of the quantum or classical systems in the absence of structural changes. Equation (IIb) shows that the maps (d) maintain the type of conjunction and disjunction. Equations (IIc) are chosen to represent assertions of the following type: ⟨the sentence "a system component is excited to the i-th level or it is not excited to the same level" is true⟩, and ⟨the sentence "a system component is excited to the i-th level and it is not excited to the same level, at the same time" is always false⟩.

Equation (IId) actually defines the actions of maps δ_t. Thus, Eq. (I) is chosen to represent a change from a certain level to another level as low as possible, just above the zero level of $\mathbf{L}$. δ_2 carries a certain level x in assertion X just above the same level in δ_1 (X), δ_3 carries the level x-which is present in assertion X–just above the corresponding level in δ_2 (X), and so on. Equation (IIe) gives the rule of composition for the maps δ_t. Equation (IIf) states that any two assertions that have equal images under all maps δ_t, are equal. Equation (IIg) states that the application of δ to the negation of proposition X leads to the negation of proposition $\delta(X)$, if $i + j = n$.

In order to have the n-valued Łukasiewicz Logic Algebra represent correctly the basic behavior of quantum systems (observed through measurements that involve a quantum system interactions with a measuring instrument—which is a macroscopic object, several of these axioms have to be significantly changed so that the resulting lattice becomes non-distributive and also (possibly) non-associative (Dalla Chiara 2004). With an appropriately defined quantum logic of events one can proceed to define Hilbert, or 'nuclear'/Frechet, spaces in order to be able to utilize the 'standard' procedures of quantum theories.

On the other hand, for classical systems, modeling with the unmodified Łukasiewicz Logic Algebra can also include both stochastic and fuzzy behaviors. For an example of such models the reader is referred to a previous publication (Baianu 1977) modeling the activities of complex genetic networks from a classical standpoint. One can also define as in (Georgescu and Vraciu 1970) the 'centers' of certain types of Łukasiewicz n-Logic Algebras; then one has the following important theorem for such Centered Łukasiewicz n-Logic Algebras which actually defines an equivalence relation.

Theorem 3.1 The Adjointness Theorem (Georgescu and Vraciu 1970). *There exists an Adjointness between the Category of Centered Łukasiewicz n-Logic Algebras*, $\mathbf{CLuk}-n$, *and the Category of Boolean Logic Algebras* ($\mathbf{Bl}$).

Note: this adjointness (actually, equivalence) relation between the Centered Łukasiewicz n-Logic Algebra Category and **Bl** has a logical basis: *non* (*non*(A)) = A in both **Bl** and **CLuk**–n.

Conjecture 3.1 *There exist adjointness relationships, respectively, between each pair of the Centered Heyting Logic Algebra*, $\mathbf{Bl}$, *and the Centered* $\mathbf{CLuk}-n$ *Categories*.

Remark 3.1 R1. Both a Boolean Logic Algebra and a Centered Łukasiewicz Logic Algebra can be represented as/are Heyting Logic algebras (the converse is, of course, generally false!).

R2. The natural equivalence logic classes defined by the adjointness relationships in the above Adjointness Theorem define a fundamental, *'logical groupoid'* structure.

Note also that the above Łukasiewicz Logic Algebra is *distributive* whereas the quantum logic requires a *non-distributive* lattice of quantum 'events'. Therefore, in order to generalize the standard Łukasiewicz Logic Algebra to the appropriate Quantum Łukasiewicz Logic Algebra, *LQL*, axiom I needs modifications, such as: $N(N(X)) = Y \neq X$ (instead of the restrictive identity $N(N(X)) = X$, and, in general, giving up its 'distributive' restrictions, such as

$$N(X\bigcup Y) = N(X)\bigcap N(Y) \text{ and } N(X\bigcap Y) = N(X)\bigcup N(Y), \tag{3.10}$$

for any X, Y in the Łukasiewicz Quantum Logic Algebra, *LQL*, whenever the context, 'reference frame for the measurements', or 'measurement preparation' interaction conditions for quantum systems are incompatible with the standard 'negation' operation N of the Łukasiewicz Logic Algebra that remains however valid for classical systems, such as various complex networks with n-states (cf. Baianu 1977).

4 Local-to-Global Problems in Spacetime Structures. Symmetry Breaking, Irreversibility and the Emergence of Highly Complex Dynamics

On summarizing in this section the evolution of the physical concepts of space and time, we are pointing out first how the views changed from homogeneity and continuity to *inhomogeneity and discreteness*. Then, we link this paradigm shift to a possible, novel solution in terms of local-to-global approaches and procedures to spacetime structures. Such solutions are enabled by the following Sect. 5 which presents in an abbreviated and simplified form the fundamental concepts and results of modern Algebraic Topology that allow the skilled mathematician and theoretical physicist to design procedures for constructing, as well as classifying, spacetime structures, and then further to obtain local-to-global solutions to highly complex dynamic problems through the application of novel theorems in Non-Abelian Algebraic Topology (NAT). These local-to-global procedures will therefore lead to a wide range of applications sketched in the later sections, such as the *emergence of higher dimensional spacetime* structures through highly complex dynamics in organismic development, adaptation, evolution, consciousness and society interactions.

4.1 Spacetime Local Inhomogeneity, Discreteness and Broken Symmetries: From Local to Global Structures

Physics, up to 1900's, involved a concept of both *continuous* and *homogeneous*, absolute space and time with strict causal (mechanistic) evolution of all physical processes ("*God does not play dice*", cf. Albert Einstein). Furthermore, up to the introduction of *quanta–discrete* portions, or packets-of energy by Ernst Planck (which was further elaborated by Einstein, Heisenberg, Dirac, Feynman, Weyl and other eminent physicists of the last century), energy was also considered to be a continuous function, though not homogeneously distributed in space and time. Einstein's Relativity theories joined together space and time into one 'new' entity—the concept of *spacetime*. Furthermore, in the improved form of General Relativity (GR), inhomogeneities caused by the presence of matter were allowed to occur in spacetime. Causality, however, remained *strict*, but also more complicated than in the Newtonian theories. Both the standard GR theory and Newtonian mechanics can be considered as Abelian theories, even though the former not only allows, but indeed, requires spacetime inhomogeneities to occur in the presence of gravitational fields, unlike Newtonian mechanics *where space is both absolute* and *homogeneous*. Recent efforts to develop *non-Abelian* GR theories—especially with an intent to develop Quantum Gravity theories—seem to have considered both possibilities of locally homogeneous or inhomogeneous spacetimes. The successes of non-Abelian gauge theories have become well known in physics since 1999, but they still await the experimental discovery of their predicted Higgs boson particles.

Although Einstein's Relativity theories incorporate the concept of *quantum of energy*, or photon, into their basic structures, they also deny such discreteness to spacetime even though the discreteness of energy is obviously accepted within

Relativity theories. The GR concept of spacetime being modified, or *distorted/ 'bent'*, by matter goes further back to Riemann, but it was Einstein's GR theory that introduced the idea of representing gravitation as the result of *spacetime distortion by matter*. Implicitly, such spacetime distortions remained continuous even though the gravitational field energy—as all energy—was allowed to vary in *discrete*, albeit very tiny portions–the gravitational quanta. So far, however, the detection of gravitons—the quanta of gravity—related to the spacetime distortions by matter—has been unsuccessful. Mathematically elegant/precise and physically 'validated' through several crucial experiments and astrophysical observations, Einstein's GR is obviously not reconcilable with Quantum theories (QTs). GR was designed as the *large-scale* theory of the Universe, whereas Quantum theories—at least in the beginning—were designed to address the problems of *microphysical* measurements at very tiny scales of space and time involving extremely small quanta of energy. We see therefore the QTs vs. GR as a local-to-global problem that has not been yet resolved in the form of an universally valid Quantum Gravity. Promising, partial solutions are suggested in the following two papers in this issue (Baianu, Brown and Glazebrook 2007b; Brown, Glazeborrk and Baianu 2007a, in this volume).

Quantum theories (QTs) were developed that are just as elegant mathematically as GR, and they were also physically 'validated' through numerous, extremely sensitive and carefully designed experiments. However, to date quantum theories have not been extended, or generalized, to a form capable of recovering the results of Einstein's GR as a quantum field theory over a GR-spacetime altered by gravity is not yet available.

Furthermore, quantum symmetries occur not only on microphysical scales, but also macroscopically in certain, 'special' cases, such as liquid ^{3}He close to absolute zero and superconductors where *extended coherence* is possible for the superfluid, Cooper electron-pairs. Explaining such phenomena requires the consideration of *symmetry breaking* (Weinberg 1995, 2000). Occasionally, symmetry breaking is also invoked as a 'possible mechanism for human consciousness' which also seems to involve some form of 'global coherence'—over most of the brain; however, the existence of such a '*quantum* coherence in the brain'—at room temperature—as it would be precisely required/defined by QTs, is a most unlikely event. On the other hand, a *quantum symmetry breaking* in a neural network considered metaphorically as a Hopfield ('spin-glass') network might be conceivable close to physiological temperatures but for the lack of existence of any requisite (electron?) spin lattice structure which is indeed an absolute requirement in such a spin-glass metaphor—if it is to be taken at all seriously!

Now comes the real, and very interesting part of the story: neuronal networks do form functional patterns and structures that possess partially 'broken', or more general symmetries than those described by quantum groups. Such *extended symmetries* can be mathematically determined, or specified, by certain *groupoids*—that were previously called '*neuro-groupoids*'. Even more generally, genetic networks also exhibit extended symmetries represented for an organismal species by a *biogroupoid* structure, as previously defined and discussed by Baianu et al. (2006b). Such biogroupoid structures can be experimentally validated, for example, at least partially through Functional Genomics observations and computer, bioinformatics processing

(Baianu 2007). We shall discuss further several such interesting groupoid structures in the following sections, and also how they have already been utilized in local-to-global procedures to construct 'global' solutions; such global solutions in quite complex (holonomy) cases can still be *unique* up to an isomorphism (*the Globalization Theorem*, as discussed in Brown, Glazebrook and Baianu 2007a, in this volume). Last-but-not-least, *holonomy* may provide a global solution, or 'explanation' for 'memory storage by 'neuro-groupoids', and we shall further discuss this possibility in the next subsection and also in Sect. 14. Uniqueness holonomy theorems might possibly 'explain' unique, persistent and resilient memories.

4.2 The Conceptual Development of Local-to-Global Problems

Related to the local-to-global problem considered here, in Mathematics, Ehresmann developed many new themes in category theory. One example is *structured categories* with principal examples those of differentiable categories, groupoids, and multiple categories. His work on these is quite distinct from the general development of the mathematical theory of categories in the 20th century, and it is interesting to search for reasons for this distinction. One must be the fact that he used his own language and notation, which has not helped with the objectivation by several other, perhaps 'competing', mathematical schools. Another is surely that his early training and motivation came from analysis, rather than from algebra, in contrast to the origins of category theory in the work of Eilenberg, Mac Lane (including Steenrod and others) centered on homology theory and algebraic topology. Part of the developing language of category theory became essential in those areas, but other parts, such as those of algebraic theories, groupoids, multiple categories, were not used till fairly recently (see the next sub-section and Brown et al. (2007). for the precise definitions of these terms). It seems likely that Ehresmann's experience in analysis led him to the major theme of *local-to-global* questions. The author Brown first learned of this term from R. Swan in Oxford in 1957–58, when as a research student Brown was writing up notes of his Lectures on the *Theory of Sheaves*. Swan explained to him that two important methods for local-to-global problems were *sheaves and spectral sequences*—he was thinking of Poincaré duality, which is discussed in the lecture notes, and the more complicated theorems of Dolbeault for complex manifolds. But in fact, such problems are central in mathematics, science and technology. They are fundamental, for example, to the theories of *differential equations and dynamical systems*. Even deducing consequences of a set of rules is a local-to-global problem: the rules are applied *locally*, but we are interested in their *global* consequences.

Brown's work on local-to-global problems arose from writing an account of the Seifert-van Kampen theorem on the fundamental group. This theorem can be given as follows, as first shown by Crowell (1959):

Theorem 4.1 Crowell (1959). *Let the space X be the union of open sets U,V with intersection W, and suppose W,U,V are path connected. Let* $x_0 \in W$. *Then the diagram of fundamental group morphisms induced by inclusions*:

$$\begin{array}{ccc} \pi_1(W, x_0) & \xrightarrow{i} & \pi_1(U, x_0) \\ {\scriptstyle j}\downarrow & & \downarrow \\ \pi_1(V, x_0) & \longrightarrow & \pi_1(X, x_0) \end{array} \tag{4.1}$$

is a pushout of groups.

Here the 'local parts' are of course *U*, *V* put together with intersection *W* and the result describes completely, under the open set and connectivity conditions, the (*non-Abelian*) *fundamental group* of the global space *X*. This theorem is usually seen as a necessary part of basic algebraic topology, but one without higher dimensional analogues. On the other hand, the generalization of the van Kampen theorem to groupoids, and subsequently, indeed to the most general case of higher homotopy/higher dimensions—as well as non-Abelian cases—was carried out by author R. Brown and his research group. Both generalized theorems are provided here as they are pertinent to the procedures discussed above, also to Sect. 3, and Sects. 5 through 12.

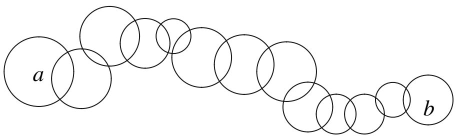
4.3 Iterates of Local Procedures Using Groupoid Structures

Often we will look for a modeling of levels regarded as highly complex systems that can be described in terms of specific categorical structures and natural transformations of functors which compare modeling diagrams or categories. A special subclass of categories is that of *groupoids*–small categories with all morphisms *invertible* (Brown 2006; Weinstein 1996). These are essential as descriptive models for the reciprocity (i.e., *morphism invertibility, or isomorphism*) in the relay of signalling that occurs in various classes of genetic, neural and bionetworks, besides providing descriptive mechanisms for *local-to-global* properties within the latter, the collection of objects of which can comprise various genera of organismic sets. Groupoid actions and certain convolution algebras of groupoids (cf. Connes 1994) were suggested to be the main carriers of *non-commutative processes*. Many types of cell systems such as those representative of neural networks or physiological locomotion, can be described in terms of equivalence classes of cells, links and inputs, etc. leading to the notion of a system's *symmetry groupoid* the breaking of which can induce a transition from one state to another (Golubitsky and Stewart 2006). This notion of classification involves equivalence relations, but the groupoid point of view extends this notion not only to say that two elements are equivalent but also to label the proofs that they are equivalent. Such an approach features in an information-based theory of interactive cognitive modules cast within the Baars global neuronal workspace (Wallace 2005). The theories of Shannon (information) and Dretske (communication) are combined in an immunology/language and network analysis/groupoid setting to describe a fundamental homology with the thermodynamic principles as derived from statistical physics. The thread of ideas may be exemplified by such cognitive disorders as *inattentional blindness* and *psycho-social stress* resulting from such factors as information distortion/overload, socio-cultural pressure, and as represented by the manifestation of network transition phases (often attributed to an *induced symmetry breaking* within the network in question). Such cognitive disorders are considered as having their analogues at the levels of culturally embedded/institutional, higher-level multi-tasking where such ailments can result in a demise or total failure of the constituent operative systems. The latter include the general areas of public health administration, (disease prevention, therapeutic practice, etc.), environmental/ecological management, to name a few. The future development of 'conscious machines', is likely to be no less prone to such failures (Wallace 2005, 2007).

The notion of *holonomy* occurs in many situations, both in physics and differential geometry. Non-trivial holonomy occurs when an iteration of local procedures which returns to the starting point can yield a change of phase, or of other more general values. Charles Ehresmann realized the notion of *local procedure* formalized by the notion of *local smooth admissible section of a smooth groupoid*, and Pradines (1966) generalized this to obtain a *global holonomy Lie groupoid* from a *locally* Lie groupoid: the details were presented in Aof and Brown (1992).

This concept of *local procedure* may be applicable to the evolution of super-complex systems/organisms for which there are apparently "missing links"—ancestors whose fossils cannot be found; when such links are genuinely missing, the evolution process can be viewed as maintaining an evolutionary trend not by virtue of analytical continuity, from point to point, but through overlapping regions from networks of genes and their expressed phenotype clusters. This idea of a local procedure applied to speciation is illustrated below, with the intermediate circles representing such possible missing links, without the need to appeal to 'catastrophes'.

In this speciation example, the following picture illustrates a chain of local procedures (COLP) leading from species *a* to species *b* via intermediates that are not 'continuous' in the analytical sense discussed above:



One would like to be able to define such a chain, and equivalences of such chains, without resource to the notion of 'path' between points. The claim is that a candidate for this lies in the constructions of Charles Ehresmann and Jean Pradines for the *holonomy groupoid*. The globalization of structure can be thus encoded in terms of the *holonomy groupoid* which for any groupoid-related system encodes the notion of the subsequent *phase transition* (and its amplitude) of the latter phase towards a new phase (Aof and Brown 1992).

One question is whether a COLP is either a fact or a description. Things evolve and change in time. We think usually of this change as a real number modeling of time. But it may be easier to see what is happening as a COLP, since each moment of time has an environment, which is carried along as things evolve.

The Aof–Brown paper, based on certain ideas of Charles Ehresmann and Jean Pradines, shows that such ideas have a mathematical reality, and that some forms of holonomy are nicely described in this framework of the globalization theorem for a locally Lie groupoid.

The generalization of the manifold/atlas structure (Brown 2006) is that of the *groupoid atlas* (Bak 2006; and Brown, Glazebrook and Baianu (2007), in this volume) which is already relevant in 'concurrent' and 'multi-agent systems' (Porter 2002). But concurrent and multi-agent systems are distinct, though they may be somehow related. Concurrency itself is a theory about many processes occurring at the same time, or, equivalently, about processes taking place in multiple times. Since time has a direction, *multiple times* have a 'multiple direction', hence the *directed spaces*. This leads to a novel descriptive and computational technique for charting informational flow and management in terms of *directed spaces, dimaps and dihomotopies* (see e.g., Goubault 2003). These may provide alternative approaches to *'iterates of local procedures'* along with key concepts such as the notion of 'scheduling of paths' with respect to a cover that can be used as a globalization technique, for instance, to recover the Hurewicz continuous fibration theorem (Hurewicz 1955) as in Dyer and Eilenberg (1988).

Ontological levels themselves will entail '*processes* of processes' for which HDA seeks to provide the general theories of transport along n-paths and subsequent n-holonomy (cf. Brown and İçen 2003 for the two-dimensional case), thus leading to a globalizaton of the dynamics of local networks of organisms across which multiple morphisms interact, and which are multiply–observable. This representation, unless further specified, may not be able, however, to distinguish between levels and multiple processes occurring on the same level.

5 Fundamental Concepts of Algebraic Topology with Potential Applications to SpaceTime Structures

5.1 Potential Applications of Novel Algebraic Topology Methods to the Fundamental Ontology Level and the Problems of Quantum Spacetime

With the advent of Quantum Groupoids–generalizing Quantum Groups, Quantum Algebra and Quantum Algebraic Topology, several fundamental concepts and new theorems of Algebraic Topology may also acquire an enhanced importance through their potential applications to current problems in theoretical and mathematical physics, such as those described in an available preprint (Baianu et al. 2006a), and also in the next article in this issue (Baianu et al. 2007). Such potential applications will be briefly outlined at the conclusion of this section as they are based upon the following ideas, algebraic topology concepts and constructions.

Traditional algebraic topology works by several methods, but all involve going from a space to some form of combinatorial or algebraic structure. The earliest of these methods was 'triangulation': a space was supposed represented as a simplicial complex, i.e. was subdivided into simplices of various dimensions glued together along faces, and an algebraic structure such as a chain complex was built out of this simplicial complex, once assigned an orientation, or, as found convenient later, a total order on the vertices. Then in the 1940s a convenient form of singular theory was found, which assigned to any space X a 'singular simplicial set' SX, using continuous mappings from $\Delta^n \to X$, where Δ^n is the standard n-simplex. From this simplicial set, the whole of the weak homotopy type could in principle be determined. An alternative approach was found by Čech, using an open covers $\mathcal{U}$ of X to determine a simplicial set $N\mathcal{U}$, and then refining the covers to get better 'approximations' to X. It was this method which Grothendieck discovered could be extended, especially combined with new methods of homological algebra, and the theory of sheaves, to give new applications of algebraic topology to algebraic geometry, via his theory of schemes.

The 600-page manuscript, *'Pursuing Stacks'* by Alexander Grothendieck (2007) was aimed at a *non-Abelian homological algebra*; it did not achieve this goal but has been very influential in the development of weak n-categories and other *higher categorical structures*.

Now if quantum mechanics is to reject the notion of a continuum, then it must also reject the notion of the real line and the notion of a path. How then is one to construct a homotopy theory?

One possibility is to take the route signalled by Čech, and which later developed in the hands of Borsuk into 'Shape Theory' (see, Cordier and Porter 1989). Thus a quite general space is studied by means of its approximation by open covers. Yet another possible approach is briefly pointed out in the next subsection.

A few fundamental concepts of Algebraic Topology and Category Theory will be introduced next that have an extremely wide range of applicability to the higher complexity levels of reality as well as to the fundamental, quantum level(s). We have omitted in this section technical details to focus on the ontologically relevant aspects; full mathematical details are however available in the companion paper in this issue that focuses on a mathematical concept framework for a formal approach to categorical ontology.

5.2 Groupoids, Topological Groupoids, Groupoid Atlases and Locally Lie Groupids

Recall that a groupoid G is a small category in which every morphism is an isomorphism.

5.2.1 Topological Groupoids

An especially interesting concept is that of a *topological groupoid* is a groupoid internal to the category Top. Further mathematical details are presented in the third paper of this issue (Brown, Glazebrook, and Baianu 2007, in this volume).

5.2.2 An Atlas of Groupoids

Motivation for the notion of a groupoid atlas comes from considering families of group actions, in the first instance on the same set. As a notable instance, a subgroup H of a group G gives rise to a group action of H on G whose orbits are the cosets of H in G. However a common situation is to have more than one subgroup of G, and then the various actions of these subgroups on G are related to the actions of the intersections of the subgroups. This situation is handled by the notion of *Global Action*, as defined in Bak (2000). A key point in this construction is that the orbits of a group action then become the connected components of a groupoid. Also this enables relations with other uses of groupoids.

The above account motivates the following. A *groupoid atlas* $\mathcal{A}$ on a set $X_{\mathcal{A}}$ consists of a family of 'local groupoids' $(\mathsf{G}_{\mathcal{A}})$ defined with respective object sets $(X_{\mathcal{A}})_{\alpha}$ taken to be subsets of $X_{\mathcal{A}}$. These local groupoids are indexed by a set $\Psi_{\mathcal{A}}$, again called the *coordinate system of* $\mathcal{A}$ which is equipped with a reflexive relation denoted by $\leq$. This data is to satisfy several conditions (Bak et al. 2006), as completely specified in the fourth paper in this issue.

5.2.3 Crossed Complexes

On the other hand, crossed complexes are equivalent to a bewildering array of other structures, which are important for applications (Brown 2004). Cat^n-groups are also equivalent to *crossed n-cubes of groups*. The construction of the equivalences and of the functors Ξ in all these cases is difficult conceptually and technically. The general philosophy is that one type of category is sufficiently geometric to allow for the formulation and proof of theorems, in a higher dimensional fashion, while another is more 'linear' and suitable for calculation. The transformations between the two forms give a kind of synaethesia. The classifying space constructions are also significant, and allow for information on the homotopy classification of maps.

From the ontological point of view, these results indicate that it is by no means obvious what algebraic data will be useful to obtain precise local-to-global results, and indeed new forms of this data may have to be constructed for specific situations. These results do not give a TOE, but do give a new way of obtaining new information not obtainable by other means, particularly when this information is in a non-commutative form. The study of these types of results is not widespread, but will surely gain attention as their power becomes better known.

In Algebraic Topology crossed complexes have several *advantages* such as:

- They are good for *modeling CW-complexes*. Free crossed resolutions enable calculations with *small CW-models* of $K(G,1)$s *and their maps* (Brown and Razak Salleh 1999).
- Also, they have an interesting relation with the Moore complex of simplicial groups and of *simplicial groupoids*.

- They *generalise groupoids and crossed modules to all dimensions*. Moreover, the natural context for the second relative homotopy groups is crossed modules of groupoids, rather than groups.
- They are convenient for *calculation*, and the functor Π is classical, involving *relative homotopy groups*.
- They provide a kind of *'linear model'* for homotopy types which includes all 2-types. Thus, although they are not the most general model by any means (they do not contain quadratic information such as Whitehead products), this simplicity makes them easier to handle and to relate to classical tools. The new methods and results obtained for crossed complexes can be used as a model for more complicated situations. For example, this is how a general *n*-adic Hurewicz Theorem was found (Brown and Loday 1987)
- Crossed complexes have a *good homotopy theory*, with a *cylinder object, and homotopy colimits*. (A *homotopy classification* result generalises a classical theorem of Eilenberg-Mac Lane).
- They are close to chain complexes with a group(oid) of operators, and related to some classical homological algebra (e.g., *chains of syzygies*). In fact if *SX* is the simplicial singular complex of a space, with its skeletal filtration, then the crossed complex Π(*SX*) can be considered as a slightly *non-commutative version of the singular chains of a space*.

For more details on these points, we refer the reader to Brown (2004).

5.2.4 Locally Lie Groupoids

We shall begin here with the important definition of the concept of a locally Lie groupoid.

A locally Lie groupoid (Pradines 1966; Aof and Brown 1992) is a pair (G, W) consisting of a groupoid G with range and source maps denoted α, β respectively, (in keeping with the last quoted literature) together with a smooth manifold *W*, such that :

(1) $\mathrm{Ob}(\mathsf{G}) \subseteq \mathrm{W} \subseteq \mathsf{G}$.
(2) $W = W^{-1}$.
(2) The set $W_\delta = \{W \times_\alpha W\} \cap \delta^{-1}(W)$ is open in $W \times_\alpha W$ and the restriction to W_δ of the difference map $\delta : \mathsf{G} \times_\alpha \mathsf{G} \longrightarrow \mathsf{G}$ given by $(g, h) \mapsto gh^{-1}$, is smooth.
(3) The restriction to *W* of the maps α, β are smooth and (α, β, *W*) admits enough smooth admissible local sections.
(4) *W* generates G as a groupoid.

We have to explain more of these terms. A *smooth local admissible section* of (α, β, *W*) is a smooth function *s* from an open subset of *U* of $X = Ob(\mathsf{G})$ to *W* such that $\alpha\, s = 1_U$ and $\beta\, s$ maps *U* diffeomorphically to its image which is open in *X*. It is such a smooth local admissible section which is thought of as a *local procedure* (in the situation defined by the locally Lie groupoid (G, W)).

There is a *composition* of those local procedures due to Charles Ehresmann of these local procedures given by $s * t(x) = s(\beta t(x)) \circ t(x)$ where $\circ$ is the composition in the groupoid G. The domain of $s * t$ is usually smaller than that of t and may even be empty. Further the codomain of $s * t$ may not be a subset of W: thus the notion of smoothness of $s * t$ may not make sense. In other words, the composition of local procedures may not be a local procedure. Nonetheless, the set $\Gamma^{\omega}(\mathsf{G}, W)$ of all compositions of local procedures with its composition * has the structure of an *inverse semigroup*, and it is from this that the Holonomy Groupoid, $\mathbf{Hol}(\mathsf{G}, W)$ is constructed as a Lie groupoid in Aof and Brown (1992), following details given personally by J. Pradines to Brown in 1981.

The motivation for this construction, due to Pradines, was to construct the *monodromy groupoid* $M(G)$ of a Lie groupoid G. The details are given in Brown and Mucuk (1995; 1996).

The monodromy groupoid has this name because of the *monodromy principle* on the extendability of local morphisms. It is a *local-to-global* construction. It has a kind of *left adjoint* property given in detail in Brown and Mucuk (1995; 1996). So it has certain properties that are analogous to a van Kampen theorem.

The holonomy construction is applied to give a Lie structure to $M(G)$. When G is the pair groupoid $X \times X$ of a manifold X, then $M(G)$ is the *fundamental groupoid* $\pi_1 X$. It is crucial that this construction of $M(X)$ is independent of paths in X, but is defined by a suitable neighbourhood of the diagonal in $X \times X$, which is in the spirit of synthetic differential geometry, and so has the possibility of being applicable in wider situations. What is *unknown* is how to extend this construction to define *higher homotopy groupoids* with useful properties.

In a real quantum system, a *unique* holonomy groupoid may represent *parallel transport* processes and the *'phase-memorizing'* properties of such remarkable quantum systems. This theme could be then further pursued by employing *locally Lie groupoids in local-to-global procedures* (cf. Aof and Brown 1992) for the construction in Quantum Spacetime of the *Holonomy Groupoid* (which is *unique*, according to the Globalization Theorem).

An alternative approach might involve the application of the more recently found fundamental theorems of Algebraic Topology—such as the Higher Homotopy generalization of the van Kampen theorem—to characterize the *topological invariants* of a higher-dimensional topological space, for example in the context of AQFT, in terms of *known invariants* of its simpler subspaces. We also mention here the recent work of Brown and Janelidze (1997) which extends the van Kampen theorem to a purely *categorical* construction, thereby facilitating novel applications such as the development of a *non-Abelian Categorical Ontology* of *spacetimes of higher dimensions*.

Thus, the *generalized* notion of a van Kampen theorem has many suggestive possibilities for both extensions and applications, and it should provide a basis for *higher dimensional, non-Abelian* methods in *local-to-global* questions in theoretical physics and Categorical Ontology, and therefore open up completely new fields. The precise content of the Higher Homotopy van Kampen theorem according to Brown (2004) is specified next.

5.3 The van Kampen Theorem and Its Generalizations to Groupoids and Higher Homotopy

The van Kampen Theorem 4.1 has an important and also anomalous rôle in algebraic topology. It allows computation of an important invariant for spaces built up out of simpler ones. It is anomalous because it deals with a nonabelian invariant, and has not been seen as having higher dimensional analogues.

However Brown (1967), found a generalization of this theorem to groupoids, as follows. In this, $\pi_1(X,X_0)$ is the fundamental *groupoid* of X on a set X_0 of base points: so it consists of homotopy classes rel end points of paths in X joining points of $X_0 \cap X$.

Theorem 5.1 The Van Kampen Theorem for the Fundamental Groupoid, (Brown 1967) *Let the space X be the union of open sets U,V with intersection W, and let* X_0 *be a subset of X meeting each path component of U,V,W. Then (C) (connectivity)* X_0 *meets each path component of X, and (I) (isomorphism) the diagram of groupoid morphisms induced by inclusions:*

$$\begin{array}{ccc} \pi_1(W,X_0) & \xrightarrow{\;i\;} & \pi_1(U,X_0) \\ \Big\downarrow{\scriptstyle j} & & \Big\downarrow{\scriptstyle k} \\ \pi_1(V,X_0) & \xrightarrow[\;l\;]{} & \pi_1(X,X_0) \end{array} \tag{5.1}$$

is a pushout of groupoids.

Theorem 4.1 discussed in Sect. 4 is the special case when $X_0 = \{x_o\}$. From Theorem 5.1 one can compute a particular fundamental group $\pi_1(X,x_0)$ using combinatorial information on the graph of intersections of path components of U,V,W. For this it is useful to develop some combinatorial groupoid theory, as in Brown (2006), and Higgins (1971). Notice two special features of this method:

(i) The computation of the *invariant* one wants to obtain—*the fundamental group*—is derived from the computation of a larger structure, and so part of the work is to give methods for computing the smaller structure from the larger one. This usually involves non-canonical choices, such as that of a maximal tree in a connected graph.

(ii) The fact that the computation can be done is surprising in two ways: (a) The fundamental group is computed *precisely*, even though the information for it uses input in two dimensions, namely 0 and 1. This is contrary to the experience in homological algebra and algebraic topology, where the interaction of several dimensions involves exact sequences or spectral sequences, which give information only up to extension, and (b) the result is a *non-commutative invariant*, which is usually even more difficult to compute precisely. Thus exact sequences by themselves cannot show that a group is given as an HNN-extension: however such a description may be obtained from a pushout of groupoids, generalizing the pushout of groupoids in diagram 5.1 (see Brown 2006).

The reason for this success seems to be that the fundamental groupoid $\pi_1(X,X_0)$ contains information in *dimensions 0 and 1*, and therefore it can adequately reflect the geometry of the intersections of the path components of U,V,W and the morphisms induced by the inclusions of W in U and V. This fact also suggested the question of whether such methods could be extended successfully to *higher dimensions*. The Higher Homotopy van Kampen Theorem is discussed in the accompanying paper by Brown, Glazebrook and Baianu (2007a, in this volume).

5.4 Local-to-Global (LG) Construction Principles consistent with Quantum 'Axiomatics'

A novel approach to QST construction in Algebraic/Axiomatic QFT involves the use of generalized fundamental theorems of algebraic topology from specialized, 'globally well-behaved' topological spaces, to arbitrary ones. In this category are the generalized, *Higher Homotopy van Kampen theorems (HHvKT)* of Algebraic Topology with novel and unique non-Abelian applications. Such theorems greatly aid the calculation of higher homotopy of topological spaces. In the case of the Hurewicz theorem, this was generalized to arbitrary topological spaces (Spanier 1966), and establishes that certain homology groups are isomorphic to 'corresponding' homotopy groups of an arbitrary topological space. Brown and coworkers (2004–2006) went further and generalized the van Kampen theorem, at first to fundamental groupoids on a set of base points (Brown 1967), and then, to higher dimensional algebras involving, for example, homotopy double groupoids and 2-categories (Brown 2004). The more sensitive *algebraic invariant* of topological spaces seems to be, however, captured only by *cohomology* theory through an algebraic *ring* structure that is not accessible either in homology theory, or in the existing homotopy theory. Thus, two arbitrary topological spaces that have isomorphic homology groups may not have isomorphic cohomological ring structures, and may also not be homeomorphic, even if they are of the same homotopy type. The corollary of this statement may lead to an interesting cohomology-based classification in a category of certain Coh topological spaces that have isomorphic ring structures and are also homeomorphic. Furthermore, several *non-Abelian* results in algebraic topology could only be derived from the Generalized van Kampen Theorem (cf. Brown 2004a), so that one may find links of such results to the expected '*non-commutative* geometrical' structure of quantized space–time (Connes 1994). In this context, the important algebraic–topological concept of a *Fundamental Homotopy Groupoid (FHG) is applied to a Quantum Topological Space (QTS)* as a "partial classifier" of the *invariant* topological properties of quantum spaces of *any* dimension; quantum topological spaces are then linked together in a *crossed complex over a quantum groupoid* (Baianu et al. 2006a), thus suggesting the construction of global topological structures from local ones with well-defined quantum homotopy groupoids. The latter theme is then further pursued through defining locally topological groupoids that can be globally characterized by applying the Globalization Theorem, which involves the *unique* construction of the Holonomy Groupoid.

We shall consider in the last paper of this issue how this concept of a Locally Lie Groupoid might be applied in the context of Algebraic/Axiomatic Quantum Field Theory (AQFT) to provide a Local-to-Global Construction of Quantum Space Times in the presence of intense gravitational fields without generating singularities as in GR, even in the presence of black holes, 'with or without hair'. The result of this construction is a *Quantum Holonomy Groupoid* (QHG) which is unique up to an isomorphism.

6 Basic Structure of Categorical Ontology and the Theory of Levels. Emergence of Higher Levels and Their Sublevels

An effective Categorical Ontology requires, or generates—in the constructive sense—a '*structure*' rather than a discrete set of items. The classification process itself generates collections of items, as well as a *hierarchy of higher-level 'items'* of items, thus facing perhaps certain possible antimonies if such collections were to be just sets that are subject to the Axiom of Choice and problems arising from the set membership concept at different levels.

The categorical viewpoint as emphasized by Lawvere, etc., is that the key structure is that of *morphisms*, seen, for example, as abstract relations, mappings, functions, connections, interactions, transformations, etc. Therefore, in this section we shall consider both the Categorical viewpoint in the Ontology of Space and Time in complex/super-complex systems, as well as the fundamental structure of Categorical Ontology, as for example in the Ontological Theory of Levels (Poli 2001a,b, 2006a,b) which will be discussed briefly in the next section.

6.1 Duality Concepts in Theology, Philosophy and Category Theory

Duality is often used with several meanings in philosophy, running from opposites or contrary forces as in Hegel's theory of internal dialectics in dynamics to complementary items as in 'Yin and Yang', and apposites, too. Western philosophy has many such 'dual' concepts: matter-energy and ideas, brain and mind, existence and non-existence, cause and effect, living and inanimate, mortal and immortal, system and environment, simple and complex, 'good' and 'evil', Deity and man, and so on. On the other hand, in some of the Eastern philosophies, such dualism is obliterated, as for example in 'the unity of oneself with the environment' as stated in some Buddhist philosophies, but others remain, such as enlightened vs. un-enlightened, kharma vs. 'potential existence', or ku', but nevertheless all are subject to 'an universal law' of nature that governs life, consciousness and everything else. *Enlightenment*—according to such ancient beliefs—corresponds to the 'sudden realization of such an universal law' and does allow one 'to escape the repeated cycle of birth and death, thus eliminating all suffering'. One may note here a possible connection with the ancient Egyptian myth of the God Ossiris (murdered by his envious brother Seth) who was then 'revived' in the Spring by his wife (the goddess Isis), and who would cyclically come back to life every Spring, with the life-giving, fertile flood by the Nile. Similarly, in ancient Greek mythology, Persepona comes back to the surface of the earth in Spring only to return in late autumn to her husband in Hades. The myths also reflect man's very long-standing preoccupation with life and death, as well as his/her wish for eternal youth and immortality. Whereas in ancient Greek mythology all men are mortal, unless on extremely rare occasions some 'very special' individuals are immortalized by the gods (as for example in the case of Persepona), in ancient Egyptian mythology there is an 'afterlife', and so it is in the case of Christian beliefs also where the soul has an independent, immortal existence from that of the body, which is mortal. On the other hand, in Buddhism, there is no soul but only kharma and 'ku'—or potential existence—and the repeated cycle of birth, death and re-birth that could only be broken through enlightenment. Unwilling to accept any such unproven and unprovable, religious beliefs, many contemporary, Western philosophers of the last two centuries, deny the existence of a Deity—in the theological sense, but remain in favor of a dualist matter–idea, Cartesian philosophy. As religion and ideology played major social roles throughout all human history, and as reification may serve to enforce such social roles of religions and ideologies, philosophy has often been called upon to take a stand in matters pertaining to religions or ideologies. It is interesting that so far religion has won last century's battle for hegemony over marxist ideologies of various 'colors and strengths' in spite of very intensive propaganda and very strong ideological enforcement practiced by the dictatorial, marxist states whose only declared acceptable philosophy was the monistic brand of materialism.

When compared with the rich groups of problems posed by philosophical duality, the categorical concept of '*duality*' is deceptively simple and straightforward as it involves just the reversal of arrows or morphisms between objects, without changing the latter. In the standard quantum approach all microscopic processes are reversible and therefore the quantum dynamics involves basically a groupoid (microscopic dynamic) structure. *Self-dual* structures sometimes exist especially for globally commutative or Abelian structures in category theory (Freyd 1964; Oberst 1969). Interestingly also all quantum observable operators for finite systems are *self-adjoint*. On the other hand, Prigogine's time and Liouville *super-operators* for quantum systems with an infinite number of degrees of freedom (such as quantum *fields*) are not self-adjoint (or Hermitian) operators. As mental processes are often thought as occurring in an *irreversible* manner, such processes may not be 'self-dual' or even self-adjoint, thus making it rather unlikely that such processes are either quantum in nature or microscopic. A semi-classical, non-Abelian approach to mental processes, and especially consciousness, on the other hand would remain possible, as for example with an underlying many-valued logic, such as one of the (non-commutative) Łukasiewicz–Moisil logics.

6.2 Towards a Formal Theory of Levels

The first subsection here will present the fundamentals of the ontological theory of levels together with its further development in terms of mathematical categories, functors and natural transformations, as well as the necessary non-commutative generalizations of Abelian categorical concepts to non-Abelian formal systems and theories.

6.2.1 Fundamentals of Poli's Theory of Levels

The ontological theory of levels (Poli 2001a,b, 2006a,b, 2008) considers a hierarchy of *items* structured on different levels of existence with the higher levels *emerging* from the lower, but usually *not* reducible to the latter, as claimed by widespread reductionism. This approach draws from previous work by Hartmann (1935, 1952) but also modifies and expands considerably both its vision and range of possibilities. Thus, Poli (1998, 2001a, 2006a, b, 2008) considers four realms or *levels* of reality: Material-inanimate/Physico-chemical, Material-living/Biological, Psychological and Social. We harmonize this theme by considering categorical models of complex systems in terms of an evolutionary dynamic viewpoint using the mathematical methods of category theory which afford describing the characteristics and binding of levels, besides the links with other theories which, *a priori*, are essential requirements. The categorical techniques which form an integral part of the discussion provide a means of describing a hierarchy of levels in both a linear and interwoven, or *entangled*, fashion, thus leading to the necessary bill of fare: emergence, higher complexity and open, non-equilibrium/irreversible systems. We further stress that the categorical methodology intended is *intrinsically 'higher dimensional'* and can thus account for 'processes between processes…' within, or between, the levels—and sub-levels—in question.

Whereas a strictly Boolean classification of levels allows only for the occurrence of *discrete* ontological levels, and also does not readily accommodate either *contingent* or *stochastic sub-levels*, the LM-logic algebra is readily extended to *continuous*, *contingent* or even *fuzzy* (Baianu and Marinescu 1968) sub-levels, or levels of reality (cf. Georgescu 2006; Baianu 1977, 1987a,b; Baianu et al. 2006b). Clearly, a Non-Abelian Ontology of Levels would require the inclusion of either Q- or LM-logics algebraic categories because it begins at the fundamental quantum level—where Q-logic reigns—and 'rises' to the emergent ultra-complex level(s) with 'all' of its possible sub-levels represented by certain LM-logics.

Poli (2006a) has stressed a need for understanding *causal and spatiotemporal* phenomena formulated within a *descriptive categorical context* for theoretical levels of reality. There are three main points to be taken into account: differing spatiotemporal regions necessitate different (levels of) causation, for some regions of reality analytic reductionism may be inadequate, and there is the need to develop a *synthetic* methodology in order to compensate for the latter, although one notes (cf. Rosen 1999) that analysis and synthesis are not the exact inverse of each other. Following Poli (2001a,b), we consider a causal dependence on levels, somewhat apart from a categorical dependence. At the same time, we address the *internal dynamics*, the *temporal rhythm, or cycles*, and the subsequent unfolding of reality. The genera of corresponding concepts such as ‘processes’, ‘groups’, ‘essence’, ‘stereotypes’, and so on, can be simply referred to as ‘*items*’ which allow for the existence of many forms of causal connection (Poli 2008). The implicit meaning is that the *irreducible multiplicity* of such connections converges, or it is ontologically integrated within a *unified synthesis*. Rejecting reductionism thus necessitates accounting for an irreducible multiplicity of ontological levels, and possibly the ontological acceptance of many worlds also. In this regard, the Brentano hypothesis is that the class of physical phenomena and the class of psychological (or spiritual) phenomena are *complementary*; in other words, physical categories were said to be ‘orthogonal’ to psychological categories (Poli 2006a, b).

As befitting the situation, there are devised *universal* categories of reality in its entirety, and also subcategories which apply to the respective sub-domains of reality. Following Poli (2001a,b), the ontological procedures in question provide:

- coordination between categories (for instance, the interactions and parallels between biological and ecological reproduction);
- modes of dependence between levels (for instance, how the co-evolution/interaction of social and mental realms depend and impinge upon the material);
- the categorical closure (or completeness) of levels.

Already we can underscore a significant component of this essay that relates the ontology to geometry/topology; specifically, if a level is defined via ‘iterates of local procedures’ (cf ‘items in iteration’, Poli (2001a,b), then we have some handle on describing its intrinsic governing dynamics (with feedback) and, to quote Poli (2001a,b), to ‘restrict the *multi-dynamic* frames to their linear fragments’.

On each level of this ontological hierarchy there is a significant amount of connectivity through inter-dependence, interactions or general relations often giving rise to complex patterns that are not readily analyzed by partitioning or through stochastic methods as they are neither simple, nor are they random connections. But we claim that such complex patterns and processes have their logico-categorical representations quite apart from classical, Boolean mechanisms. This ontological situation gives rise to a wide variety of networks, graphs, and/or mathematical categories, all with different connectivity rules, different types of activities, and also a hierarchy of super-networks of networks of subnetworks. Then, the important question arises what types of basic symmetry or patterns such super-networks of items can have, and how do the effects of their sub-networks percolate through the various levels. From the categorical viewpoint, these are of two basic types: they are either *commutative* or *non-commutative*, where, at least at the quantum level, the latter takes precedence over the former, as we shall further discuss and explain in the following sections.

6.3 Connectivity and Bionetwork Topology: Genetic Ontology and Interactomics Reconstruction

One may place special emphasis on network topology and connectivity in Genetic Ontology and Categorical Biology since these concepts are becoming increasingly important in modern biology, as realized in rapidly unfolding areas such as post-Genomic Biology, *Proteomics* and *Interactomics* that aim at relating structure and protein–protein-biomolecule interactions to biological function. The categories of the biological/genetic/ecological/ levels may be seen as more 'structured' compared with those of the cognitive/mental levels (hinging on epiphenomenalism, interactive dualism, etc.) which may be seen as 'less structured', but not necessarily having less structural power owing to the increased abstraction in their design of representation. We are here somewhat in concert with Hartmann's (1952) laws of autonomy.

6.4 Dynamic Emergence of the Higher Complexity Levels: Organisms, the Human Mind and Society

We shall be considering the question of how biological, psychological and social functions are entailed through *emergent* processes of increasing complexity in higher-dimensional spacetime structures that are essential to Life, Evolution of Species and Human Consciousness. Such emergent processes in the upper three levels of reality considered by Poli (2006b) have corresponding, defining levels of increasing dynamic complexity from biological to psychological and, finally, to the social level. It is therefore important to distinguish between the *emergent* processes of higher complexity and the underlying, component physicochemical processes, especially when the latter are said to be '*complex*' by physicists only because they occur either as a result of 'sensitivity' to initial conditions, small perturbations, etc., or because they give rise to unpredictable behavior that cannot be completely simulated on any digital computer; the latter systems with (deterministic) chaotic dynamics are *not*, however, *emergent* systems because their existence does not belong to a higher-level of reality than the simple dynamic systems that are completely predictable. We are here defending the claim that all 'true' dynamic complexity of higher order is *irreducible* to the dynamics of sub-processes—usually corresponding to a lower level of reality—and it is therefore a truly *emergent*, real phenomenon. In other words, **no emergence** ⇒ **no complexity** higher than that of physicochemical systems with chaos, whereas reductionists now attempt to reduce everything, from life to societies and ecology, to systems with just chaotic behavior.

The detailed nature of the higher-level emergence will be further developed and treated in a more formal/precise manner in Sects. 7 through 15 after introducing and developing first the novel, pre-requisite concepts that allow a vastly improved understanding of dynamic emergent processes in higher dimensions of spacetime structures.

There is an ongoing ambiguity in the current use of the term 'complex', as in 'complex dynamics and dynamical systems'—which is employed by chaotic physics reports and textbooks with a very different meaning from the one customarily employed in Relational Biology (Rosen 1987; and also earlier, the more general definitions proposed by Baianu 1968 through 1987). We propose to retain the term 'complexity'—in accord with the use adopted for the field of physico-chemical chaotic dynamics established by modern physicists and chemists. Then, in order to avoid the recurring confusion that would occur between inanimate, chaotic or robotic, systems that are 'complex' and living organisms which are at a distinctly higher-level of dynamic complexity, we propose to define the latter, higher complexity level of *biosystems* as '*supercomplex*'. Thus, we suggest that the *biological* complex systems—whose dynamics is quite distinct from that of *physical* 'complex systems'—should be called *'supercomplex'* (Baianu and Poli 2008). (Elsasser also claimed that living organisms are 'extremely complex', as discussed in a recent report (Baianu 2006)). From a reductionist's viewpoint, such a distinction may appear totally unnecessary because a reductionist does believe (*without any possibility of proof*) that all systems—complex or otherwise—ultimately obey only known physical laws, as the complex systems can be 'reduced' (by unspecified, and/or unspecifiable, procedures!) to a finite collection of the simple component systems contained in any selected complex system. For example, such a collection of parts could be assembled through a categorical *colimit*, as it will be shown in a subsequent Section 12. Note also that a categorical colimit is defined not just by its parts but also by the morphisms between the objects, which conforms with the naive view that an engine, say, is not just a collection of parts, but depends crucially on how they are put together, if it is to work! Any suggestion of alternative possibilities is regarded by the reductionist approach as an attempt to introduce either 'ghosts' or undefinable entities/relations that 'could not physically exist', according to (simple) physical principles that govern the dynamics of (simple) physical systems. Although this line of reasoning seems to satisfy Occam's razor principle—taken as an 'economy' of thought—it does exclude both life and human consciousness from having any independent, or even *emergent*, ontological existence. Taken to its ultimate extreme, this 'simple' reductionist approach would seem to demand the reduction of even human societies not only to collections of individual people but also to the 'elementary' particles and quantum-molecular fields of which humans are made of.

Interestingly, the term 'super-complex' is already in use in the computer industry for high performance digital computer systems designed with a high-degree of parallel processing, whose level of complexity is, however, much lower than that of physicochemical chaotic systems that are called 'complex' by physicists. On the other hand, in the fields of structural and molecular biology, the term 'super-complex' recently designates certain very large super-aggregates of biopolymers that are functional within a cell. Thus, our proposed use of the term ⟨*super-complex*⟩ is for the higher-level of organization—that of the *whole, functional organism*, not for the first (physicochemical) level of reality—no matter how complicated, 'chaotic' or intricate it is at the molecular/atomic/quantum level. Therefore, in our proposed terminology, *the level of supercomplex dynamics is the first emergent level*—which does correspond to the first emergent level of reality in the ontological theory of levels recently proposed by Poli (2006a, b). A more precise formulation and, indeed, resolution of such emergent complexity issues will be presented in Sect. 9.

Our approach from the perspectives of spacetime ontology and dynamic complexity thus requires a reconsideration of the question how new levels of dynamic complexity arise at both the biological and psychological levels. Furthermore, the close interdependence/two-way relations of the psychological and social levels of reality (Poli 2006a) do require a consideration of the correlations between the dynamic complexities of human consciousness and human society. The *emergence* of one is ultimately determined by the other, in what might be expressed as *iterated feedback and/or feedforward loops*, though not restricted to the engineering meaning which is usually implied by these terms. Thus, *feedforward* loops should be understood here in the sense of *anticipatory* processes, that can, for example, lead in the future to the improvement of social interactions through deliberate, conscious human planning—or even more—to the prevention of the human, and other species, extinction. Further *inter-relations* among the different ontological levels of system complexity are discussed in in Baianu and Poli (2008).

7 Categorical Representations of the Ontological Theory of Levels: From Abelian Categories to Non-Abelian Theories

General system analysis seems to require formulating ontology by means of categorical concepts (Poli 2008, TAO-1; Baianu and Poli 2008). Furthermore, category theory appears as a natural framework for any general theory of transformations or dynamic processes, just as group theory provides the appropriate framework for classical dynamics and quantum systems with a finite number of degrees of freedom. Therefore, we shall adopt here a categorical approach as the starting point, meaning that we are looking for *"what is universal"* (in some domain, or in general), and that for simple systems this involves *commutative* modeling diagrams and structures (as, for example, in Fig. 1 of Rosen 1987). Note that this ontological use of the word '*universal*' is quite distinct from the mathematical use of '*universal property*', which means that a property of a construction on particular objects is defined by its relation to *all* other objects (i.e., it is a *global* attribute), usually through constructing a morphism, since this is the only way, in an *abstract* category, for objects to be related. With the first (ontological) meaning, the most universal feature of reality is that it is *temporal*, i.e. it changes, it is subject to countless transformations, movements and alterations. In this select case of *universal temporality*, it seems that the two different meanings can be brought into superposition through appropriate formalization. Furthermore, *concrete* categories may also allow for the representation of ontological 'universal items' as in certain previous applications to *cat-neurons*—categories of neural networks (Baianu 1972; Ehresmann and Vanbremeersch 2006; Healy and Caudell 2006).

As we shall be considering here only a few special cases of modeling diagrams that include simple, reductionist systems in order to compare them with super-complex biological systems, the following discussion in Sects. 8 through 9 will require just the use of such 'concrete' categories of 'sets with structure' (e.g., groups, groupoids, crossed complexes, etc.) For general categories, however, each object is a kind of a Skinnerian black box, whose only exposure is through input and output, i.e. the object is given by its *connectivity* through various morphisms, to other objects. For example, the opposite of the category of sets has objects but these have *no structure* from the categorical viewpoint. Other types of category are important as expressing useful relationships on structures, for example *lextensive* categories, which have been used to express a general van Kampen theorem by Brown and Janelidze (1997).

This concrete categorical approach seems also to provide an elegant formalization that matches the ontological theory of levels briefly described above. The major restriction—as well as for some, attraction—of the 3-level categorical construction outlined above seems to be its built-in *commutativity* (see also Sect. 9.6 for further details). Note also how 2-arrows become '3-objects' in the meta-category, or '3-category', of functors and natural transformations. This construction has already been considered to be suitable for representing dynamic processes in a generalized Quantum Field Theory. The presence of mathematical structures is just as important for highly complex systems, such as organisms, whose organizational structure—in this mathematical and biological function/physiological sense—may be superficially apparent but difficult to relate unequivocally to anatomical, biochemical or molecular 'structures'. Thus, abstract mathematical structures are developed to define *relationships*, to deduce and calculate, to exploit and define analogies, since *analogies are between relations* between things rather than between things themselves.

As *structures* and *relations* are present at the very core of mathematical developments (Ehresmann 1965, 1966), the theories of categories and toposes distinguish at least two fundamental types of items: *objects* and *arrows* (also called suggestively '*morphisms*'). Thus, first-level arrows may represent mappings, relations, interactions, dynamic transformations, and so on, whereas categorical objects are usually endowed with a selected type of structure only in 'concrete' categories of 'sets with structure'. Note, however, that simple sets have only the 'discrete topology structure', consisting of just discrete elements, or points.

A description of a new structure is in some sense a development of part of a new language. The notion of *structure* is also related to the notion of *analogy*. It is one of the triumphs of the mathematical theory of categories in the 20th century to make progress in *unifying* mathematics through the finding of *analogies* between various behavior of structures across different areas of mathematics. This theme is further elaborated in the article by Brown and Porter (2002) which argue that many analogies in mathematics, and in many other areas, are *not* between objects themselves but *between the relations* between objects. Here, we mention as an example, only the categorical notion of a *pushout*, which we shall use later in discussing the higher homotopy, generalized van Kampen theorems. A pushout has the same definition in different categories even though the construction of pushouts in these categories may be widely different. Thus, focusing on the *constructions* rather than on the *universal properties* may lead to a failure to see the analogies. Super-pushouts, on the other hand, were reported to be involved in multi-stability and metamorphoses of living organisms (Baianu 1970). Charles Ehresmann developed new concepts and new language which have been very influential in mathematics; we mention here only those of holonomy groupoid, Lie groupoid, fibre bundles, foliations, germs and jets. There are other concepts whose time perhaps is just coming or has yet to come: included here might be ordered groupoids, *variable groupoids* and *multiple categories*. Disclosing new worlds is as worthwhile a mathematical enterprise as proving old conjectures. For example, we are also seeking *non-Abelian* methods for higher dimensional local-to-global problems in homotopy theory.

One must note in the latter case above the use of a very different meaning of the word 'structure', which is quite distinct from that of the organizational/physiological and mathematical structure introduced at the beginning of this section. Even though concrete, molecular or anatomical 'structures' could also be defined with the help of 'concrete sets with structure', the physical structures representing 'anatomy' are very different from those representing physiological–functional/organizational structures. Further aspects of this representation problem for systems with highly complex dynamics, together with their structure–functionality relationships, will be discussed in Sects. 10.2.

In reference to the above-discussion, one of the major goals of category theory is to see how the properties of a particular mathematical structure, say S, are reflected in the properties of the category $\mathsf{Cat}(S)$ of all such structures and of morphisms between them. Thus the first step in category theory is that a definition of a structure should come with a definition of a morphism of such structures. Usually, but not always, such a definition is obvious. The next step is to compare structures. This might be obtained by means of a *functor* $A : \mathsf{Cat}(S) \longrightarrow \mathsf{Cat}(T)$. Finally, we want to compare such functors $A, B : \mathsf{Cat}(S) \longrightarrow \mathsf{Cat}(T)$. This is done by means of a natural transformation η: $A \Rightarrow B$. Here η assigns to each object X of $\mathsf{Cat}(S)$ a morphism $\eta(X) : A(X) \longrightarrow B(X)$ satisfying a commutativity condition for any morphism $a : X \longrightarrow Y$. In fact we can say that η assigns to each morphism a of $\mathsf{Cat}(S)$ a commutative square of morphisms in $\mathsf{Cat}(T)$ (as shown in Diagram 12.3 here, and also Diagram 2.7 in Brown Glazebrook and Baianu (2007). This notion of *natural transformation* is at the heart of category theory. As Eilenberg-Mac Lane write: "*to define natural transformations one needs a definition of functor, and to define the latter one needs a definition of category*."

As explained in next subsection 3.3, the second level arrows, or 2-arrows ('*functors*') representing relations, or comparisons, between the first level 'concrete' categories of 'sets with structure' do not 'look inside' the 1-objects, which may appear as necessarily limiting the mathematical construction; however, the important ability to 'look inside' 1-objects at their structure, for example, is recovered by the third level arrows, or 3-arrows, in terms of natural transformations. For example, if A is an object in a mathematical category C, E is a certain 'corresponding' object in a category D and F is a covariant functor $F : \mathsf{C} \longrightarrow \mathsf{D}$, such that $F(A) = E$, then one notes that F carries the whole object A into the category D without 'looking' inside the object A at its components; in the case when A is a set the functor F does not 'look' at the elements of A when it 'transforms' the whole set A into the object E (which does not even have to be a set; a functor F, therefore, does not act like a 'mapping' on elements). On the other hand, natural transformations in the case of *concrete* categories do define mappings of objects with structure by acting first on functors, and then by imposing the condition of naturality on diagrams, such as (12.3), that also include comparisons between *functorial mappings of morphisms* under Mathematical categories, functors and natural transformations).

From the point of view of mathematical modeling, the mathematical theory of categories models the dynamical nature of reality by representing temporal changes through either *variable* categories or through *toposes*. According to Mac Lane and Moerdijk (1992) certain variable categories can also be generated as a topos. For example, the category of sets can be considered as a topos whose only generator is just a single point. A variable category of varying sets might thus have just a generator set.

The claim advanced by several recent textbooks and reports is that standard topos theory may also suit to a significant degree the needs of complex systems. Such claims, however, do not seem to draw any significant, qualitative ontological distinction between 'simple' and 'complex' systems, and furthermore, they do not satisfy also the second condition (naturality of modeling diagrams, as pointed out in Rosen 1987). As it will be shown in Sect. 9, a qualitative distinction *does exist*, however, between organisms—considered as complex systems—and 'simple', inanimate dynamical systems, in terms of the modeling process and the type of predictive mathematical models or representations that they can have (Rosen 1987, and also, previously, Baianu 1968 through 1987).

A Hierarchical, Formal Theory of Levels. Commutative and Non-Commutative Structures: Abelian Category Theory vs. Non-Abelian Theories: One could formalize-for example as outlined in Baianu and Poli (2008)–the hierarchy of multiple-level relations and structures that are present in biological, environmental and social systems in terms of the mathematical Theory of Categories, Functors and Natural Transformations (TC-FNT, see subsection in Brown et al. (2007)). On the first level of such a hierarchy are the links between the system components represented as '*morphisms*' of a structured category which are subject to several axioms/restrictions of Category Theory, such as *commutativity* and associativity conditions for morphisms, functors and natural transformations. Among such mathematical structures, *Abelian* categories have particularly interesting applications to rings and modules (Popescu 1973; Gabriel 1962) in which commutative diagrams are essential. Commutative diagrams are also being widely used in Algebraic Topology (Brown 2006; May 1999). Their applications in computer science also abound.

Then, on the second level of the hierarchy one considers '*functors*', or links, between such first level categories, that compare categories without 'looking inside' their objects/ system components.

On the third level, one compares, or links, functors using '*natural transformations*' in a 3-category (meta-category) of functors and natural transformations. At this level, natural transformations not only compare functors but also look inside the first level objects (system components) thus 'closing' the structure and establishing 'the universal links' between items as an integration of both first and second level links between items. The advantages of this constructive approach in the mathematical theory of categories, functors and natural transformations have been recognized since the beginnings of this mathematical theory in the seminal paper of Eilenberg and Mac Lane (1945). Note, however, that in general categories the objects have no 'inside', though they may do so for example in the case of 'concrete' categories.

A relevant example of applications to the natural sciences, e.g., neurosciences, would be the higher-dimensional algebra representation of processes of cognitive processes of still more, linked sub-processes (Brown 2004). Additional examples of the usefulness of such a categorical constructive approach to generating higher-level mathematical structures would be that of supergroups of groups of items, 2-groupoids, or double groupoids of items. The hierarchy constructed above, up to level 3, can be further extended to higher, *n*-levels, always in a consistent, natural manner, that is using commutative diagrams. Let us see therefore a few simple examples or specific instances of commutative properties. The type of global, natural hierarchy of items inspired by the mathematical TC-FNT has a kind of *internal symmetry* because at all levels, the link compositions are *natural*, that is, all link compositions that exist are independent on the path taken between the starting and the last object, thus yielding a path-independent composition result. This general property of such link composition chains or diagrams involving any number of sequential links is called *commutativity* (see for example Samuel and Zarisky 1957), and is often expressed as a *naturality condition for diagrams*. This key mathematical property also includes the mirror-like symmetry $x \star y = y \star x$ when x and y are operators and the symbol '$\star$' represents the operator multiplication. Then, the equality of $x \star y$ with $y \star x$ defines the statement that "the x and y operators *commute*"; in physical terms, this translates into a sharing of the same set of eigenvalues by the two commuting operators, thus leading to 'equivalent' numerical results i.e., up to a multiplication constant); furthermore, the observations X and Y corresponding, respectively, to these two operators would yield the same result if X is performed before Y in time, or if Y is performed first followed by X. This property, when present, is very convenient for both mathematical and physical applications (such as those encountered in quantum mechanics). When commutativity is global in a structure, as in an Abelian (or commutative) group, commutative groupoid, commutative ring, etc., such a structure that is commutative throughout is usually called ***Abelian***. However, in the case of category theory, this concept of Abelian structure has been extended to a special class of categories that have meta-properties formally similar to those of the category of commutative groups, *Ab*-**G**; the necessary and sufficient conditions for such 'Abelianness' of categories other than that of Abelian groups were expressed as three axioms **Ab1** to **Ab3** and their duals (Freyd 1964; see also the details in our third paper in this issue and Oort 1970). A first step towards re-gaining something like the 'global commutativity' of an Abelian group is to require that all classes of morphisms [A,B] or Hom(A,B) have the structure of commutative groups; subject to a few other general conditions such categories are called **additive**. Then, some kind of global commutativity is assured for all morphisms of *additive* categories. However, in order to ensure that an additive category is well 'modeled' by the category of Abelian groups, according to

Mitchell (1965), it must also be exact and have finite products. The exactness condition amounts to requiring that each morphism in an additive category ***A*** can be decomposed into, or expressed as the composition of, an epimorphism and monomorphism, in addition to requiring that ***A*** has kernels, cokernels, and also that it is both normal and conormal; the requirement that ***A*** is **normal** expresses the condition that every monomorphism in ***A*** is a kernel, whereas the requirement that ***A*** is ***conormal*** means that every epimorphism of ***A*** must be a cokernel. Implicitly, ***A*** has a null object, **0**, the Ab1 axiom of Freyd (1964). Moreover, one can trace back such requirements as exactness, null object, normality and conormality to (commutative) Homological Algebra (Cartan and Eilenberg 1956; Grothendieck 1957; Heller 1958; Bourbaki 1961–1964; Mac Lane 1963). Additional properties for Abelian categories were also posited for extending applications of Abelian category theory to other fields of modern mathematics (Grothendieck 1957; Grothendieck and Dieudonné 1960; Huber 1962; Roos 1967; Stenström 1968; Oberst 1969; Gablot 1971; Popescu 1973). A Homotopy theory was also formulated in Abelian categories (Kleisli, 1962). The equivalence of Abelian categories was reported by Roux, and important imbedding theorems were proved by Mitchell (1964) and by Lubkin (1960); a characterization of Abelian categories with generators and exact limits was presented by Gabriel and Popescu (1964). As one can see from both earlier and recent literature, Abelian categories have been studied in great detail, even though one cannot say that all their properties have been already found.

Unfortunately, not all operators 'commute', and not all categorical diagrams or mathematical structures are, or need be, commutative. *Non-commutativity* may therefore appear as a result of 'breaking' the 'internal symmetry' represented by commutativity. As a physical analogy, this might be considered a kind of '*symmetry breaking*' which is thought to be responsible for our expanding Universe and CPT violation, as well as many other physical phenomena such as phase transitions and superconductivity (Weinberg 1995; 2000).

The more general case is, therefore, the *non-commutative* one. On the other hand, one is used to encounter—not only in the sciences but also in the visual arts—things or patterns, or items that are considered to be 'beautiful', in the sense of being *symmetric*, perhaps with the possible exception of certain abstract paintings that ignore simple symmetries. Furthermore, with very few exceptions, the educational systems are over-emphasizing in both mathematics and physics *commutative* structures, such as Abelian Lie groups, commutative homology theory, Abelian Algebraic Topology, and *Abelian* theories such as Newtonian or GR/SR theories in physics. As an example, several standard space forms are representable in the quotient form G/K where G is a Lie group and $K \subset G$ is a closed subgroup, that is, as *homogeneous spaces* usually with the extra property of symmetry (thus *symmetric spaces*). The *n*-sphere S^n, for instance is such a symmetric space, but in the traditional Riemannian–geometric sense it is not normally considered as a 'non-commutative space' unless it is 'quantized' by some means (à la Connes 1994), and that is indeed a separate matter.

Whereas the Abelian Lie groups can be considered as 'flat', certain non-Abelian Lie groups can be viewed as the the most basic Riemannian manifolds with non-trivial *curvature* properties and, thus, might provide a useful basis for generating curved quantum supergravity spacetimes through graded Lie algebras (Weinberg 2004; see also Baianu et al. 2007b).

Thus, one may be often prejudiced to favor commutative structures and Abelian theories (Freyd 1964; Mitchell 1965; Gabriel 1962; Popescu 1973,1975) that rely heavily on 'symmetric' representations which are either attractive, seductively elegant, or simply 'beautiful', but not necessarily true to our selected subject of discourse—that is, the real spacetime in our universe. Several intriguing counter-examples are provided by certain ('non-commutative') *asymmetric* drawings by Escher such as his perpetuum water mill or his 3D-evading, illusory castle with monks 'climbing' from one level to the next at 'same-height' (that might be considered as a hint at the reductionist puzzle caused by the imposition of only one level of reality, similar to Abbott's flatland).

An example of a non-commutative structure relevant to Quantum Theory is that of the *Clifford algebra* of quantum observable operators (Dirac 1962; see also Plymen and Robinson 1994). Yet another—more recent and popular—example in the same QT context is that of C^*-algebras of (quantum) Hilbert spaces. A few early studies of non-commutative structures have been reported (Ore 1931; Croisot and Lasier 1963; Goldie 1967; Silver 1967; Knight 1970).

7.1 Non-Abelian Theories

Last-but-not least, there are the interesting mathematical constructions of non-commutative 'geometric spaces' obtained by 'deformation' introduced by Connes (1994) as possible models for the physical, quantum space–time which will be further discussed in our companion paper (Baianu et al. 2007b). Thus, the microscopic, or quantum, 'first' level of physical reality does *not* appear to be subject to the categorical naturality conditions of Abelian TC-FNT—the 'standard' mathematical theory of categories (functors and natural transformations). It would seem therefore that the commutative hierarchy discussed above is not sufficient for the purpose of a General, Categorical Ontology which considers all items, at all levels of reality, including those on the 'first', quantum level, which is non-commutative. On the other hand, the mathematical, Non-Abelian Algebraic Topology (Brown, Glazebrook and Baianu 2007, in this volume), the Non-Abelian Quantum Algebraic Topology (NA-QAT; Baianu et al. 2006a, b), and the physical, Non-Abelian Gauge theories (NAGT) may provide the ingredients for a proper foundation for Non-Abelian, hierarchical multi-level theories of a super-complex system dynamics in a General Categorical Ontology (GCO). Furthermore, it was recently pointed out (Baianu et al. 2006a, b) that the current and future development of both NA-QAT and of a quantum-based Complex Systems Biology, *a fortiori*, involve *non-commutative*, many-valued logics of quantum events, such as a modified Łukasiewicz–Moisil (LMQ) logic algebra (Baianu et al. 2006b), complete with a fully developed, novel probability measure theory grounded in the LM-logic algebra (Georgescu 2006). The latter paves the way to a new projection operator theory founded upon the *non-commutative quantum logic of events*, or dynamic processes, thus opening the possibility of a complete, *Non-Abelian Quantum theory*.

Furthermore, such recent developments point towards a paradigm shift in Categorical Ontology and to its extension to more general, *Non-Abelian theories*, well beyond the bounds of commutative structures/spaces and also free from the *logical* restrictions and limitations imposed by the Axiom of Choice to Set Theory. Additional restrictions imposed by representations using set theory also occur as a result of the 'primitive' notion of set membership, and also because of the 'discrete topology', very impoverished structure of simple sets. It is interesting that D'Arcy W. Thompson also arrived in 1941 at an ontologic "*principle of discontinuity*" which "is inherent in all our classifications, whether mathematical, physical or biological… In short, nature proceeds *from one type to another* among organic as well as inorganic forms… and to seek for stepping stones across the gaps between is to seek in vain, for ever. Our geometrical analogies weigh heavily against Darwin's conception of endless small variations; they help to show that discontinuous variations are a natural thing, that "mutations"—or sudden changes, greater or less—are bound to take place, and new "types" to have arisen, now and then." (p. 1094 of Thompson 1994, re-printed edition).

7.2 Ontological Organization of Systems in Space and Time: Classification in Categories of Items with Reference to Space and Time

Ontological classification based on items involves the organization of concepts, and indeed theories of knowledge, into a *hierarchy of categories of items at different levels of 'objective reality'*, as reconstructed by scientific minds through either a *bottom-up* (induction, synthesis, or abstraction) process, or through a *top-down* (deduction) process (Poli 2008), which proceeds from abstract concepts to their realizations in specific contexts of the 'real' world. A more formal approach to this problem will be considered in the following Sect. 9, with several ontological examples being also provided in subsequent sections and two related articles (Baianu and Poli 2008, and Baianu et al. 2007; in this volume). The conceptual foundation for such effective formulations in terms of different level categories and their higher-order relations has been already outlined in the preceding subsections.

7.2.1 Chronotopoids

The hierarchical theory of levels paves the way towards the claim that there could be different families of times and spaces, each with its own structure and dynamics, symmetric or otherwise. Following Poli (2008), one could treat the general problem of space and time as a problem of *chronotopoids* (understood jointly, or separated into '*chronoids*' *and* '*topoids*'). The guiding intuition is that each level of reality comes equipped with its own family of chronotopoids (as originally introduced by Poli 2008). Note also that the correct quantization of time may be the major required step towards a consistent quantum theory to the Planck limit, as energy is divided into quanta and frequency also changes in discrete steps in molecular, atomic and sub-atomic/nuclear systems. Thus '*chronoids*' may be thought—in a quantum sense—as consisting of *chronon regions* in the Planck limit.

7.2.2 Categorical Logics of Processes and Structures: Universal Concepts and Properties

The logic of classical events associated with either mechanical systems, mechanisms, universal Turing machines, automata, robots and digital computers is generally understood to be simple, *Boolean* logic. The same applies to Einstein's GR. It is only with the advent of quantum theories that quantum logics of events were introduced which are *non-commutative*, and therefore, also *non-Boolean*. Somewhat surprisingly, however, the connection between quantum logics (QL) and other *non-commutative* many-valued logics, such as the Łukasiewicz logic, has only been recently made (Dalla Chiara 2004 and refs. cited therein; Baianu 2004; Baianu et al. 2006a, b). The universal properties of categories of LM-logic algebras are, in general, categorical constructions that can be, in particular cases,'just universal objects'—which still involve categorical constructions; therefore such a danger of confusion does not arise at all in this context. Such considerations are of potential interest for a wide range of complex systems, as well as quantum ones, as it has been pointed out previously (Baianu 1977, 2004; Baianu et al. 2006a,b). Furthermore, both the concept of 'Topos' and that of variable category, can be further generalized by the involvement of *many-valued* logics, as for example in the case of 'Łukasiewicz–Moisil, or LM Topos' (Baianu et al. 2005). This is especially relevant for the development of theories on *non-Abelian dynamics* of complex and super-complex systems; it may also be essential for understanding human consciousness (as it will be discussed in the context of Sect. 14).

Whereas the hierarchical theory of levels provides a powerful, systematic approach through categorical ontology, the foundation of science involves *universal* models and theories pertaining to different levels of reality. Such theories are based on axioms, principles, postulates and laws operating on distinct levels of reality with a specific degree of complexity. Because of such distinctions, inter-level principles or laws are rare and over-simplified principles abound. As relevant examples, consider the Chemical/Biochemical Thermodynamics, Physical Biochemistry and Molecular Biology fields which have developed a rich structure of specific-level laws and principles, however, without 'breaking through' to the higher, emergent/integrative level of organismic biology. This does not detract of course from their usefulness, it simply renders them incomplete as theories of biological reality. With the possible exceptions of Evolution and Genetic Principles or Laws, Biology has until recently lacked other universal principles for highly complex dynamics in organisms, populations and species, as it will be shown in the following sections. One can therefore consider Biology to be at an almost 'pre-Newtonian' stage by comparison with either Physics or Chemistry.

It will be therefore worthwhile considering the structure of scientific theories and how it could be improved to enable the development of emergence principles for various complexity levels, including those of the *inter(active)-level* types.

The prejudice prevailing towards 'pure', i.e. unmixed, levels of reality, and its detrimental effects on the development of Life sciences, Psychology, Sociology and Environmental sciences will also be discussed in the next section. Then, alternatives and novel, possible solutions are presented in subsequent sections and the closing subsection of Brown, Glazebrook and Baianu (2007, in this volume).

8 Theories: Axioms, Principles, Postulates and Laws. Occam's Razor and Einstein's Dictum. Analogies and Metaphors

The more rigorous scientific theories, including those founded in Logics and Mathematics, proceed at a fundamental level from axioms and principles, followed in the case of 'natural sciences' by laws of nature that are valid in specific contexts or well-defined situations. Whereas axioms are rarely invoked in the natural sciences perhaps because of their abstract and exacting attributes, (as well as their coming into existence through elaborate processes of repeated abstraction and refinement), postulates are 'obvious assumptions' of extreme generality that do not require proof but just like axioms are accepted on the basis of their very numerous, valid consequences. Somewhat surprisingly, principles and laws, even though quite strict, may not apply under certain exceptional situations. Natural laws are applicable to well-defined zones or levels of reality, and are thus less general, or universal, than principles. Different books often interchange liberally principles for laws. Whereas Newton's "Principia" introduced 'principles', the latter are nowadays called the Laws of Mechanics by standard textbooks, as they can be expressed as simple mathematical formulae—which is often the form taken by physical laws. Principles are instead often explained in words, and tend to have the most general form attainable/acceptable in an established theory. It is interesting to note that in Greek, and later Roman antiquity, both philosophers and orators did link philosophy and logic; moreover, in medieval time, first Francis Bacon, then Newton opted for quite precise formulations of "natural philosophy" and a logical approach to 'objective' reality. In Newton's approach, the logical and precise formulation of such "natural principles" demanded the development of mathematical concepts suitable for the exact determination and quantification of the rate of a change in the "state of motion" of any mechanical body, or system. Later philosophical developments have strayed from such precise formulations and, indeed, mathematical developments seem to have lost their appeal in 'natural philosophy'.

On the other hand, it would seem natural to expect that theories aimed at different ontological levels of reality should have different principles. Furthermore, one may philosophically, and indeed ontologically, address the question of why such distinct levels of reality originated in the first place, and then developed, or emerged, both in space and time. Without reverting to any form of Newtonian or quantum-mechanical determinism, we are also pointing out in this essay the need for developing precise but nevertheless 'flexible' concepts and novel mathematical representations suitable for understanding the emergence of the higher complexity levels of reality.

Interestingly, the founder of Relational Biology, Nicolas Rashevsky (1954, 1968) proposed that physical laws and principles can be expressed in terms of *mathematical functions*, or mappings, and are thus being predominantly expressed in a *numerical* form, whereas the laws and principles of biological organisms and societies need take a more general form in terms of quite general, or abstract–mathematical and logical relations which cannot always be expressed numerically; the latter are often qualitative, whereas the former are predominantly quantitative. According to his suggested criterion, string theories may not be characteristic of the physical domain as they involve many qualitative relations and features. In this respect, one may also suggest that modern, Abstract Art, in its various forms—if considered as a distinct class of representations—has moved ahead of modern philosophy to attempt universal representations of reality in a precise but flexible manner, thus appealing to both reason and emotions combined.

8.1 Towards Biological Postulates and Principles

Often, Rashevsky considered in his Relational Biology papers, and indeed made comparisons, between established physical theories and principles. He was searching for new, more general relations in Biology and Sociology that were also compatible with the former. Furthermore, Rashevsky also proposed two biological principles that add to Darwin's natural selection and the 'survival of the fittest principle', *the emergent relational structure defining adaptive organisms*:

1. The Principle of Optimal Design, and
2. The Principle of Relational Invariance (phrased by Rashevsky as "*Biological Epimorphism*").

In essence, the 'Principle of Optimal Design' defines the 'fittest' organism which survives in the natural selection process of competition between species, in terms of an extremal criterion, similar to that of Maupertuis; the optimally 'designed' organism is that which acquires maximum functionality essential to survival of the successful species at the lowest 'cost' possible. The 'costs' are defined in the context of the environmental niche in terms of material, energy, genetic and organismic processes required to produce/entail the pre-requisite biological function(s) and their supporting anatomical structure(s) needed for competitive survival in the selected niche. Further details were presented by Robert Rosen in his short but significant book on optimality (1970). The 'Principle of Biological Epimorphism' on the other hand states that the highly specialized biological functions of higher organisms can be mapped (through an epimorphism) onto those of the simpler organisms, and ultimately onto those of a (hypothetical) primordial organism (which was assumed to be unique up to an isomorphism or *selection-equivalence*). The latter proposition, as formulated by Rashevsky (1967), is more akin to a postulate than a principle. However, it was then generalized and re-stated in the form of the existence of a *limit* in the category of living organisms and their functional genetic networks $(\mathbf{GN}^i)$, as a directed family of objects, $\mathbf{GN}^i(-t)$ projected backwards in time (Baianu and Marinescu 1968), or subsequently as a super-limit (Baianu 1970 to 1987a,b; Baianu et al. 2006b); then, it was re-phrased as the Postulate of Relational Invariance, represented by a *colimit* with the arrow of time pointing forward (Baianu et al. 2006b).

Somewhat similarly, a dual principle and colimit construction was invoked for the ontogenetic development of organisms (Baianu 1970), and also for populations evolving forward in time; this was subsequently applied to biological evolution although on a much longer time scale—that of evolution—also with the arrow of time pointing towards the future in a representation operating through Memory Evolutive Systems (MES) by Ehresmann and Vanbremeersch (1987, 2006).

An axiomatic system (ETAS) leading to higher dimensional algebras of organisms in supercategories has also been formulated (Baianu 1970) which specifies both the logical and the mathematical (π–) structures required for complete self-reproduction and self-reference, self-awareness, etc., of living organisms. To date there is no higher dimensional algebra axiomatics other than the ETAS proposed for complete self-reproduction in super-complex systems, or for self-reference in ultra-complex ones. On the other hand, the preceding, simpler ETAC axiomatics, was proposed for the foundation of 'all' mathematics, including categories (Lawvere 1966, 1969), but this seems to have occurred before the emergence of higher dimensional algebra.

8.2 Occam's razor—An 'Economy or Simplicity Principle' and Einstein's Dictum

One of the often invoked 'principles' of Science is Occam's razor: the simplest 'theory'—with the fewest hypothesis or assumptions—that explains all known facts wins over the more sophisticated, complex explanations. An even more stringent form, or actually a *disguise*, of Occam's razor is the reductionist, or physicalist, approach which aims at reducing the study of all complex systems to the investigation of their arbitrarily selected, 'component', simple dynamic systems, and provides so called 'explanations' for complex dynamical processes in terms of strict causal mechanisms. Romans have successfully employed a form of this approach (i.e., '*Divide et Impera*') in their conquests and empire building. It is also in this context that the 'local-to-global' model approach becomes relevant, as in the case of generalized van Kampen theorems (see the Brown, Glazebrook and Baianu (2007a, in this volume). for a concise presentation of the van Kampen generalized theorems), considered as a principle. A prime example of the failure of reductionism is that of the Borromean rings: the whole is *not* simply the *sum* of its parts, but, by the way it is put together, constitutes a new structure. Of course, we need to know the parts which make this structure, but knowing just the parts, *without* the construction procedure, does not allow one to assemble the Borromean rings. An essential modification of Occam's razor was suggested by Einstein who proposed that a theory should be as simple as possible but *no simpler* than the observed phenomenon or essential reality. Thus, he eliminated the ellusive 'ether' and introduced instead the concept of a topologically flexible spacetime curved by the presence of matter, thereby also eliminating gravity as an 'actual force'.

8.3 From Lower to Higher Order Theories

In accordance with replacing reductionism by appropriate complexity theories of the highly complex human mind and its supporting matter systems in the brain, one requires second order models consisting of a meta-model or meta-theory. A brief and only partial analogy as discussed in Atmanspacher and Jahn (2003) might be made with first-order engineering connecting hardware to software in AI systems; this partial analogy suffers, however, from severe, reductionist limitations. In a separate context, the expectation value of an observable defined in some limit $N \longrightarrow \infty$, which conceivably does not exist, in the second order viewpoint can be realized by studying the mean-value of the considered observation as changing in accord with functions up to finite N. In general it is erroneous to employ first-order experiments as an attempt to validate second-order models (a psychological stumbling block when it comes to "thinking about thinking", again, cf. the 'mereological fallacy', Bennett and Hacker 2003). In other words, whereas a level (n–1)-theory may be deducible from a level n-theory, the converse is not true, in so far, for instance, that a theory of neuronal assemblies cannot be used as the sole basis for the explanation of a given cognitive process. In this regard, the categorical methods we propose for (ultra) complex systems are suitably geared for the 'contraction principle' in going from level n down to level (n–1) and making the right predictions accordingly.

For example, the 'self' increases in complexity in confronting new challenges and implementing new tasks. But this categorical approach of access to level n–1 from level n is a blueprint for studying complex processes that the usual 'self' often dispenses with. Many individuals can admirably perform their secular duties, enjoy their leisure etc. in society without any due regard to the concepts and functions of their corporal metabolism, neurophysiology, and cognitive mechanisms, etc., unless illness or some other disposition causes an alert to these functions. The situation for AI and 'conscious' machines is even more pronounced. Chalmers (1996) points out the examples of Hofstadter (1979)—it is not necessary to give a system access to its low-level components—and Winograd's program SHRDLU (1972) had no knowledge of the programming language in which it was written despite its capacity to assimilate the structure of a virtual world and make inferences about it.

9 Modeling and Classification of Systems in Relation to the Categorical Theory of Levels: Simple, Complex and Super-Complex Systems. Logical Models of Higher Complexity Levels

9.1 Dynamic Systems as Stable Spacetime Structures

As defined in Baianu and Poli (2008), a system is a dynamical (whole) entity able to maintain its working conditions; the system definition is here spelt out in detail by the following, general definition, **D1**.

D1. A simple system is in general a bounded, but not necessarily closed, entity—here represented as a category of stable, interacting components with inputs and outputs from the system's environment, or as a supercategory for a complex system consisting of subsystems, or components, with internal boundaries among such subsystems. In the latter case, one may represent a *meta-level* of existence as in the case of the human mind.

As proposed by Baianu and Poli (2008) in order to define a system one therefore needs specify: (1) components or subsystems, (2) mutual interactions or links; (3) a separation of the selected system by some boundary which distinguishes the system from its environment; (4) the specification of the system's environment; (5) the specification of the system's categorical structure and dynamics; (6) a supercategory will be required when either components or subsystems need be themselves considered as represented by a category, i.e. the system is in fact a super-system of (sub) systems, as it is the case of emergent super-complex systems or organisms.

Point (2008) claims that a system should occupy a macroscopic spacetime region: a system that comes into birth and dies off extremely rapidly may be considered either a short-lived process, or rather, a 'resonance'—an instability rather than a system, although it may have significant effects as in the case of 'virtual particles', 'virtual photons', etc., as in quantum electrodynamics and chromodynamics. Note also that there are many other, different mathematical definitions of 'systems' ranging from (systems of) coupled differential equations to operator formulations, semigroups, monoids, topological groupoids and categories. Clearly, the more useful system definitions include algebraic and/or topological structures rather than simple, structureless sets, classes or their categories (cf. Baianu 1970; Baianu et al. 2006a, b). The main intuition behind this first understanding of system is well expressed by the following passage:

> The most general and fundamental property of a system is the inter-dependence of parts/components/sub-systems or variables.

As discussed by Baianu and Poli, *inter-dependence* thus consists in the existence of determinate relationships among the parts or variables as contrasted with randomness or extreme variability. In other words, inter-dependence is the presence or existence of a certain organizational order in the relationship among the components or subsystems which make up the system. It can be shown that such organizational order must either result in a stable attractor or else it should occupy a stable spacetime domain, which is generally expressed in *closed* systems by the concept of equilibrium. On the other hand, in non-equilibrium, open systems, one cannot have a static but only a *dynamic self-maintenance* in a 'state-space region' of the open system—which cannot degenerate to either an equilibrium state or a single attractor spacetime region. Thus, non-equilibrium, open systems capable of self-maintenance (seen as a form of autopoiesis) are also generic/structurally stable: their perturbation from a homeostatic maintenance regime does not result either in completely chaotic dynamics with a single attractor or the loss of their stability. It may however involve an ordered process of change—a process that follows a determinate pattern rather than random variation relative to the starting point.

9.2 Selective Boundaries and Homeostasis. Varying Boundaries vs. Horizons

Boundaries are especially relevant to *closed* systems. According to Poli (2008): “they serve to distinguish what is internal to the system from what is external to it”, thus defining the fixed, overall structural topology of a closed system. By virtue of possessing boundaries, “a whole (entity) is something on the basis of which there is an interior and an exterior. The initial datum, therefore, is that of a difference, of something/a key attribute which enables a difference to be established between the whole closed system and environment.” (cf. Baianu and Poli 2008). One notes however that a boundary, or boundaries, may change or be quite selective/directional—in the sense of dynamic fluxes crossing such boundaries—if the system is *open* and grows/develops as in the case of an organism, which will be thus characterized by a *variable* topology that may also depend on the environment, and is thus *context-dependent* as well. Perhaps the simplest example of a system that changes from *closed to open*, and thus has a *variable topology*, is that of a pipe equipped with a functional valve that allows flow in only one direction. On the other hand, a semi-permeable membrane such as a cellophane, thin-walled ’closed’ tube—that allows water and small molecule fluxes to go through but blocks the transport of large molecules such as polymers through its pores—is *selective* and may be considered as a primitive/‘simple’ example of an open, selective system. Organisms, in general, are *open systems with variable topology* that incorporate both the valve and the selectively permeable membrane boundaries—albeit much more sophisticated and dynamic than the simple/fixed topology cellophane membrane—in order to maintain their stability and also control their internal structural order, or low microscopic entropy.

The formal definition of this important concept of ‘variable topology’ will be introduced in this essay for the first time in the context of the spacetime evolution of organisms, populations and species in Sect. 11.5.

As proposed by Baianu and Poli (2008), an essential feature of boundaries in open systems is that they can be crossed by matter; however, all boundaries may be crossed by either fields or by quantum wave-particles if the boundaries are sufficiently thin, even in ‘closed’ systems. Thus, there are more open boundaries and less open ones, but they can all be crossed in the above sense. The boundaries of closed systems, however, cannot be crossed by molecules or larger particles. On the contrary, a horizon is something that one cannot reach or cross. In other words, a horizon is not a boundary. This difference between horizon and boundary might be useful in distinguishing between systems and their environment. “Since the environment is delimited by open horizons, not by boundaries capable of being crossed, it is not a system.” (cf. Luhmann 1995). We note here, however, that one can define both open horizons and varying boundaries in terms of variable topologies, but with different organization or structure. As far as open systems are concerned, the difference between inside and outside loses its common sense, or ‘spatial’ understanding. As a matter of fact, ‘inside’ doesn’t anymore mean ‘being placed within’, but it means ‘being part of’ the system. In essence, the attributes internal and external are first and foremost relative to the system, not to its actual location in physical space. The situation is, however, much less clear-cut in the case of viruses that insert themselves into the host genome and are expressed by the latter as if the viral genes 'belonged' to the host genome. Even though the host may not always recognize the viral genes as 'foreign', or 'external' to the host, their actions may become incompatible with the host organization as in the case of certain oncogenic viruses that cause the death of their host. These key attributes—internal and external—might also be taken as features describing the difference between the world of 'inanimate' things/machines and the world of organisms. In the mechanistic, 'linear' order of things or processes, the world is regarded as being made, or constituted, of entities which are outside of each other, in the sense that they exist independently in different regions of spacetime and interact through forces. By contrast, in a living organism, *each part grows in the context of the whole*, so that it does not exist independently, nor can it be said that it merely 'interacts' with the others, without itself being essentially affected in this relationship. The parts of an organism grow and develop together as a result of cell division, migration, and other related processes.

Boundaries may be fixed, clear-cut, precise, rigid, or they may be vague, blurred, mobile, varying/variable in time, or again they may be intermediate between these two typical cases, according to how the differentiation is structured. In the beginning there may be only a slightly asymmetric distribution in perhaps just one direction, but usually still maintaining certain symmetries along other directions or planes. Interestingly, for many multi-cellular organisms, including man, the overall symmetry retained from the beginning of development is bilateral—just one plane of mirror symmetry—from Planaria to humans. The presence of the head-to-tail asymmetry introduces increasingly marked differences among the various areas of the head, middle, or tail regions as the organism develops.

The formation of additional borderline phenomena occurs later as cells divide and differentiate thus causing the organism to grow and develop. Generally speaking, a closed boundary generates an internal situation characterized by limited differentiation. Open boundaries allow instead, and indeed stimulate, greater internal differentiation, and therefore, a greater degree development of the system than would occur in the presence of just closed boundaries. In its turn, a population with marked internal differentiation, that is, with a higher degree of development, in addition to having numerous internal boundaries is also surrounded by a nebula of functional and non-coincident boundaries. This non-coincidence is precisely one of the principal reasons for the dynamics of the system. Efforts to harmonize, coordinate or integrate boundaries, whether political, administrative, social, etc, generate a dynamic which constantly maintains the boundary situation at a steady-state.

Note, however, that in certain 'chaotic' systems, organized patterns of spatial boundaries do indeed occur, albeit established as a direct consequence of their 'chaotic' dynamics. The multiplicity of boundaries, and the dynamics that derive from it, generate interesting phenomena. Boundaries also tend to reinforce each other, as in the case of dissipative structures formed through coupled chemical, chaotic reactions. One may also quote Platt's view on this phenomenon: "The boundary-surface for one property… will tend to coincide with the boundary surfaces for many other properties… because the surfaces are mutually reinforcing."

According to Poli (2008) this somewhat astonishing regularity of nature has not been sufficiently emphasized in perception philosophy. It is this that makes it useful and possible for us to identify sharply defined regions of space as ‘objects’. “This is what makes a collection of properties a ‘thing’ rather than a smear of overlapping images”.

On the other hand, the underlying quantum-theoretical reason for the macroscopically sharp-definition of objects is the *decoherence* of the wave-function in many-particle systems in the presence of overwhelming thermal motions. The surfaces thus appear to be ‘mutually-reinforcing’ because their quantum phases are sharply different and vary from location to location.

The mathematician John von Neumann regarded ‘complexity’ as a measurable property of natural systems below the threshold of which systems behave ‘simply’, but above which they evolve, reproduce, self-organize, etc. Rosen (1987) proposed a refinement of these ideas by a more exact classification between ‘simple’ and ‘complex’. Simple systems can be characterized through representations which admit maximal models, and can be therefore re-assimilated via a hierarchy of informational levels. Besides, the duality between dynamical systems and states is also a characteristic of such simple dynamical systems. It was claimed that any ‘natural’ system fits this profile. But the classical assumption that natural systems are simple, or ‘mechanistic’, is too restrictive since ‘simple’ is applicable only to machines, closed physicochemical systems, computers, or any system that is recursively computable. On the other hand, an *ultra-complex* system as applied to psychological–sociological structures is describable in terms of *variable categories* or structures, and cannot be reasonably represented by a fixed state space for its entire lifespan. Replacements by limiting dynamical approximations lead to increasing system ‘errors’ and through such approximations a complex system can be viewed in its acting as a single entity, but not conversely.

Just as for simple systems, both *super-complex* and *ultra-complex* systems admit their own orders of causation, but the latter two types are different from the first—by inclusion rather than exclusion—of the mechanisms that control simple dynamical systems.

On the other hand, the reductionist approach seems to exclude the possibility of the existence of either relational laws or principles applicable only to biological organisms and/or societies that cannot be reduced to physical laws, and that are *complementary* to physical laws in the sense of being consistent with—but not reducible to—the latter. Ultimately, the ‘physicalist’ approach consists in reducing all Ontology to Physics. On the other hand, Descartes, who seems to have thought of organisms as complicated machines, drew a line between mind and matter, because he invoked thinking as ‘proof’ of one’s existence.

Super-complex (or Rosen’s ‘complex’) systems, such as those supporting neurophysiological activities, are explained only in terms of ‘circular’, or non-linear, rather than linear causality. In some way then, these systems are not normally considered as part of either traditional physics or the complex systems physics generated by ‘chaos’, which are nevertheless fully deterministic. However, super-complex (biological) systems have the potential to manifest novel and counter-intuitive behavior such as in the manifestation of ‘emergence’, development/

morphogenesis and biological evolution. Their precise meaning is formally defined for the first time in Sects. 5.3 and 9.5 to 9.10.

9.3 Historical 'Continuity' in the Evolution of Super-Complex Systems: Topological Transformations and Discontinuities in Biological Development

Anthropologists and evolutionary biologists in general have emphasized biological evolution as a '*continuous*' process, in a *historical*, rather than a topological, sense. That is, there are historical sequences of organisms—phylogeny lines—which evolved in a well-defined order from the simpler to the more complex ones, with intermediate stages becoming extinct in the process that translates 'becoming into being', as Prigogine (1980) might have said. This picture of evolution as a 'tree of life', due initially and primarily to Wallace and Darwin, subsequently supported by many evolutionists, is yet to be presented in *dynamic*, rather than historical, terms. Darwin's theory of *gradual* evolution of more complex organisms from simpler ones has been subject to a great deal of controversy which is still ongoing. The alternatives are either saltatory or catastrophic changes; the latter has been especially out of favor with biologists for a very long time. If we accept for the moment Darwin's gradual evolution of species, then we can envisage the emergence of higher and higher *sub-levels* of super-complexity through biological evolution until a transition occurs through human society co-evolution to ultra-complexity, the emergence of human consciousness. Thus, without the intervention of human society co-evolution, a smooth increase in the degree of super-complexity occurs only until a distinct/discrete transition to the (higher) ultra-complexity level becomes possible through society co-evolution. If the previous process of increasing complexity—which occurred before the transition at the super-complexity level—were to be iterated also at the ultra-complex level, one might ask how and what will be the deciding factor for the further 'co-evolution of minds' and the transition towards still higher complexity levels? Of course, one might also ask first the contingent ontology question if any such higher-level above human consciousness could at all emerge into existence. As we will show in subsequent Sects. 7–10, the emergence of levels or sub-levels of increasing higher complexity can be represented by means of *variable* structures of increasingly higher order or dimensions. There still remains the unsolved question why humans—as well as parrots—have the inherited inclination to talk whereas the apes do not. A chimpanzee pup will not talk even if brought up in a human environment, whereas a human baby will first 'babble' and than develop early a 'motherese' talk as an intermediate stage in learning the adults' language; the chimpanzee pup never babbles nor develops any 'motherese' through natural interactions with either its own biological mother or with a human, surrogate mother. These facts seem to point to the absence in apes of certain brain structures, perhaps linked to mirror neurons, that are responsible for the human baby's inheritable *inclination* to babble (Wiener 1950, 1989), which then leads to speech through learning and nurture in the human environment. Thus, one might hypothesize the existence of 'talk inclination' (TI) gene(s), both present and active, in both humans and parrots, but not in apes.

9.4 Organisms Represented as Variable Dynamic Systems: Generic States and Dynamic System Genericity

In actual fact, the super-complexity of the organism itself emerged through the generation of dynamic, variable structures which then entail variable/flexible functions, homeostasis, autopoiesis, anticipation, and so on. In this context, it is interesting that Wiener (1950, 1989) proposed the simulation of living organisms by variable machines/automata that did not exist in his time. The latter were subsequently formalized independently in two related reports (Baianu 1971a,b).

Unlike physical and chemical studies, evolutionary ones are usually limited severely by the absence of controlled experiments to yield the prerequisite data needed for a complete theory. The pace of discoveries is thus much slower in evolutionary studies than it is in either physics or chemistry; furthermore, the timescale on which evolution has occurred, or occurs, is extremely far from that of physical and chemical processes occurring on earth, despite Faraday's saying that "*life is but a delayed chemical reaction*". Such a multi-billion year timescale for evolution is a significant part of the evolution of the universe itself over some 14–20 billion years. Thus, interestingly, both Evolutionary and Cosmological studies work by quite different ontological and epistemologic means to uncover events that span across huge spacetime regions. Whereas in Cosmology the view of an *absolute and fixed* Universe prevailed for quite a long time, it is currently accepted that the Universe evolves—it changes while very rapidly expanding. The Contingent Universes are neither fixed nor absolute, they are changing/evolving and are also relative to the observer or reference frame (as discussed in Sect. 2). Similarly, Darwin's over-simplifying concepts of Natural Selection and Origin of species has survived a surprisingly long time in biology and is still considered by many biologists as 'fact' even today. On a much smaller space scale than Cosmology, biological Evolution has also 'continuously' generated a vast, increasing number of species, however, with the majority of such species becoming extinct. In this latter process, geographical location, the climate, as well as occasional catastrophes (meteorites, volcanoes, etc.), seem to have played major roles. The historical view of biological evolution proposed by Darwin stems from the fact that every organism, or living cell, originates only from another, and there is no *de nuovo* re-starting of evolution. This raises two very important, related questions: *how did life start on earth in the first place? How did the first, primordial organism emerge some 4 billion years ago?* We shall see briefly in Sects. 11 to 13 how specific organismic models may provide some partial answers to these key questions left completely unanswered by Darwin's theory, or indeed any of its reductionist alternatives by neo-Darwinists.

In D'Arcy Thompson's extensive book "On Growth and Form" (ca. 1900) there are many graphic examples of coordinate, continuous transformations (in fact *homotopies*) of anatomical structure from one species to another, rates of growth in organisms and populations, as well as a vast array of dynamic data serving as a source of inspiration in a valiant attempt to understand morphogenesis in terms of physical forces and chemical reactions. It is a remarkable, very early attempt to depart from Darwin's historical approach to evolution, and to understand organismic forms in terms of their varied and complex dynamic growth; it is often criticized for disagreeing with Darwin's theory of evolution, and also for being a physicalist attempt. Yet, some of the issues raised by D'Arcy W. Thompson are of interest even today, as he explicitly pointed out in his book that the 'morphogenetic dynamics' he is considering does not exhaust the real, *very complex dynamics* of biological development.

Separated in time by almost a century is René Thom's work on Catastrophe Theory (1980) that attempts to explain 'topologically' the presence of discontinuities and 'chaotic' behavior, such as bifurcations, 'catastrophes', etc. in organismic development and evolution. Often criticized, his book does have though the insight of *structural stability* in biodynamics via 'generic' states that when perturbed lead to other similarly stable states. The use of the term 'catastrophe' was 'gauche' as it reminds one of Cuvier's catastrophic theory for the formation of species, even though Thom's theory had no connection to the former. When analyzed from a categorical standpoint, organismic dynamics has been suggested to be characterized not only by homeostatic processes and steady state, but also by *multi-stability* (Baianu 1970). The latter concept is clearly equivalent from a dynamic/topological standpoint to super-complex system genericity, and the presence of *multiple dynamic attractors* (Baianu 1971a,b) which were categorically represented as *commutative super-pushouts* (Baianu 1970). The presence of generic states and regions in super-complex system dynamics is thus linked to the emergence of complexity through both structural stability and the *open* system attribute of any living organism that enable its persistence in time, in an accommodating niche, suitable for its competitive survival.

9.5 Simple vs. Complex Dynamics—Closed vs. Open Systems

In an early report (Baianu and Marinescu 1968), the possibility of formulating a (super-) Categorical Unitary Theory of Systems (i.e., both simple and complex, etc.) was pointed out both in terms of organizational structure and dynamics. Furthermore, it was proposed that the formulation of any model or 'simulation' of a complex system—such as living organism or a society—involves generating a first–stage *logical model* (not-necessarily Boolean), followed by a *mathematical* one, *complete with structure* (Baianu 1970). Then, it was pointed out that such a modeling process involves a diagram containing the complex system, (**CS**) and its dynamics, a corresponding, initial logical model, **L**, '*encoding*' the essential dynamic and/or structural properties of **CS**, and a detailed, structured mathematical model (**M**); this initial modeling diagram may or may not be commutative, and the modeling can be iterated through modifications of ***L***, and/or **M**, until an acceptable agreement is achieved between the behavior of the model and that of the natural, complex system (Baianu and Marinescu 1968; Comoroshan and Baianu 1969). Such an *iterative modeling* process may ultimately 'converge' to appropriate models of the complex system, and perhaps a best possible model could be attained as the categorical colimit of the directed family of diagrams generated through such a modeling process. The possible models **L**, or especially **M**, were not considered to be necessarily either numerical or recursively computable (e.g., with an algorithm or software program) by a digital computer (Baianu 1971b, 1986).

9.6 Commutative vs. Non-commutative Modeling Diagrams

Interestingly, Rosen (1987) also showed that complex dynamical systems, such as biological organisms, cannot be adequately modeled through a *commutative* modeling diagram—in the sense of digital computer simulation—whereas the simple (physical/engineering) dynamical systems can be thus numerically simulated. Furthermore, his modeling commutative diagram for a *simple dynamical system* included both the 'encoding' of the 'real' system **N** in (**M**) as well as the 'decoding' of (**M**) back into **N**:

$$\begin{array}{ccc} [N] & \xrightarrow{\textbf{(Encoding)}} & L \oplus M \\ \delta \downarrow & & \downarrow \aleph_M \\ N & \xleftarrow[\textbf{(Decoding)}]{} & [M] \end{array}$$

where δ is the real system dynamics and $\aleph_{\mathbf{M}}$ is an algorithm implementing the numerical computation of the mathematical model (**M**) on a digital computer. Firstly, one notes the ominous absence of the *Logical Model*, ***L***, from Rosen's diagram published in 1987. Secondly, one also notes the obvious presence of logical arguments and indeed (*non-Boolean*) 'schemes' related to the entailment of organismic models, such as MR-systems, in the more recent books that were published last by Robert Rosen (1994, 2001, 2004). This will be further discussed in Sects. 11–12, with the full mathematical details provided in Brown, Glazebrook and Baianu (2007a, in this volume).

Furthermore, Elsasser (1981) pointed out a fundamental, logical difference between physical systems and biosystems or organisms: whereas the former are readily represented by *homogeneous* logic classes, living organisms exhibit considerable variability and can only be represented by *heterogeneous* logic classes. One can readily represent homogeneous logic classes or endow them with 'uniform' mathematical structures, but heterogeneous ones are far more elusive and may admit a multiplicity of mathematical representations or possess variable structure. This logical criterion may thus be useful for further distinguishing simple systems from highly complex systems. The importance of *Logic Algebras*, and indeed of *Categories of Logic Algebras*, is rarely discussed in modern Ontology even though categorical formulations of specific Ontology domains such as Biological Ontology and Neural Network Ontology are being extensively developed. For a recent review of such categories of logic algebras the reader is referred to the concise presentation by Georgescu (2006); their relevance to network biodynamics was also recently assessed (Baianu 2004; Baianu and Prisecaru 2004; Baianu et al. 2006a,b).

9.7 Development of Living Organisms and Super-Complex Dynamics

Above the level of 'complex systems with chaos' considered in the non-commutative diagram of the previous section there is still a higher, super-complexity level of living organisms—which are neither machines nor simple dynamical systems, in the above sense. Biological organisms are *extremely* complex as recently discussed elsewhere in more detail (Baianu 2006) in the sense of their required, unique axiomatics (Baianu 1970), super-complex dynamics (Baianu 1970 through 2006), new biological/relational principles (Rashevsky 1968; Baianu and Marinescu 1968; Baianu 1970, 1971a,b; Rosen 1970; Baianu et al. 2006a,b) and their *non-computability* with recursive functions, digital computers or Boolean algorithms (Baianu 1986; Rosen 1987; Penrose 1994, 2007; Baianu et al. 2006a,b).

In Sects. 11 and 12 we shall explain in further detail how super-complex dynamics emerges in organisms from the *molecular and supra-molecular* levels that recently have already been claimed to exist by several experimental molecular biologists to be 'super-complex'. As shown in previous reports (Baianu 1973 through 2004; Baianu et al. 2006a,b), multi-cellular organismic development, or ontogeny, can be represented as a directed system or family of dynamic state spaces corresponding to all stages of ontogenetic development of increasing dimensionality. The *colimit* of this *directed system* of ontogenetic stages/dynamic state spaces represents the *mature* stage of the organism (Baianu 1970 through 2004; Baianu et al. 2006a,b). On the other hand, both single-cell and multi-cellular organisms can be represented in terms of variable dynamic systems, such as generalized **(M,R)**-systems (Baianu 1973; Baianu and Marinescu 1974), including dynamic realizations of **(M,R)**-systems (Rosen 1971; 1977); this was also conjectured by Norbert Wiener in 1950 (also reprinted in 1989) to be an appropriate representation of living systems, or even as a means of constructing variable 'machines' mimicking organisms, however without either any published formalization or proof by Wiener. The concept of variable automaton was formally introduced by Baianu (1971b) along with that of quantum automaton (Baianu 1971a, 1987) and quantum computation (Baianu 1971b). This emergent process involved in ontogeny as well as the becoming/'birth' of the primordial organism leads directly to *variable*, super-complex dynamics and *higher dimensional* state spaces. As an over-simplified, pictorial—but also formalizable—representation, let us consider a living cell as a topological 'cell' or simplex of a CW-complex. Then, as a multi-cellular organism develops a complete simplicial (CW) complex emerges as an over-simplified picture of the whole, mature organism. The higher dimensionality then emerges by considering each cell with its associated, *variable* dynamic state space (Baianu 1970, 1971a,b). As shown in previous reports (Baianu 1970, 1980), the corresponding variable dynamic structure representing biological relations, functionalities and dynamic transitions is an organismic supercategory, or **OS**. The time-ordered sequence of CW-complexes of increasing dimensionality associated with the development of a multi-cellular organism provides a specific example of a *variable topology*. The 'boundary conditions' or constraints imposed by the environment on the organismic development will then lead to context-dependent variable topologies that are not strictly determined by the genome or developing genetic networks. Although ontogenetic development is usually structurally stable there exist teratogenic conditions or agents that can 'de-stabilize' the developing organism, thus leading to abnormal development. One also has the possibility of abnormal organismic, or brain, development caused by altered genomes, as for example in those cases of autism caused by the fragile-X chromosome syndrome.

9.8 Super-Complex, Anticipatory Systems. Feedbacks and Feedforward. Autopoiesis

Rosen (1985, 1987) characterized a change of state as governed by a predicted future state of the organism and/or in respect of its environment. These factors appear separate from the idea of simple systems since future influence (via inputs, etc.) are not seen as compatible with causality. Since simple or mechanistic systems are not considered as anticipatory, the latter square-up well with Rosen's complex systems since, *a fortiori*, a complex system is more susceptible to external influences beyond any dynamical representation of it. Indeed, any effort to monitor a complex system through a predictive dynamic model results in a growing discrepancy between the actual function of the system and its predicative counterpart thus leading to a (global) system failure (Rosen 1987). Furthermore, *anticipatory behavior*, considered apart from any non-feedback mechanism, is realized in all levels of biological organization such as found in immune and neuronal systems (cf. Atlan 1972; Jerne 1974; Rosen 1958a,b), or the broad-scale *autopoiesis* of structurally linked systems/processes that continually inter-adjust with their environment over time (Maturana and Varela 1980). Within a social system the autopoiesis of the various components is a necessary and sufficient condition for realization of the system itself. In this respect, the structure of a society as a particular instance of a social system is determined by the structural framework of the (autopoietic) components and the sum total of collective interactive relations. Consequently, the societal framework is based upon a selection of its component structures in providing a medium in which these components realize their ontogeny. It is just through participation alone that an autopoietic system determines a social system by realizing the relations that are characteristic of that system. The descriptive and causal notions are essentially as follows (Maturana and Varela 1980, Chapter III):

(1) Relations of constitution that determine the components produced constitute the topology in which the autopoiesis is realized.
(2) Relations of specificity that determine that the components produced be the specific ones defined by their participation in the autopoiesis.
(3) Relations of order that determine that the concatenation of the components in the relations of specification, constitution and order be the ones specified by the autopoiesis.

The huge number and variety of biological organisms formed through evolution can be understood as a result of the very numerous combinatorial potentialities of *super-complex* systems, as well as the large number of different environmental factors available to organismic evolution.

Dynamical System and Automata Decompositions. Decompositions in Abelian Categories. The Open Question of Super-Complex System Decomposition.

In the case of discrete system dynamics as in the case of automata/sequential machines or Turing machines, their state space which has a semigroup (or monoid) structure can be canonically decomposed into a cyclic group subautomaton and two other types or permutation sub-semigroups (Krohn-Rhodes Basic (Algebraic) Decomposition Theorem, as for example in Arbib, 1968). Other decomposition results can be obtained via partitions of equivalent states, thus leading to a dynamic groupoid structure of 'reduced' automata which is obviously compatible with the original state-space semigroup of the 'unreduced' automaton. One suspects that somewhat similar results may hold for simple, continuous dynamical systems, though in this case an algebraic topological approach to decomposition seems unavoidable. In the case of quantum dynamical systems there is no known general decomposition of quantum state spaces. Interestingly, certain canonical decompositions seem to have been found only for Abelian categories (Riley 1962; Dickinson 1965; Popescu 1967, 1973), such as the Krull-Remak-Schmidt theorem. Although complex systems seem to have no standard decomposition, living cells may be enucleated for example and successfully manipulated in nuclear transplant experiments (Baianu and Scripcariu 1973) or animal cloning. It would seem also that somatic as well as stem cells include a cellular cyclic group with automata-like properties that 'counts' the number of cell cycles or divisions in the case of somatic cells thus acting as an internal 'clock' that determines when apoptosis should, or must, occur. Multi-cellular systems, and especially highly integrated biosystems, or organs such as the human brain, or the immune system are not readily decomposed into their components, not only in a practical, surgical sense but also in the general, theoretical sense. The opposite is true for the simpler organisms such as Planaria or nematodes that can regenerate an entire organism from a 'sufficiently large' fragment, thus suggesting the possibility of segmental decomposition, and perhaps a more commutative or Abelian structure for the development of such simpler organisms.

9.9 Comparing Systems: Similarity and General Relations between Systems. Categorical Adjointness and Functional or Genetic Homology

We have seen already in the previous Subsection 9.6 that categorical comparisons of different types of systems in diagrams provides a useful means for their classification and understanding the relations between them. From a global viewpoint, comparing categories of such different systems does reveal useful analogies, or similarities, between systems and also their universal properties. According to Rashevsky (1969), general relations between sets of biological organisms can be compared with those between societies, thus leading to more general principles pertaining to both. Using the theory of levels does indicate however that the two levels of super-complex and ultra-complex systems are quite *distinct*, and therefore, categorical diagrams that 'mix' such distinct levels also fail to commute. This may be also the implicit reason behind the Western philosophical duality between the brain and the mind, etc.

Considering dynamic similarity, Rosen (1968) introduced the concept of '*analogous*' (classical) dynamical systems in terms of categorical, dynamic isomorphisms i and j, as shown in the following commutative diagram of isomorphic state-spaces S and S':

$$\begin{array}{ccc} S & \xrightarrow{\sim i} & S' \\ \delta \downarrow & & \downarrow \delta' \\ S & \xrightarrow[\sim j]{} & S' \end{array} \tag{9.1}$$

with δ and δ' being respectively the corresponding transition functions/ dynamic laws of S and S'.

However, the extension of this concept to either complex or super-complex systems has not yet been investigated, and may be similar in importance to the introduction of the Lorentz-Poincaré group of transformations for reference frames in Relativity theory. On the other hand, one is often looking for *relational invariance* or *similarity in functionality* between different organisms or between different stages of development during ontogeny—the development of an organism from a fertilized egg. In this context, the categorical concept of '*dynamically adjoint systems*' was introduced in relation to the data obtained through nuclear transplantation experiments (Baianu and Scripcariu 1973).

Thus, extending the latter concept to super- and ultra-complex systems, one has in general, that two complex or supercomplex systems with 'state spaces' being defined respectively as A and A*, are dynamically adjoint if they can be represented naturally by the following (functorial) diagram:

$$\begin{array}{ccc} A & \xrightarrow{F} & A^* \\ F' \downarrow & & \downarrow G \\ A^* & \xrightarrow[G']{} & A \end{array} \tag{9.2}$$

with $F \approx F'$ and $G \approx G'$ being isomorphic (that is, $\approx$ representing natural equivalences between adjoint functors of the same kind, either left or right), and as above in (9.1), the two diagonals are, respectively, the state-space transition functions $\Delta: A \to A$ and $\Delta^*: A^* \to A^*$ of the two adjoint dynamical systems. It would also be very interesting to investigate dynamic adjointness in the context of quantum dynamical systems and quantum automata (as defined in Baianu 1971a).

A *left-adjoint* functor, such as the functor F in the above commutative diagram between categories representing state spaces of equivalent cell nuclei *preserves limits*, whereas the *right-adjoint* (or coadjoint) functor, such as G above, *preserves colimits*. (For precise definitions of adjoint functors the reader is referred to Brown et al. (2007) in this issue, as well as to Popescu (1973), Baianu and Scripcariu (1973), and Kan (1958)).

Thus, dynamical attractors and genericity of states are preserved for nuclei up to the blastula stage of organismic development. Subsequent stages of development can be considered only 'weakly adjoint' or partially analogous. Clearly, such concepts are relevant also to cloning and related phenomenological data. A more elaborate dynamic concept of 'homology' between the genomes of different species during evolution was also proposed (Baianu 1971a), suggesting that an entire phylogenetic series can be characterized by a topologically—rather than biologically—*homologous sequence* of genomes which preserves certain genes encoding *the essential* biological functions. A striking example was recently suggested involving the differentiation of the nervous system in the fruit fly and mice (and perhaps also man) which leads to the formation of the back, middle and front parts of the neural tube.

9.10 Emergence of Unique Ultra-Complexity through Co-Evolution of the Human Mind and Society

Higher still than the organismic level characterized by super-complex dynamics, there emerged perhaps even earlier than 400,000 years ago the *unique, ultra-complex* levels of human mind/consciousness and human society interactions, as it will be further discussed in the following sections. There is now only one species known who is capable of rational, symbolic/abstract and creative thinking as part-and-parcel of consciousness–*Homo sapiens sapiens*—which seems to have descended from a common ancestor with *Homo ergaster*, and separated from the latter some 2.2 million years ago. However, the oldest fossils of *H. sapiens* found so far are just about 400,000 years-old.

The following diagram summarizes the relationships/links between such different systems on different ontological levels of increasing complexity from the simple dynamics of physical systems to the ultra-complex, global dynamics of psychological processes, collectively known as 'human consciousness'. With the emergence of the ultra-complex system of the human mind—based on the super-complex human organism—there is always an associated progression towards higher dimensional algebras from the lower dimensions of human neural network dynamics and the simple algebra of physical dynamics, as shown in the following, essentially *non-commutative* categorical ontology diagram. This is similar—but not isomorphic—to the higher dimensionality emergence that occurs during ontogenetic development of an organism, as discussed in the previous subsection.

$$\begin{array}{ccc} [SUPER-COMPLEX] & \xrightarrow{\text{(Higher Dim)}} & ULTRA-COMPLEX \\ \Lambda \downarrow & & \downarrow onto \\ COMPLEX & \xleftarrow[\text{(Generic Map)}]{} & [SIMPLE] \end{array}$$

Note that the above-diagram is indeed not 'natural' for reasons related to the emergent higher dimensions of the super-complex (biological/organismic) and/or ultra-complex (psychological/neural network dynamic) levels in comparison with the low dimensions of either simple (physical/classical) or complex (chaotic) dynamic systems. It might be possible, at least in principle, to obtain commutativity by replacing the simple dynamical system in the diagram with a quantum system, or a quantum 'automaton' (Baianu 1971, 1987); however, in this case the diagram still does not necessarily close between the quantum system and the complex system with chaos, because it would seem that *quantum systems are 'fuzzy'*—not strictly deterministic—as complex 'chaotic' systems are. Furthermore, this categorical ontology diagram is neither recursively computable nor representable through a commutative algorithm of the kind proposed for Boolean neural networks (Healy and Caudell 2006; for an extensive review of network biodynamic modeling, 'simulations' and also non-computability issues for biological systems see Baianu 1986; 1987a and references cited therein). Note also that the top layer of the diagram has generic states and generic regions, whereas the lower layer does not; the top layer lives, the bottom one does not.

10 From Object and Structure to Organismic Functions and Relations: A Process-based Approach to Ontology

Wiener (1950, 1954, 1989) made the important remark that implementation of complex functionality in a (complicated) machine requires also the design and construction of a complex structure. A similar argument holds *mutatis mutandis*, or by induction, for variable machines, variable automata and variable dynamic systems (Baianu 1970 through 1986; Baianu and Marinescu 1974); therefore, if one represents organisms as variable dynamic systems, one *a fortiori* requires a *super-complex structure* to enable or entail *super-complex dynamics*, and indeed this is the case for organisms with their extremely intricate structures at both the molecular and *supra-molecular* levels. It is an open question how the first organism has emerged through *self-assembly*, or 'self-construction'. On the other hand, for simple automata, or machines, there is the famous mathematical result about the existence of an *unique, Universal Turing Automaton* (uUTA) that can build or construct any other automaton. Furthermore, the category of all automata, and also the category of **(M,R)**-systems have both limits and colimits (Baianu 1973; Baianu and Marinescu 1974; Baianu 1987a, b). It would seem that the uUTA is isomorphic to the *colimit construction* in the category of all automata (Baianu 1973). One can also conjecture, and indeed, perhaps even prove formally, that a certain Variable Universal Automaton (VUA) also exists which can build *any* other variable automata; one may also hypothesize the metamorphosis of a certain selected variable automaton through an evolution-like process into variable automata of higher complexity and higher dimensionality, thus mimicking ontogeny, and possibly also phylogeny. Thus, an analogy is here suggested with the primordial organism as a specially selected variable universal automaton. Furthermore, the colimit of such an evolving, or developing, *direct system of variable automata* may be conjectured to exist as a VUA structure; such a VUA would then be a universal object in the supercategory of variable automata, and *a fortiori* would also be unique.

Although the essence of super- and ultra-complex systems is in the *interactions, relations and dynamic transformations* that are ubiquitous in such higher-level ontology, surprisingly many a psychology, cognitive and an ontology approach begins with a very strong emphasis on *objects* rather than relations. It would also seem that a basic 'trick' of human consciousness is to pin a subjective sensation, perception and/or feeling on an internalized *object*, or vice-versa to represent/internalize an object in the form of an internal symbol in the mind. The example often given is that of a human child's substituting a language symbol, or image for the *mother 'object'*, thus allowing 'her permanent presence' in the child's consciousness. Clearly, however, a complete approach to ontology must also include *relations and interconnections* between items, with a strong emphasis on *dynamic processes, complexity* and *functionality* of systems, which all require an emphasis on general relations, *morphisms* and the *categorical viewpoint* of ontology.

The *process-based approach* to universal ontology is therefore essential to an understanding of the ontology of levels, hierarchy, complexity, anticipatory systems, Life, Consciousness and Universe(s). On the other hand, the opposite approach, based on objects, is perhaps useful only at the initial cognitive stages in experimental science, as the reductionist approach of 'cutting off' functional connectivities and relations, retaining the object pieces, and then attempting 'to put back together the pieces' does *not* work for complex, super-complex or ultra-complex systems. Psychologists would be horrified at the proposition of 'taking a mind to pieces and attempting to put it back together afterwards'; not only it would not work, but it would also be *highly unethical*. One could also argue that if chimpanzees are very close to humans genetically (and maybe also to some extent functionally, even though separated from a 'common', hypothetical ancestor by 5–8 million years of evolution), their use in reductionist-inspired neurophysiological 'experiments' involving cutting and poking with electrodes, thus presumably, altering their chimpanzee 'consciousness' is also unethical.

10.1 The Object-based Approach vs. Process-based (Dynamic) Ontology

In classifications, such as those developed over time in Biology for organisms, or in Chemistry for chemical elements, the *objects* are the basic items being classified even if the 'ultimate' goal may be, for example, either evolutionary or mechanistic studies. Rutherford's comment is pertinent in this context:

> There are two major types of science: physics or stamp collecting.

An ontology based strictly on object classification may have little to offer from the point of view of its cognitive content. It is interesting that many psychologists, especially behavioral ones, emphasize the object-based approach rather than the process-based approach to the ultra-complex process of consciousness occurring 'in the mind'—with the latter thought as an 'object'. Nevertheless, as early as the work of William James in 1850, consciousness was considered as a '*continuous stream that never repeats itself*'—a Heraclitian concept that does also apply to super-complex systems and life, in general. We shall see more examples of the object-based approach to psychology in Sect. 14.

On the other hand, it is often thought that the object-oriented approach can be readily converted from an ontological viewpoint into a process-based one. It would seem that the answer to this question depends critically on the ontological level selected. For example, at the quantum level, *object and process become inter-mingled*. Either comparing or moving between levels, requires ultimately a process-based approach, especially in Categorical Ontology where relations and inter-process connections are essential to developing any valid theory. At the fundamental level of ‘elementary particle physics’ however the answer to this question of process-vs. object becomes quite difficult as a result of the ‘blurring’ between the particle and the wave concepts. Thus, it is well-known that any ‘elementary quantum object’ is considered by all accepted versions of quantum theory not just as a ‘particle’ or just a ‘wave’ but both: the quantum ‘object’ is *both* wave and particle, *at the same-time*, a proposition accepted since the time when it was proposed by de Broglie. At the quantum microscopic level, the object and process are inter-mingled, they are no longer separate items. Therefore, in the quantum view the ‘object-particle’ and the dynamic process-‘wave’ are united into a single dynamic entity or item, called *the wave-particle quantum*, which strangely enough is *neither discrete nor continuous*, but both at the same time, thus ‘refusing’ intrinsically to be an item consistent with Boolean logic. Ontologically, the quantum level is a very important starting point which needs to be taken into account by any theory of levels that aims at completeness. Such completeness may not be attainable, however, simply because an ‘extension’ of Gödel’s theorem may hold here also. The fundamental quantum level is generally accepted to be dynamically, or intrinsically *non-commutative*, in the sense of the *non-commutative quantum logic* and also in the sense of *non-commuting quantum operators* for the essential quantum observables such as position and momentum.

Therefore, any ‘complete’ theory of levels, in the sense of incorporating the quantum level, is thus—*mutatis mutandis*—*non-Abelian*. Therefore, at this point, there are two basic choices in Categorical Ontology: either to include the quantum level and thus generate a non-Abelian Ontology founded upon the non-commutative quantum logic, or to exclude the ‘fundamental’ level and remain strictly Abelian, that is accepting only strict determinism/linear causality and a commutative logic for its foundation such as Boolean or Brouwer-intuitionistic logic.

Furthermore, as the non-Abelian case is the more general one, from a strictly formal viewpoint, a non-Abelian Categorical Ontology is the preferred choice. Nevertheless, from the point of view of simplicity (see Occam’s razor) or ‘economy of thought’, the *Abelian* form of Categorical Ontology may be often selected by reductionists, mathematicians or engineers, for example; the commutativity and/or symmetry present in the Abelian theory can be seen as quite attractive either from an esthetic viewpoint or from the standpoint of the rapid elaboration/development of Categorical Ontology. Regardless of the latter views, the paradigm-shift towards a *non-Abelian Categorical Ontology* has already started (Brown, Glazebrook and Baianu 2007a, in this volume: ‘*Non-Abelian Algebraic Topology*’; Baianu et al. 2006a: NA-QAT).

10.2 Physico-Chemical Structure–Function Relationships

Perhaps an adequate response to both physicalist reductionism and/or 'pure' relationalism (as defined here in the previous sections) consists in considering the integration of a concrete categorical ontology approach which considers important experimentally well- studied examples of super-complex systems of defined physico-chemical structures with organizational–relational/ logical-abstract models that are expressed in terms of related function(s). Whereas such a combined approach does address the needs of—and in fact it is essential to—the experimental science of complex/super-complex systems, it is also considerably more difficult than either physicalist reductionism or *abstract relationalism*. Moreover, because there are many alternative ways in which the physico-chemical structures can be combined within an organizational map or relational complex system, there is a *multiplicity of 'solutions'* or mathematical models that needs be investigated, and the latter are not computable with a digital computer in the case of complex/super-complex systems such as organisms (Rosen 1987). It is generally accepted at present that structure–functionality relationships are key to the understanding of super-complex systems such as living cells and organisms. This classification problem of structure–functionality classes for various organisms and various complex models is therefore a difficult and yet unresolved one, even though several paths and categorical methods may lead to rapid progress in Categorical Ontology as discussed here in Sect. 6. The problem is further compounded by the presence of structural *disorder* (in the physical structure sense) which leads to a *multiplicity* of dynamical-physicochemical structures (or 'configurations') of a biopolymer, be it a protein, enzyme, or nucleic acid in a living cell or organism that correspond, or 'realize', just a single recognizable biological function (Baianu 1980b); this complicates the assignment of a 'fuzzy' physico-chemical structure to a well-defined biological function unless extensive experimental data are available, as for example, those derived through computation from 2D-NMR spectroscopy data (Wütrich 1995), or neutron/X-ray scattering and related multi-nuclear NMR spectroscopy/relaxation data (as for example in Chapters 2–9 in Baianu et al. 1995). It remains to be seen if this approach can also be carried *in vivo* in specially favorable cases. Detailed considerations of the ubiquitous, partial disorder effects on the structure–functionality relationships were reported for the first time by Baianu (1980b). Specific aspects were also recently discussed by Wütrich (1995) on the basis of 2D-NMR analysis.

11 What is Life?

11.1 Emergence of Super-Complex Systems and Life. The 'Primordial' as the Simplest (M,R)—or Autopoietic—System

Although the distinction between living organisms and simple physical systems, machines, robots and computer simulations appears obvious at first sight, the profound differences that exist both in terms of dynamics, construction and structure require a great deal of thought, conceptual analysis, development and integration or synthesis. This fundamental, ontological question about Life occurs in various forms, possibly with quite different attempts at answers, in several books (e.g., Schrödinger 1945; Rosen 1995, 1999). In the previous Sect. 9 we have already discussed from the categorical viewpoint several key systemic differences in terms of dynamics and modeling between living and inanimate systems. The ontology of super-complex biological systems, or biosystems (BIS), has perhaps begun with Elsasser's paper (1981) who recognized that organisms are extremely complex systems, that they exhibit wide variability in behavior and dynamics, and also from a logical viewpoint, that they form—unlike physical systems—*heterogeneous classes*. (We shall use the 'shorthand' term '*biosystems*' to stand for super-complex biological systems, thus implicitely specifying the attribute super-complex within biosystems). This intrinsic BIS variability was previously recognized as *fuzziness* (Baianu and Marinescu 1968) and some of its possible origins were suggested to be found in the partial structural disorder of biopolymers and biomembranes (Baianu 1980). Yet other basic reasons for the presence of both dynamic and structural '*bio-fuzziness*' is the 'immanent' LM-logic in biosystems, such as functional genetic networks, and possibly also the Q-logic of signalling pathways in living cells. There are, however, significant differences between Quantum Logic, which is also non-commutative, and the LM-Logics of Life processes. Whereas certain reductionists would attempt to reduce Life's logics, or even human consciousness, to Quantum Logic (QL), the former are at least logically and algebraically *not reducible to QL*. Nonetheless, it may be possible to formulate QL through certain modifications of *non-commutative LM-logics* (Baianu 2005; Baianu et al. 2006b).

Perhaps the most important attributes of Life are those related to the logics 'immanent' in those processes that are essential to Life. As an example, the logics and logic-algebras associated with functioning neuronal networks in the human brain–which are different from the multi–valued (Łukasiewicz–Moisil) logics (Georgescu 2006) associated with functional genetic networks (Baianu 1977, 1987a,b; Baianu et al. 2006b) and self-reproduction (Lofgren 1968; Baianu 1970; 1987a)—were shown to be different from the simple Boolean-crysippian logic upon which machines and computers are built by humans. The former n-valued (LM) logics of functional neuronal or genetic networks are *non-commutative* ones, leading to *non-linear, supercomplex* dynamics, whereas the simple logics of 'physical' dynamic systems and machines/automata are *commutative* (in the sense of involving a commutative lattice structure). Here, we find a fundamental, logical reason why living organisms are *non-commutative*, supercomplex systems, whereas simple dynamical systems have *commutative modeling diagrams* that are based on *commutative Boolean* logic. We also have here the reason why a *commutative* Categorical Ontology of Neural networks leads to advanced robotics and AI, but has indeed little to do with the '*immanent logics*' and functioning of the living brain, contrary to the proposition made by McCulloch and Pitts (1943).

There have been several attempts at defining life in reductionistic terms and a few non-reductionist ones. Rashevsky (1968) attempted to define life in terms of the essential functional relations arising between organismic sets of various orders, i.e. organizational levels, beginning with genetic sets, their activities and products as the lowest possible order, zero, of on 'organismic set' (OS). Then he pursued the idea in terms of logical Boolean predicates (1969). Attempting to provide the simplest model possible he proposed the organismic set, or OS, as a basic representation of living systems, but he did not attempt himself to endow his OS with either a topological or categorical structure, in spite of the fact that he previously reported on the fundamental connection between Topology and Life (Rashevsky 1954). He did attempt, however, a logical analysis in terms of formal symbolic logics and Hilbert's predicates. Furthermore, his PhD student, Robert Rosen did take up the challenge of representing organisms in terms of simple categorical models—his Metabolic-Repair, (**M,R**)-systems, or (**MR**)s (Rosen 1958a, b). These two seminal papers were then followed by a series of follow up reports with many interesting, biologically relevant results and consequences in spite of the simplicity of the MR, categorical set 'structure'. Further extensions and generalizations of MR's were subsequently explored by considering abstract categories with both algebraic and topological structures (Baianu and Marinescu 1973; Baianu 1974, 1980a, 1984, 1987a,b).

Whereas simple dynamic systems, or general automata, have *canonically decomposable semigroup* state spaces (the Krone-Rhodes Decomposition Theorem), supercomplex systems do not have state spaces that are known to be canonically decomposable, or partitioned into functionally independent subcomponent spaces, that is within a living organism all organs are inter-dependent and integrated; one cannot generally find a subsystem or organ which retains organismic life—the full functionality of the whole organism. However, in some of the simpler organisms, for example in *Planaria*, regeneration of the whole organism is possible from several of its major parts. Pictorially, and typically, living organisms are not 'Frankensteins'/chimeras that can be functionally subdivided into independent smaller subsystems (even though cells form the key developmental and ontological levels of any multi-cellular organism that cannot survive independently unless transformed.) By contrast, automata do have in general such *canonical sub-automata/machine decompositions* of their state-space. It is in this sense also that recursively computable systems are 'simple', whereas organisms are not. We note here that an interesting, incomplete but computable, model of multi-cellular organisms was formulated in terms of 'cellular' or 'tessellation' automata simulating cellular growth in planar arrays with such ideas leading and contributing towards the 'mirror neuron system hypothesis' (Arbib 2002). This incomplete model of 'tessellation automata' is often borrowed in one form or another by seekers of computer-generated/algorithmic, artificial 'life'.

On the one hand, simple dynamical (physical) systems are often represented through groups of dynamic transformations. In GR, for example, these would be Lorentz–Poincaré groups of spacetime transformations/reference frames. On the other hand, super-complex systems, or biosystems, emerging through self-organization and complex aggregation of simple dynamical ones, are therefore expected to be represented mathematically—at least on the next level of complexity—through an extension, or generalization of mathematical groups, such as, for example, *groupoids*. Whereas simple physical systems with linear causality have high symmetry, a single energy minimum, and thus they possess only *degenerate* dynamics, the super-complex (living) systems emerge with lower symmetries but higher dynamic and functional/relational complexity. As symmetries get 'broken' the complexity degree increases sharply. From groups that can be considered as very simple categories that have just one object and reversible/invertible endomorphisms, one moves through 'symmetry breaking' to the structurally more complex groupoids, that are categories with many objects but still with all morphisms invertible. Dynamically, this reflects the transition from degenerate dynamics with one, or a few stable, isolated states ('degenerate' ones) to dynamic state regions of many generic states that are metastable; this multi-stability of biodynamics is nicely captured by the many objects of the groupoid and is the key to the 'flow of life' occurring as multiple transitions between the multiple metastable states of the homeostatic, living system. More details of how the latter emerge through biomolecular reactions, such as catabolic/anabolic reactions, will be presented in the next subsections, and also in the next section, especially under natural transformations of functors of biomolecular categories. As we shall see later in Sects. 14 through 15 the emergence of human consciousness as an ultra-complex process became possible through the development of the *bilaterally asymmetric* human brain, not just through a mere increase in size, but a basic change in brain architecture as well. Relationally, this is reflected in the transition to a higher dimensional structure, for example a double biogroupoid representing the bilaterally asymmetric human brain architecture, as we shall discuss further in Sect. 14.

Therefore, we shall consider throughout the following sections various groupoids as some of the 'simplest' illustrations of the mathematical structures present in super-complex biological systems and classes thereof, such as *biogroupoids* (the groupoids featuring in biosystems) and variable biogroupoids to represent evolving biological species. Relevant are here also *crossed complexes* of variable groupoids and/or *multi-groupoids* as more complex representations of biosystems that follow the emergence of ultra-complex systems (the mind and human societies, for example) from supercomplex dynamic systems (organisms).

Although Darwin's Natural Selection theory has provided for more than 150 years a coherent framework for mapping the Evolution of species, it could not attempt to explain how Life itself has emerged in the first place, or predict the rates at which evolution occurred/occurs, or even predict to any degree of detail what the intermediate 'missing links', or intervening species, looked like, especially during their ascent to man. On the other hand, Huxley, the major proponent of Darwin's Natural Selection theory of Evolution, correctly proposed that the great, 'anthropoid' apes were perhaps 10 million years ago in man's ancestral line. The other two major pieces specified here—as well as the Relational and Molecular Biologies—that are missing from Darwin's and neo-Darwinist theories, are still the subject of intense investigation. We intend to explore in the next sections some possible, and plausible, answers to these remaining questions.

We note here that part of the answer to the question how did life first emerge on earth is suggested by the modeling diagram considered in Sect. 11.2 and the evolutionary taxonomy: it must have been the simplest possible organism, i.e., one that defined the minimum conditions for the emergence of life on earth. Additional specifications of the path taken by the emergence of the first super-complex living organism on earth, the 'primordial', come from an extension of **MR** theory and the consideration of its possible molecular realizations and molecular evolution (Baianu 1984). The question still remains open: why primordial life-forms or super-complex systems no longer emerge on earth, again and again. The usual answer is that the conditions existing for the formation of the 'primordial' no longer exist on earth at this point in time. Whereas, this could be part of the answer, one could then further enquire if such conditions may not be generated artificially in the laboratory. The answer to the latter question, however, shows that we do not yet have sufficient knowledge to generate the primordial in the laboratory, and also that unlike natural evolution which had billions of years available to pseudo-randomly explore numerous possibilities, man does not have that luxury in the laboratory.

11.2 Emergence of Organisms, Essential Organismic Functions and Life

Whereas it would be desirable to have a precise definition of living organisms, the list of attributes needed for such a definition can be quite lengthy. In addition to super-complex, recursively non-computable and open, the attributes: auto-catalytic, self-organizing, structurally stable/generic, self-repair, self-reproducing, highly interconnected internally, multi-level, and also possessing multi-valued logic and anticipatory capabilities would be recognized as important. One needs to add to this list at least the following: diffusion processes, inter-cellular flows, essential thermodynamically linked, irreversible processes coupled to bioenergetic processes and (bio)chemical concentration gradients, and fluxes selectively mediated by semi-permeable biomembranes. This list is far from being complete. Some of these important attributes of organisms are inter-dependent and serve to define life categorically as a super-complex dynamic process that can have several alternate, or complementary descriptions/representations; these can be formulated, for example, in terms of variable categories, variable groupoids, generalized Metabolic-Repair systems, organismic sets, hypergraphs, memory evolutive systems (MES), organismic toposes, interactomes, organismic super-categories and higher dimensional algebra.

*11.2.1 The Primordial(s) as the simplest (**M,R**)-System. Enzyme Catalysis and Organismic Self-Repair. Auto-catalytic and Autopoietic Systems*

Organisms are thought of having all evolved from a simpler, 'primordial', proto-system or cell formed (how?) three, or perhaps four, billion years ago. Such a system, if considered to be the simplest, must have been similar to a bacterium, though perhaps without a cell wall, and also perhaps with a much smaller, single chromosome containing very few RNA 'genes' (two or, most likely, four).

We shall consider next a simple 'metaphor' of metabolic, self-repairing and self-reproducing models called (M,R)-systems, introduced by Robert Rosen (1958a,b). Such models can represent some of the organismic functions that are essential to life; these models have been extensively studied and they can be further extended or generalized in several interesting ways. Rosen's simplest MR predicts one RNA 'gene' and just one proto-enzyme for the primordial 'organism'. An extended **MR** (Baianu 1973; 1984) predicts however the primordial, PMR, equipped with a *ribozyme* (a telomerase-like, proto-enzyme), and this PMR is then also capable of ribozyme—catalized DNA synthesis, and would have been perhaps surrounded by a 'simple' lipid-bilayer membrane some 4 billion years ago. Mathematically, this can be represented as:

$$A \xrightarrow{f} B \xrightarrow{\Phi} \Re[\mathsf{A},\mathsf{B}] \xrightarrow{\beta} \Re[\mathsf{B},\Re[\mathsf{A},\mathsf{B}]\,] \xrightarrow{\gamma} \ldots \longrightarrow \infty \ldots$$

where the symbol $\Re$ is the **MR** category representing the 'primordial' organism, PMR, and $\Re$[A,B] is the class of morphisms (proto-enzymes) bewteen the metabolic input class A (substrates) and the metabolic output class B (metabolic products of proto-enzymes). The ribozyme γ is capable of both catalizing and 'reverse' encoding its RNA template into the more stable DNA duplex, ∞. One can reasonably expect that such primordial genes were conserved throughout evolution and may therefore be found through comparative, functional genomic studies. The first ribozymes may have evolved under high temperature conditions near cooling volcanoes in hot water springs and their auto-catalytic capabilities may have been crucial for rapidly producing a large population of self-reproducing primordials and their descendant, *Archea*-like organisms.

Note that the primordial **MR**, or *PMR* = $\Re$, is an auto-catalytic, self-reproducing and autopoietic system. However, its 'evolution' is not entailed or enabled as yet. For this, one needs define first a variable biogroupoid or variable category, as we shall see in the next sections.

11.3 Generalized (M,R)-Systems as Variable Groupoids

We have the important example of MR-Systems with *metabolic groupoid* structures (that is, *reversible enzyme reactions/metabolic functions–repair replication* groupoid structures), for the purpose of studying RNA, DNA, epigenomic and genomic functions. For instance, the relationship of

$$\text{METABOLISM} = \text{ANABOLISM} \Longrightarrow \Longleftarrow \text{CATABOLISM}$$

can be represented by a metabolic groupoid of '*reversible', anabolic/catabolic processes*. In this respect the simplest MR-system can be represented as a *topological groupoid* with the open neighbourhood topology defined for the entire dynamical state space of the MR-system, that is an open/generic—and thus, a structurally stable—system, as defined by the dynamic realizations of MR-systems (Rosen 1971; 1977). This necessitates a descriptive formalism in terms of *variable groupoids* following which the human MR-system would then arise as the *colimit* of its complete biological family tree expressible in terms of a family of many linked/ connected groupoids; this variable biogroupoid formalism is briefly outlined in the next section.

11.4 Evolving Species as Variable Biogroupoids

For a collection of *variable groupoids* we can firstly envisage a parametrized family of groupoids $\{\mathsf{G}_\lambda\}$ with parameter λ (which may be a time parameter, although in general we do not insist on this). This is one basic and obvious way of seeing a variable groupoid structure. If λ belongs to a set M, then we may consider simply a projection $\mathsf{G} \times M \longrightarrow M$, which is an example of a trivial fibration. More generally, we could consider a *fibration of groupoids* $\mathsf{G} \hookrightarrow Z \longrightarrow M$ (Higgins and Mackenzie 1990). However, we expect in several of the situations discussed in this paper (such as, for example, the metabolic groupoid introduced in the previous subsection) that the systems represented by the groupoid are interacting. Thus, besides systems modeled in terms of a *fibration of groupoids*, we may consider a multiple groupoid as defined as a set with a number of groupoid structures any distinct pair of which satisfy an *interchange law* which can be expressed as: each is a morphism for the other, or alternatively: there is a unique expression of the following composition:

$$\begin{bmatrix} x & y \\ z & w \end{bmatrix} \qquad \begin{array}{l} \longrightarrow j \\ \downarrow \\ i \end{array} \tag{11.2}$$

where i and j must be distinct for this concept to be well defined. This uniqueness can also be represented by the equation

$$(x \circ_j y) \circ_i (z \circ_j w) = (x \circ_i z) \circ_j (y \circ_i w). \tag{11.3}$$

This illustrates the principle that a 2-dimensional formula may be more comprehensible than a linear one!

Brown and Higgins (1981) showed that certain multiple groupoids equipped with an extra structure called *connections* were equivalent to another structure called a *crossed complex* which had already occurred in homotopy theory. We shall say more about these later.

In general, we are interested in the investigation of the applications of the inclusions

$$(\text{groups}) \subset (\text{groupoids}) \subset (\text{multiple groupoids}).$$

The applications of groups, and Lie groups, in mathematics and physics are well known. Groupoids and Lie groupoids are beginning to be applied in such areas as quantization (see Landsmann 1998). Indeed it is well known that groupoids allow for a more flexible approach to symmetry than do groups alone. There is probably a vast field open to further exploration at the doorstep.

One of the difficulties, however, is that multiple groupoids can be very complex algebraic objects. It is known for example that they model weak homotopy n-types. This allows the possibility of a revolution in algebraic topology.

Another important notion is the *classifying space* BC of a crossed complex C. This, and the monoidal closed structure on crossed complexes, have been applied by Porter and Turaev to questions on Homotopy Quantum Field Theories (these are

TQFT's with a 'background space' which can be helpfully taken to be of the form *BC* as above), and by Martins and Porter (2004), as *invariants* of interest in physics.

The *patching mechanism of a groupoid atlas* connects the iterates of local procedures (Bak et al. 2006). One might also consider in general a *stack in groupoids* (Borceux 1994), and indeed there are other options for constructing relational structures of higher complexity, such as *double, or multiple* groupoids (Brown 2004; 2006). As far as we can see, these are different ways of dealing with gluing or patching procedures, a method which goes back to Mercator!

For example, the notion of an *atlas* of structures should, in principle, apply to a lot of interesting, topological and/or algebraic, structures: groupoids, multiple groupoids, Heyting algebras, n-valued logic algebras and C^*-convolution-algebras. One might incorporate a 3-valued logic here and a 4-valued logic there, and so on. An example from the ultra-complex system of the human mind is *synaesthesia*—the case of extreme communication processes between different types of 'logics' or different levels of 'thoughts'/thought processes. The key point here is *communication*. Hearing has to communicate to sight/vision in some way; this seems to happen in the human brain in the audiovisual (neocortex) and in the Wernicke (W) integrating area in the left-side hemisphere of the brain, that also communicates with the speech centers or the Broca area, also in the left hemisphere. Because of this *dual-functional*, quasi-symmetry, or more precisely asymmetry of the human brain, it may be useful to represent all two-way communication/signalling pathways in the two brain hemispheres by a *double groupoid* as an over-simplified groupoid structure that may represent such quasi-symmetry of the two sides of the human brain. In this case, the 300 millions or so of neuronal interconnections in the *corpus callosum* that link up neural network pathways between the left and the right hemispheres of the brain would be represented by the geometrical connection in the double groupoid. The brain's overall *asymmetric* distribution of functions and neural network structure between the two brain hemispheres may therefore require a non-commutative, double-groupoid structure for its relational representation. The potentially interesting question then arises how one would mathematically represent the split-brains that have been neurosurgically generated by cutting just the *corpus callosum*—some 300 million interconnections in the human brain (Sperry 1992). It would seem that either a crossed complex of two, or several, groupoids, or indeed a direct product of two groupoids G_1 and G_2, $G_1 \times G_2$ might provide some of the simplest representations of the human split-brain. The latter, direct product construction has a certain kind of built-in commutativity: $(a, b)(c, d) = (ac, bd)$, which is a form of the interchange law. In fact, from any two groupoids G_1 and G_2 one can construct a double groupoid $G_1 \bowtie G_2$ whose objects are $\mathrm{Ob}(G_1) \times \mathrm{Ob}(G_2)$. The internal groupoid 'connection' present in the double groupoid would then represent the remaining basal/'ancient' brain connections between the two hemispheres, below the *corpum* callosum that has been removed by neurosurgery in the split-brain human patients.

The remarkable variability observed in such human subjects both between different subjects and also at different times after the split-brain (bridge-localized) surgery may very well be accounted for by the different possible groupoid representations. It may also be explained by the existence of other, older neural pathways that remain untouched by the neurosurgeon in the split-brain, and which re-learn gradually, in time, to at least partially re-connect the two sides of the human split-brain. The more common health problem—caused by the senescence of the brain—could be approached as a *local-to-global*, super-complex ageing process represented for example by the *patching* of a *topological double groupoid atlas* connecting up many local faulty dynamics in 'small' un-repairable regions of the brain neural network, caused for example by tangles, locally blocked arterioles and/or capillaries, and also low local oxygen or nutrient concentrations. The result, as correctly surmised by Rosen (1987), is a *global*, rather than local, senescence, super-complex dynamic process.

On the other hand, for 'simple' physical systems it is quite reasonable to suppose that structures associated with symmetry and transitions could well be represented by 1-groupoids, whereas transitions between *quantum* transitions, could be then represented by a special type of quantum symmetry double groupoid that we shall call here simply a *quantum double groupoid* (QDG; Baianu, Brown and Glazebrook 2007b), as it refers to *fundamental quantum* dynamic processes (cf. Werner Heisenberg, as cited by Brown 2002).

Developmental processes, and in general, ontogeny considered from a structural or anatomical viewpoint involves not only geometrical or topological transformations but more general/complex transformations of even more flexible structures such as variable groupoids. The natural generalizations of variable groupoids lead to 'variable topology' concepts that are considered in the next subsection.

11.5 Variable Topologies

Let us recall the basic principle that a *topological space* consists of a set X and a 'topology' on X where the latter gives a precise but general sense to the intuitive ideas of 'nearness' and 'continuity'. Thus the initial task is to axiomatize the notion of 'neighbourhood' and then consider a topology in terms of open or of closed sets, a compact-open topology, and so on (see Brown 2006). In any case, a topological space consists of a pair $(X, \mathcal{T})$ where $\mathcal{T}$ is a topology on X. For instance, suppose an *open set topology* is given by the set $\mathcal{U}$ of prescribed open sets of X satisfying the usual axioms (Brown 2006, Chapter 2). Now, to speak of a variable open-set topology one might conveniently take in this case a family of sets $\mathcal{U}_\lambda$ of *a system of prescribed open sets*, where λ belongs to some indexing set Λ. The system of open sets may of course be based on a system of contained neighbourhoods of points where one system may have a different geometric property compared say to another system (a system of disc-like neighbourhoods compared with those of cylindrical-type). In general, we may speak of a topological space with a *varying topology* as a pair $(X, \mathcal{T}_\lambda)$ where $\lambda \in \Lambda$. The idea of a varying topology has been introduced to describe possible topological distinctions in bio-molecular organisms through stages of development, evolution, neo-plasticity, etc. This is indicated schematically in the diagram below where we have an n-stage dynamic evolution (through complexity) of categories D_i where the vertical arrows denote the assignment of topologies $\mathcal{T}_i$ to the class of objects of the D_i along with functors $\mathcal{F}_i : \mathsf{D}_i \longrightarrow \mathsf{D}_{i+1}$, for $1 \leq i \leq n$–1:

$$\begin{array}{ccccccccc}
\mathcal{T}_1 & & \mathcal{T}_2 & & \cdots & & \mathcal{T}_{n-1} & & \mathcal{T}_n \\
\downarrow & & \downarrow & & & & \downarrow & & \downarrow \\
\mathsf{D}_1 & \xrightarrow{\mathcal{F}_1} & \mathsf{D}_2 & \xrightarrow{\mathcal{F}_2} & & \cdots & \mathsf{D}_{n-1} & \xrightarrow{\mathcal{F}_{n-1}} & \mathsf{D}_n
\end{array}$$

In this way a variable topology can be realized through such n-levels of complexity of the development of an organism. Another instance is when cell/network topologies are prescribed and in particular when one considers a categorical approach involving concepts such as *the free groupoid over a graph* (Brown 2006). Thus a varying graph system clearly induces an accompanying system of variable groupoids. As suggested by Golubitsky and Stewart (2006), symmetry groupoids of various cell networks would appear relevant to the physiology of animal locomotion, as one example.

12 Evolution and Dynamics of Systems, Networks and Organisms: Evolution as the Emergence of Increasing Organismic Complexity. Speciation and Molecular 'Evolution'

12.1 Propagation and Persistence of Organisms through Space and Time. Survival and Extinction of Species

The autopoietic model of Maturana and Varela (1980) claims to explain the persistence of living systems in time as the consequence of their structural coupling or *adaptation* as structure determined systems, and also because of their existence as *molecular* autopoietic systems with a 'closed' network structure. As part of the autopoietic explanation is the 'structural drift', presumably facilitating evolutionary changes and speciation. One notes that autopoietic systems may be therefore considered as dynamic realizations of Rosen's simple **MR**s. Similar arguments seem to be echoed more recently by Dawkins (1982) who claims to explain the remarkable persistence of biological organisms over geological timescales as the result of their intrinsic, (super-) complex adaptive capabilities.

The point is being often made that it is not the component atoms that are preserved in organisms (and indeed in 'living fosils' for geological periods of time), but the *structure–function relational pattern*, or indeed the associated organismic categories or supercategories. This is a very important point: only the functional organismic structure is 'immortal' as it is being conserved and transmitted from one generation to the next. Hence the relevance here, and indeed the great importance of the science of abstract structures and relations, i.e., Mathematics.

This was the feature that appeared paradoxical or puzzling to Erwin Schrödinger from a quantum theoretical point of view when he wrote his book "What is Life?" As individual molecules often interact through multiple quantum interactions, which are most of the time causing *irreversible*, molecular or energetic changes to occur, how can one then explain the hereditary stability over hundreds of years (*or* *occasionally, a great deal longer, NAs*) within the same genealogy of a family of men? The answer is that the 'actors change but the play does not!'. The atoms and molecules turn-over, and not infrequently, but the *structure–function patterns/ organismic categories remain unchanged*/are conserved over long periods of time through repeated repairs and replacements of the molecular parts that need repairing, as long as the organism lives. Such stable patterns of relations are, at least in principle, amenable to logical and mathematical representation without tearing apart the living system. In fact, looking at this remarkable persistence of certain gene subnetworks in time and space from the categorical ontology and Darwinian viewpoints, the *existence of live 'fossils'* (e.g., a coelacanth found alive in 1923 to have remained unchanged at great depths in the ocean as a species for 300 million years!) it is not so difficult to explain; one can attribute the rare examples of 'live fossils' to the lack of 'selection pressure in a very stable niche'. Thus, one sees in such exceptions the lack of any adaptation apart from those which have already occurred some 300 million years ago. This is by no means the only long lived species: several species of marine, giant unicellular green algae with complex morphology from a family called the *Dasycladales* may have persisted as long as 600 million years (Goodwin 1994), and so on. However, the situation of many other species that emerged through *super-complex adaptations*—such as the species of *Homo sapiens*—is quite the opposite, in the sense of marked, super-complex adaptive changes over much shorter time-scales than that of the exceptionally 'lucky' coelacanths. Clearly, some species, that were less adaptable, such as the Neanderthals or *Homo erectus*, became extinct even though many of their functional genes may be still conserved in *Homo sapiens*, as for example, through comparison with the more distant chimpanzee relative. When comparing the *Homo erectus* fossils with skeletal remains of modern men one is struck how much closer the former are to modern man than to either the *Australopithecus* or the chimpanzee (the last two species appear to have quite similar skeletons and skulls, and also their 'reconstructed' vocal chords/apparatus would not allow them to speak). Therefore, if the functional genomes of man and chimpanzee overlap by about 98%, then the overlap of modern man functional genome would have to be greater than 99% with that of *Homo erectus* of 1 million years ago, if it somehow could be actually found and measured (but it cannot be, at least not at this point in time). Thus, one would also wonder if another more recent hominin than *H. erectus*, such as *Homo floresiensis*—which is estimated to have existed between 74,000 and 18,000 years ago on the now Indonesian island of Flores—may have been capable of human speech. One may thus consider another indicator of intelligence such as the size of region 10 of the dorsomedial prefrontal cortex, which is thought to be associated with the existence of *self-awareness*; this region 10 is about the same size in *H. floresiensis* as in modern humans, despite the much smaller overall size of the brain in the former (Falk et al. 2005).

Passing the threshold towards human consciousness and awareness of the human self may have occurred—with any degree of certainty—only with the ascent of the *Cro–Magnon* man which is thought to belong to the modern species of *Homo sapiens sapiens*, (chromosomally descended from the Y haplogroup F/mt haplogroup N populations of the Middle East). This important transition seems to have taken place between 60,000 and 10,000 years ago through the formation of Cro–Magnon, human ‘societies’–perhaps consisting of small bands of 25 individuals or so sharing their hunting, stone tools, wooden or stone weapons, a fire, the food, a cave, one large territory, and ultimately reaching human consensus.

12.1.1 Biological Species

After a century-long debate about what constitutes a biological species, taxonomists and general biologists seem to have now adopted the operational concept proposed by Mayr:

> a species is a group of animals that share a common gene pool and that are reproductively isolated from other groups.

Unfortunately, this concept is not readily applicable to extinct species and their fossils, the subject of great interest to paleoanthropologists, for example. From an ontology viewpoint, the biological species can be defined as a class of equivalent organisms from the point of view of sexual reproduction and or/functional genome, or as a *biogroupoid* (Baianu et al. 2006b). Whereas satisfactory as taxonomic tools these two definitions are not directly useful for understanding evolution. The biogroupoid concept, however, can be readily extended to a more flexible concept, the *variable groupoid*, which can be then utilized in theoretical evolutionary studies, and through predictions, impact on empirical evolutionary studies, as well as possibly organismic taxonomy.

12.2 Super-Complex Network Biodynamics in Variable Biogroupoid Categories. Variable Bionetworks and their Super-Categories

This section is an extension of the previous one in which we introduced variable biogroupoids in relation to speciation and the evolution of species. The variable category concept generalizes the concept of variable groupoid which can be thought as a variable category whose morphisms are invertible. The latter is thus a more ‘symmetric’ structure than the general variable category.

We have seen that variable biogroupoid representations of biological species, as well as their categorical limits and colimits, may provide powerful tools for tracking evolution at the level of species. On the other hand, the representation of organisms, with the exception of unicellular ones, is likely to require more general structures, and super-structures of structures (Baianu 1970). In other words, this leads towards higher-dimensional algebras (HDA) representing the super-complex hierarchies present in a complex-functional, multi-cellular organism, or in a highly evolved functional organ such as the human brain. The latter (HDA) approach will be discussed in a later section in relation to neurosciences and consciousness, whereas we shall address here the question of representing biosystems in terms of variable categories that are lower in complexity than the ultra-complex human mind. A variable category and/or variable topology approach is, on the other hand, a simpler alternative to the organismic LM-topos that will be employed in Sects. 12.7 and 12.7.1 to represent the emergence and evolution of genetic network biodynamics, comparative genomics and phylogeny. In terms of representation capabilities, the range of applications for variable categories may also extend to the neurosciences, neurodynamics and brain development, in addition to the evolution of the simpler genomes and/or interactomes. Last-but-not-least, it does lead directly to the more powerful 'hierarchical' structures of higher dimensional algebra.

12.3 Evolution as a Local-to-Global Problem: The Metaphor of Chains of Local Procedures. Alternate Representations of Evolution by MES and Colimits of Transforming Species. Bifurcations, Phylogeny and the 'Tree of Life'

Darwin's 'theory' of natural selection, sometimes considered as a reductionist attempt in spite of its consideration of both specific and general biological functions such as adaptation, reproduction, heredity and survival, has been substantially enriched over the last century; this was achieved through more precise mathematical approaches to population genetics and molecular evolution which developed new solutions to the key problem of speciation (Bendall 1982; Mayr and Provine 1986; Pollard 1984; Sober 1984; Gregory 1987). Modified evolutionary theories include neo–Darwinism, the 'punctuated evolution' (Gould 1977) and the 'neutral theory of molecular evolution' of Kimura (1983). The latter is particularly interesting as it reveals that evolutionary changes do occur much more frequently in unexpressed/silent regions of the genome, thus being 'invisible' phenotypically. Therefore, such frequent changes ('silent mutations') are uncorrelated with, or unaffected by, natural selection. For further progress in completing a logically valid and experimentally based evolutionary theory, an improved understanding of speciation and species is required, as well as substantially more extensive, experimental/genomic data related to speciation than currently available. Furthermore, the ascent of man, as often proposed by evolutionary theories of *H. sapiens* beginning with that of Huxley, is apparently not the result of only natural selection but also that of co-evolution through society interactions; thus, simply put: the emergence of human speech and consciousness occurred both through selection and co-evolution, with the former not being all that 'natural' as society played a protective, as well as selective role from the very beginnings of hominin and hominid societies more than 2.2 million years ago. Somewhat surprisingly, the subject of *social selection* in human societies is rarely studied even though it may have played a crucial role in the emergence of *H. sapiens*, and occurs in every society that we know without exception. To the extent that social selection is not driven—at least not directly—by the natural environment it might be classified also as 'artificial' even though it does not involve any artificial breeding procedures, and it cannot be therefore assimilated in any way with what we call artificial selection of plants or animals.

Furthermore, there is a theory of levels, ontological question that has not yet been adequately addressed, although it has been identified: *at what level does evolution operate: species, organism or molecular (genetic)?* According to Darwin the answer seems to be the species; however, not everybody agrees because in Darwin's time a valid theory of inherited characters was neither widely known nor accepted. Moreover, molecular evolution and concerted mutations are quite recent concepts whose full impact has not yet been realized. As Brian Goodwin (1994) puts it succinctly:

> "Where has the organism disappeared in Darwin's evolutionary theory?"

The answer in both Goodwin's opinion, and also in ours, lies in the presence of key functional/relational patterns that emerged and were preserved in organisms throughout various stages over 4 billion years or so of evolution. The fundamental relations between organism, species and the speciation process itself do need to be directly addressed by any theory that now claims to explain the Evolution of species and organisms. Furthermore, an adequate consideration of the biomolecular levels and sub-levels involvement in Speciation and Evolution must also be present in any modern evolutionary theory. These fundamental questions will be addressed for the first time from the categorical ontology standpoint in this and the next section.

To date there is no complete, direct observation of the formation of even one live, new multi-cellular species through *natural* selection, in spite of the rich paleontological, indirect evidence of evolution towards organisms of increasingly higher complexity with evolutionary time. However, man has generated many new species through selective breeding/artificial selection based on a fairly detailed understanding of hereditary principles, both Mendelian and non-Mendelian. Still more species of the simpler organisms are being engineered by man through molecular genetic manipulations, often raising grave concerns to the uninitiated layman leading to very restrictive legislation, especially in Europe. There are several differences between natural and artificial selection, with the main difference being seen in the pseudo-randomness of natural selection as opposed to the sharply directed artificial selection exerted by human breeders. This is however a matter of degree rather than absolute distinction: natural selection is not a truly random process either and artificial selection does involve some trial and error as it is not a totally controllable exercise. Furthermore, natural selection operates through several mechanisms on different levels whereas artificial selection involves strictly controlled reproduction and may involve just the single organism level to start with, followed by deliberate inbreeding, as an example. Therefore one can reasonably argue that natural selection mechanisms differ from those of artificial selective breeding, with *adaptive* 'mechanisms' being largely eliminated in the latter, even though the laws of heredity are of course respected by both, but with fertilization and embryonic/organismal development being often under the breeder's control.

In this section, we shall endeavour to address the question of super-complex systems' evolution as a *local-to-global* problem and we shall seek solutions in terms of the novel categorical concepts that we introduced in the previous subsections. Thus, we shall consider biological evolution by introducing the unifying metaphor of '*local procedures*' which may represent the formation of new species that branch out to generate still more evolving species.

In his widely read book, D-Arcy W. Thompson (1994, re-printed edition) gives a large number of biological examples of organismic growth and forms analyzed at first in terms of physical forces. Then, he is successful in carrying out analytical geometry coordinate transformations that allow the continuous, homotopic mapping of series of species that are thought to belong to the same branch—phylogenetic line—of the tree of life. However, he finds it very difficult or almost impossible to carry out such transformations for fossil species, skeleton remains of species belonging to different evolutionary branches. Thus, he arrives at the conclusion that the overall evolutionary process is not a continuous sequence of organismic forms or phenotypes (see p. 1094 of his book).

Because genetic mutations that lead to new species are discrete changes as discussed above in Sects. 9 and 10, we are therefore not considering evolution as a series of continuous changes—such as a continuous curve drawn analytically through points representing species—but heuristically as a *tree of 'chains of local procedures'* (Brown 2006). Evolution may be alternatively thought of and analyzed as a *composition of local procedures*. Composition is a kind of combination and so it might be confused with a colimit, but they are substantially different concepts.

Therefore, one may attempt to represent biological evolution as an evolutionary tree, or tree of life, with its branches completed through chains of local procedures (pictured in Fig. 1 as overlapping circles) involving certain groupoids, which informally we call *variable topological biogroupoids*, and with the overlaps corresponding to 'intermediate' species or classes/populations of organisms which are rapidly evolving under strong evolutionary pressure from their environment (including competing species, predators, etc., in their niche).

A more specific formalization follows. The notion of 'local procedure' is an interpretation of Ehresmann's formal definition of a *local admissible section* s for a groupoid G in which $X = \mathrm{Ob}(G)$ is a topological space. Then s is a section of the source map $\alpha\colon G \to X$ such that the domain of s is open in X. If s, f are two such sections, their composition st is defined by $\mathsf{st}(x) = \mathsf{s}(\beta t(x)) \circ t(x)$ where $\circ$ is the composition in G. The domain of st may be empty. One may also put the additional condition that s is 'admissible', namely $\beta\mathsf{s}$ maps the open domain of $\mathbf{s}$ homeomorphically to the image of $\beta\mathsf{s}$, which itself is open in X. Then an admissible local section is invertible with respect to the above composition.

The categorical colimits of MES, that may also be heuristically thought as 'chains of local procedures' (COLP), have their vertex object at the branching point on the evolutionary tree. The entire evolutionary tree—tracked to present day—is then intuitively represented through such connected chains of local procedures beginning with the primordial(s) and ending with *Homo*, thus generating an intuitive *global colimit* in the *2-category* of all variable topological

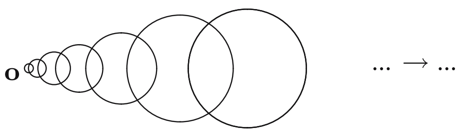


Fig. 1 Pictorial representation of Biological Evolution as a composition of local procedures involving variable biogroupoids that represent biological speciation phenomena. COLPs may form the branches of the evolutionary tree, oriented in this diagram with the time arrow pointing to the right

biogroupoids (VTBs) that correspond to all classes of evolving organisms (either dead or alive). Such VTBs have a generic–dynamic, pictorial illustration which is shown as circles in the following diagram of this global (albeit intuitive) evolutionary colimit ("lim $\hookrightarrow$"). The primordial can be selected in this context as represented by the special PMR which is (was) realized by ribozymes as described in Sect. 11.2.1.

Note also that organisms were previously represented in terms of categories of dynamic state-spaces (Baianu 1970, 1980, 1987; Baianu et al. 2006a, b) which are defined in terms of the various stages of ontogenetic development with increasing numbers of cells and functions as specialization and morphogenesis proceed in real time. This representation leads to the concept of a *direct limit* of organisms or equivalence classes of organisms of increasing complexity during evolution, as explained before. We start with the definition of a *direct system* of objects and homomorphisms or homeomorphisms, ('transformations', functors, super-functors, natural transformations, etc). Let $(\mathsf{I}, \leq)$ be a *directed poset* whose elements i are the *complexity indices* of evolving organisms; an index of complexity is defined for example in terms of the genome complexity, with genetic network dynamics represented in terms of an **LM**-logic algebra and **LM**-algebra morphisms (Baianu 1977, 1987; Baianu et al. 2006a, b). Let $O_i|i \in \mathbf{I}$ be a family of objects (organisms or organismic supercategories (Baianu 1970, 1971)) indexed by **I** and suppose we have a family of homomorphisms (or homeomorphisms, or transformations, functors/ super-functors, etc.) f_{ij}: $O_i \to O_j$ for all $i \leq j$ with the following properties:

1. f_{ii} is the identity in O_i,
2. $f_{ik} = f_{jk} \circ f_{ij}$ for all $i \leq j \leq k$.

Then the pair (O_i, f_{ij}) is called a *direct system* over **I**. The ***direct limit***, **O**, of the direct system (O_i, f_{ij}) is defined as the coproduct of the O_i's *modulo* a certain equivalence relation defined by evolutionary complexity:

$\varinjlim O_i = (\amalg_i)/[x_i \sim x_j \mid$ there exists $k \in \mathbf{I}$ such that $f_{ik}(x_i) = f_{jk}(x_j)$.

Two elements in the disjoint union can be regarded as 'equivalent' if and only if they "eventually become equal" in the direct system. Thus, one naturally obtains from this equivalence definition the corresponding *canonical* morphisms $\varphi_i : O_i \to \mathsf{O}$ sending each 'element' (organism) to its equivalence (complexity) class. The algebraic operations on O are defined via these maps in an obvious manner. A general definition is also possible (Mac Lane 2000). The *direct limit* can be defined abstractly in an arbitrary category by means of a *universal property*. Let (X_i, f_{ij}) be a direct system of objects and morphisms in a category C (same definition as above). The abstract *direct limit* of this system of evolving organisms is an object X in C together with morphisms $\varphi_i : X_i \to X$ satisfying $\varphi_i = \varphi_j \circ f_{ij}$. The pair (X, φ_i) must be *universal* with the meaning that for any other such pair (Y, ψ_i) there exists a unique morphism $u : X \to Y$ making all the "obvious" identities hold, i.e. the cocone diagram (such as 14.2) must commute for all i, j. The direct limit is often denoted as:

$$\boldsymbol{X} = \varinjlim X_i,$$

with the direct system (X_i, f_{ij}) being tacitly assumed to exist and also to be completely specified. Unlike the case of algebraic objects, the direct limit may not exist in an arbitrary category. If it does, however, it is unique in a strong sense: given any other direct limit X' there exists is a unique isomorphism $\mathsf{X}' \longrightarrow \mathsf{X}$ commuting with the canonical morphisms. One notes also that a direct system in the category C admits an alternative description in terms of functors. Any directed poset I can be regarded as a small category where the morphisms consist of arrows $i \longrightarrow j$ if and only if $i \leq j$. The direct system is then just a covariant functor $L : \mathsf{I} \longrightarrow \mathsf{C}$. Similarly, a *colimit* can be thus defined by the family of ontogenetic development stages/ dynamic state-spaces, indexed by their corresponding complexity indices at specified instants of (ontogenetic) developmental time (Δ t ε R), as fully specified in previous papers (Baianu 1970; Baianu and Scripcariu 1973; Baianu 1980, 1984).

Such constructions of ontogenetic development colimits in terms of *cocone* diagrams of objects and morphisms (see Diagram 14.2) can be viewed as specific examples of 'local procedures'. Nevertheless, in a certain specific sense, these organismic (ontogenetic) development (OOD) colimits play the role of 'local procedures' in the 2-category of evolving organisms. Thus, the global colimit of the evolutionary 2-category of organisms may be regarded as a super-colimit, or an evolutionary colimit of the OOD colimits briefly mentioned above from previous reports. A tree-graph that contains only single-species biogroupoids at the 'core' of each 'local procedure' does define precisely an evolutionary branch without the need for subdivision because a species is an 'indivisible' entity from a breeding or reproductive viewpoint. Interestingly, in this dynamic sense, biological evolution 'admits' super-colimits (Baianu and Marinescu 1968; Comoroshan and Baianu 1969; Baianu 1970, 1980, 1987; Baianu et al. 2006b), with a higher-dimensional structure which is less restrictive than either MES (Ehresmann and Vanbremeersch 1987), or simple MR's represented as categories of sets (in which case direct and inverse limits can *both* be constructed in a canonical manner, cf. Baianu 1973).

We note that several different concepts introduced by distinct ontological approaches to organismal dynamics, stability and variability *converge* here on the metaphor of (chains of) 'local procedures' for evolving organisms and species. Such distinct representations are: the dynamic genericity of organismic states which lead to structural stability—as introduced by Robert Rosen (1987) and René Thom (1980), the logical class heterogeneity of living organisms introduced by Elsasser (1981), the inherent 'bio-fuzziness' of organisms (Baianu and Marinescu 1968; also discussed by Comoroshan and Baianu 1969) in both their structure and function, or as ranges of autopoietic 'structural variability' exhibited by living systems (Maturana 1980), imposed to the organism through its coupling with a specific environmental niche.

This dynamic intuition of evolution—unlike Darwin's historical concept—may be hard to grasp at first as it involves several construction stages on different ontological levels: it begins with organisms (or even with biomolecular categories!), emerges to the level of populations/subspecies/ species that evolve into classes of species, that are then further evolving,… and so on, towards the point in time where the emergence of man's, *Homo* family of species began to separate from other hominin/hominide families of species some 5–8 million years ago. Therefore, it is not at all surprising that most students of evolutionary biology have had, or still have, difficulties in understanding the real intricacies of evolutionary processes that operate on several different levels/sublevels of reality, different time scales, and also aided by geographical barriers or geological accidents. In this case, Occam's razor may seem to patently fail as the simplest 'explanations', or the longest-lasting myths, ultimately cannot win when confronted by the reality of emerging higher levels of complexity.

Furthermore, we note also that the organisms within the species represented by VTBs have an ontogenetic development represented in the dynamic state space of the organism as a categorical colimit. Therefore, the evolutionary, global colimit is in fact a *super-colimit* of all organismic developmental colimits up to the present stage of evolution. This works to a good approximation insofar as the evolutionary changes occur on a much longer timescale than the lifespan of the 'simulation' model. Thus, the degree of complexity increases above the level of super-complexity characteristic of individual organisms, or even species (biogroupoids), to a next, evolutionary meta-level, that we shall call *evolutionary meta-complexity*. Whenever there are uncertainties concerning taxonomy one could compare the alternate evolutionary possibilities by means of pairs of functors that preserve limits or colimits, called respectively, right- and left-adjoint functors. Moreover, such adjoint functor pairs also arise in comparing different developmental stages of the same organism from the viewpoint of preserving their developmental potential (Baianu and Scripcariu 1973), *dynamic* colimits preserved by the right-adjoint functor, G, and/or the *functional*, projective limits preserved by a left-adjoint functor of G (cf. Rashevsky's Principle of Biological Epimorphism, or the more general Postulate of Relational Invariance (cf. Baianu et al. 2006b); see also the Brown, Glazebrook and Baianu (2007a, in this volume) for both relevant definitions and theorems.)

Furthermore, the concept of colimit also can be extended or generalized either via variable topology or VTBs or as a multi-valued functor (Mitchell 1965) on product OS-categories, or variable categories of VTBs.

12.4 Natural Transformations of Organismic Structures

12.4.1 Biomolecular Reaction Models in Categories

A simple introduction of molecular models in categories is based here on set-theoretical models of chemical transformations. Consider the very simple case of unimolecular chemical transformations (Bartholomay 1971):

$$T : A \times I \longrightarrow B \times I \tag{12.1}$$

with A being the original sample set of molecules and $I = [0,t]$ being defined as a finite segment of the real time axis; thus, $A \times I$ allows the indexing of each A-type molecule by the instant of time at which each molecule $a \ \varepsilon \ A$ is actually transforming into a B-type molecule (see also eq. 3 of Bartholomay 1971). $B \times I$ then denotes the set of the newly formed B-type molecules which are indexed by their corresponding instance of birth.

A molecular class, denoted as A, is specified along with $f : A \longrightarrow A$, the *endomorphisms* that form the set $H(A,A)$.

One can then consider the category, $\underline{\mathsf{M}}$, of these molecular classes and their chemical transformations and also introduce natural transformations between certain canonical (hom) functors, as shown explicitly in the section on "Natural Transformations" in Brown, Glazebrook and Baianu (2007a, in this volume). A hom-functor, h^A, indexed by a specified object A, is defined as:

$$h^A : \underline{\mathsf{M}} \longrightarrow \underline{\mathsf{Set}}$$

with its action determined by:

$$h^A(X) = H(A, X) \text{ for any } X \in \underline{\mathsf{M}}$$

and

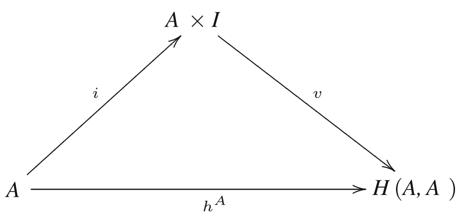
$$h^A(t) = m : H(A, A) \longrightarrow H(A, B) \text{ for any } t : A \longrightarrow B$$

where: $\mathbf{A} =$ ***Molecular Class of type A−molecules*** and $\mathbf{B} =$ ***Molecular class of reaction products or type B−molecules***

Such hom-functors—which provide representations of chemical or biochemical reactions, (that is quantum molecular transformations of **molecular class A** into **molecular class B** of reaction B-products, or molecules of type "B")—thus allow the *emergence* of the next level of organization—the natural transformations obtained through the canonical Yoneda-Grothendieck construction (as provided explicitly by the corresponding Lemma in Brown, Glazebrook and Baianu (2007, in this volume).

*12.4.2 Definition of the Molecular Class (or set) Variable, **mcv**).*

The flexible notion of a *molecular class variable* (**mcv**) is precisely represented by the morphisms **v** in the following diagram:



where morphisms v are induced by the inclusion mappings $i : A \longrightarrow A \times I$ and the commutativity conditions $h^A = v \circ i$. The naturality of this diagram simply means that such commutativity conditions hold for any functor h^A defined as above. Note also that one can define a (non-commutative) Clifford algebra (see e.g., Plymen and Robinson 1994) for the mcv-observables by endowing $A \times I$ and A with the appropriate non-commutative structures, thus generating an mcv-quantum space that is its own dual!

Simply stated, the *observable of an mcv* **B**, characterizing the chemical reaction product molecules "*B*" is defined as a morphism:

$$\gamma : H(B,B) \longrightarrow R$$

with R being the set of real numbers. This *mcv-observable* is subject to the following commutativity conditions:

$$\begin{array}{ccc} H(A,A) & \xrightarrow{\ f\ } & H(B,B) \\ {\scriptstyle e}\downarrow & & \downarrow{\scriptstyle \gamma} \\ H(A,A) & \xrightarrow[\ \delta\ ]{} & R \end{array} \qquad (12.2)$$

with $c : A_u^* \longrightarrow B_u^*$, and A_u^*, B_u^* being specially prepared *fields of states*, within a measurement uncertainty range, Δ.

On the other hand, by endowing various classes A with different Hilbert space (topological) structures one obtains mcv's that are also endowed with variable topologies determined by such 'indexing' Hilbert spaces.

The next level of complexity emerges then by extending the above representations to *multi-molecular* reactions, coupled reactions,..., stable biochemical hypercycles—as in living organisms, and also perhaps in the now extinct primeval, single-cell organism.

As shown in Brown, Glazebrook, Baianu (2007, in this volume), this extended representation then involves the *canonical functor* of category theory:

$$h : M \longrightarrow [M, \mathsf{Set}]$$

that assigns to each molecular set A the functor h^A, and to each chemical transformation $t : A \longrightarrow B$, the natural transformation $h^A \longrightarrow h^B$.

12.5 Natural Transformations as Representations of Emergent Biomolecular Reactions

12.5.1 The Category, $\underline{\mathsf{M}}$*, of Molecular Classes and their Chemical Transformations*

Let C be any category and X an object of C. We denote by $h^X : \mathsf{C} \longrightarrow \mathsf{Set}$ the functor obtained as follows: for any $Y \in \mathrm{Ob}(\mathsf{C})$ and any $f : X \longrightarrow Y$, $h^X(Y) = \mathrm{Hom}_{\mathsf{C}}(X, Y)$; if $g : Y \longrightarrow Y'$ is a morphism of C then $h^X(f) : \mathrm{Hom}_{C}(X, Y) \longrightarrow \mathrm{Hom}_{\mathsf{C}}(X, Y')$ is the map $h^X(f)(g) = fg$. One can also denote h^X as $\mathrm{Hom}_{\mathsf{C}}(X, -)$. Let us define now the very important concept of *natural transformation* which was first introduced by Eilenberg and Mac Lane (1945). Let $X \in \mathrm{Ob}(\mathsf{C})$ and let $F : \mathsf{C} \longrightarrow \mathsf{Set}$ be a covariant functor. Also, let $x \in F(X)$. We shall denote by $\eta_X : h^X \longrightarrow F$ the *natural transformation* (or *functorial morphism*) defined as follows: if $Y \in \mathrm{Ob}(\mathsf{C})$ then $(\eta_x)_Y : h^X(Y) \longrightarrow F(Y)$ is the mapping defined by the equality $(\eta_x)_Y\,(f) = F(f)(x)$; furthermore, one imposes the *naturality* (or *commutativity*) condition on the following diagram:

$$\begin{array}{ccc} F(X) & \xrightarrow{\eta_X} & F(Y) \\ {\scriptstyle F(f)}\downarrow & & \downarrow{\scriptstyle F(g)} \\ G(X) & \xrightarrow[\eta_Y]{} & G(Y) \end{array} \qquad (12.3)$$

The hom-functor, h^A, indexed by a specified object A is defined as:

$$h^A : \underline{\mathsf{M}} \longrightarrow \underline{\mathsf{Set}}$$

with its action defined as:

$$h^A(X) = H(A,X) \quad \text{for any } X \in \underline{\mathsf{M}}$$

and

$$h^A(t) = m : H(A,A) \longrightarrow H(A,B) \quad \text{for any } t : A \longrightarrow B$$

where **A = Molecular Class** and **B = Molecular class of reaction products of type "B"**, resulting from a chemical reaction.

12.6 The Representation of Unimolecular, Biochemical Reactions as Natural Transformations

The *unimolecular chemical reaction* is here represented by the natural transformations $\eta : h^A \longrightarrow h^B$, through the following commutative diagram:

$$\begin{array}{ccc} h^A(A) = H(A,A) & \xrightarrow{\eta_A} & h^B(A) = H(B,A) \\ {\scriptstyle h^A(t)}\downarrow & & \downarrow{\scriptstyle h^B(t)} \\ h^A(B) = H(A,B) & \xrightarrow[\eta_B]{} & h^B(B) = H(B,B) \end{array} \qquad (12.4)$$

with the states of the molecular sets $Au = a_1,\ldots, a_n$ and $Bu = b_1,\ldots\ b_n$ being represented by certain endomorphisms, respectively from $H(A,A)$ and $H(B,B)$.

*12.6.1 An Example of an Emerging Super-Complex System as A Quantum–Enzymatic Realization of the Simplest (**M,R**)-System*

Note that in the case of either uni-molecular or multi-molecular, *reversible* reactions one obtains a *quantum-molecular groupoid*, QG, defined as above in terms of the mcv-observables. In the case of an enzyme, E, with an activated complex, $(ES)^*$, a *quantum biomolecuar groupoid* can be uniquely defined in terms of mcv–observables for the enzyme, its activated complex $(ES)^*$ and the substrate S. Quantum tunnelling in $(ES)^*$ then leads to the separation of the reaction product and the enzyme E which enters then a new reaction cycle with another substrate molecule S', indistinguishable—or equivalent to—S. By considering a sequence of two such reactions coupled together,

$$QG_1 \leftrightarrows QG_2,$$

corresponding to an enzyme f coupled to a ribozyme ϕ, one obtains a *quantum–molecular realization of the simplest* (**M,R**)-*system* (f, ϕ) (see also the previous Sect. 11.2.1 for further details about the MR/PMR).

The non-reductionist caveat here is that the relational systems considered above are *open* ones, exchanging both energy and *mass* with the system's environment in a manner which is dependent on time, for example in cycles, as the system 'divides'-reproducing itself; therefore, even though generalized quantum-molecular observables can be defined as specified above, neither a stationary nor a dynamic Schrödinger equation holds for such examples of 'super-complex' systems. Furthermore, instead of just energetic constraints—such as the standard quantum Hamiltonian—one has the constraints imposed by the diagram commutativity related to the mcv-observables, canonical functors and natural transformations, as well as to the concentration gradients, diffusion processes, chemical potentials/activities (molecular Gibbs free energies), enzyme kinetics, and so on. Both the canonical functors and the natural transformations defined above for uni- or multi-molecular reactions represent the relational increase in complexity of the emerging, super-complex dynamic system, such as, for example, the simplest (**M,R**)-system, (f, ϕ).

12.6.2 A Simple Metabolic-Repair (**M,R**)*-System with Reverse Transcription as an example of Multi-molecular Reactions Represented by Natural Transformations*

We shall consider again the diagram corresponding to the simplest (**M, R**)-System realization of a Primordial Organism, PO.

The RNA and/or DNA duplication and cell divisions would occur by extension to the right of the simplest MR-system, (f, Φ), through the β: $H(A,B) \rightarrow H(B,H(A,B))$ and γ: $H(B,H(A,B)) \rightarrow H(H(A,B),H(B,H(A,B)))$ morphism. Note in this case, the 'closure' entailed by the functional mapping, γ, that physically represents the regeneration of the cell's *telomere* thus closing the DNA-loop at the end of the chromosome in eukaryotes. Thus γ represents the activity of a *reverse transcriptase*. Adding to this diagram an hTERT suppressor gene would provide a *feedback* mechanism for an effective control of the cell division and the possibility of cell cycle arrest in higher, multi-cellular organisms (which is present only in *somatic* cells). The other alternative—which is preferred here—is the addition of an hTERT *promoter gene* that may require to be activated in order to begin cell cycling. This also allows one to introduce simple models of carcinogenesis or cancer cells.

Rashevsky's hierarchical theory of organismic sets can also be constructed by employing mcv's with their observables and natural transformations as it was shown in a previous report (Baianu 1980).

Thus, one obtains by means of natural transformations and the Yoneda–Grothendieck construction a unified, categorical-relational theory of organismic structures that encompasses those of organismic sets, biomolecular sets, as well as the general $(\boldsymbol{M}, \boldsymbol{R})$ *-systems/autopoietic systems which takes explicitly into account both the molecular and quantum levels in terms of molecular class variables* (Baianu 1980, 1984, 1987a,b).

12.7 Łukasiewicz and LM-Logic Algebra of Genome Network Biodynamics. Quantum Genetics and Q-Logics

The representation of categories of genetic network biodynamics, **GNET**s, as subcategories of LM–Logic Algebras (**LMA**s) was recently reported (Baianu et al. 2006b) and several theorems were discussed in the context of morphogenetic development of organisms. The **GNET** section of the cited report was a review and extension of an earlier article on the 'immanent' logic of genetic networks and their complex dynamics and non-linear properties (Baianu 1977). Comparison of GNET universal properties relevant to *Genetic Ontology* can be thus carried out by colimit- and/or limit-preserving functors of GNETs that belong to adjoint functor pairs (Baianu and Scripcariu 1973; Baianu 1987a, b; Baianu et al. 2006a, b). Furthermore, evolutionary changes present in functional genomes can be monitored by natural transformations of such universal-property preserving functors, thus pointing towards evolutionary patterns that are of importance to the emergence of increasing complexity through evolution, and also to the emergence of man and ultra-complexity in the human mind. Missing from this approach is a consideration of the important effects of social, human interactions in the formation of language, symbolism, rational thinking, cultural patterns, creativity, and so on… to full human consciousness. The space, and especially time, ontology of such societal interaction effects on the development of human consciousness will also be briefly considered in the following sections.

12.7.1 The Organismic LM–Topos

As reported previously (Baianu et al. 2006a, b) it is possible to represent directly the actions of LM, many-valued logics of genetic network biodynamics in a categorical structure generated by selected LM–logics. The combined logico-mathematical structure thus obtained may have several operational and consistency advantages over the GNET-categorical approach of 'sets with structure'. Such a structure was called an 'LM-Topos' and represents a significant, non-commutative logic extension of the standard Topos theory which is founded upon a commutative, intuitionist (Heyting-Brouwer) logic. Whereas the latter topos may be more suitable for representing general dynamics of simple systems, machines, computers, robots and

AI structures, the non-commutative logic LM–topos offers a more appropriate foundation for structures, relations and organismic or societal functions that are respectively super-complex or ultra-complex. This new concept of an LM–topos thus paves the way towards a Non-Abelian Ontology of SpaceTime in Organisms and Societies regarded and treated precisely as super- or ultra-complex dynamic systems.

12.7.2 Quantum Genetics and Microscopic Entropy

Following Schrödinger's attempt (Schrödinger 1945), Robert Rosen's report in 1960 was perhaps one of the earliest quantum-theoretical approaches to genetic problems that utilized explicitly the properties of von Neumann algebras and spectral measures/self-adjoint operators (Rosen 1960). A subsequent approach considered genetic networks as *quantum automata* and genetic reduplication processes as *quantum relational oscillations* of such bionetworks (Baianu 1971a). This approach was also utilized in subsequent reports to introduce representations of genetic changes that occur during differentiation, biological development, or oncogenesis in terms of *natural transformations of organismal (or organismic) structures* (Baianu 1980, 1987a,b, 2004a,b; Baianu and Prisecaru 2004), thus paving the way to a *Quantum Relational Biology* (Baianu 1971a, 2004a). The significance of these results for quantum bionetworks was also recently considered from both a *logical and an axiomatic* viewpoint Baianu et al. 2006b).

On the other hand, the extension of quantum theories, and especially quantum statistics, to non-conservative systems, for example by Prigogine (1980) has opened the possibility of treating *irreversible*, super-complex systems that vary in time and 'escape' the constraints of unitary transformations, as discussed above in Sect. 2.4. Furthermore, the latter approach allows the consideration of functional genetic networks from the standpoint of quantum statistics and microscopic entropy. Thus, information transfer of the 'genetic messages' throughout repeated somatic cell divisions may be considered either in a modified form of Shannon's theory of communication channels in the presence of 'noise', or perhaps more appropriately in terms of Kolmogorov's concept of entropy (see Li and Vitanyi 1997). On the other hand, the preservation and/or repeated 'transmission' of genetic 'information' through germ cells—in spite of repeated quantum 'observations' of active DNA genes by replicase—is therefore an open subject that might be understood by applying the concept of microscopic entropy to Quantum Genetics.

12.8 Oncogenesis, Dynamic Programming and Algebraic Geometry (Baianu 1971a)

In this section we shall discuss changes of normal controls in cells of an organism. On an experimental basis, we argued that some specific changes of cellular controls are produced in oncogenesis through an initial abnormal human genome architecture (Baianu 1969; Baianu and Marinescu 1969).

Generally, the changes of controls in a cell may be produced through a strong localized perturbation of cellular activity (that is, through unusually strong forcing inputs), or through the prolonged action of unusual inputs at the level of chromosomes and/or mitotic spindle. These changes become permanent if in one way or another, the activity of operons or replicons is impaired, that is, if a change of basic relational oscillators of the cell has taken place. In the current language of qualitative dynamics it may be translated as a change of dominating attractors, followed by the inhibition or destruction of the former dominating attractors. This kind of change is not necessarily a mutation, that is, the change may not produce the replacement of some essential observables in the genetic system; this would however result eventually in many mutations and also alter the chromosomal architecture and modify the diploid arrangement of chromosomes in the cell nucleus. This may be the reason for which extensive research on cancer failed to discover so far a general, unique and specific alteration of the genetic system of cancer cells, *except for aneuploidy*. The change of basic relational oscillators in the genetic system may have such consequences as, for example, abnormally large nucleoli. The reason may be that a change in the subspace of the controller produces the change of dynamic programming of the whole cell. Dynamic programming consists in the existence of distinguished states, or policies in the subspace corresponding to the controller, to which correspond specific changes of trajectories in the subspace of the controlled subsystem. The appropriate mathematical concept corresponding to such situations is found in Algebraic Geometry. The fact that some basic concepts of algebraic geometry are by now currently expressed in categorical terms, allows us to make use of the mathematical formalism of categories and functors. A projective space of n dimensions will be assigned to the controlled subsystem, and a policy would be then represented by an allowable coordinate system in the projective space of the controlled subsystem. A projective space of n dimensions is defined as a set of elements S (called the points of the space) together with another set Z (the set of allowable coordinate systems in the space). Let $(a_0,\ldots, a_n)$ be an n-tuple of elements such that not all the elements $a_0,\ldots, a_n$ are zero. Two n-tuples $(a_0,\ldots, a_n)$, $(b_0,\ldots, b_n)$ are said to be right-hand equivalent if there exists an element. $(\mathbf{a_i}, \mathbf{b_i})$ of a ground field such that $a_i = b_i$? (i = 0,…,n). A set of right-hand equivalent (n + 1)-tuples is called a point of the right-hand projective number space of dimension n over the ground field K. The aggregate of such points is called a projective number space of dimension n over K, and will be denoted by PNn(K). If T denotes a correspondence among the elements of a set S and the points of PNn(K), which is an isomorphism, then, to any element A of S, there corresponds a set of equivalent $(n+1)$-tuples $(a_0,\ldots, a_n)$, where T(A) is $(a_0,\ldots, a_n)$. Any $(n+1)$–tuple of this set is called a set of coordinates of A (Thus, a set of equations written in matrix form as:

$$y = Ax \tag{12.5}$$

transforms $(n+1)$-tuples $(x_0,\ldots, x_n)$ into the set of equivalent (n + 1)-tuples $(y_0,\ldots, y_n)$. That is, Eq. (12.5) transforms a point of PN_n (K) into a point of PN_n (K). This set of equations will be called a *projective transformation* of PNn (K) into itself. If S is the set from the definition of a **projective space**, then a projective transformation leads to a change of coordinate system in S. The different coordinate systems obtained through the application of different projective transformations are called allowable coordinate systems in S. Allowable coordinate systems in S define policies of the controller. In this case the set of all policies of a controller has the structure of a group as far as the projective transformations form a group. Furthermore multiple, inter-connected controllers lead to a groupoid structure. Now, if there is an extension K_0 of the ground field K, and any h in K_0, h will be called *algebraic* if there exists a non-zero polynomial $f(x)$ in $K[x]$ such that $f(h) = 0$. The aggregate of points defined by the set of equations

$$f_1(x_o, \ldots, x_n) = 0, \tag{12.6}$$

with $f_1(x_o,\ldots, x_n)$ being a homogeneous polynomial over K, is called an **algebraic variety**. Thus, one can define a **dynamical program** in terms of algebraic varieties of a projective space corresponding to the subspace of the controlled subsystem, and with allowable coordinate systems (projective transformations) corresponding to policies in the subspace of the controller. Analytical forms used in some economical problems are only examples of metric aspects of the qualitative theory of dynamical programming. This suggests that quantitative results concerning changes of controls in oncogenesis could be eventually obtained on the basis of algebraic computations by algebraic geometrical methods. The power of such computations and the elegance of the method is improved by means of the theory of categories and functors. A quantitative result which is directly suggested by this representation is the degree of synchrony in cultured cancer cells. However, this algebraic geometrical method of representation requires further investigation.

13 Super-Complex Dynamics on Evolutionary Timescales

13.1 The Ascent of Man through Co-Evolution: Biological Evolution of Hominins (Hominides) and Their Social Interactions

Studies of the difficult problem of the emergence of man have made considerable progress over the last 50 years with several key hominide/hominin fossils (to name just a few), such as *Australopithecines*, *Homo erectus*, and *Homo habilis* being found, preserved, studied and analyzed in substantial detail. Other species considered to belong to *Homo* are: *H. habilis*, *H. rudolfensis*, *H. georgicus*, *H. ergaster* and *H. erectus*.

Hominini is defined as the tribe of *Homininae* that only includes humans (*Homo*), chimpanzees (*Pan*), and their extinct ancestors. Members of this tribe are called *hominins* (cf. Hominidae or 'hominids').

In the case of hominin species alternate names are sometimes used also for purely historical reasons. Consider, for example, the scientific classification of *Australopithecus africanus*: Kingdom: Animalia; Phylum: Chordata; Class: Mammalia; Order: Primates; Family: Hominidae; Subfamily: Homininae; Tribe: Hominini; Subtribe: Hominina; Genus: *Australopithecus* (cf. Dart in 1925) Its other closely related species are: *A. afarensis* ("Lucy"), *A. anamensis, A. bahrelghazali*, and

A. garhi. Note also that the following species were also classified formerly as *Australopithecus*, but are now classified as *Paranthropus*: *P. aethiopicus, P. robustus* and *P. boisei*.

Humans, on the other hand are: of the Kingdom: Animal; Phylum: Chordate; Class: Mammal; Order: Primate;…; Tribe: hominin. The Tribe hominini describes all the human/ human-line species that ever evolved (including the extinct ones) which excludes the chimpanzees and gorillas. On the other hand, the corresponding, old terminology until 1980 was 'hominides', now hominoides.

It would seem however that—according to the Chimpanzee Genome Project—both hominin (*Ardipithecus, Australopithecus* and *Homo*) and chimpanzee (*Pan troglodytes* and *Pan paniscus*) lineages might have diverged from a common ancestor about 5–6 million years ago, if one were to assume a *constant* rate of evolution (which does not seem to be the case). Phylogeny became complicated once more, however, when two earlier hominide fossils were found: *Sahelanthropus tchadensis*, commonly called "Toumai" which is about 7 million years old, and *Orrorin tugenensis* that lived at least 6 million years ago; both of these hominin-like 'apes' were bipedal and had possibly diverged from a common ancestor further back during evolution. Therefore, there is still considerable controversy among paleontologists about their place in human ancestry because the 'molecular clock' approach claims to show that humans and chimpanzees had an evolutionary split around 5 million years ago, i.e., at least 2 million years *after* the appearance of the "Toumai" hominins, which does not make much sense!

The overall picture completed from such paleoanthropologic and geological studies seems to indicate an accelerated biological evolution towards man between 15 million and 7 million years ago, and then perhaps even further accelerated when *Homo erectus* (the upright man) some 2 million years ago seems to have emerged from Africa as the victor over the more distant hominins. Its fossils were first found on Solo River at Trinil (in central Java) in 1890 by the Dutch anatomist Eugene Dubois and were named by him as *Pithecanthropus erectus*; similar fossils were later found also as far East as China (*Homo erectus pekinensis*). However, some paleoanthropologists believe that *H. erectus*, (discovered by Dubois and reported in 1892) is '*too derived*' an evolutionary lineage to have been the ancestor to the modern man species, *H. sapiens*. The fact remains that the *H. erectus* skull is so much closer to that of modern man than any of the found skulls of *Australophitecines* in both shape and internal capacity. *Homo erectus* (and *H. ergaster*) were probably the first hominins to form a hunter gatherer society; many anthropologists along with Richard Leakey are inclined to think that *H. erectus* was moving socially somewhat closer to modern humans than any of the other, more primitive species before it. Even though *H. erectus* used more sophisticated tools than the previous hominin species, the discovery of the Turkana boy in 1984 has produced the very surprising evidence that despite the *H. erectus*'s human-like skull and general anatomy, it was disappointingly incapable of producing sounds of the complexity required for either, ancient (before 8,000 BC) or modern, elaborate speech. Therefore, as we shall see later, it could not have topped the super-complexity threshold towards human consciousness!

13.2 The Evolution of the Human Brain and the Emergence of Human Consciousness: The Key Roles Played by Human Social Interactions

Following *Homo erectus*, however, some apparent and temporary slowing down of hominin biological evolution may have occurred over the next 1.9 million years or so for hominides other than *H. sapiens* which according to some anthropologists separated as a species from a common ancestor with *H. ergastus* about 2 million years ago. Thus, the emergence of language, and the whole social co-evolution and progression towards consciousness may have accelerated only through the unique appearance of *H. sapiens*. Stronger evidence of human speech comes only from the discoveries of the pre-historic Cro–Magnon man some 60,000 years ago. To sum up the entire sequence of paleontologic findings for the 4 billion years of biological evolution: whereas the evolution towards increasing complexity has accelerated towards the appearance of of *H. erectus* some 7–6 million years ago, it always remained within the very wide limits of super-complexity up to the emergence of *H. sapiens* 2 million years ago; the more substantial evidence from the Cro–Magnon man some 60,000 years ago allows one to assume—with a great degree of certainty—that a 'very rapid' transition either occurred or began *from super- to ultra-complexity*, from biologically based evolution to the societally based 'co-evolution' of human consciousness only after the birth of *H. sapiens*. This relatively high rate of development through *societal-based 'co-evolution'* in comparison with the very slow, preceding biological evolution is consistent with consciousness 'co-evolving' rapidly as the result of primitive societal interactions that have acted nevertheless as a powerful, and seemingly essential, 'driving force'. On the other hand, one may expect that the degree of complexity of human primitive societies which supported and promoted the emergence of human consciousness was also higher than those of hominin bands characterized by what one might call individual *hominin 'consciousness'*. Once human consciousness fully emerged along with complex social interactions within pre-historic *H. sapiens* tribes, it is likely to have acted as a positive feedback on both the human individual and society development through multiple social interactions, thus leading to an ever increasing complexity of the already ultra-complex system of the first historic human societies appearing perhaps some 10,000 years ago.

13.3 Organization in Societies: Interactions, Cooperation and Society Complex Dynamics. A Rosetta Biogroupoid of Social Interactions

Our discussion concerning the ontology of biological and genetic networks may be seen to have a counterpart in how scientific technologies, socio-political systems and cultural trademarks comprise the methodology of the planet's evolutionary development (or possibly its eventual demise!). Dawkins (1982) coined the term 'meme' as a unit of cultural information having a societal effect in an analogous way to how the human organism is genetically coded. The idea is that memes have 'hereditary' characteristics similar to how the human form, behaviour, instincts, etc. can be genetically inherited. Csikzentmihalyi (1990) suggests a definition of a meme as "any permanent pattern of matter or information produced by an act of human intentionality". A meme then is a concept auxiliary to that of the ontology of a 'level': to an extent, the latter is the result of generations of a 'memetic evolution' via the context of their ancestry. Memes occur as the result of a neuro–cognitive reaction to stimuli and its subsequent assimilation in an effective communicable form. Any type of scientific invention, however primitive, satisfies this criteria. Once a meme is created there is a subsequent inter-reaction with its inventor, with those who strive to develop and use it, and so forth (e.g., from the first four–stroke combustion engine to the present day global automobile industry). Csikzentmihalyi (1990) suggests that mankind is not as threatened by natural biological evolution as by the overall potential content of memes. This is actually straightforward to see as global warming serves as a striking example. Clearly, memetic characteristics are quite distinct from their genetic counterparts. Cultures evolve through levels and species compete. Memetic competition can be found in the conflicting ideologies of opposing political camps who defend their policies in terms of economics, societal needs, employment, health care, etc. Memes that function with the least expenditure of psychic energy are more likely to survive (as did the automobile over the horse and cart, the vacuum cleaner over the house broom). Whether we consider the meme in terms of weapons, aeronautics, whatever, its destiny reaches to as far as mankind can exploit it, and those who are likely to benefit are founding fathers of new industrial cultures, inventors and explorers alike, the reformers of political and educational systems, and so on. Unfortunately, memes can create their own (memetic) entropy: addiction, obesity and pollution are such examples. Thus to an extent memetic systems are patently complex and at ontologically different levels possessing their respective characteristic order of causality.

Related to memetic and autopoietic systems are those of *social prosthetic systems* (Kosslyn 2007) in which the limitations of the individual cognitive capacity can be extended via participation within varieties of socio-environmental networks. The premises for such a system is that "selfish" genetic programming on registering limitations on information processing, motivates reaching into the environment for positive adjustment, exploration and improved management. Loosely speaking, the brain uses the world and 'enduring relationships" as extensions of itself (Kosslyn 2007). As for many of the systems discussed in this essay, the underlying structures can be represented in terms of equivalence classes, thus leading to configurations of interacting groupoids and/or the applicability of the groupoid atlas concept itself.

13.3.1 A Rosetta Biogroupoid of Social, Mutual Interactions: The Emergence of Self

One may consider a human pre-historic society consisting of several individuals engaged in hunting and afterwards sharing their food. The ability to share food seems to be unique to humans, perhaps because of the pre-requisite *consensual* interactions, which in their turn will require similar mental abilities, as well as an understanding of the need for such sharing in order to increase the survival chances of each individual. Furthermore, it seems that the awareness of the self of the other individuals developed at first, and then, through *an extension of the others* to oneself, *self awareness* emerged in a final step. These pre-historic societal interactions that are based on consensus, and are thus mutual, lead to a natural representation of the formation of 'self' in terms of a '*Rosetta biogroupoid*' structure as depicted below (Diagram 1), but possibly with as many as 25 branches from the center, reference individual:

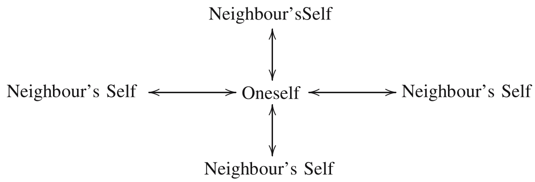


Diagram 1 *A Rosetta biogroupoid* of consensual, societal interactions leading to self-awareness, one's self and full consciousness; there could be as few as five, or as many as 25, individuals in a pre-historic society of humans; here only four are represented as branches

14 Emergence of a Higher Dimensional Algebra of Human Brain's SpaceTime Structures and Functions. Local-to-Global Relations and Hierarchical Models of Space and Time in Neurosciences

14.1 Relations in Neurosciences and Mathematics

The Greeks devised *the axiomatic method*, but thought of it in a different manner to that we do today. One can imagine that the way Euclid's Geometry evolved was simply through the delivering of a course covering the established facts of the time. In delivering such a course, it is natural to formalize the starting points, and so arranging a sensible structure. These starting points came to be called *postulates, definitions and axioms*, and they were thought to deal with real, or even ideal, objects, named points, lines, distance and so on. The modern view, initiated by the discovery of non-Euclidean geometry, is that the words points, lines, etc. should be taken as undefined terms, and that axioms give the *relations* between these. This allows the axioms to apply to many other instances, and has led to the power of modern geometry and algebra. This suggests a task for the professionals in neuroscience, in order to help a trained mathematician struggling with the literature, namely to devise some kind of glossary with clear relations between these various words and their usages, in order to see what kind of axiomatic system is needed to describe their relationships. Clarifying, for instance, the meaning to be ascribed to 'concept', 'percept', 'thought', 'emotion', etc., and above all the *relations* between these words, is clearly a fundamental but difficult step. Although relations—in their turn—can be, and were, defined in terms of sets, their axiomatic/categorical introduction greatly expands their range of applicability. Ultimately, one deals with *relations among relations* and relations of higher order as discussed next.

14.2 Memory Evolutive Systems. Global Organization of MES into Super-Complex Systems and the Brain

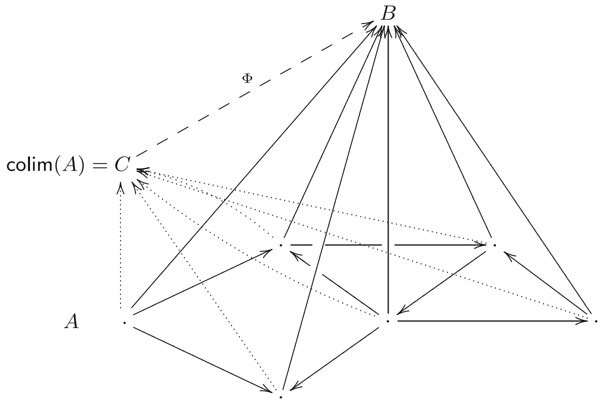
Following Ehresmann and Vanbremeersch (1987, 2006), if we have a system as represented by a graph, it is said to be *hierarchial* if the objects can be divided into specified complexity levels representative of the embeddings of contexts. The idea is to couple this with a *family of categories indexed by time*, as first proposed for biosystems by Baianu and Marinescu (1968), thus leading recently to the important concept of *Evolutionary System* (ES) (see Ehresmann and Vanbremeersch 1987). Mathematically, this requires the construction of categorical colimits, very useful 'tools' in many topological and algebraic contexts dealing, respectively, with spaces and group/groupoid symmetries, but here also incorporating time through the ES concept.

The concept of a colimit in a category generalizes that of forming the union $A \cup B$ of two overlapping sets, with intersection $A \cap B$. However, rather than concentrating on the actual sets A,B, we place them in context with the role of the union as permitting the construction of functions $f : A \cup B \longrightarrow C$, for any C, by specifying functions $f_A : A \longrightarrow C$, $f_B : B \longrightarrow C$ agreeing on $A \cap B$. Thus the union $A \cup B$ is replaced by a property which describes in terms of functions the relationship of this construction to all other sets. In practical terms it is how we might relate between input and output. In this respect, a colimit has 'input data', viz a *cocone*. For the union $A \cup B$, the cocone consists of the two functions $i_A : A \cap B \longrightarrow A$ and $i_B : A \cap B \longrightarrow B$ (see Brown et al. 2004).

If we regard objects as labeled in terms of ordered states $A < A'$, a transition functor $F(A, A') : F_A \longrightarrow F_{A'}$, represents a change in states $A \longrightarrow A'$, and satisfies

$$F(A, A'') = F(A, A') \circ F(A', A''). \tag{14.1}$$

Consider a pattern of linked objects A as a family of objects A_i with specified links (edges) between them, as well as another object B to which we can associate a collective link from A to B by a family of links $f_i : A_i \longrightarrow B$. We can picture then a cone with a base consisting of $A = \{A_1, A_2, \ldots\}$ and with B as the vertex. The pattern is said to admit a *colimit* denoted C, if there exists a collective link $A \longrightarrow C$ such that any other collective link $A \longrightarrow B$ admits a unique factorization through C. If such a colimit C exists, then locally C is well-defined by the nature of the pattern to which it is attached, and globally, C enjoys a universal property determined by the totality of the possible collective links of the pattern. In other words, C effectively binds the pattern objects while at the same time functions as the entire pattern in the sense that the collective links to B (regarded as a central processor) are in a one–to-one correspondence with those to C. Further, a category can be said to be *hierarchial* if its objects can be partitioned into different levels of complexity, with an object C of level $n + 1$ say, being the colimit of at least one pattern of linked objects of (strictly) lower levels n, n–1,…



(14.2)

In this way, colimits are instrumental for dealing with local to global properties, and the above description thus models an evolutionary autonomous system (or organism) with a hierarchy of components dealing with organized exchanges within an environment. By means of a learning network, this system re-adapts to changing conditions in that environment, thus creating a *Memory Evolutive System* (MES). The colimit C then functions as the binding agent for the respective channels for an MES modeled on some configuration of say, neural networks leading to an emergence of strictly increasing complexity. The *multiplicity principle* (MP) leads to the existence of both simple and complex links between components. In the category of quantum objects the colimit may represent an entanglement or superposition of states and the MP is satisfied at the microscopic level by the laws of quantum physics (Ehresmann and Vanbremeersch 1987, 2006).

14.3 Neuro-Groupoids and Cat-Neurons

Such categorical representations in the terminology of Ehresmann and Vanbremeersch (1987, 2006) are called 'categorical neurons' (or *cat-neurons* for short). Consciousness loops (Edelmann 1989, 1992) and the neuronal workspace of Baars (1988) (see also Baars and Franklin 2003) are among an assortment of models that have such a categorical representation. Among other things, there were proposed several criteria for studying the binding problem via the overall integration of neuronal assemblies and concepts such as *the archetypal core*: the cat-neuron resonates as an echo propagated to target concepts through series of thalamocortical loops suggesting that the thalamus is responsive to stimuli. Analogous to how neurons communicate mainly through synaptic networks, cat-neurons interact in accordance with certain linking procedures and can be studied in the context of categorical logic which in turn may be applied to semantic modeling for neural networks (Healy and Caudell 2004, 2006) and possibly the schemata of *adaptive resonance theory* (Grossberg 1999). For such interactive network systems we expect the role of *global actions* and *groupoid atlases* to play a more instrumental role such as they are realized in various types of multi-agent systems (Bak et al. 2006). But let us be aware that such models may tend to be reductionist in character and fall somewhere between simple and complex systems. Although useful for the industry of higher-level automata and robotics, they are unlikely to explain the ontology of human mind in themselves.

14.4 The Thalamacortical Model

In many regions within the various cortical zones, neuronal groups from one zone can arouse those in another so to produce a relatively organized re-projection of signals back to the former, thus creating a wave network of reverberating loops as are realized in the hippocampus, the olfactory system and cortical-thalamus. It is assumed that the synchronization of neurons occurs through resonance and periodic oscillations of the neighbouring population activity. The theories of *re-entry* and *thalamocortical looping* maps between neuron and receptor cells describe component mechanisms of the cerebral anatomy which are both endowed with and genetically coded by such networks (Edelman 1989, 1992; Edelman and Tononi 2000). Re-entry is a selective process whereby a multitude of neuronal groups interact rapidly by two-way signaling (reciprocity) where parallel signals are inter-relayed between maps; take for instance the field of reverberating/signalling cycles active within the thalamocortical meshwork which in itself is a complex system. The maps/re-entry processes comprise a representational schemata for external stimuli on the nervous system, ensuring the context dependence of local synaptic dynamics at the same time mediating conflicting signals. Thus re-entrant channels between hierarchial levels of cortical regions assist the synchronous orchestration of neural processes. Impediments and general malfunctioning of information in the re-entry processes (possibly due to some biochemical imbalance) may then be part explanation for various mental disorders such as depression and schizophrenia. The association of short-term memory with consciousness within an architecture of thalamocortical reverberating loops flowing in a wave-like fashion is proposed by Crick and Koch (1990). The reticular nucleus of the thalamus is considered by Baars and Newman (1994) as instrumental in gating attention.

14.5 Holographic, Holonomic and Hierarchical Models of Space and Time in Neurosciences

The ideas of holography/uncertainty have been further explored by Pribram (1991) in the context of neural networks and brain transition states, to some extent based upon the Gabor theory. It also hinges upon the fact that cognitive processes up to consciousness may emerge from the neural level, but this emergence necessitates the integration of lower levels as in a MES. Within neuronal systems, dendritic-processing employs analogous uncertainty in order to optimize the relay of information by micro-processing. Both time and spectral information (frequencies) are considered as stored in the brain which supposedly maintains a process of self-organization in order to minimize the uncertainty through a wide-scale regulatory system of phase transitions the origin of which involves the various computational neuroscientific mechanisms of (hyper) polarizing action potentials, spiking, bursting and phase-locking, etc. These contribute to a multitude of network cells that register and react to an incoming perceptive signal. Pribram introduced the term 'holonomic' in relationship to the principles of a 'dynamically varying hologram' since the resulting sharp phase transitions through states of chaos, enable the brain to perform its neuro-cognitive tasks. The hypothesis suggests that the neuronal functions employ *holonomic and inverse transformations* as distributing spectral information across domains of vast numbers of neurons which are later re-focused in the form of memory. This is described by a subcellular level, complex system: namely, an entirety of an axonic–teledendronic–synaptic–dendritic–perikaryonic–axonic cycle forming a distributed memory store across a 'holoscape' upon which information processing can occur. This store of information, or memory, can be accessed by the same means which developed it in the first place, that is, by the reduction of (quantum) wave forms which function as attractors (Pribram 2000). As for the cortical neuropil, the holoscape is a level of complexity within those constituting the overall operative working of the brain. However, vastly difficult questions remain such as how Pribram's 'holoscape' is linked e.g., to the 'dendron mind field' suggested by Eccles (1986), or to Stapp's quantum approach to 'neural intention' via the von Neumann–Wigner theory (Stapp 1993). Nevertheless, when viewed as the successive complexifications of a neural category, the 'holoscopic' process may be modeled by the descriptive mechanism of MES. The central memory developing in time allows for the choice of local operations. The categories evolving with time within the colimit structure representing higher brain functions such as integration are descriptive of local and temporal anticipatory mechanisms based on memory. This follows from how the MP induces and regulates the formation of higher levels from the culmination of those at lower stages. Just as chemical reactions and syntheses engage canonical functors to build up neural networks, and natural transformations between them to possibly enable 'continuous' perceptions, the various neural dynamic super-network structures—at increasingly higher levels of complexity—may allow the dynamic emergence of the *continuous, coherent and global 'flow of human consciousness'* as a new, *ultra-complex level of the mind*—as clearly distinct from the underlying human brain's localized neurophysiological processes.

15 What is Consciousness?

The problem of how mind and matter are related to each other has many facets, and it can be approached from many different starting points. Over the last 25 years considerable attention has been paid to the question of whether or not mental processes have some physical content, and if not, how do they affect physical processes. Of course, the historically leading disciplines in this respect are philosophy and psychology, which were later joined by behavioural science, cognitive science and neuroscience. In addition, the physics of complex systems and quantum physics have played stimulating roles in the discussion from their beginnings.

Regarding the issue of complexity, this is quite evident: the brain is one of the most complex systems we know. The study of neural networks, their relation to the operation of single neurons and other important topics do, and will, profit a great deal from complex systems approaches.

As regards quantum physics the situation is different. Although there can be no reasonable doubt that quantum events do occur in the brain as elsewhere in the material world, it is the subject of controversy whether quantum events are in any way efficacious and relevant for those aspects of brain activity that are correlated with mental activity. Bohm (1990), and Hiley and Pylkkännen (2005) have suggested theories of *active information* enabling 'self' to control brain functions without violating energy conservation laws. Such ideas are relevant to how quantum tunneling is instrumental in controlling the engagement of synaptic exocytosis (Beck and Eccles 1992) and how the notion of a '(dendron) mind field' (Eccles 1986) could alter quantum transition probabilities as in the case of synaptic vesicular emission (nevertheless, there are criticisms to this approach as in Wilson 1999). Active information at the quantum level plays an organizational role for the dynamic evolution of the system for which there is a quantum potential energy, namely a form of internal energy which contains information about the environment. If, accordingly, there exist quantum processes that trigger off some neural process, then these processes can in turn be influenced by some higher-level organizational process endowed with both mental and physical qualities. Thus, the mind would be understood as a new level that houses active information which would somehow affect the quantum potential energy and subsequently bring about an influence on the brain's physical process (Hiley and Pylkkännen 2005). The obvious, remaining question then arises why this phenomenon might only happen in individuals of the *H. sapiens* species?

The existence of human consciousness was admitted even by Descartes—a determined reductionist that claimed living organisms are just 'machines'. Attempting to define consciousness runs into similar problems to those encountered in attempting to define Life; there is a long list of attributes of human consciousness from which one must decide which ones are essential and which ones are derived from the primary attributes. Human consciousness is *unique*—it is neither an item nor an attribute shared with any other species on earth. It is also unique to each human being even though certain 'consensual' attributes do exist, such as, for example, *reification*. We shall return to this concept later in this section.

William James (1958) in "Principles of Psychology" considered consciousness as *"the stream of thought"* that never returns to the same exact 'state'. Both *continuity* and *irreversibility* are thus claimed as key, defining attributes of consciousness. We note here that our earlier metaphor for evolution in terms of 'chains of local (mathematical) procedures' may be viewed from a different viewpoint in the context of human consciousness—that of chains of 'local' thought processes leading to global processes of processes…, thus emerging as a 'higher dimensional' stream of consciousness. Moreover, in the monistic—rather than dualist—view of ancient Taoism the individual flow of consciousness and the flow of all life are at every instant of time interpenetrating one another; then, Tao in motion is constantly *reversing* itself, with the result that consciousness is *cyclic*, so that everything is—at some point—without fail changing into its opposite. One can visualize these cyclic patterns of Tao as another realization of the Rosetta biogroupoids that we introduced earlier in a different context—relating the self of others to one's own self. Furthermore, we can utilize our previous metaphor of 'chains of local procedures'—which was depicted in Fig. 1—to represent here the Tao "flow of all life" as a dynamic global colimit—according to Tao—not only of biological evolution, but also of the generic local processes involving sensation, perception, logical/'active' thinking and/or meditation that are part of the 'stream of consciousness' (as described above in dualist terms). There is a significant amount of empirical evidence from image persistence and complementary color tests in perception for the existence of such cyclic patterns as invoked by Tao and pictorially represented by the Rosetta biogroupoids in our Diagram 1; this could also provide a precise representation of the ancient Chinese concept of "Wu-wei"—literally 'inward quietness'—the perpetual changing of the stream of both consciousness and the unconscious into one another/each other. 'Wu', in this context, is just awareness with no conceptual thinking. Related teachings by Hui-neng can be interpreted as implying that *"consciousness of what is normally unconscious causes both the unconscious and consciousness to change/become something else than what they were before"*.

The important point here is the opposite approaches of Western (duality) and Eastern (monistic) views of Consciousness and Life. On the other hand, neither the Western nor the Eastern approaches discussed here represent the only existing views of human consciousness, or even consciousness in general. The Western 'science' of consciousness is divided among several schools of thought: *cognitive psychology*—the mainstream of academic orientation, the *interpretive psychoanalytic tradition*—emphasizing the dynamics of the *unconscious* (and its relation to the adaptive functioning of the ego) *the 'humanistic' movement*—with a focus on the creative relationship between consciousness and the unconscious, and finally, the *trans-personal psychology* which focuses on the 'inner' exploration and actualization by the human individual of 'the ultimate states' of consciousness through practicing 'mental exercises' such as meditation, prayer, relaxation and yoga, or whatever one's practice towards transcendence.

Because the spacetime ontology of man has as key items both human Life and Consciousness, the investigation/research of these two subjects should be of very high priority to society. However, as there are major difficulties encountered with studying, modeling and understanding the global functions of highly complex systems such as the human brain and the mind, society's pragmatic approach to supporting human biology and psychology studies has consistently fallen far short in modern times by comparison with the support for research in physics, chemistry or medicine. Perhaps, this is also a case of 'familiarity breeding contempt', and/or of short-term practical implications/applications winning over long-term ones? Some of the conceptual difficulties encountered in studying highly complex systems were already pointed out in Sects. 9 to 11, and they have so far severely impeded, or deterred progress in this fundamental area of human knowledge—*the cognition of our own self*. As reductionism fits very well Platonic simplicity, it has only produced a large number of 'pieces' but no valid means of putting together the puzzle of emergent complexities of the human brain and consciousness. At the other extreme, unfounded theories–that are 'not even wrong' abound. Clearly, a thorough understanding of how complex levels emerge, develop, and evolve to still higher complexity is a prerequisite for making progress in understanding the human brain and the mind; Categorical Ontology and Higher Dimensional Algebra are tools indeed equal to this hard task of intelligent and efficient learning about our own self, and also without straying into either a forest of irrelevant reductionist concepts or simply into Platonic meditation.

Thus, Categorical Ontology and HDA may not be enough for 'all' future, but it is one big, first step on the long road of still higher complexities.

15.1 Intentionality

Consciousness is always *intentional*, in the sense that it is always directed towards (or intends) ***objects*** (Pickering and Skinner 1990). Amongst the earlier theories of consciousness that have endured are the *objective self-awareness* theory and Mead's (1934) *psychology of self-consciousness*. According to the pronouncement of William James (1890, pp.272–273),

> *the consciousness of objects must come first.*

The reality of everyday human experience 'appears already objectified' in consciousness, in the sense that it is 'constituted by an '*ordering of objects*' (*lattice*) which have already been designated 'as objects' before being reflected in one's consciousness. All individuals that are endowed with consciousness live within a web, or *dynamic network*, of human relationships that are expressed through language and symbols as *meaningful objects*. One notes in this context the great emphasis placed on *objects* by such theories of consciousness, and also the need for utilizing '*concrete categories that have objects with structure'* in order to lend precision to fundamental psychological concepts and utilize powerful categorical/mathematical tools to improve our representations of consciousness. A new field of *categorical psychology* may seem to be initiated by investigating the categorical ontology of ultra-complex systems; this is a field that may link neurosciences closer to psychology, as well as human ontogeny and phylogeny. On the other hand, it may also lead to the 'inner', or '*immanent', logics* of human consciousness in its variety of forms, modalities (such as 'altered states of consciousness'-ACS) and cultures.

Furthermore, consciousness classifies different objects to different 'spheres' of reality, and is capable also of moving through such different spheres of reality. The world as 'reflected' by consciousness consists of multiple 'realities'. As one's mind moves from one reality to another the transition is experienced as a kind of 'shock', caused by the shift in attentiveness brought about by the transition. Therefore, one can attempt to represent such different 'spheres of reality' in terms of concrete categories of objects with structure, and also represent the dynamics of consciousness in terms of families of categories/‘spheres of reality’ indexed by time, thus allowing ‘transitions between spheres of reality’ to be represented by functors of such categories and their natural transformations for ‘transitions between lower-order transitions’. Thus, in this context also one finds the need for categorical colimits and MES representing coherent thoughts which assemble different spheres of reality (*as objects* 2008).

There is also a common, or *universal, intentional character of consciousness*. Related to this, is *the apprehension of human phenomena as if they were* ‘**things**’, which psychologists call ‘*reification*’. Reification can also be described as the extreme step in the process of objectivation at which the objectivated world loses its comprehensibility as an enterprise originated and established by human beings. Complex theoretical systems can be considered as reifications, but “*reification also exists in the consciousness of the man in the street*” (Pickering and Skinner 1990). Both psychological and ethnological data seem to indicate that the original apprehension of the social world (including society) is *highly reified* both ontogenetically and philogenetically.

Kant considered that the internal structure of reasoning was essential to human nature for knowledge of the world but the inexactness of empirical science amounted to limitations on the overall comprehension. Brentano considered intentional states as defined via the mental representation of objects regulated by mental axioms of reason. As it is experienced, Freeman (1997, 1999) regards intentionality as the dynamical representation of animal and human behaviour with the aim of achieving a particular state circumstance in a sense both in unity and entirety. This may be more loosely coined as ‘aboutness’, ‘goal seeking’ and or ‘wound healing’. The neurophysiological basis of intentionality according to Freeman is harbored in the limbic system: momentarily the structure of intentional action extends through the forebrain based in the fabric of cortical neuropil, a meshwork of synaptic connections interconnected by axons and dendrites within which a field of past experiences is embedded via learning. Kozma et al. (2004) use network percolation techniques to analyze phase transitions of dynamic neural systems such as those embedded within segments of neuropil. This idea of *neuro-percolation* so provides a means of passage via transition states within a neurophysiological hierarchy (viz. levels). But the actual substance of the hierarchy cannot by itself explain the quality of intention. The constitution of the latter may be in part consciousness, but actual neural manifestations, such as for example pain, are clearly not products of a finite state Turing machine (Searle 1983).

It is the olfactory system among others that presents a range of chemical sensors through which a neural process can classify its inputs—a principle of Hebbian learning (Hebb 1949)—between selected neurons a reinforced stimulus induces a strengthening of the synapses. But there remains the question how populations of neurons do actually create the patterns of neural activity that can engender intentionality which we might consider as attained through some hierarchy of structured levels—a matter that clearly warrants further investigation.

15.2 Mental Representations—The Hypothesis of A < System > of Internal Representations in Psychology and Cognitive Sciences

Mental representations are often considered in psychology and cognitive sciences (including neocognitivism, cf. Dennett 1981) as fundamental; the concept has been therefore intensely debated by philosophers of psychology, as well as psychologists, and/or cognitive scientists. The following discussion of such concepts does not imply our endorsement of any of such possible philosophical interpretations even though it is hard to see how their consideration and the mental roles they play could be either completely or justifiably avoided. The important question of how *language-like* are mental representations is one that is often debated by philosophers of the mind.

According to Harman, "thought may be regarded as consisting in large part of *operations on 'sentences under analysis'...*" (as cited in Hills 1981). However, Harman, and also Fodor (1981), claim that only some mental representations are highly language-like, and that not all of them are such.

Brentano's position regarding *intentionality* of mental representations was clearly stated as making the distinction between the physical and mental realms. Other philosophers are less supportive of this view; a cogent presentation of various positions adopted by philosophers of the mind vis a vis mental representations was provided by Field (Ch. 5 in Block 1981). As pointed out by Field, postulating the *irreducibility* of mental properties (e.g., to physical or neurophysiological ones) raises two main problems: the problem of *experiential* properties and the problem of *intentionality* raised by Brentano. Most mental properties, if not all, seem to be *relational* in nature; some for example may relate a person, or people, to certain items called "propositions" that are usually assumed not to be linguistic. Field claims however that in order to develop a psychological theory of beliefs and desires one could avoid propositions altogether and utilize "something more accessible" that he calls *sentences*. Thus, mental representations would be expressed as relations between people and 'sentences' instead of propositions. Unlike propositions then, sentences do have linguistic character, such as both syntax and semantics, or else they are sentence-analogs with significant grammatical structure, perhaps following Tarski's compositional theory. On the other hand, Harman is quite critical of those compositional semantics that regard a *knowledge* of truth-conditions as what is essential in semantics (… "*Davidson's theory would be circular*"). Furthermore, Gilbert Harman wrote: "no reason has been given for a compositional theory of meaning for whatever system of representation we think in, be it Mentalese or English", (p. 286 in Gunderson, ed., 1975).

Then, "*organisms which are sufficiently complicated for the notions of belief and desire to be clearly applicable have* ***systems of internal representations*** *(SIR) in which sentence-analogs have significant grammatical structure*", writes Field. On this hypothesis of SIR, a belief involves a ***relation between organisms and sentence-analogs in a SIR*** for organisms of 'sufficient complexity'. From a functionalism standpoint which abstracts out the physical structure of particular organisms, the problem arises how psychological properties are realized by such organisms, as well as the questions of how to define a *realization* of a psychological property, and how to define "what a psychological property itself is". Therefore, *"if you do not construe belief relationally, you need a physical realization of the belief relation"* (p. 91 of Field 1981).

15.2.1 Propositional Attitudes

Following Fodor (1968) *propositional attitudes* are assumed to ascribe or represent *relations between organisms and internal representations* (p. 45). Furthermore, they seem to be often identified with the inner speech and/or thought. According to Fodor (1981), cognitive psychology is a revival of the representational 'theory' of the mind: "*the mind is conceived as an organ whose function is the manipulation of representations, and these in turn, provide the domain of mental processes and the (immediate) objects of mental states*."

If mental representations, on the other hand, were to require the existence of an 'observer' or 'exempt internal agent' that can interpret what is being represented, one would face an infinite regress. Therefore, the claim was made that the human mind's representations related to the thinking process and/or human solving/cognition processes are in fact <*representations*> *of representations*, or even some kind of 'self-representation'. In this respect also, the human mind is *unique* by comparison with that of any lower animal, if the latter can be at all considered as a 'mind' because it clearly has only limiting boundaries and no conceivable horizon. Note the critique of the propositional attitude concept by Field in the previous subsection, and the latter's hypothesis that *sentence–analogs* in a SIR can replace propositional attitudes in psychology. The difference between the two views seems to lie in the specific nature of propositional attitudes (that may be somewhat intangible) and sentence-analogs in an SIR that may be 'tangible' in the sense of having significant grammatical structure (syntax, semantics, etc.), e.g., being more language-like. Furthermore, as attitudes are intentionality related the propositional *attitudes* may be more complex and richer than Field's sentence–analogs. One also notes that Rudolf Carnap (1938) suggested that propositional attitudes might be construed as *relations between people and sentences* they are disposed to utter. The reader may also note that in these two subsections, as well as in the next one, the emphasis is on the role of *relations* and properties—instead of objects—in the philosophy of psychology, and thus a categorical, logico-mathematical approach to SIR seems to be here fully warranted, perhaps including a Tarskian compositional semantics, but with Harman's critical *proviso* and warnings cited above!

Either representational 'theory', or hypothesis, leaves open the questions:

1. What relates internal representations to the outside world?
2. How is SIR semantically interpreted? or How does one give meaning to the system of internal representations?

Perhaps Field's proposal could be implemented along the Tarskian compositional semantics in a many-valued setting, such as the Łukasiewicz generalized topos (LGT), that was first introduced in Baianu (2004, 2005) and which can provide an adequate conceptual framework for such semantic interpretations *with nuances specified by many truth values* instead of a single one.

15.3 Intuition

There is much that can be said about intuition in a logical or mathematical sense; this precise meaning of intuition is further addressed in Brown, Glazebrook and Baianu (2007, in this volume), where the necessary, logical and mathematical concepts are also available. In this section, we shall however consider the broader meaning of intuition that is much less precise or even partially understood, as it seems to play a major role in developing new concepts, theories, or even paradigm shifts. When one speaks of the "intuitive grasp of a concept" is it that one's 'subconscious' (if that indeed exists?) registers impressions and information (from the environment, say) and the mind processes the latter while lacking linguistic functions or appropriate words that are either yet to be conceived or fully developed in order for a direct logical explanation to arise? Phrasing it another way, one may speak of intuition correlating to some form of intentionality which momentarily may not be derivable to a semantic/linguistic meaning regardless of a causal framework but may involve a 'pictorial analogy'. Perhaps this is relevant to the sign language of the deaf and 'dumb', which is three-dimensional and contains semantic elements. But intuition may also involve nuances of learning and wording towards boundaries within the overlaps of 'fuzzy nets' which, as we propose, are based on the principles of non-commutative (multi-valued) *n*- Łukasiewicz logics (cf. Baianu et al. 2006a, b; Georgescu 1971, 2006). In this respect, an intuition may be released by person *X* having the necessary faculties arising from a fully contingent evolutionary system, whereas person *Y* may not realize the same intuition as *X* because of an under-development of awareness and logico-semantic capabilities (as in the case of an infant or even a neonate), or quite simply, *X*'s intuition is wrong and *Y* may be already disposed to initiation of semantic/linguistic meaning, or even having the choice of over-riding it altogether. Ultimately, if an intuition is 'correct' or 'wrong' in the 'collective eyes of society', is determined through an *objectivation* process which pervades all human culture: it is either accepted or rejected by an intellectual majority in a specific human society. As this process is rarely based only on logic, or logics, and may also involve experiential considerations, objectivation does not have the 'permanent' character that this word may imply. Paradigm shifts in science are, in this sense, major re-considerations of objectivation of scientific concepts and theories. Perhaps one of the most important paradigm shifts and re-objectivations of all time is now occurring in the ontology of higher complexity systems and processes, currently labeled as 'Complexity Theory' or 'Complex Systems Biology' when the latter is restricted to living organisms. As expected, the clash among different intuitions of complexity leads to many debates and pitched controversies, further fuelled by the novelty and difficulty of this very important subject.

An '*intuitive space*' or *intuition* **layer** of complexity (cf. Poli 2006c; Baianu and Poli 2007) might thus appear to exist apart from, or relatively independent of, how experiences can be rationalized. Since intuition is a property attributed to the mind of humans (or to the 'autobiographical self' in the sense of Damasio 1994), it has therefore to be considered as conceptually different from 'instincts' or brain-initiated reflexes. Chalmers (1996) mentions the notion of a '*zombie*'—a physical entity having neither conscious experience nor sense of self-comprehension as may also be applicable to certain social groups or sub-populations: lacking in consciousness, neither can conceive nor react to an intuitive space. Such a 'space' might then be reasonably considered as existing somewhere between the phenomenal and noumenal worlds where the former embraces sense-intuition, and the latter those conceived by the intellect (such as the concept of 'soul'), questions of either reason or chosen religious beliefs aside.

In keeping with the previous sections of our essay, 'intuition' may thus be regarded as a by-product of an ultra-complex 'system' of processes occurring in the unique human mind, if not an essential property, or attribute, of that 'system' of processes.

15.4 Human Consciousness as an Ultra-Complex Process of Brain's Super-Complex Subprocesses: The Emergence of An Ultra-Complex Meta <System> of Processes

Most species possess subject awareness even though the individual nature of awareness differs dramatically *de facto*. Whereas states of of mind, intention, qualia etc. are ingredient factors of consciousness that instantaneously occur with subjective awareness, none of these are essential for the latter. Bogen (1995) discusses the neurophysiological aspect of this property in relationship to the intralaminar nuclei (ILN) which is a critical site when normal consciousness is impaired as the result of thalamic injury. It is suggested that the ILN provides an optimal candidate for a cerebral mechanism and subjective awareness is an emergent property of some such mechanism as subserved by the ILN.

As a working hypothesis, one can formulate a provisional (and most likely incomplete) definition of human consciousness as an *ultra-complex* process integrating numerous super-complex 'sub-processes' in the human brain that are leading to a '*higher-dimensional ontological, mental level*' capable of free will, new problem solving, and also capable of speech, logical thinking, generating new conceptual, abstract, emotional, etc., ontological structures, including—but not limited to—'awareness', self, high-level intuitive thinking, creativity, sympathy, empathy, and a wide variety of 'spiritual' or 'mental' *introspective* experiences. It may be possible to formulate a more concise definition but for operational and modeling purposes this will suffice, at least provisionally. The qualifier '*ultra-complex*' is mandatory and indicates that the ontological level of consciousness, or mental activities that occur in the conscious '(psychological) state', is *higher* than the levels of the underlying, *super-complex* neurodynamic sub-processes leading to, and supporting, consciousness. A methaporical comparison is here proposed of consciousness with the mathematical structure of a ('higher dimensional') *double* groupoid constructed from a 'single' topological groupoid—that would, through much over-simplifying, represent the topology of the human brain network processes (occurring in the two interconnected brain hemispheres) which lead to consciousness.

In order to obtain a sharper, more 'realistic' (or should one perhaps say instead, 'ideal') representation of consciousness one needs consider psychological 'states' (Ψ), 'structures' (Φ) as well as consciousness modes (CMs) in addition, or in relation to neurophysiological network structure and neural network super-complex dynamics. According to James (1890), consciousness consists in a '*continuous stream or flow*' of psychological 'states' which never repeats the same 'state' because it is continually changing through the interaction with the outer world, as well as through internal thought processes (suggested to have been metaphorically expressed by the saying of Heraclitus that '*one never steps in the same water of a flowing river*', and also by his "*Panta rhei*"–"*Everything flows!*"). However, the recurrence of patterns of thoughts, ideas, mental 'images', as well as the need for *coherence of thought*, does seem to establish certain psychological 'states' (Ψ), psychological 'structures' (Φ), and indeed at least two 'modes' of consciousness: an active mode and a *'receptive', or 'meditative'* one. Whereas the 'active' mode would be involved in biological survival, motor, speech/language, abstract thinking, space or time perception and volitional acts (that might be localized in the left-side hemisphere for right-handed people), the 'receptive' mode would be involved in muscle- or general-relaxation, meditation, imagination, intuition, introspection, and so on (i.e., mental processes that do not require direct interaction with the outside world, and that might be localized in the right-side cerebral hemisphere in right-handed people). The related issue of the obvious presence of two functional hemispheres in the human brain has been the subject of substantial controversy concerning the possible dominance of the left-side brain over the right-side, as well as the possibility of a subject's survival with just one of his/her brain's hemisphere.

An important 'structural' aspect related to the human or the chimpanzee brain's active mode, and also possibly pertinent to autism in children, is the recently discovered presence of groups of mirror neurons (*Science*, 2006). All of these related 'psi' categories and attributes are relevant to a mathematical representation of consciousness as an ultra-complex process emerging through the integration of super-complex sub-processes that have evolved as a result of both biological evolution/survival of the human organism, and also—just as importantly—through human social interactions which have both shaped and 'sharpened' human consciousness (especially over the last 5,000 years, or so).

15.5 Psychological Time, Spatial Perceptions, Memory and Anticipation

Subdivisions of space and spatiotemporal recognition cannot satisfactorily answer the questions pertaining to the brains capability to register qualia-like senses arising from representations alone (such as a sense of depth, ambiguity, incongruity, etc.) Graphic art in its many forms such as cubism, surrealism, etc. which toy around with spatial concepts, affords a range of mysterious visual phenomena often escaping a precise neuro-cognitive explanation. For instance, we can be aware of how an extra dimension (three) can be perceived and analyzed from a lower dimensional (respectively, two) dimensional representation by techniques of perceptual projection and stereoscopic vision, and likewise in the observation of holographic images. Thus any further analysis or subdivision of the perceived space would solely be a task for the 'minds–eye' (see Velmans 2000, Chapter 6 for a related discussion). Through such kaleidoscopes of cognition, the induced mental states, having no specified location, may escape a unique descriptive (spatiotemporal) category. Some exception may be granted to the creation of holographic images as explained in terms of radiation and interference patterns; but still the perceived three dimensional image is *illusory* since it depends on an observer and a light source; the former then peers into an 'artificial' space which otherwise would not have existed. However, the concept of holography heralds in one other example of the ontological significance between space–time and spectra in terms of a fundamental duality. The major mathematical concept for this analysis involves the methods of *the Fourier transform* that decompose spatiotemporal patterns into a configuration of representations of many different, single frequency oscillations by which means the pattern can be re-constructed via either summation or integration. Note, however, that visualizing a 4-dimensional space from a picture or painting, computer-generated drawing, etc., is not readily achieved possibly because the human mind has no direct perception of *spacetime*, having achieved separate perceptions of 3D-space and time; it has been even suggested that the human brain's left-hemisphere perceives time as related to actions, for example, whereas the right-hemisphere is involved in spatial perception, as supported by several split-brain and ACS tests. This may also imply that in all other species—which unlike man—have symmetric brain hemispheres temporal perception—if it exists at all—is not readily separated from space perception, at least not in terms of localization in one or the other brain hemisphere.

Gabor (1946) considered how this 'duality' may be unified in terms of phase spaces in which space–time and spectra are embedded in terms of an orthogonal pair of system components/coordinates which comprise a certain 'framing'. Gabor postulated an 'uncertainty'—a quantum of information corresponding to a limit to which both frequency modulations and spatial information can be simultaneously measured. The ensuing techniques afforded a new class of (Gabor) elementary functions along with a modification of the Weyl–Heisenberg quantization procedure. Thus was realized a representation of a one-dimensional signal in the two dimensions of (time, frequency) and hence a basic framework for holographic principles leading eventually to a theory of *wavelets*.

The purely mathematical basis relating to the topographical ideas of Pribram's work lies in part within the theory of harmonic analysis and (Lie) transformation groups. Relevant then are the concepts of (Lie) groupoids and their convolution algebras/algebroids (cf. Landsman 1998) together with species of 'localized' groupoids. Variable groupoids (with respect to time) seem then to be relevant, and thus more generally is the concept of a fibration of groupoids (see e.g., Higgins and Mackenzie 1990) as a structural descriptive mechanism.

These observations, in principle representative of the ontological theory of levels, can be reasonably seen as contributing to a synthetic methodology for which psychological categories may be posited as complementary to physical, spatiotemporal categories (cf. Poli 2007). Such theories as those of Pribram do not fully address the question of universal vs. personal mind: how, for instance, does mind evolve out of spatiotemporal awareness of which the latter may by continuously fed back into the former by cognition alone? The answer—not provided by Pribram, but by previous work by Mead (1850)–seems to be negative because human consciousness appears to have evolved through social, consensual communications that established symbolic language, self-talk and thinking leading to consciousness, as modeled above by the Rosetta biogroupoid of human/hominin social interactions. A possible, partial mechanism may have involved the stimulation of forming an increased number of specialized 'mirror neurons' that would have facilitated human consciousness and symbolism through the evoked potentials of mirror neuron networks; yet another is the *synaesthesia*, presumably occurring in the Wernicke area (W) of the left-brain, coupled to the 'mimetic mirror neurons' thus facilitating the establishment of permanent language centers (Broca) linked to the W-area, and then strongly re-enforced and developed through repeated consensual social human interactions.

In the beginning, such interactions may have involved orderly rituals and ritual, 'primitive' dances whose repetitive motions and sensory perception acts may have enforced collectively an orderly 'state' in the primitive *Homo*'s minds. Such periodic and prolonged rituals in primitive societies—as suggested by Mumford (1979)—may have served the role of ordering the mind, *prior to*, and also facilitating, the emergence of human speech. Thus a collective system of internal representations and reification in the human mind may have had its very origin in the primitive rituals and ritualistic dancing prior to the development of truly human speech. The periodic, repetitive action of ritual dancing, charged with emotional content and intentionality, may have served as a very effective *training* means in such primitive tribal societies, much the same way as human champions train today by rhythmic repetition in various sports.

Clearly, *both a positive feedback, and a feedforward (anticipatory)* mechanism were required and involved in the full development of human consciousness, and may still be involved even today in the human child's mind development and its later growth to full adult consciousness. Interestingly, even today, in certain tribes the grandfather trains the one-year old child to 'dance' thus speeding up the child's learning of speech.

A sidetrack is to regard these 'mysteries' as contributing to the (hard) problem of consciousness: such as how one can fully comprehend the emergence of non-spatial forms arising from one that is spatial (such as the brain) within the subjective manifold of human sensibility? The brain matter is insentient and does not by itself explain causal, spatiotemporal events as agents of consciousness.

The claim is made by practitioners of meditation that its goal is something beyond the bounds of our customary experience. However, there have been attempts such as those made by Austin (1998) to 'link' the brain's neurobiology with the mind in order to explain the qualities of conscious experience, in this case within a Buddhist-philosophical (strictly *non-dual* or *monistic*) context of awareness; the latter is inconsistent with the Western, *dual* approach extensively discussed in this essay, in the sense of the mind vs. the brain, organism vs. life, living systems *vs* inanimate ones, super-complex *vs* simple systems, environment *vs* system, boundary *vs* horizon, and so on, considering them all as pairs of *distinct* (and *dual/apposed*, but not opposed) ontological items. Surprisingly, reductionism shares with Buddhism a *monistic* view of the world—but coming from the other, physical extreme—and unlike Buddhism, it reduces all science to simple dynamic systems and all cognition to mechanisms. On the other hand, Buddhism aims 'higher' than the human consciousness—at *Enlightenment*, towards a completely 'spiritual', internal world without 'objectivity', and also claimed to be free of all pains accompanying the human, mortal existence, but consistently declining to recognize the existence of an immortal human 'soul'. The enlightenment is thus considered by Buddhists to be an eternal form of existence, of dimensions high above the level of human consciousness, still very rarely reachable from, but transcending, through the highest level of consciousness.

One might say that in the ancient Buddhist philosophy, the non-duality postulate translates into *'an openness of all ontic items'*, the universal 'all', indivisible and undivided multiverses, *'having neither a beginning nor an end'*—either in time or space—a philosophy which was also expounded in the West in a quantum-based form by David Bohm, a desenting quantum physicist; this is quite the opposite of the new astrophysical Cosmology of the 'Big Bang'—the inflationary theory of our Universe, or the Creationist theology.

The problems of mind vs. brain remain perplexing, however. Kantian intuitionism may reduce matters to an interplay of intellect and imagination as far as differing qualities of 'space' are concerned, but the dictum of physics, however, claims '*non-existence if it can't be measured*', even though the quantum wave function is supposed to (somehow 'magically') *collapse upon being measured*. It would thus acquire 'existence' upon being measured even though it collapses at that very instant of measurement, very much like a rabbit pulled out of a magic hat! Not surprisingly, many quantum physicists no longer subscribe to the idea of the "collapse of the wave function". (Bohm did not agree with the collapse either). Such predicaments are not new to groups of philosophers who claim metaphysical limits upon intellectually conceived representations, to the extent that definitive explanations might remain beyond the grasp of human comprehension (e.g., McGinn 1995). Others (cf. Bennett and Hacker 2003) in part echoing Gilbert Ryle's pronouncement of "categorical problems" (Ryle 1949), argue that brain science alone cannot explain consciousness owing to a plague of intrinsic (categorical) errors such as when a certain neuropsychological entity is conceived as a 'linear' superposition of it constituent parts (cf. 'the mereological fallacy'); in this regard, Bennett and Hacker (2003) spare no reductionist 'theories of neuroscience'.

To what degree the visual and auditory processes are "sharp" or "fuzzy" remains open to further research. Nevertheless, it is conceivable that certain membrane-interactive neurophysiological phenomena occur via a fuzzy, a *semi-classical* or *a quantum stochastic* process. From the "sharp" point of view, Stapp (1999) has described a dynamic/body/brain/mind schemata as a *quantum system complete with an observer* on the basis of the von Neumann–Wigner theory involving *projection operators* P as above. The intentional viewpoint interprets "*Yes*" = P and in the complementary case, "*No*" = $I - P$, where I is the *identity* operator. The projection

P is said to act on the degrees of freedom of the brain of the observer and reduces the latter as well as a universal state to one that is compatible with “Yes” or “No” reduced states:

$$(\text{``Yes''})\ S \mapsto PSP \qquad (\text{``No''})\ S \mapsto (I - P)S(I - P).$$

The actualization of a single thought creates a chain of subsequent thoughts and conscious action which might be realized by projection into the future of a component of the thought to which the body/world scheme itself becomes actualized. In turn, the neuronal processes that result from this associated body/world scheme eventually achieve the actual intention itself. As this process unfolds, consciousness is sustained through the continued interplay of fundamental neuro-cognitive processes (such as, recognition, sensory-motor responses, information management, logical inferences, learning, and so on), as well as through language/speech/communication, symbol/picture manipulation, analogies, metaphors, and last-but-not least, illusory and imaginary/virtual processes that both enable and trap the mind into performing superbly its ‘magic’ *continuity* tricks—*the creative acts* of bringing into existence many completely new things out of old ones, or simply out of ‘nothing at all’.

On the one hand, Wittgenstein claimed that we cannot expect language to help us realize the effects of language. On the other hand, Mathematics—the democratic Queen of sciences (cf. Gauss)—is, or consists to a large extent of, precise, formal type(s) of language(s), (cf. Hilbert, or more recently, the Bourbaki school) which do allow one to have ‘clear, sharp and verifiable representations of items’; these, in turn, enable one to make powerful deductions and statements through Logics, intuition and abstract thoughts, even about the undecidability of certain types of its own theorems (Gödel 1945). A misconception promoted by some mathematicians, as well as Wittgenstein, is that mathematics is merely a ‘tautological exercise’, presumably this label being reserved for ‘pure’ mathematics which is just an editorial convenience mode of operation. Perhaps, if all of mathematics could be reduced to, or based upon, only Boolean logic this might be a possibility; however, recent trends in mathematics are towards greater emphasis on the use of intuitionistic logic such as Brouwer-Heyting logic (as shown in further detail in the Brown, Glazebrook and Baianu (2007a, in this volume)), and also of many-valued logics (Georgescu 2006) in defining universal mathematical concepts.

Last-but-not-least, even though the human brain consists in a very large (approximately 100,000,000,000), yet finite, number of neurons—and also a much higher number of neuronal connections greater than 10^{29}—the power of thought enables us to construct symbols of things, or items, *apart from the things themselves*, thus allowing for our extension of representations to higher dimensions, to infinity, enlightenment, and so on, paradoxically extending the abilities of human consciousness very far beyond the apparent, finite limitations, or boundaries, of our super-complex, unique human brain. One notes here also that the psychological concept of dynamic ‘net without boundary’ occurring and moving in the ‘conscious plane’, but often with a specific focus (McCrone 1991), leads to a ‘completely open’, variable topology of the human mind. Thus, one may not be able to consider the human mind as a ‘system’ because it seems to possess no boundary—but as an *‘open multiverse of many layers, or super-patterns of processes of processes,… with a horizon’*.

By comparison, species other than *Homo sapiens*, even though they may have comparably scaled brain sizes or numbers of neurons, it would seem they have remained unable of attaining an ultra-complexity level comparable to that of the unique human consciousness. The latter is leading us either to higher dimensions and towards infinity, or else to the total destruction of life and consciousness on earth—as in a nuclear ‘accident’, or through intentional conflagration and environmental destruction. This moral and societal ‘duality’—as long as it persists—may make to us, all, the difference between “*to be or not to be?*”, which is indeed the question!

16 Human Society and Ultra-Complexity. The Human Use of Human Beings. Criticality and Decision Making

Should one consider modern society as a *‘hyper-complex’ system*, whatever that may be? Not necessarily, because the *human* $\Leftrightarrow$ *human*, social interactions may not be as intense, restrictive, or ‘strong’ as those among the living cells belonging to the whole human body, or those of the neurons in the human brain’s neural networks with their highly complex dynamic hierarchy of multiple inter-connections leading to integrated, global processes.

The overall effect of such an emergence of the unique, *ultra-complex human mind* has been the complete and uncontested *dominance* by man of all the other species on earth. Is it possible that the emergence of the highly complex society of modern man is also resulting in the eventual, complete domination of man as an individual by ‘his’ highly complex society? The historical events of the last two centuries would seem to be consistent with this possibility, without however providing certainty of such an undesirable result. Whereas the biological evolution of *H. sapiens* may typically appear to be unobservable over the last 15,000 years, the complexification and expansion of human society has occurred at a rapidly accelerating pace with the exception of several centuries during part of the Middle (‘dark’) Ages. Furthermore, as we have seen that society has strongly influenced human consciousness, indeed making possible its very emergence, what major effect(s) may the modern, highly complex society have on human consciousness? Or is it that the biological (evolutionary) limitations of the human brain which emerged in its present form some 2 million years ago (or maybe $\sim$60,000 years ago?) are preventing, or partially ‘filtering out’ the complexification pressed onto man by the hyper-complex modern societies? There are arguments that human consciousness has already changed since ancient Greece, but has it substantially changed since the beginnings of the industrial revolution? There are indications of human consciousness perhaps ‘resisting’—in spite of societal reification—changes imposed from the outside, perhaps as a result of *self-preservation of the self*. Hopefully, an improved complexity/super- and ultra-complexity theory, as well as a better understanding of spacetime ontology in both human biology and society, will provide answers to such difficult and important questions.

16.1 Society and Cybernetics: The Human Use of Human Beings

In his widely-read books on Cybernetics and Society, Norbert Wiener (1950, 1989) attempted to reconcile mechanistic views and machine control concepts with the dynamics of modern society. He also advocated the representation of living organisms in terms of *variable* machines or variable automata (formally introduced in Baianu 1971b). The problem with representing life forms as variable automata is in essence the strictly deterministic character of the latter systems, as well as their simple dynamic character determined by the semigroup, or monoid,'purely' algebraic structure of any automaton's state space. As discussed in previous sections, the variable topology is a far richer and extremely flexible structure, or system of structures, by comparison with the rigid, semigroup structure of any machine's state space. Thus, a variable topology dynamics provides a greatly improved metaphor for the dynamic 'state spaces' of living organisms which have emerged as super-complex systems precisely because of their variable topology. Therefore, they evolved as highly adaptable, autopoietic, self-reproducing, self-organizing, autonomous, etc., systems.

Wiener's serious concerns towards rigid and unjustified control of academic freedom by 'politically powerful' administration bureaucrats, as well as the repeated, gross misuses of scientific discoveries, are even more justified today than half a century ago when he first expressed them in his books and lectures; this is because the consequences of such severe controls of creative human minds by uncreative ones are always very grave indeed.

Many other society 'evolution' issues, and well-founded concerns about the human misuse of human beings, raised by Wiener are much amplified and further compounded today by major environmental issues. On the other hand, Cybernetics, in spite of its early promise, cannot help much with solving highly complex problems as those faced now by human society. It remains to be seen if complexity theories will be able to fare better than Cybernetics in addressing 'the human use of human beings' as Wiener has so aptly labeled the key problem of human societies, past and present.

17 Conclusions and Discussion

Current developments in the SpaceTime Ontology of Complex, Super-Complex and Ultra-Complex Systems were here presented covering a very wide range of highly complex systems and processes, such as the human brain and neural network systems that are supporting processes such as perception, consciousness and logical/abstract thought.

Mathematical generalizations such as higher dimensional algebra are concluded to be logical requirements of the unification between complex system and consciousness theories that would be leading towards a deeper understanding of man's own spacetime ontology, which is claimed here to be both *unique* and *universal*.

However, we have not been able to consider in detail in our essay the broader, and very interesting implications of *objectivation* processes for human societies, cultures and civilizations. To what extent the tools of Categorical Ontology and Higher Dimensional Algebra are suitable for the latter three items remains thus an open question. Furthermore, the possible extensions of our approach to investigating globally the *biosphere* and also

$$\textbf{Biosphere} \Longleftrightarrow \textbf{Environment interactions}$$

remain as a further object of study in need of developing a formal definition of the horizon concept, only briefly touched upon here.

New areas of Categorical Ontology are likely to develop as a result of the recent paradigm shift towards non-Abelian theories. Such new areas would be related to recent developments in: non-Abelian Algebraic Topology, non-Abelian gauge theories of Quantum Gravity, non-Abelian Quantum Algebraic Topology and Noncommutative Geometry, that were briefly outlined in this essay in relation to spacetime ontology.

Although the thread of the current essay strongly entails the elements of 'non-linear' and 'non-commutative' science, we adjourn contesting the above strictures. One can always adopt the Popperian viewpoint that theoretical models, at best, are approximations to the truth, and the better models (or the hardest to de-bunk *myths*, according to Goodwin 1994) are simply those that can play out longer than the rest, such as Darwin's theory on the origin of species. As Chalmers (1996) and others suggest, re-conceptualizing the origins of the universe(s) may provide an escape route towards getting closer to a definitive explanation of consciousness. Whether such new explanations will dispel the traditional metaphysical problems of the phenomenal world, that remains to be seen.

Several claims were defended in this essay regarding the spacetime ontology of emergent, highly complex systems and the corresponding ontological theory of levels of reality. Furthermore, claims were also defended concerning important consequences of non-commutative complex dynamics for human society and the Biosphere; potential non-Abelian tools and theories that are most likely to enable solutions to such ultra-complex problems were also pointed out in connection with the latter consequences. Such claims are summarized here as follows:

- The *non-commutative*, fundamentally 'asymmetric' character of Categorical Spacetime Ontology *relations and structure*, both at the top and bottom levels of reality; the origins of a paradigm shift towards non-Abelian theories in science and the need for developing a *non-Abelian Categorical Ontology*, especially a complete, non-commutative theory of levels founded in LM- and Q-logics.
- The existence of *super-complex* systems (organisms/biosystems) and highly complex processes which emerged and evolved through dynamic symmetry breaking from the molecular/quantum level, but are not reducible to their

molecular or atomic components, and/or any known physical dynamics; succinctly put: *no emergence* $\Longrightarrow$ *no real complexity*;

- The co-evolution of the unique human mind(s) and society, with the emergence of an ultra-complex level of reality; the emergence of human consciousness through such co-evolution/societal interactions and highly efficient communication through elaborate speech and symbols;
- The potential for exact, symbolic calculation of the non-commutative invariants of spacetime through logical or mathematical, precise language tools (categories of LM–logic algebras, generalized LM–toposes, HHvKT, Higher Dimensional Algebra, ETAS, and so on).
- The urgent need for *a resolution of the moral duality* between creation/creativity and destruction posed to the human mind and the current society/civilization which is potentially capable of not only self-improvement and progress, but also of total Biosphere annihilation on land, in oceans, seas and atmosphere; the latter alternative would mean the complete, rapid and irrevocable reversal of 4 billion years of evolution. Arguably, human mind and society may soon reach a completely unique cross-road–a potentially non-generic/strange dynamic attractor—unparalleled since the emergence of the first (so humble) primordial(s) on earth.
- The great importance to human society of rapid progress through fundamental, cognitive research of Life and Human Consciousness that employs highly efficient, non-commutative tools, or precise 'language', towards developing a complete, Categorical Ontology Theory of Levels and Emergent Complexity.

We have thus considered a wide range of important problems whose eventual solutions require an improved understanding of the ontology of both the space and time dimensions of 'objective' reality especially from both relational complexity and categorical viewpoints.

Among these important problems, currently of great interest in science, that we have considered here are:

- SpaceTime Structures and Local-to-Global Procedures.
- Reductionism, Occam's razor, Biological Axioms (ETAS) and Relational Principles.
- The Emergence of Life and Highly Complex Dynamics.
- What is Life and Life's multiple Logics, Biological Evolution, Global and Local aspects of Biological Evolution in terms of Variable Biogroupoids, Colimits and Compositions of Local Procedures.
- The Primordial organism models from the perspective of Generalized Metabolic-Repair Systems, Temporal and Spatial Organization in Living Cells, Organisms and Societies.
- The Ascent of Man and the Human Brain, Split-brain models and Bilateral Asymmetry of the Human Brain, the Thalamocortical Model, Colimits and the MES.
- What is Consciousness and Synaesthesia–the Extreme Communication between different 'logics' or thoughts, the Emergence of Human Consciousness through

Social Interactions and Symbolic Communication, the Mind, Consciousness and Brain Dynamics as Non-Abelian Ultra-Complex Processes.
- The emergence of higher complexity, ontological levels of reality represented by organisms, the unique human mind and societies as a dynamic consequence of iterated, symmetry breaking stemming from the *fundamental non-commutative logics* underlying reality. Related also to such LM- and Q-logics, we considered the key attributes of life, evolution/co-evolution and the human mind: multi-stability and genericity of nonlinear dynamics delimited by biofuzziness.
- How one might possibly extend in the future higher homotopy tools and apply Non-Abelian Algebraic Topology results—such as the Higher Homotopy van Kampen theorems to calculate exactly the *non-commutative invariants* of higher dimensional dynamic spaces in highly complex systems—organisms, and perhaps also for the ultra-complex 'system' of the human mind and societies.

In the following two papers (Baianu et al. 2007b; Brown, Glazebrook and Baianu (2007a, in this volume), we shall further consider spacetime ontology in the context of Astrophysics and our Universe's representations in terms of quantum algebraic topology and quantum gravity approaches founded upon the theory of categories, functors/natural transformations, quantum logics, non-Abelian Algebraic Topology and Higher Dimensional Algebra, as well as the integrated viewpoint of the Quantum Logics in a Generalized 'Topos'—a new concept that ties in closely Q-logics with many-valued, LM-logics and category theory.

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