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An Efficient Thermal Infrared Radiation Parameterization for Use in General Circulation Models

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Abstract

A detailed description of a parameterization for thermal infrared radiative transfer designed specifically for use in global climate models is presented. The parameterization includes the effects of the main absorbers of terrestrial radiation: water vapor, carbon dioxide, and ozone. While being computationally efficient, the schemes compute very accurately the clear-sky fluxes and cooling rates from the earth's surface to 0.01 mb. Errors are < 0.4 K day^{-1} in the cooling rate and $< 1\%$ in the fluxes. Since no transmittances are pre-computed, the atmospheric layers and the vertical distribution of the absorbers may be freely specified. The scheme can also account for any vertical distribution of fractional cloudiness with arbitrary optical thickness. In addition, the numerics and the FORTRAN implementation have been carefully designed to conserve both memory and computer time. This code should thus be particularly attractive to those contemplating long-term climate simulations, wishing to model the middle atmosphere, or planning to use a large number of levels in the vertical.

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1 Introduction

Thermal infrared radiation (IR) plays a crucial role in determining the earth's climate and its sensitivity. It is, therefore, important to have an accurate IR radiation parameterization in atmospheric general circulation models (GCMs) used for studying climate change. In GCMs, calculations of thermal infrared fluxes can easily take a third or more of the computing time. As the spatial resolution and vertical extent of the models increase and the treatment of physical processes improves, it becomes imperative to have a fast and accurate IR radiation parameterization.

Detailed calculation of the IR fluxes involves three integrations: a spectral integration, a vertical integration, and a directional integration. The spectral integration is the most time consuming. This is due to the narrowness of molecular absorption lines, which makes the absorption coefficient vary rapidly with wavenumber and thus requires very high spectral resolution ($\approx 10^6$ intervals for the entire IR spectrum) to obtain accurate integrated fluxes. But even this obstacle could be easily surmounted if it were not for the vertical integration. If the atmosphere were vertically homogeneous in pressure and temperature, the absorption coefficient over any vertical extent would be a function only of wavenumber, and wavenumbers with the same absorption coefficient would be radiatively identical. The spectral integration could then be greatly simplified by grouping all those spectral region with the same absorption coefficient—the k-distribution approach (see, for example, Arking and Grossman 1972). In the real atmosphere, however, the dependence of the absorption coefficient on pressure and temperature varies with wavenumber, and no two spectral intervals can be treated identically. It is this difficulty that makes the broad-band parameterization of IR fluxes such a difficult problem.

The effect of vertical variations of pressure and temperature on absorption is commonly taken into account by using one of the following approximations:

- *One-parameter scaling*, in which the effect of the variations of temperature and pressure along a path are taken into account by reducing a nonhomogeneous layer to an equivalent homogeneous layer with a "scaled" absorber amount at fixed reference temperature and pressure (e.g., Chou and Arking 1980).
- *Two-parameter scaling*, in which the effects of variations of temperature and pressure along a path are taken into account by approximating a nonhomogeneous layer as a homogeneous layer with the actual absorber amount, but at an effective temperature and an effective pressure that depend on vertical variations within the layer (e.g., Kratz and Cess,1988; Chou and Kouvaris 1991; Schwarzkopf and Fels 1991; Morcrette et al 1986; Kratz et al. 1993; Rosenfield 1991). In some cases (e.g., Schwarzkopf and Fels 1991; Rogers and Walshaw 1966), effective optical properties, such as line width and strength, are used as parameters instead of effective temperature and pressure; but the objective is the same—to account for vertical inhomogeneities in the atmosphere.

- *Modified k-distribution methods*, in which, as in the traditional k-distribution approach, wavenumbers with similar absorption coefficient are grouped, but in such a way as to account for the effects of vertical variations of temperature and pressure (Chou et al. 1993; Fu and Liou 1992; Goody et al. 1989; Lacis and Oinas 1991; Wang and Shi 1988; Zhu 1992).

At the Goddard Climate and Radiation Branch, we have developed various broad-band IR radiation parameterizations for the major water vapor, carbon dioxide, and ozone absorption bands based on all three approximations (Chou and Kouvaris 1991; Chou et al. 1993; Chou et al. 1994). These IR radiation parameterizations have been shown to be accurate and efficient in computing cooling rates in the troposphere and lower stratosphere, as well as in the middle atmosphere (up to the 0.01-mb level).

This report describes in detail how these parameterizations have been integrated into a single IR radiation package for use in GCMs. This package has been implemented and tested in the Goddard GCMs and is currently being used for both climate modeling and data assimilation applications. In Section 2 we briefly review the transfer equations for fluxes and cooling rates in clear and cloudy atmospheres. Section 3 addresses the partitioning of the spectrum into broad bands and the representations of the spectrally integrated fluxes in these bands. The forms of the transmittances used and the scheme for overlapping gaseous absorptions are given in Section 4. The vertical discretization of the transfer equations is given in Section 5. Section 6 presents some results of flux and cooling rate calculations and compares the parameterization against line-by-line calculations. The results are summarized in Section 7.

2 Infrared Transfer Equations

2.1 Radiative Transfer Through a Clear Atmosphere

Let us consider a thin atmospheric layer at pressure p' with a temperature T', which has a differential pressure thickness dp' and an optical thickness du_ν at wavenumber ν (Figure 1). The radiance emitted by this layer is $\mathcal{B}_\nu(T')\frac{du_\nu}{\mu}$, where μ is the cosine of the angle between the beam and the vertical, and $\mathcal{B}_\nu(T')$ is the Planck function. Let us further consider a higher level at pressure p where we wish to compute the upwelling radiance, and let us assume that between p and p' there is a mixture of gaseous absorbers with monochromatic optical thickness

$$u_\nu^g(p,p') = \sum_i u_{\nu,i}^g(p,p'), \qquad (1)$$

where $u^g_{\nu,i}(p,p')$ is the optical thickness of the ith absorber given by

$$u^g_{\nu,i}(p,p') = \int_p^{p'} q_i(p'')k_{\nu,i}(p'',T'')\frac{dp''}{g}. \tag{2}$$

Here q_i is the ith absorber's specific mass, $k_{\nu,i}$ is its absorption coefficient, and g is the gravitational acceleration. The quantity $\frac{dp''}{g}$ is the differential atmospheric mass per unit horizontal area.

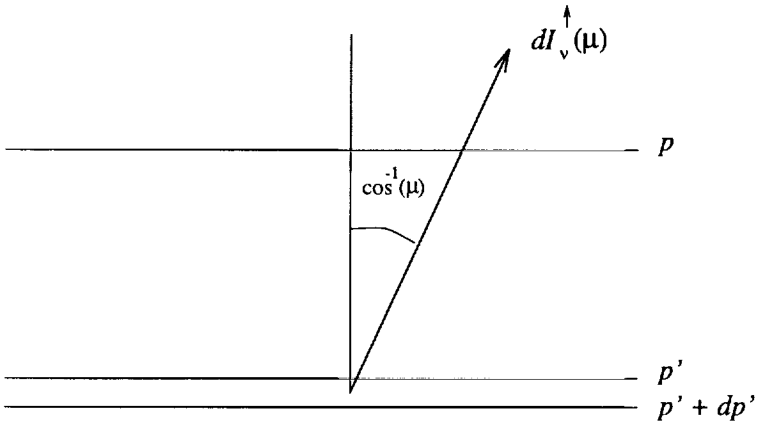

Figure 1: Contribution from a differential layer at p' to the upward radiance at p.

The transmittance between p and p' in the direction μ is then $e^{-u^g_\nu(p,p')/\mu}$, and the contribution of the layer between p' and $p'+dp'$ to the upwelling radiance at p is given by

$$dI_\nu^\uparrow(\mu) = \mathcal{B}_\nu(T')\frac{du_\nu}{\mu}e^{-u^g_\nu(p,p')/\mu}. \tag{3}$$

The total contribution of the layer to the upward flux at p can be obtained by angular integration of (3) over the hemisphere,

$$\begin{aligned} dF_\nu^\uparrow(p) &= 2\pi\int_0^1 dI_\nu^\uparrow(\mu)\mu d\mu, & \text{(4a)}\\ &= 2\pi\mathcal{B}_\nu(T')\int_0^1 \frac{du_\nu}{\mu}e^{-u^g_\nu(p,p')/\mu}\mu d\mu, & \text{(4b)}\\ &= -B_\nu(T')d\tau_\nu(p,p'), & \text{(4c)}\end{aligned}$$

where $B_\nu(T) \equiv \pi\mathcal{B}_\nu(T)$ and $\tau_\nu(p,p')$ is the flux transmittance for isotropic radiation given by

$$\tau_\nu(p,p') = 2\int_0^1 e^{-u^g_\nu(p,p')/\mu}\mu d\mu. \tag{5}$$

Equations analogous to (3)-(5) can be written for the downward radiance and flux at p due to differential layers above p.

2.2 The Effects of Clouds

To model the effects of clouds we follow the general treatment described in Harshvardhan et al. (1987). Clouds are allowed to occupy any combination of model layers, and the fractional coverage and cloud optical thickness are assumed to be specified for each layer.

Eq (5) applies to the case of a clear atmosphere, but it can be easily extended to the case of a cloudy and horizontally homogeneous (i.e., overcast) atmosphere by replacing the gaseous optical thickness with the total optical thickness:

$$u_\nu(p,p') = u^g_\nu(p,p') + u^c_\nu(p,p'), \tag{6}$$

where

$$u^c_\nu(p,p') = \sum_j u^c_{\nu,j}(p,p').$$

is the total cloud optical thickness between p and p', and the $u^c_{\nu,j}(p,p')$ is the optical thicknesses of the jth cloud layer.

If, in addition to clear and overcast layers, a single cloud layer with fractional cover A^c and optical thickness $\tilde{u}^c_\nu$ is introduced between p and p', the mean radiance transmittance in the direction μ becomes

$$(1 - A^c)\, e^{-u_\nu(p,p')/\mu} + A^c e^{-[u_\nu(p,p') + \tilde{u}^c_\nu]/\mu},$$

and the mean flux transmittance analogous to (5) is

$$\begin{aligned} \tau^*_\nu(p,p') &= 2\int_0^1 \left[(1 - A^c)\, e^{-u_\nu(p,p')/\mu} + A^c e^{-[u_\nu(p,p') + \tilde{u}^c_\nu]/\mu}\right] \mu d\mu, \\ &= \left[1 - A^c(1 - e^{-\tilde{u}^c_\nu/\overline{\mu}})\right] \tau_\nu(p,p'), \end{aligned} \tag{7}$$

where $\overline{\mu}$ is the effective mean value of μ that converts the radiance transmittance to flux transmittance. Clearly, the diffusivity factor $\frac{1}{\overline{\mu}}$ depends on both u_ν and $\tilde{u}^c_\nu$, but it is commonly taken to be constant and assigned the value 1.66.

Letting $N_\nu \equiv A^c(1 - e^{-\tilde{u}^c_\nu/\overline{\mu}})$, (7) becomes

$$\tau^*_\nu(p,p') = (1 - N_\nu)\tau_\nu(p,p'). \tag{8}$$

It is important to note that in this case the cloud fraction, A^c, and the cloud optical thickness, $\tilde{u}^c_\nu$, enter the equations only through the quantity N_ν. Since the factor $(1 - e^{-\tilde{u}^c_\nu/\overline{\mu}})$ is the flux emissivity of the cloud layer, N_ν can be regarded as the cloud fraction of an equivalent black cloud; alternatively, $(1 - N_\nu)$ can be regarded as the effective transmissivity of an equivalent overcast cloud.

When there is more than one cloud layer with fractional cover between p and p', the situation is considerably more complicated, since we need to describe *how* the clouds are overlapped. In general, we can write

$$\tau_\nu^*(p,p') = C_\nu(p,p')\tau_\nu(p,p'), \tag{9}$$

where $C_\nu(p,p')$ depends on the values of A^c and u_ν^c for the various cloud layers.

For the special case of any combination of overcast and *randomly overlapped* fractional cloud layers, we have

$$C_\nu(p,p') = \prod_j (1 - N_{\nu,j}), \tag{10}$$

where the subscript j denotes the cloud layers.

Since $\prod_j(1 - N_{\nu,j})$ is the fraction of the horizontal area that would be cloud-free if all cloud layers (including overcast layers) were assigned their equivalent black-cloud fraction and then randomly overlapped, Harshvardhan et al. (1987) referred to $C_\nu(p,p')$ as the probability of a clear line of sight between p and p'. The clear-line-of-sight interpretation of $C_\nu(p,p')$ is also valid for any overlapping of black clouds, making $C_\nu(p,p')$ easy to obtained. Obtaining $C_\nu(p,p')$ for *non-random* arrangements of *gray* clouds, however, is not as straightforward. In general, $C_\nu(p,p')$ does not depends on N_ν alone, but on more complicated combinations of the A_j^c and the $u_{\nu,j}^c$.

As an example, consider another arrangement discussed by Harshvardhan et al. (1987), that of *maximally overlapped* clouds. If the largest cloud between p and p' is black, then $C_\nu(p,p')$ is simply equal to one minus its fraction. If all clouds are allowed to be gray, however, $C_\nu(p,p')$ will depend on all cloud fractions and optical thicknesses. For a case with J maximally overlapped clouds between p and p', $C_\nu(p,p')$ may be easily computed by putting the clouds in order of increasing cloud cover and then evaluating the following recursion:

$$\mathcal{N}_\nu^{(0)} = 0, \tag{11a}$$

$$\mathcal{N}^{(j)} = N_{\nu,j} + \mathcal{N}^{(j-1)} e^{-u_{\nu,j}^c/\bar{\mu}}, \quad j = 1, \ldots, J. \tag{11b}$$

One can easily verify that

$$C_\nu(p,p') = 1 - \mathcal{N}^{(J)}. \tag{12}$$

Note that whenever a black cloud ($u_{\nu,j}^c \to \infty$) occurs in (11) all smaller clouds are eliminated from the recursion.

In the current code we have only implemented the random overlap strategy; but the computation of $C(p,p')$ is well isolated so that it can be easily replaced if maximal overlapping or some other arrangement is desired.

2.3 The Flux Calculation

Using the cloudy transmittance τ_ν^* from (9) in (4c), the contribution of the layer between p' and $p' + dp'$ to the upward flux at p becomes:

$$dF_\nu^\uparrow(p) = -B_\nu(T')d\tau_\nu^*(p,p'). \tag{13}$$

Similarly, it can be easily shown that the contribution of the earth's surface to the upward flux at level p is given by

$$\delta F_\nu^\uparrow(p) = B_\nu(T_s)\tau_\nu^*(p,p_s), \tag{14}$$

where the subscript s denotes surface quantities. The total upward flux at p is then obtained by integrating over all wavenumbers and all differential layers below p:

$$F^\uparrow(p) = \int d\nu \left[B_\nu(T_s)\tau_\nu^*(p,p_s) - \int_p^{p_s} B_\nu(T')\frac{\partial\tau^*(p,p')}{\partial p'}dp'\right]. \tag{15a}$$

Similarly, assuming no downward flux at $p = 0$, the total downward flux at p is

$$F^\downarrow(p) = \int d\nu \left[\int_0^p B_\nu(T')\frac{\partial\tau^*(p,p')}{\partial p'}dp'\right]. \tag{15b}$$

The cooling rate is simply proportional to the flux divergence:

$$-\frac{\partial T(p)}{\partial t} = \frac{g}{c_p}\frac{\partial}{\partial p}\left[F^\downarrow(p) - F^\uparrow(p)\right], \tag{16}$$

where c_p is the heat capacity of air at constant pressure.

3 Broad-Band Calculations

3.1 The 8 Bands

For computing IR fluxes due to water vapor, carbon dioxide, and ozone, we divide the spectrum into eight bands. Table 1 shows the spectral ranges of these bands, together with the absorbers involved in each band and the parameterization methods used to compute the transmittance in each band. The transmittance parameterizations are discussed in Section 4.

Water vapor line absorption covers the entire IR spectrum, while water vapor continuum absorption is included in the 540-1380 cm^{-1} spectral region (Bands 3 through 6). The absorption due to CO$_2$ is included in the 540–800 cm^{-1} region (Band 3), and the absorption due to O$_3$ is included in the 980–1100 cm^{-1} region (Band 5). The division of Band 3 into three sub-bands is discussed in Section 4.5.

Table 1: Spectral bands, absorbers, and parameterization methods.

Band	Spectral Range (cm^{-1})	Absorber	Option for Transmittance Parameterization	
			"LOW"	"HIGH"
1	0–340	H_2O line	K	T
2	340–540	H_2O line	K	T
3a	540–620	H_2O line	K	K
3b	620–720	H_2O continuum	S	S
3c	720–800	CO_2	K	T
4	800–980	H_2O line H_2O continuum	K S	K S
5	980–1100	H_2O line H_2O continuum O_3	K S T	K S T
6	1100–1380	H_2O line H_2O continuum	K S	K S
7	1380–1900	H_2O line	K	T
8	1900–3000	H_2O line	K	K

K: k-distribution method with linear pressure scaling.
T: Table look-up with temperature and pressure scaling.
S: One-parameter temperature scaling.

3.2 Planck-Weighted Band Integrals

From (15a,b), the integrated fluxes for each of the bands can be written as:

$$F_i^{\uparrow}(p) = B_i(T_s)\tau_i^*(p,p_s) - \int_p^{p_s} B_i(T')\left(\frac{\partial\tau^*(p,p')}{\partial p'}\right)_i dp', \tag{17a}$$

$$F_i^{\downarrow}(p) = \int_0^p B_i(T')\left(\frac{\partial\tau^*(p,p')}{\partial p'}\right)_i dp', \tag{17b}$$

where T' is the temperature at p',

$$F_i(p) = \int_{\Delta\nu_i} F_\nu(p)d\nu, \tag{18a}$$

$$B_i(T) = \int_{\Delta\nu_i} B_\nu(T)d\nu, \tag{18b}$$

$$\tau_i^*(p,p') = \frac{\int_{\Delta\nu_i} B_\nu(T')\tau_\nu^*(p,p')d\nu}{B_i(T')}, \tag{18c}$$

$$\left(\frac{\partial\tau^*(p,p')}{\partial p'}\right)_i = \frac{\int_{\Delta\nu_i} B_\nu(T')\frac{\partial\tau_\nu^*(p,p')}{\partial p'}d\nu}{B_i(T')} \tag{18d}$$

are the Planck-weighted band integrals, and $\Delta\nu_i$ is the width of the ith band.

Within each spectral band, either the range of B_ν is sufficiently small or the shape of B_ν is sufficiently independent of temperature that we can make the following approximations:

$$\tau_i^*(p,p') \approx \frac{\int_{\Delta\nu_i} B_\nu(T_o)\tau_\nu^*(p,p')d\nu}{B_i(T_o)}, \tag{19a}$$

$$\left(\frac{\partial\tau^*(p,p')}{\partial p'}\right)_i \approx \frac{\int_{\Delta\nu_i} B_\nu(T_o)\frac{\partial\tau_\nu^*(p,p')}{\partial p'}d\nu}{B_i(T_o)} = \frac{\partial\tau_i^*(p,p')}{\partial p'}, \tag{19b}$$

where T_o is a typical value of the atmospheric temperatures. (In parameterizing τ_i below, we use T_o = 250 K.) Figure 2 illustrates the accuracy of this approximation for water vapor line absorption. It compares the band-integrated flux transmittance of Bands 1, 2, and 3 weighted by B_ν(250K) with the same transmittances but weighted by B_ν(200K) and B_ν(300K). The calculations were done at pressures of 10 mb and 500 mb and with the water vapor amount varying from 10^{-6} g cm^{-2} to 10 g cm^{-2}. Clearly, (19a) is an excellent approximation for a wide range of temperatures, pressures, and humidities.

With the approximations (19a,b), the band-integrated fluxes reduce to:

$$F_i^{\uparrow}(p) = B_i(T_s)\tau_i^*(p,p_s) - \int_p^{p_s} B_i(T')\frac{\partial\tau_i^*(p,p')}{\partial p'}dp', \tag{20a}$$

$$F_i^{\downarrow}(p) = \int_0^p B_i(T')\frac{\partial\tau_i^*(p,p')}{\partial p'}dp'. \tag{20b}$$

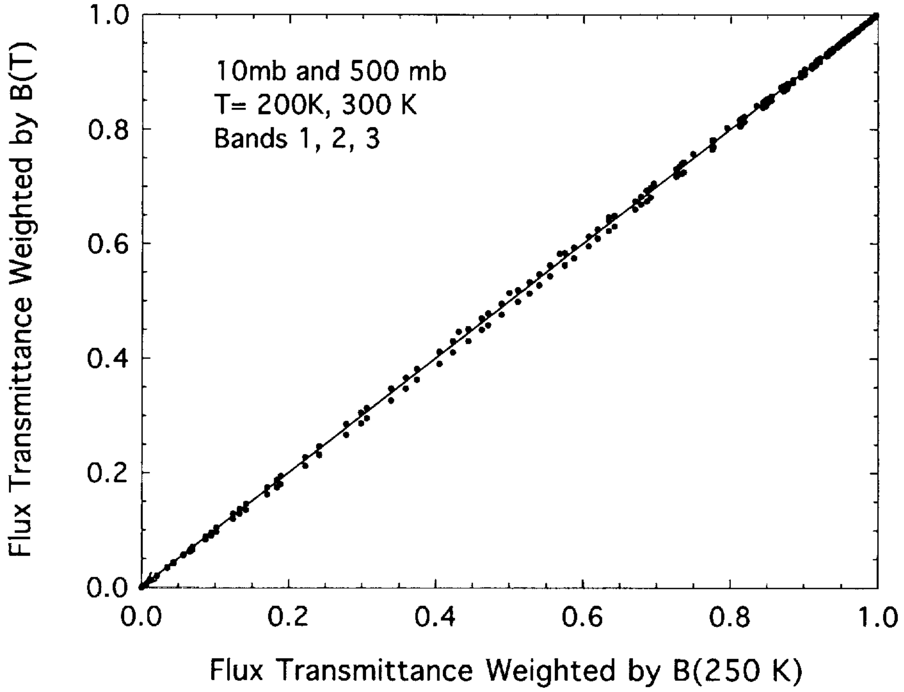

Figure 2: Relation between flux transmittances weighted by $B_\nu(250\mathrm{K})$ to those weighted by $B_\nu(300\mathrm{K})$ and $B_\nu(200\mathrm{K})$. Results plotted are for various pressures and water vapor amounts.

The parameterization problem is thus reduced to obtaining the $\tau_i^*(p,p')$ for each of the eight bands.

Unlike the gaseous absorption coefficient, the cloud absorption coefficient varies slowly with wavenumber, and the cloud emissivity can be considered constant within a band. Using (9) and the approximation (19a), the flux transmittance becomes

$$\begin{aligned} \tau_i^*(p,p') &= \frac{\int_{\Delta\nu_i} B_\nu(T_o)C_\nu(p,p')\tau_\nu(p,p')d\nu}{B_i(T_o)} \\ &\approx C_i(p,p')\frac{\int_{\Delta\nu_i} B_\nu(T_o)\tau_\nu(p,p')d\nu}{B_i(T_o)} = C_i(p,p')\tau_i(p,p'). \end{aligned} \tag{21}$$

Derivations of τ_i, the Planck-weighted flux transmittance due to gaseous absorption, are given in Section 4.

3.3 Band-Integrated Planck Functions

The spectrally integrated Planck fluxes were pre-computed for each band and then fitted by a 4th-degree polynomial in temperature:

$$B_i(T) = c_{i,0} + \sum_{n=1}^{4} c_{i,n} T^n. \tag{22}$$

When integrated over the eight bands, errors in this regression are negligible ($< 0.1\%$) for 160 K $< T <$ 345 K. The coefficients $c_{i,n}$ are listed in Table 2.

Table 2: Coefficients for computing the spectrally integrated Planck fluxes using the polynomial fits. The units of temperature are Kelvin.

Band	$c_{i,0}$	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$	$c_{i,4}$
1	−2.6844E−1	−8.8994E−2	1.5676E−3	−2.9349E−6	−2.2233E−9
2	3.7315E+1	−7.4758E−1	4.6151E−3	−6.3260E−6	3.5647E−9
3	3.7187E+1	−3.9085E−1	−6.1072E−4	1.4534E−5	−1.6863E−8
4	−4.1928E+1	1.0027E+0	−8.5789E−3	2.9199E−5	−2.5654E−8
5	−4.9163E+1	9.8457E−1	−7.0968E−3	2.0478E−5	−1.5514E−8
6	−1.0345E+2	1.8636E+0	−1.1753E−2	2.7864E−5	−1.1998E−8
7	−6.9233E+0	−1.5878E−1	3.9160E−3	−2.4496E−5	4.9301E−8
8	1.1483E+2	−2.2376E+0	1.6394E−2	−5.3672E−5	6.6456E−8

4 Transmission Functions

For accuracy and speed, the Planck-weighted flux transmittance for gaseous absorption:

$$\tau_i(p,p') = \frac{\int_{\Delta\nu_i} B_\nu(T_o)\tau_\nu(p,p')d\nu}{B_i(T_o)} \tag{23}$$

are computed using three different approaches, depending on the absorber and the spectral band:

- The *k-distribution method* with linear pressure scaling is applied to the water vapor bands. This method is also applied to the 15 μm CO_2 band if accurate cooling rate calculations in the middle atmosphere are not required.
- The transmittances due to CO_2 and O_3 absorption in Bands 3 and 5 are obtained from pre-computed transmittance tables based on *two-parameter scaling*. Because the k-distribution method with linear pressure scaling underestimates the water vapor cooling rate in the middle atmosphere, the transmittances of the three strongest water vapor absorption bands (Bands 1, 2, and 7) are also obtained from pre-computed transmittance tables if accurate computations of the water vapor cooling in the middle atmosphere are required.
- The transmittances due to water vapor continuum absorption in Bands 3 through 6 are computed using a *one-parameter scaling* approach.

The applications of these parameterizations to the different spectral bands and absorbers are summarized in Table 1. The code allows "HIGH" and "LOW" options to be specified depending on the desired accuracy in the middle atmosphere. With the "HIGH" option the more accurate, but more expensive, table look-up method is applied to CO_2 absorption and to water vapor line absorption in Bands 1, 2, and 7. With the "LOW" option these are all done with the k-distribution method. The intermediate option of doing only CO_2 by table look-up, which may be adequate for some stratospheric modeling when economy is important, is not explicitly provided but could be easily implemented.

4.1 The k-Distribution Method

As shown in Chou et al. (1993), the cooling due to water vapor in the lower atmosphere ($p > 20$ mb) is primarily attributable to the spectral regions away from the center of absorption lines—where the absorption coefficient is approximately linear in pressure and its dependence on temperature varies smoothly with wavenumber. Under these conditions, the absorption coefficient at any temperature and pressure is simply proportional to its value at a reference pressure and temperature. This scaling of the absorption coefficient is also appropriate for CO_2 absorption in the troposphere and lower stratosphere.

When this scaling is applicable, we adopt the following form for the absorption coefficient:

$$k_\nu(p,T) = k_\nu(p_r,T_r)\left(\frac{p}{p_r}\right)^m h(T,T_r), \tag{24}$$

where p_r and T_r are the reference pressure and temperature, m is an empirical constant, and $h(T,T_r)$ is the temperature scaling factor, which satisfies $h(T_r,T_r) = 1$. We use the following empirical relation for $h(T,T_r)$:

$$h(T,T_r) = 1 + \alpha(T - T_r) + \beta(T - T_r)^2, \tag{25}$$

where the coefficients α and β—constants for each absorber and spectral band—are obtained by minimizing errors in the flux transmittances. Details of their derivation are given in Chou et al. (1993). Values of p_r, T_r, m, α, and β used to scale the absorption coefficients for water vapor and CO_2 in each of the bands are given in Table 3. Note that Band 3 is divided into three sub-bands; the reasons for this are explained in Section 4.5. With the scaling of

Table 3: Scaling parameters used with the k-distribution method. Bands are defined in Table 1. The reference temperature and pressure, T_r and p_r, are those used in (24), m is the power for the pressure scaling in the same equation, and α and β are the temperature scaling coefficients that appear in (25).

	H_2O $p_r = 500$ mb, $T_r = 250$ K, $m = 1$		CO_2 $T_r = 250$ K			
Band	α (K^{-1})	β (K^{-2})	p_r (mb)	m	α (K^{-1})	β (K^{-2})
1	.0021	$-1.01e-5$				
2	.0140	$5.57e-5$				
3a	.0167	$8.54e-5$	300	.50	.0182	$1.07e-4$
3b			30	.85	.0042	$2.00e-5$
3c			300	.50	.0182	$1.07e-4$
4	.0302	$2.96e-4$				
5	.0307	$2.86e-4$				
6	.0154	$7.53e-5$				
7	.0008	$-3.52e-6$				
8	.0096	$1.64e-5$				

(24), the monochromatic flux transmittance given by (5) reduces to

$$\tau_\nu(p,p') = 2\int_0^1 e^{-k_\nu^r \tilde{q}(p,p')/\mu}\mu\, d\mu, \tag{26}$$

where $\tilde{q}(p,p')$ is the scaled absorber amount between p and p' given by

$$\tilde{q}(p,p') = \int_p^{p'} q(p'') \left(\frac{p''}{p_r}\right)^m h(T(p''),T_r)\frac{dp''}{g}, \tag{27}$$

and $k_\nu^r \equiv k_\nu(p_r, T_r)$. In (27), we have assumed a single absorber. The overlapping of gaseous absorbers is discussed in Section 4.4.

It can be seen from (26) and (27) that by scaling the absorption coefficient the dependence of τ_ν on wavenumber (through k_ν^r) is separated from its dependence on pressure and temperature (through $\tilde{q}$). Wavenumbers which have the same absorption coefficient at p_r and T_r will thus have the same transmittance at all other pressures and temperatures. Within spectral intervals narrow enough to consider the Planck function constant, these wavenumbers are radiatively identical, and the integration over wavenumber can be replaced by an integration over the absorption coefficient k at the reference temperature and pressure:

$$\int_{\Delta\nu} (\bullet) d\nu = \Delta\nu \int_0^\infty (\bullet) f^r(k) dk. \tag{28}$$

Here $f^r(k)dk$ is the the fraction of the spectrum in band $\Delta\nu$ with absorption coefficients between k and $k + dk$, so that

$$\int_0^\infty f^r(k) dk = 1. \tag{29}$$

The function $f^r(k)$ is called the k-distribution function. We have used the superscript r as a reminder that we are using the k-distribution at the reference temperature and pressure. This is the basic formalism of the k-distribution method.

Unfortunately, our 8 bands are too broad to assume a constant Planck function, and so (28) cannot be applied directly. Instead, we divide each band into a number of narrow sub-bands, for which the Planck function *can* be considered constant, and apply the k-distribution method to each sub-band. Letting $\delta\nu_j$ be the width of each sub-band, the Planck-weighted transmission function given in (23) becomes

$$\tau(\tilde{q}) = \frac{\sum_j B_j(T_o) \int_{\delta\nu_j} \tau_\nu(\tilde{q}) d\nu}{B(T_o)}, \tag{30}$$

where we have dropped the band index and (30) is understood to apply to each of the eight main bands. In (30), $B_j(T_o)$ is the constant reference Planck flux in sub-band j, and $B(T_o) = \sum_j B_j(T_o)\delta\nu_j$ is the band-integrated reference Planck flux.

Using (28), the spectral integral is replaced with an integration over all values of k:

$$\tau(\tilde{q}) = \sum_j \left[\frac{B_j(T_o)}{B(T_o)} \delta\nu_j \int_0^\infty \tau_k(\tilde{q}) f_j^r(k) dk \right], \tag{31}$$

where

$$\tau_k(\tilde{q}) = 2 \int_0^1 e^{-k\tilde{q}/\mu} \mu d\mu.$$

We use a different k-distribution function, $f_j^r(k)$, in each sub-band, but the same scaling of the absorber for all sub-bands within each band.

As shown in Chou and Arking (1980), the flux transmittance can be approximated by

$$\tau_k(\tilde{q}) \approx e^{-k\tilde{q}/\bar{\mu}}, \tag{32}$$

where $\frac{1}{\bar{\mu}}$ is the diffusivity factor taken to be 1.66. Using this approximation, (31) reduces to

$$\tau(\tilde{q}) = \sum_j \left[\frac{B_j(T_o)}{B(T_o)} \delta\nu_j \int_0^\infty e^{-k\tilde{q}/\bar{\mu}} f_j^r(k) dk \right]. \tag{33}$$

The integral over k can then be replaced by a simple quadrature over N k-intervals of width $(\delta k)_n$ located at k_n, where $n = 1, \ldots, N$. In doing this we use the same k-intervals for all sub-bands within a band, but allow the intervals to vary from band to band. The band transmittance can then be expressed as an exponential sum:

$$\tau(\tilde{q}) = \sum_{n=1}^{N} e^{-k_n \tilde{q}/\bar{\mu}} (\Delta g)_n, \tag{34}$$

where $(\Delta g)_n$ is the Planck-weighted k-distribution function for the nth k-interval given by

$$(\Delta g)_n = \sum_j f_j^r(k_n)(\delta k)_n \frac{B_j(T_o)}{B(T_o)} \delta\nu_j. \tag{35}$$

Note that for each k-interval the $(\Delta g)_n$ are summations over all sub-bands and are independent of temperature, pressure, and absorber amount; they can therefore be pre-computed for each band.

Values of $(\Delta g)_n$ were first obtained from line-by-line calculations and then slightly adjusted using regression so that the rms difference between the transmittances computed from the line-by-line method and from (34) is minimized. The adjustment was necessary because of the diffusivity approximation ($\frac{1}{\bar{\mu}} = 1.66$) and the small number of terms ($N \leq 6$) used. Details are given in Chou et al. (1993).

Calculations of the Planck-weighted flux transmission function using (34) can be done very efficiently, particularly if the k_n are appropriately chosen. If the transmittances need to be computed between $L + 1$ levels separating L contiguous layers, (34) involves $\frac{1}{2}L(L-1)N$ exponentials. Note, however, that for multi-layer regions the exponentials in each term in (34) can be obtained by multiplying together the exponential factors for the individual layers; thus, only LN exponential calculations are required. To reduce this still further, we choose the k_n so that they satisfy

$$k_n = \eta k_{n-1}, \qquad n = 2, N, \tag{36}$$

where η is a positive integer. With this choice, only a single set of L exponential operations for k_1 is needed. The other exponential terms can be obtained by raising the first to an integer power. For the values of η used, this can be done with only 3 or 4 multiplication for each succeeding exponential. This greatly accelerates the transmittance calculation.

In the parameterization, we apply the k-distribution method as just described to compute the water vapor line absorption in all bands but Band 3. The first absorption coefficient,

k_1, the constant η, and the $(\Delta g)_n$ are given in Table 4. As shown in Table 1, in Bands 1, 2, and 7 water vapor line absorption can be computed using either this method or the transmittance tables described in the next subsection; in all other bands we use only the k-distribution method. In Band 3, the 15 μm band, we use a somewhat more complicated procedure described in Section 4.5.

Table 4: Parameters used for the transmission functions due to water vapor line absorption. $\frac{k_1}{\mu}$ is the first flux absorption coefficient used with the k-distribution method. The remaining coefficients are defined using $k_n = \eta k_{n-1}$, where η is the band dependent constant given in the table. Δg is the Planck-weighted k-distribution function defined in (35). The use of the k-distribution method in Band 3 is described in Section 4.5. Units of k are $g^{-1}cm^2$; other values are non-dimensional.

	Band 1	Band 2	Band 4	Band 5	Band 6	Band 7	Band 8
$\frac{k_1}{\mu}$	2.96e+1	4.17e−1	5.25e−4	5.25e−4	2.34e−3	1.32	5.25e−4
η	6	6	6	6	8	6	16
Δg_1	.2747	.1521	.4654	.5543	.1846	.0740	.1437
Δg_2	.2717	.3974	.2991	.2723	.2732	.1636	.2197
Δg_3	.2752	.1778	.1343	.1131	.2353	.4174	.3185
Δg_4	.1177	.1826	.0646	.0443	.1613	.1783	.2351
Δg_5	.0352	.0374	.0226	.0160	.1146	.1101	.0647
Δg_6	.0255	.0527	.0140		.0310	.0566	.0183

4.2 Pre-Computed Transmittance Tables

Although the k-distribution method with linear pressure scaling is computationally very fast, it is not accurate in the middle atmosphere (0.01-20 mb), where the pressure ranges by three orders of magnitude and where the Doppler broadening of absorption lines is important. Radiative cooling in the middle atmosphere is primarily due to CO_2 in the 15 μm band (Band 3), to O_3 in the 9.6 μm band (Band 5), and to a lesser extent, to water vapor near the centers of absorption bands (Bands 1, 2, and 7).

As shown in Chou and Kouvaris (1991) and Chou et al. (1994), the transmittances in these bands can be simply derived from pre-computed transmittance tables. In this approach the tables are based on two-parameter scaling in which a non-homogeneous layer with pressure and temperature varying with height can be treated as if it were homogeneous with an effective pressure and temperature given by

$$p_{\text{eff}} = \frac{\int p dw}{\int dw}, \qquad (37)$$

$$T_{\text{eff}} = \frac{\int T dw}{\int dw}, \qquad (38)$$

where w is the absorber amount, and the integration is over the depth of a layer. It has been well recognized (e.g., Wu 1980; Chou and Kouvaris 1991) that the cooling rates depend primarily on contributions to the fluxes from nearby layers. Since pressure and temperature variations over nearby layers are generally small, the simple scaling approximations (37) and (38) produce accurate cooling rates.

With two-parameter scaling, the flux transmittance becomes a function of the absorber amount and the effective temperature and pressure. These dependencies can be accurately pre-computed from the following equation using a line-by-line method,

$$\tau(w, p_{\text{eff}}, T_{\text{eff}}) = \frac{\int_{\Delta\nu} \tau_\nu(w, p_{\text{eff}}, T_{\text{eff}}) B_\nu(T_o) d\nu}{\int_{\Delta\nu} B_\nu(T_o) d\nu}, \qquad (39)$$

where $\Delta\nu$ is the width of the entire spectral band. The band transmittance varies rapidly with pressure, but rather smoothly with temperature; thus, the size of the three-dimensional transmittance tables can be reduced to three two-dimensional tables using a quadratic fit in temperature,

$$\tau(w, p_{\text{eff}}, T_{\text{eff}}) = a(w, p_{\text{eff}}) + b(w, p_{\text{eff}})(T_{\text{eff}} - 250K) + c(w, p_{\text{eff}})(T_{\text{eff}} - 250K)^2, \qquad (40)$$

where a, b, and c are regression coefficients. This regression is valid for temperatures ranging from 170 K to 330 K; for this range it introduces only $\approx$ 1% error in the absorptance. Because of the wide range of variation of w and p_{eff}, values of the coefficients are tabled at equal intervals in $\log_{10}(w)$ and $\log_{10}(p_{\text{eff}})$. Table 5 specifies the ranges used for the tables for CO_2 absorption in the 15 μm region, O_3 absorption in the 9.6 μm region, and water vapor absorption in the three strong absorption bands. The actual tables can be obtained from the authors.

This method is simple and accurate. As shown in Chou and Kouvaris (1991) and Chou et al. (1994), it can be used to calculate fluxes and cooling rate in both the middle and lower atmosphere, from 0.01 mb to the earth's surface. However, this method can be significantly slower than the k-distribution method on computers that cannot efficiently perform table look-ups; we have therefore provided the option of using the k-distribution method for all bands, except for the computation of ozone transmittance in Band 5.

4.3 One-Parameter Scaling For Water Vapor Continuum Absorption

As shown in Table 1, water vapor continuum absorption is included in Bands 3 through 6. The water vapor continuum absorption coefficient, k_ν^c, depends on the water vapor partial

Table 5: The first values, intervals, and sizes of the pre-computed tables of the regression coefficients a, b, and c used in (40) for transmittance calculations. Units of w_{eff} are g cm^{-2} for water vapor and (cm-atm)$_{\mathrm{STP}}$ for CO_2 and O_3; p_{eff} is in mb.

	Band 1	Band 2	Band 3	Band 5	Band 7
ABSORBER	H_2O	H_2O	CO_2	O_3	H_2O
$\log_{10}(w_{\mathrm{eff}})_1$	−8	−8	−4	−6	−8
$\log_{10}(p_{\mathrm{eff}})_1$	−2	−2	−2	−2	−2
$\Delta\log_{10}(w_{\mathrm{eff}})$	0.3	0.3	0.3	0.3	0.3
$\Delta\log_{10}(p_{\mathrm{eff}})$	0.2	0.2	0.2	0.2	0.2
w-dimension	31	31	24	21	31
p-dimension	26	26	26	26	26

pressure, p_e, and on temperature. It increases with increasing partial pressure and with decreasing temperature. As shown by Roberts et al. (1976), it can be approximated by

$$k_\nu^c(p_e, T) = k_{\nu,0}^c \frac{p_e}{p_o} \exp\left[1800\left(\frac{1}{T} - \frac{1}{296}\right)\right], \tag{41}$$

where $k_{\nu,0}^c$ is the absorption coefficient when p_e is p_o (=1013 mb), and the temperature is 296 K. Values of $k_{\nu,0}^c$ are computed from the analytical representation given by Roberts et al. (1976), which is a fit to the laboratory data of D. E. Burch.

With the scaling of (41) for the absorption coefficient, the flux transmittance reduces to

$$\tau_\nu(p, p') = 2\int_0^1 e^{-k_{\nu,0}^c \hat{q}(p,p')/\mu} \mu d\mu, \tag{42}$$

where $\hat{q}$ is the scaled water vapor amount for continuum absorption given by

$$\hat{q}(p, p') = \frac{1.61}{g}\int_p^{p'} \frac{p''}{p_o} q^2(p'') e^{1800\left(\frac{1}{T(p'')} - \frac{1}{296}\right)} dp''. \tag{43}$$

Here we have used the approximation

$$p_e \approx \frac{q(p)p}{0.622}, \tag{44}$$

where q is the specific humidity.

Taking the Planck-weighted average of (42) over a band we have

$$\tau(\hat{q}) = \frac{\int_{\Delta\nu} \tau_\nu(\hat{q}) B_\nu(T_0) d\nu}{\int_{\Delta\nu} B_\nu(T_0) d\nu}, \tag{45}$$

where we note that the transmittance is a function of $\hat{q}$ only. The absorption coefficient $k^c_{\nu,0}$ varies slowly with wavenumber, except in Band 3 where it varies by a factor of 3.5; we therefore divide Band 3 into three sub-bands, as described in Section 4.5. The Planck-weighted flux transmittances for the three sub-bands and for Bands 4, 5, and 6 were computed from (45). These transmittances were then fit by

$$\tau(\hat{q}) = e^{-k^c \hat{q}/\bar{\mu}}. \tag{46}$$

The effective absorption coefficients, $\frac{k^c}{\bar{\mu}}$, that provide the best fit for each band are given in Table 6.

Table 6: The effective absorption coefficient $\frac{k^c}{\bar{\mu}}$ for water vapor continuum absorption in Bands 4-6 and the three sub-bands of Band 3. Units are $g^{-1}cm^2$.

	Band 3a	Band 3b	Band 3c	Band 4	Band 5	Band 6
$\frac{k^c}{\bar{\mu}}$	109.6	54.8	27.4	15.8	9.4	7.8

4.4 Overlapping of Absorptions

When there is more than one absorber involved in a spectral band, overlaps must be considered. The total Planck-weighted transmittance for two absorbers, can be written as

$$\tau_T = \frac{\int B_\nu \tau_1(\nu) \tau_2(\nu) d\nu}{\int B_\nu d\nu}. \tag{47}$$

where the subscripts 1 and 2 denote the two absorbers. If the transmittances due to the individual absorbers are expressed as the sum of the band-mean transmittance,

$$\bar{\tau} = \frac{\int B_\nu \tau(\nu) d\nu}{\int B_\nu d\nu}, \tag{48}$$

and the deviation from the band-mean, $\tau' = \tau - \overline{\tau}$, then (47) reduces to

$$\tau_T = \overline{\tau}_1\overline{\tau}_2 + \frac{\int B_\nu \tau_1'(\nu)\tau_2'(\nu)d\nu}{\int B_\nu d\nu}. \tag{49}$$

If the overall shapes of the absorption curves due to both absorbers are uncorrelated with each other and with B_ν, the second term on the right-hand side of (49) can be neglected and the total transmittance becomes

$$\tau_T = \overline{\tau}_1\overline{\tau}_2. \tag{50}$$

Overlapping of absorption in individual bands are shown in Table 1. As shown in Chou et al. (1993), the multiplication approximation (50) can be applied to Bands 4, 5, and 6 for the overlapping of water vapor line and continuum absorption, as well as for overlapping the water vapor and ozone absorption in Band 5 (9.6 μm). Overlapping in Band 3 (15 μm) is discussed in the next subsection.

4.5 Special Treatment of the 15 μm Band

The 15 μm band poses additional difficulties for two reasons. First, the water vapor line absorption and the continuum absorption are highly correlated in Band 3. As can be seen in the bottom two panels of Figure 3, the absorption coefficient increases with decreasing wavenumber by a factor of about 100 for water vapor line absorption and of about 10 for continuum absorption. Thus, the approximation (50) cannot be applied directly to the entire band. Second, the CO_2 absorption coefficients differ by several orders of magnitude between the band center and the wings (see the top panel of Figure 3). Rather than trying to parameterize the correlation effect or the variations in CO_2 absorption, we simply divide the band into three sub-bands (see Figure 3 and Table 1) and then combine the parameterized transmittances of the sub-bands into a single band transmittance. This transmittance is then used in the usual way to solve the transfer equations for the entire band.

The H_2O Transmittance

Water vapor line absorption in Band 3 is always done with the k-distribution method (Table 1). After dividing the band into three sub-bands, we apply the k-distribution method to each sub-band as just described for the full bands. Within each of the three sub-bands the transmittances due to line and continuum water vapor absorption are sufficiently uncorrelated that we can overlap them using the multiplication approximation (50). Letting τ_i^L and τ_i^C be the line and continuum transmittances for sub-band i, we obtain the total water vapor transmittance in Band 3 by overlapping them and taking the Planck-weighted

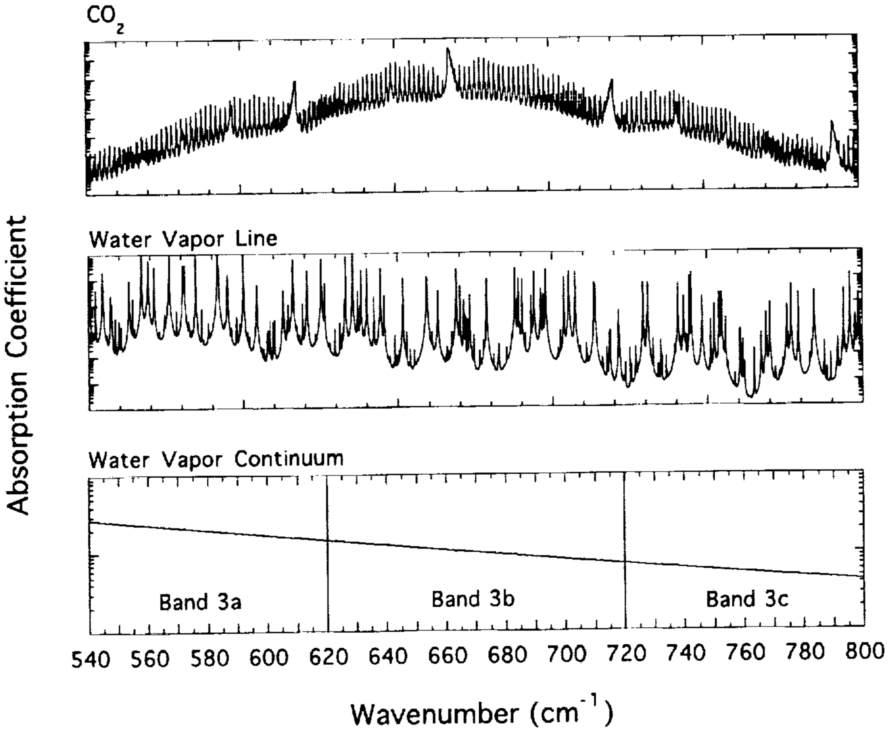

Figure 3: Schematic of the absorption coefficient for CO_2 and for water vapor line and continuum absorption in the 15μm region (Band 3). The subdivision of the band into three sub-bands is made to accurately account for the large variations of the absorption coefficients.

average:

$$\tau^{\mathrm{WV}} = \sum_{i=1}^{3} \tau_i^{\mathrm{C}} \tau_i^{\mathrm{L}} \frac{B_i(T_o)}{B(T_o)}, \tag{51}$$

where $B_i(T_o)$ is the Planck flux integrated over sub-band i, and $B(T_o) = \sum_{i=1}^{3} B_i(T_o)$ is the total Planck flux for Band 3.

Substituting for the line transmittance from (34) and for the continuum transmittance from (46) we have:

$$\begin{aligned} \tau^{\mathrm{WV}} &= \sum_{i=1}^{3} e^{-k_i^c \hat{q}/\bar{\mu}} \sum_{n=1}^{N} e^{-k_n \hat{q}/\bar{\mu}} (\Delta g)_{n,i} \frac{B_i(T_o)}{B(T_o)} \\ &= \sum_{i=1}^{3} e^{-k_i^c \hat{q}/\bar{\mu}} \sum_{n=1}^{N} e^{-k_n \hat{q}/\bar{\mu}} (\widehat{\Delta g})_{n,i}. \end{aligned} \tag{52}$$

Since we use the same scaling parameters (Table 3 and (43)) for the three sub-bands, the scaled water vapor amounts, $\tilde{q}$ and $\hat{q}$ are independent of sub-band. In addition, since we use a single set of k_n, the exponentials $e^{-k_n \hat{q}/\bar{\mu}}$ in the inner sum of (52) only need to be evaluated once.

Values for $(\widehat{\Delta g})_{n,i}$ are given in Table 7. The effective continuum absorption coefficients, k_i^c, are given in Table 6. Note that the coefficients for Sub-bands 3a and 3b are integer multiples of the coefficient for Sub-band 3a; thus only one exponentiation needs to be performed to evaluate (46) for the three sub-bands.

The CO_2 Transmittance

Unlike the water vapor line transmittance, the CO_2 transmittance may be computed either using the k-distribution method or by table look-up. When the table look-up is used, a single transmittance is computed directly for the entire band. When the k-distribution method is used, we separate Sub-band 3b (the center region) from sub-bands 3a and 3c (the wings). The optical properties of Sub-bands 3a and 3c are very similar, and so we have combined them by using the same scaling parameters (α, β, p_r and m in Table 3) and the same set of absorption coefficients (given by k_1 and η in Table 7). The band-averaged CO_2

Table 7: Parameters for computing transmittances in the three sub-bands of Band 3. The factors $\widehat{\Delta g}_n$ appear in (52) and (53). Units of k are g^{-1} cm^2 for water vapor and $(\text{cm-atm})^{-1}_{\text{STP}}$ for CO_2.

	Water Vapor			CO_2	
	Band 3a	Band 3b	Band 3c	Wings	Center
$\frac{k_1}{\mu}$	1.33e−2	1.33e−2	1.33e−2	2.66E−5	2.66E−3
η	8	8	8	8	8
$\widehat{\Delta g}_1$	.0000	.0923	.1782	.1395	.0766
$\widehat{\Delta g}_2$	.1083	.1675	.0593	.1407	.1372
$\widehat{\Delta g}_3$	.1581	.0923	.0215	.1549	.1189
$\widehat{\Delta g}_4$	.0455	.0187	.0068	.1357	.0335
$\widehat{\Delta g}_5$	.0274	.0178	.0022	.1820	.0169
$\widehat{\Delta g}_6$	.0041	.0000	.0000	.0220	.0059

transmittance for Band 3 is thus computed from

$$\tau^{\mathrm{CO}_2} = \sum_{n=1}^{N} e^{-(k_c)_n \tilde{q}_c/\overline{\mu}} (\widehat{\Delta g_c})_n + \sum_{n=1}^{N} e^{-(k_w)_n \tilde{q}_w/\overline{\mu}} (\widehat{\Delta g_w})_n, \tag{53}$$

where subscript c denotes the band "center" (Band 3b) and subscript w the "wings" (Bands 3a and 3c). Values of $(\widehat{\Delta g})_n$ are given in Table 7.

Finally, the total flux transmittance in the 15 μm band is computed from

$$\tau = \tau^{\mathrm{WV}} \tau^{\mathrm{CO}_2}. \tag{54}$$

5 Vertical Discretization

To approximate the vertical integrals in (20a,b), the atmosphere is divided into L layers numbered as shown in Figure 4. The upward and downward fluxes at level l for the ith band can be computed from

$$F^{\uparrow}_{i,l} = \sum_{l'=l}^{L+1} B_{i,l'+\frac{1}{2}} \left[\tau^*_i(l',l) - \tau^*_i(l'+1,l)\right], \quad l = 1,\ldots,L+1, \tag{55a}$$

$$F^{\downarrow}_{i,l} = \sum_{l'=1}^{l-1} B_{i,l'+\frac{1}{2}} \left[\tau^*_i(l'+1,l) - \tau^*_i(l',l)\right], \quad l = 2,\ldots,L+1, \tag{55b}$$

where the integer subscripts denote atmospheric levels and half-integer subscripts the layers between them, at which the atmospheric state variables, such as temperature and specific masses are defined. The quantity $B_{i,l'+\frac{1}{2}}$ is the spectrally integrated Planck flux of the layer between l' and $l'+1$, and $\tau^*_i(l',l)$ is the Planck-weighted flux transmittance between l and l'. The earth's surface (level $L+1$) is treated as if it were an atmospheric layer filled with black clouds, i.e., $B_{i,L+\frac{3}{2}} = B_i(T_s)$ and $\tau^*_i(L+2,l) = 0$ for $l = 1,\ldots,L+1$.

Because $\tau^*_i(l',l)$ is a symmetric matrix, calculations of the transmittance can be reduced in half by evaluating this matrix before performing the vertical integrations. If we did this, then in a vectorized computer code, where fluxes are computed simultaneously for m atmospheric columns, the $\tau^*_i(l',l)$ matrix would require storage for $\frac{m}{2}(L+1)L$ floating point numbers. This could reach 10^6 storage locations. To avoid this large storage requirement, (55a,b) are rearranged so that the calculation can be done with only m storage locations.

By expanding (55a,b) and rearranging terms, the upward and downward fluxes can be written as

$$F^{\uparrow}_{i,l} = B_{i,l+\frac{1}{2}} + \sum_{l'=l+1}^{L+1} \tau^*_i(l',l) \left[B_{i,l'+\frac{1}{2}} - B_{i,l'-\frac{1}{2}}\right], \quad l = 1,\ldots,L, \tag{56a}$$

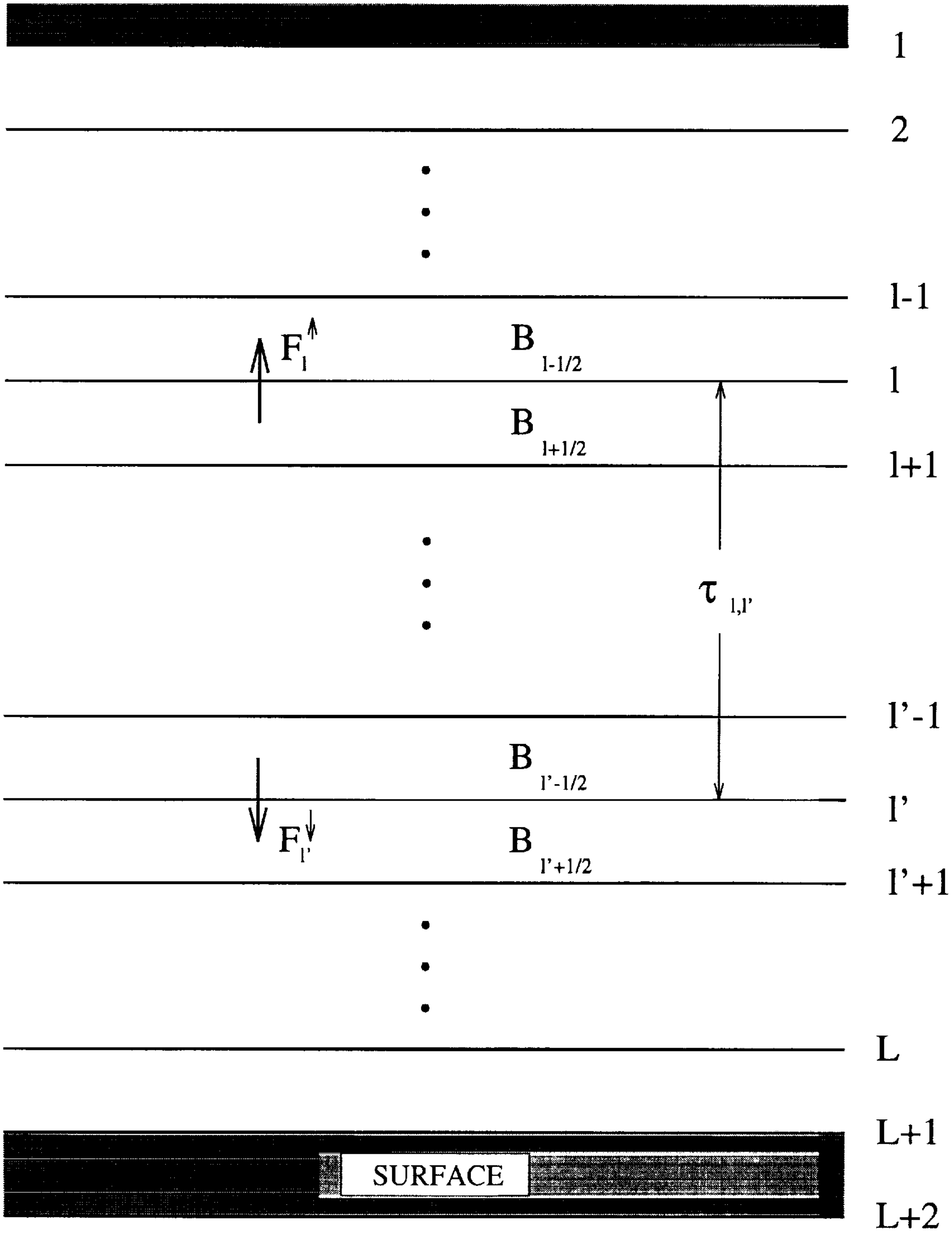

Figure 4: The vertical grid and placement of various quantities for an atmosphere consisting of L layers. Quantities defined at the layers, such as the Planck flux B, are denoted by half-integer subscripts, and quantities defined at the levels separating them, such as the upward and downward fluxes F, by integer subscripts. The transmittance shown is for a multi-layer region bounded by levels l and l'. Note that the surface is treated as a fictitious layer at $L + \frac{3}{2}$.

$$F^{\downarrow}_{i,l'} = B_{i,l'-\frac{1}{2}} - \sum_{l=1}^{l'-1} \tau_i^*(l',l)\left[B_{i,l+\frac{1}{2}} - B_{i,l-\frac{1}{2}}\right], \quad l' = 2,\ldots,L+1, \tag{56b}$$

with the boundary conditions:

$$F^{\uparrow}_{i,L+1} = B_i(T_s), \tag{57a}$$

$$F^{\downarrow}_{i,1} = 0. \tag{57b}$$

As opposed to (55a,b), only one transmittance appears in each term under the summation in (56a,b). The sums can thus be built-up term by term. When the transmittance of a layer bounded by l and l' is computed, the upward flux at the top of the layer and the downward flux at the bottom of the layer are immediately updated; in this way there is no need to store the transmittance matrix, The storage for the entire routine scales then like L, rather than L^2—a very significant advantage for models with high vertical resolution.

In practice, we do not store both the upward and downward fluxes, but keep only the total net downward flux summed over the bands:

$$F_l^{\mathrm{net}} = \sum_{i=1}^{8}(F^{\downarrow}_{i,l} - F^{\uparrow}_{i,l}). \tag{58}$$

This is computed for both "all-sky" and "clear-sky" conditions. The clear-sky flux is obtained from (56a,b) by replacing $\tau^*(l',l)$ with the gaseous transmittance, $\tau(l',l)$.

The parameterization also computes the rate of change of the $L+1$ all-sky fluxes with respect to the surface temperature, T_s. This is done because in the general circulation model the IR parameterization is called relatively infrequently (currently every 3 hours), while the boundary layer and land surface parameterizations use time steps of a few minutes. To maintain consistency between the upward radiative fluxes aloft and that at the surface when the surface temperature changes significantly between call to the IR parameterization, all fluxes are linearized about the surface temperature at the beginning of the radiation interval, and radiative heating rates are recomputed based on this linearization every few minutes, at each time step. The partial derivative of the net downward flux with respect to the surface temperature is:

$$\frac{\partial F_l^{\mathrm{net}}}{\partial T_s} = -\frac{\partial F_l^{\uparrow}}{\partial T_s} = -\sum_{i=1}^{8}\tau_i^*(L+1,l)\frac{\partial B_{i,L+\frac{3}{2}}}{\partial T_s}, \quad l = 1,\ldots,L+1, \tag{59}$$

where $\tau_i^*(L+1,L+1) = 1$, $B_{i,L+\frac{3}{2}} = B_i(T_s)$, and $\frac{\partial B_{i,L+\frac{3}{2}}}{\partial T_s}$ is obtained by differentiating (22).

6 Comparisons With Line-by-Line Calculations

In this section, we compare fluxes and cooling rates computed from the transmittance parameterizations with detailed line-by-line calculations. Our line-by-line calculations of the absorption coefficient use the Air Force Geophysical Laboratory (AFGL) 1992 edition of the molecular line parameters (Rothman et al. 1987). The line profile is assumed to follow the Voigt function. The absorption coefficient is taken to be zero at wavenumbers > 10 cm^{-1} from the line center; this is equivalent to a line-cutoff of 10 cm^{-1}. Since the Doppler line-width increases linearly with wavenumber, the spectral resolution can be increased at the higher wavenumbers. We use spectral intervals of 0.0005 cm^{-1} for Bands 1 and 2, 0.001 cm^{-1} for Band 3, and 0.002 cm^{-1} for Bands 4-8. This resolution adequately resolves individual absorption lines. As in Ridgway et al. (1991), the flux transmittance given by (5) is evaluated using table look-up.

Fluxes and cooling rates are computed for a midlatitude summer (MLS) atmosphere and a sub-arctic winter (SAW) atmosphere taken from McClatchey et al. (1972). Following the specifications of the Intercomparison of Radiation Codes in Climate Models (ICRCCM) (Ellingson et al. 1991), CO_2 concentration is fixed at 300 ppmv, and the specific humidity above the tropopause is set to $4\mathrm{x}10^{-6}$. The atmosphere is divided into 75 layers with $\delta p \approx$ 25 mb at pressures greater than 100 mb and $\delta \log_{10} p = 0.15$ at pressures less than 100 mb. In interpolating from the levels provided in McClatchey et al. (1972) to the levels used for the calculations, we assume the temperature, ozone concentration, and the logarithm of specific humidity vary linearly in the logarithm of pressure.

The total cooling rates computed using the line-by-line method (solid lines), the HIGH option of the parameterization (solid circles), and the LOW option of the parameterization (dashed lines), are shown in Figure 5. The top panels are for the midlatitude summer atmosphere and the bottom panels are for the sub-arctic winter atmosphere. Similar results for the individual bands are shown in Figures 8 – 15 in Appendix A.

The cooling rate error for the total IR spectrum (Figure 5) is ≈ 0.4 C day^{-1} in the middle atmosphere and ≈ 0.2 C day^{-1} in the troposphere and lower stratosphere. For the LOW option, the cooling rate is computed accurately in the lower atmosphere, at pressures greater than ≈ 20 mb; above the 20 mb pressure level, the cooling rate computed using the parameterization decreases rapidly to nearly zero. This is due to the pressure scaling of the absorption, which greatly underestimates the cooling in low pressure regions. It can be seen in the figures in the appendix that for the HIGH option, the cooling rate is computed accurately not only for the total cooling but also for individual bands. The maximum error of ≈ 0.4 C day^{-1} occurs in Band 3 (Figure 10) at pressures less than 1 mb. This error is small when compared with the maximum cooling of 10–12 C day^{-1} due to CO_2 at these levels. Cooling rates in bands dominated by water vapor and ozone absorption are very accurately computed, with errors of less than 0.2 C day^{-1}.

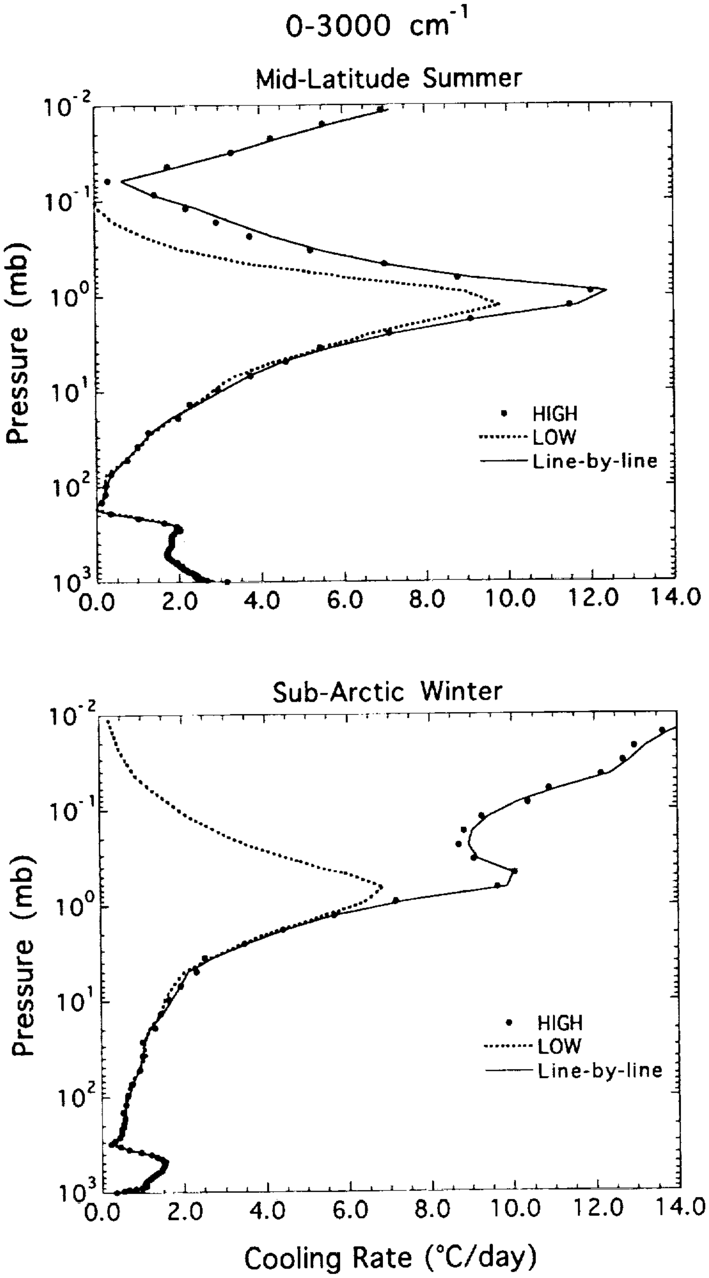

Figure 5: Cooling rates computed using the line-by-line method (solid lines), the "LOW" option (dashed lines), and the "HIGH" option (solid circles) for the total IR spectral region (0–3000 cm^{-1}). Cooling is due to water vapor molecular line and continuum absorption, as well as CO_2 and O_3 absorption.

The downward fluxes at the surface and the upward fluxes at the top of the atmosphere for the entire IR spectrum are shown in Table 8. In addition to our line-by-line results and the results for the LOW and HIGH options of the parameterization, we give the results of two other line-by-line calculations that were performed at the Goddard Laboratory for Atmospheres (GLA) and at the Geophysical Fluid Dynamics Laboratory (GFDL) as part of the ICRCCM intercomparison. These are taken from Table 2 of Ridgway et al. (1991). These two line-by-line calculations are very similar to each other, but differ from the line-by-line calculation used for the current study in important details.

Table 8: Downward fluxes at the surface, $F^{\downarrow}_{\mathrm{sfc}}$, and upward fluxes at the top of the atmosphere, $F^{\uparrow}_{\mathrm{top}}$, computed using a line-by-line method and the HIGH and LOW options of the parameterization. Also shown are the fluxes from other line-by-line calculations (GLA and GFDL) taken from Ridgway et al. (1991). Units are W m^{-2}.

	$F^{\downarrow}_{\mathrm{sfc}}$	$F^{\uparrow}_{\mathrm{top}}$
MID-LATITUDE SUMMER		
Parameterization (LOW)	342.33	292.49
Parameterization (HIGH)	342.45	293.07
line-by-line	339.93	293.10
line-by-line (GLA)	343.07	290.45
line-by-line (GFDL)	343.27	290.32
SUB-ARCTIC WINTER		
Parameterization (LOW)	161.17	203.58
Parameterization (HIGH)	160.44	204.22
line-by-line	161.51	204.39
line-by-line (GLA)	164.65	203.42
line-by-line (GFDL)	164.44	202.95

Comparing the parameterized results with our own line-by-line calculation, we find that the parameterizations can compute the fluxes to within 1% of the line-by-line calculations for the midlatitude summer atmosphere and within 0.5% for the sub-arctic winter atmosphere. Tables 9 and 10 in Appendix B give the line-by-line and parameterized fluxes for the individual bands. Errors are generally much smaller than 1 W m^{-2}.

As may be seen from Table 8, there is generally better agreement between the parameterization and our own line-by-line calculation than there is between our line-by-line calculation and those of GLA and GFDL; these two, on the other hand, are in very close agreement with each other. This disagreement between our line-by-line calculations—on which the parameterizations are based—and those used for ICRCCM were somewhat disturbing. Since the GLA calculation was done with essentially the same code as ours, but using the conditions prescribed by the intercomparison, we could easily isolate the differences. Figure 6 shows the differences for the upward and downward fluxes for the MLS and SAW atmospheres between the GLA and our line-by-line calculations. Differences in Bands 1, 5, and 7 are small. Differences in bands 2, 3, 4, and 8 are of opposite sign at the top and bottom of the atmosphere, with the upward flux at the top being larger in our calculations than in the GLA calculations; this indicated some missing absorption in our calculations in these bands. This effect is particularly large in Band 2. The difference in Band 6 is more perplexing. These differences can be explained as follows:

- The water vapor continuum absorption in the spectral region with strong molecular line absorption is not adequately understood. We therefore do not include the water vapor continuum absorption in the spectral region with wavenumber less than 540 cm^{-1}, but the water vapor continuum absorption in the spectral region 400–540 cm^{-1} was included in the previous GLA and GFDL calculations. This discrepancy accounts for most of the discrepancies in Band 2.

- The absorption due to O_3 in the 14 μm region, which was included in the previous GLA and GFDL calculations but not in the present study, contributes to the discrepancy in Band 3.

- The minor CO_2 bands located at 10.4 μm (in Band 4) and 9.4 μm (in Band 5) regions were included in the previous GLA and GFDL calculations but not in the present study. According to the line-by-line calculations of Kratz et al. (1993), each of the two minor bands causes a reduction of 0.2–0.3 W m^{-2} in the top-of-the-atmosphere flux and an increase of 0.4–0.5 W m^{-2} in the surface flux.

- The absorption due to CO_2 in the 4.3 μm band (in Band 8) was included in the previous GLA and GFDL calculations but not in the present study. Most of the discrepancies in Band 8 as shown in Figure 6 are due to the CO_2 absorption.

- The current study uses the 1992 edition of the AFGL line parameters, while the previous GLA and GFDL calculations used the 1986 edition. Integrated over 10 cm^{-1} spectral regions, the molecular line intensity of the 1992 edition is significantly higher (with a maximun of 33%) than the 1986 edition in the 950-1500 cm^{-1} spectral region (Bands 5 and 6). The large difference in the downward surface fluxes for the midlatitude summer atmosphere is due to the use of different editions of the line parameters.

These comparisons between the parameterized and line-by-line results and between the various line-by-line calculations give some idea of the sources and magnitudes of the errors that remain in the parameterization of the clear-sky IR radiation. As we can see from Table

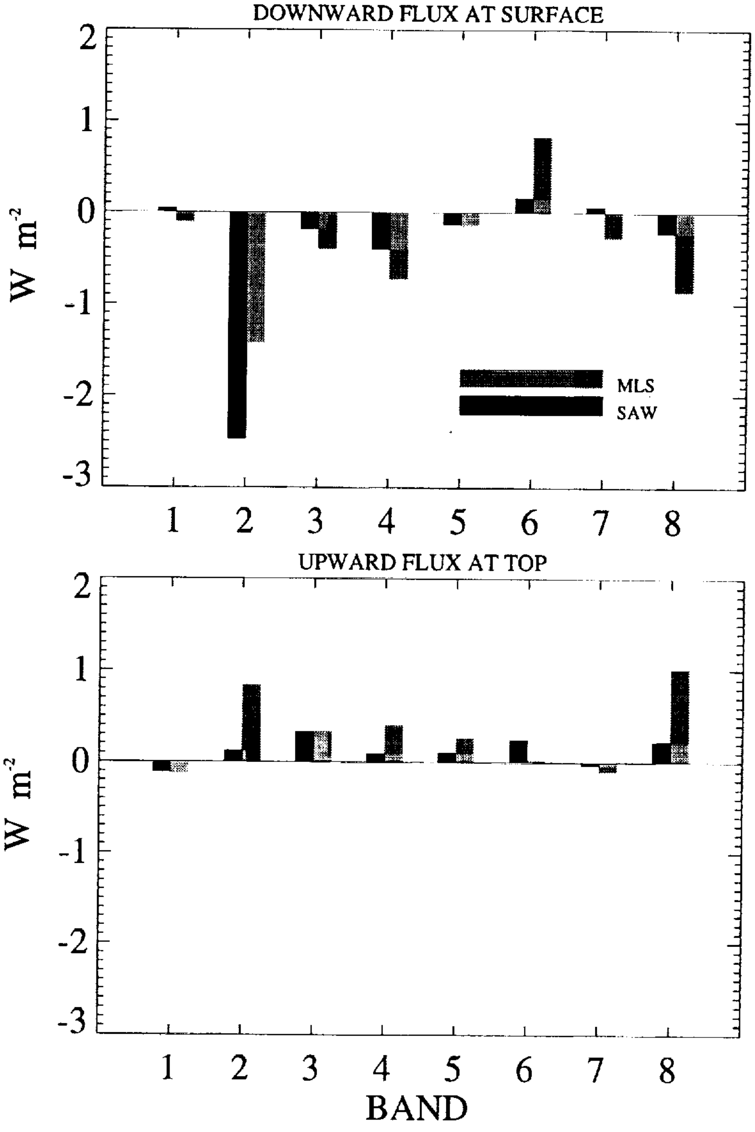

Figure 6: Differences between the GLA and our line-by-line calculations for the downward flux at the surface and the upward flux at the top of the atmosphere for the midlatitude summer (MLS) and sub-arctic winter (SAW) atmospheres. Positive values indicate our fluxes are larger than the GLA fluxes.

8, errors due to the broad-band parameterization of the line-by-line results are comparable to the differences between the line-by-line results. The effects of neglecting the minor bands of CO_2 and O_3 and of the uncertanty in modeling the water vapor continuum are significant. We have also neglected the effects of minor infrared absorbers, most notably nitrous oxide (N_2O), methane (CH_4), and the chlorofluorocarbons (CFCs). The total effect of neglecting all the minor absorption bands is an underestimate of ≈ 5 W m^{-2} in the downward flux at the surface and an overestimate of ≈ 3 W m^{-2} in the upward flux at the top of the atmosphere. We plan to include both the minor gases and the minor bands of the main gases in future versions of the parameterization.

7 Summary

To achieve both requirements in accuracy and speed, various approaches are applied to computing the transmission functions in various sections of the spectrum due to different absorbers. There are two options for flux calculations, HIGH and LOW. For the HIGH option, the k-distribution method with linear pressure-scaling is applied to those water vapor spectral regions where absorption is not strong and where the contribution to the middle atmospheric cooling is negligible. For the CO_2 and O_3 bands, as well as the water vapor spectral regions with strong absorption, cooling in the middle atmosphere is significant, and the transmittances are derived from pre-computed tables. For the LOW option, the k-distribution method is used in all situations, except for the absorption due to ozone which uses the table look-up method.

The HIGH option of the parameterization can compute accurately the cooling rate for the middle and lower atmospheres from 0.01 mb to the surface. Errors are < 0.4 C day^{-1} in cooling rates and $< 1\%$ in fluxes. The LOW option is computationally faster than the HIGH option. It can achieve the same accuracy as the HIGH option except in the regions with pressure < 20 mb. For the HIGH option of the parameterization, it takes 9.1 seconds to compute the fluxes at 75 levels for 1000 soundings on a single processor of the CRAY/C90 computer. For comparison, it takes 4.6 seconds to do the same computation with the LOW option. Figure 7 shows each band's contribution to the total time. Bands 4, 5, 6, and 8 are the same for the two options. Bands 4 and 6 differ from Band 8 only in the presence of continuum absorption, which evidently does not contribute significantly to the total cost. In Bands 1, 2, and 7, the two options differ only in the treatment of water vapor line absorption, which costs nearly four times as much when done with table look-up as when done by the k-distribution method. With the HIGH option, Band 3 is the most costly due to the use of a table look-up to compute the CO_2 transmittances and to the separation into three sub-bands used to compute the water vapor transmittances. For the LOW option, Band 5 is the most costly due to the table look-up used to compute the O_3 transmittances. A LOW option without O_3, which should be acceptable for tropospheric models, would be considerably faster.

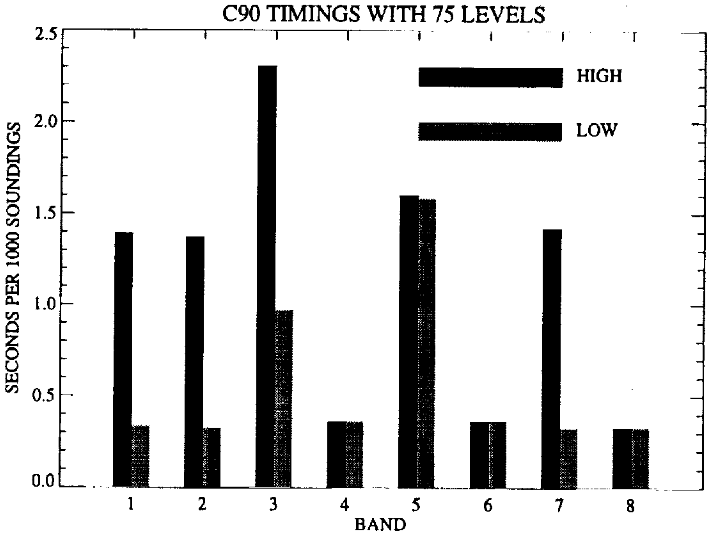

Figure 7: Execution times on a single Cray C90 processor.

In addition to the off-line testing described here, the parameterizations developed at Goddard have been implemented and tested in long-term simulations in the climate and data assimilation general circulation models at Goddard.

Acknowledgment: The authors are grateful to Dr. William Ridgway for providing them the line-by-line calculated fluxes for comparisons with the parameterization and to Mr. Michael Yan for assistance in testing the computer code. This work was supported by the Global Atmospheric Modeling and Analysis Program, Office of Mission to Planet Earth, NASA Headquarters.

Appendix A: Band-by-Band Reults

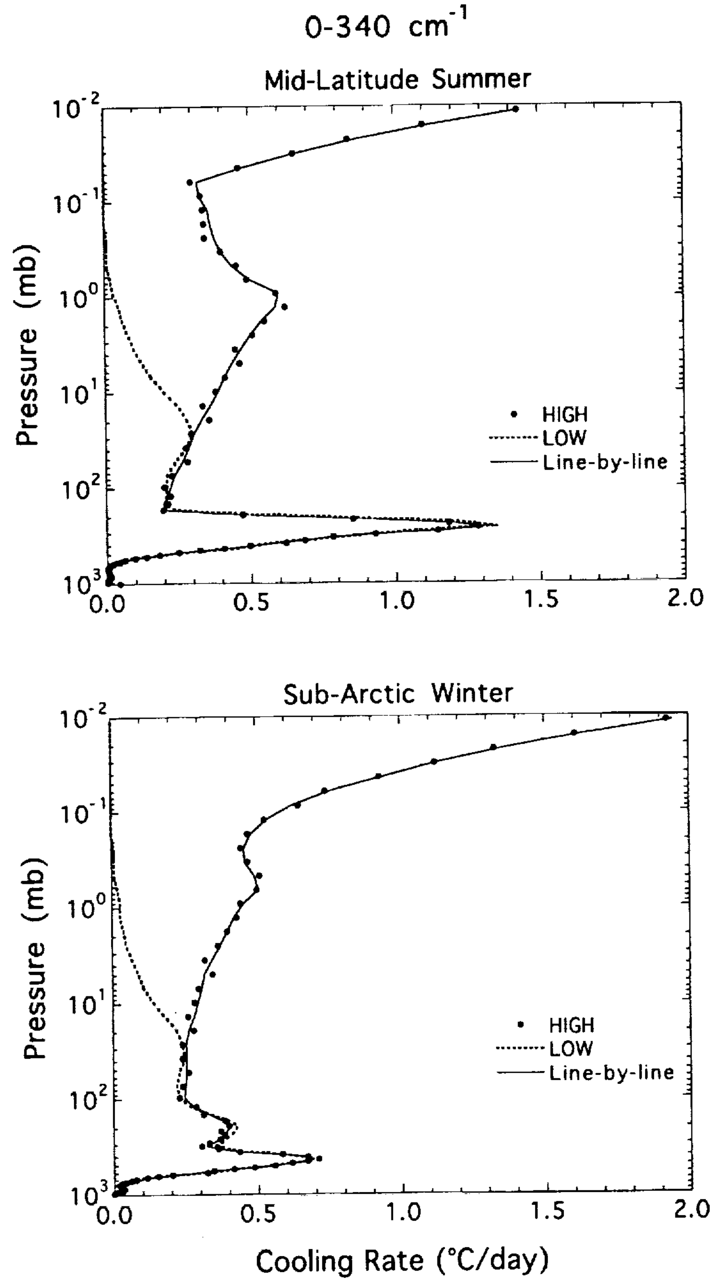

Figure 8: Cooling rates computed using the line-by-line method (solid lines), the "LOW" option (dashed lines), and the "HIGH" option (solid circles) for the 0–340 cm^{-1} band. Cooling is due to water vapor molecular line absorption. Options for the transmittance parameterization are outlined in Table 1

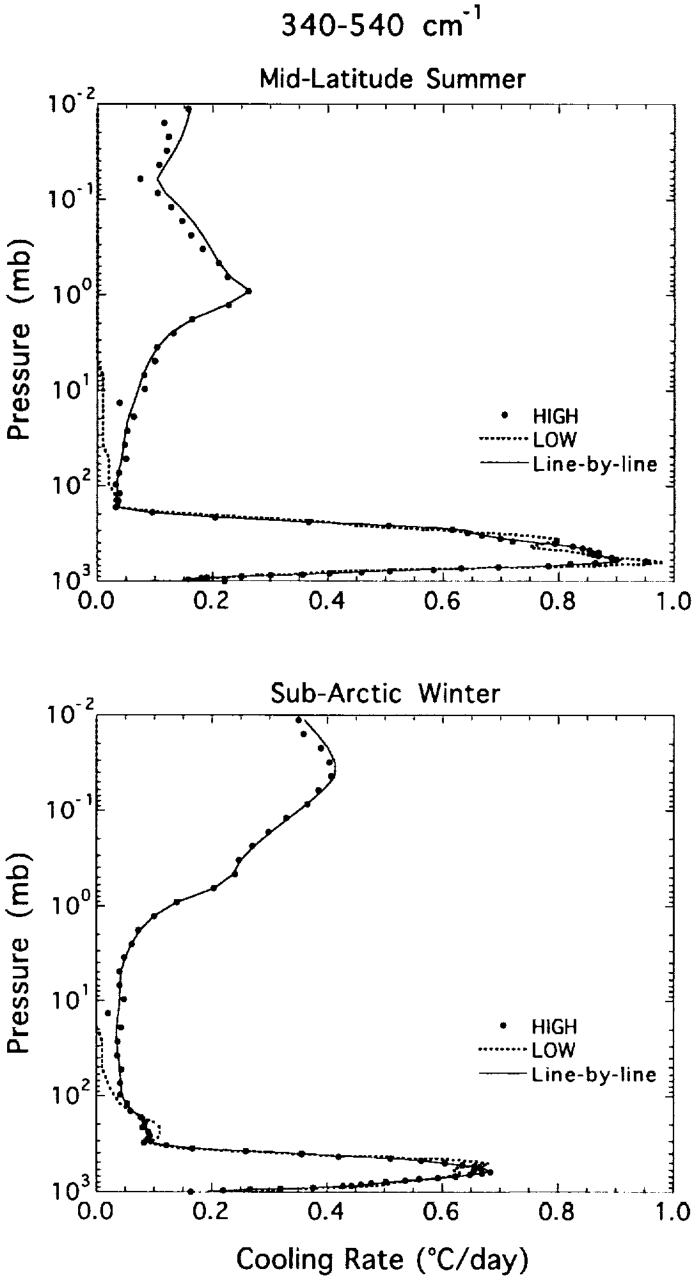

Figure 9: Same as Figure 8, except for the 340–540 cm^{-1} band.

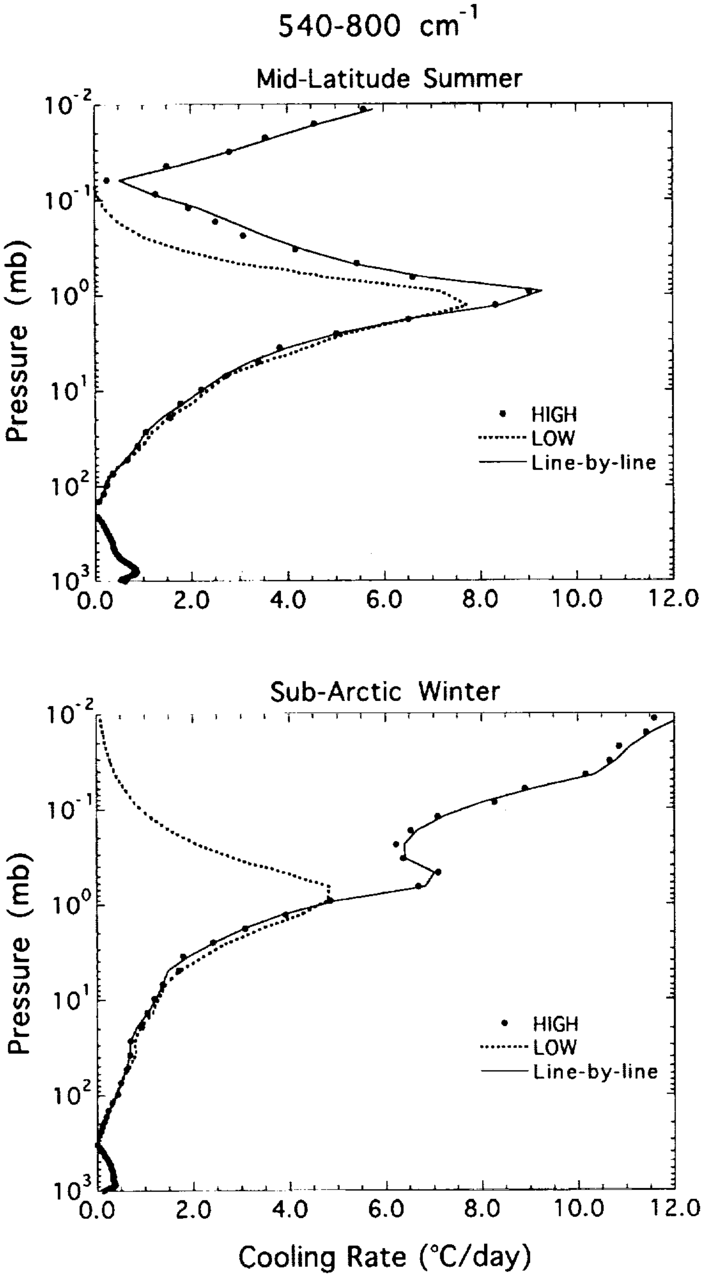

Figure 10: Same as Figure 8, except for the 540–800 cm^{-1} band. Cooling is due to water vapor molecular line and continuum absorption, as well as CO_2 absorption.

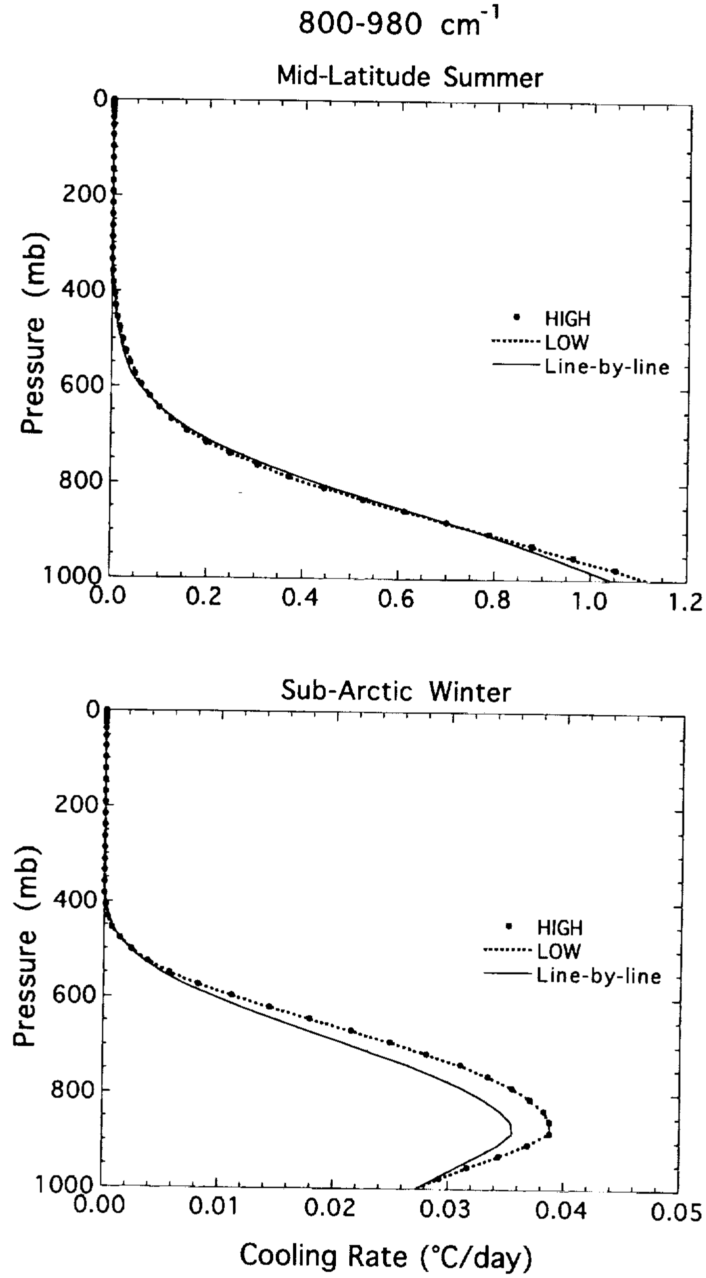

Figure 11: Same as Figure 8, except for the 800–980 cm^{-1} band. Cooling is due to water vapor molecular line and continuum absorption. Note that the HIGH and LOW options are identical in this band.

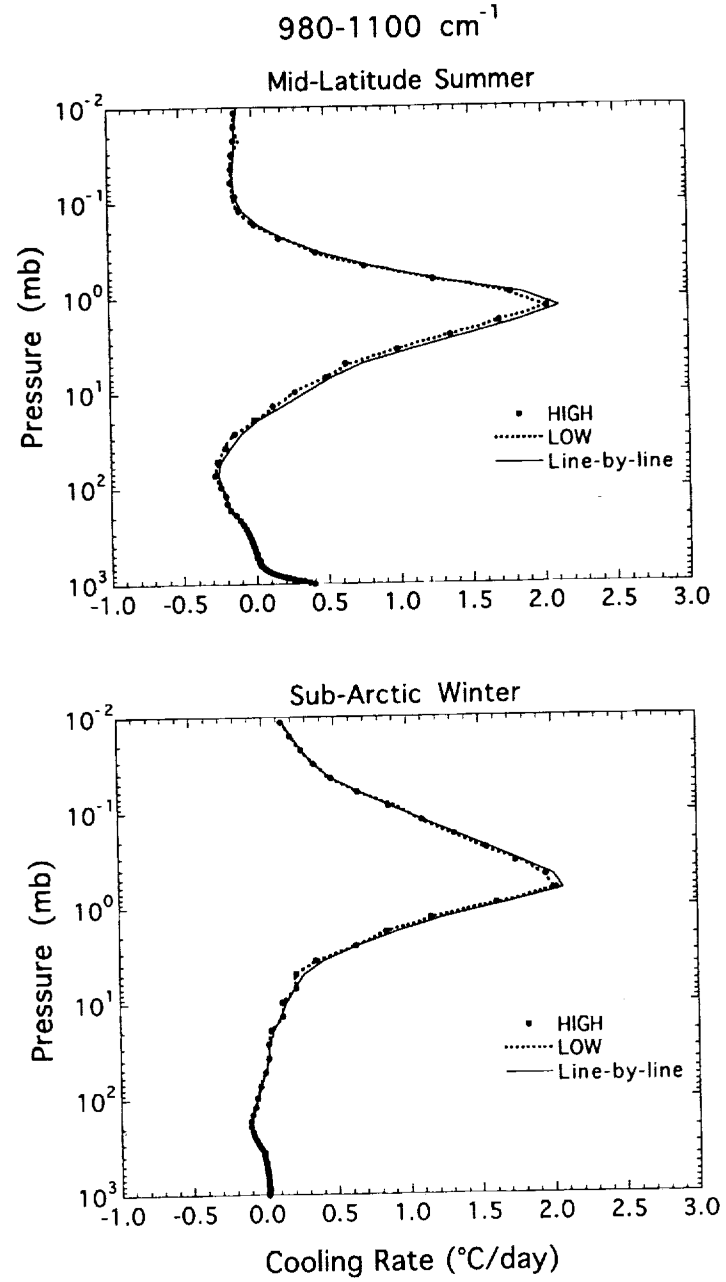

Figure 12: Same as Figure 8, except for the 980–1100 cm^{-1} band. Cooling is due to water vapor molecular line and continuum absorption, as well as O_3 absorption.

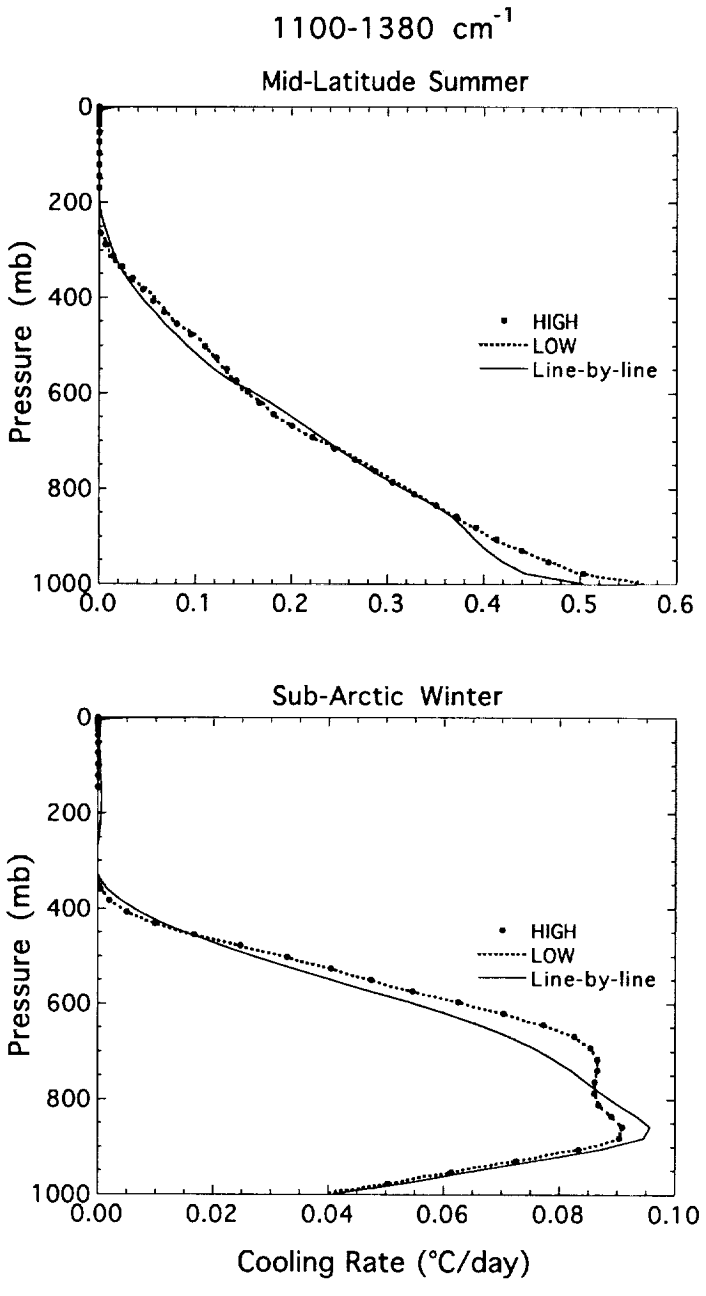

Figure 13: Same as Figure 8, except for the 1100–1380 cm^{-1} band. Cooling is due to water vapor molecular line and continuum absorption.

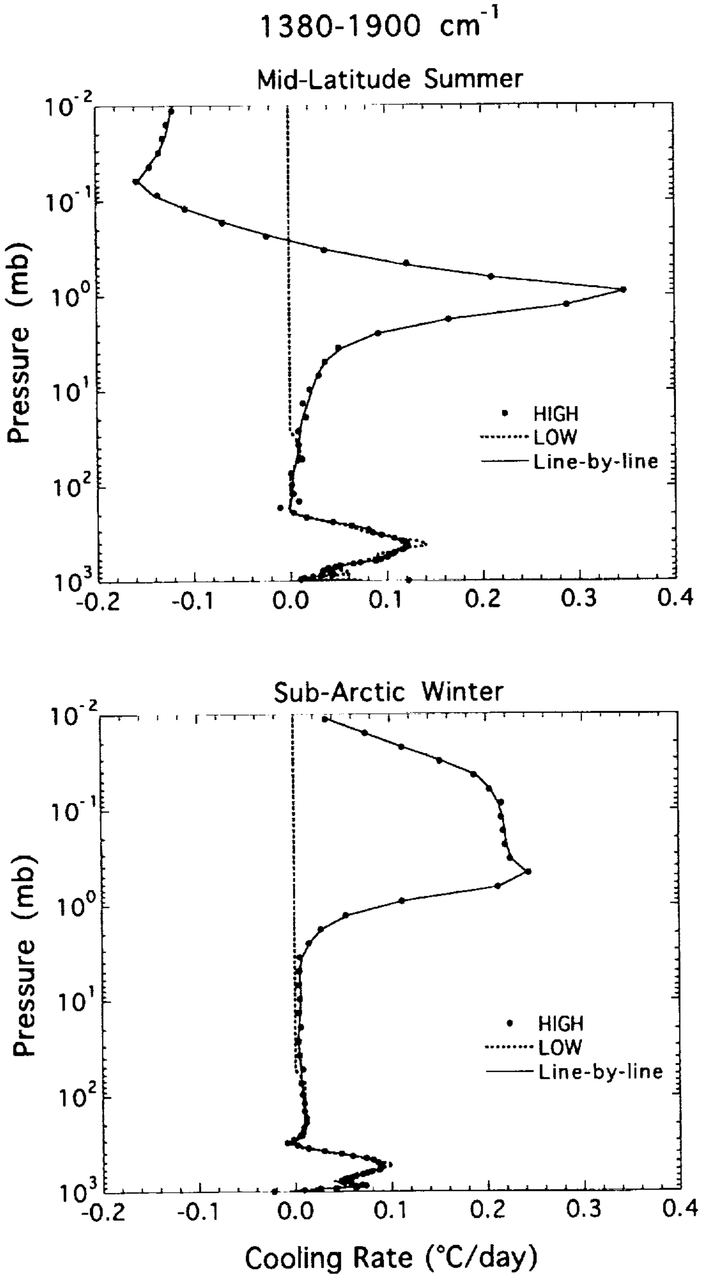

Figure 14: Same as Figure 8, except for the 1380–1900 cm^{-1} band.

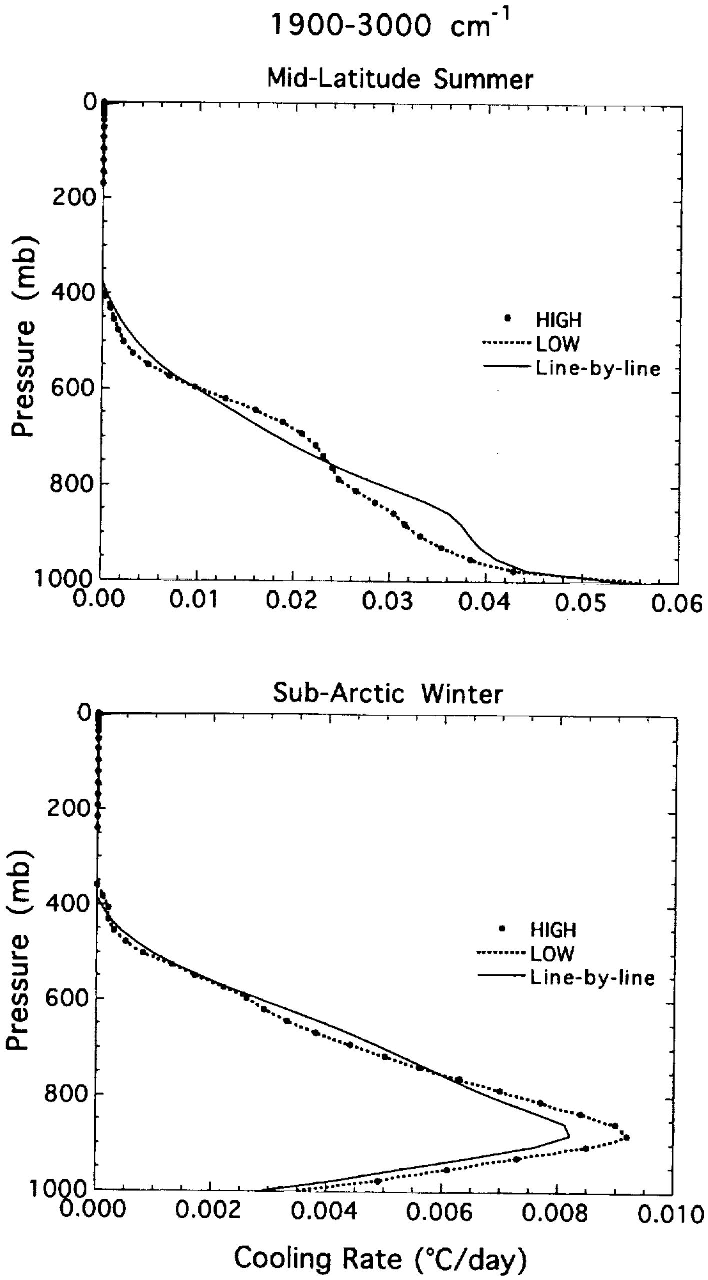

Figure 15: Same as Figure 8, except for the 1900–3000 cm^{-1} band.

Table 9: Downward fluxes at the surface, $F^{\downarrow}_{sfc}$, and upward fluxes at the top of the atmosphere, $F^{\uparrow}_{top}$, for the mid-latitude summer atmosphere computed using a line-by-line method and the HIGH and LOW options of the parameterization. Units are W m^{-2}.

Spectral Band (cm^{-1})		$F^{\downarrow}_{sfc}$	$F^{\uparrow}_{top}$
0 – 340			
	line-by-line	50.96	34.25
	Parameterization (HIGH)	50.97	34.04
	Parameterization (LOW)	50.97	33.92
340 – 540			
	line-by-line	80.72	60.51
	Parameterization (HIGH)	81.23	60.01
	Parameterization (LOW)	81.28	60.03
540 – 800			
	line-by-line	105.98	68.03
	Parameterization (HIGH)	107.58	68.40
	Parameterization (LOW)	107.43	67.74
800 – 980			
	line-by-line	27.97	58.49
	Parameterization (HIGH)	28.35	58.50
	Parameterization (LOW)	28.34	58.50
980 – 1100			
	line-by-line	12.80	22.28
	Parameterization (HIGH)	12.86	21.81
	Parameterization (LOW)	12.86	21.82
1100 – 1380			
	line-by-line	28.14	37.18
	Parameterization (HIGH)	27.95	38.21
	Parameterization (LOW)	27.95	38.21
1380 – 1900			
	line-by-line	30.30	7.27
	Parameterization (HIGH)	30.35	7.22
	Parameterization (LOW)	30.33	7.40
1900 – 3000			
	line-by-line	3.06	5.09
	Parameterization (HIGH)	3.16	4.88
	Parameterization (LOW)	3.16	4.88

Table 10: Same as Table 9, except for the sub-arctic winter atmosphere.

Spectral Band (cm^{-1})	$F^{\downarrow}_{sfc}$	$F^{\uparrow}_{top}$
0 – 340		
line-by-line	40.39	32.10
Parameterization (HIGH)	40.40	31.93
Parameterization (LOW)	40.40	31.82
340 – 540		
line-by-line	47.35	52.01
Parameterization (HIGH)	47.30	51.74
Parameterization (LOW)	48.09	51.78
540 – 800		
line-by-line	53.17	51.38
Parameterization (HIGH)	51.84	51.65
Parameterization (LOW)	51.90	51.06
800 – 980		
line-by-line	1.45	32.85
Parameterization (HIGH)	1.63	32.86
Parameterization (LOW)	1.63	32.86
980 – 1100		
line-by-line	3.14	10.99
Parameterization (HIGH)	3.23	10.87
Parameterization (LOW)	3.22	10.87
1100 – 1380		
line-by-line	5.49	18.80
Parameterization (HIGH)	5.49	18.98
Parameterization (LOW)	5.50	18.98
1380 – 1900		
line-by-line	10.11	4.90
Parameterization (HIGH)	10.13	4.87
Parameterization (LOW)	10.02	4.90
1900 – 3000		
line-by-line	0.41	1.36
Parameterization (HIGH)	0.42	1.32
Parameterization (LOW)	0.43	1.32

Appendix B: The Code

The following is a listing of the FORTRAN implementation of the parameterization. This code follows the "plug-compatible" rules of Kalnay et al. (1988), which should make it easy to use in climate models. The parameterization is accessed by calling subroutine IRRAD. All information needed by this subroutine is passed through the argument list and all results are returned through the argument list. Its arguments are described in detail in the comments at the top of the code.

The inputs to the routine are a two-dimensional (M by N) array of soundings, with each sounding consisting of NP layers. The temperature, the specific humidity, the ozone mixing ratio, and the cloud fraction and optical thickness are specified at the NP layers, while the pressure is specified at the NP-1 interfaces and at the lower and upper boundaries. The surface temperature is also required. The CO_2 concentration is assumed constant throughout the atmosphere.

Although the routine processes M by N soundings, all input and output arrays are dimensioned M by NDIM, where NDIM $\geq$ N. This is done so that a large array can be stripmined over the outer dimension without the need to make copies. Such stripmining is useful in parallel applications and when trying to minimize storage, since the scratch space required by the routine is proportional to M*N.

The outputs of the routine are the net downward all-sky and clear-sky fluxes and the rate of change of the all-sky flux with respect to surface temperature. The routine also returns the upward flux at the surface, i.e., the approximate form of σT_s^4 used in the parameterization. This allows the calling program to maintain energetic consistency by using this value in the surface energy budget.

7.1 Subroutine IRRAD

```
********************* CLIRAD IR   Date: October, 1994 ****************
*
      subroutine irrad (m,n,ndim,np,taucl,ccld,pl,ta,wa,oa,co2,ts,
     *                  high,flx,flc,dfdts,st4)
*
**********************************************************************
*
* This routine computes ir fluxes due to water vapor, co2, and o3.
*   Clouds in different layers are assumed randomly overlapped.
*
* This is a vectorized code.  It computes fluxes simultaneously for
*   (m x n) soundings, which is a subset of (m x ndim) soundings.
*   In a global climate model, m and ndim correspond to the numbers of
*   grid boxes in the zonal and meridional directions, respectively.
*
* Detailed description of the radiation routine is given in
*   Chou and Suarez (1994).
*
* There are two options for computing cooling rate profiles.
*
*   if high = .true., transmission functions in the co2, o3, and the
*   three water vapor bands with strong absorption are computed using
*   table look-up.  cooling rates are computed accurately from the
*   surface up to 0.01 mb.
*   if high = .false., transmission functions are computed using the
*   k-distribution method with linear pressure scaling.  cooling rates
*   are not calculated accurately for pressures less than 20 mb.
*   the computation is faster with high=.false. than with high=.true.
*
* The IR spectrum is divided into eight bands:
*
*   bnad     wavenumber (/cm)  absorber         method
*
*    1           0 - 340           h2o            K/T
*    2         340 - 540           h2o            K/T
*    3         540 - 800       h2o,cont,co2       K,S,K/T
*    4         800 - 980       h2o,cont           K,S
*    5         980 - 1100      h2o,cont,o3        K,S,T
*    6        1100 - 1380      h2o,cont           K,S
*    7        1380 - 1900          h2o            K/T
*    8        1900 - 3000          h2o            K
*
* Note : "h2o" for h2o line absorption
*        "cont" for h2o continuum absorption
*        "K" for k-distribution method
*        "S" for one-parameter temperature scaling
*        "T" for table look-up
*
* The 15 micrometer region (540-800/cm) is further divided into
*   3 sub-bands :
```

```
*
*   subbnad   wavenumber (/cm)
*
*    1          540 - 620
*    2          620 - 720
*    3          720 - 800
*
*---- Input parameters                                    units     size
*
*   number of soundings in zonal direction (m)             n/d       1
*   number of soundings in meridional direction (n)        n/d       1
*   maximum number of soundings in
*                  meridional direction (ndim)             n/d       1
*   number of atmospheric layers (np)                      n/d       1
*   cloud optical thickness (taucl)                        n/d     m*ndim*np
*   cloud cover (ccld)                                  fraction   m*ndim*np
*   level pressure (pl)                                    mb      m*ndim*(np+1)
*   layer temperature (ta)                                 k       m*ndim*np
*   layer specific humidity (wa)                           g/g     m*ndim*np
*   layer ozone mixing ratio by mass (oa)                  g/g     m*ndim*np
*   surface temperature (ts)                               k       m*ndim
*   co2 mixing ratio by volumn (co2)                       ppmv      1
*   high                                                             1
*
* pre-computed tables used in table look-up for transmittance calculations:
*
*   c1 , c2, c3: for co2 (band 3)
*   o1 , o2, o3: for  o3 (band 5)
*   h11,h12,h13: for h2o (band 1)
*   h21,h22,h23: for h2o (band 2)
*   h71,h72,h73: for h2o (band 7)
*
*---- output parameters
*
*   net downward flux, all-sky   (flx)                  w/m**2     m*ndim*(np+1)
*   net downward flux, clear-sky (flc)                  w/m**2     m*ndim*(np+1)
*   sensitivity of net downward flux
*       to surface temperature (dfdts)                  w/m**2/k   m*ndim*(np+1)
*   emission by the surface (st4)                       w/m**2     m*ndim
*
* Notes:
*
*   (1)  Water vapor continuum absorption is included in 540-1380 /cm.
*   (2)  Scattering by clouds is not included.
*   (3)  Clouds are assumed "gray" bodies.
*   (4)  The diffuse cloud transmission is computed to be exp(-1.66*taucl).
*   (5)  If there are no clouds, flx=flc.
*   (6)  plevel(1) is the pressure at the top of the model atmosphere, and
*        plevel(np+1) is the surface pressure.
*   (7)  Downward flux is positive, and upward flux is negative.
*   (8)  dfdts is always negative because upward flux is defined as negative.
*   (9)  For questions and coding errors, please contact with Ming-Dah Chou,
*        Code 913, NASA/Goddard Space Flight Center, Greenbelt, MD 20771.
```

```
*       Phone: 301-286-4012, Fax: 301-286-1759,
*       e-mail: chou@climate.gsfc.nasa.gov
*
c-----parameters defining the size of the pre-computed tables for transmittance
c     calculations using table look-up.
c
c     "nx" is the number of intervals in pressure
c     "no" is the number of intervals in o3 amount
c     "nc" is the number of intervals in co2 amount
c     "nh" is the number of intervals in h2o amount
c     "nt" is the number of copies to be made from the pre-computed
c         transmittance tables to reduce "memory-bank conflict"
c         in parallel machines and, hence, enhancing the speed of
c         computations using table look-up.
c         If such advantage does not exist, "nt" can be set to 1.
c***************************************************************************
      implicit none
      integer nx,no,nc,nh,nt,m,n,ndim,np
      integer i,j,k,ip,iw,it,ib,ik,iq,isb,k1,k2
      parameter (nx=26,no=21,nc=24,nh=31,nt=7)

c---- input parameters ------

      real*8 co2
      real*8 taucl(m,ndim,np),ccld(m,ndim,np),pl(m,ndim,np+1),
     *       ta(m,ndim,np),wa(m,ndim,np),oa(m,ndim,np),ts(m,ndim)

      logical high

c---- output parameters ------

      real*8 flx(m,ndim,np+1),flc(m,ndim,np+1),dfdts(m,ndim,np+1),
     *       st4(m,ndim)

c---- static data -----

      real*8  cb(5,8)

c---- temporary arrays -----

      real*8 pa(m,n,np),dt(m,n,np)
      real*8 sh2o(m,n,np+1),swpre(m,n,np+1),swtem(m,n,np+1)
      real*8 sco3(m,n,np+1),scopre(m,n,np+1),scotem(m,n,np+1)
      real*8 dh2o(m,n,np),dcont(m,n,np),dco2(m,n,np),do3(m,n,np)
      real*8 th2o(m,n,6),tcon(m,n,3),tco2(m,n,6,2)
      real*8 h2oexp(m,n,np,6), conexp(m,n,np,3), co2exp(m,n,np,6,2)
      real*8 clr(m,n,0:np+1),fclr(m,n)
      real*8 blayer(m,n,0:np+1),dbs(m,n)
      real*8 trant(m,n)

      logical oznbnd
      logical co2bnd
      logical h2otbl
```

```
      logical conbnd

      real*8 c1 (nx,nc,nt),c2 (nx,nc,nt),c3 (nx,nc,nt)
      real*8 o1 (nx,no,nt),o2 (nx,no,nt),o3 (nx,no,nt)
      real*8 h11(nx,nh,nt),h12(nx,nh,nt),h13(nx,nh,nt)
      real*8 h21(nx,nh,nt),h22(nx,nh,nt),h23(nx,nh,nt)
      real*8 h71(nx,nh,nt),h72(nx,nh,nt),h73(nx,nh,nt)

      real*8 dp,xx,trantc

c-----the following coefficients (table 2 of chou and suarez, 1995) are for
c      computing spectrally integtrated planck fluxes of the 8 bands using
eq. (22)

       data cb/
     1 -2.6844e-1,-8.8994e-2, 1.5676e-3,-2.9349e-6, 2.2233e-9,
     2  3.7315e+1,-7.4758e-1, 4.6151e-3,-6.3260e-6, 3.5647e-9,
     3  3.7187e+1,-3.9085e-1,-6.1072e-4, 1.4534e-5,-1.6863e-8,
     4 -4.1928e+1, 1.0027e+0,-8.5789e-3, 2.9199e-5,-2.5654e-8,
     5 -4.9163e+1, 9.8457e-1,-7.0968e-3, 2.0478e-5,-1.5514e-8,
     6 -1.0345e+2, 1.8636e+0,-1.1753e-2, 2.7864e-5,-1.1998e-8,
     7 -6.9233e+0,-1.5878e-1, 3.9160e-3,-2.4496e-5, 4.9301e-8,
     8  1.1483e+2,-2.2376e+0, 1.6394e-2,-5.3672e-5, 6.6456e-8/

c-----copy tables to enhance the speed of co2 (band 3), o3 (band5),
c      and h2o (bands 1, 2, and 7 only) transmission calculations
c      using table look-up.


      logical first
      data first /.true./

      include "h2o.tran3"
      include "co2.tran3"
      include "o3.tran3"

      save c1,c2,c3,o1,o2,o3
      save h11,h12,h13,h21,h22,h23,h71,h72,h73

      if (first) then

c-----tables co2 and h2o are only used with 'high' option

       if (high) then

        do iw=1,nh
         do ip=1,nx
          h11(ip,iw,1)=1.0-h11(ip,iw,1)
          h21(ip,iw,1)=1.0-h21(ip,iw,1)
          h71(ip,iw,1)=1.0-h71(ip,iw,1)
         enddo
        enddo
```

```
        do iw=1,nc
         do ip=1,nx
          c1(ip,iw,1)=1.0-c1(ip,iw,1)
         enddo
        enddo

c-----tables are replicated to avoid memory bank conflicts

        do it=2,nt
         do iw=1,nc
          do ip=1,nx
           c1 (ip,iw,it)= c1(ip,iw,1)
           c2 (ip,iw,it)= c2(ip,iw,1)
           c3 (ip,iw,it)= c3(ip,iw,1)
          enddo
         enddo
         do iw=1,nh
          do ip=1,nx
           h11(ip,iw,it)=h11(ip,iw,1)
           h12(ip,iw,it)=h12(ip,iw,1)
           h13(ip,iw,it)=h13(ip,iw,1)
           h21(ip,iw,it)=h21(ip,iw,1)
           h22(ip,iw,it)=h22(ip,iw,1)
           h23(ip,iw,it)=h23(ip,iw,1)
           h71(ip,iw,it)=h71(ip,iw,1)
           h72(ip,iw,it)=h72(ip,iw,1)
           h73(ip,iw,it)=h73(ip,iw,1)
          enddo
         enddo
        enddo

       endif

c-----always use table look-up for ozone transmittance

        do iw=1,no
         do ip=1,nx
          o1(ip,iw,1)=1.0-o1(ip,iw,1)
         enddo
        enddo

        do it=2,nt
         do iw=1,no
          do ip=1,nx
           o1 (ip,iw,it)= o1(ip,iw,1)
           o2 (ip,iw,it)= o2(ip,iw,1)
           o3 (ip,iw,it)= o3(ip,iw,1)
          enddo
         enddo
        enddo

       first=.false.
```

```
      endif

c-----compute layer pressure (pa) and layer temperature minus 250K (dt)

      do k=1,np
       do j=1,n
        do i=1,m
         pa(i,j,k)=0.5*(pl(i,j,k)+pl(i,j,k+1))
         dt(i,j,k)=ta(i,j,k)-250.0
        enddo
       enddo
      enddo

c-----compute layer absorber amount

c     dh2o : water vapor amount (g/cm**2)
c     dcont: scaled water vapor amount for continuum absorption (g/cm**2)
c     dco2 : co2 amount (cm-atm)stp
c     do3  : o3 amount (cm-atm)stp
c     the factor 1.02 is equal to 1000/980
c     factors 789 and 476 are for unit conversion
c     the factor 0.001618 is equal to 1.02/(.622*1013.25)
c     the factor 6.081 is equal to 1800/296

      do k=1,np
       do j=1,n
        do i=1,m

         dp           = pl(i,j,k+1)-pl(i,j,k)
         dh2o (i,j,k) = 1.02*wa(i,j,k)*dp+1.e-10
         dco2 (i,j,k) = 789.*co2*dp+1.e-10
         do3  (i,j,k) = 476.0*oa(i,j,k)*dp+1.e-10

c-----compute scaled water vapor amount for h2o continuum absorption
c     following eq. (43).

         xx=pa(i,j,k)*0.001618*wa(i,j,k)*wa(i,j,k)*dp
         dcont(i,j,k) = xx*exp(1800./ta(i,j,k)-6.081)+1.e-10

c-----compute effective cloud-free fraction, clr, for each layer.
c     the cloud diffuse transmittance is approximated by using a
c     diffusivity factor of 1.66.

         clr(i,j,k)=1.0-(ccld(i,j,k)*(1.-exp(-1.66*taucl(i,j,k))))

        enddo
       enddo
      enddo

c-----compute column-integrated h2o amoumt, h2o-weighted pressure
c     and temperature.  it follows eqs. (37) and (38).

       if (high) then
```

```
        call column(m,n,np,pa,dt,dh2o,sh2o,swpre,swtem)

       endif

c-----the surface (with an index np+1) is treated as a layer filled with
c     black clouds.

      do j=1,n
       do i=1,m
        clr(i,j,0)    = 1.0
        clr(i,j,np+1) = 0.0
        st4(i,j)      = 0.0
       enddo
      enddo

c-----initialize fluxes

      do k=1,np+1
       do j=1,n
        do i=1,m
         flx(i,j,k)  = 0.0
         flc(i,j,k)  = 0.0
         dfdts(i,j,k)= 0.0
        enddo
       enddo
      enddo

c-----integration over spectral bands

      do 1000 ib=1,8

c-----if h2otbl, compute h2o (line) transmittance using table look-up.
c     if conbnd, compute h2o (continuum) transmittance in bands 3, 4, 5 and 6.
c     if co2bnd, compute co2 transmittance in band 3.
c     if oznbnd, compute  o3 transmittance in band 5.

       h2otbl=high.and.(ib.eq.1.or.ib.eq.2.or.ib.eq.7)
       conbnd=ib.ge.3.and.ib.le.6
       co2bnd=ib.eq.3
       oznbnd=ib.eq.5

c-----blayer is the spectrally integrated planck flux of the mean layer
c     temperature derived from eq. (22)

       do k=1,np
        do j=1,n
         do i=1,m
          blayer(i,j,k)=ta(i,j,k)*(ta(i,j,k)*(ta(i,j,k)
     *                 *(ta(i,j,k)*cb(5,ib)+cb(4,ib))+cb(3,ib))
     *                 +cb(2,ib))+cb(1,ib)
         enddo
        enddo
```

```
      enddo

c-----the earth's surface, with an index "np+1", is treated as a layer

      do j=1,n
       do i=1,m
        blayer(i,j,0)   = 0.0
        blayer(i,j,np+1)=ts(i,j)*(ts(i,j)*(ts(i,j)
     *                   *(ts(i,j)*cb(5,ib)+cb(4,ib))+cb(3,ib))
     *                   +cb(2,ib))+cb(1,ib)

c-----dbs is the derivative of the surface planck flux with respect to
c     surface temperature (eq. 59).

         dbs(i,j)=ts(i,j)*(ts(i,j)*(ts(i,j)
     *           *4.*cb(5,ib)+3.*cb(4,ib))+2.*cb(3,ib))+cb(2,ib)

        enddo
       enddo

c-----compute column-integrated absorber amoumt, absorber-weighted
c     pressure and temperature for co2 (band 3) and o3 (band 5).
c     it follows eqs. (37) and (38).

c-----this is in the band loop to save storage

      if( high .and. co2bnd) then

        call column(m,n,np,pa,dt,dco2,sco3,scopre,scotem)

      endif

      if(oznbnd) then

        call column(m,n,np,pa,dt,do3,sco3,scopre,scotem)

      endif

c-----compute the exponential terms (eq. 32) at each layer for
c     water vapor line absorption when k-distribution is used

      if( .not. h2otbl) then

        call h2oexps(ib,m,n,np,dh2o,pa,dt,h2oexp)

      endif

c-----compute the exponential terms (eq. 46) at each layer for
c     water vapor continuum absorption

      if( conbnd) then

        call conexps(ib,m,n,np,dcont,conexp)
```

```
      endif


c-----compute the  exponential terms (eq. 32) at each layer for
c     co2 absorption

      if( .not.high .and. co2bnd) then

        call co2exps(m,n,np,dco2,pa,dt,co2exp)

      endif

c-----compute transmittances for regions between levels k1 and k2
c     and update the fluxes at the two levels.

      do 2000 k1=1,np

c-----initialize fclr, th2o, tcon, and tco2

        do j=1,n
         do i=1,m
          fclr(i,j)=1.0
         enddo
        enddo

c-----for h2o line absorption

      if(.not. h2otbl) then
        do ik=1,6
         do j=1,n
          do i=1,m
           th2o(i,j,ik)=1.0
          enddo
         enddo
        enddo
      endif

c-----for h2o continuum absorption

       if (conbnd) then
         do iq=1,3
          do j=1,n
           do i=1,m
            tcon(i,j,iq)=1.0
           enddo
          enddo
         enddo
       endif

c-----for co2 absorption when using k-distribution method.
c     band 3 is divided into 3 sub-bands, but sub-bands 3a and 3c
c     are combined in computing the co2 transmittance.
```

```
      if (.not. high .and. co2bnd) then
        do isb=1,2
         do ik=1,6
          do j=1,n
           do i=1,m
            tco2(i,j,ik,isb)=1.0
           enddo
          enddo
         enddo
        enddo
      endif

c-----loop over the bottom level of the region (k2)

      do 3000 k2=k1+1,np+1

          do j=1,n
           do i=1,m
            trant(i,j)=1.0
           enddo
          enddo

       if(h2otbl) then

c-----compute water vapor transmittance using table look-up

          if (ib.eq.1 ) then

           call tablup(k1,k2,m,n,np,nx,nh,nt,sh2o,swpre,swtem,
     *                 -8.0,-2.0,0.3,0.2,h11,h12,h13,trant)

          endif
          if (ib.eq.2 ) then

           call tablup(k1,k2,m,n,np,nx,nh,nt,sh2o,swpre,swtem,
     *                 -8.0,-2.0,0.3,0.2,h21,h22,h23,trant)

          endif
          if (ib.eq.7 ) then

           call tablup(k1,k2,m,n,np,nx,nh,nt,sh2o,swpre,swtem,
     *                 -8.0,-2.0,0.3,0.2,h71,h72,h73,trant)

          endif

        else

c-----compute water vapor transmittance using k-distribution.

         call wvkdis(ib,m,n,np,k2-1,h2oexp,conexp,th2o,tcon,trant)

        endif
```

```
       if(co2bnd) then

        if( high )  then

c-----compute co2 transmittance using table look-up method

          call tablup(k1,k2,m,n,np,nx,nc,nt,sco3,scopre,scotem,
     *                 -4.0,-2.0,0.3,0.2,c1,c2,c3,trant)

        else

c-----compute co2 transmittance using k-distribution method

          call co2kdis(m,n,np,k2-1,co2exp,tco2,trant)

        endif

       endif

c-----compute o3 transmittance using table look-up

       if (oznbnd) then

          call tablup(k1,k2,m,n,np,nx,no,nt,sco3,scopre,scotem,
     *                 -6.0,-2.0,0.3,0.2,o1,o2,o3,trant)

       endif

c-----fclr is the clear line-of-sight between levels k1 and k2.
c     in computing fclr, clouds are assumed randomly overlapped
c     using eq. (10).

      do j=1,n
       do i=1,m
        fclr(i,j) = fclr(i,j)*clr(i,j,k2-1)
       enddo
      enddo

c-----compute upward and downward fluxes


c-----add "boundary" terms to the net downward flux.
c     these are the first terms on the right-hand-side of
c     eqs. (56a) and (56b).
c     downward fluxes are positive.

      if (k2 .eq. k1+1) then
       do j=1,n
        do i=1,m

         flc(i,j,k1)=flc(i,j,k1)-blayer(i,j,k1)
         flc(i,j,k2)=flc(i,j,k2)+blayer(i,j,k1)
```

```
         flx(i,j,k1)=flx(i,j,k1)-blayer(i,j,k1)
         flx(i,j,k2)=flx(i,j,k2)+blayer(i,j,k1)

        enddo
       enddo
      endif

c-----add flux components involving the four layers above and below
c     the levels k1 and k2.  it follows eqs. (56a) and (56b).

      do j=1,n
       do i=1,m
        xx=trant(i,j)*(blayer(i,j,k2-1)-blayer(i,j,k2))
        flc(i,j,k1) =flc(i,j,k1)+xx
        flx(i,j,k1) =flx(i,j,k1)+xx*fclr(i,j)

        xx=trant(i,j)*(blayer(i,j,k1-1)-blayer(i,j,k1))
        flc(i,j,k2) =flc(i,j,k2)+xx
        flx(i,j,k2) =flx(i,j,k2)+xx*fclr(i,j)

       enddo
      enddo

 3000 continue

c-----compute the partial derivative of fluxes with respect to
c     surface temperature (eq. 59).

      do j=1,n
       do i=1,m
        dfdts(i,j,k1) =dfdts(i,j,k1)-dbs(i,j)*trant(i,j)*fclr(i,j)
       enddo
      enddo

 2000 continue

c-----add contribution from the surface to the flux terms at the surface.

      do j=1,n
       do i=1,m
        dfdts(i,j,np+1) =dfdts(i,j,np+1)-dbs(i,j)
       enddo
      enddo

      do j=1,n
       do i=1,m
        flx(i,j,np+1)=flx(i,j,np+1)-blayer(i,j,np+1)
        flc(i,j,np+1)=flc(i,j,np+1)-blayer(i,j,np+1)
        st4(i,j)=st4(i,j)-blayer(i,j,np+1)
       enddo
      enddo
```

```
1000 continue

     return
     end
```

7.2 Subroutine COLUMN

```
c*****************************************************************************
      subroutine column (m,n,np,pa,dt,sabs0,sabs,spre,stem)
c
c*****************************************************************************
c-----compute column-integrated (from top of the model atmosphere)
c     absorber amount (sabs), absorber-weighted pressure (spre) and
c     temperature (stem).
c     computations of spre and stem follows
c     eqs. (37) and (38) of Chou and Suarez (1994).
c
c--- input parameters
c   number of soundings in zonal direction (m)
c   number of soundings in meridional direction (n)
c   number of atmospheric layers (np)
c   layer pressure (pa)
c   layer temperature minus 250K (dt)
c   layer absorber amount (sabs0)
c
c--- output parameters
c   column-integrated absorber amount (sabs)
c   column absorber-weighted pressure (spre)
c   column absorber-weighted temperature (stem)
c
c--- units of pa and dt are mb and k, respectively.
c    units of sabs are g/cm**2 for water vapor and (cm-atm)stp for co2 and o3
c*****************************************************************************
      implicit none
      integer m,n,np,i,j,k

c---- input parameters -----

      real*8 pa(m,n,np),dt(m,n,np),sabs0(m,n,np)

c---- output parameters -----

      real*8 sabs(m,n,np+1),spre(m,n,np+1),stem(m,n,np+1)

c*************************************************************************
        do j=1,n
         do i=1,m
          sabs(i,j,1)=0.0
          spre(i,j,1)=0.0
          stem(i,j,1)=0.0
         enddo
        enddo

        do k=1,np
         do j=1,n
          do i=1,m
           sabs(i,j,k+1)=sabs(i,j,k)+sabs0(i,j,k)
```

```
    spre(i,j,k+1)=spre(i,j,k)+pa(i,j,k)*sabs0(i,j,k)
    stem(i,j,k+1)=stem(i,j,k)+dt(i,j,k)*sabs0(i,j,k)
   enddo
  enddo
 enddo

return
end
```

7.3 Subroutine TABLUP

```
c**********************************************************************
      subroutine tablup(k1,k2,m,n,np,nx,nh,nt,sabs,spre,stem,w1,p1,
     *                  dwe,dpe,coef1,coef2,coef3,tran)
c**********************************************************************
c   compute water vapor, co2, and o3 transmittances between levels k1 and k2
c   using table look-up for m x n soundings.
c
c   Calculations follow Eq. (40) of Chou and Suarez (1994)
c
c---- input ---------------------
c  indices for pressure levels (k1 and k2)
c  number of grid intervals in zonal direction (m)
c  number of grid intervals in meridional direction (n)
c  number of atmospheric layers (np)
c  number of pressure intervals in the table (nx)
c  number of absorber amount intervals in the table (nh)
c  number of tables copied (nt)
c  column-integrated absorber amount (sabs)
c  column absorber amount-weighted pressure (spre)
c  column absorber amount-weighted temperature (stem)
c  first value of absorber amount (log10) in the table (w1)
c  first value of pressure (log10) in the table (p1)
c  size of the interval of absorber amount (log10) in the table (dwe)
c  size of the interval of pressure (log10) in the table (dpe)
c  pre-computed coefficients (coef1, coef2, and coef3)
c
c---- updated ---------------------
c  transmittance (tran)
c
c  Note:
c   (1) units of sabs are g/cm**2 for water vapor and (cm-atm)stp for co2
and o3.
c   (2) units of spre and stem are, respectively, mb and K.
c   (3) there are nt identical copies of the tables (coef1, coef2, and
c       coef3).  the prupose of using the multiple copies of tables is
c       to increase the speed in parallel (vectorized) computations.
C       if such advantage does not exist, nt can be set to 1.
c
c**********************************************************************
      implicit none
      integer k1,k2,m,n,np,nx,nh,nt,i,j,k

c---- input parameters -----

      real*8 w1,p1,dwe,dpe
      real*8 sabs(m,n,np+1),spre(m,n,np+1),stem(m,n,np+1)
      real*8 coef1(nx,nh,nt),coef2(nx,nh,nt),coef3(nx,nh,nt)

c---- update parameter -----
```

```
      real*8 tran(m,n)

c---- temporary variables -----

      real*8 x1,x2,x3,we,pe,fw,fp,fm,pa,pb,pc,ax,ba,bb,t1,ca,cb,t2
      integer iw,ip,nn

c**********************************************************************

      do j=1,n
       do i=1,m

        nn=mod(i,nt)+1

        x1=sabs(i,j,k2)-sabs(i,j,k1)
        x2=(spre(i,j,k2)-spre(i,j,k1))/x1
        x3=(stem(i,j,k2)-stem(i,j,k1))/x1

        we=(log10(x1)-w1)/dwe
        pe=(log10(x2)-p1)/dpe

        we=max(we,w1-2.*dwe)
        pe=max(pe,p1)

        iw=int(we+1.5)
        ip=int(pe+1.5)

        iw=min(iw,nh-1)
        iw=max(iw, 2)

        ip=min(ip,nx-1)
        ip=max(ip, 1)

        fw=we-float(iw-1)
        fp=pe-float(ip-1)

c-----linear interpolation in pressure

        pa = coef1(ip,iw-1,nn)*(1.-fp)+coef1(ip+1,iw-1,nn)*fp
        pb = coef1(ip,iw,  nn)*(1.-fp)+coef1(ip+1,iw,  nn)*fp
        pc = coef1(ip,iw+1,nn)*(1.-fp)+coef1(ip+1,iw+1,nn)*fp

c-----quadratic interpolation in absorber amount

        ax = (-pa*(1.-fw)+pc*(1.+fw)) *fw*0.5 + pb*(1.-fw*fw)

c-----linear interpolation in temperature

        ba = coef2(ip,iw,  nn)*(1.-fp)+coef2(ip+1,iw,  nn)*fp
        bb = coef2(ip,iw+1,nn)*(1.-fp)+coef2(ip+1,iw+1,nn)*fp
        t1 = ba*(1.-fw) + bb*fw

        ca = coef3(ip,iw,  nn)*(1.-fp)+coef3(ip+1,iw,  nn)*fp
```

```
          cb = coef3(ip,iw+1,nn)*(1.-fp)+coef3(ip+1,iw+1,nn)*fp
          t2 = ca*(1.-fw) + cb*fw

c-----update the total transmittance between levels k1 and k2

          tran(i,j)= (ax + (t1+t2*x3) * x3)*tran(i,j)

         enddo
        enddo

        return
        end
```

7.4 Subroutine WVKDIS

```
c***********************************************************************
      subroutine wvkdis(ib,m,n,np,k,h2oexp,conexp,th2o,tcon,tran)
c***********************************************************************
c   compute water vapor transmittance between levels k1 and k2 for
c   m x n soundings using the k-distribution method.
c
c   computations follow eqs. (34), (46), (50)
c   and (52) of Chou and Suarez (1994).
c
c---- input parameters
c  spectral band (ib)
c  number of grid intervals in zonal direction (m)
c  number of grid intervals in meridional direction (n)
c  number of levels (np)
c  current level (k)
c  exponentials for line absorption (h2oexp)
c  exponentials for continuum absorption (conexp)
c
c---- updated parameters
c  transmittance between levels k1 and k2 due to
c    water vapor line absorption (th2o)
c  transmittance between levels k1 and k2 due to
c    water vapor continuum absorption (tcon)
c  total transmittance (tran)
c
c***********************************************************************
      implicit none
      integer ib,m,n,np,k,i,j

c---- input parameters ------

      real*8 conexp(m,n,np,3),h2oexp(m,n,np,6)

c---- updated parameters -----

      real*8 th2o(m,n,6),tcon(m,n,3),tran(m,n)

c---- static data -----

      integer ne(8)
      real*8  fkw(6,8),gkw(6,3)

c---- temporary arrays -----

      real*8 trnth2o

c-----fkw is the planck-weighted k-distribution function due to h2o
c     line absorption given in table 4 of Chou and Suarez (1995).
c     the k-distribution function for the third band, fkw(*,3), is not used
```

```
      data fkw / 0.2747,0.2717,0.2752,0.1177,0.0352,0.0255,
     2           0.1521,0.3974,0.1778,0.1826,0.0374,0.0527,
     3           6*1.00,
     4           0.4654,0.2991,0.1343,0.0646,0.0226,0.0140,
     5           0.5543,0.2723,0.1131,0.0443,0.0160,0.0000,
     6           0.1846,0.2732,0.2353,0.1613,0.1146,0.0310,
     7           0.0740,0.1636,0.4174,0.1783,0.1101,0.0566,
     8           0.1437,0.2197,0.3185,0.2351,0.0647,0.0183/

c-----gkw is the planck-weighted k-distribution function due to h2o
c     line absorption in the 3 subbands (800-720,620-720,540-620 /cm)
c     of band 3 given in table 7.  Note that the order of the sub-bands
c     is reversed.

      data gkw/  0.1782,0.0593,0.0215,0.0068,0.0022,0.0000,
     2           0.0923,0.1675,0.0923,0.0187,0.0178,0.0000,
     3           0.0000,0.1083,0.1581,0.0455,0.0274,0.0041/



c-----ne is the number of terms used in each band to compute water vapor
c     continuum transmittance (table 6).

      data ne /0,0,3,1,1,1,0,0/


c-----tco2 are the six exp factors between levels k1 and k2
c     tran is the updated total transmittance between levels k1 and k2


c-----th2o is the 6 exp factors between levels k1 and k2 due to
c     h2o line absorption.

c-----tcon is the 3 exp factors between levels k1 and k2 due to
c     h2o continuum absorption.

c-----trnth2o is the total transmittance between levels k1 and k2 due
c     to both line and continuum absorption computed from eq. (52).

          do j=1,n
           do i=1,m
            th2o(i,j,1) = th2o(i,j,1)*h2oexp(i,j,k,1)
            th2o(i,j,2) = th2o(i,j,2)*h2oexp(i,j,k,2)
            th2o(i,j,3) = th2o(i,j,3)*h2oexp(i,j,k,3)
            th2o(i,j,4) = th2o(i,j,4)*h2oexp(i,j,k,4)
            th2o(i,j,5) = th2o(i,j,5)*h2oexp(i,j,k,5)
            th2o(i,j,6) = th2o(i,j,6)*h2oexp(i,j,k,6)
           enddo
          enddo


       if (ne(ib).eq.0) then
```

```
         do j=1,n
          do i=1,m

           trnth2o      =(fkw(1,ib)*th2o(i,j,1)
     *                  + fkw(2,ib)*th2o(i,j,2)
     *                  + fkw(3,ib)*th2o(i,j,3)
     *                  + fkw(4,ib)*th2o(i,j,4)
     *                  + fkw(5,ib)*th2o(i,j,5)
     *                  + fkw(6,ib)*th2o(i,j,6))

          tran(i,j)=tran(i,j)*trnth2o

          enddo
         enddo

      elseif (ne(ib).eq.1) then


         do j=1,n
          do i=1,m

           tcon(i,j,1)= tcon(i,j,1)*conexp(i,j,k,1)

           trnth2o      =(fkw(1,ib)*th2o(i,j,1)
     *                  + fkw(2,ib)*th2o(i,j,2)
     *                  + fkw(3,ib)*th2o(i,j,3)
     *                  + fkw(4,ib)*th2o(i,j,4)
     *                  + fkw(5,ib)*th2o(i,j,5)
     *                  + fkw(6,ib)*th2o(i,j,6))*tcon(i,j,1)

          tran(i,j)=tran(i,j)*trnth2o

          enddo
         enddo

      else

         do j=1,n
          do i=1,m

           tcon(i,j,1)= tcon(i,j,1)*conexp(i,j,k,1)
           tcon(i,j,2)= tcon(i,j,2)*conexp(i,j,k,2)
           tcon(i,j,3)= tcon(i,j,3)*conexp(i,j,k,3)

           trnth2o      = (  gkw(1,1)*th2o(i,j,1)
     *                     + gkw(2,1)*th2o(i,j,2)
     *                     + gkw(3,1)*th2o(i,j,3)
     *                     + gkw(4,1)*th2o(i,j,4)
     *                     + gkw(5,1)*th2o(i,j,5)
     *                     + gkw(6,1)*th2o(i,j,6) ) * tcon(i,j,1)
     *                  + (  gkw(1,2)*th2o(i,j,1)
     *                     + gkw(2,2)*th2o(i,j,2)
```

```
     *                     + gkw(3,2)*th2o(i,j,3)
     *                     + gkw(4,2)*th2o(i,j,4)
     *                     + gkw(5,2)*th2o(i,j,5)
     *                     + gkw(6,2)*th2o(i,j,6) ) * tcon(i,j,2)
     *                  + (  gkw(1,3)*th2o(i,j,1)
     *                     + gkw(2,3)*th2o(i,j,2)
     *                     + gkw(3,3)*th2o(i,j,3)
     *                     + gkw(4,3)*th2o(i,j,4)
     *                     + gkw(5,3)*th2o(i,j,5)
     *                     + gkw(6,3)*th2o(i,j,6) ) * tcon(i,j,3)

          tran(i,j)=tran(i,j)*trnth2o

          enddo
         enddo

      endif



      return
      end
```

7.5 Subroutine CO2KDIS

```
c**********************************************************************
      subroutine co2kdis(m,n,np,k,co2exp,tco2,tran)
c**********************************************************************
c   compute co2 transmittances between levels k1 and k2 for m x n soundings
c   using the k-distribution method with linear pressure scaling.
c
c   computations follow eq. (34) of Chou and Suarez (1994).
c
c---- input parameters
c   number of grid intervals in zonal direction (m)
c   number of grid intervals in meridional direction (n)
c
c---- updated parameters
c   transmittance between levels k1 and k2 due to co2 absorption
c     for the various values of the absorption coefficient (tco2)
c   total transmittance (tran)
c
c**********************************************************************
      implicit none
      integer m,n,np,k,i,j

c---- input parameters -----

      real*8 co2exp(m,n,np,6,2)

c---- updated parameters -----

      real*8 tco2(m,n,6,2),tran(m,n)

c---- static data -----

      real*8 gkc(6,2)

c---- temporary arrays -----

      real*8 xc

c-----gkc is the planck-weighted co2 k-distribution function
c     in the band-wing and band-center regions given in table 7.
c     for computing efficiency, sub-bands 3a and 3c are combined.

      data gkc/  0.1395,0.1407,0.1549,0.1357,0.0182,0.0220,
     2           0.0766,0.1372,0.1189,0.0335,0.0169,0.0059/

c-----tco2 is the 6 exp factors between levels k1 and k2.
c     xc is the total co2 transmittance given by eq. (53).

         do j=1,n
          do i=1,m
```

```
c-----band-wings

            tco2(i,j,1,1)=tco2(i,j,1,1)*co2exp(i,j,k,1,1)
            xc=              gkc(1,1)*tco2(i,j,1,1)

            tco2(i,j,2,1)=tco2(i,j,2,1)*co2exp(i,j,k,2,1)
            xc=xc+gkc(2,1)*tco2(i,j,2,1)

            tco2(i,j,3,1)=tco2(i,j,3,1)*co2exp(i,j,k,3,1)
            xc=xc+gkc(3,1)*tco2(i,j,3,1)

            tco2(i,j,4,1)=tco2(i,j,4,1)*co2exp(i,j,k,4,1)
            xc=xc+gkc(4,1)*tco2(i,j,4,1)

            tco2(i,j,5,1)=tco2(i,j,5,1)*co2exp(i,j,k,5,1)
            xc=xc+gkc(5,1)*tco2(i,j,5,1)

            tco2(i,j,6,1)=tco2(i,j,6,1)*co2exp(i,j,k,6,1)
            xc=xc+gkc(6,1)*tco2(i,j,6,1)

c-----band-center region

            tco2(i,j,1,2)=tco2(i,j,1,2)*co2exp(i,j,k,1,2)
            xc=xc+gkc(1,2)*tco2(i,j,1,2)

            tco2(i,j,2,2)=tco2(i,j,2,2)*co2exp(i,j,k,2,2)
            xc=xc+gkc(2,2)*tco2(i,j,2,2)

            tco2(i,j,3,2)=tco2(i,j,3,2)*co2exp(i,j,k,3,2)
            xc=xc+gkc(3,2)*tco2(i,j,3,2)

            tco2(i,j,4,2)=tco2(i,j,4,2)*co2exp(i,j,k,4,2)
            xc=xc+gkc(4,2)*tco2(i,j,4,2)

            tco2(i,j,5,2)=tco2(i,j,5,2)*co2exp(i,j,k,5,2)
            xc=xc+gkc(5,2)*tco2(i,j,5,2)

            tco2(i,j,6,2)=tco2(i,j,6,2)*co2exp(i,j,k,6,2)
            xc=xc+gkc(6,2)*tco2(i,j,6,2)

            tran(i,j)=tran(i,j)*xc

          enddo
        enddo

      return
      end
```

7.6 Subroutine H2OEXPS

```
c**********************************************************************
      subroutine h2oexps(ib,m,n,np,dh2o,pa,dt,h2oexp)
c**********************************************************************
c   compute exponentials for water vapor line absorption
c   in individual layers.
c
c---- input parameters
c  spectral band (ib)
c  number of grid intervals in zonal direction (m)
c  number of grid intervals in meridional direction (n)
c  number of layers (np)
c  layer water vapor amount for line absorption (dh2o)
c  layer pressure (pa)
c  layer temperature minus 250K (dt)
c
c---- output parameters
c  6 exponentials for each layer  (h2oexp)
c
c**********************************************************************
      implicit none
      integer ib,m,n,np,i,j,k,ik

c---- input parameters ------

      real*8 dh2o(m,n,np),pa(m,n,np),dt(m,n,np)

c---- output parameters -----

      real*8 h2oexp(m,n,np,6)

c---- static data -----

      integer mw(8)
      real*8 xkw(8),aw(8),bw(8)

c---- temporary arrays -----

      real*8 xh


c-----xkw  are the absorption coefficients for the first
c     k-distribution function due to water vapor line absorption
c     (tables 4 and 7).  units are cm**2/g

      data xkw / 29.55  , 4.167e-1, 1.328e-2, 5.250e-4,
     *           5.25e-4, 2.340e-3, 1.320e-0, 5.250e-4/

c-----mw are the ratios between neighboring absorption coefficients
c     for water vapor line absorption (tables 4 and 7).
```

```
      data mw /6,6,8,6,6,8,6,16/

c-----aw and bw (table 3) are the coefficients for temperature scaling
c     in eq. (25).

      data aw/ 0.0021, 0.0140, 0.0167, 0.0302,
     *         0.0307, 0.0154, 0.0008, 0.0096/
      data bw/ -1.01e-5, 5.57e-5, 8.54e-5, 2.96e-4,
     *          2.86e-4, 7.53e-5,-3.52e-6, 1.64e-5/

c**********************************************************************
c    note that the 3 sub-bands in band 3 use the same set of xkw, aw,
c    and bw.  therefore, h2oexp for these sub-bands are identical.
c**********************************************************************


        do k=1,np
         do j=1,n
          do i=1,m

c-----xh is   the scaled water vapor amount for line absorption
c     computed from (27).

           xh = dh2o(i,j,k)*(pa(i,j,k)*0.002)
     1        * ( 1.+(aw(ib)+bw(ib)* dt(i,j,k))*dt(i,j,k) )

c-----h2oexp is the water vapor transmittance of the layer (k2-1)
c     due to line absorption

           h2oexp(i,j,k,1) = exp(-xh*xkw(ib))

          enddo
         enddo
        enddo

        do ik=2,6

         if(mw(ib).eq.6) then

          do k=1,np
           do j=1,n
            do i=1,m
             xh = h2oexp(i,j,k,ik-1)*h2oexp(i,j,k,ik-1)
             h2oexp(i,j,k,ik) = xh*xh*xh
            enddo
           enddo
          enddo

         elseif(mw(ib).eq.8) then

          do k=1,np
           do j=1,n
            do i=1,m
```

```
       xh = h2oexp(i,j,k,ik-1)*h2oexp(i,j,k,ik-1)
       xh = xh*xh
       h2oexp(i,j,k,ik) = xh*xh
      enddo
     enddo
    enddo

  else

    do k=1,np
     do j=1,n
      do i=1,m
       xh = h2oexp(i,j,k,ik-1)*h2oexp(i,j,k,ik-1)
       xh = xh*xh
       xh = xh*xh
       h2oexp(i,j,k,ik) = xh*xh
      enddo
     enddo
    enddo

  endif
 enddo


return
end
```

7.7 Subroutine CONEXPS

```
c***********************************************************************
      subroutine conexps(ib,m,n,np,dcont,conexp)
c***********************************************************************
c   compute exponentials for continuum absorption in individual layers.
c
c---- input parameters
c  spectral band (ib)
c  number of grid intervals in zonal direction (m)
c  number of grid intervals in meridional direction (n)
c  number of layers (np)
c  layer scaled water vapor amount for continuum absorption (dcont)
c
c---- output parameters
c  1 or 3 exponentials for each layer (conexp)
c
c***********************************************************************
      implicit none
      integer ib,m,n,np,i,j,k,iq

c---- input parameters ------

      real*8 dcont(m,n,np)

c---- updated parameters -----

      real*8 conexp(m,n,np,3)

c---- static data -----

      integer ne(8)
      real*8 xke(8)


c-----xke are the absorption coefficients for the first
c     k-distribution function due to water vapor continuum absorption
c     (table 6).  units are cm**2/g

      data xke /  0.00,    0.00,   27.40,   15.8,
     *            9.40,    7.75,     0.0,    0.0/


c-----ne is the number of terms in computing water vapor
c     continuum transmittance (Table 6).
c     band 3 is divided into 3 sub-bands.

      data ne /0,0,3,1,1,1,0,0/


c***********************************************************************
```

```
          do k=1,np
           do j=1,n
            do i=1,m
             conexp(i,j,k,1) = exp(-dcont(i,j,k)*xke(ib))
            enddo
           enddo
          enddo

         if (ib .eq. 3) then

c-----the absorption coefficients for sub-bands 3b (iq=2) and 3a (iq=3)
c     are, respectively, double and quadruple that for sub-band 3c (iq=1)
c     (table 6).

          do iq=2,3
           do k=1,np
            do j=1,n
             do i=1,m
              conexp(i,j,k,iq) = conexp(i,j,k,iq-1) *conexp(i,j,k,iq-1)
             enddo
            enddo
           enddo
          enddo

        endif

      return
      end
```

7.8 Subroutine CO2EXPS

```
c**********************************************************************
      subroutine co2exps(m,n,np,dco2,pa,dt,co2exp)
c
c**********************************************************************
c   compute co2 exponentials for individual layers.
c
c---- input parameters
c  number of grid intervals in zonal direction (m)
c  number of grid intervals in meridional direction (n)
c  number of layers (np)
c  layer co2 amount (dco2)
c  layer pressure (pa)
c  layer temperature minus 250K (dt)
c
c---- output parameters
c  6 exponentials for each layer (co2exp)
c**********************************************************************
      implicit none
      integer m,n,np,i,j,k

c---- input parameters -----

      real*8 dco2(m,n,np),pa(m,n,np),dt(m,n,np)

c---- output parameters -----

      real*8 co2exp(m,n,np,6,2)

c---- static data -----

      real*8 xkc(2),ac(2),bc(2),pm(2),prc(2)

c---- temporary arrays -----

      real*8 xc

c-----xkc is the absorption coefficients for the
c     first k-distribution function due to co2 (table 7).
c     units are 1/(cm-atm)stp.

      data xkc/2.656e-5,2.656e-3/

c-----parameters (table 3) for computing the scaled co2 amount
c     using (27).

      data prc/  300.0,   30.0/
      data pm /    0.5,   0.85/
      data ac / 0.0182, 0.0042/
      data bc /1.07e-4,2.00e-5/
```

```
c***********************************************************************

        do k=1,np
         do j=1,n
          do i=1,m

c-----compute the scaled co2 amount from eq. (27) for band-wings
c     (sub-bands 3a and 3c).

           xc = dco2(i,j,k)*(pa(i,j,k)/prc(1))**pm(1)
     1             *(1.+(ac(1)+bc(1)*dt(i,j,k))*dt(i,j,k))

c-----six exponential by powers of 8 (table 7).

           co2exp(i,j,k,1,1)=exp(-xc*xkc(1))

           xc=co2exp(i,j,k,1,1)*co2exp(i,j,k,1,1)
           xc=xc*xc
           co2exp(i,j,k,2,1)=xc*xc

           xc=co2exp(i,j,k,2,1)*co2exp(i,j,k,2,1)
           xc=xc*xc
           co2exp(i,j,k,3,1)=xc*xc

           xc=co2exp(i,j,k,3,1)*co2exp(i,j,k,3,1)
           xc=xc*xc
           co2exp(i,j,k,4,1)=xc*xc

           xc=co2exp(i,j,k,4,1)*co2exp(i,j,k,4,1)
           xc=xc*xc
           co2exp(i,j,k,5,1)=xc*xc

           xc=co2exp(i,j,k,5,1)*co2exp(i,j,k,5,1)
           xc=xc*xc
           co2exp(i,j,k,6,1)=xc*xc

c-----compute the scaled co2 amount from eq. (27) for band-center
c     region (sub-band 3b).

           xc = dco2(i,j,k)*(pa(i,j,k)/prc(2))**pm(2)
     1             *(1.+(ac(2)+bc(2)*dt(i,j,k))*dt(i,j,k))

           co2exp(i,j,k,1,2)=exp(-xc*xkc(2))

           xc=co2exp(i,j,k,1,2)*co2exp(i,j,k,1,2)
           xc=xc*xc
           co2exp(i,j,k,2,2)=xc*xc

           xc=co2exp(i,j,k,2,2)*co2exp(i,j,k,2,2)
           xc=xc*xc
           co2exp(i,j,k,3,2)=xc*xc

           xc=co2exp(i,j,k,3,2)*co2exp(i,j,k,3,2)
```

```
        xc=xc*xc
        co2exp(i,j,k,4,2)=xc*xc

        xc=co2exp(i,j,k,4,2)*co2exp(i,j,k,4,2)
        xc=xc*xc
        co2exp(i,j,k,5,2)=xc*xc

        xc=co2exp(i,j,k,5,2)*co2exp(i,j,k,5,2)
        xc=xc*xc
        co2exp(i,j,k,6,2)=xc*xc

       enddo
      enddo
     enddo

    return
    end
```

7.9 Sample Program

```
c*************** source program for CLIRAD IR: October 1994  ************

      implicit none
      integer m,n,ndim,np,i,j,k
      parameter (m=2,n=2,ndim=250,np=75)

c-----input parameters

      real*8 pl(m,ndim,np+1),ta(m,ndim,np),wa(m,ndim,np),
     * oa(m,ndim,np),ts(m,ndim),taucl(m,ndim,np),ccld(m,ndim,np)
      real*8 co2

c-----output parameters

      real*8 flx(m,ndim,np+1),flc(m,ndim,np+1),dfdts(m,ndim,np+1)
      real*8 coolr(m,ndim,np),st4(m,ndim)

      real*8 xx

      logical high
      data high/.true./
c     data high/.false./

c-----mid-latitude summer profiles

      data (pl(1,1,i),i=1,np+1)/.00,  .0006244,.0008759,  .001229,
     1 .001723,  .002417,  .003391,  .004757,  .006672,  .009359,
     2  .01313,   .01842,   .02583,   .03623,   .05082,   .07129,
     3    0.10,     0.14,     0.20,     0.28,     0.39,     0.54,
     4    0.76,     1.07,     1.50,     2.10,     2.95,     4.14,
     5    5.80,     8.14,    11.42,    16.01,    22.46,    31.51,
     6   44.20,    62.00,    85.78,   109.55,   133.32,   157.10,
     7  180.88,   204.65,   228.43,   252.20,   275.98,   299.75,
     8  323.52,   347.30,   371.08,   394.85,   418.63,   442.40,
     9  466.17,   489.95,   513.72,   537.50,   561.28,   585.05,
     a  608.83,   632.60,   656.38,   680.15,   703.92,   727.70,
     b  751.47,   775.25,   799.03,   822.80,   846.58,   870.35,
     c  894.13,   917.90,   941.67,   965.45,   989.22,  1013.00/
c
      data (ta(1,1,i),i=1,np)/209.86,   210.20,   210.73,
     1    211.27,   211.81,   212.35,   212.89,   213.44,   213.98,
     2    214.53,   215.08,   215.62,   216.17,   216.74,   218.11,
     3    223.20,   230.04,   237.14,   244.46,   252.00,   259.76,
     4    267.70,   274.93,   274.60,   269.38,   262.94,   256.45,
     5    250.12,   244.31,   238.96,   233.74,   228.69,   224.59,
     6    221.75,   219.10,   216.64,   215.76,   215.75,   215.78,
     7    216.22,   219.15,   223.79,   228.29,   232.45,   236.33,
     8    239.92,   243.32,   246.53,   249.56,   252.43,   255.14,
     9    257.69,   260.11,   262.39,   264.57,   266.66,   268.67,
     a    270.60,   272.48,   274.29,   276.05,   277.75,   279.41,
```

```
     b    281.02,    282.59,    284.09,    285.53,    286.86,    288.06,
     c    289.13,    290.11,    291.03,    291.91,    292.76,    293.59/
c
      data (wa(1,1,i),i=1,np)/                              15*0.400E-05,
     1   0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,
     2   0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,
     3   0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,
     4   0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.400E-05,0.406E-05,
     5   0.520E-05,0.115E-04,0.275E-04,0.572E-04,0.107E-03,0.166E-03,
     6   0.223E-03,0.285E-03,0.360E-03,0.446E-03,0.547E-03,0.655E-03,
     7   0.767E-03,0.890E-03,0.103E-02,0.118E-02,0.136E-02,0.159E-02,
     8   0.190E-02,0.225E-02,0.264E-02,0.306E-02,0.351E-02,0.399E-02,
     9   0.450E-02,0.504E-02,0.560E-02,0.619E-02,0.680E-02,0.742E-02,
     a   0.805E-02,0.869E-02,0.935E-02,0.100E-01,0.107E-01,0.113E-01/

c
      data (oa(1,1,i),i=1,np)/
     1  0.6427E-07,  0.2022E-06,  0.2458E-06,  0.2896E-06,  0.3337E-06,
     2  0.3779E-06,  0.4224E-06,  0.4671E-06,  0.5120E-06,  0.5572E-06,
     3  0.6026E-06,  0.6482E-06,  0.6940E-06,  0.7401E-06,  0.7934E-06,
     4  0.1009E-05,  0.1313E-05,  0.1635E-05,  0.1976E-05,  0.2336E-05,
     5  0.2716E-05,  0.3117E-05,  0.3590E-05,  0.4645E-05,  0.5897E-05,
     6  0.7649E-05,  0.9102E-05,  0.9598E-05,  0.9944E-05,  0.1008E-04,
     7  0.9900E-05,  0.8525E-05,  0.7098E-05,  0.5761E-05,  0.4231E-05,
     8  0.2604E-05,  0.1523E-05,  0.1025E-05,  0.7859E-06,  0.5983E-06,
     9  0.4483E-06,  0.3518E-06,  0.3021E-06,  0.2516E-06,  0.2117E-06,
     a  0.1932E-06,  0.1761E-06,  0.1599E-06,  0.1466E-06,  0.1366E-06,
     b  0.1270E-06,  0.1180E-06,  0.1094E-06,  0.1028E-06,  0.9748E-07,
     c  0.9238E-07,  0.8828E-07,  0.8458E-07,  0.8098E-07,  0.7782E-07,
     d  0.7490E-07,  0.7205E-07,  0.6939E-07,  0.6706E-07,  0.6480E-07,
     e  0.6258E-07,  0.6069E-07,  0.5928E-07,  0.5789E-07,  0.5651E-07,
     f  0.5517E-07,  0.5398E-07,  0.5281E-07,  0.5167E-07,  0.5054E-07/

        ts(1,1)=294.0

c-----m x n is the number of atmospheres

       do j=1,n
        do i=1,m
         ts(i,j)=ts(1,1)
        enddo
       enddo

c-----np is the number of atmospheric layers

       do k=1,np
        do j=1,n
         do i=1,m
          pl(i,j,k)=pl(1,1,k)
          ta(i,j,k)=ta(1,1,k)
          wa(i,j,k)=wa(1,1,k)
          oa(i,j,k)=oa(1,1,k)
         enddo
```

```
         enddo
        enddo

c-----np+1 is the index for the surface level

         do j=1,n
          do i=1,m
           pl(i,j,np+1)=pl(1,1,np+1)
          enddo
         enddo

c-----assign co2 amount. units are parts/part by volumn

        co2=300.e-6

c-----specify cloud cover and optical thickness

        do k=1,np
         do j=1,n
          do i=1,m
           ccld(i,j,k)=0.0
           taucl(i,j,k)=0.0
          enddo
         enddo
        enddo

          do k=46,49
           do j=1,n
            do i=1,m
            ccld(i,j,k)=0.5
            taucl(i,j,k)=2.5
           enddo
          enddo
         enddo

c-----compute ir fluxes due to water vapor, co2, and o3

         call irrad (m,n,ndim,np,taucl,ccld,pl,ta,wa,
     *              oa,co2,ts,high,flx,flc,dfdts,st4)

c-----compute cooling rate profile

        do k=1,np
         do j=1,n
          do i=1,m
           coolr(i,j,k)=(flx(i,j,k+1)-flx(i,j,k))*8.441874/
     *              (pl(i,j,k+1)-pl(i,j,k))
          enddo
         enddo
        enddo

c-----print out input data
```

```
      write (7,100)
      write (7,102) ts(1,1),co2
      write (7,105)
      write (7,108)
      write (7,110)

     do j=1,1
      do i=1,1
       do k=1,np
        write (7,115) k,pl(i,j,k),  flx(i,j,k),flc(i,j,k),dfdts(i,j,k)
        write (7,116) ta(i,j,k),wa(i,j,k),oa(i,j,k),ccld(i,j,k),
   *                 taucl(i,j,k),       coolr(i,j,k)
       enddo
      enddo
     enddo

       i=1
       j=1
       k=np+1
       write (7,115) k,pl(i,j,k),  flx(i,j,k),flc(i,j,k),dfdts(i,j,k)
       write (7,117) st4(i,j)


100 format (/,'  ****  Sample mid-latitude summer atmosphere ****',/)
102 format (/,'  ts=', f8.2,' K',/,'  co2=',e9.3,' ppmv',/)
105 format (/,' ******  INPUT DATA  ******',28x,
   *          '******  RESULTS  ******',/)
108 format (10x,'p',6x,'T',7x,'q',8x,'o3',6x,'ccld',2x,'taucl',
   *         3x,'flx',6x,'flc',4x,'dfdts',2x,'cooling rate')
110 format (8x,'(mb)',4x,'(k)',4x,'(g/g)',5x,'(g/g)',16x,'(W/m^2)',
   *        1x,'(W/m^2)'1x,'(W/m^2/K)',1x,'(C/day)',/)
115 format (i4,f10.4,  40x,f7.2,2x,f7.2,1x,f6.2)
116 format (14x,f7.2,2e10.3,f5.2,f6.2, 25xf8.2)
117 format (/,2x,'st4=',f8.2, ' W/m**2')

      stop
      end
```

7.10 Verification Output of Sample Program

```
 ****  Sample mid-latitude summer atmosphere ****


 ts=  294.00 K
 co2=0.300E-03 ppmv


******  INPUT DATA  ******                                ******  RESULTS  ******

         p       T       q          o3      ccld   taucl   flx      flc    dfdts  cooling rate
        (mb)    (k)    (g/g)       (g/g)                 (W/m^2) (W/m^2) (W/m^2/K) (C/day)

  1   0.0000                                            -191.22  -293.07  -0.08
              209.86 0.400E-05 0.643E-07 0.00  0.00                                91.63
  2   0.0006                                            -191.21  -293.07  -0.08
              210.20 0.400E-05 0.202E-06 0.00  0.00                                24.69
  3   0.0009                                            -191.21  -293.07  -0.08
              210.73 0.400E-05 0.246E-06 0.00  0.00                                32.02
  4   0.0012                                            -191.21  -293.06  -0.08
              211.27 0.400E-05 0.290E-06 0.00  0.00                                22.13
  5   0.0017                                            -191.21  -293.06  -0.08
              211.81 0.400E-05 0.334E-06 0.00  0.00                                22.57
  6   0.0024                                            -191.20  -293.06  -0.08
              212.35 0.400E-05 0.378E-06 0.00  0.00                                15.31
  7   0.0034                                            -191.20  -293.06  -0.08
              212.89 0.400E-05 0.422E-06 0.00  0.00                                14.91
  8   0.0048                                            -191.20  -293.06  -0.08
              213.44 0.400E-05 0.467E-06 0.00  0.00                                 9.93
  9   0.0067                                            -191.20  -293.05  -0.08
              213.98 0.400E-05 0.512E-06 0.00  0.00                                 7.08
 10   0.0094                                            -191.19  -293.05  -0.08
              214.53 0.400E-05 0.557E-06 0.00  0.00                                 7.29
 11   0.0131                                            -191.19  -293.05  -0.08
              215.08 0.400E-05 0.603E-06 0.00  0.00                                 5.53
 12   0.0184                                            -191.19  -293.05  -0.08
              215.62 0.400E-05 0.648E-06 0.00  0.00                                 4.28
 13   0.0258                                            -191.18  -293.04  -0.08
              216.17 0.400E-05 0.694E-06 0.00  0.00                                 3.13
 14   0.0362                                            -191.18  -293.04  -0.08
              216.74 0.400E-05 0.740E-06 0.00  0.00                                 1.93
 15   0.0508                                            -191.18  -293.03  -0.08
              218.11 0.400E-05 0.793E-06 0.00  0.00                                 0.28
 16   0.0713                                            -191.18  -293.03  -0.08
              223.20 0.400E-05 0.101E-05 0.00  0.00                                 1.50
 17   0.1000                                            -191.17  -293.03  -0.08
              230.04 0.400E-05 0.131E-05 0.00  0.00                                 2.19
 18   0.1400                                            -191.16  -293.02  -0.08
              237.14 0.400E-05 0.164E-05 0.00  0.00                                 3.12
 19   0.2000                                            -191.14  -293.00  -0.08
              244.46 0.400E-05 0.198E-05 0.00  0.00                                 3.88
 20   0.2800                                            -191.10  -292.96  -0.08
              252.00 0.400E-05 0.234E-05 0.00  0.00                                 5.24
 21   0.3900                                            -191.03  -292.89  -0.08
              259.76 0.400E-05 0.272E-05 0.00  0.00                                 7.18
 22   0.5400                                            -190.91  -292.77  -0.08
              267.70 0.400E-05 0.312E-05 0.00  0.00                                 8.92
```

```
23    0.7600                                                  -190.67  -292.54  -0.08
              274.93 0.400E-05 0.359E-05 0.00  0.00                                      12.15
24    1.0700                                                  -190.23  -292.10  -0.08
              274.60 0.400E-05 0.465E-05 0.00  0.00                                      11.64
25    1.5000                                                  -189.63  -291.51  -0.08
              269.38 0.400E-05 0.590E-05 0.00  0.00                                       9.29
26    2.1000                                                  -188.97  -290.87  -0.08
              262.94 0.400E-05 0.765E-05 0.00  0.00                                       7.26
27    2.9500                                                  -188.24  -290.16  -0.08
              256.45 0.400E-05 0.910E-05 0.00  0.00                                       5.72
28    4.1400                                                  -187.44  -289.39  -0.08
              250.12 0.400E-05 0.960E-05 0.00  0.00                                       4.85
29    5.8000                                                  -186.48  -288.48  -0.08
              244.31 0.400E-05 0.994E-05 0.00  0.00                                       4.03
30    8.1400                                                  -185.37  -287.44  -0.08
              238.96 0.400E-05 0.101E-04 0.00  0.00                                       3.19
31   11.4200                                                  -184.13  -286.30  -0.08
              233.74 0.400E-05 0.990E-05 0.00  0.00                                       2.56
32   16.0100                                                  -182.73  -285.04  -0.08
              228.69 0.400E-05 0.853E-05 0.00  0.00                                       2.24
33   22.4600                                                  -181.02  -283.52  -0.08
              224.59 0.400E-05 0.710E-05 0.00  0.00                                       1.51
34   31.5100                                                  -179.40  -282.17  -0.08
              221.75 0.400E-05 0.576E-05 0.00  0.00                                       1.27
35   44.2000                                                  -177.50  -280.66  -0.08
              219.10 0.400E-05 0.423E-05 0.00  0.00                                       1.00
36   62.0000                                                  -175.40  -279.08  -0.08
              216.64 0.400E-05 0.260E-05 0.00  0.00                                       0.58
37   85.7800                                                  -173.75  -278.09  -0.08
              215.76 0.400E-05 0.152E-05 0.00  0.00                                       0.43
38  109.5500                                                  -172.54  -277.40  -0.08
              215.75 0.400E-05 0.102E-05 0.00  0.00                                       0.39
39  133.3200                                                  -171.45  -276.75  -0.08
              215.78 0.406E-05 0.786E-06 0.00  0.00                                       0.28
40  157.1000                                                  -170.66  -276.39  -0.08
              216.22 0.520E-05 0.598E-06 0.00  0.00                                       0.10
41  180.8800                                                  -170.38  -276.51  -0.09
              219.15 0.115E-04 0.448E-06 0.00  0.00                                       0.48
42  204.6500                                                  -169.02  -275.53  -0.09
              223.79 0.275E-04 0.352E-06 0.00  0.00                                       1.16
43  228.4300                                                  -165.76  -272.66  -0.09
              228.29 0.572E-04 0.302E-06 0.00  0.00                                       1.80
44  252.2000                                                  -160.69  -268.06  -0.09
              232.45 0.107E-03 0.252E-06 0.00  0.00                                       2.18
45  275.9800                                                  -154.55  -262.55  -0.09
              236.33 0.166E-03 0.212E-06 0.00  0.00                                       2.32
46  299.7500                                                  -148.02  -256.92  -0.09
              239.92 0.223E-03 0.193E-06 0.50  2.50                                      19.25
47  323.5200                                                   -93.83  -251.64  -0.17
              243.32 0.285E-03 0.176E-06 0.50  2.50                                       6.42
48  347.3000                                                   -75.74  -246.54  -0.34
              246.53 0.360E-03 0.160E-06 0.50  2.50                                      -2.61
49  371.0800                                                   -83.10  -241.45  -0.67
              249.56 0.446E-03 0.147E-06 0.50  2.50                                     -12.85
50  394.8500                                                  -119.28  -236.35  -1.31
              252.43 0.547E-03 0.137E-06 0.00  0.00                                      -0.15
51  418.6300                                                  -119.71  -231.24  -1.32
              255.14 0.655E-03 0.127E-06 0.00  0.00                                       0.02
52  442.4000                                                  -119.66  -226.19  -1.32
              257.69 0.767E-03 0.118E-06 0.00  0.00                                       0.12
53  466.1700                                                  -119.31  -221.29  -1.32
```

```
              260.11 0.890E-03 0.109E-06 0.00  0.00                           0.22
54  489.9500                                        -118.68  -216.44  -1.33
              262.39 0.103E-02 0.103E-06 0.00  0.00                           0.29
55  513.7200                                        -117.88  -211.61  -1.33
              264.57 0.118E-02 0.975E-07 0.00  0.00                           0.36
56  537.5000                                        -116.87  -206.82  -1.33
              266.66 0.136E-02 0.924E-07 0.00  0.00                           0.42
57  561.2800                                        -115.67  -201.99  -1.34
              268.67 0.159E-02 0.883E-07 0.00  0.00                           0.50
58  585.0500                                        -114.26  -196.97  -1.34
              270.60 0.190E-02 0.846E-07 0.00  0.00                           0.56
59  608.8300                                        -112.70  -191.82  -1.35
              272.48 0.225E-02 0.810E-07 0.00  0.00                           0.63
60  632.6000                                        -110.93  -186.36  -1.36
              274.29 0.264E-02 0.778E-07 0.00  0.00                           0.67
61  656.3800                                        -109.03  -180.95  -1.37
              276.05 0.306E-02 0.749E-07 0.00  0.00                           0.74
62  680.1500                                        -106.95  -175.29  -1.38
              277.75 0.351E-02 0.721E-07 0.00  0.00                           0.79
63  703.9200                                        -104.72  -169.63  -1.39
              279.41 0.399E-02 0.694E-07 0.00  0.00                           0.86
64  727.7000                                        -102.29  -163.82  -1.41
              281.02 0.450E-02 0.671E-07 0.00  0.00                           0.92
65  751.4700                                         -99.69  -157.89  -1.44
              282.59 0.504E-02 0.648E-07 0.00  0.00                           1.02
66  775.2500                                         -96.81  -151.75  -1.46
              284.09 0.560E-02 0.626E-07 0.00  0.00                           1.09
67  799.0300                                         -93.74  -145.49  -1.50
              285.53 0.619E-02 0.607E-07 0.00  0.00                           1.21
68  822.8000                                         -90.34  -139.01  -1.54
              286.86 0.680E-02 0.593E-07 0.00  0.00                           1.31
69  846.5800                                         -86.64  -132.32  -1.59
              288.06 0.742E-02 0.579E-07 0.00  0.00                           1.38
70  870.3500                                         -82.75  -125.49  -1.66
              289.13 0.805E-02 0.565E-07 0.00  0.00                           1.41
71  894.1300                                         -78.77  -118.64  -1.75
              290.11 0.869E-02 0.552E-07 0.00  0.00                           1.44
72  917.9000                                         -74.72  -111.85  -1.86
              291.03 0.935E-02 0.540E-07 0.00  0.00                           1.50
73  941.6700                                         -70.49  -104.80  -2.01
              291.91 0.100E-01 0.528E-07 0.00  0.00                           1.58
74  965.4500                                         -66.03   -97.57  -2.24
              292.76 0.107E-01 0.517E-07 0.00  0.00                           1.70
75  989.2200                                         -61.23   -90.04  -2.64
              293.59 0.113E-01 0.505E-07 0.00  0.00                           2.22
76 1013.0000                                         -54.99   -81.15  -5.76

st4= -423.62 W/m**2
```

References

Arking, A., and K. Grossman, 1972: The influence of line shape and band structure on temperatures in planetary atmospheres. *J. Atmos. Sci.*, **29**, 937–949.

Chou, M.-D., and A. Arking, 1980: Computation of infrared cooling rates in the water vapor bands. *J. Atmos. Sci.*, **37**, 856–867.

Chou, M.-D., and L. Kouvaris, 1991: Calculations of transmission functions in the IR CO_2 and O_3 bands. *J. Geophy. Res.*, **96**, 9003–9012.

Chou, M.-D., W. Ridgway, and M.-H. Yan, 1993: One-parameter scaling and exponential-sum fitting for water vapor and CO_2 infrared transmission functions. *J. Atmos. Sci.*, **50**, 2294–2303.

Chou, M.-D., W. Ridgway, and M.-H. Yan, 1994: Parameterizations for water vapor IR radiative transfer in the middle atmosphere. *J. Atmos. Sci.*, in press.

Ellingson, R. G., J. Ellis and S. Fels, 1991: The Intercomparison of Radiation Codes in Climate Models: Long wave results. *J. Geophy. Res.*, **96**, 8929–8953.

Fu, Q, and K.-N. Liou, 1992: On the correlated *k*-distribution method for radiative transfer in nonhomogeneous atmospheres. *J. Atmos. Sci.*, **49**, 2153–2170.

Goody, R. M., R. West, L. Chen, and D. Crisp, 1989: The correlated-*k* method for radiation calculation in nonhomogeneous atmospheres. *J. Quant. Spectrosc. Radiat. Transfer.*, **42**, 539–550.

Harshvardhan, D. A. Randall and T. G. Corsetti, 1987: A fast radiation parameterization for atmospheric circulation models. *J. Geophy. Res.*, **92**, 1009–1016.

Kalnay, E., M. Kanamitsu, J. Pfaendtner, J. Sela, M. Suarez, J. Stackpole, J. Tuccillo, L. Umscheid, and D. Williamson, 1989: Rules for the interchange of physical parameterizations, *Bull. Am Met. Soc.*, **70**, 620-622.

Kratz, D. P., and R. D. Cess, 1988: Infrared radiation models for atmospheric ozone. *J. Geophy. Res.*, **93**, 7047–7054.

Kratz, D. P., M.-D. Chou, and M. M.-Y. Yan, 1993: Infrared radiation parameterizations for the minor CO_2 bands and for several CFC bands in the window region. *J. Climate*, in press.

Lacis, A., and V. Oinas, 1991: A description of the correlated *k*-distribution method for modeling non-gray gaseous absorption, thermal emission, and multiple scattering in vertically inhomogeneous atmospheres. *J. Geophy. Res.*, **96**, 9027–9063.

McClatchey, R. A., R. W. Fenn, J. E. A. Selby, F. E. Volz and J. S. Garing, 1972: Optical properties of the atmosphere, 3rd ed. AFCRL-72-0497, 108 pp. [NTIS N7318412].

Morcrette, J.-J., L. Smith and Y. Fouquart, 1986: Pressure and temperature dependence of the absorption in longwave radiation parameterizations. *Beitr. Phys. Atmos.*, **59**, 455–469.

Ridgway, W. L., Harshvardhan and A. Arking, 1991: Computation of atmospheric cooling rates by exact and approximate methods. *J. Geophy. Res.*, **96**, 8969–8984.

Roberts, R. E., J. E. A. Selby and L. M. Biberman, 1976: Infrared continuum absorption by atmospheric water vapor in the 8–12 μm window. *Appl. Opt.*, **15**, 2085–2090.

Rogers, C. D., and C. D. Walshaw, 1966: The computation of infra-red cooling rate in planetary atmospheres. *Q. J. R. Meteorol. Soc.*, **92**, 67–92.

Rosenfield, J. E., 1991: A simple parameterization of ozone infrared absorption for atmospheric heating rate calculations. *J. Geophy. Res.*, **96**, 9065–9074.

Rothman, L. S., R. R. Gamache, A. Barbe, A. Goldman, L. R. Brown, R. A. Toth, H. M. Pickett, R. L. Poynter, J.-M. Flaud, C. Camy-Peyret, A. Barbe, N. Husson, C. P. Rinsland, and M. A. Smith, 1987: The HITRAN data base: 1986 edition. *Appl. Opt.*, **26**, 4058–4097.

Schwarzkopf, M. D., and S. B. Fels, 1991: The simplified exchange method revisited: An accurate, rapid method for computation of infrared cooling rates and fluxes. *J. Geophy. Res.*, **96**, 9075–9096.

Wang, W.-C., and G. Y. Shi, 1988: Total band absorptance and k-distribution function for atmospheric gases. *J. Quant. Spectrosc. Radiat. Transfer*, **39**, 387–397.

Wu, M.-L. C., 1980: The exchange of infrared energy in the troposphere. *J. Geophy. Res.*, **85**, 4084-4090.

Zhu, X., 1992: Calculation of CO_2 15-μm band atmospheric cooling rates by Curtis matrix interpolation of correlated-k coefficients. *J. Geophy. Res.*, **97**, 12787–12797.

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A detailed description of a parameterization for thermal infrared radiative transfer designed specifically for use in global climate models is presented. The parameterization includes the effects of the main absorbers of terrestrial radiation: water vapor, carbon dioxide, and ozone. While being computationally efficient, the schemes compute very accurately the clear-sky fluxes and cooling rates from the Earth's surface to 0.01 mb. This combination of accuracy and speed makes the parameterization suitable for both tropospheric and middle atmospheric modeling applications. Since no transmittances are precomputed, the atmospheric layers and the vertical distribution of the absorbers may be freely specified. The scheme can also account for any vertical distribution of fractional cloudiness with arbitrary optical thickness. These features make the parameterization very flexible and extremely well suited for use in climate modeling studies. In addition, the numerics and the FORTRAN implementation have been carefully designed to conserve both memory and computer time. This code should thus be particularly attractive to those contemplating long-term climate simulations, wishing to model the middle atmosphere, or planning to use a large number of levels in the vertical.

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