

The importance of theoretical tools as guidance for turbulence measurements in the Atmospheric Boundary Layer

Nelson Luís Dias¹

¹Department of Environmental Engineering, Federal University of Paraná, Brazil

✉: nldias@ufpr.br

🔗: www.nldias.github.io

FLUXNET-ECN Webinar Nov 05 2021

Thanks

For inviting me here.

Contents

Introduction	5
Atmospheric Turbulence	8
Letting the equations talk	16
Conclusions	24

Introduction

Intersection of surface flux measurement interests

Many scientific communities have interest in surface fluxes:

Meteorologists Because they need the surface boundary conditions for weather and climate,

Hydrologists Because evapotranspiration is a missing link in Hydrology,

Ecologists Because NEE and GPP are key variables to understand ecosystems,

Agronomists Because evapotranspiration is essential for crop productivity,

Environmental Engineers Because of air pollution,

etc.

Intersection of surface flux measurement interests

Many scientific communities have interest in surface fluxes:

Meteorologists Because they need the surface boundary conditions for weather and climate,

Hydrologists Because evapotranspiration is a missing link in Hydrology,

Ecologists Because NEE and GPP are key variables to understand ecosystems,

Agronomists Because evapotranspiration is essential for crop productivity,

Environmental Engineers Because of air pollution,

etc.

Turbulence measurements in the ABL can bring important contributions to many fields

My approach in this talk

This talk consists of:

My approach in this talk

This talk consists of:

A few examples (a recipe?) of successful efforts to understand the Atmospheric Boundary-Layer (ABL)

My approach in this talk

This talk consists of:

A few examples (a recipe?) of successful efforts to understand the Atmospheric Boundary-Layer (ABL)

- Good data sets.

My approach in this talk

This talk consists of:

A few examples (a recipe?) of successful efforts to understand the Atmospheric Boundary-Layer (ABL)

- Good data sets.
- Correct and as thorough as possible application and interpretation of the *governing equations*

My approach in this talk

This talk consists of:

A few examples (a recipe?) of successful efforts to understand the Atmospheric Boundary-Layer (ABL)

- Good data sets.
- Correct and as thorough as possible application and interpretation of the *governing equations*

Examples drawn from contributions to the field (based on my personal experience).

Atmospheric Turbulence

Different approaches

- **Theoretical** The ABL is a “natural laboratory” for turbulent flows because of the very high Reynolds numbers.
- **Applied** Boundary-Layer Meteorology has important applications in many fields, as we have seen (Weather prediction, Climate Simulation, Agronomy, Ecology, Hydrology, Air Pollution, etc.).



Above: The Chaitén Eruption in Chile, (O Globo Newspaper, May 07 2008)

An optimistic view of atmospheric turbulence, c. 1970

(Monin and Yaglom, 1971, v. 1, p. 22):

The fact is, that the atmosphere, which von Kármán himself (1934) called “a giant laboratory for ‘turbulence research,’ ” possesses *very valuable properties* which make it especially suitable for the verification of the deductions of modern statistical theory. We have already observed that atmospheric motion is usually characterized by *far larger Reynolds numbers* than flows created in the laboratory, and therefore is far more convenient for investigating specific laws relating to the case of very large Re. Moreover, the geometrical conditions of atmospheric turbulence (namely, the conditions of a two-dimensional flow in a half-space bounded by a rigid wall, ... *where in many cases the “wall” may be considered as plane and homogeneous; ...*) *are simpler than in most laboratory experiments*. The *only additional complication*, which arises on transition from laboratory to atmosphere, is the necessity of taking into account *the thermal stratification...*

But

(And this is a list of but a few of the problems that remain)

- There are flows with lower Reynolds numbers under stable conditions (laminar?)
- Horizontal inhomogeneity (land cover changes), topographical effects.
- Vertical inhomogeneity (what is the effect of the transport terms in the 2nd-order equations?)
- Non-stationarity and the difficulty of taking representative time averages.
- The increasing role of more and more scalars, such as CO₂, CH₄, N₂O, O₃, VOCs, etc., and the need to “partition” CO₂ into respiration and photosynthesis, and H₂O into transpiration and evaporation, etc..

Fundamental progress (a little history)

This is a personal choice of favorites! Many works will be (unfairly) left out

- **Reynolds (1895)** The first derivation of the TKE: the birth of the statistical approach to turbulence (following **Maxwell (1867)**, but much earlier than **Einstein (1905)**'s paper on Brownian motion.
- **Richardson (1920)** The birth of the Richardson number
- **Kolmogorov (1941)** The K41 Theory: $E_e(k) = \alpha_e \epsilon_e^{2/3} k^{-5/3}$, microscales
- **Kolmogorov (1941 1991)** The 4/5 law: $\overline{[u(x_1 + r_1) - u(x_1)]^3} = -\frac{4}{5} \epsilon_e r_1$
- **Obukhov (1946 1971)** The Monin-Obukhov Similarity Theory (MOST)
- **Obukhov (1949), Corrsin (1951)** The scalar spectrum
- **Batchelor (1959)** The Batchelor microscale

Kolmogorov's theory and MOST together

Stewart and Townsend (1951), Grant *et al.* (1962), Gibson and Schwarz (1963), Wyngaard and Coté (1972); Kaimal *et al.* (1972)

$$nC_{wa}(n)/\overline{wa} = \mathcal{A}(\mathcal{B}f)/[1 + (\mathcal{B}f)^{7/3}], \quad \mathcal{B} = \mathcal{B}(\zeta)$$

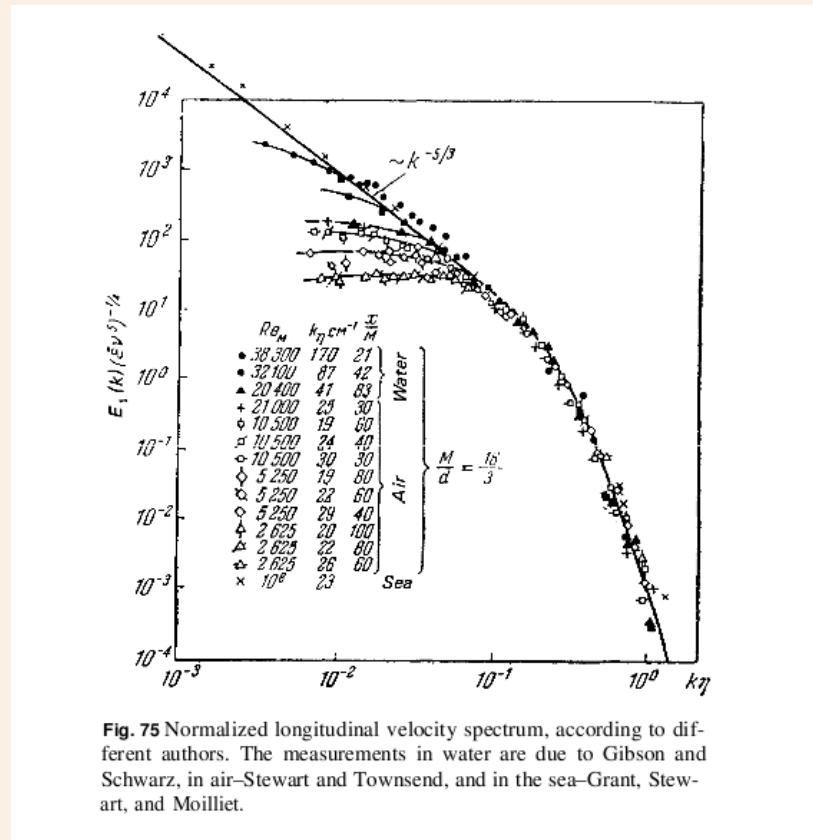


Fig. 75 Normalized longitudinal velocity spectrum, according to different authors. The measurements in water are due to Gibson and Schwarz, in air—Stewart and Townsend, and in the sea—Grant, Stewart, and Moilliet.

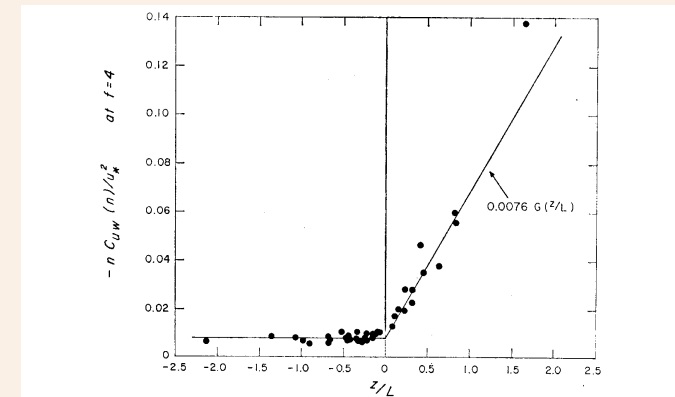


Figure 10. Normalized logarithmic uw cospectral estimates at $f = 4$ compared with empirical formula in Eq. (12).

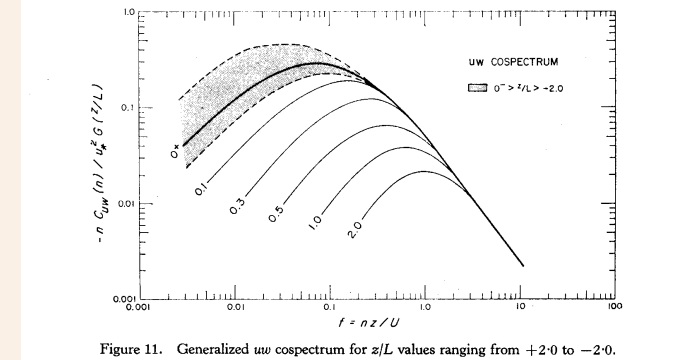
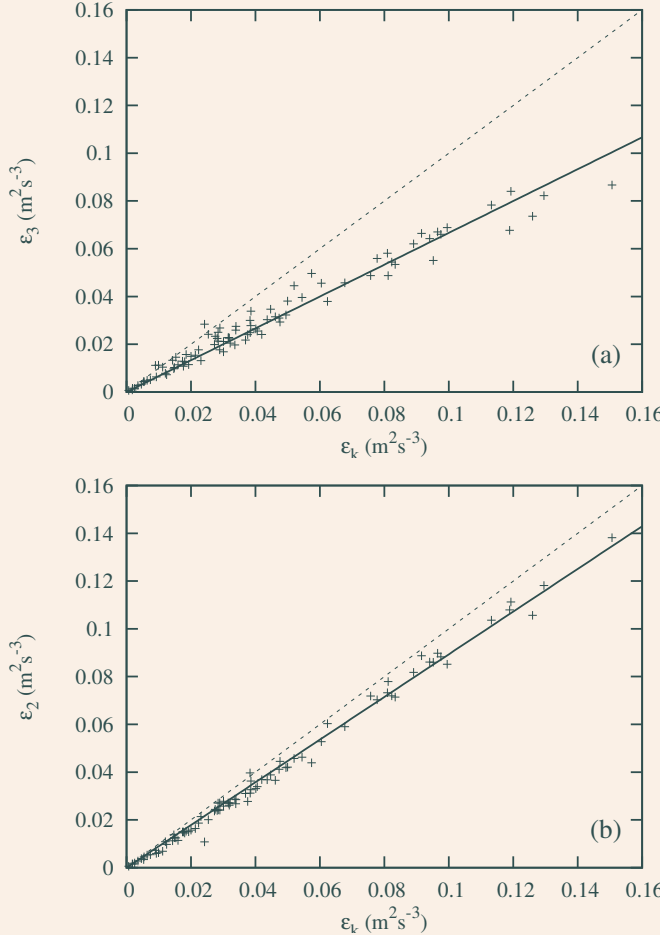
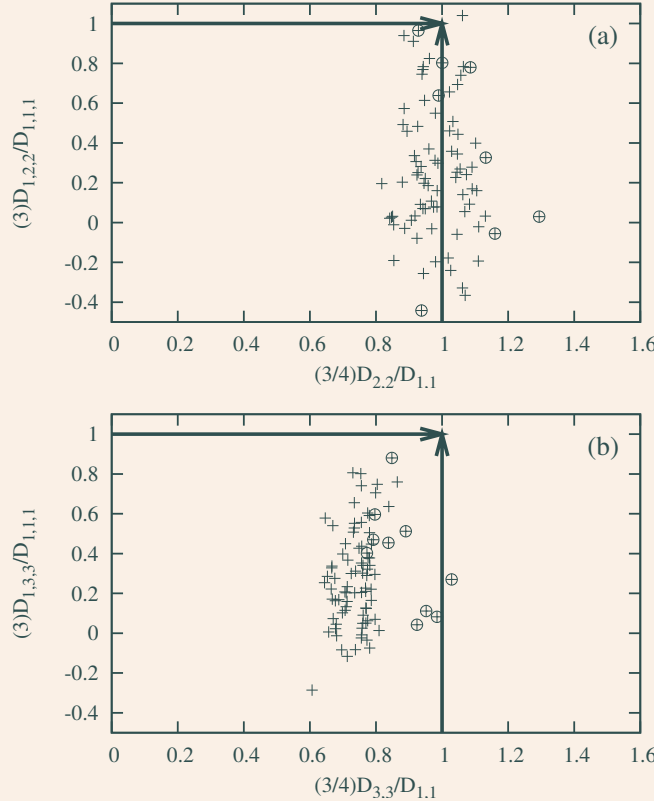


Figure 11. Generalized uw cospectrum for z/L values ranging from $+2.0$ to -2.0 .

But some questions are only partially answered to this day

Isotropy of structure functions in K41 and rate of dissipation of TKE (Chamecki and Dias, 2004): the 4/5 law.



Further things to do

- **(very difficult)** Make progress towards directly measuring

$$\epsilon_e \approx \nu \overline{\frac{\partial u'_i}{\partial x_k} \frac{\partial u'_i}{\partial x_k}}$$

- Understand deviations from the idealized 4/5 law conditions ([Danaila et al., 2001](#)):

$$-\overline{(\Delta u'_1)^3} + 6\nu \frac{d\overline{(\Delta u'_1)^2}}{dr} + \frac{6}{r^4} \int_0^r y^4 \left[-\frac{\partial u'_3 \overline{(\Delta u'_1)^2}}{\partial x_3} \right] dy = \frac{4}{5} \epsilon_e r$$

- Extend analysis to scalars

Letting the equations talk

A systematic approach

- Try to start at the *governing equations*
- Make clear physical approximations
- Make good experiments
- Support your analysis with statistics

Example 1: the Brutsaert theory for the scalar roughness length

Problem: turbulence in the interfacial sublayer and how you parameterize the scalar flux **Brutsaert (1965, 1975a,b)**.

Key: use **Danckwerts (1951)**'s surface renewal theory, but parameterize the renewal rate with the thickness of the interfacial (roughness) sublayer h . Also, match the top of the interfacial sublayer to the bottom of the inertial sublayer, eliminating \bar{c}_h .

$$s \propto \left(u_*^3 / (v\kappa(h - d_0)) \right)^{1/2}$$

$$F = \bar{\rho} (v_c s)^{1/2} (\bar{c}_0 - \bar{c}_h)$$

$$z_{0c} = z_0 \exp \left[-\kappa \left(7.3 \text{Re}_0^{1/4} \text{Sc}^{1/2} - 5 \right) \right]$$

$$F = \bar{\rho} \frac{\kappa^2}{\left[\ln \frac{z_a - d_0}{z_0} - \Psi_m(\zeta_a) \right] \left[\ln \frac{z_a - d_0}{z_{0c}} - \Psi_F(\zeta_a) \right]} \bar{u}_a (\bar{c}_0 - \bar{c}_a)$$

Example 2: similarity of scalars

- If two scalars a and b are perfectly similar in the ABL, then $a' = kb'$.
- a' may be much easier to measure than $b' \Rightarrow$ get eddy diffusivities from a , apply to b .
- Perfect similarity is often assumed in applications. Example: Model to partition the H_2O and CO_2 fluxes between evaporation and transpiration, and between respiration and photosynthesis (Scanlon and Sahu, 2008).

Dias and Brutsaert (1996):

$$\begin{aligned}
 -2\overline{w'a'}\frac{\partial\bar{a}}{\partial z} - \frac{\partial\overline{w'a'a'}}{\partial z} &= 2\epsilon_{aa}, \\
 -2\overline{w'b'}\frac{\partial\bar{b}}{\partial z} - \frac{\partial\overline{w'b'b'}}{\partial z} &= 2\epsilon_{bb}, \\
 -\overline{w'a'}\frac{\partial\bar{b}}{\partial z} - \overline{w'b'}\frac{\partial\bar{a}}{\partial z} - \frac{\partial\overline{w'a'b'}}{\partial z} &= 2\epsilon_{ab},
 \end{aligned}$$

If the transport terms can be neglected, this leads to

- Equality of the MOST dimensionless gradients for a and b ,
- Perfect correlation between the fluctuations, meaning $a' = kb'$.

With zero transport

$$\begin{aligned} \phi_H = \phi_{\epsilon_{\theta\theta}} \quad \phi_E = \phi_{\epsilon_{qq}} \quad \phi_H + \phi_E = 2\phi_{\epsilon_{\theta q}} \\ \frac{\epsilon_{\theta q}^2}{\epsilon_{\theta\theta}\epsilon_{qq}} = \frac{\left(\frac{v_{\theta}+v_q}{2}\right)^2 \left(\overline{\frac{\partial\theta'}{\partial x_k} \frac{\partial q'}{\partial x_k}}\right)^2}{v_{\theta}v_q \left(\overline{\frac{\partial\theta'}{\partial x_k} \frac{\partial\theta'}{\partial x_k}}\right) \left(\overline{\frac{\partial q'}{\partial x_k} \frac{\partial q'}{\partial x_k}}\right)} = \frac{\phi_{\epsilon_{\theta q}}^2}{\phi_{\epsilon_{\theta\theta}}\phi_{\epsilon_{qq}}} = 1.008 r_{\nabla\theta\nabla q}^2 \approx r_{\nabla\theta\nabla q}^2 \end{aligned}$$

$$x + y = z, \quad z^2 = r^2 xy \quad \Rightarrow \quad \frac{x}{z} = \frac{r^2 \pm \sqrt{r^4 - r^2}}{r^2} \Rightarrow x = y = z; \quad r^2 = 1.$$

And

$$r_{\nabla\theta\nabla q}^2 = 1 \Rightarrow r_{\theta q}^2 = 1.$$

Production and vertical transport need to be investigated further, but mean scalar gradients need good calibration, and 3rd moments have large errors.

Example 3: the alignment between mean wind \bar{u} and Reynolds stress tensor τ

Problem (**Bernardes and Dias, 2010**): after rotation, $\bar{u} = (\bar{u}, 0)$, $\tau = \rho(\overline{u'w'}, \overline{v'w'})$ and $\bar{u} \nparallel \tau$.

So how do you calculate u_* :

$$u_* = [\overline{u'w'^2} + \overline{v'w'^2}]^{1/4}$$

or

$$u_* = [-\overline{u'w'}]^{1/2}?$$

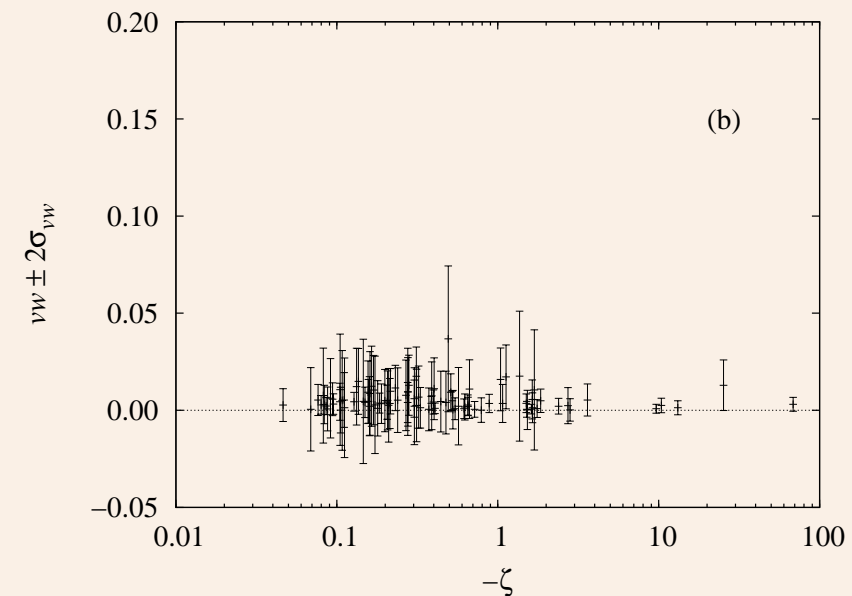
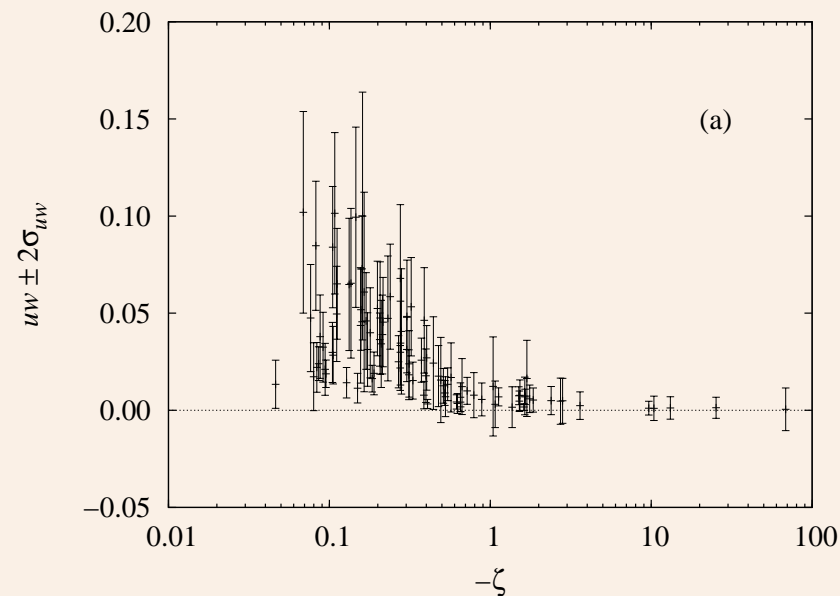
Example 3: the alignment between mean wind \bar{u} and Reynolds stress tensor τ

Problem (Bernardes and Dias, 2010): after rotation, $\bar{u} = (\bar{u}, 0)$, $\tau = \rho(\overline{u'w'}, \overline{v'w'})$ and $\bar{u} \nparallel \tau$.
So how do you calculate u_* :

$$u_* = [\overline{u'w'^2} + \overline{v'w'^2}]^{1/4}$$

or

$$u_* = [-\overline{u'w'}]^{1/2}?$$



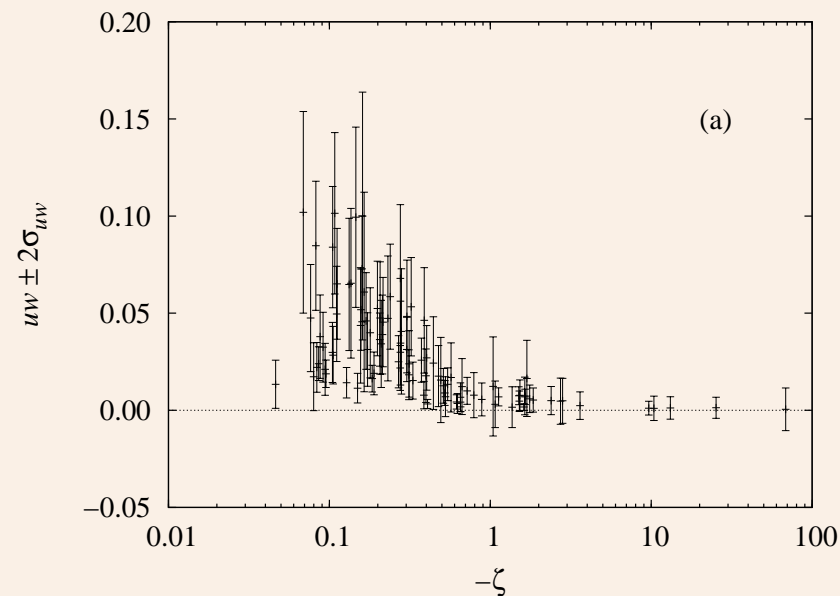
Example 3: the alignment between mean wind \bar{u} and Reynolds stress tensor τ

Problem (Bernardes and Dias, 2010): after rotation, $\bar{u} = (\bar{u}, 0)$, $\tau = \rho(\overline{u'w'}, \overline{v'w'})$ and $\bar{u} \nparallel \tau$.
So how do you calculate u_* :

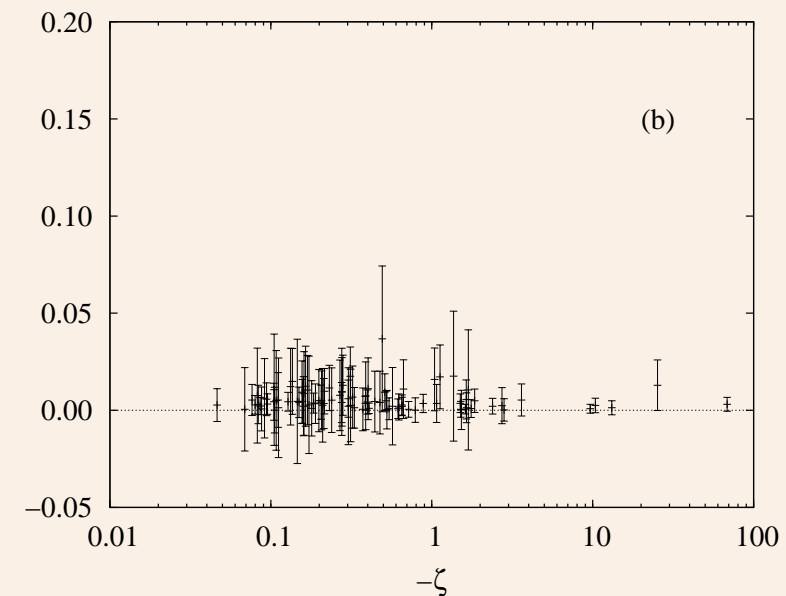
$$u_* = [\overline{u'w'^2} + \overline{v'w'^2}]^{1/4}$$

or

$$u_* = [-\overline{u'w'}]^{1/2}?$$



$-\zeta \gtrsim 1$: $\overline{u'w'}$ indistinguishable from 0



$-\zeta > 0$: $\overline{v'w'}$ indistinguishable from 0

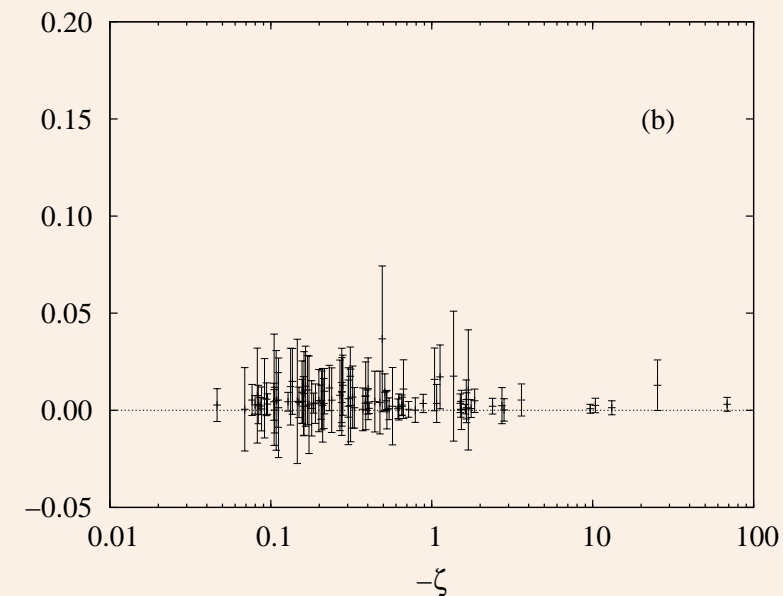
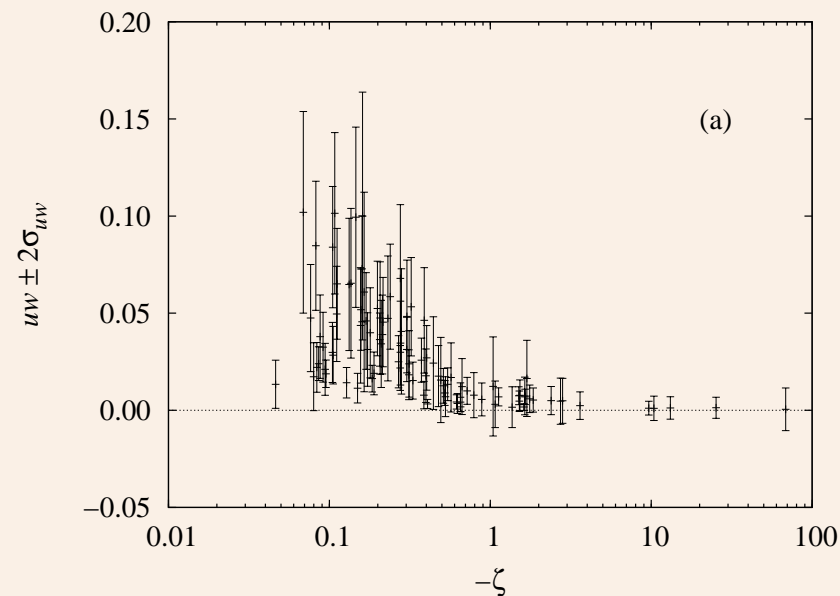
Example 3: the alignment between mean wind \bar{u} and Reynolds stress tensor τ

Problem (Bernardes and Dias, 2010): after rotation, $\bar{u} = (\bar{u}, 0)$, $\tau = \rho(\overline{u'w'}, \overline{v'w'})$ and $\bar{u} \nparallel \tau$.
So how do you calculate u_* :

$$u_* = [\overline{u'w'^2} + \overline{v'w'^2}]^{1/4}$$

or

$$u_* = [-\overline{u'w'}]^{1/2}?$$



$-\zeta \gtrsim 1$: $\overline{u'w'}$ indistinguishable from 0

$-\zeta > 0$: $\overline{v'w'}$ indistinguishable from 0

But this is Wyngaard *et al.* (1971)'s prediction for local convection!

Example 4: The reduced TKE budget

From [Chamecki *et al.* \(2018\)](#):

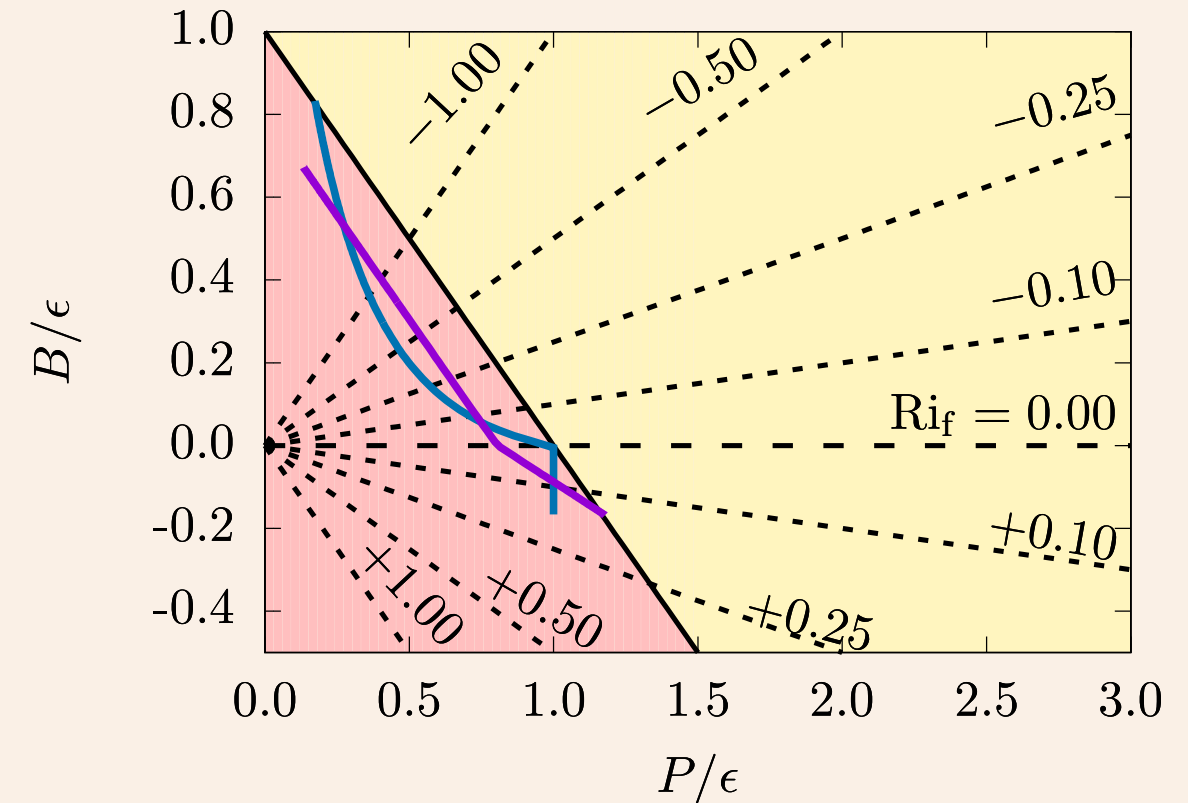
$$\underbrace{-\overline{u'w'} \frac{\partial \bar{u}}{\partial z}}_P + \underbrace{\frac{g}{\theta} \overline{w'\theta'}}_B - \epsilon_e = \underbrace{\frac{\partial \bar{e}_k}{\partial t} + \bar{u} \frac{\partial \bar{e}_k}{\partial x} + \frac{1}{\rho} \frac{\partial \overline{u'_i p'}}{\partial x_i} + \frac{\partial \overline{u'_i e'_k}}{\partial x_i}}_R,$$

$$\frac{P}{\epsilon_e} + \frac{B}{\epsilon_e} - 1 = \frac{R}{\epsilon_e}$$

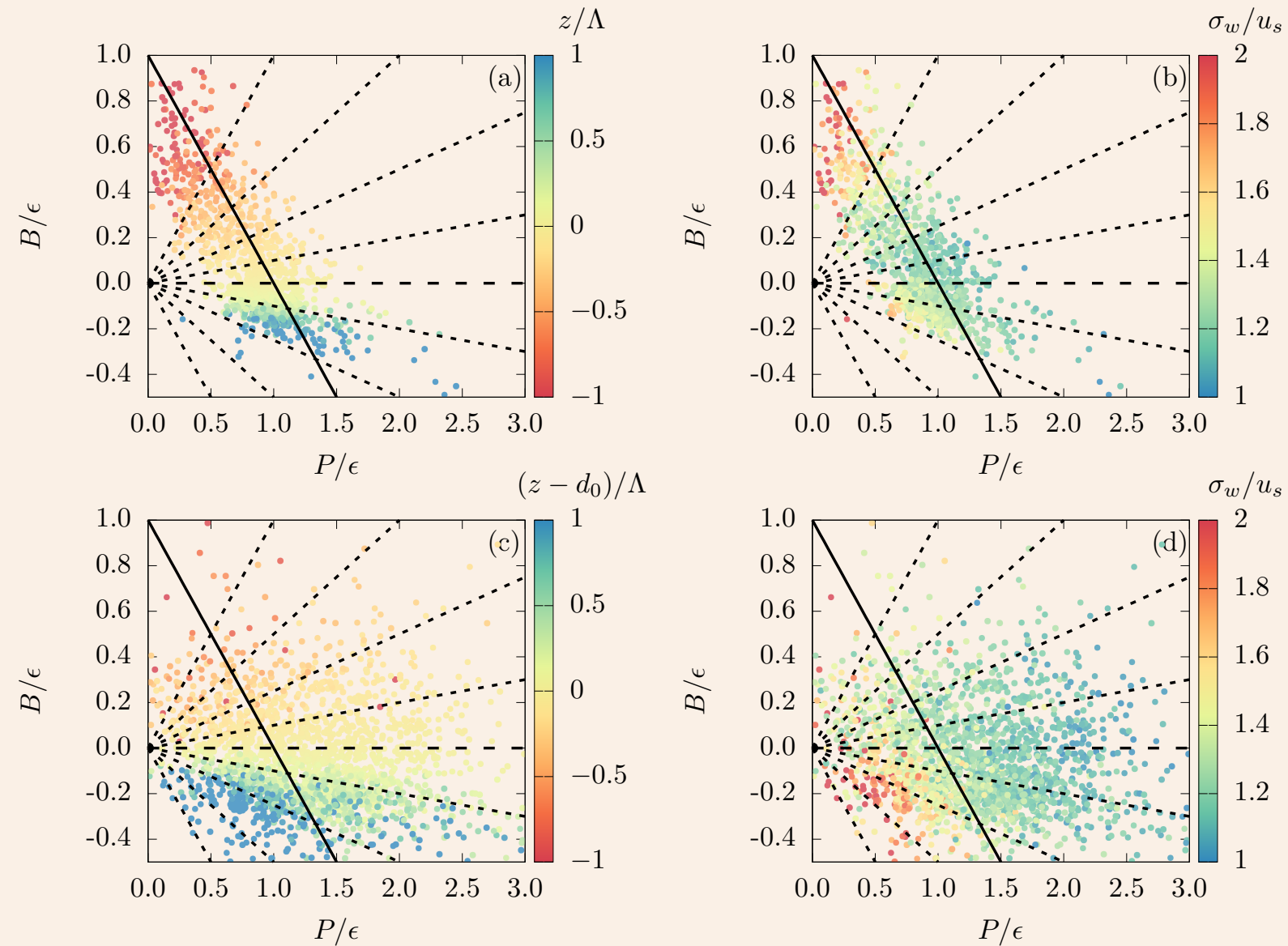
$$\frac{P}{\epsilon_e} = \frac{\phi_m(\zeta)}{\phi_{\epsilon_e}(\zeta)},$$

$$\frac{B}{\epsilon_e} = -\frac{\zeta}{\phi_{\epsilon_e}},$$

$$\zeta = \zeta(\text{Ri}_f).$$



Data from AHATS (above) and GoAmazon (below)



Conclusions

A road to progress

- Understanding the “discarded” terms in MOST (time rate of change, advection, transport term, return to isotropy – see *Freire et al. (2019)*) is essential.
- Often much can be learned by starting from “ideal” conditions and “perturbing”.
- Better field measurements, and better models and theories, are always in need.

A road to progress

- Understanding the “discarded” terms in MOST (time rate of change, advection, transport term, return to isotropy – see *Freire et al. (2019)*) is essential.
- Often much can be learned by starting from “ideal” conditions and “perturbing”.
- Better field measurements, and better models and theories, are always in need.

Wrapping up:

- Progress requires universality.

A road to progress

- Understanding the “discarded” terms in MOST (time rate of change, advection, transport term, return to isotropy – see *Freire et al. (2019)*) is essential.
- Often much can be learned by starting from “ideal” conditions and “perturbing”.
- Better field measurements, and better models and theories, are always in need.

Wrapping up:

- Progress requires universality.
- Universality is encapsulated in the governing equations and sound theory.

A road to progress

- Understanding the “discarded” terms in MOST (time rate of change, advection, transport term, return to isotropy – see *Freire et al. (2019)*) is essential.
- Often much can be learned by starting from “ideal” conditions and “perturbing”.
- Better field measurements, and better models and theories, are always in need.

Wrapping up:

- Progress requires universality.
- Universality is encapsulated in the governing equations and sound theory.
- Let them lead us to new discoveries.

A road to progress

- Understanding the “discarded” terms in MOST (time rate of change, advection, transport term, return to isotropy – see *Freire et al. (2019)*) is essential.
- Often much can be learned by starting from “ideal” conditions and “perturbing”.
- Better field measurements, and better models and theories, are always in need.

Wrapping up:

- Progress requires universality.
- Universality is encapsulated in the governing equations and sound theory.
- Let them lead us to new discoveries.

Thanks!

References

- G. K. Batchelor. Small-scale variation of convected quantities like temperature in turbulent fluid. part 1. general discussion and the case of small conductivity. *J Fluid Mech*, 5:113–133, 1959.
- M. Bernardes and N. L. Dias. The alignment of the mean wind and stress vectors in the unstable surface layer. *Boundary-Layer Meteorol*, 134:41–59, 2010.
- W. Brutsaert. A model for evaporation as a molecular diffusion process into a turbulent atmosphere. *J Geophys Res*, 70:5017–5024, 10 1965.
- W. Brutsaert. a theory for local evaporation (or heat transfer) from rough and smooth surfaces at ground level. *Water Resour Res*, 11:543–550, 1975.
- W. Brutsaert. the roughness length for water vapor, sensible heat and other scalars. *J Atmos Sci*, 32:2028–2031, 1975.
- M. Chamecki and N. L. Dias. The local isotropy assumption and the turbulent kinetic energy dissipation rate in the atmospheric surface layer. *Q J Roy Meteorol Soc*, 130(603):2733–2752, 2004.
- Marcelo Chamecki, Nelson L Dias, and Livia S Freire. A tke-based framework for studying disturbed

atmospheric surface layer flows and application to vertical velocity variance over canopies. *Geophys Res Lett*, 45:6734–6740, 2018.

S. Corrsin. On the spectrum of isotropic temperature fluctuations in isotropic turbulence. *J Appl Phys*, 22:469, 1951.

L Danaila, F Anselmet, Tongming Zhou, and RA Antonia. Turbulent energy scale budget equations in a fully developed channel flow. *Journal of Fluid Mechanics*, 430:87, 2001.

P. V. Danckwerts. Significance of liquid-film coefficients in gas absorption. *Ind Eng Chem*, 43:1460–1467, 1951.

N. L. Dias and W. Brutsaert. Similarity of scalars under stable conditions. *Boundary-Layer Meteorol*, 80:355–373, 1996.

A. Einstein. Über die von der molekularkinetischen theorie der wärme geforderte bewegung von in ruhenden flüssigkeitenssigkeiten suspendierten teilchen. *Annalen der Physik*, 322(8):549–560, 1905.

Livia S Freire, Marcelo Chamecki, Elie Bou-Zeid, and Nelson L. Dias. Critical flux richardson number for kolmogorov turbulence enabled by tke transport. *Q J R Meteorol Soc*, 145:1551–1558, 2019.

- C. H. Gibson and W. H. Schwarz. The universal equilibrium spectra of turbulent velocity and scalar fields. *J Fluid Mech*, 16:365–385, 1963.
- H. L. Grant, R. W. Stewart, and A. Moilliet. Turbulence spectra from a tidal channel. *J Fluid Mech*, 12:241–268, 1962.
- J C. Kaimal, J C. Wyngaard, Y. Izumi, and O. R. Coté. Spectral characteristics of surface-layer turbulence. *Q J R Meteorol Soc*, 98:563–589, 1972.
- Andrey Nikolaevich Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers (in Russian). *Proceedings of the USSR Academy of Sciences*, 30(299–303), 1941.
- Andrey Nikolaevich Kolmogorov. Dissipation of energy in the locally isotropic turbulence. *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, 434(1890):15–17, 1941 1991.
- J. Clerk Maxwell. On the dynamical theory of gases. *Philosophical Transactions of the Royal Society of London*, 157:pp. 49–88, 1867.
- A. S. Monin and A. M. Yaglom. *Statistical fluid mechanics: Mechanics of turbulence*, volume 1. MIT Press, Cambridge, Massachusetts, 1971.

- A. M. Obukhov. Turbulence in an atmosphere with non-uniform temperature. *Boundary-Layer Meteorol*, 2:7–29, 1946 1971.
- A. M. Obukhov. Structure of the temperature field in turbulent flow. *Izv. Akad. Nauk. SSSR, Ser. Geogr. Geofiz.*, 13:59–69, 1949.
- O. Reynolds. On the dynamical theory of incompressible viscous fluids and the determination of the criterion. *Philos. Trans. R. Soc. Lond. A*, 186:123–164, 1895.
- Lewis F. Richardson. The supply of energy from and to atmospheric eddies. *Proceedings of the Royal Society of London. Series A*, 97(686):354–373, 1920.
- Todd M Scanlon and Parameswar Sahu. On the correlation structure of water vapor and carbon dioxide in the atmospheric surface layer: A basis for flux partitioning. *Water Resources Research*, 44(10), 2008.
- R. W. Stewart and A. A. Townsend. Similarity and self-preservation in isotropic turbulence. *Philos Trans of the R Soc A*, A243:359–386, 1951.
- J C. Wyngaard and O. R. Coté. Cospectral similarity in the atmospheric surface layer. *Q J Roy Meteorol Soc*, 98:590–603, 1972.

J C. Wyngaard, O. R. Coté, and Y. Izumi. Local free convection, similarity, and the budgets of shear stress and heat flux. *J Atmos Sci*, 28:1171–1182, 1971.