EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 6.8

Structures in Class 4 Systems

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The next page shows three typical examples of class 4 cellular automata. In each case the initial conditions that are used are completely random. But after just a few steps, the systems organize themselves to the point where definite structures become visible.

Most of these structures eventually die out, sometimes in rather complicated ways. But a crucial feature of any class 4 systems is that there must always be certain structures that can persist forever in it.

So how can one find out what these structures are for a particular cellular automaton? One approach is just to try each possible initial condition in turn, looking to see whether it leads to a new persistent structure. And taking the code 20 cellular automaton from the top of the next page, the page that follows shows what happens in this system with each of the first couple of hundred possible initial conditions.

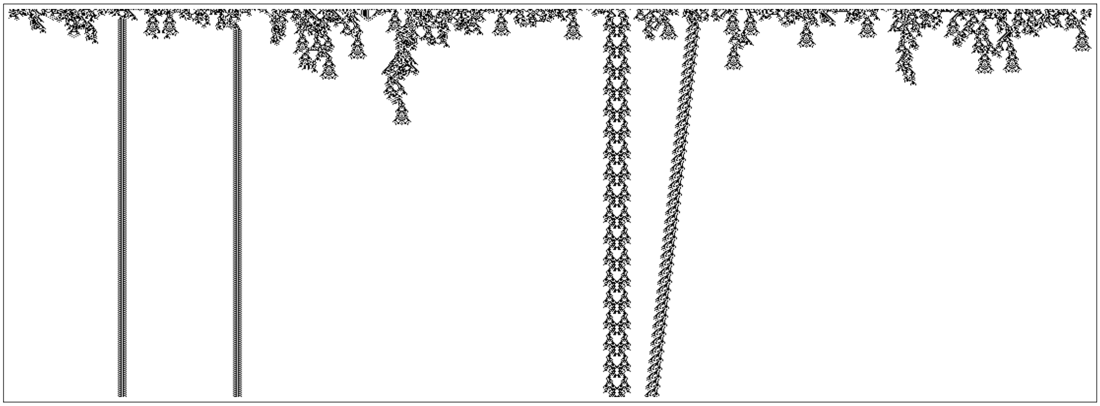
In most cases everything just dies out. But when we reach initial condition number 151 we finally see a structure that persists.

This particular structure is fairly simple: it just remains fixed in position and repeats every two steps. But not all persistent structures are that simple. And indeed at initial condition 187 we see a considerably more complicated structure, that instead of staying still moves systematically to the right, repeating its basic form only every 9 steps.

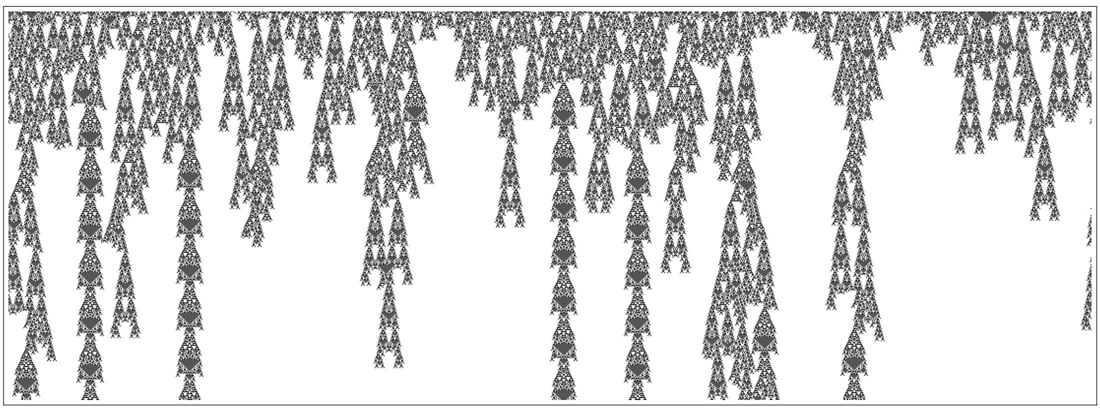
The existence of structures that move is a fundamental feature of class 4 systems. For as we discussed on page 252, it is these kinds of structures that make it possible for information to be communicated from one part of a class 4 system to another—and that ultimately allow the complex behavior characteristic of class 4 to occur.

But having now seen the structure obtained with initial condition 187, we might assume that all subsequent structures that arise in the code 20 cellular automaton must be at least as complicated. It turns out, however, that initial condition 189 suddenly yields a much simpler structure—that just stays unchanged in one position at every step.

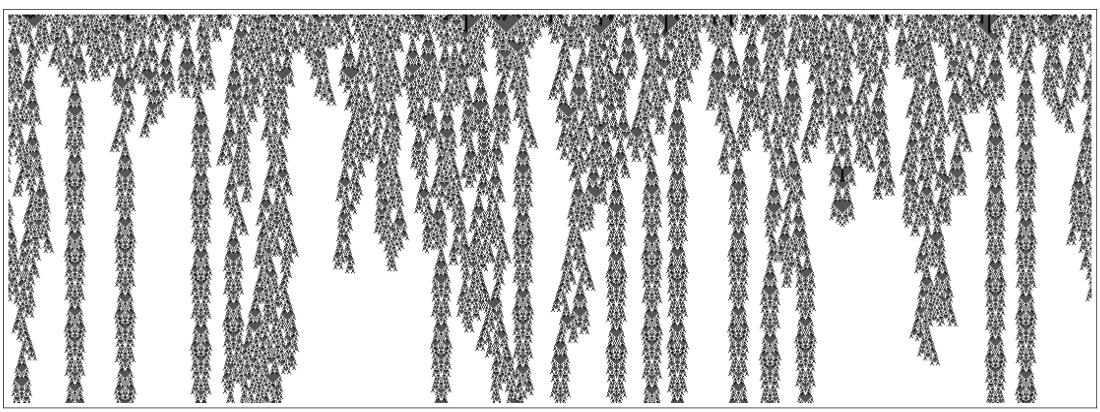
But going on to initial condition 195, we again find a more complicated structure—this time one that repeats only every 22 steps.



2 colors, next-nearest neighbors, code 20

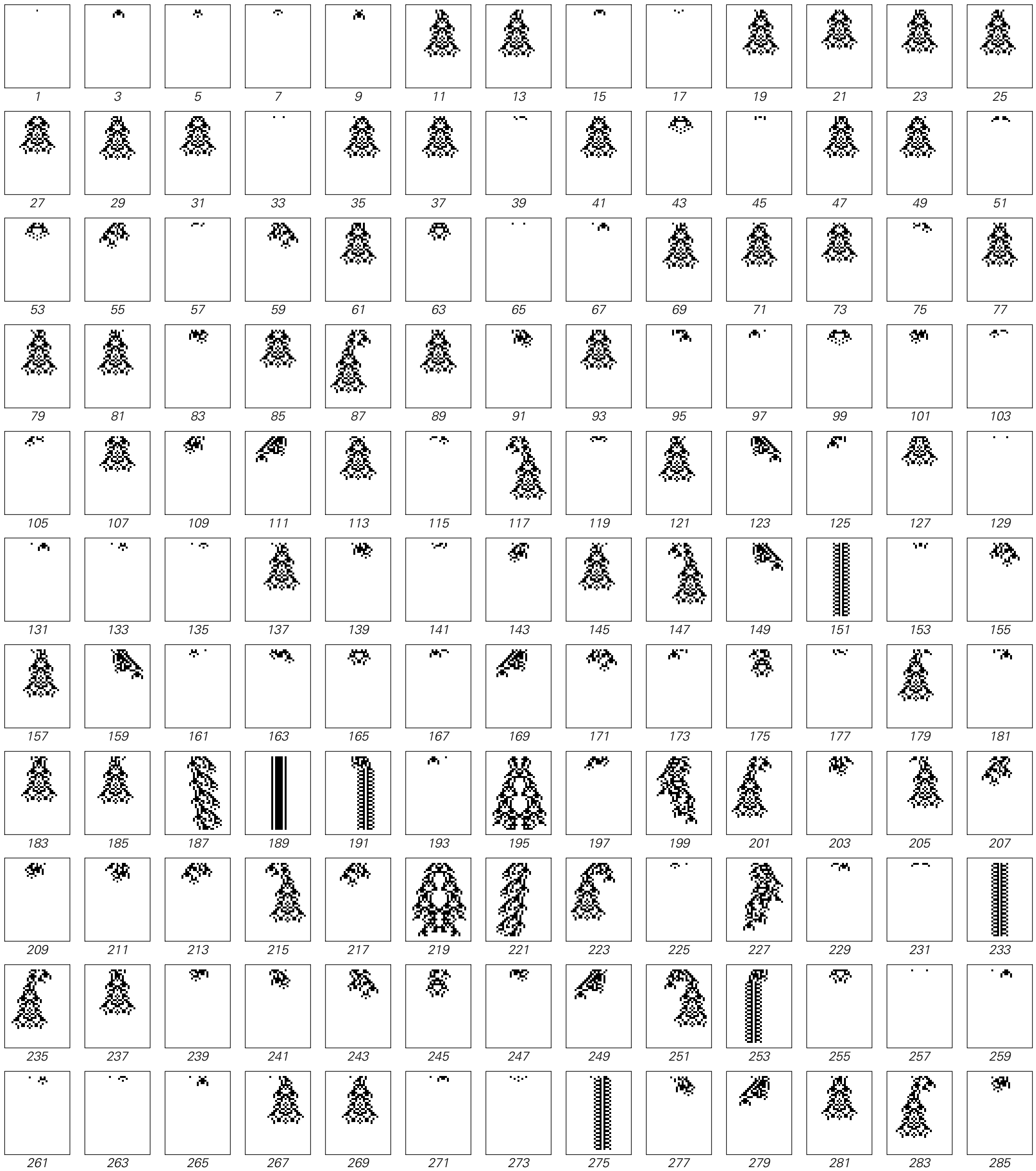


3 colors, nearest neighbors, code 357



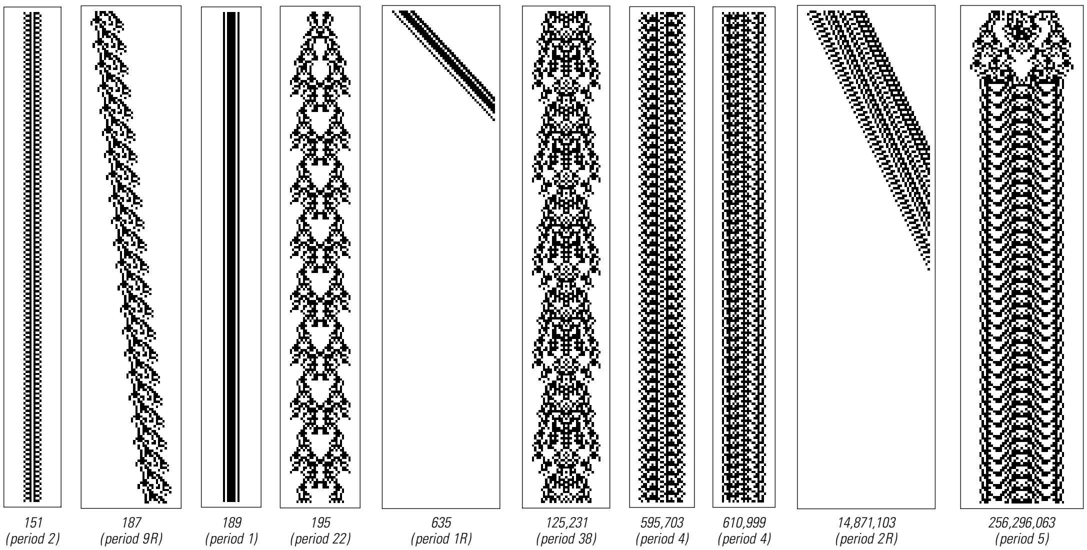
3 colors, nearest neighbors, code 1329

Three typical examples of class 4 cellular automata. In each case various kinds of persistent structures are seen.



The behavior of the code 20 cellular automaton from the top of the facing page for all initial conditions with black cells in a region of size less than nine. In most cases the patterns produced simply die out. But with some initial conditions, persistent structures are formed. Each initial condition is assigned a number whose base 2 digit sequence gives the configuration of black and white cells in that initial condition. Note that initial conditions 195 and 219 both yield the period 22 persistent structure shown on the next page.

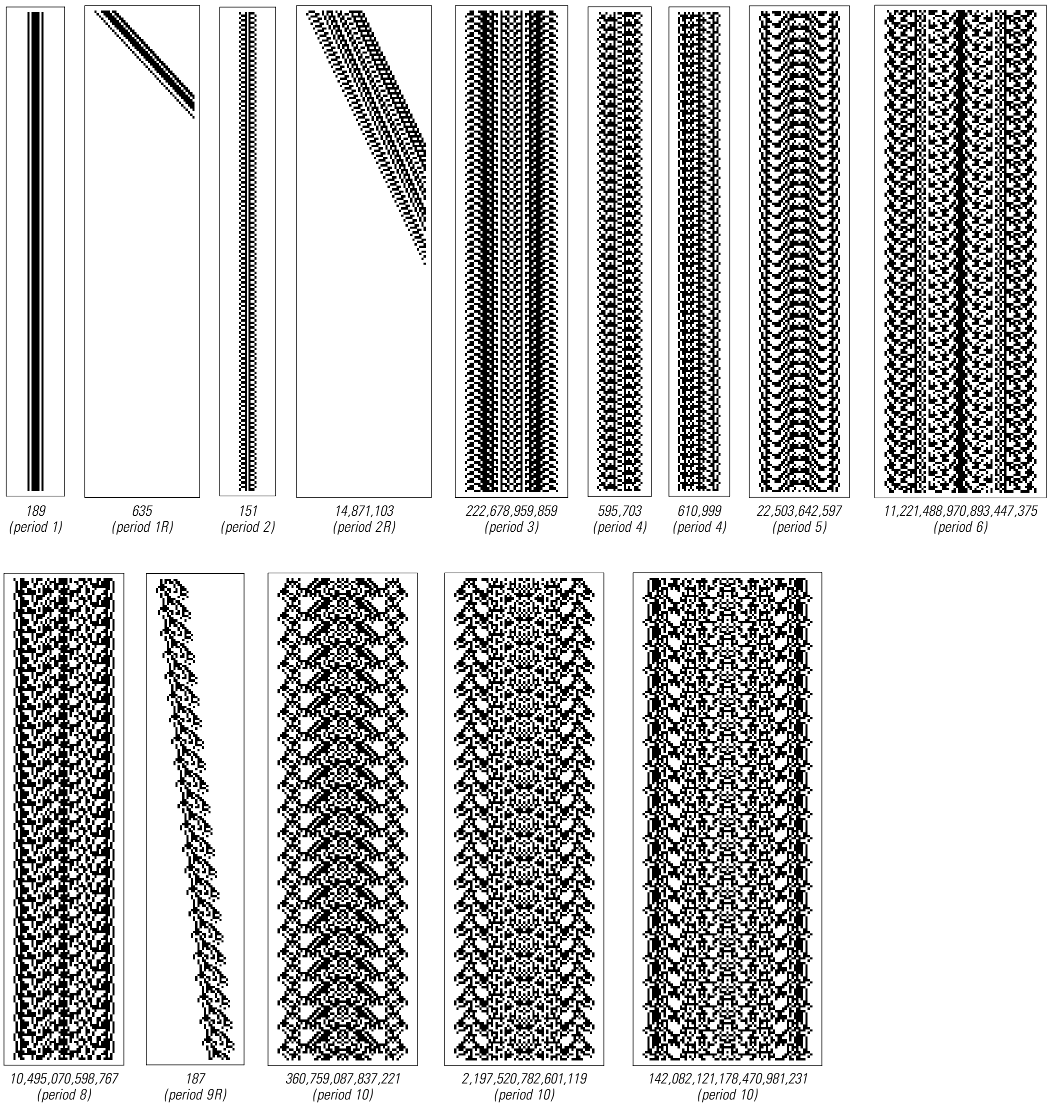
So just what set of structures does the code 20 cellular automaton ultimately support? There seems to be no easy way to tell, but the picture below shows all the structures that I found by explicitly looking at evolution from the first twenty-five billion possible initial conditions.



Persistent structures found by testing the first twenty-five billion possible initial conditions for the code 20 cellular automaton shown on the previous page. Note that reflected versions of the structures shown are also possible. The base 2 digit sequences of the numbers given correspond to the initial conditions in each case, as on the previous page.

Are other structures possible? The largest structure in the picture above starts from a block that is 30 cells wide. And with the more than ten billion blocks between 30 and 34 cells wide, no new structures at all appear. Yet in fact other structures are possible. And the way to tell this is that for small repetition periods there is a systematic procedure that allows one to find absolutely all structures with a given period.

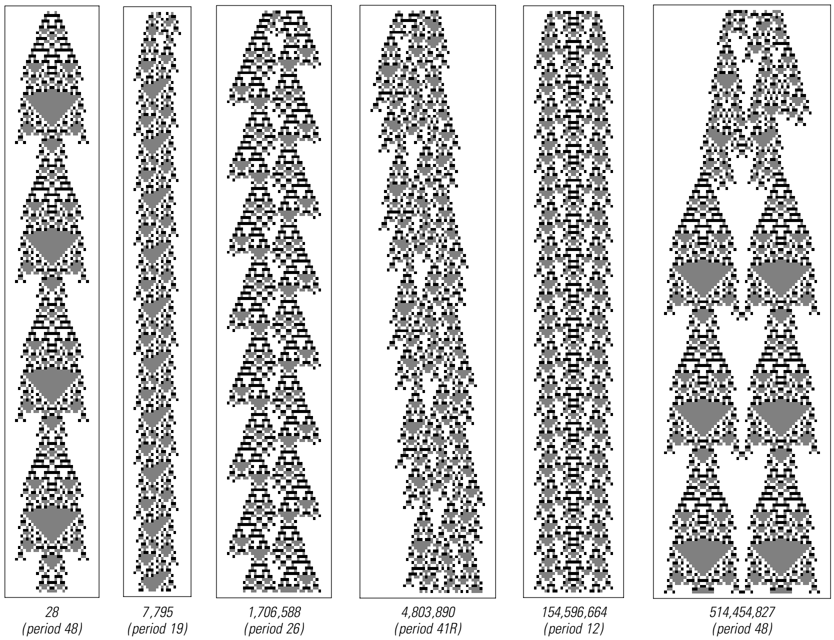
The picture on the facing page shows the results of using this procedure for repetition periods up to 15. And for all repetition periods up to 10—with the exception of 7—at least one fixed or moving structure ultimately turns out to exist. Often, however, the smallest structures for a given period are quite large, so that for example in the case of period 6 the smallest possible structure is 64 cells wide.



All the persistent structures with repetition periods up to 15 steps in the code 20 cellular automaton. The structures shown were found by a systematic method similar to the one used to find all sequences that satisfy the constraints on page 268.

So what about other class 4 cellular automata—like the ones I showed at the beginning of this section? Do they also end up having complicated sets of possible persistent structures?

The picture below shows the structures one finds by explicitly testing the first two billion possible initial conditions for the code 357 cellular automaton from page 282.



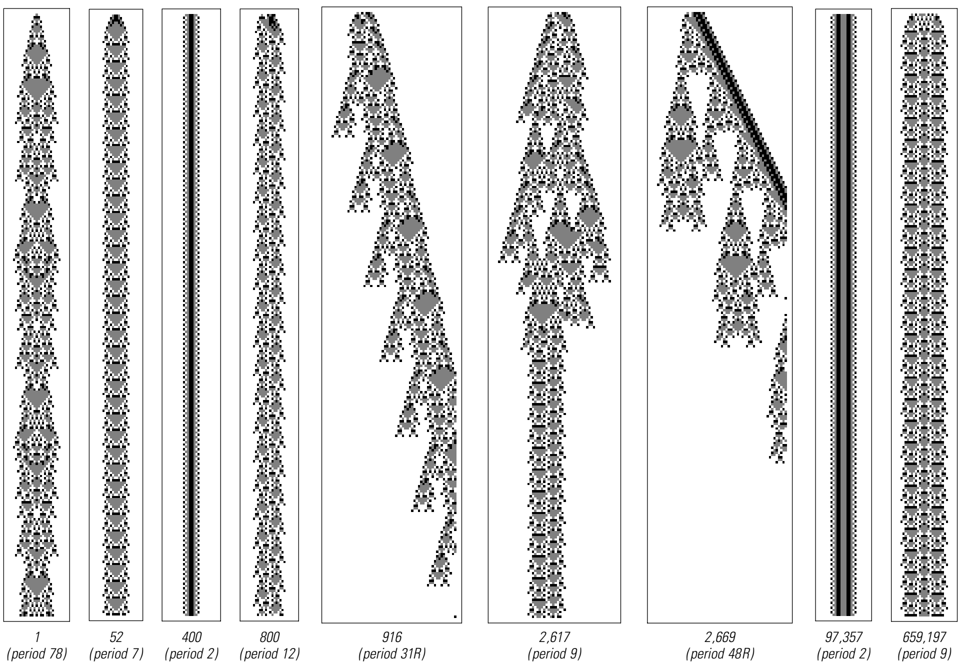
Persistent structures in the code 357 cellular automaton from page 282 obtained by testing the first two billion possible initial conditions. This cellular automaton allows three possible colors for each cell; the initial conditions thus correspond to the base 3 digits of the numbers given. No persistent structures of any size exist in this cellular automaton with repetition periods of less than 5 steps.

Already with initial condition number 28 a fairly complicated structure with repetition period 48 is seen. But with all the first million initial conditions, only one other structure is produced, and this structure is again one that does not move.

So are moving structures in fact possible in the code 357 cellular automaton? My experience with many different rules is that whenever sufficiently complicated persistent structures occur, structures that move can eventually be found. And indeed with code 357, initial condition 4,803,890 yields just such a structure.

So if moving structures are inevitable in class 4 systems, what other fundamentally different kinds of structures might one see if one were to look at sufficiently many large initial conditions?

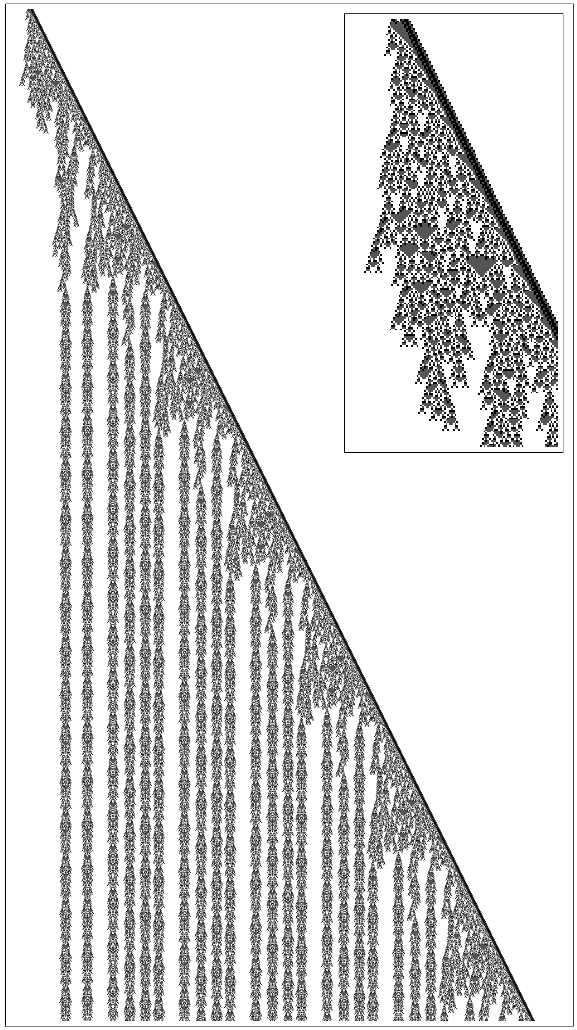
The picture below shows the first few persistent structures found in the code 1329 cellular automaton from the bottom of page 282. The smallest structures are stationary, but at initial condition 916 a structure is found that moves—all much the same as in the two other class 4 cellular automata that we have just discussed.



Persistent structures in the code 1329 cellular automaton shown on page 282.

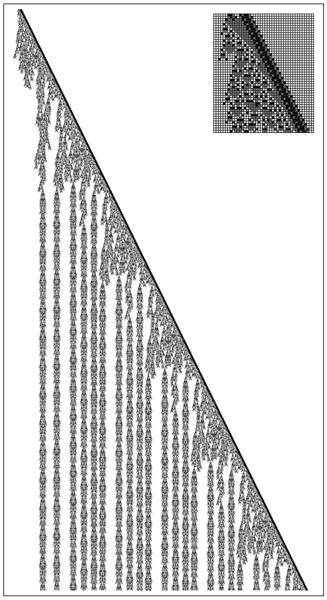
But when initial condition 54,889 is reached, one suddenly sees the rather different kind of structure shown on the next page. The right-hand part of this structure just repeats with a period of 256 steps, but as this part moves, it leaves behind a sequence of other persistent structures. And the result is that the whole structure continues to grow forever, adding progressively more and more cells.

Unbounded growth in code 1329. The initial condition contains a block of 10 cells. The right-hand side of the pattern repeats every 256 steps, and as it moves it leaves behind an infinite sequence of persistent structures.

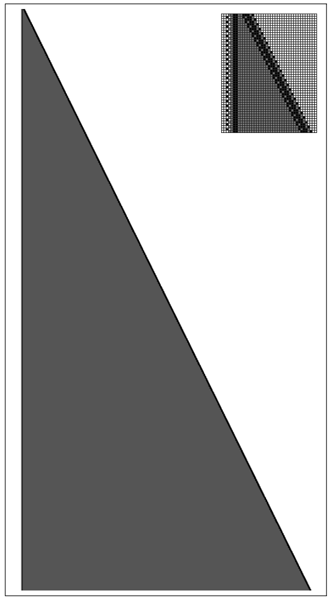


initial condition number 54,889

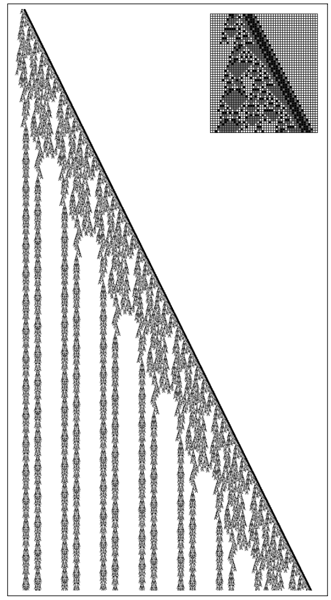
Yet looking at the picture above, one might suppose that when unlimited growth occurs, the pattern produced must be fairly complicated. But once again code 1329 has a surprise in store. For the facing page shows that when one reaches initial condition 97,439 there is again unlimited growth—but now the pattern that is produced is very simple. And in fact if one were just to see this pattern, one would probably assume that it came from a rule whose typical behavior is vastly simpler than code 1329.



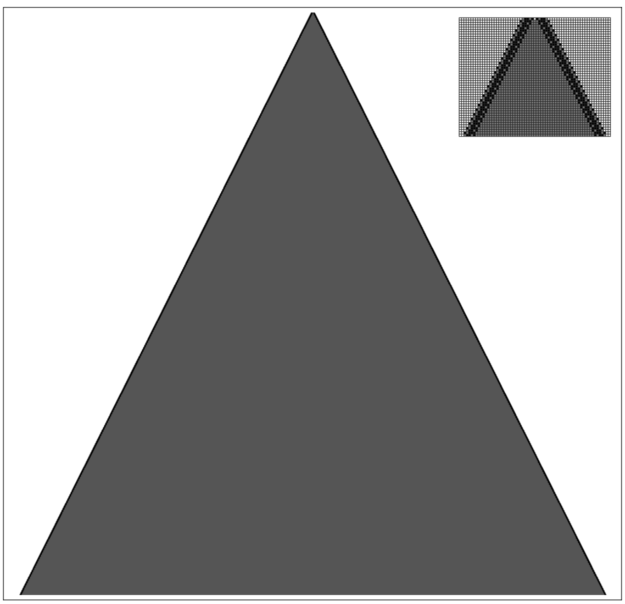
initial condition 54,889



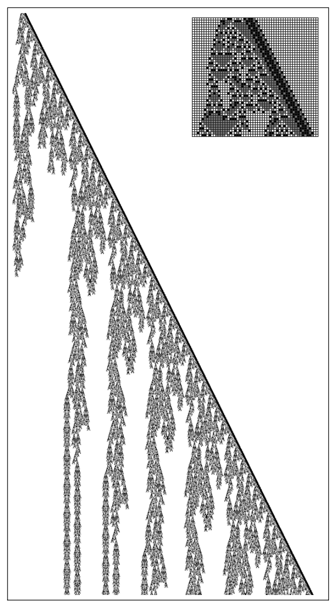
initial condition 97,439



initial condition 166,426



initial condition 115,396



initial condition 2,069,116

Further examples of unbounded growth in code 1329. Most of the patterns produced are complex—but some are simple.

A typical example of the behavior of the rule 110 cellular automaton with random initial conditions. The background pattern consists of blocks of 14 cells that repeat every 7 steps.

Indeed, it is a general feature of class 4 cellular automata that with appropriate initial conditions they can mimic the behavior of all sorts of other systems. And when we discuss computation and the notion of universality in Chapter 11 we will see the fundamental reason this ends up being so. But for now the main point is just how diverse and complex the behavior of class 4 cellular automata can be—even when their underlying rules are very simple.

And perhaps the most striking example is the rule 110 cellular automaton that we first saw on page 32. Its rule is extremely simple—involving just nearest neighbors and two colors of cells. But its overall behavior is as complex as any system we have seen.

The facing page shows a typical example with random initial conditions. And one immediate slight difference from other class 4 rules that we have discussed is that structures in rule 110 do not exist on a blank background: instead, they appear as disruptions in a regular repetitive pattern that consists of blocks of 14 cells repeating every 7 steps.

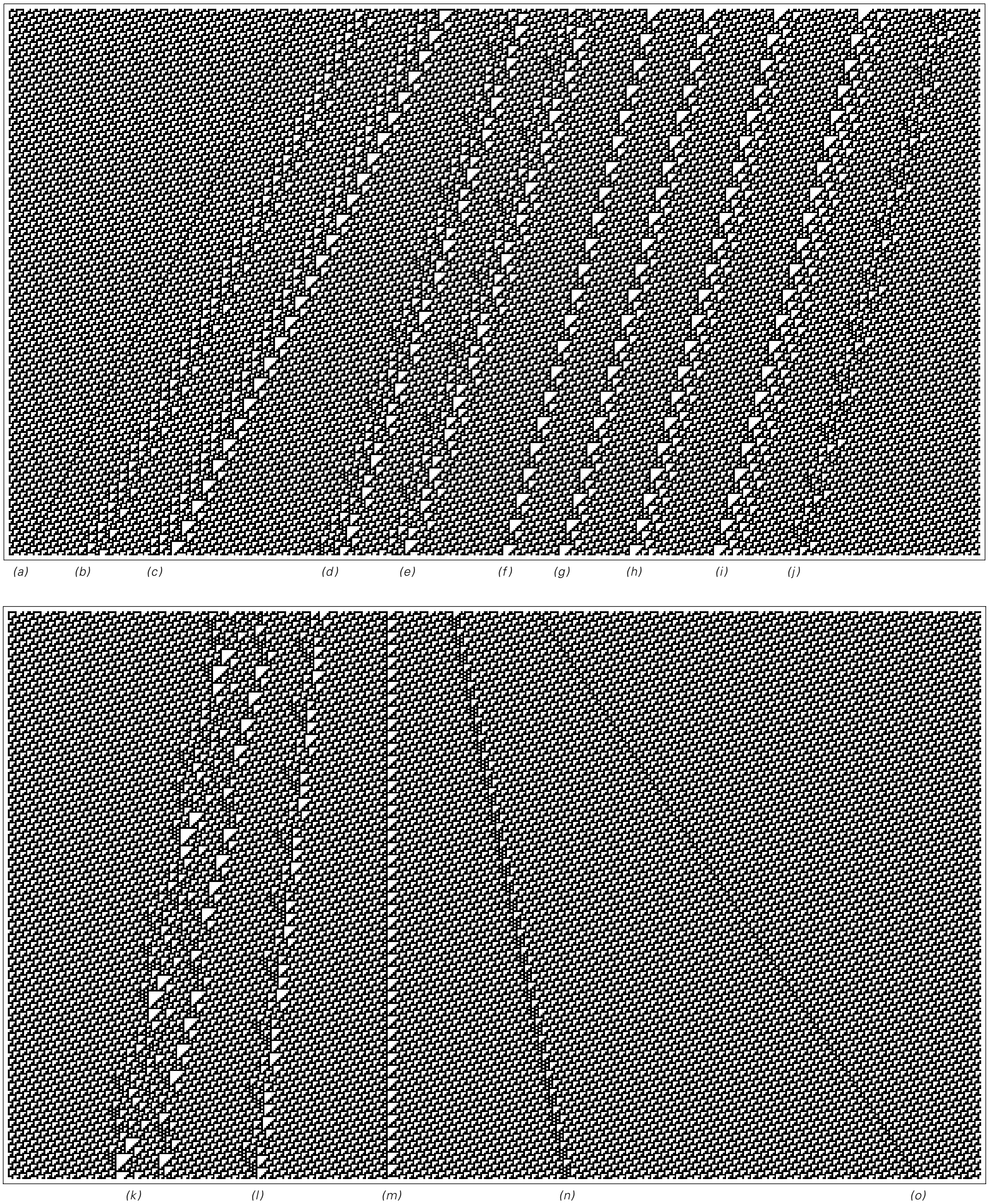
The next page shows the kinds of persistent structures that can be generated in rule 110 from blocks less than 40 cells wide. And just like in other class 4 rules, there are stationary structures and moving structures—as well as structures that can be extended by repeating blocks they contain.

So are there also structures in rule 110 that exhibit unbounded growth? It is certainly not easy to find them. But if one looks at blocks of width 41, then such structures do eventually show up, as the picture on page 293 demonstrates.

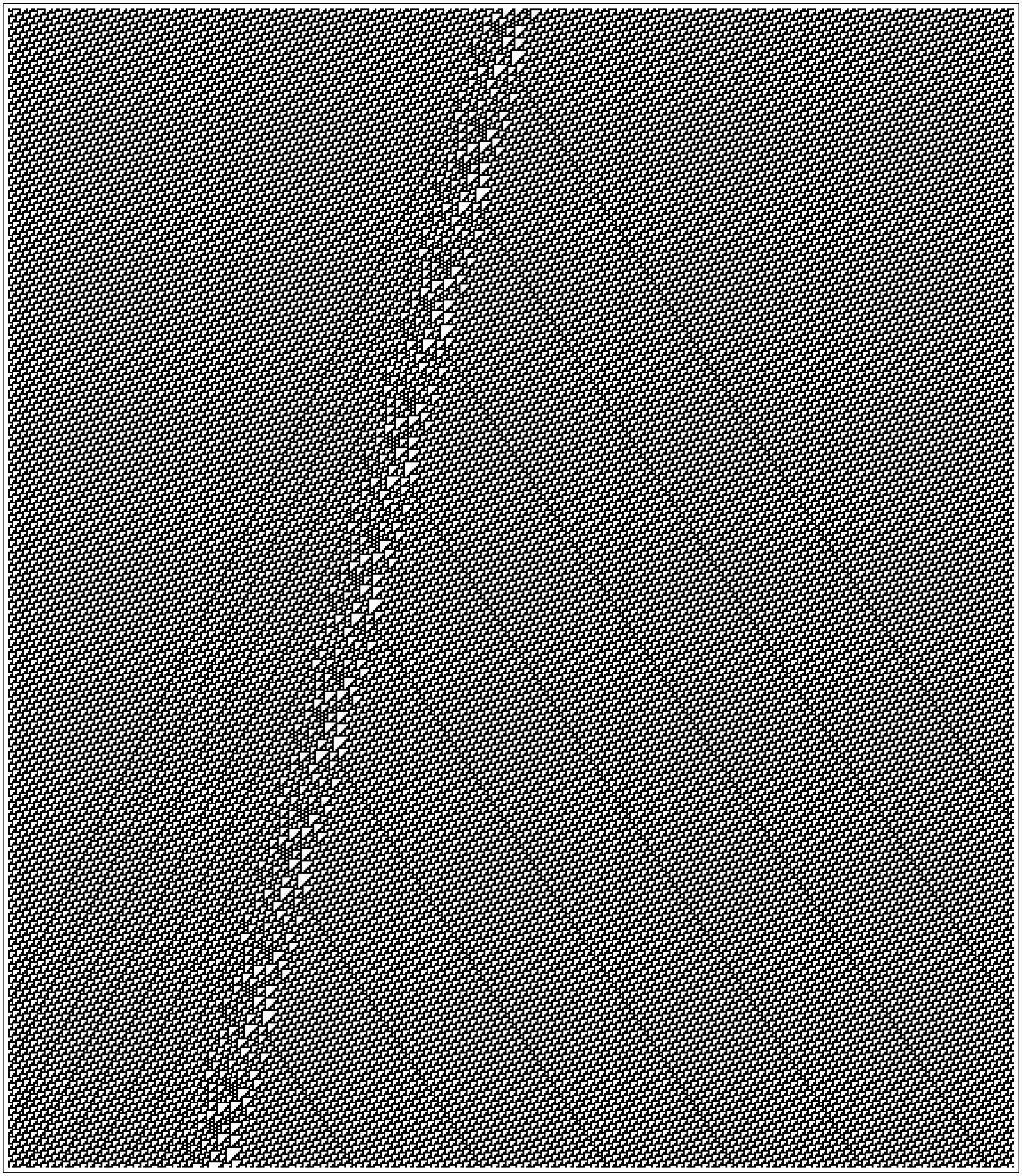
So how do the various structures in rule 110 interact? The answer, as pages 294–296 demonstrate, can be very complicated.

In some cases, one structure essentially just passes through another with a slight delay. But often a collision between two structures produces a whole cascade of new structures. Sometimes the outcome of a collision is evident after a few steps. But quite often it takes a very large number of steps before one can tell for sure what is going to happen.

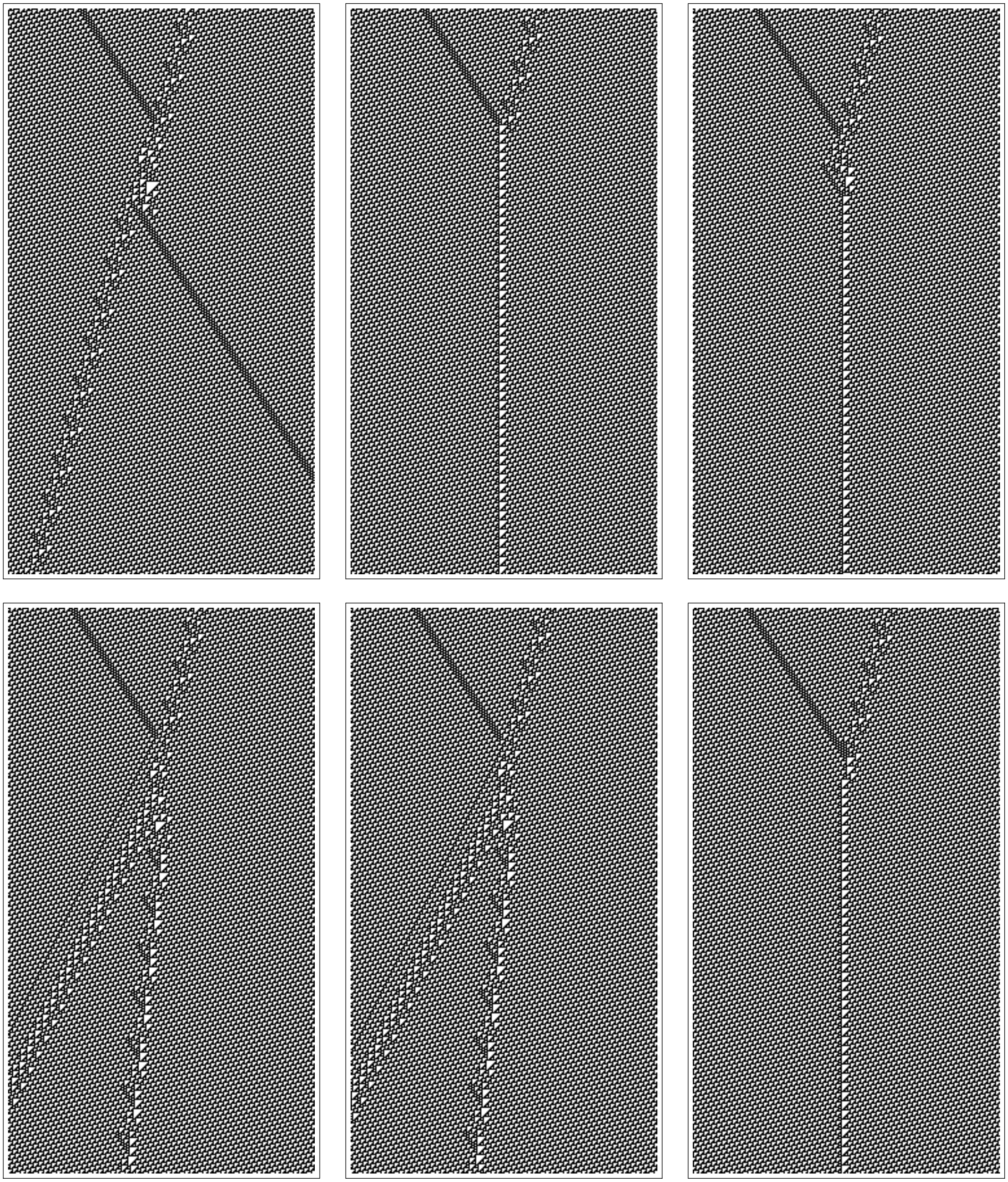
So even though the individual structures in class 4 systems like rule 110 may behave in fairly repetitive ways, interactions between these structures can lead to behavior of immense complexity.



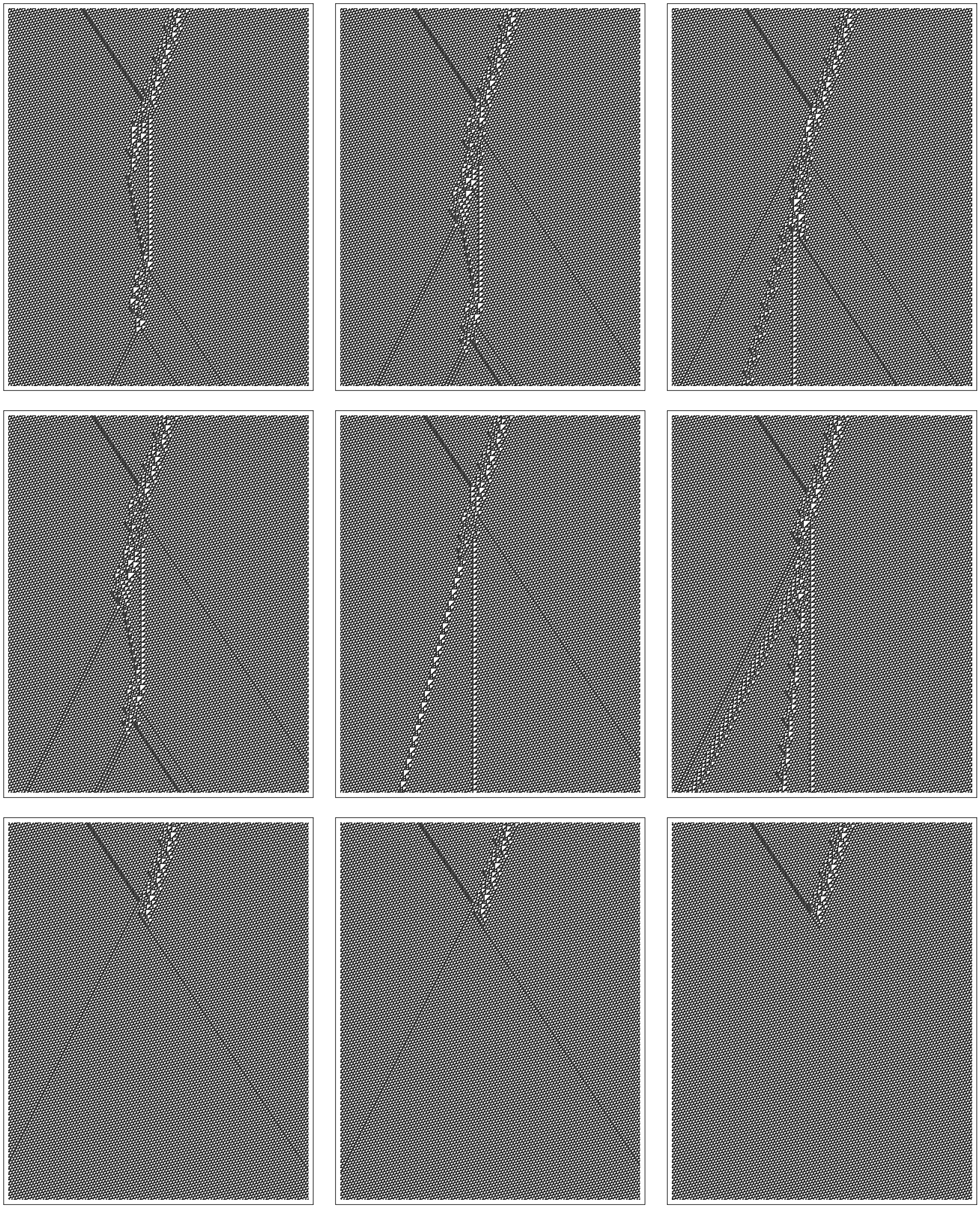
Persistent structures found in rule 110. Extended versions exist of all but structures (a) and (j). Structures (m) and (n) also exist in alternate forms shifted with respect to the background.



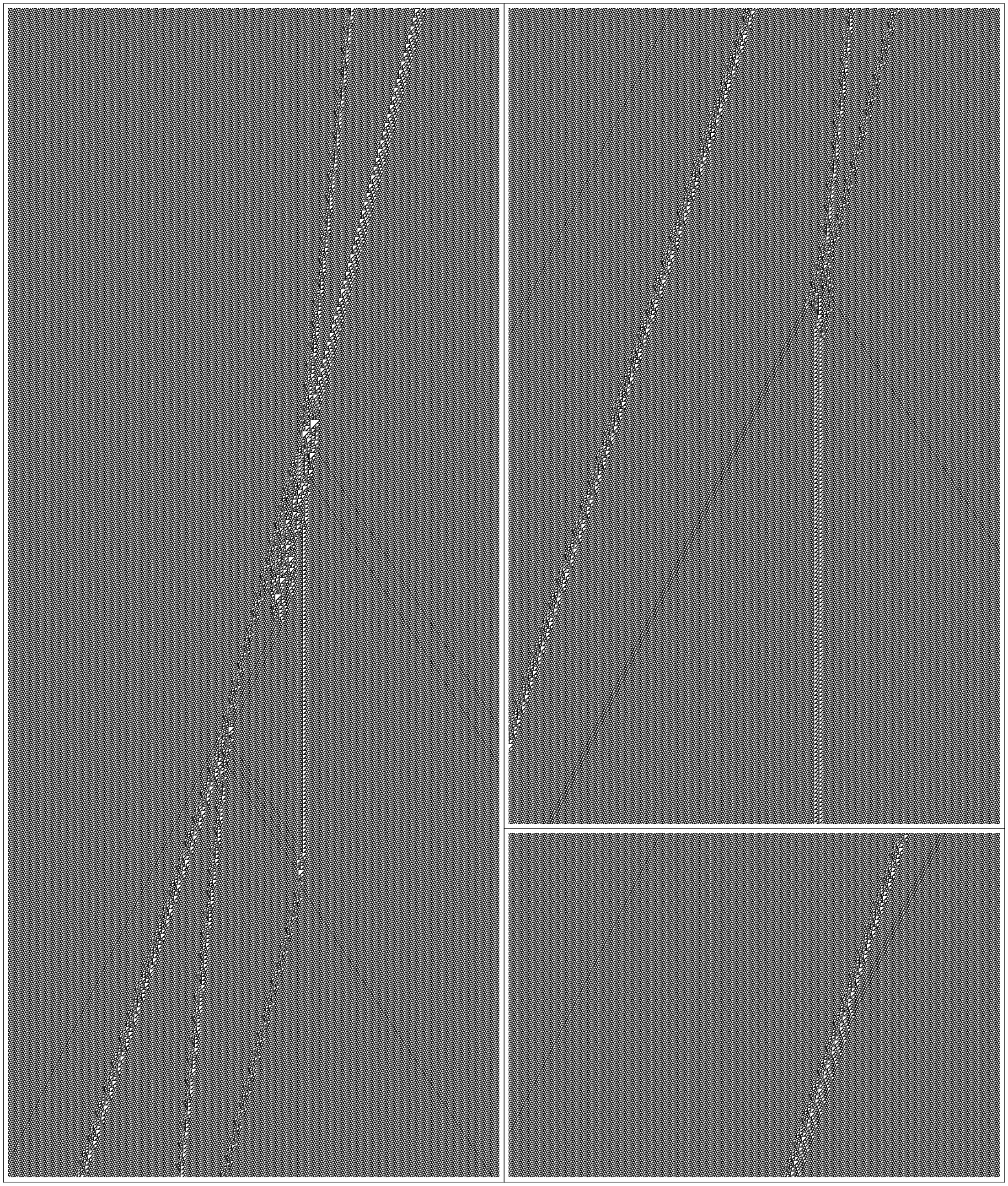
An example of unbounded growth in rule 110. The initial condition consists of a block of length 41 inserted between blocks of the background. New structures on both left and right are produced every 77 steps; the central structure moves 20 cells to the left during each cycle so that the structures on the left are separated by 37 steps while those on the right are separated by 107 steps.



Collisions between persistent structures (o) and (j) from page 292. (The first structure is actually an extended form containing four copies of structure (o) from page 292.) Each successive picture shows what happens when the original structures are started progressively further apart.



Collisions between structures (e) and (o) from page 292.



A collision between structures (l) and (i) from page 292. It takes more than 4000 steps for the final outcome involving 8 separate structures to become clear. The height of the picture corresponds to 2000 steps, and the third picture ends at step 4300.