EXCERPTED FROM

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A NEW KIND OF SCIENCE

SECTION 6.2

Four Classes of Behavior

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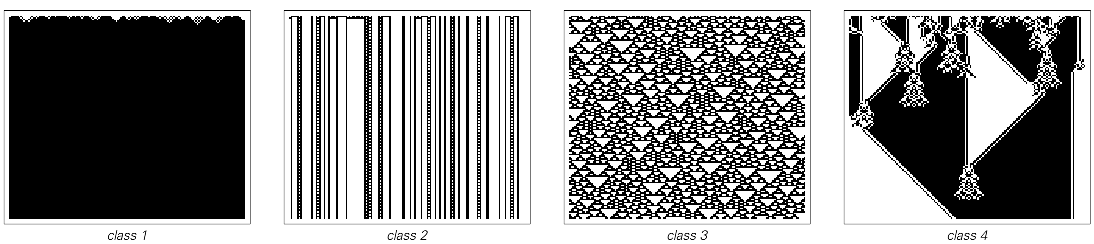
In the previous section we saw what a number of specific cellular automata do if one starts them from random initial conditions. But in this section I want to ask the more general question of what arbitrary cellular automata do when started from random initial conditions.

One might at first assume that such a general question could never have a useful answer. For every single cellular automaton after all ultimately has a different underlying rule, with different properties and potentially different consequences.

But the next few pages show various sequences of cellular automata, all starting from random initial conditions.

And while it is indeed true that for almost every rule the specific pattern produced is at least somewhat different, when one looks at all the rules together, one sees something quite remarkable: that even though each pattern is different in detail, the number of fundamentally different types of patterns is very limited.

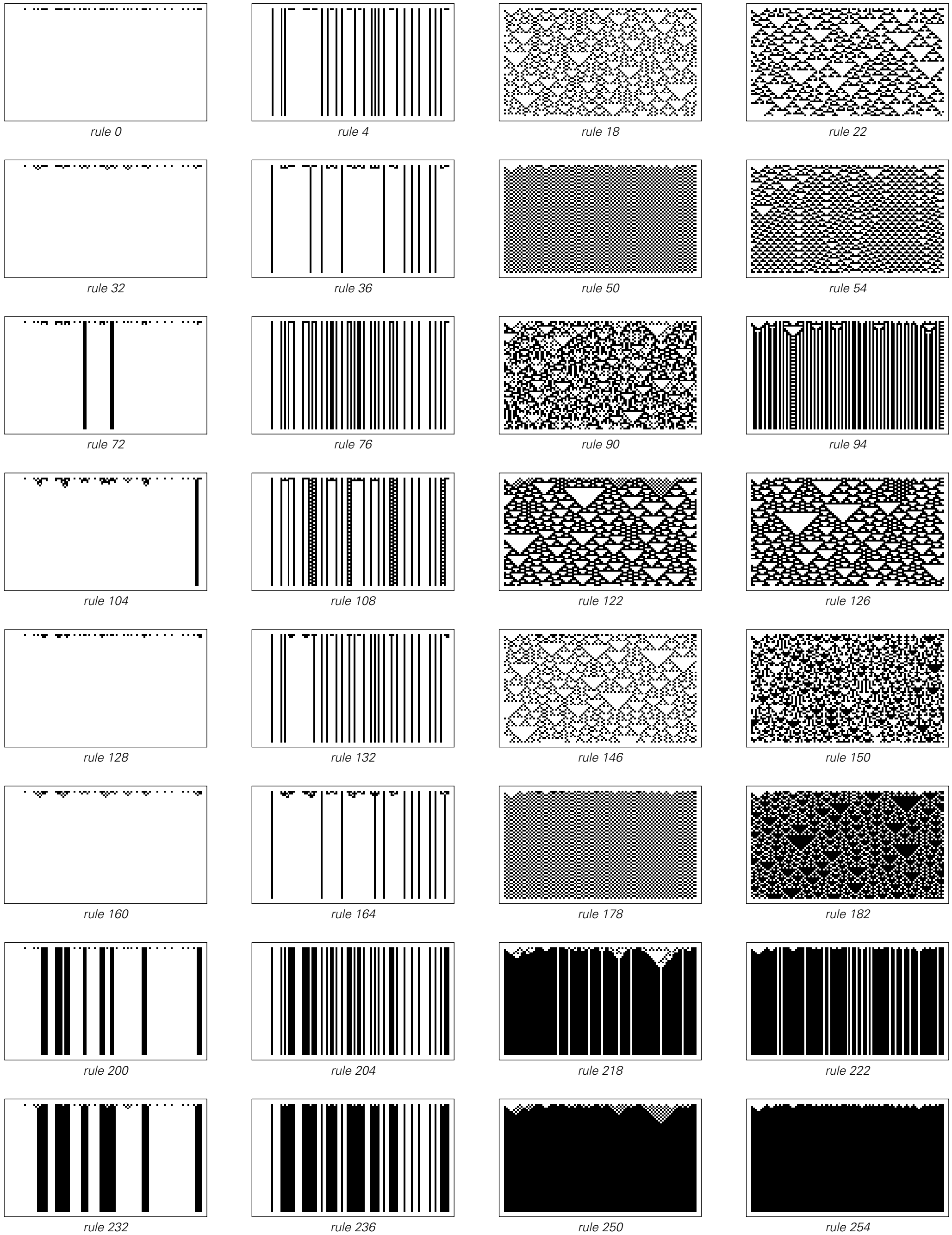
Indeed, among all kinds of cellular automata, it seems that the patterns which arise can almost always be assigned quite easily to one of just four basic classes illustrated below.



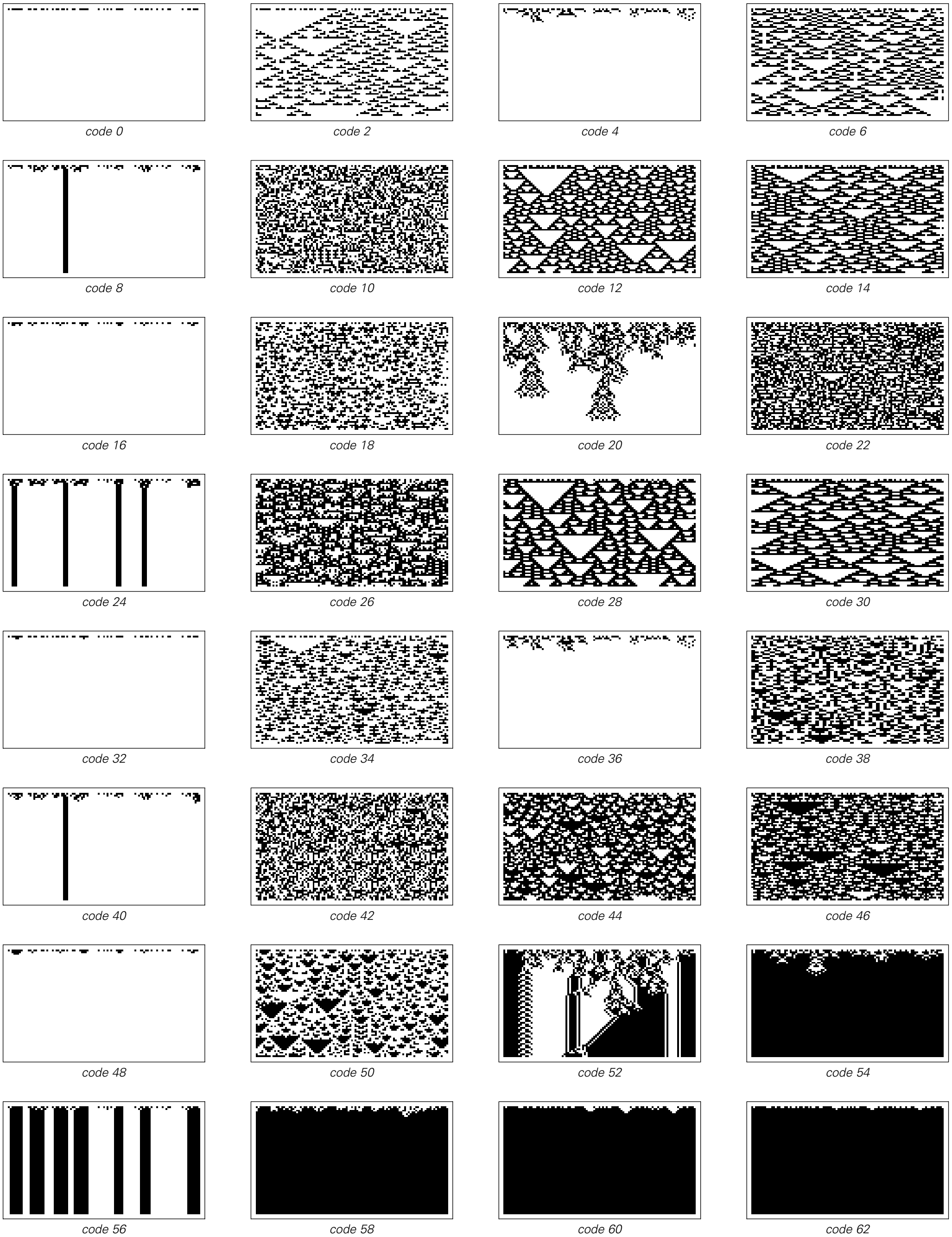
Examples of the four basic classes of behavior seen in the evolution of cellular automata from random initial conditions. I first developed this classification in 1983.

These classes are conveniently numbered in order of increasing complexity, and each one has certain immediate distinctive features.

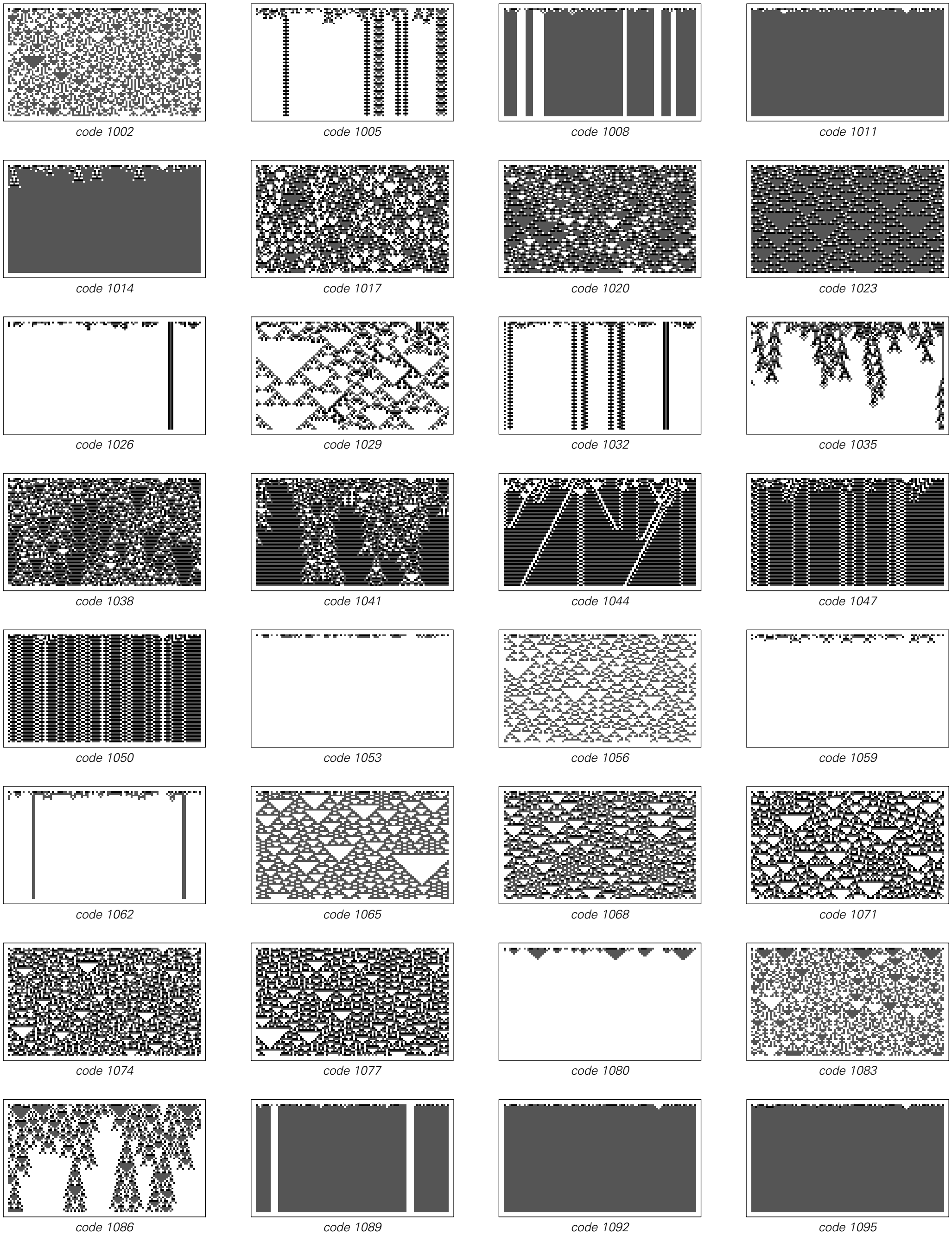
In class 1, the behavior is very simple, and almost all initial conditions lead to exactly the same uniform final state.



The behavior of all cellular automata that involve only nearest neighbors in a symmetrical way, have two possible colors for each cell, and leave states consisting only of white cells unchanged.



Totalistic cellular automata whose rules involve nearest and next-nearest neighbors, and where each cell has two possible colors.



A sequence of totalistic cellular automata with rules that involve only nearest neighbors, but where each cell can have three possible colors.

In class 2, there are many different possible final states, but all of them consist just of a certain set of simple structures that either remain the same forever or repeat every few steps.

In class 3, the behavior is more complicated, and seems in many respects random, although triangles and other small-scale structures are essentially always at some level seen.

And finally, as illustrated on the next few pages, class 4 involves a mixture of order and randomness: localized structures are produced which on their own are fairly simple, but these structures move around and interact with each other in very complicated ways.

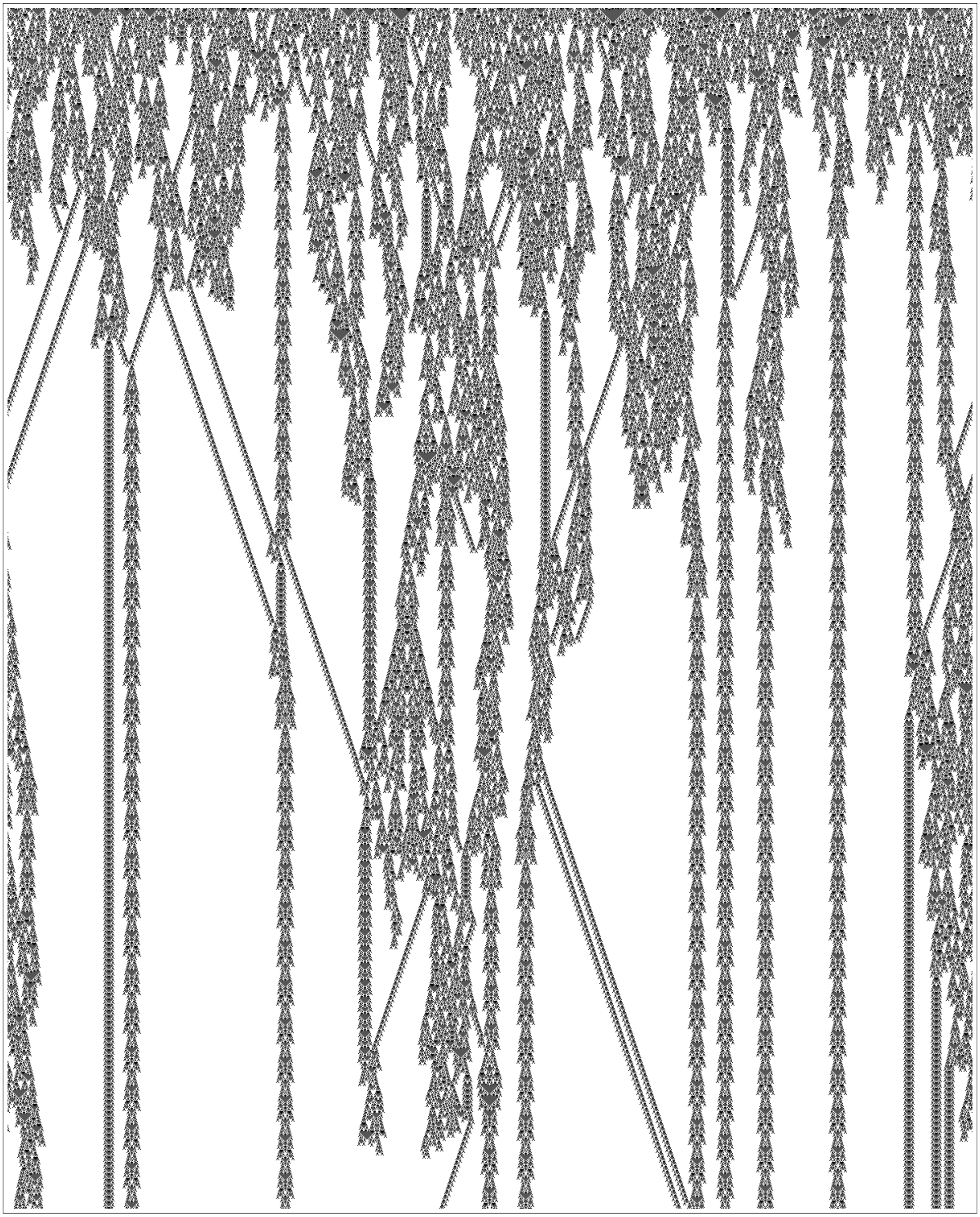
I originally discovered these four classes of behavior some seventeen years ago by looking at thousands of pictures similar to those on the last few pages. And at first, much as I have done here, I based my classification purely on the general visual appearance of the patterns I saw.

But when I studied more detailed properties of cellular automata, what I found was that most of these properties were closely correlated with the classes that I had already identified. Indeed, in trying to predict detailed properties of a particular cellular automaton, it was often enough just to know what class the cellular automaton was in.

And in a sense the situation was similar to what is seen, say, with the classification of materials into solids, liquids and gases, or of living organisms into plants and animals. At first, a classification is made purely on the basis of general appearance. But later, when more detailed properties become known, these properties turn out to be correlated with the classes that have already been identified.

Often it is possible to use such detailed properties to make more precise definitions of the original classes. And typically all reasonable definitions will then assign any particular system to the same class.

Examples of class 4 cellular automata with totalistic rules involving nearest neighbors and three possible colors for each cell. Each picture shows 1500 steps of evolution from random initial conditions. ▸



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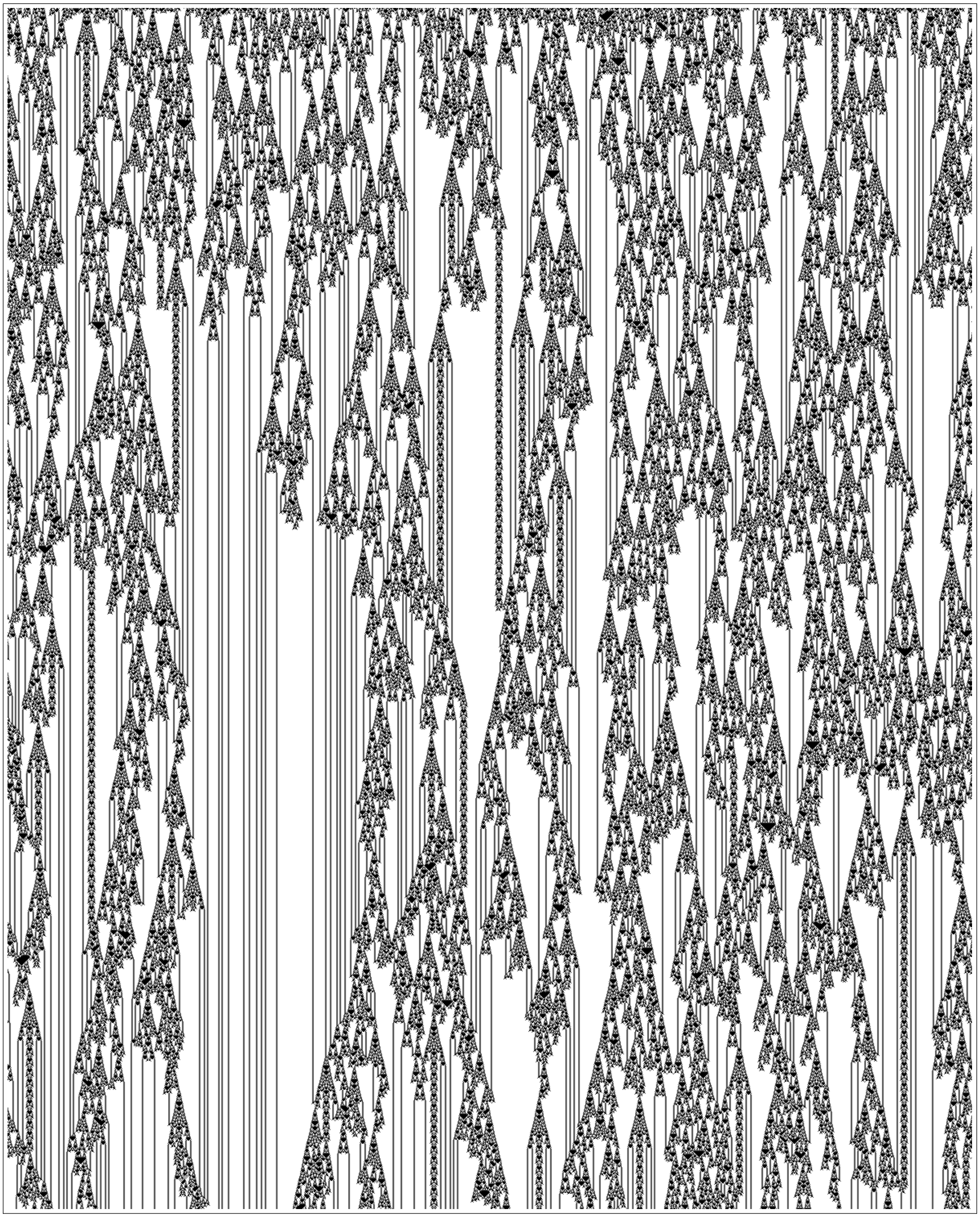

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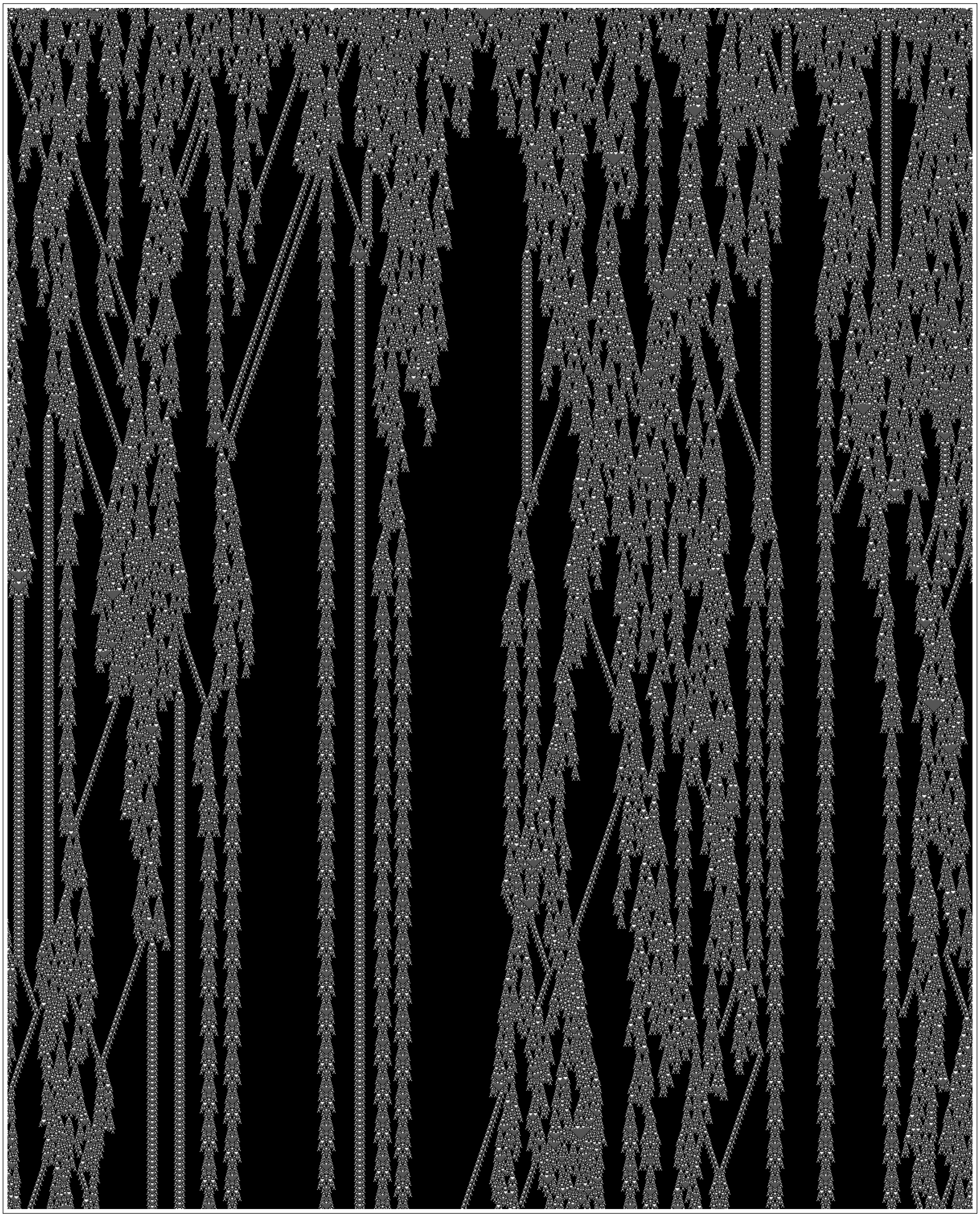

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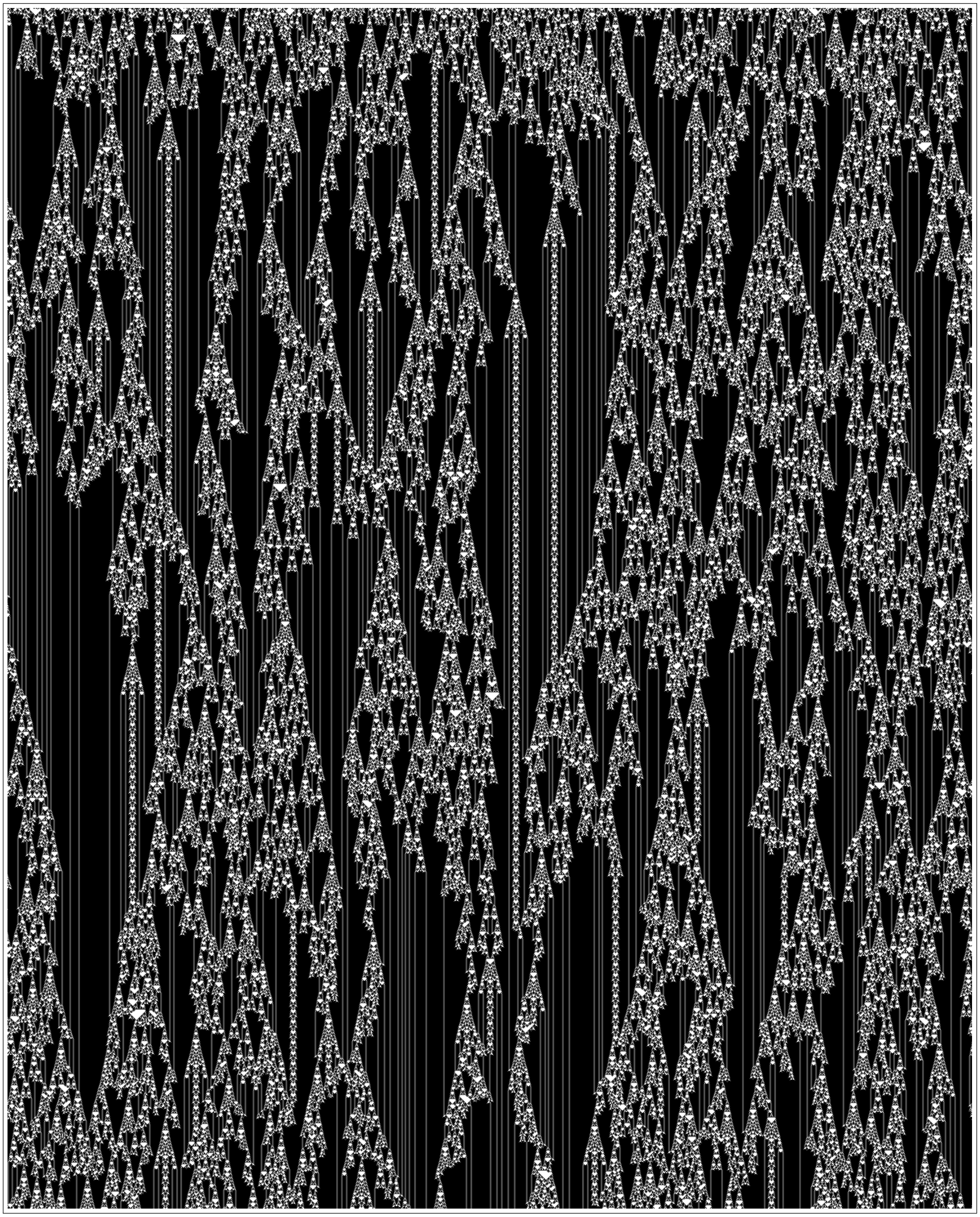

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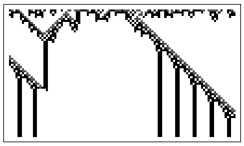
But with almost any general classification scheme there are inevitably borderline cases which get assigned to one class by one definition and another class by another definition. And so it is with cellular automata: there are occasionally rules like those in the pictures below that show some features of one class and some of another.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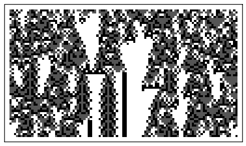
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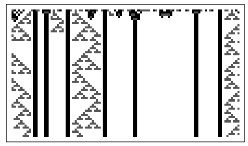
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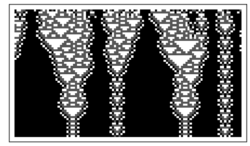
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Rare examples of borderline cellular automata that do not fit squarely into any one of the four basic classes described in the text. Different definitions based on different specific properties will place these cellular automata into different classes. The rules shown are totalistic ones involving nearest neighbors and three possible colors for each cell. The first rule can be either class 2 or class 4, the second class 3 or 4, the third class 2 or 3 and the fourth class 1, 2 or 3.

But such rules are quite unusual, and in most cases the behavior one sees instead falls squarely into one of the four classes described above.

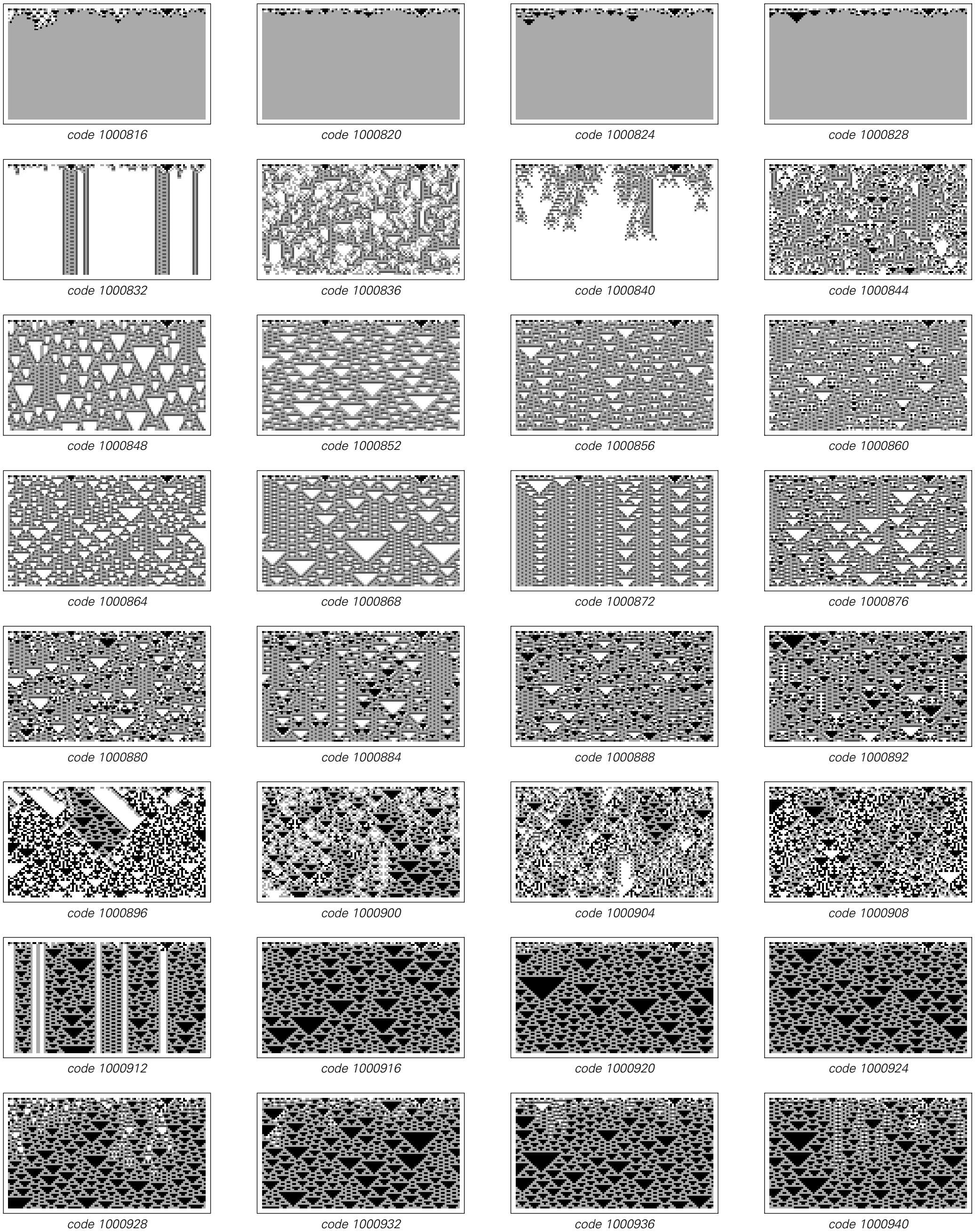
So given the underlying rule for a particular cellular automaton, can one tell what class of behavior the cellular automaton will produce?

In most cases there is no easy way to do this, and in fact there is little choice but just to run the cellular automaton and see what it does.

But sometimes one can tell at least a certain amount simply from the form of the underlying rule. And so for example all rules that lie in the first two columns on page 232 can be shown to be unable ever to produce anything besides class 1 or class 2 behavior.

In addition, even when one can tell rather little from a single rule, it is often the case that rules which occur next to each other in some sequence have similar behavior. This can be seen for example in the pictures on the facing page. The top row of rules all have class 1 behavior. But then class 2 behavior is seen, followed by class 4 and then class 3. And after that, the remainder of the rules are mostly class 3.

The fact that class 4 appears between class 2 and class 3 in the pictures on the facing page is not uncommon. For while class 4 is above class 3 in terms of apparent complexity, it is in a sense intermediate



A sequence of totalistic rules involving nearest neighbors and four possible colors for each cell chosen to show transitions between rules with different classes of behavior. Note that class 4 seems to occur between class 2 and class 3.

between class 2 and class 3 in terms of what one might think of as overall activity.

The point is that class 1 and 2 systems rapidly settle down to states in which there is essentially no further activity. But class 3 systems continue to have many cells that change at every step, so that they in a sense maintain a high level of activity forever. Class 4 systems are then in the middle: for the activity that they show neither dies out completely, as in class 2, nor remains at the high level seen in class 3.

And indeed when one looks at a particular class 4 system, it often seems to waver between class 2 and class 3 behavior, never firmly settling on either of them.

In some respects it is not surprising that among all possible cellular automata one can identify some that are effectively on the boundary between class 2 and class 3. But what is remarkable about actual class 4 systems that one finds in practice is that they have definite characteristics of their own—most notably the presence of localized structures—that seem to have no direct relation to being somehow on the boundary between class 2 and class 3.

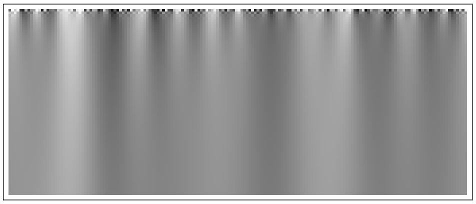
And it turns out that class 4 systems with the same general characteristics are seen for example not only in ordinary cellular automata but also in such systems as continuous cellular automata.

The facing page shows a sequence of continuous cellular automata of the kind we discussed on page 155. The underlying rules in such systems involve a parameter that can vary smoothly from 0 to 1.

For different values of this parameter, the behavior one sees is different. But it seems that this behavior falls into essentially the same four classes that we have already seen in ordinary cellular automata. And indeed there are even quite direct analogs of for example the triangle structures that we saw in ordinary class 3 cellular automata.

But since continuous cellular automata have underlying rules based on a continuous parameter, one can ask what happens if one smoothly varies this parameter—and in particular one can ask what sequence of classes of behavior one ends up seeing.

The answer is that there are normally some stretches of class 1 or 2 behavior, and some stretches of class 3 behavior. But at the transitions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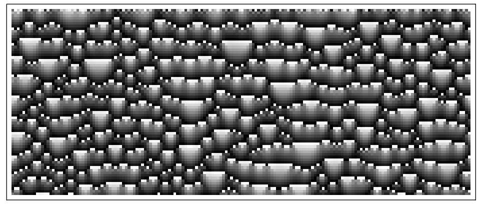
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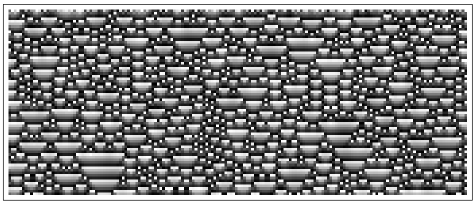
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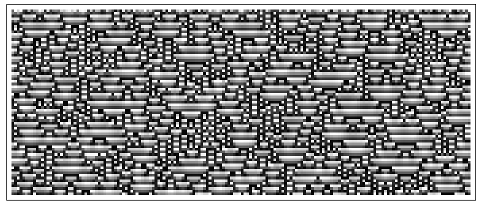
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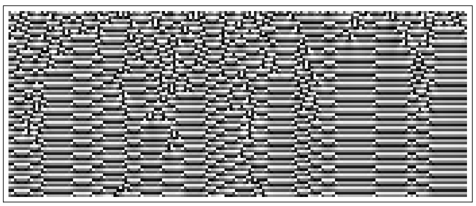
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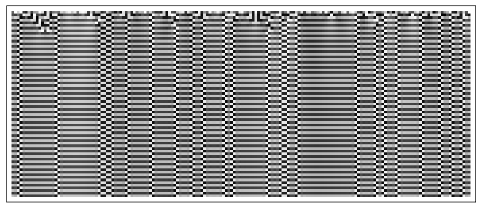
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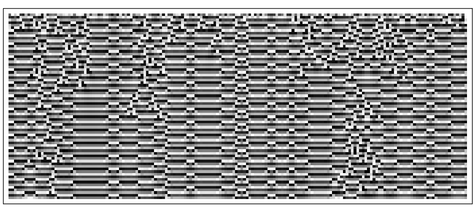
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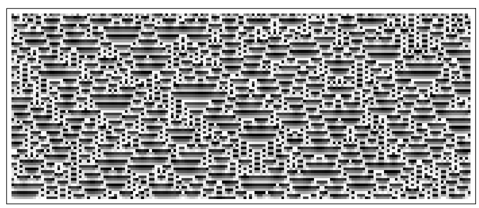
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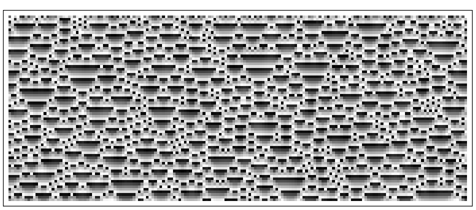
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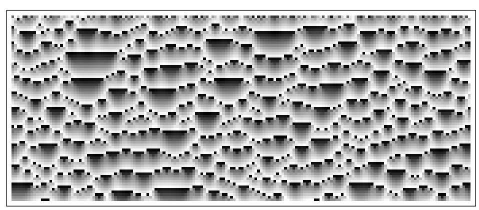
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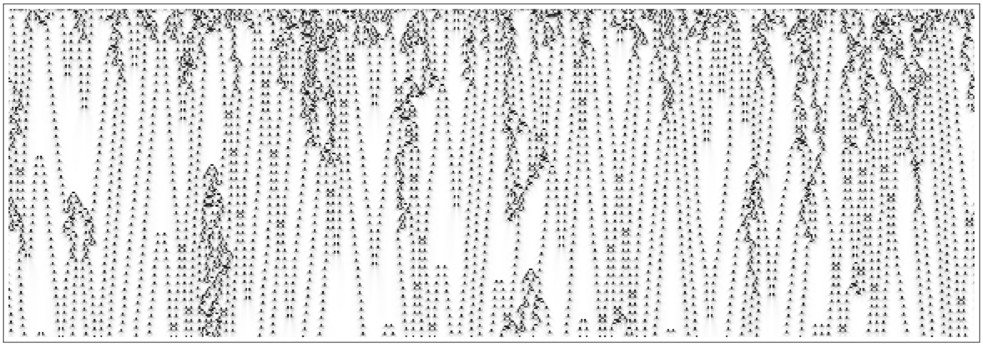
Examples of the evolution of continuous cellular automata from random initial conditions. As discussed on page 155, each cell here can have any gray level between 0 and 1, and at each step the gray level of a given cell is determined by averaging the gray levels of the cell and its two neighbors, adding the specified constant, and then keeping only the fractional part of the result. The behavior produced once again falls into distinct classes that correspond well to the four classes seen on previous pages in ordinary cellular automata.



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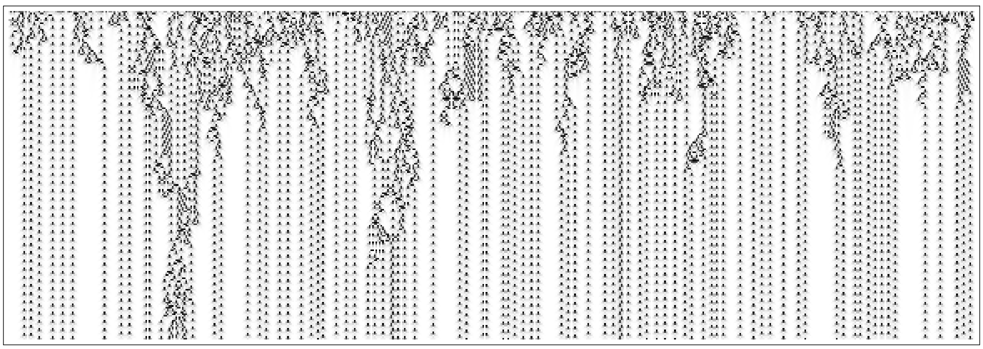

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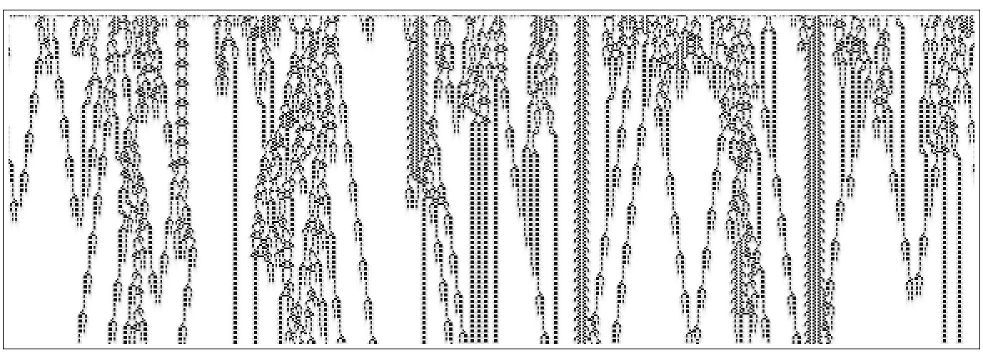


{0.5, 1.13}

Examples of continuous cellular automata that exhibit class 4 behavior. The rules are of the same kind as in the previous picture, except that in the third case shown here, the gray level of each neighboring cell is multiplied by 1.13 before the average is done. In addition, the actual gray levels in these pictures are obtained by taking the difference between the gray level of each cell and its neighbor, thus removing the uniform stripes visible in the previous picture. It is remarkable that class 4 behavior with discrete localized structures can still occur in the continuous systems shown here.

it turns out that class 4 behavior is typically seen—as illustrated on the facing page. And what is particularly remarkable is that this behavior involves the same kinds of localized structures and other features that we saw in ordinary discrete class 4 cellular automata.

So what about two-dimensional cellular automata? Do these also exhibit the same four classes of behavior that we have seen in one dimension? The pictures on the next two pages show various steps in the evolution of some simple two-dimensional cellular automata starting from random initial conditions. And just as in one dimension a few distinct classes of behavior can immediately be seen.

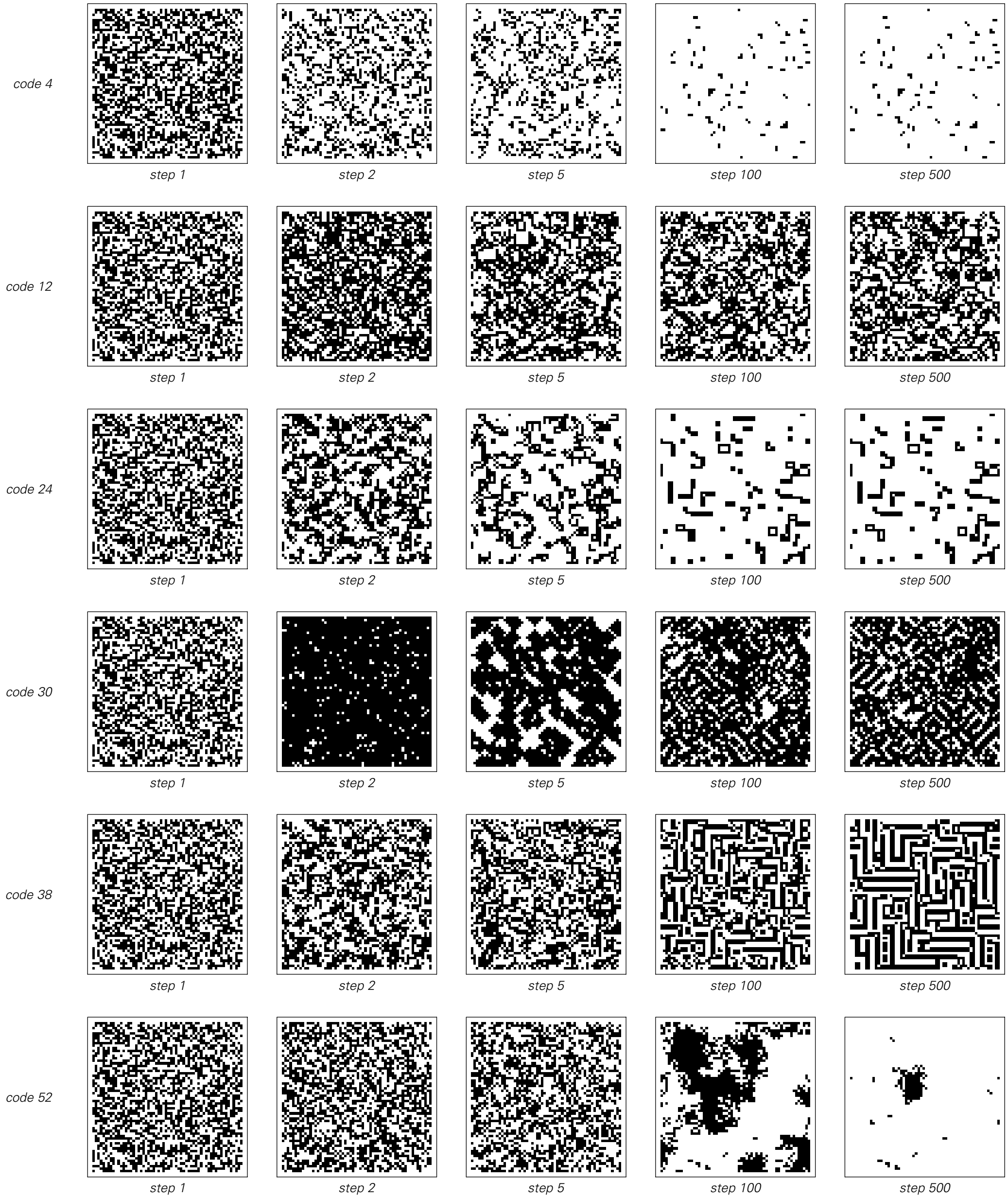
But the correspondence with one dimension becomes much more obvious if one looks not at the complete state of a two-dimensional cellular automaton at a few specific steps, but rather at a one-dimensional slice through the system for a whole sequence of steps.

The pictures on page 248 show examples of such slices. And what we see is that the patterns in these slices look remarkably similar to the patterns we already saw in ordinary one-dimensional cellular automata. Indeed, by looking at such slices one can readily identify the very same four classes of behavior as in one-dimensional cellular automata.

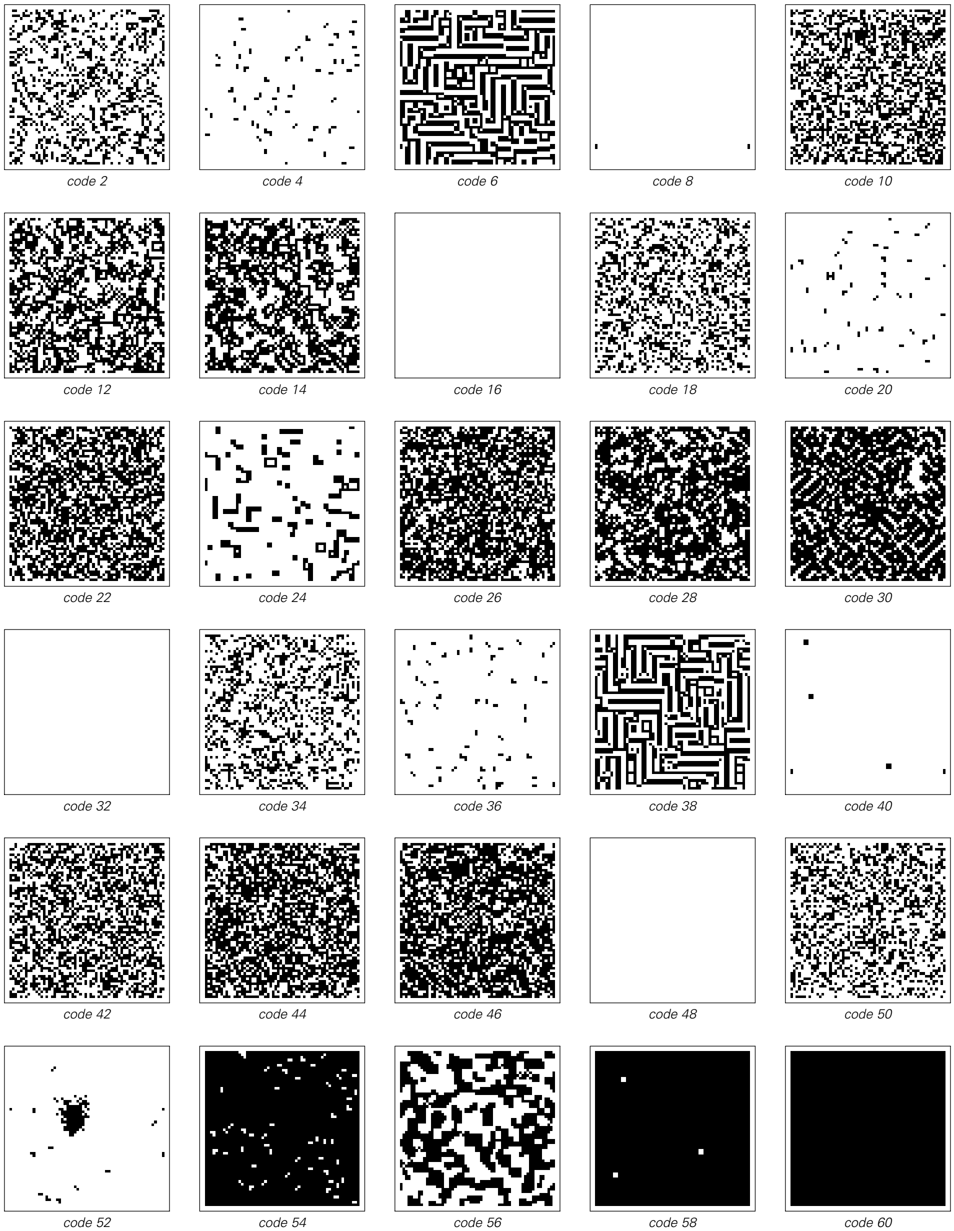
So in particular one sees class 4 behavior. In the examples on page 248, however, such behavior always seems to occur superimposed on some kind of repetitive background—much as in the case of the rule 110 one-dimensional cellular automaton on page 229.

So can one get class 4 behavior with a simple white background? Much as in one dimension this does not seem to happen with the very simplest possible kinds of rules. But as soon as one goes to slightly more complicated rules—though still very simple—one can find examples.

And so as one example page 249 shows a two-dimensional cellular automaton often called the Game of Life in which all sorts of localized structures occur even on a white background. If one watches a movie of the behavior of this cellular automaton its correspondence to a one-dimensional class 4 system is not particularly obvious. But as soon as one looks at a one-dimensional slice—as on page 249—what one sees is immediately strikingly similar to what we have seen in many one-dimensional class 4 cellular automata.

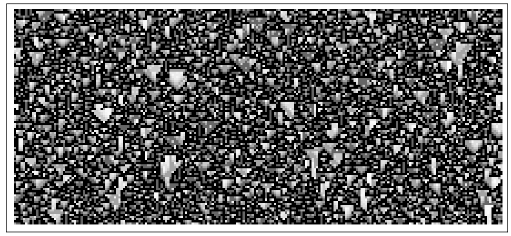


Examples of the evolution of two-dimensional cellular automata with various totalistic rules starting from random initial conditions. The rules involve a cell and its four immediate neighbors. Each successive base 2 digit in the code number for the rule gives the outcome when the total of the cell and its four neighbors runs from 5 down to 0.



Patterns produced after 500 steps in the evolution of a sequence of two-dimensional cellular automata starting from random initial conditions. The rules shown are of the same kind as on the facing page, and include most of the 64 possibilities that leave a state that contains only white cells unchanged.

code 4



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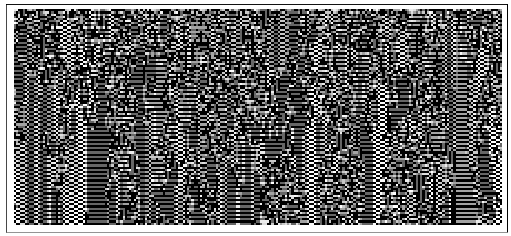
code 24



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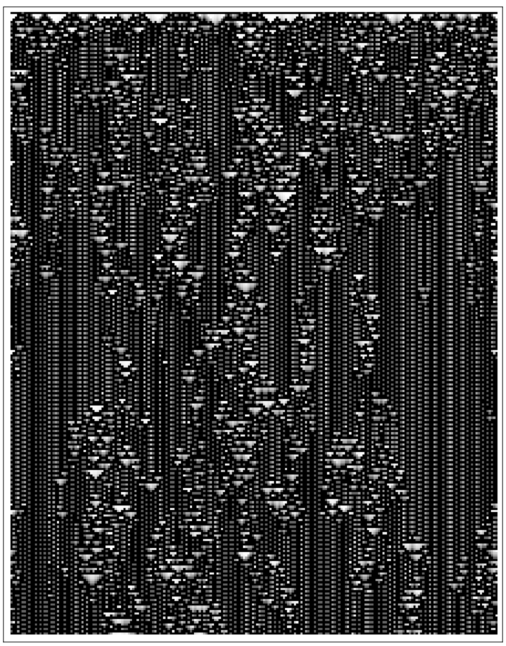

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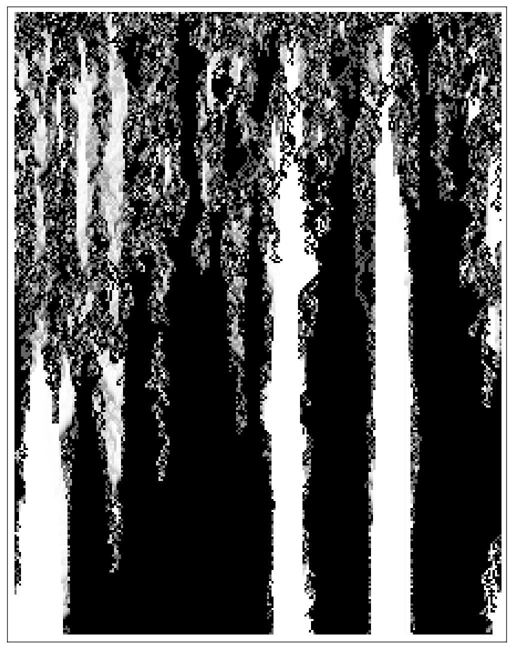
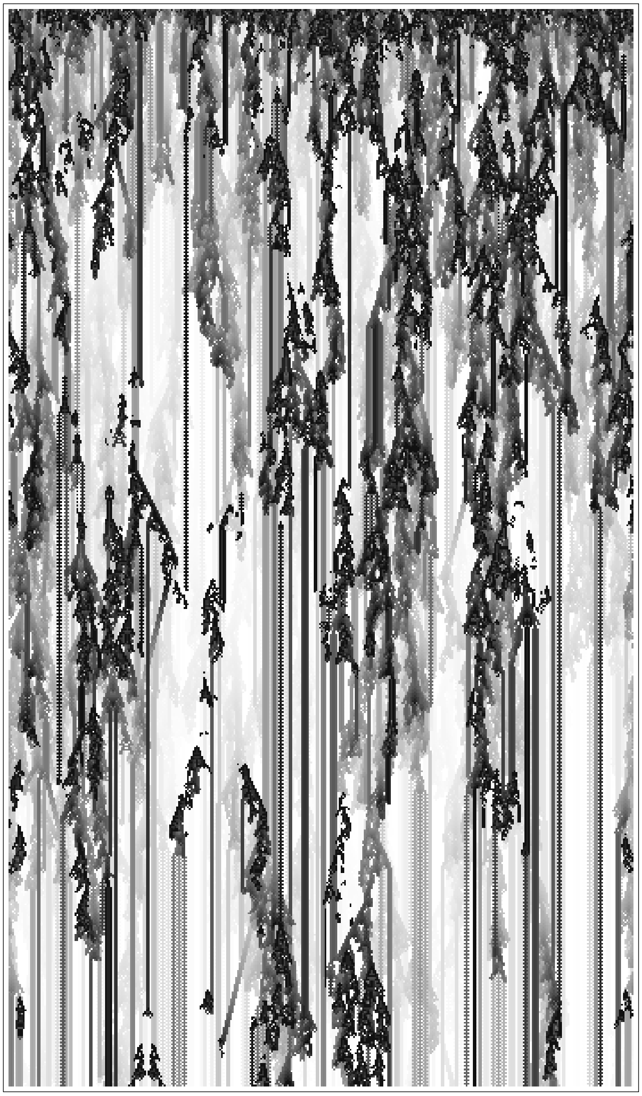
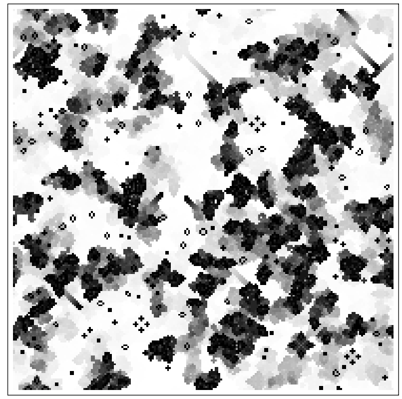


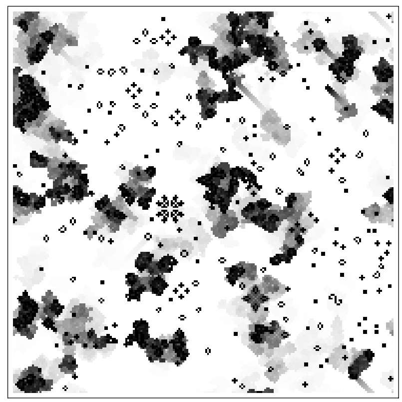
code 52

One-dimensional slices through the evolution of various two-dimensional cellular automata. In each picture black cells further back from the position of the slice are shown in progressively lighter shades of gray, as if they were receding into a kind of fog. Note the presence of examples of both class 3 and class 4 behavior that look strikingly similar to examples in one dimension.

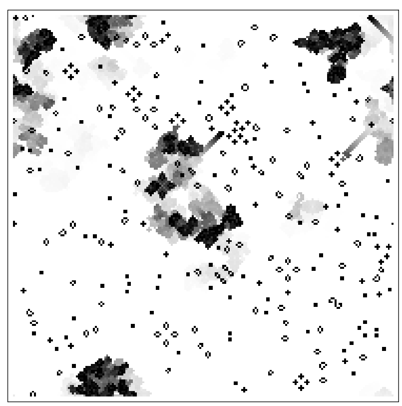




step 200



step 500



step 1000

The behavior of a class 4 two-dimensional cellular automaton often known in recreational computing as the Game of Life. Localized structures that move (so-called gliders) show up as streaks in the pictures given here. The rule for this cellular automaton considers the 8 neighbors of a cell (including diagonals): if two of these neighbors are black, then the cell stays the same color as before; if three are black, then the cell becomes black; and if any other number of neighbors are black, then the cell becomes white. This rule is outer totalistic 9-neighbor code 224. The pictures on the right show cells that were black on preceding steps in progressively lighter shades of gray.