A Problem in Geometric Probability:

Buffon's Needle Problem

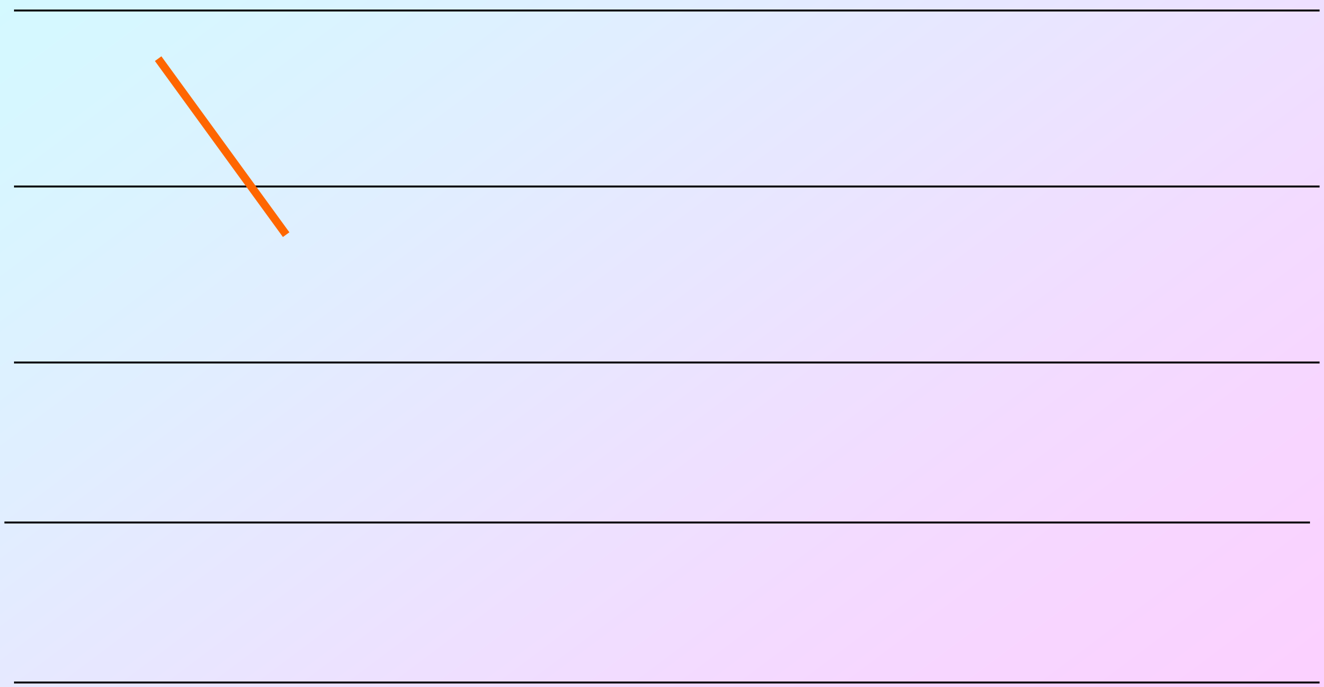
The Plan

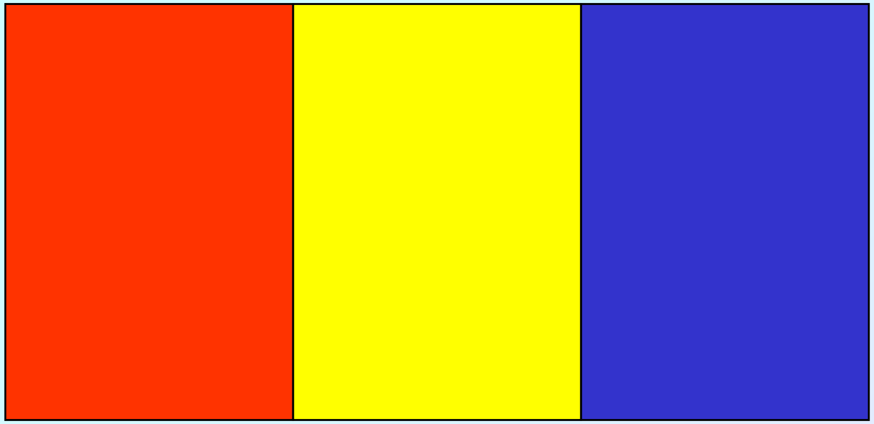
- Introduction to problem
- Some simple ideas from probability
- Set up the problem
- Find solution
- An approximation
- Generalization (solution known)
- Other generalizations (solutions known?)

Buffon's Needle Problem

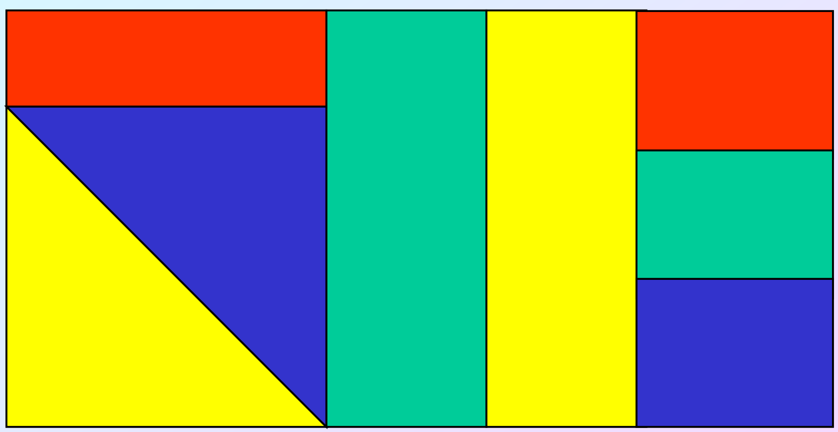
Stated in 1733 solution published 1777

by Geroges Louis Leclerc, Comte de Buffon (1707-1788)



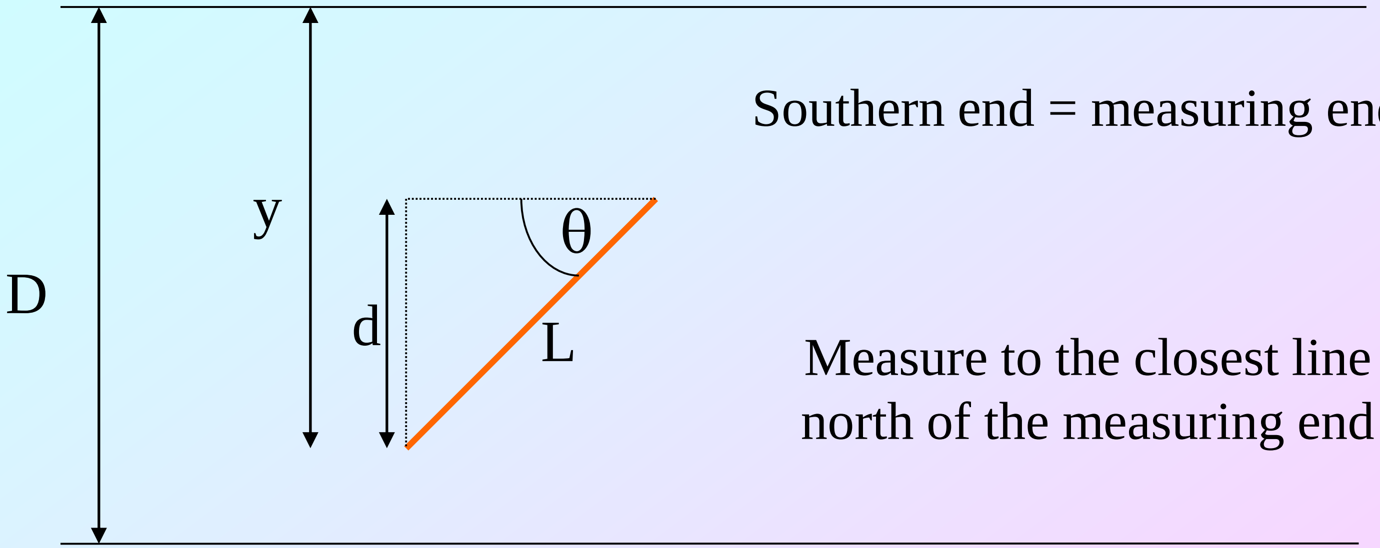


$$P(\text{landing on red}) = \frac{\text{red area}}{\text{total area}}$$



$$P(\text{landing on c}) = \frac{\text{area covered by c}}{\text{total area}}$$

The Set Up



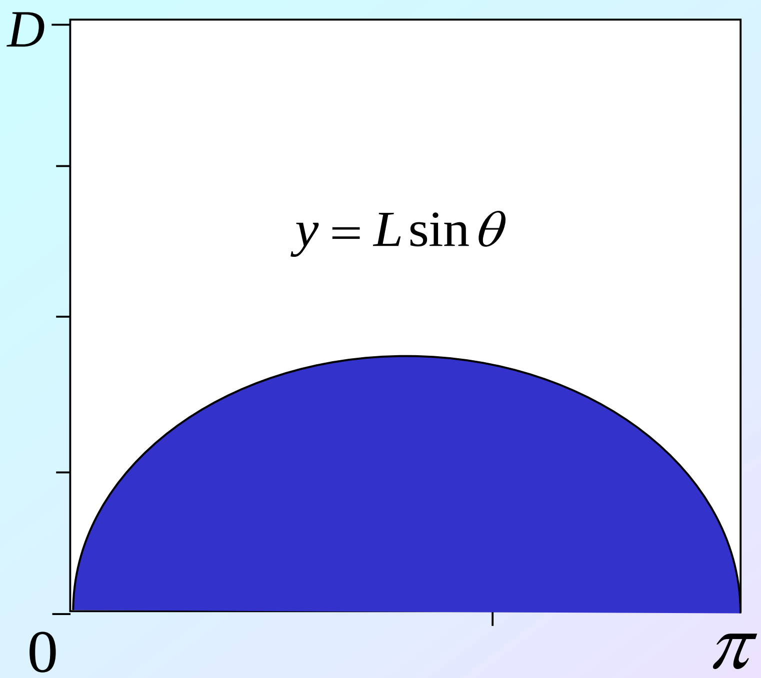
Southern end = measuring end

Measure to the closest line north of the measuring end

$$d = L\sin\theta \qquad 0 \le \theta \le \pi \qquad 0 \le y \le D$$

We have a crossing if $y \le L\sin\theta$

The Solution!



$$\text{blue area} = L\int_0^{\pi} \sin\theta \, d\theta$$

$$= L(-\cos\pi + \cos 0)$$

$$= 2L$$

$$\text{total area} = D\pi$$

$$\text{p(crossing)} = \frac{\text{blue area}}{\text{total area}} = \frac{2L}{D\pi}$$

Polyá, George (1887, 1985)

… a good teacher should understand and impress on his students the view that no problem whatever is completely exhausted.

How to Solve It. Princeton: Princeton University Press. 1945.

Something different

Let $L=1$ and $D=4$, then we have

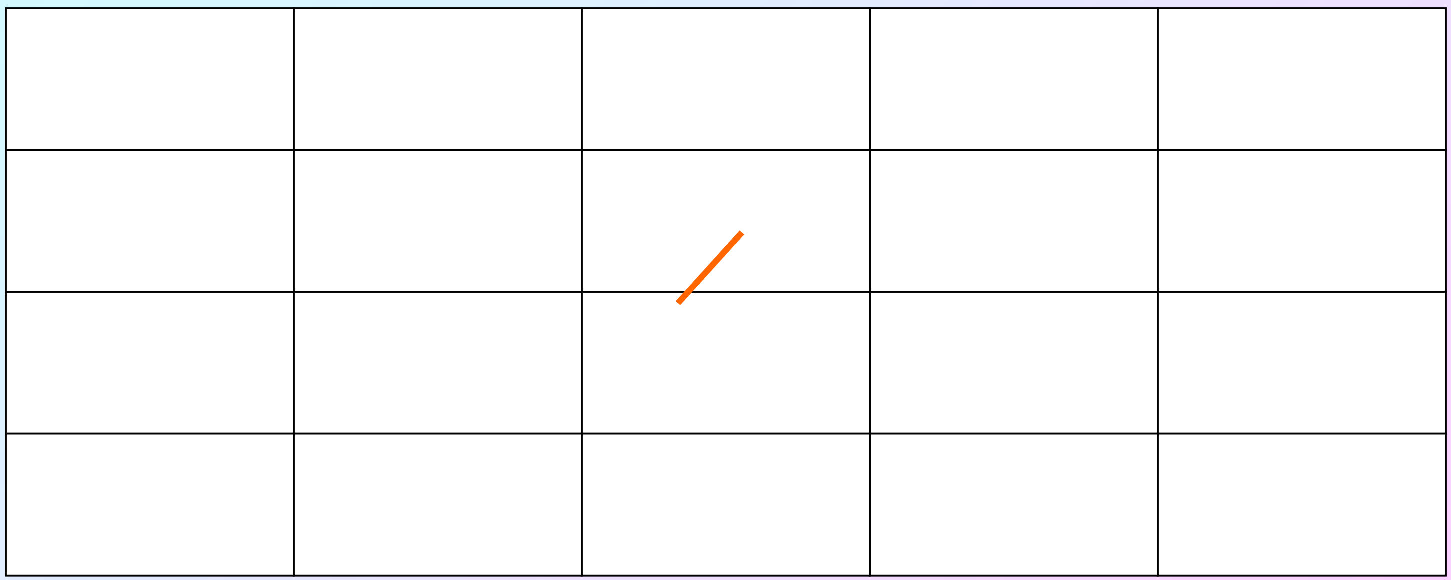
$$P(\text{crossing}) = \frac{2L}{D\pi} = \frac{2(1)}{4\pi} = \frac{1}{2\pi}$$

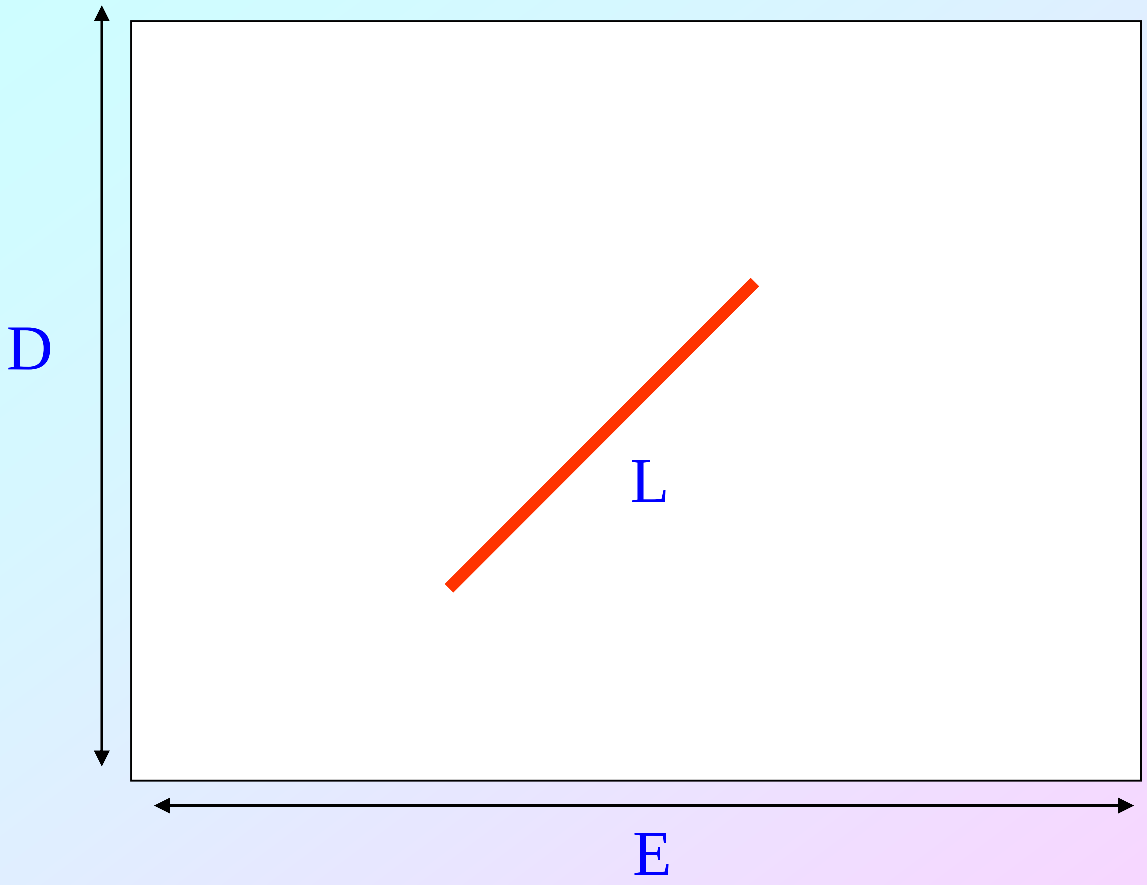
We also know that $P(\text{crossing}) \approx \dfrac{\text{number of actual crossings}}{\text{number of throws}}$

$$\pi \approx \frac{\text{number of throws}}{2\ \text{number of crossings}}$$

Simulation

Generalizations

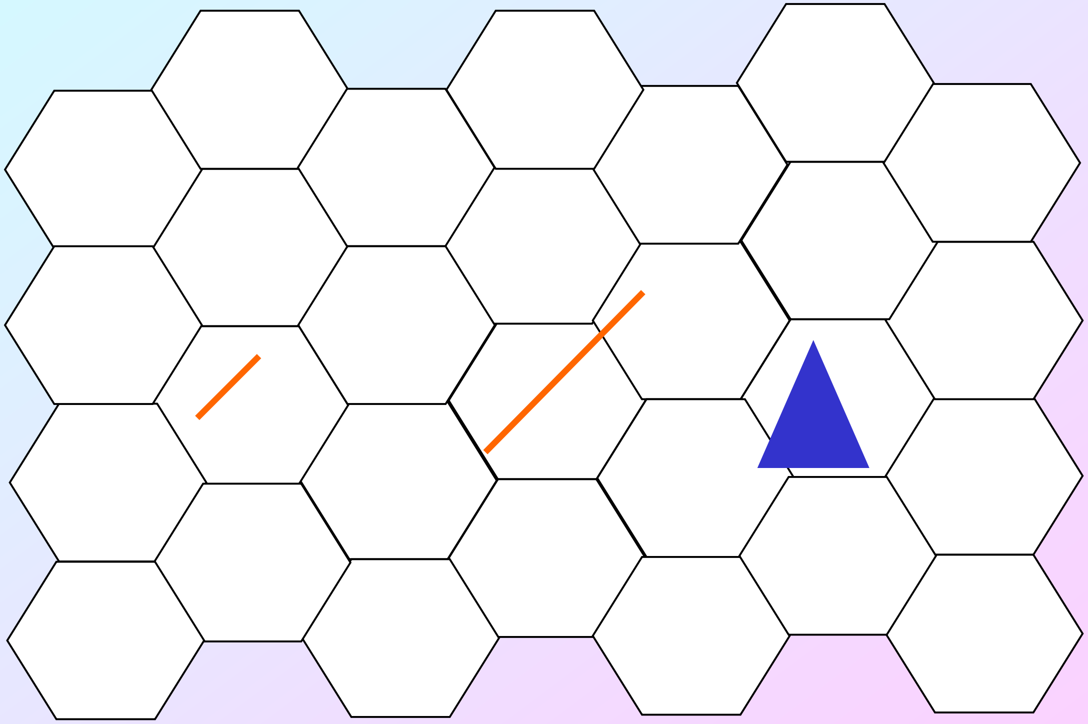




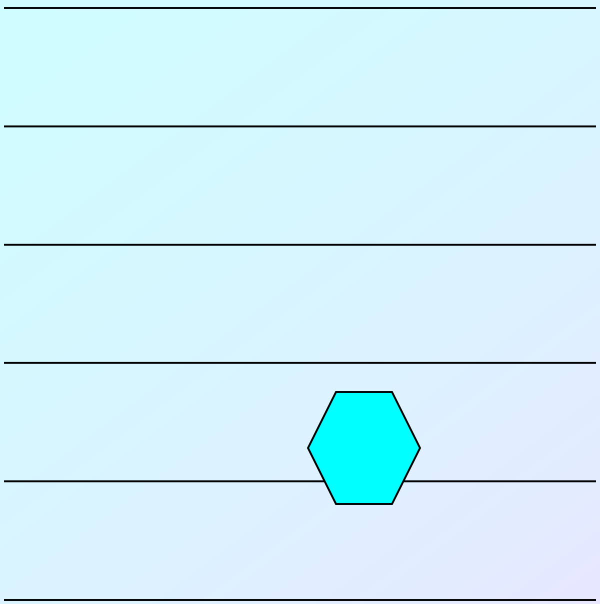
$$P(\text{crossing at least on line}) = \frac{2L(D+E) - L^2}{\pi DE}$$

$$\lim_{E \to \infty} \frac{2L(D+E) - L^2}{\pi DE} = \frac{2L}{\pi D}$$

Other Generalizations



Any Uses?



$$P(\text{crossing}) = \frac{P}{D\pi}$$

$$P(\text{crossing}) \approx \frac{\text{number of crossings}}{\text{total number of throws}}$$

$$\text{Total number of throws} \approx \frac{D\pi}{P}\left(\text{number of crossings}\right)$$

Counting white blood cells!

MATLAB CODE

```
function p=buffon(L,D,n)
cnt=0;
for i=1:n
   x=rand*(pi/2);
   y=rand*D;
   if y <= (L*sin(x))
      cnt=cnt+1;
   end
end
p=cnt/n;
```

OUTPUT

EDU»pi	ans=3.14159265
EDU» buffon(1,4,100)	ans=2.9412
EDU» buffon(1,4,1000)	ans= 2.8409
EDU» buffon(1,4,3000)	ans= 3.1646
EDU» buffon(1,4,10000)	ans= 3.1586
EDU» buffon(1,4,100000)	ans= 3.1342