Worst-Case Performance Analysis in ℓ_1-norm for an Automated Heavy Vehicle Platoon

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Abstract. Based on model set identification and unfalsification, robust performance measured in peak-to-peak gain is analyzed for heterogeneous platoons, inter-vehicle communication delays and actuator uncertainties. The goal is to demonstrate that safe platooning with acceptable performance can be achieved by utilizing the services already available on every commercial heavy truck with automated gearbox. Experimental verification of a three vehicle platoon is also presented.

Keywords: Vehicle platoons, peak-to-peak gain, performance unfalsification

1 Introduction

Safe control of vehicle platoons requires strict guaranteed bounds on inter-vehicle spacing errors. In order to avoid collision the sampled errors are best measured by their ℓ_∞ norm, so the bounds represent the worst-case peaks of the spacing errors. Consistent identification tools are the set membership methods in the ℓ_1 setting, see e.g. [2, 5, 6]. The identified model sets are employed for on-line model validation and a priori analysis of the control performance measured by the worst-case spacing errors.

Controllers for autonomous vehicle platoons usually consist of two levels of feedback controllers. At the lower level a local, vehicle specific controller is responsible for performing acceleration demands. The higher level control law is common for all vehicles, it is designed for satisfying string stability requirements of the entire platoon. Very short safety gaps can be guaranteed under certain constraints on lead vehicle maneuvers, when detailed engine, gearbox and brake system models are available, see, e.g., in references [1, 4, 9]. There is, however, some difficulty in the widespread applicability of these control methods. The required engine/gearbox/brake system models are usually not available and not reliable for all commercial heavy trucks. In addition these controllers try to directly excite the brake cylinder pressures and the throttle valve of the engine, which could also conflict with the existing control units, such as Electronic Brake System (EBS) and Engine Control Unit (ECU).

In the paper the goal is to explore the performance of an automated vehicle string where, in contrast to the former solutions, only the standardized and general services of the EBS and ECU are used. This work is an extension of the research that was presented in the conference paper [11], where the focus was placed on model set identification problems and the analysis of the spacing error bounds subject to heterogeneity in vehicle dynamics. A method for computing unfalsified performance in order to analyze the effect of actuator uncertainties is also presented. An illustration is shown for the brake system.

In Section 2 the mathematical model of the platoon is presented. The vehicle model set identification method is provided in Sections 3. The performance of a heterogeneous platoon and the effect of actuator uncertainties are analyzed in Section 4. The experimental results are shown in Section 5.

Basic notations. The peak norm of a sequence $u(k)$ is denoted by $\|u\|_\infty = \sup_k |u(k)|$, ℓ_∞ denotes the space of sequences of finite peak norm. The peak-to-peak norm of a system H is defined by $\|H\|_1 = \sup_{u \neq 0} \frac{\|Hu\|_\infty}{\|u\|_\infty}$.

2 State-space model of vehicle platoons

In this section a discrete-time, linear time-varying state-space model for the controlled platoon is briefly summarized.

The longitudinal dynamics of a single vehicle is approximated by the following first order nominal model with sampling time T_s

$$\hat{a}_i(k+1) = \theta^*_{i1}\hat{a}_i(k) + \theta^*_{i2}u_i(k), \quad i = 0,1,\ldots,n \tag{1}$$

$$a_i(k) = \hat{a}_i(k) + \nu_i(k) \tag{2}$$

where a_i and u_i denote the acceleration and acceleration demand of vehicle i, $\hat{a}_i$ denotes the acceleration output of the nominal model, θ^*_{i1} and θ^*_{i2} denote constant parameters, ν_i denotes additive disturbance representing actuator uncertainties. The spacing error of the ith follower vehicle and relative speed of vehicle i and $i-1$ are defined by

$$e_i(k) = x_i(k) + L_i - x_{i-1}(k) \tag{3}$$

$$\delta_i(k) = v_i(k) - v_{i-1}(k) \tag{4}$$

where L_i denotes the desired intervehicular space. Without loss in generality L_i can be assumed to be zero in the analysis. The position and forward speed of the ith vehicle are denoted by x_i and v_i, respectively. By using Euler approximation of integrators,

$$e_i(k+1) = e_i(k) + T_s\delta_i(k) \tag{5}$$

$$\delta_i(k+1) = \delta_i(k) + T_s(a_i(k) - a_{i-1}(k)) \tag{6}$$

the spacing error dynamics can be written for each follower vehicle as follows

$$\begin{bmatrix} e_i(k+1) \\ \delta_i(k+1) \\ \hat{a}_i(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & T_s \\ 0 & 0 & \theta^*_{i1} \end{bmatrix} \begin{bmatrix} e_i(k) \\ \delta_i(k) \\ a_i(k) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ -T_s & 0 & -T_s & T_s \\ 0 & \theta^*_{i2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_{i-1}(k) \\ u_i(k) \\ \nu_{i-1}(k) \\ \nu_i(k) \end{bmatrix} \tag{7}$$

The open-loop model of the entire platoon

$$x(k+1) = Ax(k) + Bu(k) + B_\nu \nu(k) + E_d r(k)$$

is constructed by introducing the state vector $x^T = [\hat{a}_0\ e_1\ \delta_1\ \hat{a}_1 \cdots e_n\ \delta_n\ \hat{a}_n]$, control input vector $u^T = [u_1 \cdots u_n]$, disturbance vector $\nu^T = [\nu_0\ \cdots\ \nu_n]$ and reference signal $r = u_0$.

The platoon controller is a modified version of the constant spacing strategy presented in [14, Section 3.3.4]. The modification resides in that, instead of measured acceleration, control input is transmitted through the network. Consequently, the gear change has lower impact in the control signal than in the acceleration, so each vehicle can change gear without deceiving the followers; the vehicles react quicker to maneuver changes; and no need for filtering the rather noisy acceleration measurements. The control strategy in a general form is defined by the following equations

$$u(k) := u_L(k) + \hat{u}_N(k) \tag{8}$$
$$u_L(k) = K_L x(k) \tag{9}$$
$$u_N(k) = K_N x(k) + G_N r(k) + S u(k) + H_N \nu(k) \tag{10}$$

where u_L contains the locally available radar information. Gain matrix K_L can be constructed based on the following definition

$$u_{L,1}(k) = -k_1 \delta_1(k) - k_2 e_1(k) \tag{11}$$
$$u_{L,i}(k) = -k_{1\beta} \delta_i(k) - k_{2\beta} e_i(k), \ \ i > 1 \tag{12}$$

Control signal u_N is constructed from the information received from the communication network

$$u_{N,1}(k) = u_0(k) \tag{13}$$
$$u_{N,i}(k) = \frac{1}{1+q_3} u_{i-1}(k) + \frac{q_3}{1+q_3} u_0(k) - k_{1\alpha} \sum_{j=0}^{i} \delta_j(k) - k_{2\alpha} \sum_{j=0}^{i} e_j(k), \ \ i > 1 \tag{14}$$

where k_1, k_2, $k_{1\alpha}$, $k_{2\alpha}$, $k_{1\beta}$ and $k_{2\beta}$ are design parameters, see [10] for a possible choice. Matrices K_N, G_N, H_N and S can be constructed based on (11)-(14).

The communication network has a sampling time of $T = NT_s$ and the packet is transmitted after $h < T$ constant delay. If $u_N(k)$ denotes the variable to be transmitted at the network input, then

$$\hat{u}_N(k) = \begin{cases} u_N(k-h) \text{ if } \frac{k-h}{N} \text{ is an integer} \\ \hat{u}_N(k-1) \text{ otherwise} \end{cases} \tag{15}$$

denotes the network output at the receiver.

The closed-loop system with the delayed communication is derived in [10]. The local part u_L of the controllers run with the faster sampling rate T_s. By closing the loop with u_L, re-sampling with NT_s, then closing the loop with $\hat{u}_N$ and assuming $r(k) = r(k+1) = \ldots = r(k+N-1)$ and $\nu(k) = \nu(k+1) = \ldots = \nu(k+N-1)$ we arrive at the following closed-loop model with augmented state vector

$$z(k+N) = A_z z(k) + B_{\nu,z} \nu(k) + E_z r(k), \qquad z(k) = \begin{bmatrix} x(k) \\ u_N(k-N) \end{bmatrix} \tag{16}$$

where

$$A_z = \begin{bmatrix} A_L^N + B_0(K_N + SK_L) & B_1 + B_0 S \\ K_N + SK_L & S \end{bmatrix}, \; E_z = \begin{bmatrix} E_N + B_0 G_N \\ G_N \end{bmatrix}, \; B_{\nu,z} = \begin{bmatrix} B_{\nu N} + B_0 H_N \\ H_N \end{bmatrix}$$

$$B_1 := \sum_{j=0}^{h-1} A_L^{N-1-j} B, \qquad B_0 := \sum_{j=h}^{N-1} A_L^{N-1-j} B, \qquad A_L := A + BK_L,$$

$$E_N = \sum_{j=0}^{N-1} A_L^{N-1-j} E, \qquad B_{\nu N} = \sum_{j=0}^{N-1} A_L^{N-1-j} B_\nu,$$

Notice the dependence of B_1 and B_0 on communication delay h. The spacing errors can be observed through matrixes C_i defined by $e_i(k) = C_i z(k)$, $i = 1, 2, \ldots n$.

3 Identification of nominal vehicle models

Nominal vehicle models defined by (1) and (2) are identified in the worst-case setting. Two circumstances motivate the application of this identification approach. Both the brake system and the drive-line are functioning as unknown nonlinear, hybrid systems with many thousands of program rows organizing finite state machines. An adequate description of noise statistics is not available and only reduced order models can be considered. It seems to be reasonable to assume only strict bounds on disturbances and unmodelled dynamics. Strict bounds are also useful in the worst-case analysis of spacing error bounds. On the other hand, available performance analysis tools for model sets with unmodelled dynamics may result in conservative performance bounds. Uncertainty modelling is, therefore, confined to disturbance modelling only. The corresponding peak-to-peak system norm computation for LTI systems is sufficiently accurate.

In order to obtain a preliminary view of the amount of uncertainty in the vehicle dynamics and actuators including EBS and ECU softwares, uncertainty descriptions of several different structures are identified in the section. The first one is an ARX-type model structure with time-varying parameters. The basic concept originating in the papers [8, 7] is briefly presented in the following subsection. Then, the results are extended for obtaining minimal worst-case prediction error in Section 3.2. In the second method an output error (OE) model structure is identified in Section 3.5. Both methods are applied to the experimental data of a heavy truck. The OE model structure is also applicable for the performance analysis method presented in Section 4.2.

3.1 Identification of the smallest unfalsified parameter sets for SISO transfer functions

Consider the following discrete-time linear single input single output model structure

$$G(q) = \frac{\sum_{i=1}^{m} b_i q^{-i}}{1 + \sum_{i=1}^{m} a_i q^{-i}}, \qquad \theta := [a_1, \ldots, a_m, b_1, \ldots, b_m]^T \in P_\theta(\theta^*, \varepsilon) \tag{17}$$

where q is the forward shift operator. Time-varying parameter vector θ is defined in the cube $P_\theta(\theta^*, \varepsilon_\theta) := \{\theta : \; \|W(\theta^* - \theta)\|_\infty \leq \varepsilon\}$, where the a priori given diagonal matrix

$W = \mathrm{diag}\{\frac{1}{\varepsilon_{\theta,1}},\ldots,\frac{1}{\varepsilon_{\theta,2m}}\}$ defines the shape of the cube with edges of length $2\varepsilon_{\theta,i}$. Given input output data set $\{u(k),y(k)\}_{k=1}^{l}$, the problem is to find the central parameter θ^* and the minimal size ε of the cube such that for every $k=m,\ldots,l$ there exists a parameter $\theta \in P_\theta(\theta^*,\varepsilon)$ not invalidated by the measurements, i.e.

$$P_\theta(\theta^*,\varepsilon)\cap D_k \neq \emptyset \qquad \forall k=m,\ldots,l \tag{18}$$

where $D_k := \{\theta :\ y(k)=\varphi^T(k)\theta(k)\}$ and $\varphi^T(k)=[-y(k-1),\ldots,-y(k-m),u(k-1),\ldots,u(k-m)]$. This problem can be solved by minimizing a convex function as follows

$$\varepsilon = \min_{\theta^*}\max_{m\le k\le l}\frac{|y(k)-\varphi^T(k)\theta^*|}{\|W^{-1}\varphi(k)\|_1} \tag{19}$$

In the following subsection the model structure is augmented by an additive disturbance term, and the worst case prediction error is minimized while an optimal shape of the parameter cube and a bound for the disturbance are determined.

3.2 Unfalsified ARX model set of minimal prediction error in ℓ_∞

With the notation of the previous section we can define the following ARX type model structure, denoted by $\mathcal{M}$

$$\mathcal{M} = \{\ y(k)=\varphi^T(k)\theta(k)+\nu(k), \theta(k)\in P_\theta(\theta^*,\varepsilon_\theta), \nu(k)\in P_\nu(\varepsilon_a),\ k=1,\ldots,l\ \} \tag{20}$$

where $\varepsilon_\theta = [\varepsilon_{\theta 1},\ldots,\varepsilon_{\theta 2m}]^T$, $W = \mathrm{diag}\left(\frac{1}{\varepsilon_{\theta,1}},\ldots,\frac{1}{\varepsilon_{\theta,2m}}\right)$ and

$$P_\theta(\theta^*,\varepsilon_\theta) = \{\theta :\ \|W(\theta^*-\theta)\|_\infty \le 1\}, \tag{21}$$

$$P_\nu(\varepsilon_a) = \{\nu :\ |\nu| \le \varepsilon_a\} \tag{22}$$

The shape and size of the uncertainty set characterized by ε_θ and ε_a are unknown parameters. The only information given a priori is the data set $\{u(k),y(k)\}_{k=1}^{l}$.

In order to characterize consistency of the model set with the data, define hyperplane D_k in the $n+1$ dimensional extended parameter space of $p := [\theta^T\ \ \nu]^T$

$$D_k := \{p :\ y(k) = [\varphi^T(k)\ \ \nu(k)]p\}$$

Let $P(\theta^*,\varepsilon_\theta,\varepsilon_a) := \{p=[\theta^T\ \nu]^T :\ \theta\in P_\theta(\theta^*,\varepsilon_\theta),\ \nu\in P_\nu(\varepsilon_a)\}$ denote the parameter set defining model set $\mathcal{M}$ in the extended parameter space.

Definition 1 (Consistency) *Parameter set $P(\theta^*,\varepsilon_\theta,\varepsilon_a)$ can reproduce the data if*

$$P(\theta^*,\varepsilon_\theta,\varepsilon_a)\cap D_k \neq \emptyset \qquad \forall k=m,\ldots,l \tag{23}$$

For given data $\varphi(k)$ and model set parameters θ^*, ε_θ and ε_a the output $y(k)$ that the model set can generate lies between the bounds, $\overline{y}(k)$ and $\underline{y}(k)$

$$\overline{y}(k) := \max_{\theta\in P_\theta(\theta^*,\varepsilon_\theta)}\varphi^T(k)\theta+\varepsilon_a \le y(k) \le \underline{y}(k) := \min_{\theta\in P_\theta(\theta^*,\varepsilon_\theta)}\varphi^T(k)\theta-\varepsilon_a \tag{24}$$

With these bounds, the parameter set identification problem can be formulated as follows.

Problem 1 *Assume that a data set* $\{u(k), y(k)\}_{k=1}^{l}$ *is given. Find a model set characterized by* θ^*, ε_θ *and* ε_a *such that* (23) *is satisfied and that minimizes* $\gamma := \frac{1}{2}\|\overline{y}(k) - \underline{y}(k)\|_\infty$.

3.3 Solution via linear programming

It will be shown that Problem 1 leads to the solution of a linear programming (LP) problem. In contrast to the solution of [8], where for each D_k a minimum necessary size parameter $\varepsilon = \varepsilon(D_k, \theta^*)$ is determined for a given θ^*, we characterize consistency with the help of the output bounds

Lemma 1. *Consistency condition* (23) *is satisfied if and only if there exist* θ^*, ε_θ *and* ε_a *such that*

$$y(k) \leq \varphi^T(k)\theta^* + |\varphi^T(k)|\varepsilon_\theta + \varepsilon_a,\ k = m, ..., l \tag{25}$$

$$y(k) \geq \varphi^T(k)\theta^* - |\varphi^T(k)|\varepsilon_\theta - \varepsilon_a,\ k = m, ..., l \tag{26}$$

where $|.|$ *element-wise takes the absolute value of the argument.*

Proof. We only need to show that $\max_{\theta \in P_\theta(\theta^*, \varepsilon_\theta)} \varphi^T(k)\theta = \varphi^T(k)\theta^* + |\varphi^T(k)|\varepsilon_\theta$ and $\min_{\theta \in P_\theta(\theta^*, \varepsilon_\theta)} \varphi^T(k)\theta = \varphi^T(k)\theta^* - |\varphi^T(k)|\varepsilon_\theta$, then the statement follows from the definitions. The linear function $\varphi^T(k)\theta$ over a convex polytope takes up its extreme values at the vertices of the polytope. Let the vertex set of $P_\theta(\theta^*, \varepsilon_\theta)$ be denoted by $\mathcal{V}$,

$$\mathcal{V} = \left\{ \theta :\ \theta = \theta^* + \begin{bmatrix} \pm\varepsilon_{\theta,1} \\ \vdots \\ \pm\varepsilon_{\theta,2m} \end{bmatrix} \right\}$$

where $\pm$ means all combinations. From this the claims follow.

The following theorem summarizes our results.

Theorem 1. *The model set* $\mathcal{M}$ *which is consistent with the data set* $\{u(k), y(k)\}_{k=1}^{l}$ *and minimizes* $\gamma = \frac{1}{2}\|\overline{y}(k) - \underline{y}(k)\|_\infty$ *is the solution of the following LP problem.*

$$\min_{\theta^*, \varepsilon_\theta, \varepsilon_a} \gamma \qquad \text{subject to (25), (26) and} \quad \gamma \geq |\varphi^T(k)|\varepsilon_\theta + \varepsilon_a,\ \ k = m, ..., l \tag{27}$$

The problem involves $4m+2$ variables and $3(l-m+1)$ inequality constraints, and can be efficiently solved by rutin CLP in the MPT toolbox for Matlab, [3].

3.4 Identification of ARX vehicle models

Several braking experiments have been carried out with a Volvo FH, 24 ton three-axle truck. ARX models of order $m = 1$ are identified in the following.

The LP method of Theorem 1 is applied to the model structure

$$a(k) = a(k-1)\theta_1(k) + u(k-1)\theta_2(k) + \nu(k) \tag{28}$$

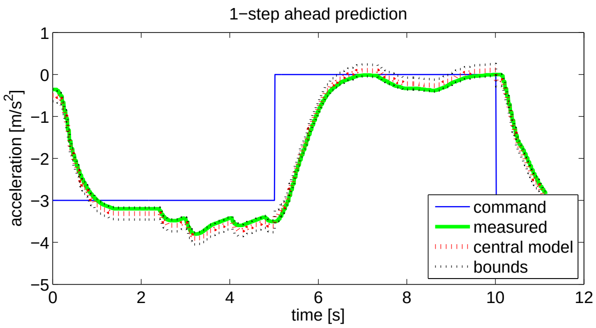

Fig. 1: One step ahead prediction with the central model with parameter θ^* in a braking experiment. Bounds for the prediction, $\bar{y}$ and $\underline{y}$, are also plotted (thin dotted black lines)

where $\theta(k) := [\theta_1(k)\ \theta_2(k)]^T \in P_\theta(\theta^*, \varepsilon_\theta)$, $\|\nu(k) - \nu^*\|_\infty \leq \varepsilon_a$, and $a(k)$ denotes the longitudinal acceleration and $u(k)$ denotes the acceleration demand. An offset error of the measurements can be taken into consideration with parameter ν^*. The unknown parameters of the model are the central parameters θ^* and ν^*, and the bounds of the parameter and noise variation, ε_θ and ε_a, respectively.

The one-step ahead prediction of the optimal model is plotted in Figure 1. The central parameters θ_1^* and θ_2^* correspond to a time constant of 1.13s and a gain of 9.5 when the model is transformed to continuous time by zero order hold ($T_s = 0.01s$). For the parameter variation $\varepsilon_\theta^T = [0.18\ \ 0.20] \cdot 10^{-12}$ is obtained.

By fixing the maximum allowed noise level ε_a, the optimization can be performed in the remaining variables. Figures 2 and 3 show the dependence of the prediction error bound and the optimal parameters on the chosen noise levels, respectively. It can be seen that forcing the model set to represent uncertainty by the time-variation of parameters will result in overly conservative models. At the optimum, the uncertainty is described almost entirely by the noise term. A more sophisticated uncertainty description is necessary which will be provided in Section 4.2.

3.5 Identification of OE models of minimal error in ℓ_∞

In this section an output error model structure is identified with the smallest error in ℓ_∞. Suppose, we are given a data set $\{u(k), y(k)\}_{k=1}^{l}$ and the model structure of LTI SISO systems in the form

$$\hat{y}(k) = G(q)u(k), \qquad G(q) = \frac{\sum_{i=1}^{m} b_i q^{-i}}{1 + \sum_{i=1}^{m} a_i q^{-i}} \tag{29}$$

$$y(k) = \hat{y}(k) + \nu(k) \tag{30}$$

The set of parameters is divided as $\theta_a = [a_1, \ldots, a_m]$ and $\theta_b = [b_1, \ldots, b_m]$. We are looking for θ_a and θ_b that minimize $\gamma := \|y(k) - \hat{y}(k)\|_\infty$. This optimization problem is nonlinear in parameter θ_a, therefore a nonlinear programming method can be applied. In case of small noises, good initialization for θ_a and determination of the model order can be

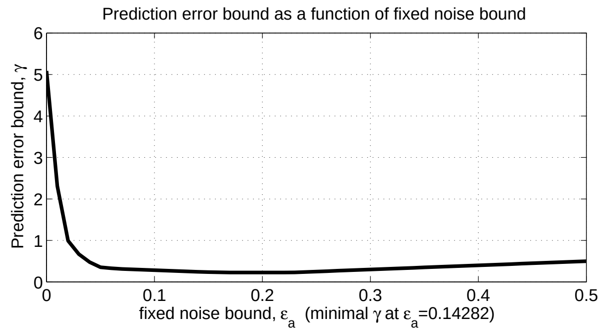

Fig. 2: Worst case prediction error as a function of fixed noise bound ε_a in the ARX model structure

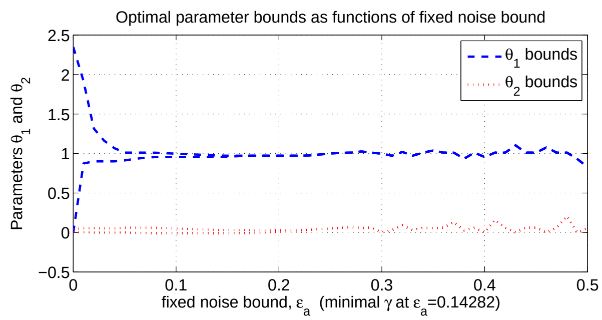

Fig. 3: Parameter bounds as functions of fixed noise bound ε_a in the ARX model structure

attained by using the recent result [13]. Once θ_a is fixed, θ_b can be computed by linear programming as follows.

1. Simulate $y_j(k) = \frac{q^{-i}}{1+\sum_{i=1}^{m} a_i q^{-i}} u(k)$, $i = 1, \ldots, m$ and let $Y(k) = [y_1(k), \ldots, y_m(k)]^T$. From this, $\hat{y}(k) = \theta_b^T Y(k)$.
2. Solve the LP problem

$$\min_{\theta_b} \gamma \quad \text{s.t.} \quad -\gamma \leq y(k) - \theta_b^T Y(k) \leq \gamma, \;\; k = m, \ldots, l \tag{31}$$

3.6 Identification of OE vehicle models

Experimental data used in Section 3.4 is applied now for identification of the OE model structure

$$a(k) = a(k-1)\theta_1 + u(k-1)\theta_2 + \nu(k) - \nu(k-1)\theta_1, \qquad \|\nu(k) - \nu^*\|_\infty \leq \varepsilon_a \tag{32}$$

The LP method presented in Section 3.5 is applied for identifying θ_2, while θ_1 is determined by simple line search. The optimal parameters correspond to a time constant of

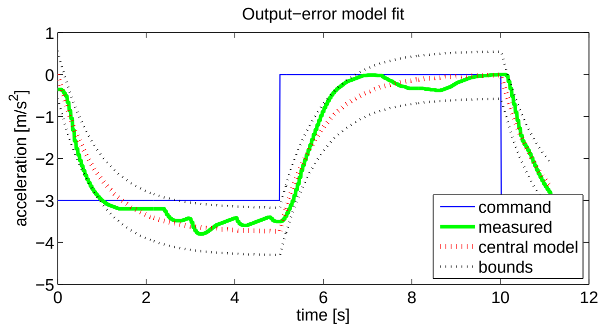

Fig. 4: Fit of the OE model with parameter to the measurements in a braking experiment. Bounds for the error are also plotted (thin dotted black lines)

0.9s and a gain of 1.25 when the model is transformed to continuous time by zero order hold ($T_s = 0.01$). The fit of the model and the error bounds are plotted in Figure 4. This model can serve as nominal models in the performance analysis of the platoon.

4 Performance analysis

4.1 Effects of platoon heterogeneity

For the case of heterogeneous platoons with nominal LTI models, ℓ_∞-bounds on spacing errors are analyzed.Assume that the allowable reference input $r = u_0$ satisfies $\|u_0\|_\infty \leq u_{max}$, where u_{max} is a given bound and there are no actuator uncertainties, $\nu_i = 0$. Then, the worst-case peaks of the spacing errors, as functions of communication delays, can be computed as follows

$$\varepsilon_i := \|e_i\|_\infty = \sum_{j=0}^{\infty} |C_i A_z^j E_z| u_{max}, \quad i = 1, \ldots, n \tag{33}$$

In the following numerical analysis ε_i, $i = 1, \ldots, n$, are computed when the platoon is not homogeneous in nominal vehicle parameters θ_i^*. It is assumed that both $\theta_{i,1}^*$ and $\theta_{i,2}^*$ may differ from vehicle to vehicle

$$\begin{aligned} &\Theta_{\tau g} := [\theta_{1,1}^* \; \theta_{1,2}^* \; \theta_{2,1}^* \; \theta_{2,2}^* \; \ldots \; \theta_{n,1}^* \; \theta_{n,2}^*], \quad \theta_{i,1}^* = 1 - \tfrac{T_s}{\tau_i}, \;\; \theta_{i,2}^* = \tfrac{T_s g_i}{\tau_i}, \\ &\tau_i \in \{0.6, 0.8\}, \qquad g_i \in \{0.9, 1.1\} \end{aligned} \tag{34}$$

where time constant τ_i and gain g_i are parameters of the continuous-time vehicle models and may take up their extremal values. It can be shown that the worst-case platoon configuration is the case when the vehicle model parameters are extremal and alternating in order. This means that if the platoon is of length $n+1$, it is enough to compute (33) for 4^{n+1} systems. Taking the maximum and minimum for the 4^{n+1} systems, Figure 5 shows the worst-case and best-case bounds as functions of the vehicle index i for $d_{max} = 2m/s^2$. The lower bounds are achieved in case of homogeneous platoons.

Upper bounds correspond to platoons of alternating vehicle dynamics. For a given set of allowable maneuvers, this analysis directly provides hints on choosing safety gaps between the vehicles in the different control modes, such as $L_i > \varepsilon_i$, assuming zero initial conditions. The analysis is carried out for a range of network delays from $h = 0$ to $h = 8T_s$, but network delay of this range has negligible impact on the bounds.

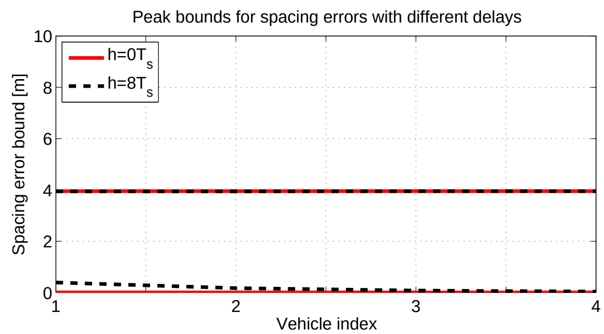

Fig. 5: Lower and upper bounds on spacing errors, ε_i for different network delays. Uncertainty is defined by (34). Lower bounds (around zero) correspond to homogeneous platoons.

In the case when gain coefficients are estimated on-line, for example with the help of parameter adaptation methods described in [14], acceleration demand can always be scaled so that θ_{i2} parameters can be set to $g_i = 1$. Then, for the uncertainty set characterized by

$$\Theta_\tau := [\theta^*_{1,1}\ \theta^*_{2,1}\ \dots\ \theta^*_{n,1}],\quad \theta^*_{i,1} = 1 - \frac{T_s}{\tau_i},\ \ \theta^*_{i,2} = \frac{T_s}{\tau_i}, \qquad \tau_i \in \{0.6, 0.8\} \tag{35}$$

the spacing errors are bounded as shown in Figure 6. The bounds reduced to about one meter.

4.2 Effects of actuator uncertainties

In this section, homogeneous platoons are assumed and merely the effects of brake actuator uncertainties are estimated. The appropriate contribution to the spacing errors is defined by

$$\varepsilon_{\nu,i} := \sum_{j=0}^{\infty} \sum_{l=0}^{n} |C_i A_z^j B_{\nu,z,l}| \nu_{l,max} \tag{36}$$

where the allowable disturbances satisfy $\|\nu_i\|_\infty \le \nu_{i,max}$, $i = 0, \dots, n$ and $B_{\nu,z,l}$ denotes column l of $B_{\nu,z}$. It can be shown that the general case can be approximated by the sum of bounds $\varepsilon_{\nu,i}$ and ε_i obtained in this and the previous sections, respectively. For driving experiments, the case is a bit more complicated, see [12].

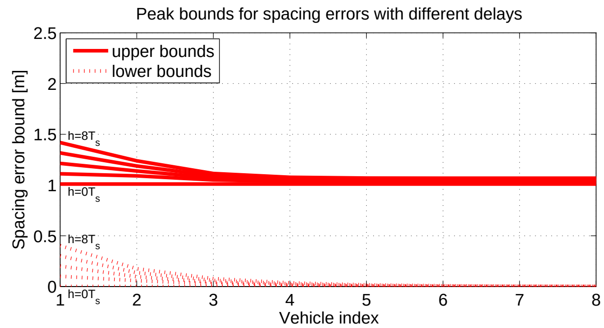

Fig. 6: Lower and upper bounds on spacing errors, ε_i for different network delays. Uncertainty is defined by (35)

For some vehicle $\nu_j \leq \gamma_j = 1.13$ is obtained by the method presented in Section 3.6 for the identified ℓ_∞-bound on the output of the nominal model. By assuming that the same bound holds for every vehicle, (36) is calculated for $i = 0, \ldots, n$. Figure 7 presents, with black solid line, the calculated spacing error bounds corresponding to this disturbance model. The bound about $19m$ indicates that an amplitude bounded but otherwise arbitrary additive disturbance might be a too conservative model for evaluating spacing performance. Assume, therefore, that brake actuator disturbance is generated by the model

$$\nu_i(k) = W_{\nu i}(q)\xi_i(k), \qquad \|\xi_i\|_\infty \leq 1, \qquad i = 0, .., n \tag{37}$$

where $W_{\nu i}$ is a bounded, stable and stable invertible operator satisfying

$$\|W_{\nu i}^{-1}(q)(a_i(k) - V_i(q)u_i(k))\|_\infty \leq 1, \tag{38}$$

i.e. a consistency condition with available experimental data $\{a_i(k), u_i(k)\}_{k=0}^N$. A subset of all consistent models can be finitely parameterized, for example, via finite impulse response representation, by using Laguerre or Kautz bases or by pole-zero-gain parametrization of fixed order. Let θ_{ν_i} denote the parameter vector of model $W_{\nu i}(q, \theta_{\nu_i})$. Then, performance of the platoon, $\varepsilon_\nu := \sum_{i=1}^n \varepsilon_{\nu,i}$, not falsified by measurement data can be obtained as the solution of the following optimization problem

$$\varepsilon_\nu := \inf_{\theta_{\nu_i}, i=0,\ldots,n} \sum_{i=1}^n \sum_{l=0}^n \|P_{\nu,il}(q) W_{\nu l}(q, \theta_{\nu_l})\|_1 \text{ s.t. (38)} \tag{39}$$

where $P_{\nu,il}(q)$ denotes the transfer function from disturbance ν_l to spacing error i. By using a pole-zero-gain parametrization for $W_{\nu i}(q, \theta_{\nu_i})$ with two real and a complex pair of poles and zeros, respectively, confined to a stable sector of the unit disc, the optimization provided a significant reduction of spacing error bounds to 6m, see red dotted line in Figure 7.

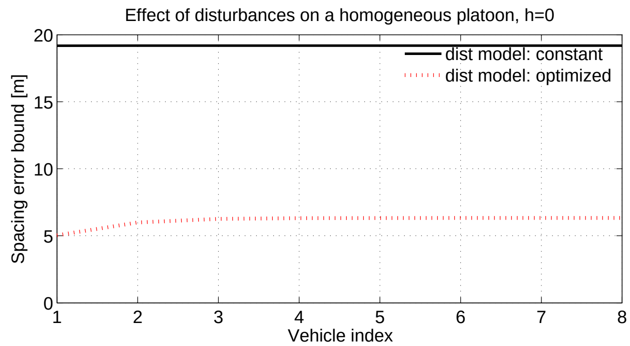

Fig. 7: Spacing error bounds ε_{vi} in *braking* experiments.

Fig. 8: Experimental vehicles in project TruckDAS

5 Experimental results

The control strategy presented in Section 2 is implemented on a platoon of three heavy trucks and tested on a 3km long runway. The leader vehicle, driven by a driver, is a 18 ton MAN TGA two-axle tractor with load cage. The second vehicle is a 24 ton Volvo FH three-axle truck. The third one is a 18 ton Renault Magnum two-axle tractor with a semitrailer, See Figure 8. All vehicles are equipped with automatic gear change. The communication network consists of radio transceivers operating on the open 868MHz ISM narrow-band.

The experimental scenario is started with a 'joining in' maneuver in which the leader vehicle passes the others which are travelling at constant speed. When the last vehicle in the platoon is caught by the radar of the joining vehicle and its driver enables autonomous mode, the joining vehicle is accelerated and braked by given constant values and for sufficient time so that the vehicle arrives approximately at the prescribed distance and speed close to that of the platoon. After the braking period the spacing controller is switched on. When both joining maneuvers are finished, the leader vehicle can freely accelerate and decelerate.

Nine experiments of similar maneuvers were carried out on a 3km long road. One of them is shown in Figure 9. The maximum spacing error was not greater than 3*m* during braking maneuvers. During driving maneuvers, the maximum leg was not greater than 8*m*.

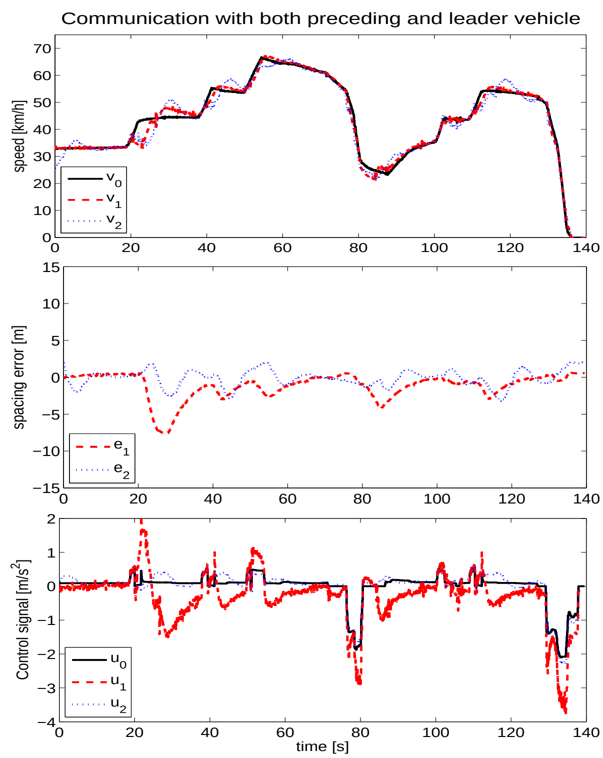

Fig. 9: Platoon control experiment

6 Conclusions

Spacing error analysis of heterogeneous platoons with inter-vehicle communication and actuator uncertainties has been presented. The acceleration and deceleration commands provided by the implemented controller have been carried out by the external demand services of ECU and EBS, respectively. According to our experiences in both unfalsification based model analysis and experimental tests with a platoon of three vehicles with different types and properties, we can conclude that a safety gap of 8*m* can be safe if the acceleration/deceleration of the leader vehicle is not greater than $2m/s^2$.

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