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RESEARCH ARTICLE

Model-based $\mathcal{H}_2/\mathcal{H}_\infty$ control design of integrated vehicle tracking systems

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Abstract

The paper presents the design of an integrated heavy vehicle system which consists of the driveline, the brake, the suspension and the steering components. The purpose of the integration is to create an adaptive cruise control which is able to keep the distance from the preceding vehicle and track the path on roads. The vehicle model consists of the dynamics of the sprung mass and the unsprung masses and handles the effects of road and wind disturbances. The design of the adaptive cruise-control system is based on the $\mathcal{H}_2/\mathcal{H}_\infty$ *control method. The operation of the controlled system is illustrated through simulation examples.*

Keywords

integrated vehicle control · $\mathcal{H}_2/\mathcal{H}_\infty$ *control design · heavy vehicle*

Acknowledgement

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1 Introduction

The increasing number of automotive electrical components in automobiles poses several significant problems. They have not only electrical reasons but also come from the necessity that individual control systems must work in cooperation. Although the integrated vehicle control is able to create a balance between control components, it is still in a research phase. Researchers are faced with several problems. First the integration has a large number of theoretical difficulties. Second it is difficult to determine how responsibility is shared among component suppliers. The industry solves the communication task between these components with various communication platforms.

Several researchers have focused on the integration of control systems. A combined use of brakes and rear-steer to augment the driver's front-steer input in controlling the yaw dynamics is proposed by [3, 5, 9]. An integrated control that involves both four-wheel steering and yaw moment control is proposed by [4, 14]. Active steering and suspension controllers are also integrated to improve yaw and roll stability [7]. An integration possibility of steering, suspension and brake is proposed in [13]. Several papers deal with the design of adaptive cruise control systems (ACC). A fault tolerant control design of ACC is presented in [12]. [6] introduces the design and analysis of a safe longitudinal control for ACC systems.

The motivation of the research is to design multiple input and multiple output control which is able to handle several actuators in an integrated way. The purpose of the integrated control methodologies is to combine and supervise all controllable subsystems affecting vehicle dynamic responses. In case of heavy vehicles the task is to track a leader vehicle (e.g. in a platoon), or perform trajectory tracking. This task is a step in the direction of future autonomous vehicles. It is also necessary to ensure the safe traveling of certain vehicles. It requires the use of safety systems (roll-over prevention, ESP, ABS); the goal of the research is to exploit the advantages of integrated automotive control design.

The paper focuses on the design principles of the integrated vehicle control of heavy vehicles. In a complex control system several components are taken into consideration such as the driveline, the brake, the steering and the active suspension. In the control design the vehicle must achieve different performances, whose priorities are also different. Because of safety regulations it is necessary to ensure accurate path (yaw-rate) tracking and the control must guarantee robustness against worst-case disturbances. This performance is formulated as an $\mathcal{H}_\infty$ optimal task. There is another group of performances, in which robustness is not necessary (e.g. traveling comfort, velocity tracking and roll stability). These performances are also important, but compared to yaw-rate tracking they are less important dynamic parameters. These performances are formulated as $\mathcal{H}_2$ optimal tasks. The joint handling of the two different performances is possible by using $\mathcal{H}_2/\mathcal{H}_\infty$ control.

This paper is organized as follows: Section 2 contains a vehicle model for heavy vehicles and the performances of the vehicle for the design of integrated vehicle control. Section 3 presents the $\mathcal{H}_2/\mathcal{H}_\infty$ control design method. Section 4 shows simulation results and the last section summarizes the achievements.

2 Vehicle model and performance specification

The design of an integrated vehicle dynamic controller requires the formalization of the dynamics of the vehicle, see Fig. 1. During the formalization of the dynamics of the vehicle in the longitudinal, lateral and vertical directions forces, moments and external disturbances must be taken into consideration. The control-oriented modeling is based on the nonlinear equations of the full-car model, see e.g. [8, 10, 16]. The movements of the vehicle are rotation angles (pitch, roll, yaw) and the vertical displacements of the masses. The lateral dynamics of the vehicle is modeled by using the bicycle-model, while the side-slip angle of the vehicle is denoted by β. The control inputs of the vehicle are the front wheel steering, the differential brake moment, the longitudinal traction/braking force and the active suspension forces. The disturbances of model are the road excitations and wind forces.

The $\mathcal{H}_2/\mathcal{H}_\infty$ control method used in the paper requires a linear model of the system, thus the nonlinearities of the vehicle dynamics must be ignored. The deviation between the model and the real plant is taken into consideration through an uncertainty model.

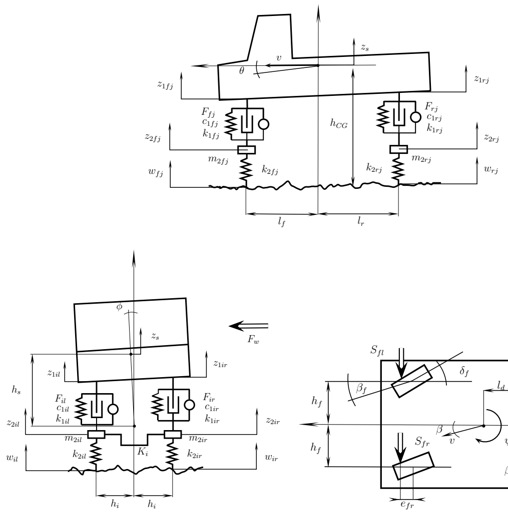


Fig. 1. Vehicle model

Several simplifications are assumed. First, the lateral tire forces in the direction of the wheel-ground-contact velocity are approximated proportionally to the tire side-slip angle β. Second, the dynamics of the unsprung masses is also neglected, i.e., $m_{2ij} = 0$ and $k_{2ij} = \infty$. Thus, the dynamics of the unsprung masses can be considered as uncertainty in the system. The computation of the vertical movement of suspension z_{1ij} requires the knowledge of θ, ϕ, w_{ij} and $\dot{w}_{ij}$. However, the number of states can be reduced if F_{1ij} is considered as disturbances.

After the description of vehicle model it is necessary to define the longitudinal distance model between a leader and a follower vehicle. The scheme of the elementary model is shown in Fig. 2. The linear dynamical equations are $F_1 = m_1\ddot{x}_1$ and $F_2 = m_2\ddot{x}_2$, where m_i is the full mass of the vehicle, x_i is the displacement and F_i is the tracking/braking force of vehicles.

The state space representation of the model contains 6 states: $x = \begin{bmatrix} \dot{\theta} & \dot{\varphi} & \beta & \dot{\psi} & \dot{z}_s & \dot{d} \end{bmatrix}^T$. These states are the pitch and roll rate of the chassis, the side-slip angle of the vehicle, the yaw rate, the vertical velocity of the chassis and the relative speed between the vehicles. The state space representation is as follows:

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

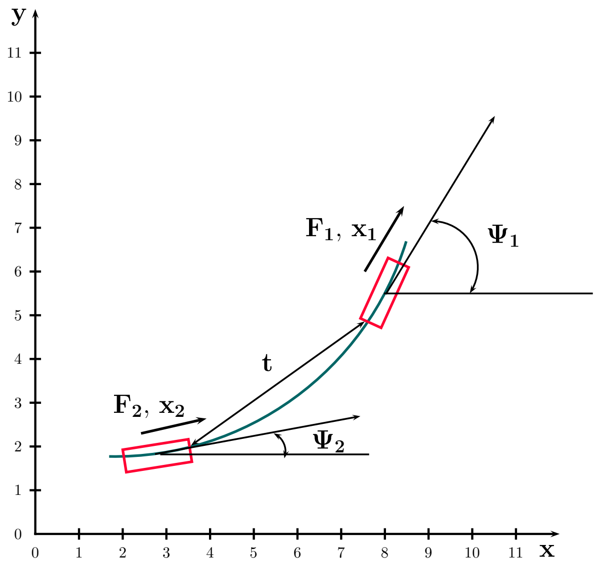


Fig. 2. Illustration of the vehicle tracking

where A, B and C are system matrices.

In the design of an integrated autonomous vehicle dynamic controller it is necessary to formalize the performances of the system. Note that the priorities of the different performance requirements are different. The tracking of the predefined path is crucial because it is related to the road holding of the vehicle. Because of the importance of road holding it is necessary to guarantee the robustness of yaw-rate tracking. Since the lateral dynamics of the vehicle is non-linear and the road-wheel contact is uncertain, the behavior of vehicle differs from the nominal vehicle model. Guaranteeing robustness requires that yaw-rate error must be minimized using the robust $\mathcal{H}_\infty$ optimal control. Vertical acceleration ($\ddot{z}_s$) is related to traveling comfort. Reducing vertical acceleration is important in terms of passenger comfort and the protection of cargo. Vertical acceleration affects the stress on machine elements. The roll of the chassis (ϕ) has an important role in the roll stability of the vehicle. [3] shows that the minimization of the chassis roll angle increases roll stability. This performance has an important role in heavy vehicles, which have relatively high center of gravity. Keeping distance and velocity $\dot{d}_{ref}$ is also important to avoid collision with other vehicles. Although these three performances have significant influence on vehicle stability, they are less important than path tracking, therefore the robustness of these performances is not required. The effect of disturbance from the road (wheels) should be minimized using $\mathcal{H}_2$ optimal control.

3 Robust optimal mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control design

The main purpose of the control design is to ensure that the system output follows a reference command signal with an acceptable error. Based on state space representation the control task is the distance between the two vehicles, which must be kept by a predefined constant value. To design an integrated vehicle control system it is necessary to operate the actuators: the traction force, the braking force, the steering and the active suspension.

The measured signals of the vehicle are the states of suspension compressions at all four suspensions, wheel rotational speeds (all of the wheels) the vertical acceleration of the chassis, pitch rate of the chassis, and the yaw rate of the vehicle. Based on these equations the integrated control can be designed.

In the following section, based on the works of [1, 2], the method of mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control design is summarized. Consider the linear plant G with input u, disturbance $w = \begin{bmatrix} w_w & w_n \end{bmatrix}^T$ (where w_w is the disturbances of vehicle dynamics e.g. wind and road disturbances, w_n is sensor noise), performance outputs z_∞ and z_2, feedback output y. The input is generated by output feedback, using the control K. The signal z_∞ is the performance associated with the $\mathcal{H}_\infty$ constraint, the signal z_2 is the performance associated with the $\mathcal{H}_2$ criterion. The closed-loop interconnection structure is illustrated in Fig. 3.

In the design of robust control weighting functions are applied. Usually the purpose of weighting function $W_{p\infty}$ is to define the robust performance specifications in such a way that a trade-off is guaranteed between them. They can be considered as penalty functions, i.e. weights should be large where small signals are desired and small where large performance outputs can be tolerated. z_∞ performance outputs are the yaw-rate tracking and roll angle of the chassis. W_{p2} is the weighting function of quadratic performances. z_2 signals are velocity tracking and the vertical acceleration of the chassis. The purpose of the weighting functions W_w and W_n is to reflect the disturbance and sensor noises. Δ block contains the uncertainties of the system, such as unmodelled dynamics and parameter uncertainty.

In the control problem four performance signals are applied, i.e. $W_{p\infty} = [W_{\text{ref}}]$ and $W_{p2} = [W_{\text{roll}} \quad W_{\text{dist}} \quad W_{\text{zs}}]^T$. The purpose of weighting functions W_{ref} and W_{roll} are to track the yaw-rate and the distance reference signal with an acceptable small error. This is important in the low frequencies because the lateral and longitudinal dynamics of vehicle cause low frequency dynamics. The purpose of weighting function W_{roll}, W_{zs} are to keep roll and vertical velocities of the chassis small over the desired operation range. It is necessary to consider that the bandwidth of the system determines its operation. It is recommended to choose the W_{ref} weighting function in a form at which the value of W_{ref} is not minimal. It guarantees suitable nominal performance in the operation frequency range of the vehicle. The formalized vehicle model approximates the vehicle chassis with a rigid body model. In case of heavy vehicles the vehicle chassis has bending and torsional vibrations. The natural frequencies of these effects increase at higher frequencies.

The performances of the system are classified in $\mathcal{H}_\infty$ and $\mathcal{H}_2$ groups. The $\mathcal{H}_2$ performance outputs and the $\mathcal{H}_\infty$ performance outputs are the following:

$$z_2 = \begin{bmatrix} \phi & \dot{d}_{\text{ref}} - \dot{d} & \ddot{z}_s \end{bmatrix}^T \tag{3a}$$

$$z_\infty = \begin{bmatrix} \dot{\psi}_{\text{ref}} - \dot{\psi} \end{bmatrix} \tag{3b}$$

The state space representation of the control system is formalized in the following way:

$$\dot{x}_{cl} = A_{cl} x_{cl} + B_{cl} w \tag{4a}$$

$$z_\infty = C_{cl1} x + D_{cl1} w \tag{4b}$$

$$z_2 = C_{cl2} x + D_{cl2} w \tag{4c}$$

The objective of mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control is to minimize the $\mathcal{H}_2$-norm of the closed-loop transfer function $T_{z_2 w}$, while constraining the $\mathcal{H}_\infty$-norm of the transfer function $T_{z_\infty w}$ to be less than some specified levels. More precisely, the problem can be stated as follows.

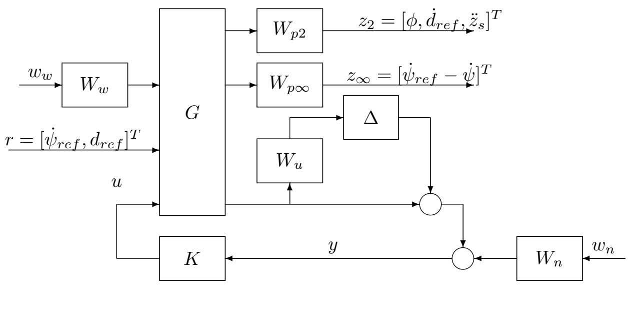


Fig. 3. Closed-loop interconnection structure for the mixed H_2/H_∞ control

The LMI problem of $\mathcal{H}_\infty$ performance is formalized as: the closed-loop RMS gain from w to z_∞ does not exceed γ if and only if there exists a symmetric matrix X_∞ such that

$$\begin{bmatrix} A_{cl}\chi_\infty + \chi_\infty A_{cl}^T & B_{cl} & X_\infty C_{cl1}^T \\ B_{cl}^T & -I & D_{cl1}^T \\ C_{cl1}\chi_\infty & D_{cl1} & -\gamma^2 I \end{bmatrix} < 0 \tag{5}$$

The LMI problem of $\mathcal{H}_2$ performance is formalized as: the $\mathcal{H}_2$ norm of the closed-loop transfer function from w to z_2 does not exceed ν if and only if $D_{cl2} = 0$ and there exists two symmetric matrices χ_2 and Q such that

$$\begin{bmatrix} A_{cl}\chi_2 + \chi_2 A_{cl}^T & B_{cl} \\ B_{cl}^T & -I \end{bmatrix} < 0 \tag{6a}$$

$$\begin{bmatrix} Q & C_{cl2}\chi_2 \\ \chi_2 C_{cl2}^T & \chi_2 \end{bmatrix} > 0 \tag{6b}$$

$$Trace(Q) < \nu^2 \tag{6c}$$

For the system P, find an admissible control K which satisfies the following design criteria:

- the closed-loop system must be asymptotically stable,
- the closed-loop transfer function from w to z_∞ satisfies the constraint:

$$\left\| T_{z_\infty w}(s) \right\|_\infty < \gamma, \tag{7}$$

  for a given real positive value γ,
- the closed-loop transfer function from w to z_2 must be minimized

$$\min \left\| T_{z_2 w}(s) \right\|_2 . \tag{8}$$

The task is to parameterize all suboptimal $\mathcal{H}_\infty$ dynamic controls that stabilize the closed-loop system and satisfy the $\mathcal{H}_\infty$ constraint, and to find among them the control that minimizes the standard $\mathcal{H}_2$ norm, [1, 2, 11, 15].

4 Simulation results

In the $\mathcal{H}_2/\mathcal{H}_\infty$ control design it is necessary to define four performance weighting functions: the $\mathcal{H}_\infty$ performance is the yaw-rate tracking, and there are three $\mathcal{H}_2$ performances such as chassis roll minimization, distance/velocity holding, and minimization of the vertical acceleration of the chassis. Weighting functions chosen for the simulations are depicted in Fig. 4.

At low frequencies it is necessary to ensure the appropriate yaw-rate, distance/velocity tracking and roll minimization. It means that at a low frequency range the values of W_{inf}, W_{dist} and W_{roll} must be high. At a high frequency range the effects of the longitudinal and lateral dynamics are lesser, thus their weights are small. In terms of disturbances and traveling comfort the situation is similar. The disturbances from model uncertainties W_u and from the road W_{zs} may be high, road disturbances must be rejected and robustness is critical

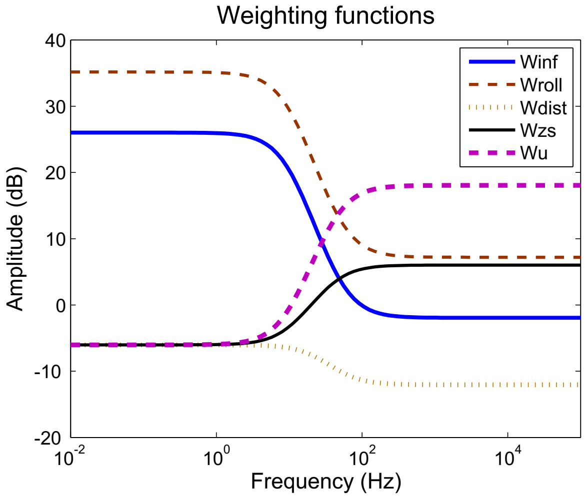


Fig. 4. Weighting functions in the control design

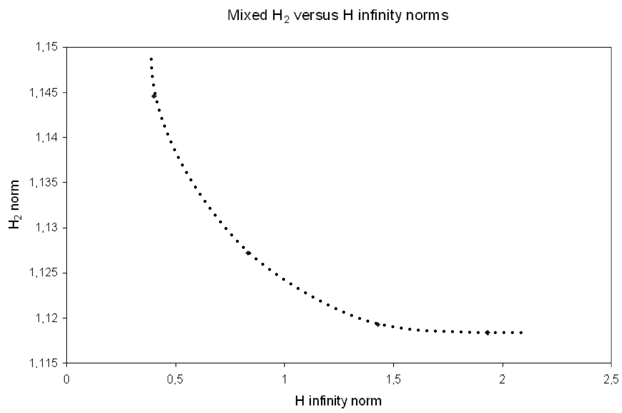


Fig. 5. Mixed H_2 and H_∞ performances

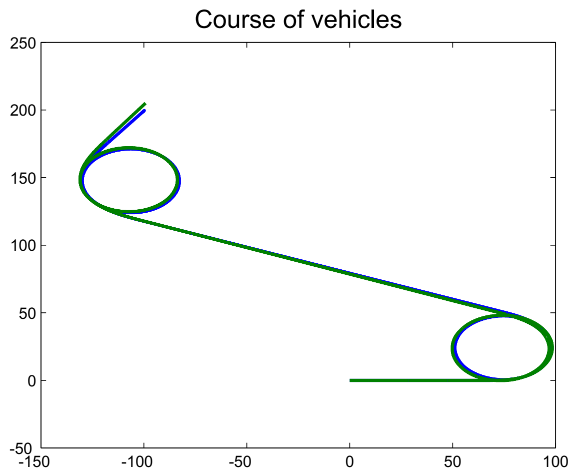


Fig. 6. 8-shaped path vehicle maneuver

in this frequency range. Therefore the value of W_{zs} and W_u are lower at low frequency, and higher at high frequency.

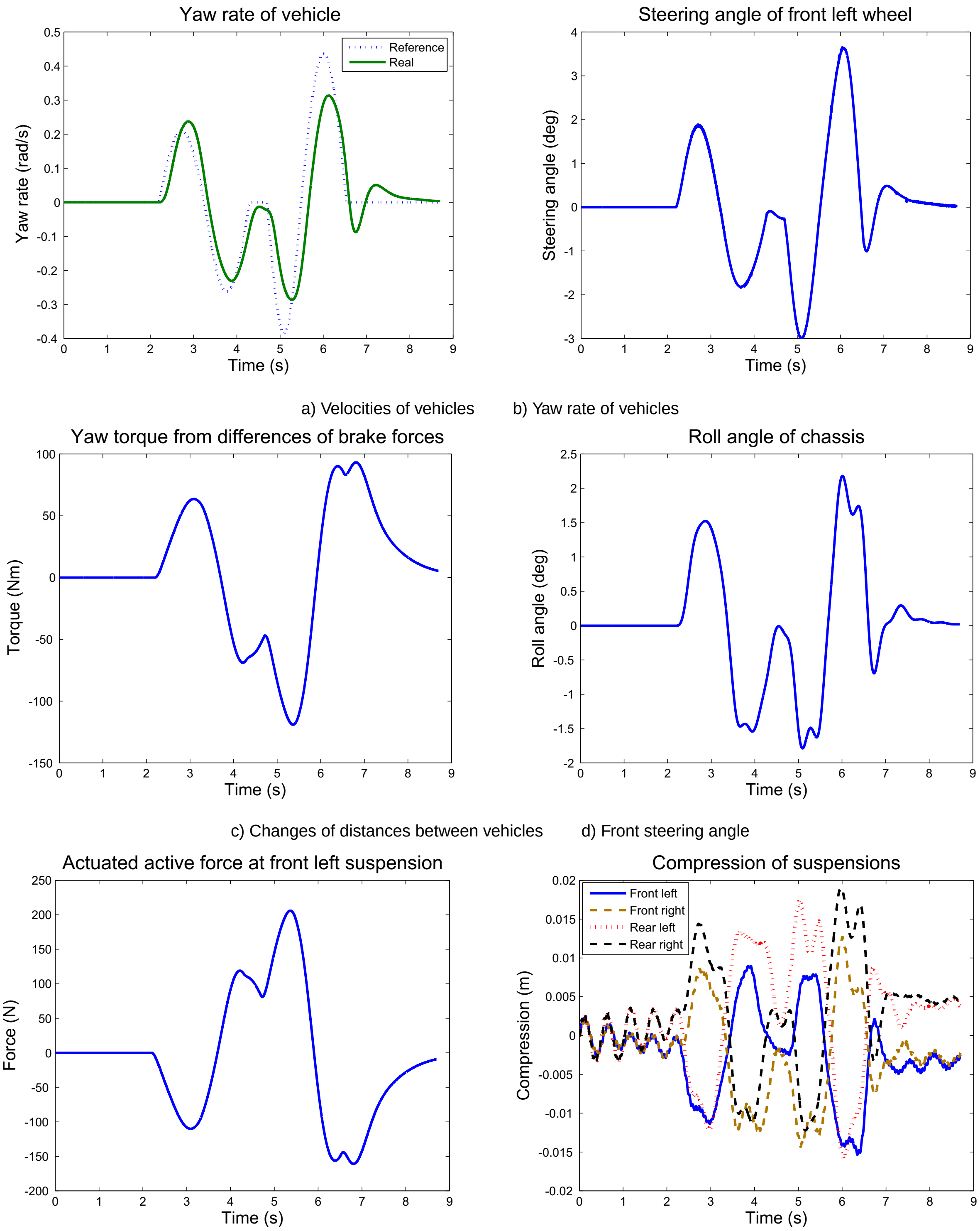
The cost function of $\mathcal{H}_2/\mathcal{H}_\infty$ control design can also be formalized by $a\|T_\infty\|_\infty^2 + b\|T_2\|_2^2 \to min$, where a and b are weighting parameters. By modifying these design parameters the $\|T_\infty\|_\infty$ and $\|T_2\|_2$ norms of the controlled system change as it is shown in Fig. 5. There are several optimal solutions, which results in different norm properties. For reasons of robustness it is recommended to choose a control, in which $\gamma = \|T_\infty\|_\infty < 1$, and simultaneously $\|T_2\|_2$ as low as possible. In these simulations the chosen control guarantees the following norms $\|T_\infty\|_\infty = 0.83$ and $\|T_2\|_2 = 1.13$.

In the presentation of the control method a full-weight pick-up vehicle is used. Two simulation cases are analyzed: an 8-shaped path test and a double-lane-changing maneuver. The 8-shaped path simulation case is a complex simulation, which is used for analyzing the integrated control system, see Fig. 6.

In this simulation the maneuvers of two vehicles are presented. The task is that the second vehicle must follow the leading vehicle with a reference distance. The time responses of the maneuver are illustrated in Fig. 7. The velocities of the two vehicles change as Fig. 7(a) shows. The second vehicle tracks the leader vehicle with an acceptable tracking error. Both the yaw-rates of vehicles and the distance-holding are acceptable, see Fig. 7(b) and Fig. 7(c). The longitudinal force of the tracking vehicle approximates that of the leader vehicle well, which guarantees the appropriate distance-holding.

The two important actuators in the cornering are the front steering and the yaw torque, see Fig. 7(d) and Fig. 7(e). These figures show well the efficiency of the integration. The cornering steering angle increases to 6° and yaw torque is 200 Nm at the same time. When the velocity increases the yaw torque also increases, therefore the actuator signals must be modified. The integrated control decreases the steering angle to 4.5° and simultaneously the yaw torque from the brake force differences increases to 700 Nm. The co-operation of the actuators shows the benefit of the integration. The fourth integrated actuator is the active suspension. The plots of the actuated vertical forces are shown in Figures 7(f) and Fig. 7(g). The effect of active suspension is that each of the wheel-chassis distance decreases, e.g. in case of abrupt braking.

The second example shows a double-lane change test. The time responses are shown in Fig. 8. In the test the vehicle is travelling in the corridor at 90 km/h velocity and moves along without throttling. The vehicle uses an integrated control with the front steering, brakes and active suspensions. The path of the vehicle and the yaw rates are shown in Figs. 9(a)



a) Velocities of vehicles b) Yaw rate of vehicles

c) Changes of distances between vehicles d) Front steering angle

e) Yaw torque from brake differences f) Front left suspension force

Fig. 7. Analysis in the time domain of the 8-shaped maneuver

and 9(b). The steering angle and the yaw torque are shown in Figs. 9(c) and 9(d). The roll of the vehicle is shown in Fig. 9(e). The simulation example shows that the integrated control ensures roll stability using active suspensions, see Fig. 9 (f), and guarantees acceptable path tracking.

5 Conclusion

In this paper an integrated adaptive cruise control system has been designed. The integrated control designed is able to perform a trajectory tracking, keep distance from another vehicle and track its yaw rate. In addition, the integrated control incorporates active suspension actuators, braking forces,

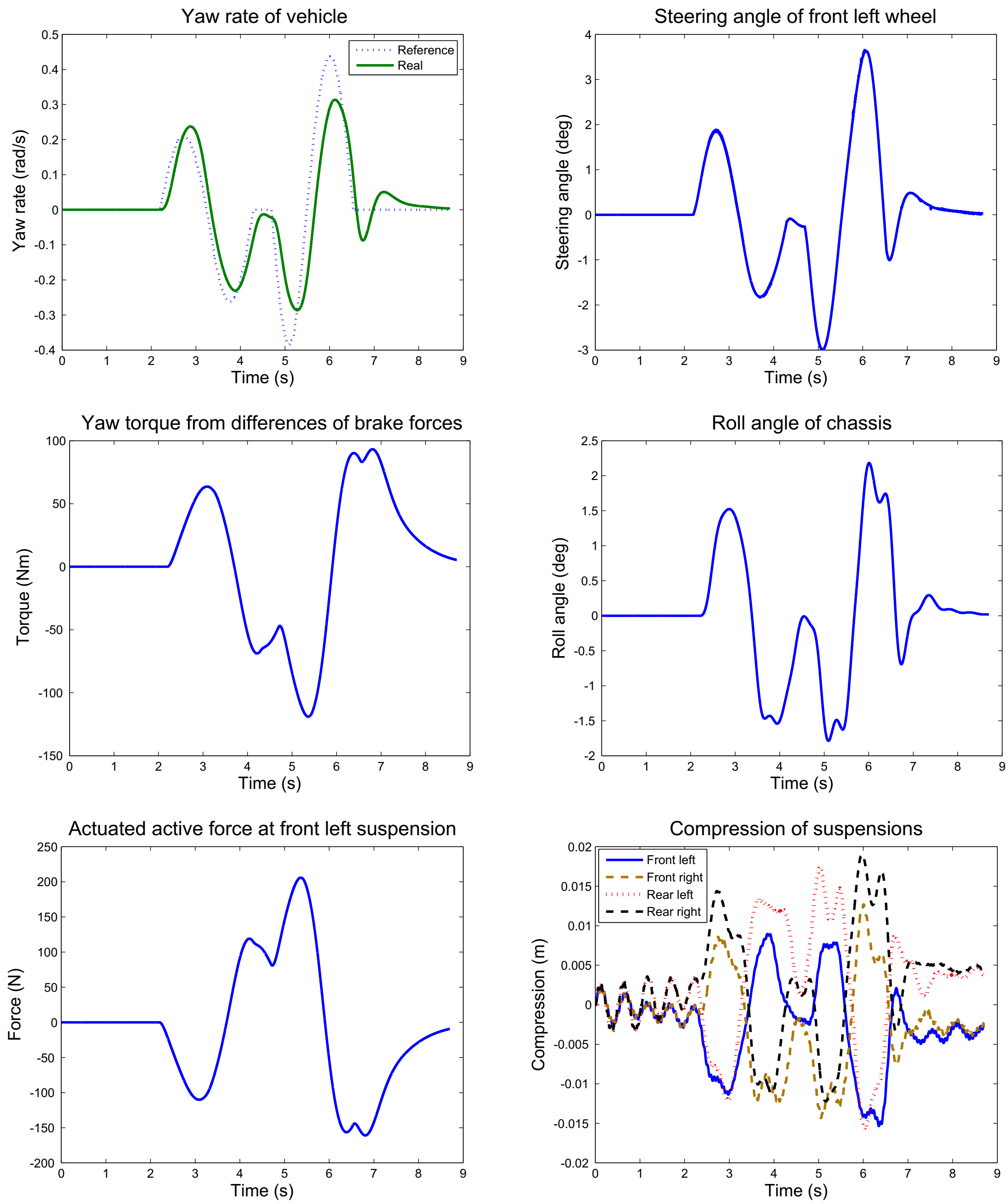


Fig. 9. Analysis in the time domain of the double-lane change maneuver

tractive force and front wheel steering. The integration of the different systems is performed by using an optimal mixed $\mathcal{H}_2/\mathcal{H}_\infty$ control. This control strategy meets $\mathcal{H}_\infty$ criterion (yaw-rate tracking) and $\mathcal{H}_2$ criteria (distance holding, roll-stability and traveling comfort) simultaneously. The actuators of different vehicle systems are able to guarantee performances and reduce conflict between them. Several traffic scenarios have been simulated. In these simulations it has been presented that the integrated control is able to handle the effects of disturbances coming from the environment and the driver. It can be stated that the integrated control enhances safety and reliability in traffic.

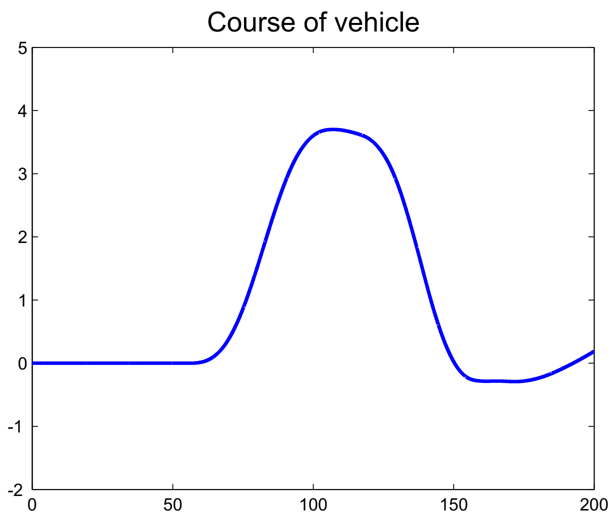


Fig. 8. Double-lane change vehicle maneuver

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