Related-Key Cryptanalysis of FUTURE The Full Round Distinguishing Attack

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Abstract. In Africacrypt 2022, Gupta et al. introduced a 64-bit lightweight MDS matrix-based SPN-like block cipher designed to encrypt data in a single clock cycle with minimal implementation cost, particularly when unrolled. While various attack models were discussed, the security of the cipher in the related-key setting was not addressed. In this work, we bridge this gap by conducting a security analysis of the cipher under related-key attacks using MILP (Mixed Integer Linear Programming) based techniques. Our model enables a related-key distinguishing attack on 8 rounds of FUTURE, requiring 2⁶⁴ plaintexts, 2⁶³ XOR operations, and negligible memory. Additionally, we present a 10-round boomerang distinguisher with a probability of 2^{-45} , leading to a distinguishing attack with 2^{46} plaintexts, 2^{46} XOR operations, and negligible memory. This result demonstrates a full break of the cipher's 64-bit security in the related-key setting.

Keywords: Related key cryptanalysis · Boomerang attack · FUTURE.

1 Introduction

In recent years, the demand for cryptographic solutions optimized for resourceconstrained environments–such as RFID tags, sensor networks, and contactless smart cards–has led to the development of lightweight cryptographic primitives. Unlike traditional cryptographic methods like AES [21], SHA-256 [33], and RSA [35], which are designed for systems with substantial processing power and memory, lightweight cryptography prioritizes efficiency across various metrics including hardware cost, power utilization, and latency. Block ciphers, which can be thought of a pseudo-random permutations to transform plaintext into ciphertext blocks of fixed lengths, are mainly categorized into Feistel structures and substitution-permutation networks (SPNs). Feistel structures, used in ciphers like TWINE [45] and Piccolo [39], are cost-effective but require more rounds to ensure security, while SPNs offer robust security but can be more resource-intensive. The field of lightweight cryptography has expanded significantly, with ciphers such as PRESENT [16], KATAN [19], SIMON & SPECK [6], PRINCE [17], MANTIS [7], LED [24], MIDORI [4], and GIFT [5] being optimized for parameters like code size, latency, and energy consumption. Moreover, tweakable block ciphers like SKINNY [7], CRAFT [8], and QARMA [3], enhance encryption modes and authentication. Additionally, CRAFT addresses challenges such as resistance to Differential Fault Analysis (DFA) attacks.

Several lightweight block ciphers, including LED, MIDORI, and SKINNY, build on the fundamental structure of the AES round function, modifying its components to enhance performance. AES employs MDS (Maximum Distance Separable) matrices in its round function to achieve strong diffusion, which is essential for robust security against various cryptographic threats. However, incorporating MDS matrices into lightweight block ciphers poses a challenge due to their high implementation cost. This often necessitates additional rounds in these ciphers to maintain security against attacks such as differential and linear attacks. As a result, many lightweight block ciphers opt for lighter components, such as near-MDS matrices and bit-permutations, to avoid the high costs associated with MDS matrices. This approach helps manage implementation costs while still aiming to achieve effective diffusion, even though MDS matrices offer superior diffusion benefits.

Mixed Integer Linear Programming (MILP) is a well-established optimization technique used to find the optimal solution for a linear objective function subject to a set of linear constraints. In 2011, Mouha *et al.* introduced an automated differential path search method utilizing MILP [31], which helps generate lower bounds for the number of active S-boxes. At that time, the method could not account for the differential properties of the S-box, limiting its application to bit-oriented ciphers like PRESENT and LS-designs [30]. This limitation was later addressed by Sunet al. [44, 43], who developed two distinct methods to model the differential propagation of S-boxes using systems of inequalities. The first approach uses logical conditions to represent differential properties through linear inequalities. The second approach employs a geometric method to capture all possible input-output difference transitions through an n-bit S-box, computing the H-representation (convex hull) of this set using the SageMath inequality generator function and simplifying constraints with a greedy approach. Sasaki and Todo [37] further advanced this technique by incorporating a MILP-based optimization phase to achieve a more compact representation of S-boxes with fewer constraints. Additionally, Boura et al. [18] enhanced the method by reducing the number of constraints needed to capture the differential properties of an S-box by adding related constraints from the set of constraints generated by the SageMath inequality generator function.

In 2022, Gupta et al. introduced a new 64-bit lightweight block cipher known as FUTURE [25], which stands out for its exceptionally low implementation cost compared to other block ciphers, particularly when implemented in an unrolled fashion. Notably, FUTURE is one of the few lightweight ciphers where all the round components are new, and it employs an MDS matrix for its diffusion layer. The internal functions of the cipher are designed for high hardware efficiency, with the MDS matrix and S-box being specifically optimized to minimize hardware costs. The S-box used is reported to match the cryptographic quality of those in SKINNY and Piccolo. Hardware benchmarks on FPGA and ASIC platforms demonstrated that FUTURE outperforms several well-known lightweight ciphers in terms of size, critical path, and throughput, achieving superior results across multiple metrics.

Researchers have explored various attack methods on the FUTURE cipher in single-key settings. In [26], a bit-based Mixed integer linear programming (MILP) approach was used to identify both differential and linear distinguishers, revealing distinguishers up to five rounds with probabilities of 2 **[−]**⁵⁸ and 2 **[−]**62, respectively. In [38], a meet-in-the-middle (MITM) technique combined with MILP demonstrated a key recovery attack with data, time, and memory complexities of 2^{64} , 2^{126} , and 2^{34} , respectively. Lin *et al.* [29] also employed a MILP-aided MITM attack, achieving complexities of 2^{64} for data, 2^{124} for time, and 2^{48} for memory complexities. Additionally, Roy *et al.* [36] conducted an attack based on biclique structures with data, time, and memory complexities of 2^{48} , $2^{125.54}$, and 2^{32} , respectively. Furthermore, Mondal *et al.* [30] applied Yoyo techniques in the secret-key settings to distinguish up to five and six rounds with data complexities of $2^{9.83}$ and $2^{58.83}$, respectively.

Despite the theoretical attacks on FUTURE in single-key settings, there has been no related-key cryptanalysis attempted on this cipher, and the design proposal did not include any related-key analysis. This paper addresses this gap by providing a detailed examination of related-key cryptanalysis. We develop both word and bit-oriented MILP models to identify improved differential characteristics. Although [26] outlines a bit-oriented MILP model for searching differential characteristics in single-key settings for the FUTURE cipher, the description is incomplete for related-key models. In this work, we provide a comprehensive description of how to build both word and bit-oriented MILP framework for the FUTURE cipher, which can also be useful for constructing MILP frameworks for other SPN-like ciphers. To search for differential characteristics, our approach first employs the word-oriented model and then utilizes a bit-based model based on the optimal input-output difference patterns obtained from the word-oriented model. When searching for related-key differences, the basic word-oriented model is insufficient because it fails to capture optimal difference patterns due to the potential cancellation of intermediate word differences when both the state and key words are active. To address this, we introduce a new non-linear constraint to improve the identification of better difference patterns for optimal differential characteristics. We apply this enhanced search technique to the FUTURE cipher, covering up to 7 rounds.

Our Contributions. Our contributions are three-fold, as follows:

– We propose an extensive bit-based related-key MILP model for the FU-TURE cipher, which can be helpful for building MILP models for any MDS matrix-based SPN ciphers. We revisit Boura *et al.* [18]'s work due to insuffcient information in their proposed algorithm to generate optimal number of constraints to capture the behavior of DDT. From our understanding, we provide a revised algorithm that produces the same results as Boura et al. [18, Algorithm 1], but with a larger set of final constraints. Additionally,

we provide a detailed explanation of how to construct a primitive representation of MDS (or near-MDS) matrices using a companion matrix approach, which is compatible with the cipher's structure.

- Utilizing this technique, we demonstrate an 8-round related-key differential characteristic for FUTURE with a probability of 2 **−**63.4 , which leads to a distinguisher with 2^{64} data, 2^{63} time, and negligible memory complexities.
- Additionally, we develop a full-round related-key boomerang distinguisher with practical complexities, indicating a full-round break of the cipher. A detailed comparison of the previous attack methods and their complexities is provided in Table 1.

Attack	Types	$#$ Rounds	Settings	Prob.		Complexity						
					Data	Time	Memory	Reference				
Differential		5	Single-key	$2 - 58$	$\overline{}$			$[26]$				
MITM	Key Recovery	10	Single-key	$\overline{}$	2^{64}	2^{126}	2^{34}	$[38]$				
MITM	Key Recovery	10	Single-key	$\overline{}$	2^{64}	2124	2^{48}	$[29]$				
Biclique	Key Recovery	10	Single-key	$\overline{}$	$< 2^{48}$	2125.53	2^{32}	$[36]$				
Yovo	Distinguisher	6	Single-key	$\overline{}$	258.83			$[30]$				
		8	Known-key	-	2 ¹⁵							
Differential	Distinguisher	8	Related-key	$2 - 63.4$	2^{64}	2^{63} XOR	Negligible	This Work				
Boomerang	Distinguisher	10	Related-key	2^{-45}	246	2^{46} XOR	Negligible	This Work				

Table 1: A Comparison of Different Attacks on FUTURE

Outline of the Paper. The paper is structured as follows: Section 2 provides an overview of the FUTURE cipher. In Section 3, we present a brief introduction to related-key differential and boomerang cryptanalysis. Section 4 explains the bit-oriented MILP model used for the analysis. In Section 5, we apply this model to construct related-key differential and boomerang distinguishers for the FUTURE cipher. Finally, Section 8 concludes the paper with remarks and suggestions for future work.

2 Description of FUTURE

FUTURE is an SPN-based 64-bit lightweight block cipher designed to have applications on low hardware cost and latency. It has a key size of 128-bit.

The Round Function. The round structure of the FUTURE cipher consists of four operations: SubCell, MixColumn, ShiftRow, and AddRoundKey, as illustrated in Figure 2. Notably, the MixColumn operation is omitted in the final round. The cipher processes a 64-bit input state S arranged as a 4×4 matrix, where each cell is a nibble (i.e., $s_i \in \{0, 1\}^4$ for $0 \le i \le 15$), as shown in the Figure 1a. Furthermore, the round structure is depicted in the following Figure 2.

SubCell. The nonlinear transformation in the round function is defined by the SubCell operation, which applies a 4-bit S-box to each cell of the state matrix. This transformation is depicted in Figure 1b.

Fig. 1: The State Representation and S-box Table of FUTURE Cipher

	${\bf SR}$	
MC SC	ARK	
	>>	

Fig. 2: Round Function

MixColumn. The linear operation is represented by the finite field matrix multiplication involving the MDS (maximum distance separable) matrix (μ) and the state matrix, where the matrix elements are in $GF(2⁴)$. The MDS matrix is illustrated in Figure 3a. Matrix and vector multiplications are performed in the field \mathbb{F}_{2^4} , defined by the primitive polynomial $x^{\hat{4}} + x + 1$.

(a) The MixColumn Matrix (b) The ShiftRow Operation

Fig. 3: The MixColumn Matrix and ShiftRow Operation of FUTURE Cipher

ShiftRow. Each row ($row(i)$, $i = 0, 1, 2, 3$) of the state matrix is rotated to the right by i positions following the MixColumn operation. This process is illustrated in Figure 3b.

AddRoundKey. The 64-bit round keys (sub-keys) SK_i , $i = 0, 1, ..., 10$ are XORed to the state S in each round. Additionally, the final round sub-key SK_{10} is XORed with the state before producing the ciphertext.

Key Schedule. In FUTURE encryption, a 128-bit secret key K is divided into two halves k_0 and k_1 for generating round and whitening keys. k_0 acts as the whitening key and generates each round sub-keys SK_i , $i = 0, 1, ..., 10$ depends

Fig. 4: FUTURE Encryption Scheme

on whether *i* is even or odd. If *i* is even, k_0 is left-rotated by $5 \cdot \frac{i}{2}$ $rac{1}{2}$ bits; if *i* is odd, k_1 undergoes the same left rotation. Left rotation involves circularly shifting bits. Additionally, except for the $5th$ and $10th$ rounds, a single '1' bit is XORed into specific positions within 4-bit cells during each encryption round, with these operations defined by round constants.

3 Related Key Cryptanalysis

A related-key attack [9] involves analyzing a cipher using multiple keys with known mathematical relationships between them. The attacker has access to encryption or decryption functions with these keys and aims to determine the actual secret keys. The simplest form uses a constant (Δ) XOR relation between keys, such as $K_2 = K_1 \oplus \Delta$. Related-key attacks offer more freedom compared to other attacks but can be more challenging to implement. Resistance to such attacks is crucial, as exemplified by the design goals of AES cipher. This paper employs differential attacks and boomerang attacks in a related-key context.

Related-Key Differential Cryptanalysis. Differential cryptanalysis is an effective method for analyzing and attacking symmetric-key ciphers by examining the differences between pairs of plaintexts and ciphertexts, known as "differential". Introduced by Biham and Shamir [13] in 1990, this technique seeks to identify specific patterns in these differences, termed "differential characteristics", that are unique to the encryption algorithm. By studying these patterns,

cryptanalysts can infer the internal state of the cipher and, with sufficient data, uncover the secret key. This approach can lead to distinguishing attacks and key-recovery attacks.

In related-key attack settings, an attacker can establish or enforce a relationship between multiple keys and has access to the corresponding encryption and decryption functions for all these keys. Consider a tuple $(\Delta_{\text{in}}, \Delta_K, \Delta_{\text{out}})$ as an n round related-key differential for a keyed round function f_K , where f_k^{ℓ} κ represents the output after the *i*-th round for $i = 0, 1, ..., n-1$. This differential is valid if, for some plaintext P and key K, the equation f_K^{n-1} K **(**P**) ⊕** ƒ n**−**1 $\chi_{\oplus\Delta_{\mathcal{K}}}^{\cdot n-1}(P\oplus\Delta_{\mathrm{in}})=\Delta_{\mathrm{out}}$ holds. Let $S_{P,K}^i$ denote the internal state of the round function at round i with inputs P and K. The tuple $(\Delta_{\text{in}}, \Delta_K, \Delta S_0, \ldots, \Delta S_{n-1} = \Delta_{\text{out}})$ is an n-round related-key differential characteristic if $(\Delta_{\text{in}}, \Delta_{\text{out}}, \Delta_K)$ is an n-round relatedkey differential and for all i , $S_{P,K}^i \oplus S_{P \oplus \Delta_{in}, K \oplus \Delta_{K}}^i = \Delta S_i$.

Let $p = Pr[(\Delta_{\text{in}}, \Delta_K) \rightarrow \Delta_{\text{out}}]$ represent the probability that the related-key differential $(\Delta_{\text{in}}, \Delta_K, \Delta_{\text{out}})$ holds. This implies that if $\frac{1}{p}$ number of plaintexts \overline{P} and keys **K** are selected uniformly at random, the equation $f_K(P) \oplus f_{K \oplus \Delta_K}(P \oplus$ Δ_{in}) = Δ_{out} will be satisfied at least once.

Related-Key Boomerang Attack. The boomerang attack, introduced by Wagner in [47], is a differential cryptanalysis method that combines two highprobability differentials to enhance the chances of breaking a cipher. This is described in Figure 5a. For a block cipher $E = E_1 \circ E_0$, with differentials $\Delta_0 \xrightarrow{\rho} \Delta_1$ for E_0 and $\nabla_0 \xrightarrow{q} \nabla_1$ for E_1 , the attack checks if differential relationships hold, with an expected probability of success given by:

$$
\Pr(E^{-1}(E(x) \oplus \nabla_1) \oplus E^{-1}(E(x \oplus \Delta_0) \oplus \nabla_1) = \Delta_0] = p^2 \cdot q^2.
$$

The procedure for mounting the distinguisher in adaptive settings is as follows:

- 1. Request the ciphertexts $C_0 = E(P_0)$ and $C_1 = E(P_1)$, where $P_1 = P_0 \oplus \Delta_0$.
- 2. Request the plaintexts $P_2 = E^{-1}(C_2)$ and $P_3 = E^{-1}(C_3)$, where $C_2 =$ $C_0 \oplus \nabla_1$ and $C_3 = C_1 \oplus \nabla_1$.
- 3. Verify if $P_2 \oplus P_3 = \Delta_0$.

To amplify this attack, the amplified boomerang attack [27] was proposed which works in a non-adaptive (chosen-plaintext attack) scenario. In this attack, the expected probability to get a right quartet will be $p^2 \cdot q^2 \cdot 2^{-n}$. Furthermore, in [10, 11], they have pointed out that any value of Δ_1 and ∇_0 can be considered as long as $\Delta_1 \neq \nabla_0$. As a result, the probability of the right quartet is increased to $2^{-n} \cdot \hat{\rho}^2 \cdot \hat{q}^2$, where $\hat{\rho} = \sqrt{\sum_i \rho_i^2}$ $\overline{\text{Pr}^2(\Delta_0 \rightarrow \Delta_1^i)}$ and $\hat{q} = \sqrt{\sum_i$ $\overline{\text{Pr}^2(\nabla_0^j \rightarrow \nabla_1)}.$ Note that this amplification can be done in the adaptive setting to increase the

probability to $\hat{p}^2 \cdot \hat{q}^2$ from $p^2 \cdot q^2$. The sandwich attack [22] further refines this approach by decomposing the cipher into three parts and using the Boomerang Connectivity Table [20] (BCT) to systematically analyze the connections between

Fig. 5: The Boomerang Framework

input and output differences, improving the probability approximation of the distinguisher.

The related-key boomerang attack [12], depicted in Figure 5b, utilizes both key and plaintext differences. It assumes that the upper sub-cipher E_0 follows a differential characteristic $\Delta_0 \xrightarrow{p} \Delta_1$ under a key difference $\alpha = K_0 \oplus K_1 =$ $K_2 \oplus K_3$, while the lower sub-cipher E_1 has a differential characteristic $\nabla_0 \stackrel{q}{\rightarrow} \nabla_1$ under a key difference $\beta = K_0 \oplus K_2 = K_1 \oplus K_3$. A related-key distinguisher is built using four different unknown keys: K_0 , $K_1 = K_0 \oplus \alpha$, $K_2 = K_0 \oplus \beta$, and $K_3 = K_1 \oplus \beta$. The related-key boomerang distinguisher in the adaptive scenario is executed as follows:

- 1. Request the ciphertext pairs (C_0, C_1) , where $C_0 = E_{K_0}(P_0)$ and $C_1 =$ **E**_{K₁}(P₁), with $P_0 \oplus P_1 = \Delta_0$, $K_0 \oplus K_1 = \alpha$.
- 2. Request the plaintexts pairs (P_2, P_3) , where $P_2 = E_{K_2}^{-1}$ $\frac{1}{K_2}(C_2)$ and $P_3 =$ E^{-1}_{ν} $K_3^{-1}(C_3)$, with $K_2 \oplus K_3 = \alpha$, $C_2 = C_0 \oplus \nabla_1$ and $C_3 = C_1 \oplus \nabla_1$. 3. Verify if $P_2 \oplus P_3 = \Delta_0$.

4 Mixed-Integer Linear Programming models

Mixed-integer linear programming (MILP) has been successfully utilized to develop automated search algorithms for differential and linear cryptanalysis. Two primary modeling approaches exist for implementing ciphers: the word-oriented model and the bit-oriented model. In the word-oriented model, the cipher state is treated as a sequence of words, with each word represented as a binary variable. In contrast, the bit-oriented model represents each bit of the cipher state as a binary variable, ensuring the generation of the most optimal and valid differential characteristics without any inconsistencies in the trail. The MILP constraints introduced in Mouha et al.'s method are insufficient to fully capture the differential propagation behavior in linear diffusion layers built from non-MDS codes. In [42], the authors first proposed a bit-oriented model specifically for SPN ciphers that utilize bit permutation-based linear layers.

In this section, we model the FUTURE cipher components as constraints to construct a bit-based MILP model for analyzing differential characteristics. To build this model, the S-box, permutation, and matrix multiplication over a finite field are represented by linear inequalities with binary variables. In [26], the authors present a bit-based MILP model for the FUTURE cipher aimed at searching for single-key differential and linear characteristics. However, the details provided are incomplete, particularly regarding the generation of linear inequalities for the S-box and the conversion of the MDS matrix to a binary matrix using the companion matrix representation. By employing linear inequalities, one can construct a comprehensive bit-based MILP model that automatically identifies differential characteristics.

4.1 Constraints for SubCell Operation

For differential cryptanalysis using bit-based MILP, the goal is to generate a minimal number of constraints involving input and output bits of an S-box to capture the actual behavior of the differences according to the difference distribution table (DDT). Let us assume that, $(\mathbf{x}_0, \ldots, \mathbf{x}_{n-1})$ and $(\mathbf{y}_0, \ldots, \mathbf{y}_{n-1})$ represent the input and output bit differences of an n**×**n S-box respectively. The problem corresponds to modeling the fact that $(x_0, \ldots, x_{n-1}) \rightarrow (y_0, \ldots, y_{n-1})$ is a possible difference transition in a DDT. In this regard, two different approaches were proposed in 2014 by Sun et al. [44, 43]. The first is a geometrical one and consists of computing the H-representation of the convex hull of the set of possible transitions. The second one is based on logical condition modeling. The first approach is to use the Sagemath inequality generator by taking all the valid difference transition points from the DDT and it generates the number of linear inequalities which satisfies all the valid difference transition points. However, the number of inequalities using Sagemath is typically quite high with many redundant inequalities. The authors of [44] applied a greedy algorithm to reduce the number of constraints. In this approach, the algorithm adds to the solution set the best possible inequality which can remove the highest number of impossible difference transition points among those that have not been removed yet. Later, Sasaki and Todo in [37] proposed a new reduction algorithm to further reduce the number of constraints compared to the greedy approach. They proposed to model the problem of minimizing the set of inequalities that remove all the impossible difference transition points as a MILP problem itself and solve it by some solver. More precisely, their method consists of assigning a binary variable z_i to each inequality in which $z_i = 1$ denotes that inequality i is included in the system. Then for each impossible difference transition point j, add the corresponding constraints in the list \mathcal{L}_j which leads to the inequality as $\sum_{i \in \mathcal{L}_i} z_i \geq 1$. Finally, the MILP solver is used for minimizing $\sum_i z_i$ giving a solution to optimize the constraints to capture the DDT of an S-box. However,

Algorithm 1 Revised Boura et al.'s Approach to Compute a Set of Inequalities from DDT of an S-box.

1: **procedure** COMPUTECONSTRAINTS(VDP , $k(≥ 2)$)
2: $H_{sef} ← inequality_generator(VDP)$ 2: Hset **←** inequality_generator(VDP) 3: $C_{\text{set}} \leftarrow \mathcal{H}_{\text{set}}$ 4: $\mathcal{D}_{\text{set}} \leftarrow \{\}$ 5: **for** all $\alpha \in VDP$ do 6: $\mathcal{H}_{\text{set}}^{\alpha} = \{ C \in \mathcal{H}_{\text{set}} | C(\alpha) = 0 \}$ 7: **for** all $\mathcal{H}_{\text{set}}^{\alpha}$, $\alpha \in \mathcal{VDP}$ do 8: if $k \geq |\mathcal{H}_{set}^{\alpha}|$ then 9: for all $\{C_1, \dots, C_k\} \subseteq \mathcal{H}_{\text{set}}^{\alpha}$ do 10: $C_{new} = C_1 + \cdots + C_k$ 11: $\qquad \qquad \text{Add } C_{new} \text{ into } \mathcal{D}_{set}$ 12: **for** all constraints $C_i \in \mathcal{H}_{set}$, $1 \leq i \leq |\mathcal{H}_{set}|$ do 13: Construct the set $S_i = \{ \beta \in \mathcal{IDP} | C_i(\beta) < 0 \}$ 14: $\mathcal{L} = [S_1, \cdots, S_{|\mathcal{H}_{\text{set}}]}]$ \triangleright A list of sets 15: **for** all constraints $C_i \in \mathcal{D}_{set}$, $1 \leq j \leq |\mathcal{D}_{set}|$ do 16: Construct the set $S = \{ \beta \in \mathcal{IDP} | C_i(\beta) < 0 \}$ 17: **if** \vec{A} any S_i from \mathcal{L} such that $S \subseteq S_i$ then 18: Add S in the list $\mathcal L$ 19: Add C_i in C_{set} 20: Apply Sasaki et al.'s [37] approach from C_{set} to finally get the optimal number of constraints to capture the DDT of S-box

there are other works [46, 28, 34] that further reduce the number of constraints to capture DDT of S-box by proposing new approaches to generate additional inequalities, surpassing the Sagemath inequality generator or using Boura et al.'s approach. In this work, we follow the method outlined by Boura et al. [18] to generate the constraints that capture the DDT of a FUTURE S-box. The DDT of the FUTURE S-box is provided in Table 3a (in Appendix A). The algorithm presented by Boura et al. [18, Algorithm 1] for deriving a set of inequalities from the DDT of an S-box lacks certain details. Specifically, in step 7, the authors state that if C_{new} removes a new set of impossible transitions, C_{new} should be added as new constraints to capture the DDT of the S-box. However, the precise meaning of C_{new} is unclear. It is not explicitly explained whether the set S of impossible transitions from C_{new} should contain elements distinct from the elements of S_i , for all *i*, where each S_i is a set of impossible transitions for every constraint in C_{set} . To clarify this, we revisited their approach and outlined the complete steps necessary to generate the S-box constraints based on our understanding.

Suppose, a difference transition $X \rightarrow Y$, $X, Y \in \mathbb{F}_2^4$ ⁴/₂ through the S-box can be seen as a vector of \mathbb{F}_2^{2n} , involving $2n$ binary variables represented as (x_0, \ldots, x_{n-1}) , y_0, \ldots, y_{n-1} or as (x_0, \ldots, x_{2n-1}) . Let VDP and IDP denote a set of valid and impossible difference transition points according to the DDT of FUTURE

S-box. For example, $0x12 \in VDP$ and $0x11 \in IDP$ are the valid and invalid difference points according to the DDT in Table 3a. Given VDP to the inequality_generator() function in the sage.geometry.polyhedron class of Sagemath returns a list of inequalities as the H-set representation of the convex hull of all possible transitions in a DDT. For FUTURE S-box, we get returns 214 inequalities. We denote this list of inequalities as $\mathcal{H}_{\mathsf{set}}$. This set $\mathcal{H}_{\mathsf{set}}$ has the following properties: (1.) each $\alpha \in VDP$ must satisfy all the constraints in $\mathcal{H}_{\mathsf{set}}$ and (2.) each $\beta \in \mathcal{IDP}$ will not be satisfied by at least one of the constraints in $\mathcal{H}_{\mathsf{set}}$. The interesting point here is that the addition of any number of constraints always maintains the above two properties. Although, adding the constraints by choosing all the subsets (of cardinality k) of constraints from $\mathcal{H}_{\mathsf{set}}$ and then append it to H_{set} can increase the list H_{set} remarkably high. Instead, for each $\alpha \in VDP$, the authors choose the constraints from $\mathcal{H}_{\mathsf{set}}$ which satisfies by the point α and store them in another list $\mathcal{H}_{set}^{\alpha}$. Then, for each set $\mathcal{H}_{set}^{\alpha}$, choose $\left(\frac{H_{\text{set}}^{\alpha}}{k}\right)$ constraints, denoted C_1, \ldots, C_k , and add $C_{\text{new}} = C_1 + \cdots + C_k$ to a set \mathcal{D} set. A list of sets, \mathcal{L} , is constructed, where each entry contains the impossible transition points for each constraint in $\mathcal{D}_{\mathsf{set}}$. Finally, the constraints from $\mathcal{D}_{\mathsf{set}}$ are filtered and added to \mathcal{L} , ensuring that the set of impossible transition points S is not a subset of any set already in L. After this filtration, Sasaki and Todo's method is applied to the selected constraints to generate the optimal constraints for capturing the DDT of the S-box. These steps are detailed in Algorithm 1.

Using this algorithm with $k = 2$, we obtain 971 constraints for C_{set} and ultimately reduce this to 17 constraints using Sasaki et al.'s approach. In comparison, according to [18, Algorithm 1], the authors generated approximately 500 constraints for the PRESENT S-box with $k = 2$, which were reduced to 17 constraints using Sasaki et al.'s method. However, by applying the revised approach outlined in Algorithm 1, we generated 1138 constraints for C_{new} for the PRESENT S-box, which were similarly reduced to 17 constraints using Sasaki et al.'s method. Using the revised Algorithm 1, we obtained 17 constraints for both the FUTURE and Present S-boxes to capture the DDT. These constraints are depicted in Figure 16 and Figure 17 (in Appendix A), respectively.

4.2 Constraints for MixColumn Operation

For word-based MILP modeling, Mouha et al. [31] modeled the MixColumn matrix multiplication using its branch number, i.e., a lower bounds on the number of active S-boxes. Whereas for a bit-based model, an MDS (or near MDS) matrix μ must be converted to a binary matrix over the base field \mathbb{F}_2 , which is called the primitive representation of M . In [40], the authors give a short description of the primitive representation of μ using a companion matrix. However, in [41], Sun et al. provided a method to obtain a primitive representation using linear maps with matrix representation. In this work, we thoroughly explore how to efficiently compute a primitive representation of μ using a companion matrix that is compatible with the cipher's bit format, whether it follows a least significant bit (LSB) to the most significant bit (MSB) order or an MSB to LSB order. Finally, to model matrix multiplication in MILP, we might need several binary XOR operations.

For 1-XOR operation, $c = a \oplus b$, $a, b, c \in \{0, 1\}$, Mouha *et al.* [31] modeled it using 4 constraints and 3 variables as $a + b + c \geq 2d_1$, $d_1 \geq a$, b, c, where d_1 is a dummy variable. Similarly, for $d = a \oplus b \oplus c$, $a, b, c, d \in \{0, 1\}$, known as a 2-XOR operation, the approach requires 8 constraints and 5 variables. Yin et al. [48] showed a method to model it using 8 constraints and 4 variables. However, Fu et al. [23] efficiently model the 1-XOR operation using only one constraint $a+b+c+d = 2d_1$, $a, b, c, d, d_1 \in \{0, 1\}$. Based on this approach, the authors in [26] extend this approach to model the n -XOR operations $\alpha_0 \oplus \ldots \oplus \alpha_{n-1} = b$, as follows.

$$
a_0 + \ldots + a_{n-1} + b = \begin{cases} (n+2)d_1 - (nd_2 + (n-2)d_3 + \ldots + 2d_{\frac{n}{2}+1}) & \text{if } n \text{ is even} \\ (n+1)d_1 - ((n-1)d_2 + (n-3)d_3 + \ldots + 2d_{\frac{n-1}{2}+1}) & \text{if } n \text{ is odd} \end{cases}
$$

In our model, we adopt this approach for n -XOR operations for MixColumn matrix multiplication. In FUTURE cipher, the multiplication by 4 **×** 4 MDS matrix μ is performed over $GF(2^4)$, defined by the primitive polynomial $x^4 + x +$ **1**. Let, α be a primitive element, serving as a root of the polynomial $x^4 + x + 1$. The MDS matrix mu includes field elements 1, 2, 3, 8, and 9 from $GF(2⁴)$. To model the multiplication by μ for bit-oriented MILP, we need to convert the 4×4 MDS matrix μ over $GF(2^4)$ into a primitive representation of μ , i.e., a 16 **×** 16 binary matrix over the base field F2. Using linear maps with matrix representation, the authors [26] express the corresponding 4**×** 4 binary matrices of these field elements in Figure 6.

$1 =$	F10001 0 1 0 0 10010 100011	$2 =$	ר0 1 0 0 1 1 0 0 0 1	$3 =$	F1 1 0 0 1	$8 =$	LI 0 0 1 1 1100 100101	$9 =$	1100 1	
-------	--	-------	-------------------------	-------	------------	-------	------------------------------	-------	---------------	--

Fig. 6: 4×4 binary matrix representation of the field elements in μ

Note that, the primitive representation of μ by replacing the corresponding field elements 1, 2, 3, 8, and 9 is compatible with cipher representation from MSB to LSB. However, this primitive representation would not be compatible with the cipher representation from LSB to MSB. Here we will describe how to construct the primitive representation of μ using a companion matrix to model the cipher which would be compatible in both ways. We know that $2 = 0010 \in$ $GF(2^4)$ is the root α of the primitive polynomial $x^4 + x + 1$ over $Gf(2^4)$. Let us assume that, the state of the cipher represented from MSB to LSB, i.e., $S = S_{63}||S_{62}||...||S_0$. In this case, the companion matrix representation of α of the monic primitive polynomial $c_0 + c_1x + c_2x^2 + c_3x^3 + x^4$, $c_i \in \mathbb{F}_2$ can be written as

$$
2=\alpha=\begin{bmatrix}c_3&1&0&0\\c_2&0&1&0\\c_1&0&0&1\\c_0&0&0&0\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&1&0\\1&0&0&1\\1&0&0&0\end{bmatrix}\text{ with }1=\alpha^0=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}.
$$

The other field elements of μ can be computed as

$$
3=\alpha+1=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \, 8=\alpha^3=\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \, 9=\alpha^3+1=\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}.
$$

Thus the 16×16 binary matrix M_1 , representing the primitive form of μ , corresponds to the cipher's bit representation from MSB to LSB over \mathbb{F}_2 is given in Figure 13 (in Appendix A). On the other hand, if the cipher is represented from LSB to MSB, i.e., $S = s_0 ||s_1|| \dots ||s_{63}$, then the companion matrix representation of α of the monic primitive polynomial $c_0 + c_1x + c_2x^2 + c_3x^3 + x^4$, $c_i \in \mathbb{F}_2$ can be written as

$$
2=\alpha=\begin{bmatrix}0&0&0&c_0\\1&0&0&c_1\\0&1&0&c_2\\0&0&1&c_3\end{bmatrix}=\begin{bmatrix}0&0&0&1\\1&0&0&1\\0&1&0&0\\0&0&1&0\end{bmatrix} \text{ with } 1=\alpha^0=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}.
$$

Similarly, the other field elements of μ can be computed as

$$
3=\alpha+1=\begin{bmatrix}1&0&0&1\\1&1&0&1\\0&1&1&0\\0&0&1&1\end{bmatrix}, 8=\alpha^3=\begin{bmatrix}0&1&0&0\\0&1&1&0\\0&0&1&1\\1&0&0&1\end{bmatrix}, 9=\alpha^3+1=\begin{bmatrix}1&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}.
$$

Thus the 16×16 binary matrix M_2 , serving as the primitive representation of μ , corresponds to the bit order from LSB to MSB over \mathbb{F}_2 is given in Figure 13 (in Appendix A). Apart from these two companion matrix representations, using any other form of companion matrix in the model either by transposing it or by reordering the rows/columns of the above two matrices would not be compatible with the cipher representation. This is because altering the companion matrix used to construct the primitive representation M would change the bit sequences. As a result, multiplying M (a 16×16 matrix) by the state (a 16×4 matrix) would not produce a correct state consistent with the cipher's structure.

Finally, the 4**×**4 state matrix of FUTURE cipher can be further deduced to 16×4 binary matrix. Let, the 16-bit column vectors as $y = (y_0, y_1, \dots, y_{15})^T$ and $t = (t_0, t_1, \dots, t_{15})^T$, where $t = M \cdot y$. The 16 constraints corresponding to one column transformation of the state after the MixColumn operation are given in Figure 14 (see Appendix A). Therefore, for all four columns of the state, a total of $16 \cdot 4 = 64$ constraints are required to represent the differential propagation through the MixColumn operation.

4.3 Constraints for ShiftRow Operation

The ShiftRow operation performs a row-wise shift at the nibble level, which can be represented as a bit-wise permutation π : $\{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$. To model this operation, the binary variables resulting from the MixColumn step are permuted by ShiftRow. After that, 64 new binary variables are introduced and assigned to these permuted values. If x_i and y_i represent the input and output binary variables respectively, the constraint $y_i = \pi(x_i)$ is added to the MILP model. To reduce the number of constraints, the output binary variables can be directly permuted according to the ShiftRow bit-wise permutation π while modeling the MixColumn operation.

4.4 Constraints for AddRoundKey Operation

The AddRoundKey operation directly XORs the state bits with the round keys. In the bit-oriented related-key model, the state difference is XORed directly with the sub-key difference. To model the XOR operation between the key and state differences $(c = a \oplus b)$, we use the following constraints without introducing dummy variables: $c \ge a - b$, $c \ge b - a$, $c \le a + b$, $c \le 2 - a - b$.

4.5 Construction of the Objective Function

The objective function of an MILP model can be designed to minimize the number of active S-boxes. In a bit-oriented MILP model, there will be no inconsistencies in the propagation of bit differences through rounds, provided the S-box constraints accurately represent its DDT. To account for an active S-box in a bit-based model, we introduce a dummy variable along with four additional constraints for each S-box. Let the input bit differences of an S-box be represented by $(\delta x_3, \delta x_2, \delta x_1, \delta x_0)$ and define a new binary dummy variable, d_0 . This dummy variable d_0 will determine whether the S-box is active or inactive

based on the following constraints: n^{−1}₇ **=**0 $\delta x_i \geq d_0$, $d_0 \geq \delta x_i$, $i = 0, 1, 2, 3$. The

objective function is then to minimize the sum of the dummy variables d_i for each S-box position in the rounds. To calculate the probability of the differential trail produced by the model, the probability of each active S-box from the DDT must be checked, and the overall probability of the differential characteristic is obtained by multiplying these values. For a clearer understanding of the MILP model applied to the FUTURE cipher, we provide our MILP model implementation in [2].

5 Results

This section presents an analysis of the differential characteristics of FUTURE in the related key attack setting. The differential characteristics are determined using the methodology in Section 4.

Algorithm 2 Distinguishing Attack against FUTURE Reduced to 8 Rounds

	1: procedure $D_{\text{ISTINGUISHER}}(\Delta P = 0 \times 0000800011800008 \Delta S K_0$														
	$0 \times 0000800011800008$, $\Delta S K_1$ = 0x0000020000200002)														
	$\frac{8 \text{ round differential}}{4}$ $\Delta C = 0 \times 01442 a0000899108$)														
2:	Randomly choose a key $K=SK_0 SK_1 \overset{\$}{\leftarrow} \{0,1\}^{128}.$														
3:	Form another key $K' = K \oplus \Delta SK_0 \Delta SK_1$. \triangleright Steps 2 and 3 are chosen by the														
	oracle.														
4:	Choose 2^{63} distinct plaintexts $P_i,\,i=1,2,\cdots,2^{63}$														
5:	for $i = 1$ to 2^{63} do														
6:	Query P_i to the encryption oracle under the key K and obtain the														
	corresponding ciphertext $C_i = E_K(P_i)$.														
7:	Query $P'_i = P_i \oplus \Delta P$ to the encryption oracle under the key K' and obtain														
	the corresponding ciphertext $C_i' = E_{K'}(P_i')$.														
8:	if $C_i \oplus C'_i = \Delta C$ then														
9:	Return 1 \triangleright The oracle is the FUTURE reduced to 8 rounds.														
10:	\triangleright The oracle is a random permutation. Return 0														

5.1 Related-Key Differential Distinguishers

To search for the differential characteristics of FUTURE in the related-key setting, we constructed an MILP model using the Gurobi Python API [1]. The necessary constraints for building the model across rounds are outlined in Section 4. A summary of the related key characteristics for different rounds, along with their probabilities, is presented in Table 2. This model enables us to search for related-key differential characteristics up to 7 rounds. However, due to the large number of constraints and variables, the model struggles to complete the search for 6 and 7 rounds. For the 7-round case, we identified several differential characteristics with 22 active S-boxes and a probability of approximately 2 **−**48 . The 7-round differential characteristic is shown in Figure 15 in Appendix ??. To confirm the individual probabilities for each S-box, the DDT, and inverse DDT are provided in Table 3a and Table 3b, respectively. Additionally, we identified three distinct clustering effects for the 7-round differential characteristic (see Table 2) from 50 different solutions generated by the model, where the characteristics share the same input and output. This clustering further increases the probability of the differential characteristic to 3 **·** 2 **[−]**⁴⁸ **≈** 2 **−**46.4 . Furthermore, for 8 rounds, the solver could not reach a near-optimal solution due to the large number of constraints and variables. Therefore, we extended the 7-round differential characteristic by adding an additional round. Using the MILP model, we verified that seven S-boxes are active in the final round, with a probability of 2 **[−]**17. Consequently, the overall probability for the 8-round differential characteristic becomes $2^{-46.4} \cdot 2^{-17} = 2^{-63.4}$. This can be directly leveraged to mount an attack on the security notion of indistinguishability against FU-TURE reduced to 8 rounds. The attack procedure is detailed in Algorithm 2. In this distinguisher, the attacker requires 2^{63} plaintext pairs, effectively exhausting the entire plaintext space. The offline time complexity amounts to 2⁶³

XOR operations. The attack does not necessitate storing intermediate values, ϵ except for one ciphertext when $C_i \oplus C'_i$ $\mathcal{L}_{i} = \Delta C$ is satisfied. Therefore, the memory complexity is minimal, or effectively negligible.

Experimental Verification. As previously mentioned, the bit-oriented MILP model guarantees no inconsistencies in the solutions it returns. In our experiments, we successfully verified differential characteristics with a probability greater than 2^{-32} . The implementation used to verify these characteristics is available in [2].

Fig. 7: Eight Round Related Key Differential Characteristic of FUTURE Cipher

5.2 Related Key Boomerang Distinguisher

In this section, we construct boomerang distinguishers for FUTURE over different rounds. Using our automated search model, we identify two distinct relatedkey differential characteristics for five rounds each, corresponding to the upper and lower halves of the boomerang. These characteristics have probabil*upper Trait* Δ_0 \longrightarrow Δ_1 and \longrightarrow Δ_2 \longrightarrow Δ_3 and \longrightarrow Δ_4 and ∇_0 Lower Trail ∇_1 denote the differential characteristics for the upper and lower five rounds of the full boomerang, respectively. Additionally, let α and β represent the differences in the round keys of the upper and lower trails. The full round boomerang structure is illustrated in Figure 8. Thus, the distinguishing probability for this boomerang is given by $(2^{-14})^2 \cdot (2^{-16})^2 = 2^{-60}$, which can be utilized to perform a distinguishing attack on the full-round FUTURE cipher under adaptively chosen plaintext and ciphertext (ACPC) settings. The detailed attack procedure is presented in Algorithm 3. In this distinguisher, the

$#$ Rounds	#Active	Differential	Probability	
	S-box	Input Differences	Output Difference	
		$AP = 0x2300001000010000$		
$\overline{4}$	$\overline{2}$	$\Delta K_0 = 0 \times 2300 0010 0001 0000$	$\Delta C = 0 \times 000040000440008c$	$2 - 5$
		$\Delta K_1 = 0 \times 0004 0000 0000 0000$		
		$AP = 0 \times 120101c00000000$		
5	6	$\Delta K_0 = 0 \times 0000$ 01 c 0 0000 0000	$\Delta C = 0 \times 0008000080000802$	2^{-14}
		$\Delta K_1 = 0 \times 0000002000200002$		
		$AP = 0xc84000000000005$		
6	11	$\Delta K_0 = 0 \times 84000000000005$	$AC = 0 \times 020000100001d420$	$2 - 27$
		$\Delta K_1 = 0 \times 0000 0000 0000 2480$		
		$AP = 0 \times 0000800011800008$		
$\overline{7}$	22	$\Delta K_0 = 0 \times 0000800011800008$	$\Delta C = 0 \times 00000000710020000$	2^{-48}
		$\Delta K_1 = 0 \times 0000 0000 000b 0000$		
		$AP = 0 \times 0000800011800008$		
8	22	$\Delta K_0 = 0 \times 0000800011800008$	$\Delta C = 0 \times 01442a0000899108$	$2 - 63.4$
		$\Delta K_1 = 0 \times 0000 0000 000b 0000$		

Table 2: Related Key Differentials for Different Rounds of FUTURE using Bit-Oriented MILP Model

attacker needs 2^{60} plaintext pairs, which corresponds to 2^{61} plaintexts in total. The offline time complexity is $2 \cdot 2^{60} = 2^{61}$ XOR operations. The attack does not require storing intermediate values, except for one plaintext P_i when $P_2^i \oplus P_3^i = \Lambda_0$ is satisfied. As a result, the memory complexity is negligible.

Checking Incompatibilities in the Boomerang. In boomerang-style attacks, selecting compatible differential characteristics for $\boldsymbol{E_0}$ and $\boldsymbol{E_1}$ is crucial, as independent choices can lead to incompatibility and reduce the probability of generating a right quartet to zero. Murphy [32] highlighted that dependencies between characteristics can benefit attackers. Biryukov et al. introduced the middle-round S-box trick [14], and later Biryukov and Khovratovich [15] proposed techniques like the ladder and S-box switch to improve probabilities. These ideas were formalized by Dunkelman et al. as the sandwich attack [22], which divides the cipher into three parts, enhancing the overall probability. Further, to evaluate the middle part efficiently and systematically, the authors [20] introduced a boomerang connectivity table (BCT) for a single round.

Suppose that the middle layer at the fourth round of the given boomerang (Figure 8) is composed of 16 S-box layers independently. For more clarity, we only chose one s-box layer which is depicted in Figure 9. According to Figure 9, the BCT [20] is defined in the following way.

$\text{BCT}(\Delta_i, \nabla_o) = \{x \in \{0, 1\}^4 : S^{-1}(S(x) \oplus \nabla_o) \oplus S^{-1}(S(x \oplus \Delta_0) \oplus \nabla_o) = \Delta_0\}.$

This BCT provides a unified representation of existing observations on checking the inconsistency as well as the dependencies to further increase the probability of the boomerang for a single round.

Incompatibility. Incompatibility occurs when, as shown in Figure 9, the boomerang connection table (BCT) entry $\text{BCT}(\Delta_i, \nabla_o) = 0$, meaning the

Fig. 8: Full Round Boomerang Distinguisher

Fig. 9: Single S-box Layer in the Middle of the Boomerang

boomerang cannot be formed. If $\text{BCT}(\Delta_i, \nabla_o) \neq 0$, the differential characteristics are compatible to form the quartet for the boomerang.

Ladder Switch. The ladder switch, introduced in [15], occurs when $\Delta_i \neq$ 0 and $\nabla_0 = 0$, resulting in BCT(Δ_i , ∇_0) = 2⁴, i.e., $Pr[\Delta'_i]$ $i = \Delta_i$] = 1. Geometrically, when $\nabla_{\mathbf{o}} = \mathbf{0}$, the upper planes coincide, and the input pairs (x_3, x_4) on the opposite plane are directly replaced by (x_1, x_2) . In a similar fashion, if $\Delta_i = 0$ and $\nabla_o \neq 0$, the lower planes coincide, and the input pairs (y_1, y_3) on the opposite plane are directly replaced by (y_2, y_4) .

Algorithm 3 Boomerang Distinguishing Attack against the FUTURE Cipher

- 2: Randomly choose a key $K_0 \stackrel{\$}{\leftarrow} \{0, 1\}^{128}$. *▶* Steps 2 and 3 are chosen by the oracle. 3: Form another keys $K_1 = K_0 \oplus \alpha$, $K_2 = K_0 \oplus \beta$, and $K_3 = K_0 \oplus \alpha \oplus \beta$. 4: Choose 2⁶⁰ distinct plaintext pairs as $(P_0^i, P_1^i \oplus \Delta_0)$, $i = 1, 2, ..., 2^{60}$ 5: **for** $i = 1$ to 2^{60} do 6: Query P_0^i to the encryption oracle under the key K_0 and obtain the corresponding ciphertext $C_0^i = E_{K_0}(P_0^i)$.
- 7: Query $P_1^i = P_0^i \oplus \Delta_0$ to the encryption oracle under the key K_1 and obtain the corresponding ciphertext $C_1^i = E_{K_1}(P_1^i)$.
- 8: Compute $C_2^i = C_0^i \oplus \nabla_1$ and $C_3^i = C_1^i \oplus \nabla_1$.
- 9: Query C_2^i to the decryption oracle under the key K_2 and obtain the corresponding plaintext $P_2^i = E_{K_2}(C_2^i)$.
- 10: Query C_3^i to the decryption oracle under the key K_3 and obtain the corresponding plaintext $P_3^i = E_{K_3}(C_3^i)$.

S-box Switch. The S-box switch, introduced in [15], occurs when $DDT(\Delta_i, \Delta_o) \neq$ **0** and $\Delta_o = \nabla_o$, resulting in BCT(Δ_i , ∇_o) = DDT(Δ_i , Δ_o), i.e., Pr[Δ_i' **=** Δ_i] = $\frac{\text{DDT}(\Delta_i, \Delta_o)}{2^4}$. Geometrically, when $\Delta_o = \nabla_o$, the upper planes interchange their input pairs, i.e., the input pairs (x_3, x_4) on the opposite plane are directly replaced by $(\mathbf{X}_2, \mathbf{X}_1)$, consistent with $\text{DDT}(\Delta_i, \Delta_o)$.

Based on the switch techniques and the BCT, we verified the compatibility of the full round boomerang distinguisher shown in Figure 8. In this distinguisher, the S-box layer in the fifth round (Round 4) is chosen as the middle layer. We examine the state difference at the Round 4 S-box layer for the upper differential trail and the state difference at the Round 5 S-box layer for the lower trail. This setup is illustrated in Figure 10. As shown, only the third S-box is active in the upper trail, while all S-box nibbles are active in the lower trail. Consequently, all nibbles except the third in the middle layer fall under the ladder switch category, resulting in a probability of 1. For the third nibble position, $\Delta_i = 0 \times 07$ and $\nabla_{\mathbf{o}} = 0 \times 0 \mathbf{c}$, where we confirmed that BCT(0×07 , $0 \times 0 \mathbf{c}$) = 2, validating the compatibility of our differential characteristics to form the full-round boomerang distinguisher.

Refinements to the Boomerang Distinguisher. In the previous paragraph, we demonstrated the compatibility of the two differential characteristics necessary to form a full-round boomerang using middle-round switch effects. Now, we will delve into a more detailed analysis of how these switching effects can be leveraged to significantly enhance the boomerang probability. As shown in Figure 10, there are three active S-boxes at positions 12, 14, and 15 in the lower trail during round 5. Tracing this lower trail backward, the first column $((0, 0, 0, 8)^T)$ contains a single active nibble, 0×08 , at the third position following the inverse ShiftRow operation. This nibble difference, 0×08 , arises from the difference $0x09$ after the inverse S-box operation in round 5. Notably, the other two active S-boxes in round 5 do not impact the first column after the inverse ShiftRow and MixColumn operations, as depicted in Figure 11.

Fig. 10: Middle Round Switching Effects

According to the Figure 11, if we chose all possible differences δ from 0×9 through S-box inverse, i.e., $0 \times 9 \xrightarrow{\text{inverse DDT}} \{0 \times 1, 0 \times 7, 0 \times 8, 0 \times a, 0 \times c, 0 \times e\}$ (see Table 3b), we get different $\delta_3(=\eta_3) \in \{0 \times \mathcal{C}, 0 \times 8, 0 \times \mathcal{C}, 0 \times \mathcal{C}, 0 \times \mathcal{C}\}$ through inverse ShiftRow and MixColumn operations. Finally, we checked that if $\delta_3(=\eta_3) \in \{0 \times \mathbb{C}, 0 \times \mathbb{C}, 0 \times \mathbb{9}\},\$ then $\mathrm{BCT}(\zeta(=0 \times 7), \eta) \neq 0$. This demonstrates that the two differential characteristics are compatible for forming the quartet in the boomerang if the output differences are $\{0x8, 0xc, 0xe\}$ from the input difference 0×09 through the inverse S-box operation at round 5 (at position 15 in the lower half). This increases the probability from 2^{-3} (= $Pr[0 \times 9 \xrightarrow{S^{-1}} 0 \times 8]$ to 2^{-1} . Furthermore, any possible output differences from the input differences 0×05 and 0×02 (i.e., Pr**[** $0 \times 5 \xrightarrow{S^{-1}} * 1 = 1$, Pr**[** $0 \times 2 \xrightarrow{S^{-1}}$ ∗**] =** 1) do not affect the first column after the ShiftRow and MixColumn inverse operations, increasing the probability from 2^{-5} to 1. Similarly, the output difference corresponding to the active S-box at round 4 for the upper half can be arbitrary, i.e., $0x7 \xrightarrow{\text{DDT}}$ ★. This also increases the probability from 2^{−2} to 1. As a result, for the one lower half, the probability improves by a factor of 2^7 . Thus, for the two parallel lower halves, the probability improves by a factor of $(2^7)^2 = 2^{14}$. For upper halves, the probability improves by a factor of $(2^2)^2 = 2^4$. Additionally, we account for the probability that $\text{BCT}(7, \delta_3) \neq 0$ for the middle-round switch at the round 4 S-box operation.

Since $\delta_3 \in 0 \times \mathbb{C}$, $0 \times \mathbb{Q}$, the probability of BCT(7, δ_3) $\neq 0$ is lower bounded by the minimum of their respective probabilities, i.e.,

$$
Pr[\text{BCT}(7, \delta_3) \neq 0] \ge \frac{\min\{\text{BCT}(7, 0x8), \text{BCT}(7, 0xa), \text{BCT}(7, 0x9)\}}{2^4}
$$

$$
= \frac{\min\{2, 4, 4\}}{2^4} = 2^{-3},
$$

where $\text{BCT}(7, 0x8) = 2$, $\text{BCT}(7, 0xa) = 4$, and $\text{BCT}(7, 0xc) = 4$. Finally, the refined probability for the boomerang becomes $2^4 \cdot 2^{14} \cdot 2^{-60} \cdot 2^{-3} = 2^{-45}$. This scenario can be further mapped to a Sandwich attack $(E = E_1 \circ E_m \circ E_0)$ with probability $\bar{p}^2 \cdot r \cdot \bar{q}^2$, where $\bar{p} = 2^{-12}$, $r = 2^{-3}$, and $\bar{q} = 2^{-9}$. As a result, the data, time, and memory complexities of the distinguishing attack are reduced to 2⁴⁶ plaintexts, 2⁴⁶ XOR operations, and negligible memory, respectively.

Fig. 11: Middle Round Switching Effects using Truncated Differences

Experimental Verification. For this boomerang distinguisher, we have experimentally verified both the upper and lower differential characteristics along with their corresponding probabilities. The implementation used for verification is provided in [2].

6 Conclusion and Future Works

In this work, we present a comprehensive implementation of bit-oriented MILP models for the FUTURE lightweight block cipher in related-key settings. This approach can be extended to model MDS (or near-MDS) based SPN ciphers in the future. Utilizing this model, we explored related-key differential characteristics across different rounds, identifying a seven-round differential characteristic with a probability of 2^{-46.4}. We further extended this characteristic by adding an extra round, providing a distinguisher with data complexity of 2^{64} , time complexity of 2^{63} XOR operations, and negligible memory requirements. Additionally, we developed a full-round boomerang distinguisher with a probability

of 2 **[−]**⁶⁰ based on the round-reduced differential characteristics. By applying a one-round middle switch effect, we refined the boomerang's probability from 2^{−60} to 2^{−45}. Consequently, the complexities of the attack are improved to 2⁴⁶ plaintexts, 2 ⁴⁶ XOR operations, and negligible memory.

In future work, it would be valuable to explore optimizing the probability of the distinguisher, rather than focusing solely on the number of active S-boxes. This could potentially enhance the overall probability of the distinguisher. Additionally, recent advancements in automated tools for cryptanalysis present an opportunity to develop a tool for conducting truncated differential and sandwich attacks, capturing more dependencies in the middle rounds, and further improving the probabilities of differential and boomerang distinguishers. Lastly, another interesting direction for future research would be to propose an efficient key recovery attack based on the distinguishers presented in this work.

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Appendix A

M¹ **=**

	1	0	0	1	Ω	Ω	0	1	1	0	0	0	1	n	Ω	1	
	1	1	0	0	1	Ω	0	0	0	1	0	0	$\mathbf{1}$	1	Ω	0	
	0	$\mathbf{1}$	1	Ω	0	$\mathbf{1}$	0	0	0	0	$\overline{1}$	0	0	$\mathbf{1}$	$\mathbf{1}$	0	
	$\mathbf 0$	0	1	0	0	0	$\mathbf{1}$	$\mathbf{1}$	0	0	0	$\mathbf{1}$	0	0	1	0	
	$\overline{1}$	$\overline{1}$	0	0	0	1	0	0	0	0	0	1	0	0	0	1	
	$\mathbf 0$	$\overline{1}$	1	Ω	0	0	$\mathbf{1}$	0	1	0	0	0	$\mathbf{1}$	Ω	Ω	0	
	$\overline{1}$	0	1	1	1	0	0	$\mathbf{1}$	0	$\mathbf{1}$	0	0	0	$\mathbf{1}$	0	0	
	$\overline{1}$	0	0	1	1	0	0	0	0	0	$\overline{1}$	1	0	0	1	1	
	0	1	Ω	0	1	1	0	0	1	0	0	1	Ω	Ω	Ω	1	
	0	0	1	0	0	1	$\mathbf{1}$	0	$\overline{1}$	1	0	0	1	0	0	0	
	$\overline{1}$	0	0	$\mathbf{1}$	1	0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf 0$	$\mathbf{1}$	$\overline{1}$	0	0	$\mathbf{1}$	0	0	
	1	0	0	0	1	0	0	1	0	0	1	0	0	0	1	1	
	0	0	0	1	0	0	0	$\mathbf{1}$	$\overline{1}$	0	0	1	$\mathbf{1}$	0	0	0	
	$\overline{1}$	0	Ω	Ω	1	0	0	0	1	$\overline{1}$	0	0	0	$\mathbf{1}$	0	0	
	$\mathbf 0$	$\overline{1}$	Ω	Ω	0	1	0	0	0	1	$\overline{1}$	0	0	0	1	0	
	0	0	1		0	Ω	1	1	0	0	1	0	0	0	0	1	

Fig. 12: The Primitive Representation of μ When Cipher's State is Represented from MSB to LSB

	O	1	n	Ω	ı	1	Ω	0	٦	0	0	Ω	Ω	1	Ω	
	0	1	1	0	0	Ω	1	0	0	1	0	Ω	0	1	1	0
	0	0	1	1	0	0	0	1	0	0	1	0	0	0	1	1
	$\overline{1}$	0	Ω	1	1	Ω	Ω	Ω	0	0	0	$\mathbf{1}$	1	0	Ω	1
	$\overline{1}$	0	0	1	0	0	0	1	1	$\overline{1}$	0	0	$\overline{1}$	$\overline{1}$	0	0
	$\overline{1}$	$\overline{1}$	Ω	1	1	Ω	0	1	0	0	1	Ω	Ω	0	$\mathbf{1}$	0
	0	$\overline{1}$	1	0	0	1	0	0	0	0	0	$\mathbf{1}$	0	0	0	1
	0	Ω	1	1	0	Ω	$\mathbf{1}$	0	$\overline{1}$	0	0	0	$\overline{1}$	0	Ω	0
$M_2 =$	$\mathbf 0$	0	0	1	1	0	0	1	0	1	0	0	$\overline{1}$	1	0	0
	$\overline{1}$	$\mathbf 0$	0	1	1	1	0	1	0	1	1	0	0	0	$\mathbf{1}$	0
	0	$\mathbf{1}$	0	0	0	1	$\mathbf{1}$	0	0	0	$\overline{1}$	$\mathbf{1}$	0	0	0	1
	$\mathbf 0$	0	1	0	0	0	1	1	1	0	0	$\overline{1}$	$\overline{1}$	0	Ω	0
	$\overline{1}$	$\overline{1}$	0	0	1	1	0	0	0	1	0	0	1	0	0	0
	0	0	1	0	0	0	1	0	0	1	1	0	0	$\mathbf{1}$	0	Ω
	0	0	Ω	1	0	0	0	1	0	0	1	1	0	0	1	0
	$\mathbf{1}$	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1

Fig. 13: The Primitive Representation of μ When Cipher's State is Represented from LSB to MSB

Fig. 14: 16 Constraints Correspond to One Column Transformation After the MixColumn Operation

(a) DDT of S-box

(b) Inverse DDT of S-box

Round 0	State(x)					Key0					State(x)		$Pr = 1$		State(x)								
	$\boldsymbol{0}$	8	$\mathbf{1}$	$\boldsymbol{0}$		$\boldsymbol{0}$	8	$\mathbf 1$	$\mathbf{0}$	$\boldsymbol{0}$	$\mathbf{0}$	$\bf{0}$	$\mathbf{0}$		$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$					
	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{1}$	$\boldsymbol{0}$	\oplus	$\bf{0}$	0	$\mathbf{1}$	$\bf{0}$	$\boldsymbol{0}$	0	$\boldsymbol{0}$	$\bf{0}$	SC	$\boldsymbol{0}$	$\bf{0}$	0	$\bf{0}$	SR o MC				
	$\boldsymbol{0}$	0	8	$\boldsymbol{0}$		$\boldsymbol{0}$	$\bf{0}$	8	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$		$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\bf{0}$					
	$\mathbf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	8		$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$	8	$\mathbf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\mathbf 0$	$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$					
Round 1																							
		State(x)					Key1					State(x)				State(x)							
	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$		$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$Pr = 2^{-3}$	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$					
	0	0	0	$\boldsymbol{0}$	\oplus	0	0	$\boldsymbol{0}$	$\bf{0}$	0	0	$\boldsymbol{0}$	0	SC	$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\bf{0}$	SR o MC				
	$\boldsymbol{0}$	0	0	$\bf{0}$		0	0	0	0	$\bf{0}$	0	0	0		$\boldsymbol{0}$	$\bf{0}$	0	0					
	$\boldsymbol{0}$	0	0	0		0	0	b	$\bf{0}$	0	$\boldsymbol{0}$	b	$\boldsymbol{0}$		$\bf{0}$	$\boldsymbol{0}$	2	$\boldsymbol{0}$					
Round 2		State(x)						Key2				State(x)		$Pr = 1$		State(x)							
	$\boldsymbol{0}$	Ω	3	$\boldsymbol{0}$		$\boldsymbol{0}$	Ω	3	$\bf{0}$	$\boldsymbol{0}$	Ω	$\boldsymbol{0}$	Ω		$\boldsymbol{0}$	Ω	Ω	$\boldsymbol{0}$					
	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$1\,$	\oplus	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	SC	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$					
	$\,1\,$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$1\,$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\boldsymbol{0}$					
	$\boldsymbol{0}$	$\overline{2}$	$\boldsymbol{0}$	$\bf{0}$		$\boldsymbol{0}$	$\overline{2}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\bf{0}$		$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$					
Round 3																							
		State(x)					Key3					State(x)		P_T		State(x)							
	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\boldsymbol{0}$	$\mathbf{0}$	$\bf{0}$	$\bf{0}$	0	$\mathbf{0}$	$\boldsymbol{0}$	$\mathbf{0}$	$=2^{\mathrm{-}5}$	$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\boldsymbol{0}$					
	$\boldsymbol{0}$	$\boldsymbol{0}$	0	$\boldsymbol{0}$	\oplus	$\boldsymbol{0}$	$\mathbf{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\mathbf 0$	$\mathbf{1}$	$\mathbf{0}$	SC	$\boldsymbol{0}$	$\bf{0}$	$\mathbf a$	$\mathbf{0}$	SR o MC				
	$\bf{0}$	$\mathbf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\bf{0}$	$\mathbf{0}$	6	$\bf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	6	$\boldsymbol{0}$		$\bf{0}$	$\bf{0}$	5	$\bf{0}$					
	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\boldsymbol{0}$	$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		0	$\bf{0}$	$\mathbf{0}$	$\bf{0}$					
Round 4																							
		State(x)						Key4				State(x)		$Pr = 2^{-16}$		State(x)							
	$\bf{0}$	$\mathbf{0}$	0	$\bf{0}$		$\boldsymbol{0}$	$\mathbf{0}$	$\boldsymbol{0}$	$\overline{2}$	$\boldsymbol{0}$	$\boldsymbol{0}$	0	$\overline{2}$		0	$\bf{0}$	$\bf{0}$	C					
	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	C	\oplus	$\sqrt{2}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\bf{0}$	$\,2$	$\boldsymbol{0}$	0	C	SC	$\mathbf{1}$	$\bf{0}$	$\mathbf{0}$	8	SR o MC				
	3	0	$\bf{0}$	$\bf{0}$		$\boldsymbol{0}$	$\overline{2}$	$\boldsymbol{0}$	$\boldsymbol{0}$	3	4	0	$\boldsymbol{0}$		4	f	$\bf{0}$	$\bf{0}$					
	$\boldsymbol{0}$	b	$\bf{0}$	$\boldsymbol{0}$		$\bf{0}$	3	$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$	d	$\boldsymbol{0}$	$\boldsymbol{0}$		$\bf{0}$	C	$\bf{0}$	$\bf{0}$					
Round 5																							
		State(x)						Key5				State(x)		$Pr = 2^{-26}$		State(x)							
	d	$\bf{0}$	$\bf{0}$	e		$\bf{0}$	$\mathbf{0}$	2	$\bf{0}$	d	$\mathbf{0}$	2	e		7	$\mathbf{0}$	e	8					
	$\overline{4}$	$\mathbf{0}$	b	$\boldsymbol{0}$	\oplus	$\boldsymbol{0}$	$\mathbf{0}$	C	$\bf{0}$	$\overline{4}$	$\mathbf{0}$	7	$\bf{0}$	SC	f	$\bf{0}$	9	$\mathbf{0}$	SR o MC				
	$\boldsymbol{0}$	$\boldsymbol{0}$	5	$\overline{4}$		$\mathbf{0}$	$\boldsymbol{0}$	$\mathbf{0}$	$\bf{0}$	$\bf{0}$	0	5	$\overline{4}$		$\boldsymbol{0}$	$\bf{0}$	7	$\overline{4}$					
	b	$\bf{0}$	$\overline{2}$	f		$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\bf{0}$	b	$\boldsymbol{0}$	2	f		b	$\bf{0}$	7	$\mathbf{1}$					
Round 6																							
	State(x)				Key6					State(x)		$Pr = 2^{-8}$		State(x)					State(x)				
	$\overline{4}$	$\boldsymbol{0}$	e	$\boldsymbol{0}$		$\sqrt{2}$	$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$	$\boldsymbol{0}$	$\bf{0}$	e	$\bf{0}$		$\boldsymbol{0}$	$\bf{0}$	9	$\bf{0}$		$\boldsymbol{0}$	$\boldsymbol{0}$	$\overline{7}$	3
	$\boldsymbol{0}$	8	$\bf{0}$	$\boldsymbol{0}$	\oplus	$\bf{0}$	8	$\bf{0}$	0	0	0	$\bf{0}$	$\bf{0}$	SC	$\boldsymbol{0}$	$\bf{0}$	$\bf{0}$	$\bf{0}$	SR o MC	$\mathbf{1}$	$\bf{0}$	$\boldsymbol{0}$	9
	$\boldsymbol{0}$	$\mathbf C$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\boldsymbol{0}$	c	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$		$\boldsymbol{0}$	1	$\boldsymbol{0}$	$\boldsymbol{0}$
	$\bf{0}$	$\boldsymbol{0}$	3	f		$\bf{0}$	$\boldsymbol{0}$	$\sqrt{4}$	$\boldsymbol{0}$	$\boldsymbol{0}$	$\boldsymbol{0}$	7	f		$\bf{0}$	$\boldsymbol{0}$	$\overline{\mathbf{c}}$	\overline{c}		$\bf{0}$	f	$\,2$	$\boldsymbol{0}$

Fig. 15: Seven Round Related Key Differential Characteristic of FUTURE Cipher

Fig. 16: 17 Number of Constraints to Capture DDT of FUTURE S-box

Fig. 17: 17 Number of Constraints to Capture DDT of Present S-box